

Quantum Detectors as Autonomous Machines: Assessing the Nonequilibrium Thermodynamics of Information Acquisition

Emanuel Schwarzhans^{1,2,*} Tony J. G. Apollaro^{1,†} Ilia Khomchenko^{1,‡} Maximilian P. E. Lock^{2,3,§}
Mark T. Mitchison^{4,5,||} and Marcus Huber^{2,3,¶}

¹*Department of Physics, University of Malta, Msida MSD 2080, Malta*

²*Atominstytut, TU Wien, 1020 Vienna, Austria*

³*Institute for Quantum Optics and Quantum Information (IQOQI), Austrian Academy of Sciences, 1090 Vienna, Austria*

⁴*School of Physics, Trinity College Dublin, College Green, Dublin 2, D02 K8N4, Ireland*

⁵*Department of Physics, King's College London, Strand, London, WC2R 2LS, United Kingdom*



(Received 5 September 2025; accepted 14 May 2026; published 1 July 2026)

We formulate a minimal model of a quantum particle detector as an autonomous quantum thermal machine. Our goal is to establish how entropy production, which is needed to maintain the detector out of equilibrium, is linked to the quality of the measurement process. Using our model, we perform a detailed investigation of the detector's key performance characteristics: namely, detection efficiency, gain, jitter, dead time, and dark counts. We find that entropy production constrains both the efficiency and temporal precision of the detection process, in the sense that improved performance generally requires more dissipation. We also find that reducing either the detection jitter or dead time unavoidably increases the rate of dark counts. Our work establishes a quantitative connection between entropy production and the quality of the irreversible detection process, highlights fundamental tradeoffs in the performance of particle detectors, and provides a framework for further investigations of the non-equilibrium thermodynamics of quantum measurement and amplification.

DOI: [10.1103/wm5p-tjtg](https://doi.org/10.1103/wm5p-tjtg)

I. INTRODUCTION

The process of measurement is an essential step in any form of information processing. In quantum systems, measurement requires information about a microscopic property to be reproduced by a macroscopic detector so that it can be recorded or broadcast for further use. And, as with all information processing, measurement comes with thermodynamic costs and limitations. An early example was Landauer's discovery that erasing one bit of information necessarily entails a minimum amount of heat dissipation [1]. While this idealized bound is not reachable for practical finite-time processes [2–7], it nonetheless implies a fundamental work cost for any quantum measurement,

which must be invested to either initially prepare [8] or ultimately erase [9,10] the memory that stores the outcome. In recent years, other thermodynamic limitations of quantum measurement have been studied intensively, including the work cost of correlating the system and the memory [11–14], the importance of a phase transition to amplify the measurement outcome [15,16], and the irreversible nature of “objectification” [17,18]: the process whereby information becomes redundantly encoded across many classical degrees of freedom [19,20].

The vast majority of prior investigations into the thermodynamics of measurement have taken place against the backdrop of a single thermal environment. In this context, any energetic or entropic costs are associated with the initial departure from, and eventual return to, thermal equilibrium by the system or measurement apparatus. This paradigm fails to capture the basic working principles of many real detectors, whose operation requires them to be maintained in a nonequilibrium steady state (NESS). Whether considering charge detection by a quantum point contact [21], microwave-field detection using a quantum-limited linear amplifier [22], or photodetection by an avalanche photodiode [23], any continuous quantum measurement with high gain demands a steady flux of energy through the detector. Indeed, it is known that a

*Contact author: emanuel.schwarzhans@um.edu.mt

†Contact author: tony.apollaro@um.edu.mt

‡Contact author: ilia.khomchenko@um.edu.mt

§Contact author: maximilian.paul.lock@tuwien.ac.at

||Contact author: mark.mitchison@kcl.ac.uk

¶Contact author: marcus.huber@tuwien.ac.at

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

quantum-limited detector in the linear-response regime exhibits strong non-reciprocity between its input and output [24], thus violating the Onsager reciprocity conditions and implying that a good detector must be far from equilibrium [25]. This suggests an intriguing connection between entropy production within the detector and the quality of the measurement process. However, the general relationship between heat dissipation and detector performance remains largely unexplored.

Here, we address this problem by constructing a minimal model of a detector that is fully self-contained and autonomous, allowing us to capture all thermodynamic resources necessary for its operation. A similar strategy has proven fruitful in establishing fundamental limits on timekeeping [26,27] and refrigeration [28,29]. So let us set out by defining the minimal properties that any such detector model must exhibit:

- P.1. It must be based on a physical interaction between a target and a measurement device [30,31].
- P.2. It must be autonomous, i.e. without external (temporal) control to mediate the measurement.
- P.3. It should be usable repeatedly or continuously.
- P.4. It needs to *amplify* the signal for a macroscopically noticeable state change to occur.

We focus on detectors that produce a discrete, macroscopic “click” when a quantum particle is present, e.g. a photodetector, as opposed to those that produce a continuous signal, e.g. a quantum-limited field amplifier or heterodyne detector. To characterize the performance of such particle detectors, we identify four desiderata that an “ideal” detector would have to fulfill [32]:

Desiderata

- D.1. It clicks *only* when a quantum excitation is present; clicks from fluctuations are called *dark counts*, and are given by the dark count rate R_{dc} .
- D.2. These clicks should happen at the *exact time* a quantum excitation is present; uncertainty in this timing is called *detection jitter* Δt_D .
- D.3. It should *always* click when a quantum excitation is present; the actual success rate is called *detection efficiency* η_D .
- D.4. After each detection, it should be *immediately* ready for the next; the minimal recovery time is called *dead time* D .

In Sec. II, we outline a minimal model that exhibits the properties P.1.–P.4. As illustrated in Fig. 1, our setup exploits a quantum thermal absorption machine [33] to drive a gain medium into a metastable state, which is triggered by an incoming particle to produce an amplified

output current with high energy gain. We then define metrics that quantify how well each of the desiderata D.1.–D.4. is satisfied in Sec. III, before applying them to analyze the performance of our minimal detector model in Sec. IV. We demonstrate that higher detection efficiency comes at the price of increased entropy production. We also find a tradeoff between the detector’s dark count rate and detection jitter, where the latter can be decreased at the expense of the former by increased heat dissipation. These results establish a quantitative link between the irreversible nature of the detection process and the dissipation of heat in the non-equilibrium steady state of the detector. We discuss the consequences of these findings for the foundations of quantum thermodynamics and quantum measurement theory in Sec. V.

II. THE MODEL

Real-world detectors typically consist of many-body systems comprising multiple energy levels that contribute to generating an amplified signal with macroscopically detectable energy. To achieve this, independent of their explicit physical implementation, all detectors share a common feature—a metastable high-energy initial state.

To facilitate a tractable analysis of fundamental thermodynamic limitations, we abstract these many-body dynamics into a minimal effective model that requires no external control to operate (no active on/off switching). The minimal model must consist of two ingredients: a *work source*, modeled as a two-qubit quantum thermal machine operating in the heat engine regime (i.e., inducing a population inversion) [34], utilizing a heat flow from a hot to a cold bath to provide work to *gain medium*, which is modeled as a three-level system. The *work source* drives the *gain medium* out of equilibrium into a high-energy state, which functions as a “metastable” energy level (see Fig. 1). The task of the gain medium is twofold: it facilitates interaction with the quantum particle that triggers the detection event (e.g., the photon), and it amplifies the signal produced by such a detection to become macroscopically detectable.

Reducing the size of the model further is not feasible, since amplification requires the gain medium to feature at least three distinct energetic states: the “ground state” $|0\rangle_G$, the “metastable” (or “detection ready”) state $|1\rangle_G$ which is at a high energy compared to the energy of the quantum particle, and the “activated” state $|2\rangle_G$ which can only be accessed via interaction with a quantum particle at the right energy.

Ideally, a detection event lifts the gain medium from the “detection-ready” state into the “activated” state after which it decays via the detection channel into the ground state. The energy dissipated in this manner corresponds to the *threshold energy* and is assumed to be macroscopically detectable (or sufficiently large to trigger a von Neumann chain [31]). Once the gain medium reaches the ground

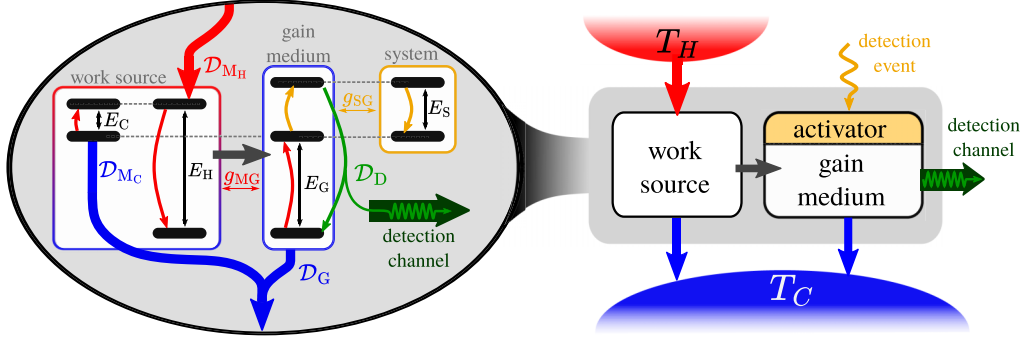


FIG. 1. Illustration of the minimal model for single-event detection. The right side depicts a rough overview of the necessary elements for autonomous single-event amplification, i.e. a *gain medium* and a *work source*, and a quantum system to be detected (indicated by the vertical yellow wavy arrow). The left side depicts the detailed energy level structure of the minimal model that achieves this task, and all parameters that determine a detector's performance, i.e. the energy gaps E_C , E_H of the machine's cold and hot qubit, $E_G = E_H - E_C$ of the gain medium's first gap and the energy E_S of the system to be detected (yellow box), which is resonant with the gain medium's second gap, the machine-gain medium coupling g_{MG} , the gain medium-system coupling g_{SG} and the dissipator coupling to the respective baths and the detector channel $\mathcal{D}_{MH}$, $\mathcal{D}_{MC}$, $\mathcal{D}_G$ and $\mathcal{D}_D$.

state, the work source lifts it back into the detection-ready state, in which it remains until the next quantum particle arrives, as depicted in Fig. 1. However, since every part of the detector is necessarily embedded in a thermal environment, thermal fluctuations introduce noise, reducing the detector's performance.

