

Real-Time Dynamics in Two Dimensions with Tensor Network States via Time-Dependent Variational Monte Carlo Method

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Reliably simulating two-dimensional many-body quantum dynamics with projected entangled pair states (PEPS) has long been a difficult challenge. In this work, we overcome this barrier for low-energy quantum dynamics by developing a stable and efficient time-dependent variational Monte Carlo (tVMC) framework for PEPS. By analytically removing all gauge redundancies of the PEPS manifold and exploiting tensor locality, we obtain a numerically well-conditioned tVMC equation, yielding robust solutions using the efficient Cholesky decomposition. This enables long-time evolution in previously inaccessible regimes. We explain how the difficulties in the traditional approach, particularly those associated with gauge redundancies, are resolved within tVMC. We demonstrate the power and generality of the method through five representative real-time local quench dynamics in two dimensions: (I) chiral edge propagation in a free-fermion Chern insulator; (II) vison propagation in a pure $\mathbb{Z}_2$ gauge theory; (III) vison confinement dynamics in a $\mathbb{Z}_2$ lattice gauge theory coupled to Higgs field; (IV) fractionalized charge transport in a fractional Chern insulator; and (V) superfluidity and critical velocity in interacting bosons. All simulations are performed on $\geq 10 \times 10$ lattices with evolution times beyond $T = 10$ using modest computational resources. Where exact benchmarks exist (case I), PEPS-tVMC matches free-fermion dynamics with high accuracy up to $T = 12$. In addition, we also simulate the paradigmatic dynamics of the Ising model following a global quench at the critical transverse field, and obtain, with modest bond dimension, agreement with previous results. These results demonstrate PEPS-tVMC as a practical and versatile tool for real-time quantum dynamics in two dimensions. The method significantly extends the reach of classical tensor-network simulations for studying elementary excitations in quantum many-body systems in real time and provides a valuable computational counterpart to emerging quantum simulators. As a by-product in the development, we also present a new form of minSR, which is more stable and offers a new perspective on tVMC.

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I. INTRODUCTION

Understanding real-time dynamics of interacting quantum many-body systems in two dimensions (2D) is a fundamental challenge that spans condensed-matter physics, particle physics, and quantum simulation. While powerful classical methods exist for ground states—including tensor networks [1,2] and quantum Monte Carlo method [3,4], real-time evolution in 2D remains notoriously difficult. Exact diagonalization is limited to small clusters; quantum Monte Carlo method suffers from a dynamical sign problem. Recently, the

combination of time-dependent variational Monte Carlo (tVMC) method [5–8], a continuous- and real-time generalization of the stochastic reconfiguration (SR) method [9–11], with neural quantum states (NQS) [12] has emerged as a promising alternative [13–16]. Here, we refer to tVMC as a Monte Carlo formulation of the time-dependent variational principle (TDVP) [5,6]. NQS provides remarkable variational flexibility [17–19], while tVMC offers a principled formulation of real-time evolution. The combination has also motivated alternative approaches to dynamics within the VMC framework [14,16,20,21]. At present, extending NQS-tVMC to large 2D systems remains an active area of research, where one must balance expressive parametrizations with efficient and stable optimization [22].

Projected entangled pair states (PEPS) [23] provide a complementary variational family whose locality and multi-linearity make them particularly appealing for tVMC. Within VMC, only a single layer of PEPS tensors

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needs to be contracted, making the method substantially more efficient than conventional PEPS update schemes [24–26]. Furthermore, VMC allows parallel accumulation of samples, making the approach well suited for modern GPU architectures. In fact, PEPS-VMC has recently achieved considerable success for ground states, giving competitive results for the ground states of frustrated spins [27–29], the Fermi-Hubbard model [30], and lattice gauge theory [31], owing to a foundational breakthrough in the sampling method of PEPS [32].

Although PEPS obey an area law for entanglement entropy [33] and cannot represent volume-law states produced by fully thermalizing dynamics, they are well suited for *local quenches* in which the injected energy does not scale with system size. In this latter regime, the energy density remains zero (in the thermodynamic limit) and we expect the dynamics to preserve area-law entanglement [34,35]. Recent theoretical studies also suggest area-law entanglement during local quenches [36,37]. Thus, PEPS naturally targets low-energy local-quench dynamics—precisely the regime relevant for elementary excitations and real-time response functions, and directly related to experiments including scanning tunneling microscopy [38], angle-resolved photoemission spectroscopy [39], particle scattering processes, and more. A controlled method capable of simulating such dynamics provides a valuable tool for studying 2D quantum systems.

While dynamical simulations with matrix product states (MPS) are enormously successful in 1D [40–43], the situation with PEPS in 2D is much more challenging. In contrast to the extensive 1D literature, only a small number of works—represented by Refs. [44–55]—have reported real-time simulations using PEPS or infinite PEPS (iPEPS), illustrating the general difficulty of the problem. Traditional approaches to dynamical PEPS have relied on full-update-like methods based on double-layer contraction [24–26], which face several challenges: (a) high cost of double-layer contraction, (b) difficulty in fixing the PEPS gauge freedom, and (c) loss of positivity in the quantum metric approximately obtained from the contraction of the double-layer environment. In Sec. III F, we explain that, within PEPS-tVMC, the last two issues are naturally resolved while the first one is significantly alleviated, at the expense of manageable statistical noise. We also note that PEPS time-evolution, based on simple updates (SU) [56], has recently experienced an exciting renaissance due to a new interpretation and improvement via belief propagation [57–59], giving impressive results on complex dynamics problems in tandem with recent experiments [60–64]. We expect such progress to continue, co-growing with the development of quantum computers.

Despite the long development of PEPS [1], their combination with tVMC for real-time dynamics has received virtually no attention, aside from our recent exploratory attempt in a simple setting [31]. Here, we

develop PEPS-tVMC systematically, analyze its structure, and demonstrate its ability to simulate large-scale 2D dynamics. Our approach builds on time-dependent variational Monte Carlo method [5,6]. We identify two ways to remove gauge redundancies in PEPS dynamics: one in the standard TDVP form, and another one in its dual minSR form [65]. After gauge removal, the TDVP equation exhibits remarkable numerical stability: the TDVP residual remains as low as 10^{-9} to 10^{-25} in the examples studied. As a result, PEPS—long viewed as difficult for real-time evolution—can in fact provide a powerful and controllable variational framework for 2D low-energy dynamics. This resolves long-standing challenges that have hindered dynamical simulations with PEPS since their introduction [23]. We illustrate the versatility of PEPS-tVMC through six physically motivated examples, which illustrate that the method is not specialized to a single model class.

The paper is organized as follows. In Sec. II, we review the tVMC framework and set up the notation. In Sec. III, we analyze the structure of PEPS-tVMC and present the ingredients necessary for its stability and efficiency. In Sec. IV, we demonstrate the method through six representative examples. In Secs. V and VI, we summarize our findings and outline future directions.

II. TIME-DEPENDENT VARIATIONAL MONTE CARLO METHOD

Consider a many-body lattice quantum system with computational basis $\mathbf{s} = (s_1, \dots, s_N)$ with local Hilbert space dimension d . To perform a variational simulation, we adopt a holomorphic parametrization of the *unnormalized* quantum state $\Psi_{\boldsymbol{\theta}}(\mathbf{s})$ with N_p complex variational parameters $\boldsymbol{\theta} = \{\theta_{\alpha} : \alpha = 1, 2, \dots, N_p\}$. At time t , the exact evolution under a Hamiltonian H for a short interval dt induces an evolution from $\boldsymbol{\theta}(t)$ to $\boldsymbol{\theta}(t + dt)$ that minimizes the Fubini-Study distance [66] between $\Psi_{\boldsymbol{\theta}(t+dt)}$ and $e^{-iHdt}\Psi_{\boldsymbol{\theta}(t)}$. For infinitesimal dt , this yields the TDVP equations [5,40]:

$$\sum_{\beta=1}^{N_p} S_{\alpha\beta} \dot{\theta}_{\beta} = -i g_{\alpha}, \quad (1)$$

where the Gram matrix or quantum geometric tensor (QGT) $S_{\alpha\beta}$ and the energy gradient g_{α} are given by (up to normalization factors)

$$S_{\alpha\beta} = \langle \partial_{\alpha} \Psi | \partial_{\beta} \Psi \rangle - \langle \partial_{\alpha} \Psi | \Psi \rangle \langle \Psi | \partial_{\beta} \Psi \rangle, \quad (2)$$

$$g_{\alpha} = \langle \partial_{\alpha} \Psi | H | \Psi \rangle - \langle \partial_{\alpha} \Psi | \Psi \rangle \langle \Psi | H | \Psi \rangle. \quad (3)$$

