

Stark Many-Body Localization in One-Dimensional Bose Gas under the Continuous Limit

Zihan Zhao^{1,2}, Zhongchi Zhang², Huaichuan Wang², Ken Deng², Wenlan Chen^{2,*}
and Jiazhong Hu^{1,†}

¹*Quantum Science Center of Guangdong-Hong Kong-Macao Greater Bay Area, Shenzhen 518045, China*

²*Department of Physics and State Key Laboratory of Low Dimensional Quantum Physics, Tsinghua University, Beijing 100084, China*



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Many-body localization, an emergent quantum phenomenon, challenges the fundamental principles of thermodynamics and has attracted significant attention. In this study, we demonstrate Stark many-body localization in a one-dimensional Bose gas under the continuous limit, a scenario that extends well beyond the conventional tight-binding model commonly employed in most discussions. This localization occurs within a background harmonic trap without any random disorder, where the potential gradient effectively halts thermalization. Contrary to prior studies on Stark localization, our findings reveal that an increasing gradient does not necessarily lead to greater localization. Instead, we observe a complex transition within this continuous interacting system. Another surprising result is that a higher initial density, which typically correlates with a higher collisional rate and thermalization rate, actually induces stronger localization in this one-dimensional model. Additionally, we observe a density plateau with zero compressibility and low-density fluctuations, which may signal an insulating behavior associated with prethermalization during nonequilibrium dynamics.

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I. INTRODUCTION

Many-body localization (MBL) has emerged as a topic of significant interest, drawing extensive investigation [1,2]. It counterintuitively breaks ergodicity and halts thermalization in the presence of interactions. In interacting systems, MBL can occur with or without disorder. The former type is the conventional one where a random potential is applied to the system, which induces the localization [3–9]. The latter, termed Stark MBL [10–15], occurs when a gradient potential is applied instead of a random one. This finding suggests that random disorder is not a prerequisite for inducing MBL, thereby motivating further research in this domain. Utilizing the platforms of quantum simulations, both types of MBL have been experimentally demonstrated in a variety of setups [6–12], particularly in recent experiments of Stark MBL supporting the nonessentiality of disorder.

In the vast array of existing research, the exploration of MBL has primarily focused on tight-binding models, such as the Ising chain [9–11,16–18] and the Hubbard model [4–8,12–15]. In these models, each particle is treated as an occupation at a lattice site and is described by a localized excitation, with a finite hopping or coupling rate connecting adjacent sites separated by only one or two lattice constants. Under these conditions, it is crucial that the chemical potentials remain significantly smaller than the trap depth to ensure the validity and applicability of the tight-binding model. This model serves as a highly effective approach for addressing quantum mechanical problems with periodic potentials in most cases. However, in some real systems, long-range couplings between particles are non-negligible [19–21]. In such cases, continuing to approximate particle wave functions as localized orbitals near each lattice site becomes incorrect, and the tight-binding approximation breaks down. At present, research on systems beyond the tight-binding approximation is still insufficient, although many works in recent years have begun to explore them [22–25]. We believe it is highly necessary to investigate transport or thermodynamic properties in such systems and to explore whether MBL still exists. However, due to the complexities inherent in numerical calculations, discussions of models under the continuous limit pose significant challenges

*Contact author: cwlaser@ultracold.cn

†Contact author: hujiazhong01@ultracold.cn

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[26]. Consequently, the question of whether MBL can occur in such systems remains to be a mystery.

In this manuscript, we present a comprehensive study of Stark many-body localization in a one-dimensional (1D) Bose gas under the continuous limit. The term “continuous” refers to two aspects: one is an extremely shallow lattice trap, and the other is a large atom number in our system, both of them make the system far beyond the conventional Bose-Hubbard model. Our study reveals several notable distinctions compared to Stark MBL in previous studies. First, we investigate Stark MBL in the weak-lattice limit, where the chemical potentials are either above or close to the lattice trap depth. Therefore, the tunneling rates derived from the band structures, which go beyond the tight-binding model and now serve as a reference for comparison, are nearly equivalent to those of free particles without lattices. This is significantly different from previous studies conducted under the Hubbard model based on the tight-binding approximation. Second, our atomic system contains thousands of atoms, which is about two orders of magnitude higher than in previous studies. Recently, Stark MBL has been experimentally observed in

an ion chain containing 15–25 particles [10] and a superconducting system with 29 qubits [11]. However, unlike these studies, our experiment leverages a background harmonic trap to demonstrate Stark MBL with a total of several thousand atoms. The initial average particle number per lattice site is approximately in the tens, which is significantly larger than in prior research [10–12]. Based on the above two points, the atomic system is nearly in a continuous space, and the periodic lattice potential can be regarded as a perturbation to the system. Furthermore, since the atom number is very large, in principle quantum fluctuations in the atom number can be ignored. This leads us to believe that the mean-field approximation is a good approach to describe our system [27]. However, in the following, we will show the failure of the mean-field description in explaining our experimental results. Third, we observe a complex crossover between oscillation behavior and stationary behavior in quasi-interaction-free system in a short-time evolution, which can be explained within the framework of mean-field approach by the intra- and interband dynamics. Then, starting from the stationary region in the absence of interactions, by increasing