We model the quantum particle to be detected as a two-level system that interacts coherently with the gain medium, where $|0\rangle_S$ means no particle is present and $|1\rangle_S$ means there is a particle.

The total Hilbert space structure of our model is given as $\mathcal{H} = \mathcal{H}_{MC} \otimes \mathcal{H}_{MH} \otimes \mathcal{H}_G \otimes \mathcal{H}_S$, where $\mathcal{H}_{MC}$ and $\mathcal{H}_{MH}$ denote the space of the hot and the cold qubit (jointly the machine subspace $\mathcal{H}_M = \mathcal{H}_{MC} \otimes \mathcal{H}_{MH}$), $\mathcal{H}_G$ denotes the gain medium and $\mathcal{H}_S$ denotes the space of the quantum particle (e.g. that of a photon).

The free Hamiltonian of the total model is given as

$$H_M = E_C|1\rangle\langle 1|_{MC} + (E_C + E_G)|1\rangle\langle 1|_{MH}, \quad (1)$$

$$H_G = E_G|1\rangle\langle 1|_G + (E_G + E_S)|2\rangle\langle 2|_G, \quad (2)$$

$$H_S = E_S|1\rangle\langle 1|_S, \quad (3)$$

$$H_0 = H_M + H_G + H_S. \quad (4)$$

The energy level structure of each of the model's components is illustrated in Fig. 1.

To ensure a complete bookkeeping of all thermodynamic resources, we require energy-conserving interactions, i.e. that $[H, H_0] = 0$ (this also avoids the additional noise otherwise mandated by the Wigner-Araki-Yanase theorem for symmetry-breaking observables [35]). Therefore, the transition $|0\rangle_G$ to $|1\rangle_G$, driven by the quantum thermal machine, must be resonant with the virtual qubit of the thermal machine. The transition from

$|1\rangle_G$ to $|2\rangle_G$ needs to be resonant with the energy of the quantum particle. This leaves us with the total interaction Hamiltonian

$$H_I = g_{MG}|101\rangle\langle 010|_{MC,MH,G} + g_{SG}|20\rangle\langle 11|_{G,S} + \text{H.c.} \quad (5)$$

The total Hamiltonian is $H = H_0 + H_I$. The dynamics of the total system coupling to a hot and a cold bath, as indicated by Fig. 1, is given by the Lindblad master equation

$$\mathcal{L}\rho = -i[H, \rho] + (\mathcal{D}_{MC} + \mathcal{D}_{MH} + \mathcal{D}_G + \mathcal{D}_D)\rho, \quad (6)$$

where the dissipators and the corresponding dissipation rates are given explicitly in Appendix A. $\mathcal{D}_{MC/H}$ are Lindblad dissipators connecting the hot and cold machine qubits with their respective baths at temperatures T_C and T_H , $\mathcal{D}_G$ acts to thermalize the gain medium with the environment, which we assume to be at the cold bath temperature T_C , and the detection channel is implemented via the dissipator

$$\mathcal{D}_D = \Gamma_D^+(T_C)\mathcal{D}[L_D^+] + \Gamma_D^-(T_C)\mathcal{D}[L_D^-]. \quad (7)$$

Here, $\mathcal{D}[L]\rho = L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}$ with $L_D^+ = |2\rangle\langle 0|_G$ and $L_D^- = |0\rangle\langle 2|_G$, while $\Gamma_D^\pm(T_C)$ are the temperature-dependent gain and loss rates. This is the channel which provides information about the detection; it couples the lowest level of the gain medium to its maximally excited energy level, i.e. $|0\rangle_G$ and $|2\rangle_G$.

Note that our description in terms of a local Lindblad master equation is valid so long as we consider weak internal coupling between the detector, the machine, and the system. In particular, these couplings should be much smaller than the correlation time of any environment, so

that these environments do not resolve the small energy splittings on the order of g_{MG} and g_{SG} [36]. This is the appropriate regime for the particle detector setup that we consider, where the goal is to register the presence of an excitation rather than to measure its precise energy. Increasing the internal couplings beyond the validity of the local Lindblad regime would generally lead to additional inefficiency due to heat leaks that can emerge in multi-bath scenarios [37]. In the remainder of this work, therefore, we focus exclusively on the regime of weak internal coupling where Eq. (6) is valid.

III. TRANSIENT BEHAVIOR AND DETECTION PERFORMANCE

Under the dynamics given by Eq. (6), the detector will evolve toward the unique non-equilibrium steady state ρ^{ss} , defined by $\mathcal{L}\rho^{ss} = 0$. This can be understood as the detection-ready state.

Note that the dynamics do not include incoming particles. To trigger a detection event, we thus initialize the detector in the state ρ^{ss} and then bring in a particle, corresponding to the update

$$\rho_0 = \frac{\sigma_S^+ \rho^{ss} \sigma_S^-}{\text{Tr}(\sigma_S^- \sigma_S^+ \rho^{ss})} = \text{Tr}_S[\rho^{ss}] \otimes |1\rangle\langle 1|_S, \quad (8)$$

where $(\sigma_S^-)^\dagger = \sigma_S^+ = |1\rangle\langle 0|_S$, thus replacing S with an excited state. We can see this as conditioning on the case where a particle is present (see [38], Sec. II C). We then let the total system evolve via Eq. (6) until the detector has returned to the detection-ready state ρ^{ss} .

The detector's performance, quantified by how well it achieves the desiderata D.1.–D.4., is determined by the transient behavior of the average net number of excitations emitted via the detection channel $\mathcal{D}_D$, i.e. the average *detection current*

$$J_D(t) = \text{Tr}[\mathcal{J}_D \rho(t)] = \text{Tr}[(\Gamma_D^- |2\rangle\langle 2|_G - \Gamma_D^+ |0\rangle\langle 0|_G) \rho(t)]. \quad (9)$$

The first term above corresponds to the emission of an amplified excitation by the detector, and thus a successfully registered detection event. The second term describes unwanted absorption by the detector from the thermal background. $\mathcal{J}_D$ is the superoperator describing quantum jumps in the detection channel [38], defined as

$$\mathcal{J}_D \rho = \Gamma_D^-(T_C) L_D^- \rho L_D^+ - \Gamma_D^+(T_C) L_D^+ \rho L_D^-, \quad (10)$$

and $\Gamma_D^\pm(T_C)$ are the temperature-dependent coupling rates (see Appendix A). The emitted excitations are greater in energy than that of the incoming particle which triggered them. The magnitude of this amplification is given by the *gain* $G = \frac{E_G}{E_S}$ (see Appendix E).

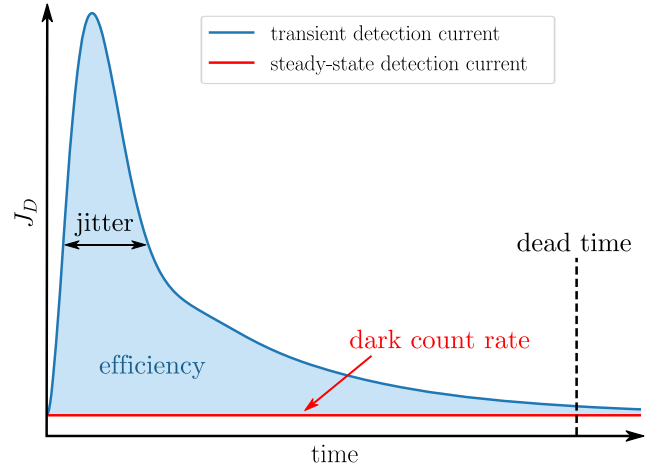


FIG. 2. Schematic dynamics of one detection event. The blue line shows the transient current in the detection channel $\mathcal{D}_D$. The red line indicates the dark current given by the steady-state current J_D^{ss} . Integrating the excess detection current gives the detection efficiency η_D , as indicated by the blue area. The detection jitter Δt_D is given by the variance of the excess detection current, as indicated by the horizontal arrows in the blue area. The dead time D is the timescale of returning to the detection-ready state ρ^{ss} , indicated by the dotted vertical line.

We treat consecutive detection events as independent; in particular, we assume that the detector reaches ρ^{ss} again before the next particle arrives and that there are no memory effects between individual runs. Thus, the average of a single event is equivalent to averaging over many consecutive detection events, allowing us to consider only one run.

We can now formalize the figures of merit corresponding to the desiderata. Figure 2 provides a schematic example of the detection current for one detection event and the corresponding figures of merit.

The *dark count rate* (D.1.) is given as the steady-state detection current of the detector, representing the false-positive detection rate

$$R_{\text{dc}} = J_D^{ss} = \text{Tr}[\mathcal{J}_D \rho^{ss}]. \quad (11)$$

The *detection jitter* (D.2.) is given as the temporal variance of the normalized excess detection current $p_{\tilde{J}_D}(t) = \frac{1}{\eta_D} (J_D(t) - J_D^{ss})$:

$$\Delta t_D = \text{Var}(t)_{p_{\tilde{J}_D}} = \int_0^\infty dt t^2 \frac{\tilde{J}_D(t)}{\eta_D} - \left[\int_0^\infty dt t \frac{\tilde{J}_D(t)}{\eta_D} \right]^2, \quad (12)$$

where $\tilde{J}_D(t) = J_D(t) - J_D^{ss}$ and η_D is the *detection efficiency* (D.3.), given as the total (excess) number of jumps through the detection channel

$$\eta_D = \int_0^\infty dt \tilde{J}_D(t). \quad (13)$$

Finally, the *dead time* (D.4.) is defined as the timescale at which the detector returns to the steady state (i.e. the time it takes to reset), which we take to be the slowest timescale in the dynamics, given by the smallest non-zero eigenvalue of the Liouvillian, denoted λ_1 :

$$D \sim -\frac{1}{\text{Re}(\lambda_1)} \quad (14)$$

note that $\text{Re}(\lambda_1) < 0$.