These quantities can be written in terms of expectation values over the Born distribution $P_{\boldsymbol{\theta}}(\mathbf{s}) = |\Psi_{\boldsymbol{\theta}}(\mathbf{s})|^2 / \sum_{\mathbf{s}} |\Psi_{\boldsymbol{\theta}}(\mathbf{s})|^2$

$$S_{\alpha\beta} = [O_\alpha^*(\mathbf{s})O_\beta(\mathbf{s})] - [O_\alpha^*(\mathbf{s})][O_\beta(\mathbf{s})], \quad (4)$$

$$g_\alpha = [O_\alpha^*(\mathbf{s})H_{\text{loc}}(\mathbf{s})] - [O_\alpha^*(\mathbf{s})][H_{\text{loc}}(\mathbf{s})]. \quad (5)$$

Here $[\cdot]$ denotes a statistical average over $P_\theta(\mathbf{s})$, $H_{\text{loc}}(\mathbf{s}) = \langle \mathbf{s} | H | \Psi \rangle / \langle \mathbf{s} | \Psi \rangle$ is the local energy, and

$$O_\alpha(\mathbf{s}) \equiv \frac{\partial \log \Psi_\theta(\mathbf{s})}{\partial \theta_\alpha} \quad (6)$$

is the log-derivative.

In practice, S and $\mathbf{g}$ are evaluated stochastically by sampling configurations $\{\mathbf{s}\}$ from P_θ . Given N_s samples, we form an $N_s \times N_p$ matrix O with elements $O_{s\alpha} = O_\alpha(\mathbf{s})$ and an N_s column vector $\mathbf{h}$ with entries $h_s = H_{\text{loc}}(\mathbf{s})$. Subtracting the sample average O_{avg} and h_{avg} ,

$$\mathcal{O} = O - O_{\text{avg}}, \quad \mathcal{h} = \mathbf{h} - h_{\text{avg}}, \quad (7)$$

the QGT and the energy gradient can be approximated as

$$S \approx \frac{1}{N_s} \mathcal{O}^\dagger \mathcal{O}, \quad \mathbf{g} \approx \frac{1}{N_s} \mathcal{O}^\dagger \mathcal{h}, \quad (8)$$

where the approximation reflects finite-sample statistical fluctuations. The centering in Eq. (7) removes the component parallel to the wave function, ensuring projective invariance of the quantum metric.

The tVMC algorithm therefore consists of: (1) sampling configurations $\mathbf{s} \sim P_\theta$; (2) constructing S and $\mathbf{g}$; (3) solving Eq. (1) for $\dot{\theta}$; and (4) integrating the parameter flow on the variational manifold via a Runge-Kutta (RK) integrator [67].

A major challenge in tVMC is that the QGT S is often severely ill-conditioned, making the linear problem in Eq. (1) difficult to solve stably. For ground-state searches, where the precise time trajectory is irrelevant, one may add Tikhonov regularization $S \rightarrow S + \epsilon I$ with typically $\epsilon \sim 10^{-3}$. However, in real-time dynamics a large ϵ introduces an unphysical bias, whereas too small a value leaves the system unstable. In NQS-tVMC, several sophisticated regularization schemes have been proposed [13,15], but none fully resolve the problem [22].

In PEPS, the main source of instability can be identified by the gauge redundancy, which introduces many null vectors into O and hence into S . The crucial feature, however, is that these null directions are *analytically identifiable*. As we will show in Sec. III, they can be removed *before* solving the TDVP equation, providing an exact preconditioning. After this gauge-removal step, the TDVP equation becomes remarkably well conditioned, and in all examples of local quench dynamics studied we find it can be solved with exceptional numerical accuracy. This stability is a key factor in the success of PEPS-tVMC.

III. STOCHASTIC RECONFIGURATION WITH PEPS

In this section, we derive the TDVP equations for PEPS, analyze the structure of their null space, and describe how gauge redundancy can be eliminated efficiently in closed form, in both the TDVP and its reduced variant based on the push-through identity (referred to as minSR) [65].

A PEPS defines a variational wave function $\Psi_\theta(\mathbf{s})$ parametrized by the tensor elements $A[\mathbf{x}]_{lrdu}^p$ at each lattice site $\mathbf{x}$, as illustrated in Figs. 1(a) and 1(b). Each index tuple $(\mathbf{x}, p, l, r, d, u)$ corresponds to one variational parameter, replacing the abstract index α introduced earlier. The physical index p labels the local Hilbert-space configuration (e.g., $p = 0, 1$ for vacuum or occupation), while the virtual indices (l, r, d, u) take values from 1 to D (except on boundaries). The PEPS in this paper all have open boundary conditions.

a. Gauge redundancy. The PEPS representation contains a large local gauge freedom associated with internal virtual bonds. As shown in Fig. 1(c), the transformation

$$A_{1lrdu}^p \rightarrow \sum_{r'=1}^D A_{1lr'du}^p X_{r'r}, \quad (9)$$

$$A_{2lrdu}^p \rightarrow \sum_{l'=1}^D (X^{-1})_{ll'} A_{2l'rdu}^p, \quad (10)$$

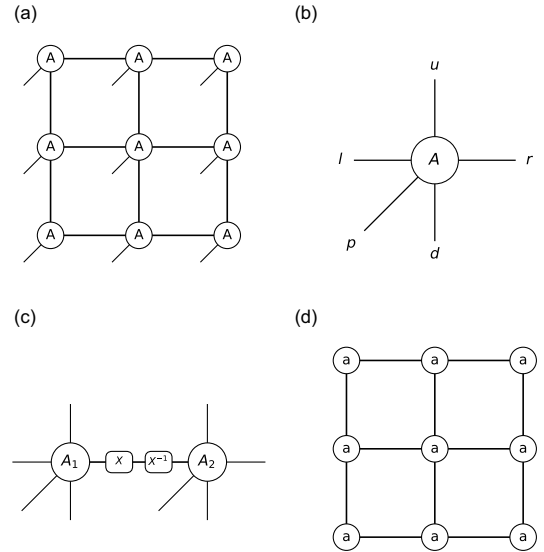


FIG. 1. PEPS structure and gauge freedom. (a) A 3×3 PEPS network with local tensors A . (b) A single tensor A with physical index p and virtual indices (l, r, d, u) . (c) Local gauge transformation on a virtual bond: inserting an invertible matrix X and its inverse X^{-1} between two adjacent tensors leaves the global PEPS wave function invariant. (d) Wave function amplitude $\langle \mathbf{s} | \Psi \rangle$, computed via the contraction of the single layer network of sliced PEPS tensors: $a[\mathbf{x}]_{lrdu}^{p=\mathbf{s}[\mathbf{x}]} = A[\mathbf{x}]_{lrdu}^p$.

leaves the global wave function invariant for any invertible $D \times D$ matrix X . The infinitesimal generators of such transformations define *gauge vectors*. Each gauge vector $\mathbf{v}$ is a null vector of O :

$$\sum_{\alpha} O_{s\alpha} v_{\alpha} = 0 \quad \forall s.$$

Hence every gauge vector also lies in the null space of $S = O^{\dagger}O$, and must be removed before solving the TDVP equation.

For an $L \times L$ PEPS there are $2L(L-1)$ internal bonds, yielding $2L(L-1)D^2$ gauge vectors. Let E^{ij} be the standard basis of $\mathbb{C}^{D \times D}$ with a single nonzero element at (i, j) . For the bond in Fig. 1(c), the corresponding gauge vector $\mathbf{v}$ has components

$$\mathbf{v}[\mathbf{x}_1]_{lrdu}^p = \sum_{r'} A_{lr'du}^p E_{r'r}^{ij}, \quad (11)$$

$$\mathbf{v}[\mathbf{x}_2]_{lrdu}^p = -\sum_{r'} E_{lr'r}^{ij} A_{2lr'du}^p, \quad (12)$$

and vanishes for all other parameters.

b. Linear dependence of gauge vectors. The $2L(L-1)D^2$ gauge vectors are not linearly independent. Consider a closed loop of virtual bonds and replace E^{ij} with the identity matrix on each bond. The sum of the corresponding gauge vectors vanishes, indicating linear dependence. These relations are generated by the $(L-1)^2$ elementary plaquettes of the square lattice. We denote the associated constraint vectors by $\mathbf{w}$, which are known exactly in the basis of the gauge vectors and independent of tensor content.

c. Additional null vectors. In addition to gauge redundancy, two further null transformations of O exist: (i) a global rescaling $A \rightarrow aA$, reflecting the projective nature of quantum mechanics; and (ii) for fixed particle number M , a relative rescaling $A^{p=0} \rightarrow aA^{p=0}$ and $A^{p=1} \rightarrow bA^{p=1}$ with $a^{L^2-M}b^M = 1$. We denote their generators as $\mathbf{u}_1$ and $\mathbf{u}_2$. Note that $\mathbf{u}_1$ is not a null direction of O , but only of $\mathcal{O}$.