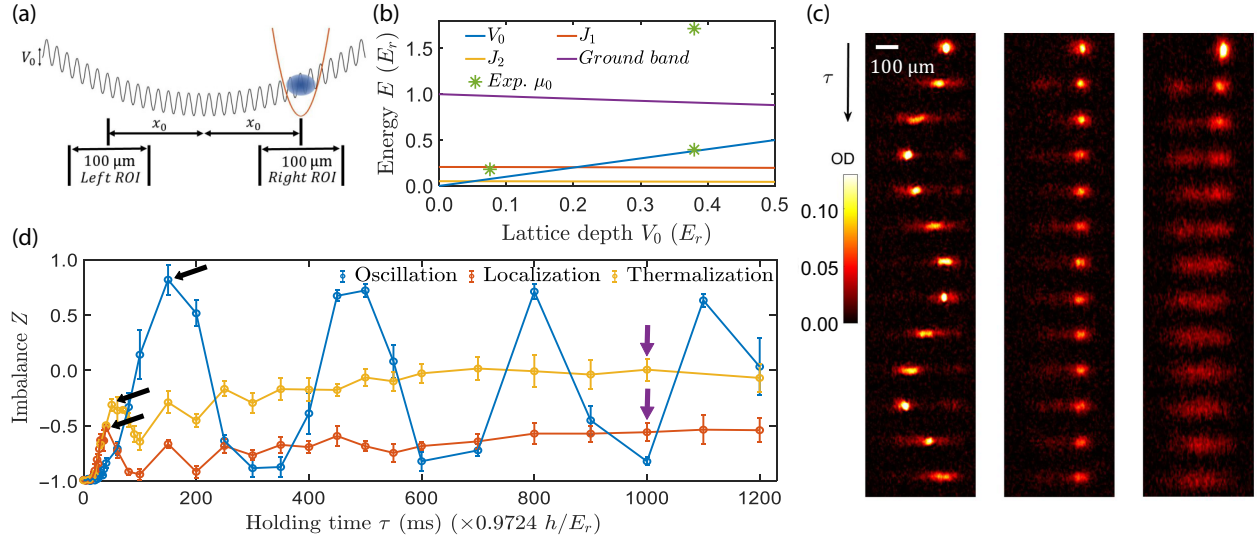


FIG. 1. (a) The experimental setup and the definition of the imbalance Z to characterize the dynamics. The red and tighter harmonic trap is the 1064-nm trap, determining the initial position and density of the atoms. The black and looser trap is the 1D 1560-nm trap with a very shallow lattice. (b) The typical energy scales in our system, specifically, the nearest-neighbor tunneling rate of the ground band (J_1 , the red line), the next-nearest-neighbor tunneling rate of the ground band (J_2 , the yellow line), the width of the ground band (the purple line), and the central chemical potential μ_0 of the initial state of the experimental data in (c) [the green stars, with the bottom left, bottom right, and top right corresponding to the left-to-right sequence in (c), respectively], vs the lattice depth V_0 . For comparison, we also plot the change of the trap depth in the same panel ($E = V_0$ the blue line). Here the tunneling rates can be obtained by fitting the ground band or the integral between different Wannier functions. Since our model is beyond the tight-binding model, the rates only serve as a reference to compare with the cases under the tight-binding model. (c) Three typical behaviors that appear in our experiment. Each column, from top to bottom, shows the time evolution of spatial optical depth vs the holding time τ , where the total time shown here is 500 ms. From left to right, it corresponds to three typical behaviors: oscillation, localization, and thermalization. (d) The time evolution of Z from 0 to 1.2 s. Black arrows point out the maximum of the first ripples of imbalance, which are used to assess the competition between oscillation and stationary behaviors. Purple arrows point out the equilibrium behaviors of localization and thermalization regimes after holding 1 s.

the scattering length, we observe the transition of the system from a localized state to a thermalized state after a long-term evolution. However, increasing the atomic density reverses this process. This contradicts the mean-field prediction, as the contributions of the scattering length and atomic density to the interactions are expected to be the same [28]. Last, we identify an insulating behavior, which may be associated with prethermalization. This insulating behavior is supported by the incompressibility and low density-fluctuations based on the spatial density measurements [29,30]. Notably, the particle occupation at each lattice site significantly exceeds that of conventional Mott insulators [31–33], potentially indicating the existence of novel phases stemming from nonequilibrium dynamics, thereby meriting further exploration.

II. RESULTS

A. Experimental setup

We perform our experiments utilizing two optical dipole traps with rubidium-85 atoms. The first trap is a running wave generated by a 1560-nm laser, with a Gaussian beam waist of 25 μm . A small fraction (about 0.25%) of the laser power is reflected back to interfere with the incident beam, forming a very shallow 1D optical lattice. This trap is used to confine the atoms along a single dimension, setting the stage for subsequent experiments. When the incident power is set to $p_0 = 37$ mW, the trap generates a harmonic potential that serves as a background for the atoms, with a longitudinal vibrational frequency of $\omega_{\parallel} = 2\pi \times 3.0(1)$ Hz and a radial vibrational frequency of $\omega_{\perp} = 2\pi \times 230(1)$ Hz. Additionally, the interference creates a shallow optical lattice with a trap depth of $V_0 = 0.072(5)E_r$, where $E_r = \hbar^2/(2m\lambda^2) = \hbar \times 972.4$ Hz is the

recoil energy, with $\lambda = 1560$ nm. Here, $\hbar$ is the Planck constant and m is the mass of rubidium-85 atom. During the experiments, we can adjust the incident power p_i from p_0 up to $10p_0$. The second optical dipole trap is a running wave with no reflection, generated by a 1064-nm laser that crosses the first trap perpendicularly, with a Gaussian beam waist of 100 μm . The intersection point of the two beams is located at the waist of the second trap but is offset by a distance x_0 from the waist of the first trap. This offset is typically around 100 μm and can be adjusted during the experiments.

During the preparation stage, we load rubidium-85 atoms into the crossed dipole trap and polarize them to the $|F = 2, m_F = -2\rangle$ hyperfine level, then cool them to a Bose-Einstein condensate (BEC) through gravity-forced evaporative cooling [34]. The scattering length is maintained at $280a_0$ during the cooling process. Subsequently, we spend 500 ms to ramp the powers of the two traps and the scattering length (using Feshbach resonance [35,36]) to the desired values used for our experiments. The atom number is approximately 1×10^4 , with a typical condensate size of around 20 μm . It is worth noting that after the preparation stage, the atomic ensemble is positioned nearly at the center of the 1064-nm trap [Fig. 1(a)]. Upon commencing the experiments, we suddenly turn off the 1064-nm optical dipole trap, allowing the atoms to slide down toward the center of the 1D trap formed by the 1560-nm laser. Given that the trap depth of the 1D lattice is very shallow and always less than one recoil energy E_r , it is reasonable to conjecture that the atoms will oscillate in this trap with an angular frequency $\omega_{\parallel}$, similar to that of a harmonic oscillator. However, in our experiments, we observe three typical behaviors, as shown in Fig. 1(c). The first is an oscillation behavior (left panel), where the