For further details on the figures of merit, please refer to Appendix B.

IV. PERFORMANCE TRADE-OFF RELATIONS

Any detector requires some resources to operate. As mentioned, our detector acquires energy from a work source, which it inevitably loses, either due to the necessary coupling to the environment or due to its conversion into a macroscopically detectable signal. The environmental coupling is responsible for these energy losses or dissipation, which inevitably result in entropy production. Increasing the precision of the output of a stochastic process typically comes at the cost of increased entropy production [39–42]. Thus, one would generally expect the performance of detectors, and thus their capacity to fulfill the desiderata D.1.–D.4. to come at an entropic cost. Additionally, optimizing one figure of merit can, in principle, come at the cost of worsening others, resulting in trade-off relations between some of these quantities.

In the following, we will present such trade-offs revealed by numerical sampling of the entire parameter space (see Fig. 1), for a given fixed energy gain $G = 9$ (see Appendix E for scaling behavior with G). This, in turn, allows us to make informed proposals for analytic relations (see [43] for all datasets and the complete code used for simulation).

The missing ingredient for understanding thermodynamic costs related to measurements is the total entropy dissipated during a single detection event Σ_{tot} . It can be split into two parts: the entropy production rate required to keep the detector ready,

$$\dot{\Sigma}_{ss} = \frac{1}{T_C} \text{Tr}[(E_C \mathcal{J}_{M_C} + E_G \mathcal{J}_{G_{1-0}} + E_S \mathcal{J}_{G_{2-1}}) \rho^{ss}], \quad (15)$$

where $\mathcal{J}_{M_C}$, $\mathcal{J}_{L_{1-0}}$ and $\mathcal{J}_{L_{2-1}}$ denote the current operators related to dissipation into the cold bath, and the additional entropy produced by the transient dynamics

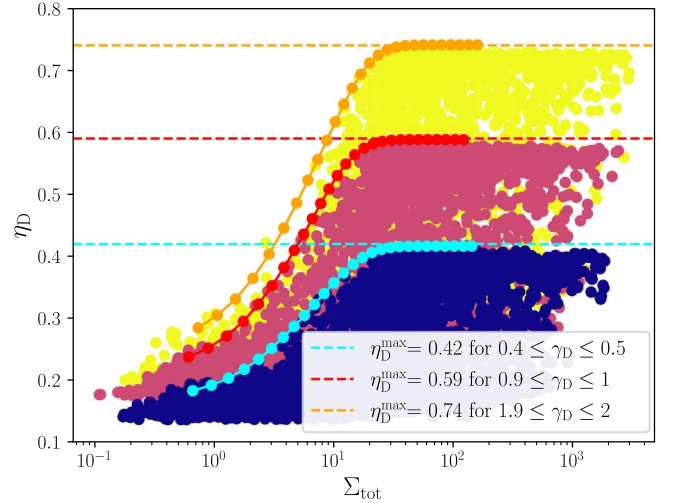


FIG. 3. Detection efficiency η_D over total entropy production Σ_{tot} for different values of detection channel coupling $\gamma_D \in \{0.45, 0.95, 1.95\}$ (selecting points in a range ± 0.05 around these values for the scatter plots) and fixed bath coupling $\gamma_G = 0.7$. The horizontal dashed lines indicate the maximal efficiency for the given coupling rates given by Eq. (18). The lines with dots are along $T_V = \frac{-T_C E_H}{E_C} (1 + \varepsilon)$, where $\varepsilon = 10^{-6}$, i.e. close to the maximal virtual temperature for which the thermal machine still operates in the engine regime [34]. The parameter ranges for the scatter plots are: $T_C \in [0.05, 1]$, $T_V \in [-10, \frac{-T_C E_H}{E_C}]$, $g_{SG} \in [0.1, 1]$, $g_{MG} \in [0.1, 2]$, $\gamma_M \in [0.1, 2]$ and $f_{E_C} \in [0.1, 2]$ (where $E_C = f_{E_C} E_G$).

$$\Sigma_{\text{trans}} = \frac{1}{T_C} \int_0^\infty dt (E_C \tilde{J}_{M_C}(t) + E_G \tilde{J}_{G_{1-0}}(t) + E_S \tilde{J}_{G_{2-1}}(t)), \quad (16)$$

where $\tilde{J}_x(t) = J_x(t) - J_x^{ss}$ is the excess current through channel $\mathcal{D}_x$ with index G_{1-0} and G_{2-1} corresponding to the channels of the first and second gap of G . Accordingly, we write

$$\Sigma_{\text{tot}} \sim -\frac{\dot{\Sigma}_{ss}}{\text{Re}(\lambda_1)} + \Sigma_{\text{trans}}, \quad (17)$$

where the entropy production due to the steady-state entropy production rate is calculated as $\int_0^D dt \dot{\Sigma}_{ss} \sim -\frac{\dot{\Sigma}_{ss}}{\text{Re}(\lambda_1)}$ [see Eq. (14)]. For further details, see Appendix B.

Now we can ask: *Does the entropy production impose a constraint on the efficiency η_D ?* Within our model, this question can be answered positively: to achieve a given efficiency, the detector must necessarily dissipate a minimal amount of entropy. This trade-off is depicted in Fig. 3 via scatter plots over all parameters (see Fig. 1), except for the coupling constants of the detector to the detection channel, γ_D , and of the gain medium to its environment, γ_G , whose ratio is kept fixed. We see that there is a regime in

which increased efficiency requires an increase in entropy production per detection event. Increasing the entropy production beyond that will yield reduced efficiency gains, eventually resulting in a plateau. In Appendix C, we conjecture that the maximal efficiency is given by:

$$\eta_{\max} = \frac{1}{1 + \frac{\gamma_G}{\gamma_D}}, \quad (18)$$

which is achieved in the limit $T_C \rightarrow 0$ (where the coupling rates tend to the temperature independent coupling constants $\Gamma_D^-(T_C) \rightarrow \gamma_D$ and $\Gamma_G^-(T_C) \rightarrow \gamma_G$, see Appendix A for their definitions), thus requiring diverging entropy production. This is supported by the numerical results, which can be found in Fig. 3. This bound has been experimentally observed, for example, in quantum-limited measurements of levitating nanoparticles, as shown in Eq. S24 of the supplemental material of [44]. The rationale behind this hypothesis is the following: In the conditional branch where a particle is present, i.e. where the detector is in a state given by Eq. (8), the excitation from the system can coherently be transferred between the system and the gain medium, or dissipatively leave the gain medium-system subspace spanned by $\mathbb{1}_M|1\rangle_G \otimes |1\rangle_S$ and $\mathbb{1}_M|2\rangle_G \otimes |0\rangle_S$. Thus, the excitation can only leave via the detection channel $\mathcal{D}_D$ or the thermalizing channel $\mathcal{D}_G$, which leads us to the guess that the coupling rates of these channels determine the efficiency. We observe numerically that this heuristic does not hold in the finite temperature case, but it does in the limit $T_C \rightarrow 0$. Since γ_G is small if the detector is well isolated from its environment and γ_D

depends on our ability to couple it to the monitored environment, the maximally achievable detection efficiency is limited by our ability to isolate the gain medium from the environment, even if the environment is at zero temperature.

High detection efficiency alone, however, is not sufficient for most information processing tasks. These often require precise temporal information (i.e. low *detection jitter* Δt_D) and a low rate of counting false positives (i.e. low *dark count rate* R_{dc}). Given a highly efficient detector, we find that jitter and dark count rate can, in fact, not be improved independently—they follow a trade-off relation of the form (see Fig. 4)

$$\Delta t_D \propto \frac{1}{R_{dc}}. \quad (19)$$

Although it is well known that the dark count rate can be mitigated through gating, provided the jitter is low enough [32], this shows that a trade-off relationship exists independent of such strategies. At the same time, reducing the detection jitter incurs increased energetic costs of maintaining the detector at a “detection ready” state, i.e. increased steady-state entropy production rate $\dot{\Sigma}_{ss}$ as can be seen from the color bar plot of Fig. 4.

Intuitively, this twofold trade-off between dark count rate, entropy production rate and jitter can be understood by noting that a sharp peak in detection current (facilitating low jitter) requires a highly populated metastable state at high energy. Driving the detector to this state and keeping it there can only be achieved with a strong driving bias, which requires negative virtual temperatures close to zero and thus incurs a high entropy production rate. At the same time, a highly excited metastable state is more susceptible to thermal excitations, resulting in an increased likelihood of dark counts.

Finally, the speed at which information can be processed is intricately related to the rate at which it can be acquired. This rate is ultimately limited by the time it takes the detector to reset, i.e. the dead time [Eq. (14)]. We find that decreasing the dead time comes at the cost of increased dark count rate, shown numerically in Fig. 5. This final trade-off thus takes the form

$$R_{dc} \propto -\text{Re}(\lambda_1) \sim \frac{1}{D}, \quad (20)$$

where the exact proportionality factor depends on specifics of the detector, e.g. the coupling strength between system and gain medium, as depicted by the color coding in Fig. 5.