A. Gauge fixing within TDVP

We now outline the procedure for eliminating null vectors within the TDVP formulation. First, we construct a projection matrix onto the subspace orthogonal to the constraint vectors $\mathbf{w}$, thereby reducing the number of independent gauge vectors to

$$N_{\text{gv}} = 2L(L-1)D^2 - (L-1)^2.$$

We then form an $N_p \times (N_{\text{gv}} + 2)$ matrix T whose columns consist of these independent gauge vectors together with $\mathbf{u}_1$ and $\mathbf{u}_2$. Performing a *complete* QR decomposition $T = \tilde{Q}R$, we define Q as the last $N_p - N_{\text{gv}} - 2$ columns

of $\tilde{Q}$. Since T has full column rank, the remaining columns of $\tilde{Q}$ span the orthogonal complement to the column space of T . By construction, Q projects onto the subspace orthogonal to all null directions.

The TDVP equation is then solved for $\mathbf{y}$ in the reduced space:

$$Q^{\dagger}O^{\dagger}OQ\mathbf{y} = -iQ^{\dagger}O^{\dagger}\mathcal{H}, \quad (13)$$

and the parameter update is obtained as $\dot{\boldsymbol{\theta}} = Q\mathbf{y}$. Only a single QR decomposition is required, which is numerically stable and GPU efficient.

B. Gauge fixing within minSR

When the number of samples is less than the column rank of O , i.e. $N_s < N_p - N_{\text{gv}} - 2$, the TDVP equation can be equivalently solved in the dual minSR form obtained using the push-through identity [65]:

$$O\mathcal{O}^{\dagger}\mathbf{y} = \mathcal{H}, \quad \dot{\boldsymbol{\theta}} = \mathcal{O}^{\dagger}\mathbf{y}. \quad (14)$$

We point out that minSR automatically achieves gauge removal, as the number of samples now determines the rank of the QGT matrix. That is, tVMC provides, via stochasticity, a surprising manner to eliminate gauge redundancies, which is entirely absent in the traditional approach to tensor networks.

Assuming decorrelation between samples, the matrix $\mathcal{O}^{\dagger}$ has a single null vector $[1, 1, \dots, 1]^T$, which we remove with the same QR projection described above. More generally, the null vector of $\mathcal{O}^{\dagger}$ is the weight vector W used in performing the statistical average: $\langle A \rangle \approx \sum_{s=1}^{N_s} W_s A_{\text{loc}}(s) / N_s$, which in our case is simply the identity weight. A non-trivial weight vector would arise if reweighting is used in the Monte Carlo estimate. The removal of this null vector may also be beneficial to NQS-tVMC in the minSR form.

C. minSR using O instead of $\mathcal{O}$

We present a new form of minSR which does not even have the zero mode $[1, 1, \dots, 1]^T$.

Lemma 1. If $\mathbf{x}$ satisfies

$$O\mathbf{x} = \mathbf{h}, \quad (15)$$

then it satisfies the tVMC equation $\mathcal{O}^{\dagger}O\mathbf{x} = \mathcal{O}^{\dagger}\mathcal{H}$.

Proof. If $\sum_{\alpha} O_{s\alpha} x_{\alpha} = h_s$, then

$$\langle O \rangle \mathbf{x} = \frac{1}{N_s} \sum_s \sum_{\alpha} O_{s\alpha} x_{\alpha} = \frac{1}{N_s} \sum_s h_s = \langle h \rangle. \quad (16)$$

Thus, $\mathcal{O}\mathbf{x} = (O - \langle O \rangle)\mathbf{x} = \mathbf{h} - \langle h \rangle = \mathcal{H}$. Applying $\mathcal{O}^{\dagger}$ to both sides gives the tVMC equation. ■

In the TDVP regime, $N_s > N_p$, a solution $\mathbf{x}$ to Eq. (15) generally does not exist, so one cannot solve the TDVP

equation via Eq. (15). In the minSR regime, $N_s < N_p$ and Eq. (15) is under-determined, and one generally can find many solutions to it. Via the fundamental theorem of linear algebra: $\ker(O)^\perp = \text{im}(O^\dagger)$, we pick the solution orthogonal to the kernel of O : $x = O^\dagger y$ and solve for y :

$$OO^\dagger y = h, \quad x = O^\dagger y. \quad (17)$$

In contrast to the conventional minSR matrix $OO^\dagger$, $OO^\dagger$ generally does not have any zero modes at all. We use Eq. (17) to perform minSR in the examples.

Equation (15) also has a nice interpretation. It can be “derived” directly from the Schrödinger equation. If quantum states are parametrized with variational parameters θ_α and satisfy the Schrödinger equation, then for *any* configuration $\mathbf{s}$,

$$\sum_\alpha \langle \mathbf{s} | \partial_\alpha \Psi \rangle \delta \theta_\alpha = -i \delta t \langle \mathbf{s} | H | \Psi \rangle. \quad (18)$$

Dividing by $\langle \mathbf{s} | \Psi \rangle$ gives one row of Eq. (15) where $\mathbf{x} = i\dot{\theta}$, for sample $\mathbf{s}$. Thus, the Schrödinger equation requires Eq. (18) for every configuration $\mathbf{s}$, while Eq. (15) requires it for a stochastically selected set of configurations.

D. Small- o trick in minSR

In the minSR case, a small- o trick can be specifically applied to PEPS to alleviate the memory cost in storing the full matrix O . The key observation is that the *locality* of PEPS forces most elements O_{sa} to vanish. For a given sample configuration $\mathbf{s}$,

$$O_{sa} = \frac{\partial \log \Psi(\mathbf{s})}{\partial A[\mathbf{x}]_{lrdu}^p} = 0 \quad \text{if } p \neq s(\mathbf{x}). \quad (19)$$

Thus each site contributes only the physical slice compatible with the sample. We therefore define the “small- o ” object

$$o[\mathbf{x}]_{lrdu}(\mathbf{s}) \equiv O[\mathbf{x}]_{lrdu}^{p=s(\mathbf{x})}(\mathbf{s}), \quad (20)$$

from which the full minSR matrix $OO^\dagger$ can be reconstructed with the memory cost of o instead of O (see Supplemental Material [68] for the explicit GPU-efficient construction).

In tVMC, the memory bottleneck is storing O or S , which can be a serious issue on GPUs. For a PEPS with local Hilbert space dimension d , this “small- o ” trick reduces memory cost by a factor of d . For the $\mathbb{Z}_N$ gauge-invariant PEPS used in Sec. IV C, the reduction is very substantial—a factor of dN^3 . In practice, this saving often determines whether a large-scale GPU simulation is feasible at all.

E. TDVP versus minSR

We comment on the choice between TDVP and minSR form. As the system size increases, N_p eventually overtakes N_s and one should use minSR for efficiency. However, it is always advised to study a problem first at small size and bond dimension and ideally benchmark against exact methods, in which case typically $N_s > (N_p - \text{gauge redundancy})$ and it is only correct to use standard TDVP. In fact, the examples we will present are all benchmarked first against exact diagonalization at small system sizes. Thus, the TDVP form and its explicit gauge removal are very important in the workflow. Even though the results presented may be of large system size simulated with minSR, many calculations were done in the TDVP form before that.