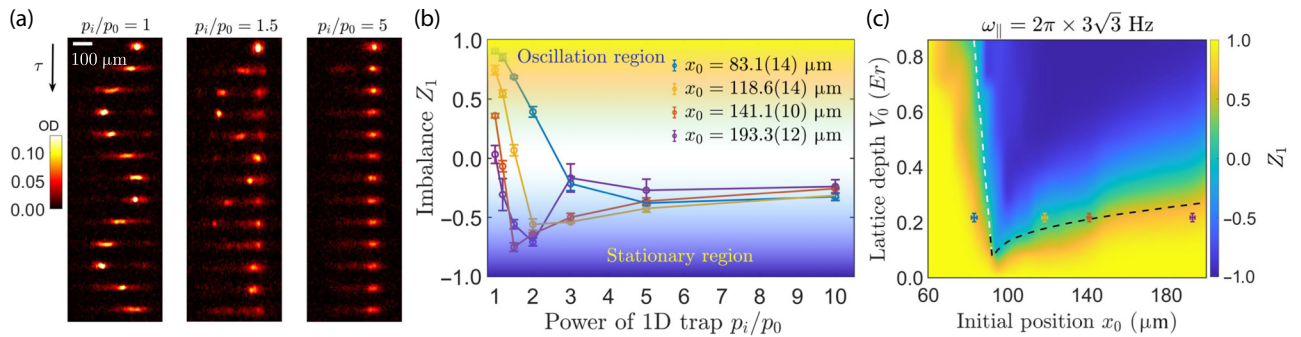


FIG. 2. (a) Three typical behaviors: oscillation ($x_0 = 118.6$ μm , $p_i/p_0 = 1$), mixture of oscillation and stationary behavior ($x_0 = 118.6$ μm , $p_i/p_0 = 1.5$), and stationary behavior ($x_0 = 118.6$ μm , $p_i/p_0 = 5$). The total holding time shown here is 500 ms. (b) Experimental data of Z_1 vs different p_i/p_0 and x_0 . Here the vibrational frequency $\omega_{\parallel}$ along the 1D trap is proportional to $\sqrt{p_i/p_0}$ while $\omega_{\parallel} = 2\pi \times 3.0(1)$ Hz for $p_i/p_0 = 1$. The solid lines are guide for the eye and drawn by simply connecting different data points. (c) The theoretical calculated Z_1 vs the lattice trap depth V_0 and the distance x_0 with $\omega_{\parallel} = 2\pi \times 3.0(1) \times \sqrt{3}$ Hz = $2\pi \times 5.20(17)$ Hz in the case of $p_i/p_0 = 3$. For a fixed lattice trap depth, an increasing potential gradient (larger x_0) will first cause Z_1 become a negative value (stationary) and then bounce back (less stationary). The white (black) dashed line represents the boundary between oscillation and stationary regions based on intraband (interband) dynamics calculation (see Sec. IV of the Supplemental Material [37]). They together determine the regions of different behaviors in the system.

atoms oscillate within the 1D trap. The second is a thermalized behavior (right panel), in which the atoms form a thermal cloud symmetrically centered around the trap. The last is a localized behavior (mid panel), where the majority of atoms neither oscillate nor thermalize, but instead remain in their initial position throughout. The oscillation behavior on the left is observed when both the scattering length and the power of the 1D trap are very small. By increasing the power of the 1D trap, the system tends to shift toward the localized behavior in the middle. If the scattering length is further increased, the system transitions toward the thermalized behavior shown in the right panel. It is worth noting that, as we will see later, these transitions are not monotonic, and the power of the 1064-nm trap also plays a significant role in influencing the behavior of the system. Notably, the chemical potentials of these initial states are all above or near the lattice trap depth [Fig. 1(b)].

To quantitatively assess the differences among these three behaviors, we introduce the imbalance parameter Z to characterize the dynamics [Fig. 1(a)] [10,13]. We define a region of interest (ROI) spanning $100\text{ }\mu\text{m}$, centered at the initial position of the atoms, which we refer to as the right ROI. Similarly, we define another region on the opposite side of the trap center, referred to as the left ROI. For each measurement, we extract the atom numbers N_R and N_L from the right and left ROIs, respectively. The imbalance Z is then defined as

$$Z = \frac{N_L - N_R}{N_L + N_R}. \quad (1)$$

Thus, for all behaviors, Z initially starts at -1 . After a holding time τ [ranging from 0 to 1.2 s, as depicted in Fig. 1(d)], the imbalance Z of the oscillation behavior fluctuates between -1 and 1 . In contrast, the Z values for the other behaviors initially exhibit small ripples before stabilizing at a quasi-steady value. Specifically, the steady value is zero for the thermalized behavior, while it remains negative for the localized one.

B. Quasi-interaction-free systems

We first perform measurements on quasi-interaction-free particles by setting the scattering length to $a_s = 7.5(18)a_0$, where a_0 is the Bohr radius. Here, we study the short-term behavior of the atomic system in order to distinguish whether the atoms are oscillating (oscillation behavior) or not (stationary behavior). This distinction is necessary because, later on, we must begin our investigation of the long-term behavior of atomic systems starting from those in the stationary regime, which will allow us to differentiate between localized state and thermalized state. During these measurements, we vary two parameters: the incident power p_i of the 1D trap and the distance x_0 from the trap's center, which affects the slope of the potential

at the atoms' location. In the following, we will use the dimensionless parameter p_i/p_0 to characterize the power of the 1D trap. It needs to be mentioned that, an augmentation in the incident power p_i/p_0 will lead to a proportional increase in both the depth of the shallow lattice and the potential gradient F , which can be calculated using the trap parameters (see Sec. III of the Supplemental Material [37]) in the form of

$$F = \frac{x_0 (\mu\text{m})}{100} \times p_i/p_0 \times \begin{cases} 0.0060(4) & (E_r/a), \\ 0.031(2) & (J_1/a), \end{cases} \quad (2)$$

where $a = \lambda/2 = 780\text{ nm}$ is the lattice spacing, and J_1 is the nearest-neighbor tunneling rate of the ground band. Concurrently, $\omega_{\parallel}$ and $\omega_{\perp}$ also increase proportionally to $\sqrt{p_i/p_0}$. However, the tunneling rates derived from the band structures remain virtually unchanged in such shallow lattices [Fig. 1(b)], which justifies treating our system as a Bose gas under the continuous limit and the breakdown of tight-binding approximation.

Three representative data obtained at $a_s = 7.5(18)a_0$ are shown in Fig. 2(a). The left panel exhibits the oscillation behavior, while the right panel showcases the stationary behavior. Notably, an intermediate behavior is also observed, characterized by partial oscillation of some atoms while others remain stationary. To quantitatively assess the transition and competition between oscillation and stationary behaviors, noting that the imbalance exhibits oscillation behavior initially in all cases in Fig. 1(d) (this oscillation behavior within the initial hundred-millisecond timescale can be explained by the Landau-Zener tunneling mechanism, as detailed in Sec. IV of the Supplemental Material [37]), we use the maximum value of the first ripple in the imbalance, denoted as Z_1 , as a descriptor of these distinctions, which is illustrated by the black arrows in Fig. 1(d). This parameter serves as a direct indicator of the proportion of atoms undergoing oscillating motion versus those exhibiting stationary, since at this moment, it is very easy to distinguish between these two parts of atoms, because all the oscillating atoms have just reached the other side of the 1D trap. The variations of Z_1 across different incident powers p_i/p_0 and distances x_0 are depicted in Fig. 2(b). As p_i/p_0 increases, which corresponds to a higher potential gradient and deeper lattice, Z_1 rapidly decreases to a negative value, signifying the crossover to the stationary regime. Following this, Z_1 exhibits a rebound, signaling a partial transition back to oscillation. Similarly, by adjusting the parameter x_0 , we find that an increase in the potential gradient (larger x_0 while keeping p_i/p_0 constant) initially leads to stationary behavior, followed by the emergence of partial oscillation (such as $p_i/p_0 = 3$).