Combining Eqs. (20) and (19), we find a threefold relation between R_{dc} , Δt_D and D , which is depicted in Fig. 6. The third relation, given by $\Delta t_D \propto D$, is implied by the former two, where g_{SG} determines the proportionality constant. In Appendix D, we show numerical evidence for this relation. Moving along the vertical axis of this trade-

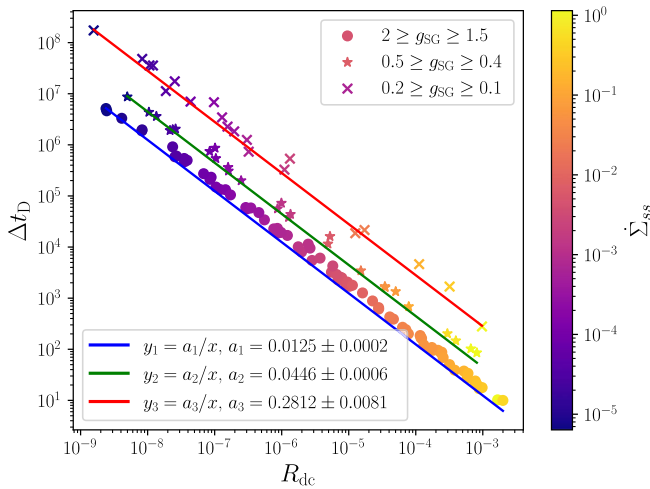


FIG. 4. Detection jitter Δt_D over dark count rate R_{dc} close to the maximum efficiency $0.72 < \eta_D < \eta_D^{\max} \approx 0.74$. The color map shows the steady-state entropy production rate. The solid lines are fits through these data points of $y = a/x$ where $y = R_{dc}$ and $x = -1/\text{Re}(\lambda_1)$. The parameter range is the same as in Fig. 3, except for $\gamma_D \in [0.1, 2]$ and $g_{SG} \in \{[0.1, 0.2], [0.4, 0.5], [1.5, 2]\}$.

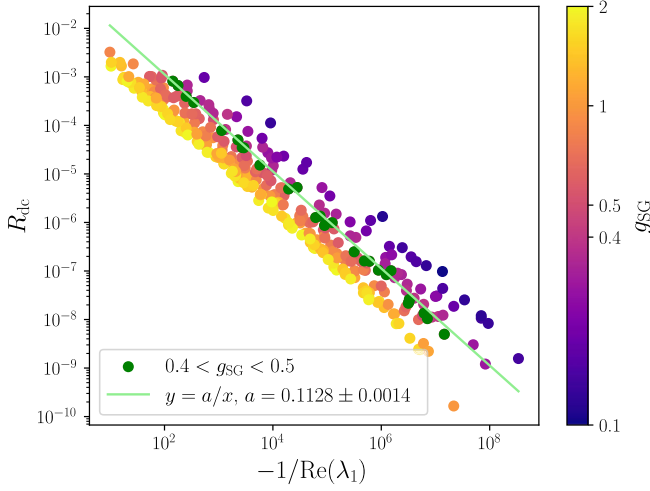


FIG. 5. The dark count rate R_{dc} over $-1/\text{Re}(\lambda_1)$, which is proportional to the dead time (i.e. $-1/\text{Re}(\lambda_1) \propto D$). The color map shows different values of g_{SG} . We consider data points close to maximal efficiency $0.72 < \eta_D < \eta_D^{\max} \approx 0.74$. The green line indicates a fit of $y = a/x$ where $x = 1/\lambda_1$ and $y = R_{dc}$. Decreasing the dead time comes at the cost of increased dark count rate. The parameter ranges for the scatter plots are the same as in Fig. 3 except $g_{SG} \in [0.1, 2]$.

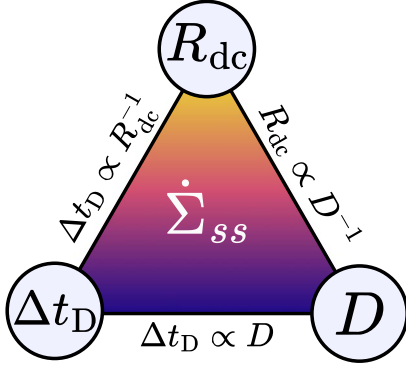


FIG. 6. Illustration of the trade-off relations between dark count rate R_{dc} , jitter Δt_D and dead time D . Increased R_{dc} comes with increased entropy production rate $\dot{\Sigma}_{ss}$ as indicated by the color gradient. Because R_{dc} is inversely proportional to both Δt_D and D , proportionality between the latter two is implied (for numerical evidence for this see Appendix D).

off triangle, i.e. changing R_{dc} incurs a change in entropy production rate $\dot{\Sigma}_{ss}$, where increasing R_{dc} increases $\dot{\Sigma}_{ss}$.

V. CONCLUSION

In this work, we have put forward a minimal autonomous model of a quantum particle detector. Our approach highlights that any such detector must be an open quantum system maintained in a metastable, non-equilibrium steady state (NESS) by a dissipative energy flux. As a result, good detectors irreversibly produce entropy. We have

demonstrated that this entropy production constrains detection efficiency, in the sense that efficiency cannot be increased beyond a certain point without also increasing the total entropy produced per detection event. We have also shown that reducing the detection timing jitter requires increasing the steady-state entropy production rate, i.e. the “housekeeping heat” needed to maintain the NESS [45]. These results establish how the quality of the detection process is determined by thermodynamic irreversibility within the detector.

While it was known that linear-response detectors must be out of equilibrium [25], our work is the first to explore the quantitative relation between entropy production and detector performance for particle detection: an inherently nonlinear process. Our overall conclusions add to the consensus [8–18] that the acquisition of information comes at a thermodynamic cost. Here, however, our analysis uniquely emphasizes that the cost of measurement is fundamentally linked to the far-from-equilibrium nature of the detector. We also note that, while our trade-offs are consistent with the intuition behind thermodynamic uncertainty relations (TURs) [39–42], they are not implied by any existing TURs. The latter constrain the relative *variance* of integrated currents—typically in the steady state—whereas the figures of merit we consider involve the transient response of the *average* current to an incoming particle [Eq. (9)]. Our results therefore represent genuinely new thermodynamic constraints on detector performance, with significant consequences for quantum thermodynamics, e.g., accurate measurements of work may themselves consume an amount of work far in excess of the measured work [46].

Thus, our results also bear on a broader theme in quantum thermodynamics, where measurements are often employed as a resource to extract work or drive thermodynamic tasks [47–49]. Our work highlights that such treatments should be approached with care: as we have shown, the measurement process itself inevitably comes with a thermodynamic cost. A complete accounting of these costs may therefore significantly alter the conclusions drawn from approaches that treat measurement as a free resource.

To identify the fundamental limits, our model assumes only limited complexity, i.e., with the minimal number of involved states and transitions. Real detectors are, of course, more complex and will exhibit many additional sources of parasitic dissipation that do not contribute to improving detector performance. However, given the recent insights that quantum clocks can circumvent entropic costs in accurate timing [50], one may wonder whether more complex detector designs could exploit coherent many-body dynamics to achieve higher thermodynamic efficiency. We also note that here we focus on the thermodynamic cost of amplification once the particle has been captured by the detector, whereas the process of

capturing an excitation (related to P.1.) may itself carry additional costs and inefficiencies.

Much work is still needed on the foundations of the measurement process and its energetic implications. While the formalism of Lindblad master equations we use is fully compatible with an “all-unitary picture of the measurement process, any stochastic collapse due to modifications of quantum mechanics [51,52] would either leave our conclusions unchanged or add extra sources of entropy production [53]. Unitarity of the process at the level of system and environment is also fully compatible with the measurement-equilibration hypothesis [17,18], and an explicit connection between the two approaches can likely be made through the equivalence of quantum master equations with closed-system equilibration [54].

While we focus our thermodynamic analysis on the detection of a definite excitation, an attractive feature of our approach is that it can naturally deal with fields in a superposition of vacuum and a particle excitation, $a|0\rangle + b|1\rangle$. In that case, the proportion of trajectories with a detector click is equal to the efficiency multiplied by the squared coefficient of superposition $|b|^2$. This seamlessly encapsulates the Born rule, which follows from our model without any further fine-tuning or assumptions.

ACKNOWLEDGMENTS

The authors thank Pharnam Bakhshinezhad, Florian Meier and Gabriel Landi for fruitful discussions. M.H. acknowledges support from ERC-2021-COG 101043705 “Cocoquest.” This research was funded in whole or in part by the Austrian Science Fund (FWF) [grant DOI: 10.55776/I6949, 10.55776/COE1] and the European Union—NextGenerationEU. M.T.M. is supported by a Royal Society University Research Fellowship. This project is co-funded by the European Union (Quantum Flagship project ASPECTS, Grant Agreement No. 101080167) and UK Research and Innovation (UKRI). The authors acknowledge the TU Wien Bibliothek for financial support through its Open Access Funding Program.

Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the European Union, Research Executive Agency or UKRI. Neither the European Union nor UKRI can be held responsible for them.

DATA AVAILABILITY

The data that support the findings of this article are openly available in [43].