F. Comparison with double-layer methods

We briefly discuss the difficulties of double-layer methods mentioned in the introduction: computational cost, gauge fixing, and positivity of the quantum metric. Although manifested differently, they reflect the same underlying difficulty.

a. Computational cost. Consider a PEPS with bond dimension D . Let D' denote the bond dimension of the boundary MPS (bMPS) used to contract a single-layer PEPS in VMC, cf. Fig. 1(d), and likewise the boundary matrix product operator (bMPO) used in full-update double-layer contraction. The dominant computational costs scale as $D^4 D'^2$ for VMC [32] and $D^4 D'^3$ for full update (FU) [26]. Typical choices are $D' \sim D$ to D^2 for VMC, and $D' \sim D^2$ to D^3 for full update, leading to effective complexities of D^6 – D^8 and D^{10} – D^{13} , respectively. This gap is substantial. VMC additionally incurs sampling overhead, but even choosing D' larger than D^3 does not reliably stabilize the full-update procedure due to the next two issues.

b. Gauge fixing. Unlike MPS in 1D [1], PEPS does not possess a canonical form that fixes the gauge freedoms. Reference [26] introduces local gauge fixing for full-update gate applications, but no globally consistent gauge-fixing procedure is known. In fact, as an alternative, a variational subfamily—*isometric* PEPS (isoPEPS) [48,55,69]—is introduced to explicitly preserve the isometric gauge at the expense of expressiveness. However, the necessary shifting of isometric centers introduces significant errors and leads to suboptimal results. In contrast, PEPS-tVMC achieves global gauge fixing efficiently through a single QR projection or simply using the minSR formulation, without any loss of representational power.

c. Positivity. Because PEPS are not in an isometric form, to perform a variational update, one needs to also form the non-trivial quantum metric of the parameters, induced from the inner product of the Hilbert space. This quantum

metric is also known as the norm matrix N in Ref. [26]. In practice, approximate contraction leads to violations of positivity of N , which destabilizes the generalized eigenvalue problems used in variational updates, especially in time evolution. Tiny errors in the boundary contraction can lead to large deviations in the time trajectory. tVMC avoids this issue entirely: normalization is enforced simply by subtracting O_{avg} and h_{avg} in Eq. (7), and the S matrix in Eq. (4), serving as the quantum metric, is manifestly positive semi-definite.

d. Abelian symmetry. In VMC, any conservation law that is diagonal in the computational basis can be directly implemented by sampling configurations in the target charge sector. This is equivalent to acting on the state with a global projector P_M onto the space with desired total charge M :

$$|\Psi\rangle = P_M |\text{PEPS}_\theta\rangle, \quad (21)$$

where only the underlying PEPS is parametrized. If written out explicitly, the global projector P_M is very expensive: it is a snake MPO with a bond dimension linear to system size. The VMC framework, however, allows such a projector not to incur any computational or memory cost, significantly increasing the representability of the underlying bare PEPS.

IV. RESULTS

We now apply PEPS-tVMC to study the real-time dynamics of six distinct two-dimensional models—bosonic and fermionic, free and interacting, and including systems with gauge symmetry or nontrivial topology. PEPS configurations are sampled using the sequential sampling algorithm, which was first introduced in Ref. [32], and later proved to satisfy detailed balance in Ref. [70]. The local energies and log-derivatives are computed via the contraction of the single-layer network shown in Fig. 1(d) (see Refs. [31,32,70] for methodological details). For Hamiltonians with no explicit time-dependence, the energies are observed to be well-conserved, with relative drift of 2×10^{-4} to 2×10^{-3} for the entire trajectory. We show the energy evolution in Supplemental Material [68]. The examples are all on squares with open boundary conditions. In examples IVA–IVE, the initial states of the dynamics are all obtained with ground-state PEPS VMC with the same D and $D' = 3D$ for the Chern insulator and the superfluid, $4D$ for the Hofstadter model, and $3\frac{D}{2}$ for the $\mathbb{Z}_2$ lattice gauge theories.

A. Free-fermion Chern insulator

As a first test, to obtain nontrivial exact benchmark, we consider a 12×12 free-fermion Hofstadter model with flux $\phi = 2\pi/3$,

$$H_{\text{chern}} = - \sum_{x,y} c_{x,y}^\dagger c_{x+1,y} + e^{-i\frac{2\pi}{3}} c_{x,y}^\dagger c_{x,y+1} + \text{H.c.}$$

We fix the particle number to $2/3$ filling, populating two of the three magnetic subbands. With open boundaries, the system hosts gapless chiral edge modes. Using fermion PEPS [70–72], we prepare the ground state in a weak attractive potential (strength -1) placed at the upper-left corner, which creates a localized edge-density bump. The system is then evolved in real time under H_{chern} with the potential removed.

As shown in Fig. 2, the excitation propagates unidirectionally along the edge. PEPS-tVMC faithfully reproduces the edge motion, its speed, and its spatial profile, in excellent agreement with the exact free-fermion numerics. For context, isoPEPS simulations of a similar chiral edge motion (e.g., Fig. 6(b) of Ref. [55]) already provided

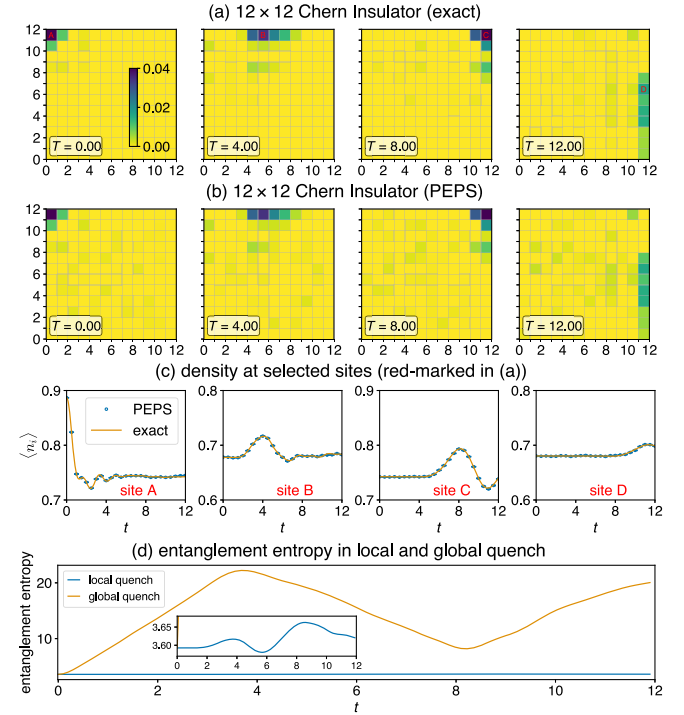


FIG. 2. Chiral edge dynamics in a 12×12 Chern insulator. (a) Exact real-time evolution obtained from free-fermion numerics. The color scale shows the density deviation $\langle n_i(t) \rangle - \langle n_i \rangle_{\text{gs}}$. A density perturbation created near the boundary at $T = 0$ propagates unidirectionally along the edge at later times $T = 4, 8, 12$. (b) Same protocol computed using PEPS-tVMC. (c) Time evolution of the density at four selected sites red-marked in (a), chosen so that the chiral pulse passes them at $T = 0, 4, 8, 12$. The error bars are of order 7×10^{-4} . All panels share the same vertical scale of 0.2. (d) Comparison of the entanglement entropy of the left half of the system during local and global quench dynamics, computed with free fermion numerics. The local quench is the same as in (a). The global quench is the dynamics under H_{chern} with the magnetic field removed.

state-of-the-art results at the time; however, their accuracy was ultimately limited by the errors due to the shifting of isometric centers. The present PEPS-tVMC approach achieves noticeably higher accuracy. We further note that the numerical difficulty of tensor network simulations is largely dictated by the smaller of L_x and L_y . Thus, although the isoPEPS study used a 9×18 lattice, the 12×12 system considered here is in fact the more challenging geometry, making this benchmark particularly stringent.

We emphasize that the PEPS used in tVMC is entirely agnostic to the Gaussianity in the free fermion state. Thus, despite the model being noninteracting, its non-trivial dynamics makes a demanding test for PEPS, especially at this lattice size and timescale.

In the end, taking advantage of the free-fermion numerics, we compute the entanglement entropy of the left half system during the dynamics. We compare it to the entanglement growth in a global quench from the same initial state, but evolving under H_{chem} with the magnetic flux removed, i.e. all hoppings being real and positive. As seen in Fig. 2(d), the entanglement growth in the global quench is much faster, peaking at around $t \approx 3.8$ due to the entanglement quasi-particle [73] hitting the edge of the system. The local quench we simulate has a mild entanglement growth and is thus more suitable for simulation with PEPS, as discussed in the introduction.