To elucidate this behavior, we utilize the one-dimensional Gross-Pitaevskii equation (1D GPE) [38–40] (see Sec. I of the Supplemental Material [37]) to calculate

the value of Z_1 [Fig. 2(c)]. In our numerical simulations, to reveal the underlying mechanisms behind the observed data, the lattice depth is varied independently of the power of the 1D trap. As shown in Fig. 2(c), with $\omega_{\parallel} = 2\pi \times 5.20$ Hz corresponding to the case of $p_i/p_0 = 3$, a two-dimensional scan is performed across the lattice depth V_0 and the distance x_0 to acquire the values of Z_1 . The experimental data corresponding to $p_i/p_0 = 3$ in Fig. 2(b) are reproduced in Fig. 2(c) with respective colors. The blue and purple points exhibit larger Z_1 values, which are qualitatively consistent with the numerical simulations under a small scattering length of $7.5a_0$. The numerical results for other values of $\omega_{\parallel}$ are presented in Sec. II of the Supplemental Material [37].

In fact, the observed oscillation-stationary-oscillation behavior has been reported in many previous work with linear gradient potential that is explained by resonant-enhanced tunneling (RET) [41–43]. However, our experiment utilizes a harmonic trap as the external force, which causes different energy shifts at different lattice sites, leading to the failure of RET. Given that our system contains multiple bands, the intraband (Bloch oscillation) and interband (Landau-Zener tunneling) dynamics should be considered. If the external force is relatively small, the atoms will be confined to the current energy band, performing Bloch oscillations. On the other hand, if the external force is large, the atoms will undergo Landau-Zener tunneling from the current band into higher energy bands. The “V”-shaped boundary in Fig. 2(c) shows a significant difference between harmonic and linear external forces. Only considering the combined effect of Bloch oscillation and Landau-Zener tunneling can explain our experimental results. A detailed analytical theory has been provided in the Supplemental Material [37] (Sec. IV) to explain both experimental data and the GPE results.

C. Many-body interacting systems

We then focus on the scenario involving interacting particles, exploring their long-term equilibrium behaviors. For this analysis, we use a holding time of $\tau = 1$ s ($\sim 972.4 \hbar/E_r$) to assess the variations in Z , as indicated by the purple arrows in Fig. 1(d), which we denote as Z_2 . This holding time corresponds to $J_1\tau \sim 190$, $J_2\tau \sim 47$, and $J_3\tau \sim 21$, which is sufficient for the system to reach equilibrium. Furthermore, the powers of the 1D trap p_i/p_0 are selected to ensure that the atoms remain in the stationary regime and that the imbalance Z approaches a near-stable value. If atoms that exhibit stationary behavior in the short term remain in their initial positions after long-term evolution, without tunneling to lattice sites with lower energy, these atoms are considered localized.

In Fig. 3(a), we present our data for different scattering lengths a_s and various powers of the 1D trap p_i/p_0 , while keeping x_0 fixed at $123 \mu\text{m}$. For each specific trap

power, we find that localization persists over a finite range of interaction strengths, from $7.5a_0$ to $61a_0$. As the interaction strength increases, the distribution becomes more uniform until Z_2 reaches zero, indicating a thermalized state. It needs to be mentioned that this thermalization is not attributed to heating effects caused by three-body losses [44–47]. Our measurements reveal negligible variation in atom number (less than 10%) throughout the entire evolution process after initially turning off the 1064-nm trap (Sec. V of the Supplemental Material [37]). Figure 3(b) shows the spatial distributions for different interaction strengths at $p_i/p_0 = 3$. It is evident that for small values of a_s , the atomic density exhibits a pronounced peak on the right side, approximately at $118 \mu\text{m}$, corresponding to the initial position. Additionally, we observe that atoms with the same a_s always show greater localization for smaller p_i/p_0 values (ranging from 3 to 10). This suggests that increasing the potential gradient does not always enhance localization effects, which is consistent with the earlier argument regarding oscillation-stationary-oscillation behavior. The consistency between the long-time evolution behavior and the short-time evolution behavior indicates that the system retains some local information about its initial state even after undergoing long-term evolution.

In this context, as shown in Fig. 3(c), a mean-field numerical simulation using the 1D GPE is conducted for the interacting particles. However, under all experimental parameters employed, the imbalance Z predicted by the GPE ultimately converges to zero in long-term equilibrium behaviors, suggesting that the system should not exhibit any localization under such a shallow lattice and such a large atom number. So the localization observed in our experiments cannot be fully described by mean-field theory. It is worth noting that the above-mentioned discrepancy between the 1D GPE and the experimental results for all parameters applies only to parameters lying in the stationary region of Fig. 2(b) (as stated in the first paragraph of this section, all parameters chosen here are in the stationary region, because only in this region can the atomic cloud exhibit localization). For atomic clouds in the oscillation region, their behaviors match the 1D GPE results even after a long-time evolution of 1 s (see Sec. VI of the Supplemental Material [37]). Moreover, the spatial distributions obtained via GPE, depicted in Fig. 3(d), are symmetric around the trap center, consistent with the dephasing behavior of classical harmonic oscillators [48,49] (see Sec. VIII of the Supplemental Material [37]).

D. Unconventional many-body localization

Based on the measurements discussed earlier, a plausible argument can be made that stronger interactions lead to greater thermalization and reduced localization. In the previous discussions, we manipulated the scattering length