APPENDIX A: LINDBLADIAN

The dynamics of the detector model is given via a GKLS master equation of the form

$$\mathcal{L}\rho = -i[H, \rho] + (\mathcal{D}_{\text{M}_c} + \mathcal{D}_{\text{M}_h} + \mathcal{D}_G + \mathcal{D}_D)\rho, \quad (\text{A1})$$

where H is the Hamiltonian of the system as given in the main text. The dissipators are given as follows:

The machine qubits are, respectively, interacting with the hot and the cold baths via

$$\mathcal{D}_{\text{M}_{c/h}} = \sqrt{\Gamma_{\text{M}}^+(T_{c/h})} \mathcal{D}[\sigma_{\text{M}_{c/h}}^+] + \sqrt{\Gamma_{\text{M}}^-(T_{c/h})} \mathcal{D}[\sigma_{\text{M}_{c/h}}^-], \quad (\text{A2})$$

where $(\sigma_{\text{M}_{c/h}}^-)^\dagger = \sigma_{\text{M}_{c/h}}^+ = |1\rangle\langle 0|_{\text{M}_{c/h}}$. The gain medium interacts with the cold bath (a result of imperfect isolation in real-world settings) via

$$\mathcal{D}_G = \sqrt{\Gamma_{\text{G}_{0-1}}^-(T_c)} \mathcal{D}[|0\rangle\langle 1|_G] + \sqrt{\Gamma_{\text{G}_{0-1}}^+(T_c)} \mathcal{D}[|1\rangle\langle 0|_G] \quad (\text{A3})$$

$$+ \sqrt{\Gamma_{\text{G}_{1-2}}^-(T_c)} \mathcal{D}[|1\rangle\langle 2|_G] + \sqrt{\Gamma_{\text{G}_{1-2}}^+(T_c)} \mathcal{D}[|2\rangle\langle 1|_G], \quad (\text{A4})$$

and the gain medium interacts with the detection channel via

$$\mathcal{D}_D = \sqrt{\Gamma_D^+(T_c)} \mathcal{D}[|2\rangle\langle 0|_G] + \sqrt{\Gamma_D^-(T_c)} \mathcal{D}[|0\rangle\langle 2|_G]. \quad (\text{A5})$$

Each dissipator is given in terms of the jump L operators as $\mathcal{D}[L]\rho = L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}$. The corresponding temperature-dependent coupling rates are given by

$$\Gamma_{\text{M}}^+(T_{c/h}) = \gamma_{\text{M}} \frac{e^{-E_{c/h}/T_{c/h}}}{\mathcal{Z}_{c/h}}, \quad \Gamma_{\text{M}}^-(T_{c/h}) = \frac{\gamma_{\text{M}}}{\mathcal{Z}_{c/h}} \quad (\text{A6})$$

$$\Gamma_{\text{G}_{0-1}}^+(T_c) = \gamma_G \left(\frac{e^{-E_G/T_c}}{\mathcal{Z}_{\text{G}_{0-1}}} \right), \quad \Gamma_{\text{G}_{0-1}}^-(T_c) = \gamma_G \left(\frac{1}{\mathcal{Z}_{\text{G}_{0-1}}} \right) \quad (\text{A7})$$

$$\Gamma_{\text{G}_{1-2}}^+(T_c) = \gamma_G \left(\frac{e^{-E_S/T_c}}{\mathcal{Z}_{\text{G}_{1-2}}} \right), \quad \Gamma_{\text{G}_{1-2}}^-(T_c) = \gamma_G \left(\frac{1}{\mathcal{Z}_{\text{G}_{1-2}}} \right) \quad (\text{A8})$$

$$\Gamma_{\text{G}_D}^+(T_c) = \gamma_D \left(\frac{e^{-(E_S+E_G)/T_c}}{\mathcal{Z}_{\text{G}_D}} \right), \quad \Gamma_{\text{G}_D}^-(T_c) = \gamma_D \left(\frac{1}{\mathcal{Z}_{\text{G}_D}} \right) \quad (\text{A9})$$

with the partition functions corresponding to the respective energy gaps E_x at bath temperature T_x given as $\mathcal{Z}_x = 1 + e^{-E_x/T_x}$.

APPENDIX B: FIGURES OF MERIT

As outlined in the introduction, an ideal detector should fulfill desiderata D.1., D.2., D.3., and D.4. These properties determine the figures of merit typically quantifying the performance of detectors (see e.g. for photon detection [32]): *dark count rate*, *timing jitter* and *detection efficiency*.

As a prerequisite, we would like to remind the reader of the following notation: In order to simplify notation, we will transform all operators into vectors in Liouville space, stacking their columns in the following way

$$\text{vec}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a \\ c \\ b \\ d \end{pmatrix} \quad (\text{B1})$$

using the notation

$$|\rho\rangle\rangle = \text{vec}(\rho), \quad (\text{B2})$$

and all superoperators into operators using the convention

$$\text{vec}(A\rho C) = (C^T \otimes A)\text{vec}(\rho). \quad (\text{B3})$$

Using this, the current through some channel x can be expressed in Liouville space via the superoperator for current $\mathcal{J}$ as

$$J_x(t) = \langle\langle 1 | \mathcal{J}_x | \rho(t) \rangle\rangle, \quad (\text{B4})$$

where $\langle\langle 1 |$ is the vectorized identity operator. The time-dependent state can be written as

$$|\rho(t)\rangle\rangle = |\rho^{ss}\rangle\rangle + \sum_{i>0} e^{\lambda_i t} |x_i\rangle\rangle \langle\langle y_i | \rho_0 \rangle\rangle, \quad (\text{B5})$$

where $|x_i\rangle\rangle$ and $\langle\langle y_i |$ are the right and left eigenvectors of the Liouvillian corresponding to eigenvalue λ_i (with $\text{Re}(\lambda_i) < 0$), fulfilling $\langle\langle y_i | x_j \rangle\rangle = \delta_{ij}$ (note that these are not complex conjugations of each other as the Liouvillian is non-hermitian, so $|x_i\rangle\rangle^\dagger \neq \langle\langle y_i |$), and $|\rho_0\rangle\rangle$ is the initial state.

The Drazin pseudo-inverse of the Liouvillian is defined as

$$\mathcal{L}^+ := \sum_{i>0} \frac{1}{\lambda_i} |x_i\rangle\rangle \langle\langle y_i |. \quad (\text{B6})$$

We will denote the transient “excess current” through channel z as $\tilde{J}_z(t)$ and rewrite in the Liouville space as

$$\begin{aligned} \tilde{J}_z(t) &= J_z(t) - J_z^{ss} = \langle\langle 1 | \mathcal{J}_z | \rho(t) \rangle\rangle - \langle\langle 1 | \mathcal{J}_z | \rho^{ss} \rangle\rangle \\ &= \langle\langle 1 | \mathcal{J}_z \sum_{i>0} e^{\lambda_i t} |x_i\rangle\rangle \langle\langle y_i | \rho_0 \rangle\rangle \end{aligned} \quad (\text{B7})$$

1. Detection efficiency

The *detection efficiency* η_D is defined as the ratio of counts recorded to the total number of counts that could have been detected [32]

$$\eta_D = \frac{\text{\#detection events}}{\text{\#incident events}} = \frac{N_D}{N_I}. \quad (\text{B8})$$

The number of counts detected N_D can be equivalently expressed as the number of total incident events N_I times $P_{\text{Detection}}$, the probability per incident event to have a detection event, i.e.

$$N_D = N_I P_{\text{Detection}} = N_I \int_0^\infty dt (J_D(t) - J_D^{ss}), \quad (\text{B9})$$

where the second equality comes from each detection event being i.i.d. in our model (based on the assumption that there are very few quantum particles around and the detector always resets perfectly after each detection event). As a consequence, the detection efficiency for a single incident event is given as

$$\eta_D = P_{\text{Detection}} = \int_0^\infty dt (J_D(t) - J_D^{ss}) = \int_0^\infty dt \tilde{J}_D(t), \quad (\text{B10})$$

where $J_D(t)$ is the detection current, J_D^{ss} is the steady state detection current (and thus the current flowing when there is no particle). To understand this one can think of the “net detection current” $J_D(t) - J_D^{ss}$ as a rate of detection whose integral gives exactly the average number of jumps in the detection channel that exceed the number of jumps that would be present in the steady state. Using Eqs. (B7) and (B6) the efficiency can be written as

$$\begin{aligned} \eta_D &= \int_0^\infty \tilde{J}_D(t) dt = \int_0^\infty dt \langle\langle 1 | \mathcal{J}_D \sum_{i>0} e^{\lambda_i t} |x_i\rangle\rangle \langle\langle y_i | \rho_0 \rangle\rangle \\ &= \langle\langle 1 | \mathcal{J}_D \sum_{i>0} \int_0^\infty dt e^{\lambda_i t} |x_i\rangle\rangle \langle\langle y_i | \rho_0 \rangle\rangle \\ &= -\langle\langle 1 | \mathcal{J}_D \sum_{i>0} \frac{1}{\lambda_i} |x_i\rangle\rangle \langle\langle y_i | \rho_0 \rangle\rangle = -\langle\langle 1 | \mathcal{J}_D \mathcal{L}^+ | \rho_0 \rangle\rangle, \end{aligned} \quad (\text{B11})$$

(B12)

where we used $\int_0^\infty dt t^n e^{\lambda_i t} = (-\lambda_i)^{1-n} n!$ for $n \in \mathbb{N} \setminus \{0\}$, and $\langle\langle y_i | x_j \rangle\rangle = \delta_{ij}$. Thus, the detection efficiency is given by

$$\eta_D = -\langle\langle 1 | \mathcal{J}_D \mathcal{L}^+ | \rho_0 \rangle\rangle \quad (\text{B13})$$

and requires only the diagonalization of the Liouvillian.