B. Pure $\mathbb{Z}_2$ lattice gauge theory

Next, to highlight the improvement due to the analysis above, we revisit our exploratory calculation in Ref. [31], which solved the TDVP equation using conjugate-gradient without any gauge removal. We considered a pure $\mathbb{Z}_2$ gauge theory on a 10×10 lattice:

$$H_{\mathbb{Z}_2} = -2 \sum_p B_p + 2g \sum_l (1 - Z_l) \quad (22)$$

subject to the Gauss-law constraint $\prod_{l \in i} Z_l = 1$. Here $B_p = \prod_{l \in p} X_l$ measures the magnetic flux through plaquette p , and a violation of $B_p = 1$ defines a *vison* excitation [74]. X_l, Z_l are the Pauli x, z matrices for the gauge field on link l . We consider the deconfined phase with $g = 0.1$, where the visons are the elementary excitations of the system and are expected to propagate with a finite life-time.

Following the same setting as in Ref. [31], we create a vison at the lower-left corner of the lattice by acting on the ground state with a Z_l operator on the link right to the site $(0, 0)$. We then simulate the subsequent tVMC dynamics under $H_{\mathbb{Z}_2}$ with gauge-invariant PEPS, using the same bond dimension ($D = 8$), sample size ($N_b = 10240$), time step ($dt = 0.005$), and integrator (2nd order Runge-Kutta) as in Ref. [31]. Here, we solve the TDVP equation with

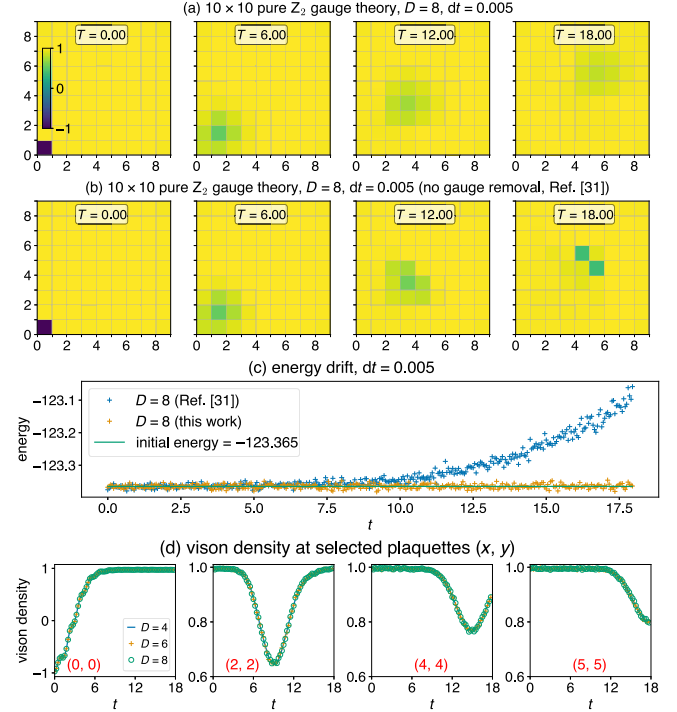


FIG. 3. Real-time vison propagation in the pure $\mathbb{Z}_2$ lattice gauge theory with $g = 0.1$ on a 10×10 lattice. (a) Snapshots of the plaquette operator $B_p(t)$, which highlights magnetic-flux (vison) excitations created at $T = 0$, simulated with gauge removal. (b) Data reported in Ref. [31]. (c) Energy evolution during the simulations. (d) Vison densities at various bond dimensions.

Cholesky decomposition and gauge removal discussed above. As seen in Fig. 3(c), with the current method, the energy conservation is significantly better and the vison propagation also looks more physical. Figures 3(a) and 3(b) reveal a noticeable difference in the vison densities at late time, highlighting the importance of gauge removal in PEPS-tVMC.

In fact, the entanglement in this dynamics is quite low, and the simulations are well-converged already at $D = 4$. We demonstrate the convergence in vison density with $D = 4, 6$, and 8 in Fig. 3(d). We note that the gauge-theory PEPS has a subtle structure [31] and the computational cost for $\mathbb{Z}_N$ gauged PEPS with bond dimension D is equivalent to that of a normal PEPS with bond dimension $\frac{D}{N}$. Thus, the $D = 4, 6, 8$ in Fig. 3 only incurs computational cost of $D = 2, 3, 4$ in the normal case.

C. $\mathbb{Z}_2$ lattice gauge theory coupled to Higgs field

Next, for a much more challenging gauge theory dynamics, using the gauge-invariant PEPS-VMC framework introduced in Ref. [31], we examine the confinement dynamics in the $\mathbb{Z}_2$ lattice gauge theory coupled to a Higgs field,

$$H_{\text{igt}} = -\sum_i \sigma_i^z - \sum_p B_p - J \sum_l \sigma_{l_-}^x X_l \sigma_{l_+}^x - g \sum_l Z_l$$

subject to the Gauss-law constraint $\sigma_i^z \prod_{l \in i} Z_l = 1$. σ_i^x, σ_i^z are the Pauli matrices on the site i , representing the Higgs field. $l_{\pm}$ are the sites at the ends of link l . The system has three ground state phases: confined (small J and large g), Higgs (large J and small g), and deconfined (small J and small g) phases [75]. When $g = 0$, the phase transition point between the deconfined phase and the Higgs phase is at $J_c \approx 0.328$ [76]. In the Higgs phase, the visons are expected to have a linearly attractive interaction, i.e. vison confinement. In the deconfined phase, the visons are expected to propagate freely, i.e. vison deconfinement.

We create a pair of visons on top of the ground state in both the Higgs and the deconfined regimes, and simulate the subsequent dynamics under H_{igt} , shown in Fig. 4. In the deconfined phase, the initially localized visons rapidly spread into extended flux clouds that delocalize across the lattice, consistent with free, deconfined excitations and the absence of a confining string. In the Higgs phase, the visons remain bound. The absence of spreading reflects a finite string tension and the confining nature of the Higgs phase. The vison densities, converged at $D = 8$

and 10, are shown in Figs. 4(c) and 4(d) with different bond dimensions.

Recently, considerable progress has been made in implementing real-time gauge-theory dynamics on quantum devices [77–79]. Our results highlight that classical tensor-network approaches can serve as a highly effective and complementary platform for these studies. It would be interesting to explore whether PEPS-tVMC can probe the intricate dynamics of glueballs in the $(2+1)$ D Yang-Mills theory, after the gauge-invariant PEPS-VMC [31] is extended to the non-Abelian case.

D. Bosonic fractional quantum Hall effect

Next we study the remarkable phenomena of fractional quantum Hall effect [80,81]. We consider a bosonic Hofstadter model with flux $\phi = \pi/2$ and hardcore constraint, on a 12×12 lattice

$$H_{\text{fqhe}} = -\sum_{x,y} b_{x,y}^\dagger b_{x+1,y} + e^{i\frac{\pi}{2}} b_{x,y}^\dagger b_{x,y+1} + \text{H.c.}$$

We consider the $1/8$ density, where the lowest magnetic band is half filled, corresponding to filling $\nu = 1/2$. With the filling belonging to the Jain sequence [82], composite fermion theory [83,84] predicts that the ground state is a bosonic analog of the Laughlin state [81] with quasi-particle charge $1/2$. The lattice structure [85] also complicates the theory which was initially developed for the continuum. We now explicitly check the fractional transport in real time.

To account for the finite boundary, in practice, we use particle number $(L_x - 1)(L_y - 1)/8 \approx 15$ [86]. As shown in Fig. 5(a), the ground-state VMC gives a charge distribution that is uniformly $1/8$ in the bulk.

To probe the fractional charge transport, we thread an external time-dependent flux $\Phi(t)$ through the central plaquette, cf. the red $\otimes$ in Fig. 5(a), increasing linearly from 0 to 2π over a period $T_{\text{flux}} = 10$. The external flux is implemented via a Peierls phase on the red horizontal bonds shown in Fig. 5(a). This generates a steady electromotive force around any closed loop enclosing the plaquette and induces a Hall transport from the middle of the system to the edge. If the system is indeed a fractional quantum Hall state, a quantized charge of $\Delta Q = \nu = 1/2$ is expected to be transported per pumping cycle [87,88]. Indeed, Fig. 5(b) shows that PEPS-tVMC confirms this quantized transfer of charge. A linear fit of the charges within a central region, cf. the blue circle in Fig. 5(a), versus t/T_{flux} gives a slope of 0.5053, close to the expected Hall response of $1/2$.