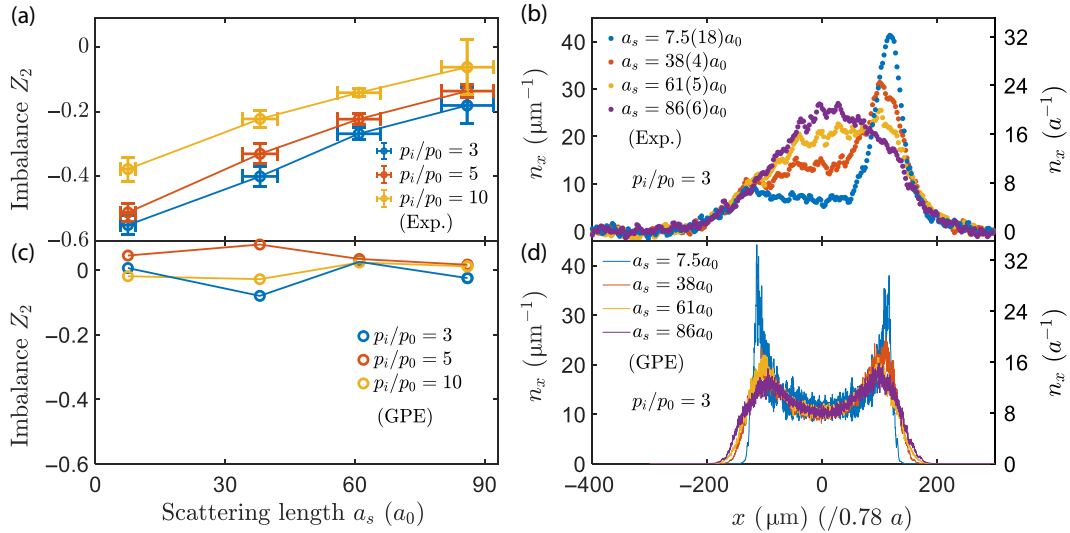


FIG. 3. (a) Z_2 , the imbalance after 1-s ($\sim 972.4 \hbar/E_r$) evolution, vs different scattering lengths a_s and the incident powers of the 1D dipole trap p_i/p_0 . (b) The 1D spatial density distribution vs different a_s , and $a = 780$ nm is the lattice constant. The trap center is at $x = 0$. Then, we perform the GPE to theoretically simulate Z_2 [panel (c)] and the spatial density distribution [panel (d)], while the distributions are symmetric around the trap center. The solid lines in panel (a) and (c) are guide for the eye and drawn by simply connecting different data points.

a_s to increase the interaction strength. However, within the framework of mean-field theory [28], the total interaction energy is given by $E_{\text{int}} = (g/2) \int n^2(\vec{r}) d\vec{r}$, where $g = 4\pi \hbar^2 a_s/m$ represents the interaction strength, and $n(\vec{r})$ is the density. Consequently, it is conceivable that an increase in the initial density would also contribute to enhanced thermalization. Contrary to this expectation, in the following, we will demonstrate that the dependence on atomic density exhibits an entirely opposite trend, where a higher initial density leads to greater localization, rather than thermalization. Furthermore, we also observe the presence of a density plateau, indicative of incompressibility.

During the sample preparation stage, we increase the power of the 1064-nm trap to compress the atomic cloud, thereby achieving a higher initial density of the condensate. Following the same procedure, we then suddenly turn off the 1064-nm trap, allowing the atoms to slide in the 1D trap. We use the parameter M_{1064} to represent the multiplication factor of the increase in the power of the 1064-nm laser. After a 1-s evolution time, we carefully examine the spatial density distributions of the atoms. In Fig. 4(a), we present the density distributions with the 1D trap set at $p_i/p_0 = 3$ and the scattering length at $a_s = 38(4)a_0$. A comparative analysis of these distributions reveals the emergence of a plateau in the central region of the trap. As the initial density increases, the height of the plateau decreases, causing a larger fraction of atoms to remain near their initial position, resulting in a more localized state. Since the primary disparity lies in the central region rather than in the relative proportions of the left and right wings, we introduce an alternative parameter, C , to quantitatively

assess the variations in localization. Specifically, let $N_C = N - N_L - N_R$ represent the atom number in the central region, where N is the total atom number. Then, C is defined as $C = (N_L + N_C - N_R)/(N_L + N_C + N_R)$. In this context, a higher value of C indicates that the atoms are more thermalized and symmetrically distributed around the trap center, while a lower value of C reflects a more localized distribution, with fewer atoms migrating toward the center. In Fig. 4(b), we present a plot of C as a function of initial compression and the scattering length a_s . The trend observed with respect to the scattering length aligns with previous measurements, indicating that an increase in the scattering length facilitates thermalization. However, we observe a contrasting dependence on density, where a higher initial density results in greater localization within the system. As previously mentioned, an increase in density directly correlates with higher collision rates and an increase in mean-field interaction energy, as the local interaction energy is proportional to $a_s n^2(\vec{r})$. Following this logic, a higher initial density ought to provide more opportunities for atoms to thermalize, a phenomenon confirmed in earlier studies conducted in tight-binding optical lattices [6,7]. Therefore, it is the first time here to notice such an unconventional behavior that a higher density leads to greater localization in a model under the continuous limit. Furthermore, as anticipated, in Fig. 4(c), the 1D mean-field theory used in earlier discussions fails to capture this unique behavior. Another noteworthy point is that in our experiment the radial vibrational frequency of the 1D trap is only $2\pi \times 230 \times \sqrt{p_i/p_0}$ Hz (for $p_i/p_0 = 1-10$). Therefore, the atoms may transit to the radial excited states