2. Dark count rate

As the dynamics do not feature any process by which the system can be excited stochastically, i.e. no external

excitations with energy E_S can occur, once the detector reaches its non-equilibrium steady state, we assume that the remaining current in the detection channel can not be the result of actual detection events but has to come from thermal fluctuations. The rate at which these “false-positive” detection counts occur is called *dark count rate* and is given as

$$R_{dc} = J_D^{ss} = \langle \mathbb{1} | \mathcal{J}_D | \rho^{ss} \rangle. \quad (\text{B14})$$

3. Detection jitter

Detection of quantum particles amounts to answering the question: Was there a quantum particle present at the *location of the detector* at a certain *point in time*? Detection thus corresponds to the localization of an event in space and time. Ideally, a detector clicks exactly at the moment in time when the particle hits it. However, in practice, the detection time fluctuates. These fluctuations can be quantified via the *detection jitter*, which is typically benchmarked for real-world detectors using the full width at half maximum (FWHM) of the detector’s instrument response function [32]. Here we will refer to the jitter as the variance of the excess current $\tilde{J}_D(t)$.

$$\Delta t_D = \text{var}(t)_{p_{j_D}} = \int_0^\infty dt t^2 \frac{\tilde{J}_D(t)}{\eta_D} - \left[\int_0^\infty dt t \frac{\tilde{J}_D(t)}{\eta_D} \right]^2 \quad (\text{B15})$$

$$= \langle \mathbb{1} | \frac{\mathcal{J}_D}{\eta_D} \sum_{i>0} \int_0^\infty dt (e^{\lambda_i t} t^2) | x_i \rangle \langle y_i | \rho_0 \rangle - \left[\langle \mathbb{1} | \frac{\mathcal{J}_D}{\eta_D} \sum_{i>0} \int_0^\infty dt (e^{\lambda_i t} t) | x_i \rangle \langle y_i | \rho_0 \rangle \right]^2 \quad (\text{B16})$$

$$= -2 \langle \mathbb{1} | \frac{\mathcal{J}_D}{\eta_D} (\mathcal{L}^+)^3 | \rho_0 \rangle - \left[\langle \mathbb{1} | \frac{\mathcal{J}_D}{\eta_D} (\mathcal{L}^+)^2 | \rho_0 \rangle \right]^2, \quad (\text{B17})$$

where we used the same integration as in Eq. (B12). The factor $\frac{1}{\eta_D}$ is needed since the variance requires a correctly normalized distribution to be calculated. Since $\tilde{J}_D(t)$ is not normalized, we normalize it: $\tilde{J}_D(t) / \int_0^\infty dt \tilde{J}_D(t) = \tilde{J}_D(t) / \eta_D$. This leaves us with the detection jitter

$$\Delta t_D = -2 \langle \mathbb{1} | \frac{\mathcal{J}_D}{\eta_D} (\mathcal{L}^+)^3 | \rho_0 \rangle - \left(\langle \mathbb{1} | \frac{\mathcal{J}_D}{\eta_D} (\mathcal{L}^+)^2 | \rho_0 \rangle \right)^2, \quad (\text{B18})$$

which again depends solely on properties of the Liouvillian.

4. Dead time

For any task that requires a high rate of measurements, it is important to quantify the minimal time at which

measurements can be reliably performed, i.e. the *dead time* of the detector D . Once the detection event has happened, the detector returns to its equilibrium state, ready for the next detection event. The time this takes is generally related to the first gap of the Liouvillian. One can write the time-evolved state in vectorized notation as

$$|\rho(t)\rangle\rangle = \sum_{i>0} e^{\lambda_i t} |x_i\rangle\rangle \langle\langle y_i | \rho_0 \rangle\rangle + |\rho^{ss}\rangle\rangle \quad (\text{B19})$$

since $\text{Re}(\lambda_{i+1}) \leq \text{Re}(\lambda_i)$ and $\text{Re}(\lambda_i) \leq 0 \quad \forall i$, we can ignore all eigenvalues (and corresponding modes) larger than the second one (the first is zero and corresponds to the steady state) as they all decay faster, and define the dead time via the slowest decaying mode as

$$D \sim -\frac{1}{\text{Re}(\lambda_1)}. \quad (\text{B20})$$

5. Total entropy production

In a similar fashion to the efficiency, we can calculate the transient excess entropy production Σ_{trans} . For weak interactions within the system compared to the interaction with the baths (to have well-defined energies transported in each jump), it is given by

$$\begin{aligned} \Sigma_{\text{trans}} &= \frac{1}{T_C} \int_0^\infty dt (E_C \tilde{J}_{M_C}(t) + E_G \tilde{J}_{G_{1-0}}(t) + E_S \tilde{J}_{G_{2-1}}(t)) \\ &= \frac{1}{T_C} \langle \mathbb{1} | (E_C \mathcal{J}_{M_C} + E_G \mathcal{J}_{G_{1-0}} + E_S \mathcal{J}_{G_{2-1}}) \mathcal{L}^+ | \rho_0 \rangle \rangle. \end{aligned} \quad (\text{B21})$$

Keeping the detector ready for detection requires it to be constantly driven out of equilibrium, which comes at its own cost, which we call the steady-state entropy production rate $\dot{\Sigma}_{ss}$ and is given by

$$\dot{\Sigma}_{ss} = \frac{1}{T_C} \langle \mathbb{1} | (E_C \mathcal{J}_{M_C} + E_G \mathcal{J}_{G_{1-0}} + E_S \mathcal{J}_{G_{2-1}}) | \rho^{ss} \rangle \rangle. \quad (\text{B22})$$

The total entropy production per detection event Σ_{tot} can be calculated by taking the entropy produced in the timescale over which one detection event takes place, given by $D \dot{\Sigma}_{ss} \sim -\dot{\Sigma}_{ss} / \text{Re}(\lambda_1)$, and adding the transient excess entropy production, i.e.

$$\Sigma_{\text{tot}} \sim -\frac{\dot{\Sigma}_{ss}}{\text{Re}(\lambda_1)} + \Sigma_{\text{trans}}. \quad (\text{B23})$$

APPENDIX C: EFFICIENCY BOUND HYPOTHESIS

In this section, we present an argument for the validity of the upper bound in Eq. (18). First note that the efficiency, as defined in Eq. (B8), can be written as

$$\eta_D = \frac{N_D}{N_D + N_{\text{Lost}}}, \quad (\text{C1})$$

where N_{Lost} counts all the particles that were not detected, such that $N_I = N_D + N_{\text{Lost}}$. The only way an excitation can leave the subspace spanned by $|2\rangle_G \otimes |0\rangle_S$ and $|1\rangle_G \otimes |1\rangle_S$ is via the detection channel $\mathcal{D}_D$ inducing the jump $|2\rangle_G \rightarrow |0\rangle_G$ or via the dissipative coupling to the cold bath $|2\rangle_G \rightarrow |1\rangle_G$. Thus, all counts in N_{Lost} must be accounted for in the current between the $|1\rangle - |2\rangle$ gap in the gain medium and the bath, given by

$$J_{G_{1-2}}(t) = \text{Tr}[(\Gamma_{G_{1-2}}^+ |1\rangle\langle 1|_G - \Gamma_{G_{1-2}}^- |2\rangle\langle 2|_G) \rho(t)], \quad (\text{C2})$$

minus the corresponding steady state current $J_{G_{1-2}}^{ss}$, i.e. $N_{\text{Lost}} = \int_0^\infty (J_{G_{1-2}}(t) - J_{G_{1-2}}^{ss}) dt$. The total efficiency can thus be written as

$$\eta_D = \frac{\int_0^\infty dt (J_D(t) - J_D^{ss})}{\int_0^\infty dt (J_D(t) - J_D^{ss}) + \int_0^\infty dt (J_{G_{1-2}}(t) - J_{G_{1-2}}^{ss})}. \quad (\text{C3})$$

We will write these two integrals in simplified notation as

$$N_D = \int_0^\infty dt (J_D(t) - J_D^{ss}) = I_2 \Gamma_D^- - I_0 \Gamma_D^+, \quad (\text{C4})$$

$$N_{\text{Lost}} = \int_0^\infty dt (J_{G_{1-2}}(t) - J_{G_{1-2}}^{ss}) = I_2 \Gamma_{G_{1-2}}^- - I_1 \Gamma_{G_{1-2}}^+, \quad (\text{C5})$$

where $I_2 \Gamma_{D/G_{1-2}}^- = \int_0^\infty dt (P_2(t) - P_2^{ss}) \Gamma_{D/G_{1-2}}^- dt$ and $I_{0/1} \Gamma_{D/G_{1-2}}^+ = \int_0^\infty dt (P_{0/1}(t) - P_{0/1}^{ss}) \Gamma_{D/G_{1-2}}^+ dt$, and $P_i(t)$ (P_i^{ss}) correspond to $\text{Tr}(\rho(t)|i\rangle\langle i|_G)$ ($\text{Tr}(\rho^{ss}|i\rangle\langle i|_G)$). Thus, the efficiency can be expressed as

$$\frac{1}{1 + \frac{I_2 \Gamma_{G_{1-2}}^- - I_1 \Gamma_{G_{1-2}}^+}{I_2 \Gamma_D^- - I_0 \Gamma_D^+}}. \quad (\text{C6})$$

In the zero temperature case, i.e. when $T_C \rightarrow 0$, using the coupling rates from Appendix A, we have

$$\eta_D \rightarrow \frac{1}{1 + \frac{I_2 \Gamma_{G_{1-2}}^-}{I_2 \Gamma_D^-}} = \frac{1}{1 + \frac{\gamma_G}{\gamma_D}} \quad (\text{C7})$$

which is equivalent to Eq. (18). In the following, we show that this is, in fact, an upper bound for the efficiency. To show this, we must show that

$$\frac{1}{1 + \frac{I_2 \Gamma_{G_{1-2}}^- - I_1 \Gamma_{G_{1-2}}^+}{I_2 \Gamma_D^- - I_0 \Gamma_D^+}} \leq \frac{1}{1 + \frac{\gamma_G}{\gamma_D}} \quad (\text{C8})$$

or equivalently that

$$I_1 \frac{\Gamma_{G_{1-2}}^+}{\gamma_G} \leq I_0 \frac{\Gamma_D^+}{\gamma_D} \quad (\text{C9})$$

substituting in the coupling rates, and bringing all of them to the right, and noting, that $\frac{\gamma_G \Gamma_D^+}{\Gamma_{G_{1-2}}^+ \gamma_D} = e^{-E_G/T_C} \leq 1$ we get the condition that

$$I_1 \leq I_0 e^{-E_G/T_C} \leq I_0. \quad (\text{C10})$$

To see why we believe that this inequality should hold (apart from numerical evidence), consider the initial state of the system:

$$\rho_0 = \rho_{\text{MG}}^{ss} \otimes |1\rangle\langle 1|_S, \quad (\text{C11})$$

where ρ_{MG}^{ss} is the steady state of the machine-gain medium subsystem, and the system qubit is prepared in its excited state. This choice of initial state creates a strong excitation imbalance relative to the global steady-state, with all population initially in level $|1\rangle_S$, and none in $|0\rangle_S$.