The density profile during the pumping is shown in Fig. 5(c). Because the flux is time-dependent, different Peierls paths are not gauge-equivalent at the level of time evolution, and our choice breaks the C_4 symmetry of the lattice. Although the instantaneous density flow seems

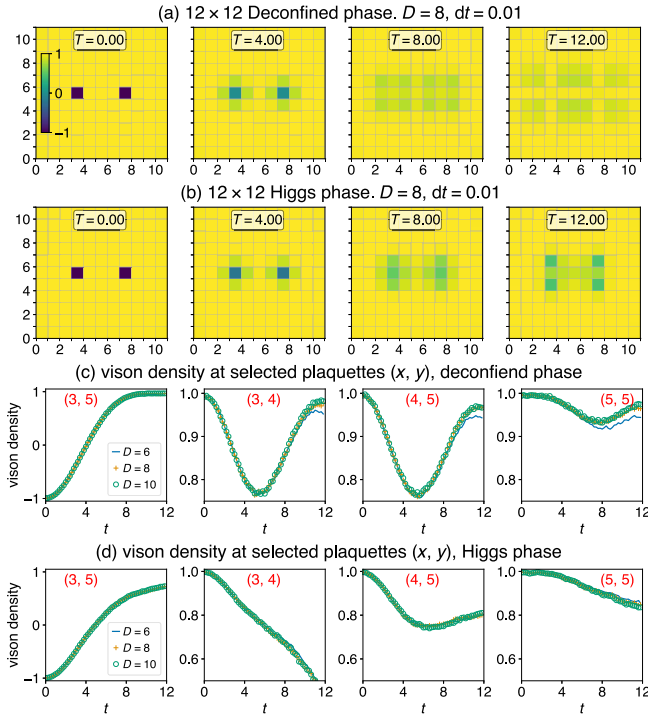


FIG. 4. Real-time vison dynamics in the $\mathbb{Z}_2$ lattice gauge theory on a 12×12 lattice. Shown are snapshots of the plaquette operator $B_p(t)$, which highlights magnetic-flux (vison) excitations created at $T = 0$. (a) Deconfined phase ($J = 0.1, g = 0.1$). (b) Higgs phase ($J = 0.5, g = 0.1$). (c),(d) Vison densities at various bond dimensions.

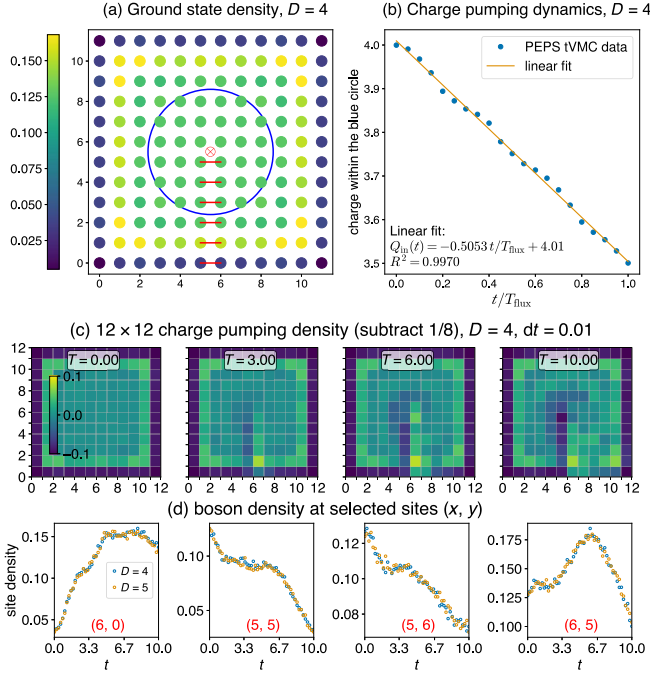


FIG. 5. Charge pumping in the bosonic Hofstadter model at flux $\phi = \pi/2$. (a) Ground-state charge distribution on a 12×12 lattice with total particle number 15. A localized external flux is threaded through the central plaquette (marked by $\otimes$). The red horizontal bonds indicate the Peierls-phase path along which the time-dependent vector potential $A_{ij}(t)$ is ramped. The blue circle denotes the region over which the enclosed charge Q_{in} is monitored. (b) Total charge within the blue circle during one flux-insertion cycle of duration $T_{\text{flux}} = 10$. (c) Evolution of real-space density, subtracted by $1/8$, during the pump. (d) Boson densities at various bond dimensions.

structureless, the integrated pumped charge remains amazingly quantized, attesting to the robustness of the Hall response to microscopic details of the system. The boson densities are converged with bond dimension $D = 4$ and 5 , as shown in Fig. 5(d).

This result provides a direct real-time confirmation of the composite-fermion prediction of fractional quasiparticle charge. While previous numerical studies of fractional Chern insulators relied primarily on static diagnostics, such as entanglement spectra or density profiles of pinned quasiholes, PEPS-tVMC enables us to directly visualize the transport of fractional charge in two dimensions. This establishes a dynamical counterpart to the usual static signatures of topological order. It would be exciting to study anyonic dynamics in other settings with PEPS-tVMC in the future.

E. Superfluid bosons

Next we study real-time flow in a superfluid of hardcore bosons on a 13×13 lattice,

$$H_{\text{sf}} = - \sum_{\langle ij \rangle} b_i^\dagger b_j \quad (23)$$

at half-filling. This system is known to be in the superfluid phase due to Bose-Einstein condensation. Although its static properties are well understood from quantum Monte Carlo method [89,90], its real-time dynamics—including the onset of dissipation and the critical velocity—remain much less explored, especially in two dimensions.

Here we study the dynamics of the bosons in the presence of an obstacle, interacting via a repulsive potential:

$$H_{\text{obs}}(t) = 2 \sum_i b_i^\dagger b_i \exp\left(-\frac{|\mathbf{r}_i - \mathbf{R}(t)|^2}{2}\right), \quad (24)$$

where $\mathbf{R}(t)$ is the position of the obstacle at time t . By Galilean invariance, we keep the superfluid at rest and drag the obstacle rightward at constant velocity: $\mathbf{R}(t) = \mathbf{R}_0 + \mathbf{v}t$. $H_{\text{obs}}(t)$ can also be realized via a laser in cold-atom simulations. The initial state is the ground state of $H_{\text{sf}} + H_{\text{obs}}(0)$. As shown in Figs. 6(a) and 6(b), for slow motion $|\mathbf{v}| = 0.5$, the density perturbation remains localized and approximately symmetric around the impurity, with no pronounced wake, consistent with dissipationless flow below the critical velocity. For fast motion $|\mathbf{v}| = 5.0$, the impurity leaves behind a pronounced density wake and radiates excitations throughout the system above the critical velocity. In particular, the density depletion caused by the obstacle closely follows the obstacle when $\mathbf{v}$ is small, while it lags far behind when $\mathbf{v}$ is large. The superfluid densities are converged with $D = 4$ and 5 as shown in Figs. 6(d) and 6(e).

Figure 6(c) shows the time evolution of $\langle H_{\text{sf}} + H_{\text{obs}}(t) \rangle$. At $|\mathbf{v}| = 5$, significant work is done to the boson-obstacle system, leading to pronounced dissipation. In contrast, for a slow velocity, $|\mathbf{v}| = 0.5$, the total energy remains approximately conserved over an extended time interval, consistent with frictionless superfluid motion. These qualitative differences between the two velocities signal the presence of a critical superfluid velocity, as anticipated by Landau [91].

In the future, it would be exciting to apply PEPS-tVMC for the dynamical studies of phase coherence, vortex nucleation, and other hallmarks of superfluidity and superconductivity.