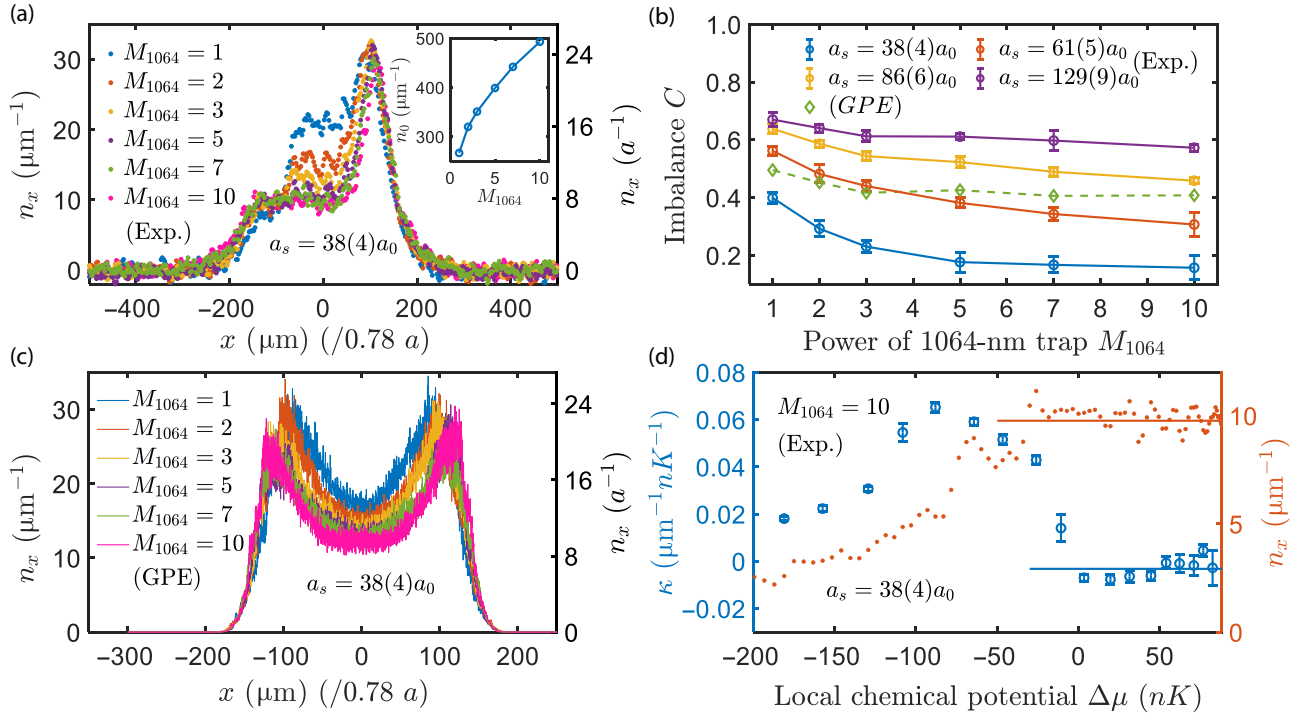


FIG. 4. (a) The 1D spatial density distributions under $a_s = 38(4)a_0$ and $p_i/p_0 = 3$ after the 1-s evolution, and $a = 780$ nm is the lattice constant. Here we use different powers of the 1064-nm trap to prepare samples with different initial densities, where the relative power change is characterized by M_{1064} . The initial peak density n_0 vs M_{1064} is shown in the inset, where n_0 is calculated by solving the ground state of 1D GPE (see Sec. VII of the Supplemental Material [37]). Here we may find that the density shows a plateau for a large M_{1064} such as purple, green, and pink points. (b) C vs a_s and M_{1064} . When the scattering length a_s becomes larger, there are more atoms leaving the initial position and trying to thermalize. Oppositely, when the initial density becomes higher, there are more atoms remaining at the initial position and keeping the original spatial information. The green dashed line is the theoretical calculation based on GPE for C vs M_{1064} . Here the results of different a_s are very close to each other, so we only plot one line ($a_s = 38a_0$) as an example. (c) We simulate this dynamics with the GPE, but the spatial distribution is always symmetric around the trap center ($x = 0$). (d) Based on the spatial distribution n_x and the local density approximation, we extract out the compressibility $\kappa = \partial n_x / \partial \mu$. Here $\Delta\mu$ characterizes the local chemical potential difference comparing to the initial BEC. We may find κ stays near zero for a large range from 0 to 100 nK (2 kHz). The detailed derivations can be found in Sec. X of the Supplemental Material [37].

during their dynamical evolution, making the 1D GPE no longer hold strictly. To address this issue and quantify the influence of three-dimensional effects on the system's evolution, we use the three-dimensional Gross-Pitaevskii equation (3D GPE) to simulate our experimental system. The results are shown in Sec. IX of the Supplemental Material [37]. The calculated density distributions are also nearly symmetric around the trap center, showing no peaks at the initial position, which is not able to reproduce the experimental results, suggesting that the underlying mechanism extends beyond mean-field descriptions and requires consideration of many-body correlations. However, current numerical methods for solving the time evolution of quantum many-body systems [26], such as exact diagonalization, time-dependent density matrix renormalization group, and quantum Monte Carlo, can typically only handle systems with relatively few particles—generally no more than 100—due to the challenges in computation time and cost. However, our atomic system contains thousands

of atoms, making it difficult to apply these more precise numerical methods. So we believe our findings can inspire theorists to conduct more in-depth studies on the quantum many-body behavior of large-scale atomic systems in the weak-lattice limit.

Besides this unconventional dynamics, we also identify the emergence of a density plateau in the spatial density distributions [Fig. 4(a)]. This plateau spans approximately 200 μm , corresponding to a variation in chemical potential of around 100 nK (2 kHz). In the presence of a harmonic trap, the existence of such a density plateau indicates that the compressibility, defined as $\kappa = \partial n / \partial \mu$, equals to zero [Fig. 4(d)], suggesting the presence of an insulating behavior. Moreover, one can invoke the fluctuation-dissipation theorem to determine that incompressibility implies a low density-fluctuation [50]. The result can be found in Sec. XI of the Supplemental Material [37]. However, under our experimental parameters, this insulating behavior should not appear. Within the framework of the GP equation, the

atomic system is treated as hydrodynamics. While under the tight-binding approximation and Bose-Hubbard model, both the ratio of chemical potential to on-site interaction strength μ/U and the ratio of hopping rate to on-site interaction strength J/U exceed the order of 10^2 due to such a shallow lattice and the carriers are rubidium-85 atoms, while each atom is 1.5×10^5 times heavier than an electron. This suggests that the insulating behavior we observe is fundamentally different from that of a conventional Mott insulator. Moreover, we examine whether the atomic system still remains in the quantum-degenerate state after 1 s of evolution. The results are shown in Sec. XII of the Supplemental Material [37]. We find that the atoms in the right peak, regarded as localized, are still in the quantum-degenerate state, whereas the atoms at the plateau on the bottom of the 1D trap are already thermal. This stands that the insulating behavior may be associated with prethermalization during nonequilibrium dynamics and provides a clearer direction for future theoretical work that a fully rigorous theory should include a description of the thermal atoms and involve the complex interactions between the cold and hot components. We will learn more about the properties of this interesting plateau in our future work, for example, how this plateau is formed, on what time scale it begins to form, and so on.

III. CONCLUSIONS

In this work, we have investigated Stark many-body localization in a one-dimensional Bose gas under the continuous limit with a very shallow lattice and a large atom number, situated in a new regime that is far from the tight-binding regime used in previous work. In this model, a complex crossover between oscillation and stationary behaviors is observed for the case of short-time evolution, which can be explained by the combined effect of intra- and interband dynamics under the framework of mean-field approach. Localized behavior is also observed after a long-term evolution with relatively small scattering length. Additionally, we find that the system exhibits stronger localization as the initial density of the atomic cloud increases, which is contrary to the mean-field prediction. Notably, a density plateau emerges in the presence of a harmonic trap, indicating an insulating behavior that may be associated with prethermalization. Our experimental results demonstrate that Stark many-body localization persists even when the tight-binding approximation breaks down, which will stimulate deeper investigations into quantum many-body systems beyond both tight-binding and mean-field approximations. We would like to emphasize that although the lattice used in our experiments is very shallow (much less than $1\text{ }Er$), its presence is crucial. Without any lattice, the trap would become a purely harmonic potential. In a harmonic trap, the atoms' potential and kinetic energies transform into each other, and their

motion resembles the harmonic oscillation of a spring. We believe that our work will deepen the understanding of Stark many-body localization and inspire further research in this domain.

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Zihan Zhao and Zhongchi Zhang contributed equally to this work.