At early times, this imbalance drives population transfer from $|1\rangle_G$ to $|2\rangle_G$ due to the coherent coupling with the gain medium. As a result, the population $P_2(t)$ rises rapidly above its steady-state value, while $P_1(t)$ drops sharply below its steady-state value. Since $|2\rangle_G$ decays into $|0\rangle_G$ via the detection channel, the surplus in P_2 directly leads to a transient surplus in $P_0(t)$, i.e., $P_0(t) > P_0^{ss}$ for an extended period. At the same time, the transient depletion of $P_1(t)$ is counteracted by the thermal two-qubit engine, which is pumping the excess population in $P_0(t)$ that occurred due to decay via the detection channel $|2\rangle \rightarrow |0\rangle$ into $|1\rangle$. This feedback process counteracts the depletion in P_1 but occurs over longer timescales due to this conditional nature. As a result, while $P_1(t)$ initially drops below P_1^{ss} and eventually recovers, the time-integrated deviation $(P_1(t) - P_1^{ss})$ remains negative for a larger portion of the evolution compared to $(P_0(t) - P_0^{ss})$, which is predominantly positive. Therefore, we hypothesize that Eq. (C10) is satisfied.

The equations of motion for the populations of the gain medium are given by

$$\begin{aligned} \dot{P}_2(t) = & 2g_{\text{SG}} \text{Im}(\rho_{11,20}^{\text{SG}}) - \Gamma_D^- P_2 + \Gamma_D^+ P_0 - \Gamma_{G_{1-2}}^- P_2 \\ & + \Gamma_{G_{1-2}}^+ P_1, \end{aligned} \quad (\text{C12})$$

$$\begin{aligned} \dot{P}_1(t) = & 2(g_{\text{MG}} \text{Im}(\rho_{010,101}^{\text{MG}}) - g_{\text{SG}} \text{Im}(\rho_{11,20}^{\text{SG}})) + \Gamma_{G_{1-2}}^- P_2 \\ & - \Gamma_{G_{1-2}}^+ P_1 - \Gamma_{G_{0-1}}^- P_1 + \Gamma_{G_{0-1}}^+ P_0, \end{aligned} \quad (\text{C13})$$

$$\begin{aligned} \dot{P}_0(t) = & -2g_{\text{MG}} \text{Im}(\rho_{010,101}^{\text{MG}}) + \Gamma_D^- P_2 - \Gamma_D^+ P_0 \\ & + \Gamma_{G_{0-1}}^- P_1 - \Gamma_{G_{0-1}}^+ P_0, \end{aligned} \quad (\text{C14})$$

$$P_0(t) + P_1(t) + P_2(t) = 1 \quad (\text{C15})$$

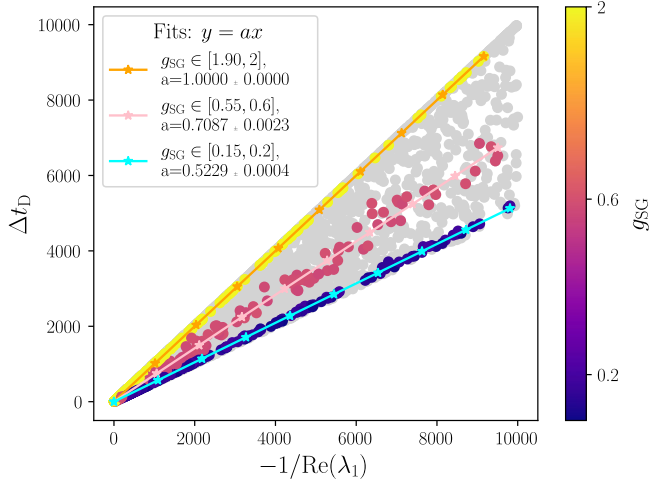


FIG. 7. Jitter over the inverse first gap of the Liouvillian $1/\lambda_1$. The gray points display the full dataset where we restrict to cases where $\eta_D > 0.4$. The colored points indicate the data points selected for the fit by restricting g_{SG} . The lines with stars indicate the linear fits.

APPENDIX D: DETECTION JITTER-DEAD TIME RELATION

This appendix aims to highlight the relationship between dead time and detection jitter. Intuitively, it is

clear that detection jitter must always be smaller than the timescale at which equilibration happens (which corresponds to the dead time, see Eq. (14) and the preceding paragraph), since the peak of the detection current must be within this time. Indeed, we find that jitter and inverse first gap $1/\lambda_1$ follow a linear scaling relation in this model, which is shown in the scatter plot in Fig. 7. The slope of this linear relationship increases with the system-gain medium interaction strength g_{sl} , approaching a maximal value of 1. Using the fact that the dead time is bounded by the equilibration time, which is of the order of $1/\lambda_1$, our numerical evidence points us to the relation

$$\Delta t_D \propto -\frac{1}{\text{Re}(\lambda_1)} = D. \quad (\text{D1})$$

APPENDIX E: ENTROPIC COST OF INCREASING THE GAIN

In this section, we discuss the scaling of entropy costs of increased energy gains. In Fig. 8 we show numerical evidence for a linear scaling of the total entropy production $\Sigma_{\text{tot}} = \Sigma_{\text{trans}} + \dot{\Sigma}_{ss}/\lambda_1$ with the gain $G = \frac{E_G}{E_S}$, i.e. the amount of energy amplification in this model, while leaving the efficiency unchanged for large enough gain values.

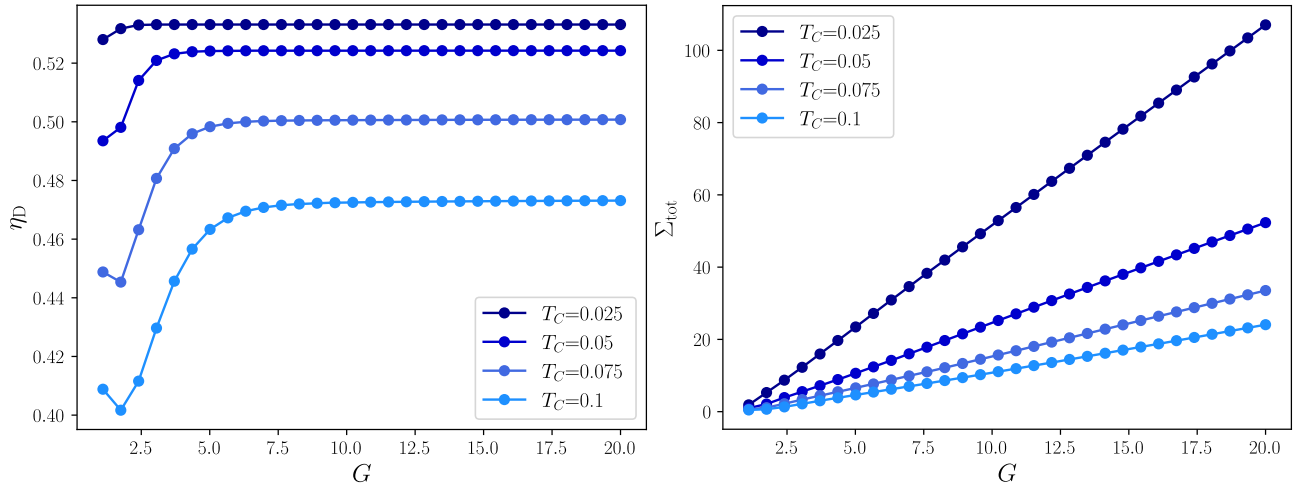


FIG. 8. Left panel: Detection efficiency η_D over gain G for different values of T_C . The initial lower efficiency can be explained by increased thermal fluctuations for a smaller gap size. As the first gap increases, these fluctuations are suppressed. For low enough T_C , efficiency and gain are independent of each other, i.e., increasing the gain does not affect the efficiency. Right panel: Total entropy production Σ_{tot} over energy gain G for varying cold bath temperatures T_C . Increasing the gain comes with a linear increase in total entropy production. Here we fixed E_S and increased E_G . Both plots were generated for one detector realization with parameters $g_{SG} = 1$, $g_{MG} = 1$, $\gamma_D = 0.8$, $\gamma_G = 0.7$, $\gamma_M = 1$, $T_V = -3$, $f_{E_C} = 0.2$ ($E_C = f_{E_C} E_L$).