F. Quantum Ising global quench

Finally we study the dynamics in the quantum Ising model following a global transverse field quench: $h = \infty \rightarrow h_c$ in

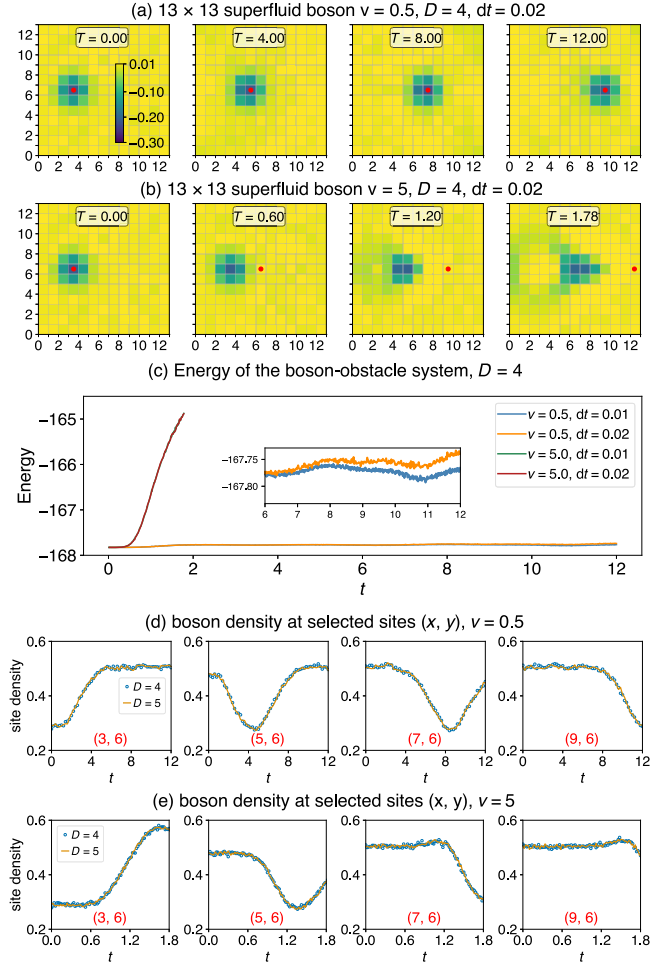


FIG. 6. (a) Real-time density response of a 2D bosonic superfluid to a moving obstacle (red dot) with velocity $|\mathbf{v}| = 0.5$. Snapshots show the deviation of the local density from 0.5 on a 13×13 lattice for different times T as the obstacle is dragged along the x direction. (b) Same as (a), but with $|\mathbf{v}| = 5$. (c) Total energy $\langle H_{\text{sf}} + H_{\text{obs}}(t) \rangle$ as a function of time for both velocities. To assess the effect of the time step on the energy evolution, results obtained with $dt = 0.01$ and 0.02 are shown. For $|\mathbf{v}| = 0.5$, the two time steps agree to within a relative energy difference of 2×10^{-4} over the entire evolution. (d),(e) Superfluid densities at various bond dimensions.

$$H_{\text{ising}} = -\sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x. \quad (25)$$

While global quenches are not the main application target of PEPS-tVMC, this Ising dynamics [45] has become a paradigmatic example in the simulation of two-dimensional real-time dynamics, which makes it interesting for PEPS-tVMC to compare. On a 11×11 lattice, we prepare the initial state as the product state polarized along positive x -axis, and evolve it under the critical H_{ising} with $h_c = 3.04438$ [92].

In Fig. 7(a), we report the transverse magnetization $\langle \sigma_{(5,5)}^x \rangle$ of the middle site during the dynamics. Figure 7(b)

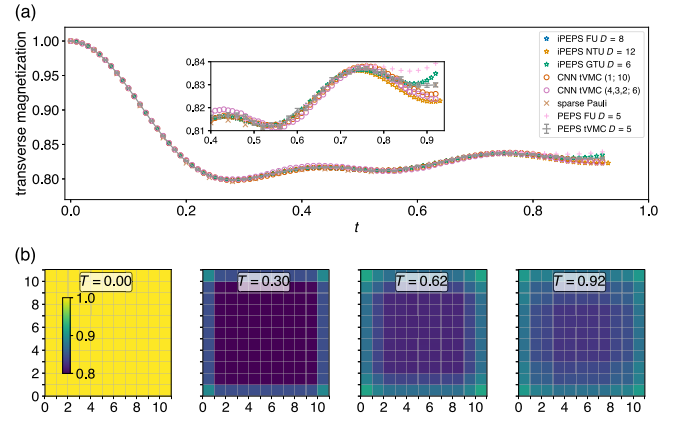


FIG. 7. Transverse magnetization during the Ising global quench dynamics. The references include iPEPS FU [45], neighborhood update (NTU) [49], gradient tensor update (GTU) [50], convolutional neural network (CNN) tVMC [13], and sparse Pauli operator dynamics [93]. PEPS FU is done by us at $D = 5$ following Ref. [26]. (a) Ising global quench dynamics $h: \infty \rightarrow h_c$. (b) 11×11 Ising $\langle \sigma^x \rangle$, $D = 5$, $dt = 0.01$.

shows the distribution of $\langle \sigma^x \rangle$ in real-space and suggests that the finite size effect at this timescale is small for the middle site. This dynamics starts from a product state, which is tricky to represent with a $D = 5$ PEPS, so we use FU [24] to evolve the state first to $T = 0.4$, and relay to tVMC to evolve until $T = 0.92$. As a comparison, we also evolve the FU fully to $T = 0.92$. We use bond dimension 125 for the bMPO of the FU, and 20 for the bMPS of the tVMC. Comparing with previous results, tVMC is likely to be the more accurate of the two.

At the final time $T = 0.92$, various simulations [13,45,49,50,93], including ours, show a discrepancy of around 0.005 to 0.01 in $\langle \sigma^x \rangle$. We can in fact evolve the PEPS-tVMC trajectory past $T = 0.92$; the results are not shown because, beyond this time, the available methods begin to disagree and it is no longer clear which calculation should be taken as the benchmark. It will remain a collective effort of the community to close out this discrepancy in the future.

Finally, we comment on a practical strategy to scale up the PEPS-tVMC to high bond dimensions for this quench. Currently all PEPS tensors are allowed to vary independently, amounting to a large number of parameters: $N_p = 2L^2D^4$, incurring a large memory cost if the sample size is also large. It is possible to variationally constrain the tensors within the boundary to be the same, reducing N_p to order $O(D^4)$. In the presence of an open boundary, this will introduce a bias. However, due to locality, one expects that observables deep in the bulk are unaffected by such bias before a certain timescale and system size. This would possibly allow the PEPS-tVMC to be scaled up to significantly higher bond dimension and system sizes, which we leave for future work. Such higher-bond-dimension calculations should also help resolve the

discrepancies among different methods beyond the present time window of agreement.

G. Error and stability in solving the TDVP equation

An important factor underlying the accuracy and efficiency of PEPS-tVMC is the numerical stability with which the TDVP equation—or its reduced minSR variant—can be solved. All simulations in this work solve the TDVP equation using a Cholesky decomposition [94] of the QGT matrix with a very small Tikhonov regulator (10^{-7} or 10^{-8}).

Compared to the rank-revealing singular value decomposition (SVD) or pseudo-inverse solver, the Cholesky solver lacks the robustness of SVD or pseudo-inverse when applied to ill-conditioned matrices, but is substantially more parallelizable and faster on GPUs.

For example, on a GPU card with 141 GB memory and 34 TFLOPS for FP64, for a 20480×20480 positive complex128 matrix S , solving the linear equation $S\mathbf{x} = \mathbf{g}$ takes 146 s with the SVD solver, but only 0.28 s with the Cholesky solver. At 40960×40960 , the Cholesky solver takes 2.1 s, while the SVD solver throws an out-of-memory error. Such equations need to be solved k times in a k th order RK integrator. See repository [95] for the benchmark details.

Recent NQS-tVMC studies often resort to the SVD solver [13,15,22], trading computational efficiency for numerical robustness. The fact that Cholesky remains stable in all our simulations indicates that once gauge redundancies are analytically removed, the PEPS variational manifold becomes exceptionally stable for real-time dynamics, which gives PEPS-tVMC a significant wall time reduction.

To quantify the accuracy of the Cholesky solution, we monitor the tVMC residual with the regulator removed,

$$\varepsilon = \frac{\|\dot{S}\dot{\theta} - (-i\mathbf{g})\|^2}{\|\mathbf{g}\|^2}, \quad (26)$$

which measures the relative error in satisfying the TDVP equation. As shown in Fig. 8, the residual remains

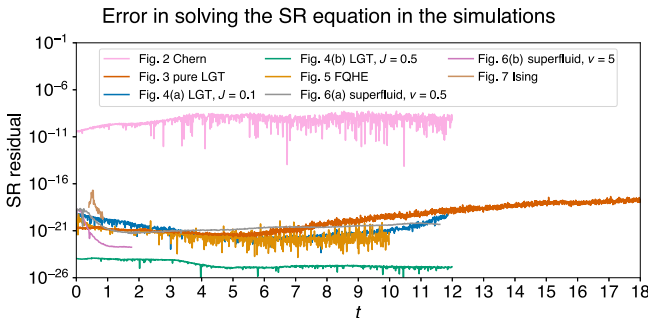


FIG. 8. Residual error $\varepsilon = \|\dot{S}\dot{\theta} - i\mathbf{g}\|^2 / \|\mathbf{g}\|^2$ during PEPS-tVMC time evolution for the models considered in this work.

exceptionally small—typically between 10^{-9} and 10^{-25} , depending on the model. These results confirm the effectiveness of our robust strategies to solve the linear TDVP equations, ensuring that the tVMC step does not contribute appreciable error during real-time evolution.