DATA AVAILABILITY

The data that support the findings of this article are openly available [51].

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- [1] Rahul Nandkishore and David A. Huse, Many-body localization and thermalization in quantum statistical mechanics, *Ann. Rev. Condens. Matter Phys.* **6**, 15 (2015).
 - [2] Dmitry A. Abanin, Ehud Altman, Immanuel Bloch, and Maksym Serbyn, *Colloquium: Many-body localization, thermalization, and entanglement*, *Rev. Mod. Phys.* **91**, 021001 (2019).
 - [3] L. Fleishman and P. W. Anderson, Interactions and the Anderson transition, *Phys. Rev. B* **21**, 2366 (1980).
 - [4] L. Fallani, J. E. Lye, V. Guarrera, C. Fort, and M. Inguscio, Ultracold atoms in a disordered crystal of light: Towards a Bose glass, *Phys. Rev. Lett.* **98**, 130404 (2007).
 - [5] S. S. Kondov, W. R. McGehee, W. Xu, and B. DeMarco, Disorder-induced localization in a strongly correlated atomic Hubbard gas, *Phys. Rev. Lett.* **114**, 083002 (2015).
 - [6] Michael Schreiber, Sean S. Hodgman, Pranjali Bordia, Henrik P. Lüschen, Mark H. Fischer, Ronen Vosk, Ehud Altman, Ulrich Schneider, and Immanuel Bloch, Observation of many-body localization of interacting fermions in a quasirandom optical lattice, *Science* **349**, 842 (2015).
 - [7] J. Y. Choi, Sebastian Hild, Johannes Zeiher, Peter Schauf, Antonio Rubio-Abadal, Tarik Yefsah, Vedika Khemani, David A. Huse, Immanuel Bloch, and Christian Gross, Exploring the many-body localization transition in two dimensions, *Science* **352**, 1547 (2016).
 - [8] Alexander Lukin, Matthew Rispoli, Robert Schittko, M. Eric Tai, Adam M. Kaufman, Soonwon Choi, Vedika Khemani, Julian Léonard, and Markus Greiner, Probing entanglement in a many-body-localized system, *Science* **364**, 256 (2019).
 - [9] J. Smith, A. Lee, P. Richerme, B. Neyenhuis, P. W. Hess, P. Hauke, M. Heyl, D. A. Huse, and C. Monroe, Many-body localization in a quantum simulator with programmable random disorder, *Nat. Phys.* **12**, 907 (2016).
 - [10] W. Morong, F. Liu, P. Becker, K. S. Collins, L. Feng, A. Kyprianidis, G. Pagano, T. You, A. V. Gorshkov, and

- C. Monroe, Observation of Stark many-body localization without disorder, *Nature* **599**, 393 (2021).
- [11] Qiujiang Guo, Chen Cheng, Hekang Li, Shibo Xu, Pengfei Zhang, Zhen Wang, Chao Song, Wuxin Liu, Wenhui Ren, Hang Dong, Rubem Mondaini, and H. Wang, Stark many-body localization on a superconducting quantum processor, *Phys. Rev. Lett.* **127**, 240502 (2021).
- [12] Sebastian Scherg, Thomas Kohlert, Pablo Sala, Frank Pollmann, Bharath Hebhe Madhusudhana, Immanuel Bloch, and Monika Aidelsburger, Observing non-ergodicity due to kinetic constraints in tilted Fermi-Hubbard chains, *Nat. Commun.* **12**, 4490 (2021).
- [13] M. Schulz, C. A. Hooley, R. Moessner, and F. Pollmann, Stark many-body localization, *Phys. Rev. Lett.* **122**, 040606 (2019).
- [14] Evert van Nieuwenburg, Yuval Baum, and Gil Refael, From Bloch oscillations to many-body localization in clean interacting systems, *Proc. Natl. Acad. Sci. U.S.A.* **116**, 9269 (2019).
- [15] Hristiana Atanasova, André Erpenbeck, Emanuel Gull, Yevgeny Bar Lev, and Guy Cohen, Stark many-body localization in interacting infinite dimensional systems, *Phys. Rev. Lett.* **132**, 166301 (2024).
- [16] Haoyuan Li, Jia Wang, Xia-Ji Liu, and Hui Hu, Many-body localization in Ising models with random long-range interactions, *Phys. Rev. A* **94**, 063625 (2016).
- [17] M. Serbyn, M. Knap, S. Gopalakrishnan, Z. Papić, N. Y. Yao, C. R. Laumann, D. A. Abanin, M. D. Lukin, and E. A. Demler, Interferometric probes of many-body localization, *Phys. Rev. Lett.* **113**, 147204 (2014).
- [18] Shuo Liu, Shi-Xin Zhang, Chang-Yu Hsieh, Shengyu Zhang, and Hong Yao, Discrete time crystal enabled by Stark many-body localization, *Phys. Rev. Lett.* **130**, 120403 (2023).
- [19] Hajime Nakanotani, Taro Furukawa, Kei Morimoto, and Chihaya Adachi, Long-range coupling of electron-hole pairs in spatially separated organic donor-acceptor layers, *Sci. Adv.* **2**, e1501470 (2016).
- [20] Gyunghun Kim, Joseph Suh, Dayeong Lee, Namkyoo Park, and Sunkyu Yu, Long-range-interacting topological photonic lattices breaking channel-bandwidth limit, *Light: Sci. Appl.* **13**, 189 (2024).
- [21] Haijun Tang, Can Huang, Yuhan Wang, Xiong Jiang, Ruiheng Jin, Yue Cui, Shumin Xiao, and Qinghai Song, Dynamically tunable long-range coupling enabled by bound state in the continuum, *Light: Sci. Appl.* **14**, 278 (2025).
- [22] N. S. Israel and L. Ramunno, in *2023 Photonics North (PN)* (IEEE, New York City, USA, 2023), pp. 1–2.
- [23] Changqing Xu, Gang Wang, Zhi Hong Hang, Jie Luo, C. T. Chan, and Yun Lai, Design of full-k-space flat bands in photonic crystals beyond the tight-binding picture, *Sci. Rep.* **5**, 18181 (2015).
- [24] Stefano Longhi, Non-Hermitian skin effect beyond the tight-binding models, *Phys. Rev. B* **104**, 125109 (2021).
- [25] Zhoufei Liu, Peng Jin, Min Lei, Chengmeng Wang, Fabio Marchesoni, Jian-Hua Jiang, and Jiping Huang, Topological thermal transport, *Nat. Rev. Phys.* **6**, 554 (2024).
- [26] Holger Fehske, Ralf Schneider, and Alexander Weisse, *Computational Many-Particle Physics* (Springer, Berlin, Germany, 2007), Vol. 739.
- [27] Oliver Morsch and Markus Oberthaler, Dynamics of Bose-Einstein condensates in optical lattices, *Rev. Mod. Phys.* **78**, 179 (2006).
- [28] Hui Zhai, *Ultracold Atomic Physics* (Cambridge University, Cambridge, United Kingdom, 2021).
- [29] Nathan Gemelke, Xibo Zhang, Chen-Lung Hung, and Cheng Chin, *In situ* observation of incompressible Mott-insulating domains in ultracold atomic gases, *Nature* **460**, 995 (2009).
- [30] Jacob F. Sherson, Christof Weitenberg, Manuel Endres, Marc Cheneau, Immanuel Bloch, and Stefan Kuhr, Single-atom-resolved fluorescence imaging of an atomic Mott insulator, *Nature* **467**, 68 (2010).
- [31] Markus Greiner, Olaf Mandel, Tilman Esslinger, Theodor W. Hensch, and Immanuel Bloch, Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms, *Nature* **415**, 39 (2002).
- [32] Thilo Stöferle, Henning Moritz, Christian Schori, Michael Köhl, and Tilman Esslinger, Transition from a strongly interacting 1D superfluid to a Mott insulator, *Phys. Rev. Lett.* **92**, 130403 (2004).
- [33] W. S. Bakr, A. Peng, M. E. Tai, R. Ma, J. Simon, J. I. Gillen, S. Fölling, L. Pollet, and M. Greiner, Probing the superfluid-to-Mott insulator transition at the single-atom level, *Science* **329**, 547 (2010).
- [34] Chen-Lung Hung, Xibo Zhang, Nathan Gemelke, and Cheng Chin, Accelerating evaporative cooling of atoms into Bose-Einstein condensation in optical traps, *Phys. Rev. A* **78**, 011604 (2008).
- [35] N. R. Claussen, S. J. J. M. F. Kokkelmans, S. T. Thompson, E. A. Donley, E. Hodby, and C. E. Wieman, Very-high-precision bound-state spectroscopy near a ^{85}Rb Feshbach resonance, *Phys. Rev. A* **67**, 060701 (2003).
- [36] Cheng Chin, Rudolf Grimm, Paul Julienne, and Eite Tiesinga, Feshbach resonances in ultracold gases, *Rev. Mod. Phys.* **82**, 1225 (2010).
- [37] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/ygdy-hrgj> for the additional experimental measurements and the details of the relevant theory, which includes Refs. [52–54].
- [38] M. M. Cerimele, M. L. Chiofalo, F. Pistella, S. Succi, and M. P. Tosi, Numerical solution of the Gross-Pitaevskii equation using an explicit finite-difference scheme: An application to trapped Bose-Einstein condensates, *Phys. Rev. E* **62**, 1382 (2000).
- [39] Weizhu Bao, Dieter Jaksch, and Peter A. Markowich, Numerical solution of the Gross-Pitaevskii equation for Bose-Einstein condensation, *J. Comput. Phys.* **187**, 318 (2003).
- [40] Xavier Antoine, Weizhu Bao, and Christophe Besse, Computational methods for the dynamics of the nonlinear Schrödinger/Gross-Pitaevskii equations, *Comput. Phys. Commun.* **184**, 2621 (2013).
- [41] C. Sias, A. Zenesini, H. Lignier, S. Wimberger, D. Ciampini, O. Morsch, and E. Arimondo, Resonantly enhanced tunneling of Bose-Einstein condensates in periodic potentials, *Phys. Rev. Lett.* **98**, 120403 (2007).