- [1] R. Landauer, *Irreversibility and heat generation in the computing process*, *IBM J. Res. Dev.* **5**, 183 (1961).
- [2] K. Proesmans, J. Ehrich, and J. Bechhoefer, *Finite-time Landauer principle*, *Phys. Rev. Lett.* **125**, 100602 (2020).
- [3] Harry J. D. Miller, Giacomo Guarnieri, Mark T. Mitchison, and John Goold, *Quantum fluctuations hinder finite-time information erasure near the Landauer limit*, *Phys. Rev. Lett.* **125**, 160602 (2020).
- [4] Yi-Zheng Zhen, Dario Egloff, Kavan Modi, and Oscar Dahlsten, *Universal bound on energy cost of bit reset in finite time*, *Phys. Rev. Lett.* **127**, 190602 (2021).
- [5] Tan Van Vu and Keiji Saito, *Finite-time quantum Landauer principle and quantum coherence*, *Phys. Rev. Lett.* **128**, 010602 (2022).
- [6] Philip Taranto, Faraj Bakhshinezhad, Andreas Bluhm, Ralph Silva, Nicolai Friis, Maximilian P. E. Lock, Giuseppe Vitagliano, Felix C. Binder, Tiago Debarba, Emanuel Schwarzhans, Fabien Clivaz, and Marcus Huber, *Landauer versus Nernst: What is the true cost of cooling a quantum system?*, *PRX Quantum* **4**, 010332 (2023).
- [7] Alberto Rolandi and Martí Perarnau-Llobet, *Finite-time Landauer principle beyond weak coupling*, *Quantum* **7**, 1161 (2023).
- [8] Yelena Guryanova, Nicolai Friis, and Marcus Huber, *Ideal projective measurements have infinite resource costs*, *Quantum* **4**, 222 (2020).
- [9] Philippe Faist, Frédéric Dupuis, Jonathan Oppenheim, and Renato Renner, *The minimal work cost of information processing*, *Nat. Commun.* **6**, 7669 (2015).
- [10] Yaoming Chu and Jianming Cai, *Thermodynamic principle for quantum metrology*, *Phys. Rev. Lett.* **128**, 200501 (2022).
- [11] Takahiro Sagawa and Masahito Ueda, *Minimal energy cost for thermodynamic information processing: Measurement and information erasure*, *Phys. Rev. Lett.* **102**, 250602 (2009).
- [12] Léo Granger and Holger Kantz, *Thermodynamic cost of measurements*, *Phys. Rev. E* **84**, 061110 (2011).
- [13] Kurt Jacobs, *Quantum measurement and the first law of thermodynamics: The energy cost of measurement is the work value of the acquired information*, *Phys. Rev. E* **86**, 040106 (2012).
- [14] Camille L. Latune and Cyril Elouard, *A thermodynamically consistent approach to the energy costs of quantum measurements*, *Quantum* **9**, 1614 (2025).
- [15] Armen E. Allahverdyan, Roger Balian, and Theo M. Nieuwenhuizen, *Quantum measurement as a driven phase transition: An exactly solvable model*, *Phys. Rev. A* **64**, 032108 (2001) publisher: American Physical Society.
- [16] A. E. Allahverdyan, R. Balian, and Th. M. Nieuwenhuizen, *Curie–Weiss model of the quantum measurement process*, *Europhys. Lett.* **61**, 452 (2003).
- [17] Emanuel Schwarzhans, Felix C. Binder, Marcus Huber, and Maximilian P. E. Lock, *Quantum measurements and equilibration: The emergence of objective outcomes via entropy maximization*, *Phys. Rev. Res.* **7**, 043279 (2025).
- [18] Sophie Engineer, Tom Rivlin, Sabine Wollmann, Mehul Malik, and Maximilian P. E. Lock, *Equilibration of objective observables in a dynamical model of quantum measurements*, *Phys. Rev. A* **113**, 032205 (2026).
- [19] Wojciech H. Zurek, *Quantum Darwinism, classical reality, and the randomness of quantum jumps*, *Phys. Today* **67**, 44 (2014).
- [20] R. Horodecki, J. K. Korbicz, and P. Horodecki, *Quantum origins of objectivity*, *Phys. Rev. A* **91**, 032122 (2015).
- [21] Florian Vigneau, Federico Fedele, Anasua Chatterjee, David Reilly, Ferdinand Kuemmeth, M. Fernando Gonzalez-Zalba, Edward Laird, and Natalia Ares, *Probing quantum devices with radio-frequency reflectometry*, *Appl. Phys. Rev.* **10**, 021305 (2023).
- [22] Alexandre Blais, Arne L. Grimsmo, S. M. Girvin, and Andreas Wallraff, *Circuit quantum electrodynamics*, *Rev. Mod. Phys.* **93**, 025005 (2021).
- [23] Yang Shi, Xiang Li, Guanyu Chen, Mingjie Zou, Hongjun Cai, Yu Yu, and Xinliang Zhang, *Avalanche photodiode with ultrahigh gain–bandwidth product of 1,033 GHz*, *Nat. Photon.* **18**, 610 (2024).
- [24] A. A. Clerk, S. M. Girvin, and A. D. Stone, *Quantum-limited measurement and information in mesoscopic detectors*, *Phys. Rev. B* **67**, 165324 (2003).
- [25] A. A. Clerk, M. H. Devoret, S. M. Girvin, Florian Marquardt, and R. J. Schoelkopf, *Introduction to quantum noise, measurement, and amplification*, *Rev. Mod. Phys.* **82**, 1155 (2010).
- [26] Paul Erker, Mark T. Mitchison, Ralph Silva, Mischa P. Woods, Nicolas Brunner, and Marcus Huber, *Autonomous quantum clocks: Does thermodynamics limit our ability to measure time?*, *Phys. Rev. X* **7**, 031022 (2017).
- [27] Emanuel Schwarzhans, Maximilian P. E. Lock, Paul Erker, Nicolai Friis, and Marcus Huber, *Autonomous temporal probability concentration: Clockworks and the second law of thermodynamics*, *Phys. Rev. X* **11**, 011046 (2021).
- [28] Noah Linden, Sandu Popescu, and Paul Skrzypczyk, *How small can thermal machines be? The smallest possible refrigerator*, *Phys. Rev. Lett.* **105**, 130401 (2010).
- [29] Amikam Levy and Ronnie Kosloff, *Quantum absorption refrigerator*, *Phys. Rev. Lett.* **108**, 070604 (2012).
- [30] Howard M. Wiseman and Gerard J. Milburn, *Quantum Measurement and Control*, 1st ed. (Cambridge University Press, Cambridge, UK, 2009), 10.1017/CBO9780511813948.
- [31] John von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Springer Berlin Heidelberg, Berlin, Heidelberg, 1932).
- [32] Robert H. Hadfield, *Single-photon detectors for optical quantum information applications*, *Nat. Photonics* **3**, 696 (2009).
- [33] Mark T. Mitchison, *Quantum thermal absorption machines: Refrigerators, engines and clocks*, *Contemp. Phys.* **60**, 164 (2019).
- [34] Nicolas Brunner, Noah Linden, Sandu Popescu, and Paul Skrzypczyk, *Virtual qubits, virtual temperatures, and the foundations of thermodynamics*, *Phys. Rev. E* **85**, 1 (2012).
- [35] Masanao Ozawa, *Conservation laws, uncertainty relations, and quantum limits of measurements*, *Phys. Rev. Lett.* **88**, 050402 (2002).
- [36] Patrick P. Potts, Alex Arash Sand Kalaei, and Andreas Wacker, *A thermodynamically consistent Markovian master equation beyond the secular approximation*, *New J. Phys.* **23**, 123013 (2021).

- [37] Luis A. Correa, José P. Palao, and Daniel Alonso, *Internal dissipation and heat leaks in quantum thermodynamic cycles*, *Phys. Rev. E* **92**, 032136 (2015).
- [38] Gabriel T. Landi, Michael J. Kewming, Mark T. Mitchison, and Patrick P. Potts, *Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics*, *PRX Quantum* **5**, 020201 (2024).
- [39] Andre C. Barato and Udo Seifert, *Thermodynamic uncertainty relation for biomolecular processes*, *Phys. Rev. Lett.* **114**, 158101 (2015).
- [40] Todd R. Gingrich, Jordan M. Horowitz, Nikolay Perunov, and Jeremy L. England, *Dissipation bounds all steady-state current fluctuations*, *Phys. Rev. Lett.* **116**, 120601 (2016).
- [41] Yoshihiko Hasegawa, *Quantum thermodynamic uncertainty relation for continuous measurement*, *Phys. Rev. Lett.* **125**, 050601 (2020).
- [42] Jordan M. Horowitz and Todd R. Gingrich, *Thermodynamic uncertainty relations constrain non-equilibrium fluctuations*, *Nat. Phys.* **16**, 15 (2020).
- [43] Emanuel Schwarzhans, *Quantum detectors as autonomous machines—assessing the thermodynamics of information acquisition—code*, <https://github.com/Emanuel-Schwarzhans/Quantum-detectors-as-autonomous-machines-assessing-the-thermodynamics-of-information-acquisit-co>.
- [44] Lorenzo Magrini, Philipp Rosenzweig, Constanze Bach, Andreas Deutschmann-Olek, Sebastian G. Hofer, Sungkun Hong, Nikolai Kiesel, Andreas Kugi, and Markus Aspelmeyer, *Real-time optimal quantum control of mechanical motion at room temperature*, *Nature (London)* **595**, 373 (2021).
- [45] Udo Seifert, *Stochastic thermodynamics, fluctuation theorems and molecular machines*, *Rep. Progress Phys.* **75**, 126001 (2012).
- [46] Tiago Debarba, Gonzalo Manzano, Yelena Guryanova, Marcus Huber, and Nicolai Friis, *Work estimation and work fluctuations in the presence of non-ideal measurements*, *New J. Phys.* **21**, 113002 (2019).
- [47] Cyril Elouard, David Herrera-Martí, Benjamin Huard, and Alexia Auffèves, *Extracting work from quantum measurement in Maxwell demon engines*, *Phys. Rev. Lett.* **118**, 260603 (2017).
- [48] Cyril Elouard, David A. Herrera-Martí, Maxime Clusel, and Alexia Auffèves, *The role of quantum measurement in stochastic thermodynamics*, *npj Quantum Inf.* **3**, 9 (2017).
- [49] Lorenzo Buffoni, Andrea Solfanelli, Paola Verrucchi, Alessandro Cuccoli, and Michele Campisi, *Quantum measurement cooling*, *Phys. Rev. Lett.* **122**, 070603 (2019).
- [50] Florian Meier, Yuri Minoguchi, Simon Sundelin, Tony J. G. Apollaro, Paul Erker, Simone Gasparinetti, and Marcus Huber, *Precision is not limited by the second law of thermodynamics*, *Nat. Phys.* **21**, 1147 (2025).
- [51] G. C. Ghirardi, A. Rimini, and T. Weber, *Unified dynamics for microscopic and macroscopic systems*, *Phys. Rev. D* **34**, 470 (1986).
- [52] Jonathan Oppenheim, *A postquantum theory of classical gravity?*, *Phys. Rev. X* **13**, 041040 (2023).
- [53] Adán Cabello, Mile Gu, Otfried Gühne, Jan-Åke Larsson, and Karoline Wiesner, *Thermodynamical cost of some interpretations of quantum theory*, *Phys. Rev. A* **94**, 052127 (2016).
- [54] Peter O'Donovan, Philipp Strasberg, Kavan Modi, John Goold, and Mark T. Mitchison, *Quantum master equation from the eigenstate thermalization hypothesis*, *Phys. Rev. B* **112**, 014312 (2025).