- [42] Alessandro Zenesini, Carlo Sias, Hans Lignier, Yeshpal Singh, Donatella Ciampini, Oliver Morsch, Riccardo Mannella, Ennio Arimondo, Andrea Tomadin, and Sandro Wimberger, Resonant tunneling of Bose-Einstein condensates in optical lattices, *New J. Phys.* **10**, 053038 (2008).
- [43] Florian Meinert, Manfred J. Mark, Emil Kirilov, Katharina Lauber, Philipp Weinmann, Michael Gröbner, Andrew J. Daley, and Hanns-Christoph Nägerl, Observation of many-body dynamics in long-range tunneling after a quantum quench, *Science* **344**, 1259 (2014).
- [44] P. O. Fedichev, M. W. Reynolds, and G. V. Shlyapnikov, Three-body recombination of ultracold atoms to a weakly bound s level, *Phys. Rev. Lett.* **77**, 2921 (1996).
- [45] B. D. Esry, Chris H. Greene, and James P. Burke, Recombination of three atoms in the ultracold limit, *Phys. Rev. Lett.* **83**, 1751 (1999).
- [46] Tino Weber, Jens Herbig, Michael Mark, Hanns-Christoph Nägerl, and Rudolf Grimm, Three-body recombination at large scattering lengths in an ultracold atomic gas, *Phys. Rev. Lett.* **91**, 123201 (2003).
- [47] P. A. Altin, G. R. Dennis, G. D. McDonald, D. Döring, J. E. Debs, J. D. Close, C. M. Savage, and N. P. Robins, Collapse and three-body loss in a ^{85}Rb Bose-Einstein condensate, *Phys. Rev. A* **84**, 033632 (2011).
- [48] Zhao-Di Liu, Henri Lyyra, Yong-Nan Sun, Bi-Heng Liu, Chuan-Feng Li, Guang-Can Guo, Sabrina Maniscalco, and Jyrki Piilo, Experimental implementation of fully controlled dephasing dynamics and synthetic spectral densities, *Nat. Commun.* **9**, 3453 (2018).
- [49] M. Kolesik and J. V. Moloney, Numerical discreteness and dephasing in high-harmonic calculations in solids, *Phys. Rev. B* **108**, 115433 (2023).
- [50] Nathan Gemelke, Xibo Zhang, Chen-Lung Hung, and Cheng Chin, *In situ* observation of incompressible Mott-insulating domains in ultracold atomic gases, *Nature* **460**, 995 (2009).
- [51] Zihan Zhao, Stark many-body localization in one-dimensional Bose gas under the continuous limit, 2025. <https://doi.org/10.5281/zenodo.17966182>.
- [52] G. M. Muslu and H. A. Erbay, A split-step Fourier method for the complex modified Korteweg-de Vries equation, *Comput. Math. Appl.* **45**, 503 (2003).
- [53] J. A. C. Weideman and B. M. Herbst, Split-step methods for the solution of the nonlinear Schrödinger equation, *SIAM J. Numer. Anal.* **23**, 485 (1986).
- [54] Clarence Zener and Ralph Howard Fowler, Non-adiabatic crossing of energy levels, *Proc. R. Soc. London, Ser. A* **137**, 696 (1932).