H. Heuristic reason for the stability

We give a preliminary understanding of why PEPS-tVMC is stable related to the small- o trick. Suppose we are in the minSR regime, then the stability of solving the tVMC equation is determined by the condition number of $OO^\dagger$, which is a measure how “orthogonal” the row vectors of O are to each other. For two row vectors corresponding to sample $\mathbf{s}$ and $\mathbf{s}'$, due to Eq. (19), their overlap only receives contributions from the sites $\mathbf{x}$ for which $\mathbf{s}(\mathbf{x}) = \mathbf{s}'(\mathbf{x})$:

$$\sum_{\alpha} O_{\alpha}^*(\mathbf{s}) O_{\alpha}(\mathbf{s}') = \sum_{\mathbf{x}: \mathbf{s}(\mathbf{x}) = \mathbf{s}'(\mathbf{x})} o^{\dagger}[\mathbf{x}](\mathbf{s}) o[\mathbf{x}](\mathbf{s}').$$

Roughly speaking, out of the $\sim L^2 D^4 d$ parameters of a PEPS, only $\frac{L^2}{d} D^4$ number of them contribute to the pairwise overlap of the row vectors of O .

In general, if each sample only activates $\frac{1}{d}$ of total parameters of an ansatz, then roughly $\frac{1}{d^2}$ of total parameters will contribute to the off-diagonal elements of $OO^\dagger$, while $\frac{1}{d}$ of them to the diagonal.

Amusingly, the above mechanism is reminiscent of mixture-of-experts (MoE) models [96–99]. In a MoE model, the input $\mathbf{s}$, via a router, selects a subset of experts E , the output is a weighted sum of the selected expert outputs,

$$\text{MoE}(\mathbf{s}) = \sum_{a \in E} g_a(\mathbf{s}) E_a(\mathbf{s}). \quad (27)$$

In the small- o PEPS representation, on each site, one PEPS tensor may be viewed as d local experts with disjoint parameters. A sample $\mathbf{s}$ selects exactly one local expert, and instead of a weighted sum, the final output is the single layer contraction of the selected L^2 experts out of a total of dL^2 experts,

$$\Psi(\mathbf{s}) = \text{tTr} \left(\prod_{\mathbf{x}} o[\mathbf{x}](\mathbf{s}) \right). \quad (28)$$

I. Simulation parameters

We document the simulation details in Table I. The integrator is 2nd order RK for pure $\mathbb{Z}_2$ gauge theory and 4th order RK for all other examples. The Tikhonov regulator in solving the TDVP or the minSR equation using the Cholesky decomposition is 10^{-8} everywhere, except for the $D = 4$ pure $\mathbb{Z}_2$ gauge theory which is 10^{-7} .

TABLE I. Simulation parameters. L : side length. D : PEPS bond dimension. D' : bMPS bond dimension. dt : time step. T : evolution time. N_s : sample size. N_p : parameter size in thousands. t_{wall} : wall time per dt in seconds; $\times n$ means the simulation ran on n GPU cards. The wall time is measured on GPUs with 141 GB memory and 34 TFLOPS for FP64. We use the standard TDVP if $N_s > N_p$, and minSR otherwise. Identical parameters for the same model are not shown.

Model	L	D	D'	dt	T	N_s	N_p	t_{wall}/s
Chern	12	4	16	1×10^2	12	40960	28 k	399×1
Pure LGT	10	4	8	5×10^3	10	40960	8 k	17×1
			6				42 k	13×1
			8				134 k	38×1
Higgs LGT	12	6	12	1×10^2	12	20480	134 k	167×1
			8				422 k	328×1
			10				1027 k	560×2
Hofstadter	12	4	24	1×10^2	10	20480	52 k	207×2
			5				129 k	538×4
Superfluid	13	4	16	2×10^2	12	20480	63 k	222×1
			5				154 k	713×1
Ising	11	5	20	1×10^2	0.92	40960	105 k	430×2

For the $\mathbb{Z}_2$ lattice gauge theory, despite the large N_p due to $D = 10$, the computational complexity is at the level of a $D = 5$ normal PEPS. The time saving is from our findings in Ref. [31], while the memory saving is due to the small- o trick. If one stores the full 20480×1027320 complex128 O matrix, the memory cost would be 336 GB, which is a significant burden.

V. CONCLUSION

In this work we have shown that projected entangled pair states, when combined with time-dependent variational Monte Carlo method, provide a stable and efficient framework for simulating real-time low-energy dynamics in two-dimensional quantum many-body systems. The key technical insight is that the ill-conditioning traditionally associated with PEPS—stemming from gauge redundancy—can be removed analytically in closed form. Once gauge directions are eliminated, the TDVP equation can be solved efficiently with high numerical stability over long evolution times. This stability, together with the locality and single-layer contraction structure of PEPS within VMC, makes PEPS-tVMC a powerful classical method for studying low-energy dynamics in 2D.

The examples explored in Sec. IV demonstrate the breadth of physical phenomena accessible with this approach. These results establish PEPS-tVMC as a general-purpose tool for simulating elementary excitations, transport phenomena, and real-time response in a wide range of quantum systems with realistic computational resources. The improvements developed here apply equally to ground-state PEPS-VMC calculations [27–30].

An important conceptual question concerns the regime of validity of PEPS for real-time dynamics. While PEPS cannot represent volume-law states, our results indicate that for local quenches and low-energy excitations the entanglement growth remains compatible with PEPS bond dimensions accessible on current hardware. A more systematic understanding of entanglement-light-cone structure in 2D local quenches, and of the interplay between PEPS bond dimension and dynamical fidelity, would provide valuable theoretical guidance. In addition, studying late-time thermalization or hydrodynamic transport within PEPS-tVMC may reveal whether approximate area-law manifolds can capture emergent long-wavelength physics even when the exact state exhibits extensive entanglement.

Overall, our work demonstrates that PEPS, long regarded as difficult for time evolution, can in fact support accurate and stable simulations of large-scale 2D quantum dynamics at low energy. We hope that the ideas introduced here can inspire new developments in both variational algorithms and tensor-network theory, ultimately expanding the reach of classical simulation deep into the low-energy real-time dynamics of quantum matter.

VI. OPEN QUESTION AND FUTURE DIRECTION

We close by listing a few concrete open questions and future directions regarding the broad scope of PEPS-tVMC.

Open questions:

- (1) *Support collapse.* tVMC has a known problem of “support collapse” in evolving a state acted on by a projective operator [20]. For example, the action of $c_i^\dagger$ annihilates the sampling probability of any configuration with $s_i = 0$, which introduces a bias in the stochastic estimation of the QGT [100].
- (2) *Product state.* Unlike SU or FU, tVMC cannot grow bond dimension dynamically. If an evolution starts from a product state, one cannot simply use a $D = 1$ PEPS due to the entanglement later in the dynamics. The naive zero-padding of a product state into a higher D PEPS will not work as the dynamics will be trapped in the local stationary points of the TDVP. We mention two possible strategies: (i) one may use the standard PEPS time evolution algorithm such as SU or FU for early time with a small bMPO and switch to tVMC later when the required bMPO becomes too expensive, as already used in Fig. 7; (ii) one may prepare the product state by a ground-state VMC of its parent Hamiltonian made of one-site terms.
- (3) *Amplitude evaluation.* PEPS-VMC hinges on an accurate estimate of the amplitude $\langle s | \Psi \rangle$. While the bMPS method already does quite well, can one do better?

- (4) *PEPS versus NQS.* What features of PEPS and NQS control the conditioning of the tVMC equations? How is this related to their different entanglement structures, including the ability of NQS to represent volume-law states? At the same time, PEPS offer several practical advantages as a tVMC ansatz: the accuracy can be systematically improved by increasing the bond dimension, and the method connects naturally to established PEPS time-evolution techniques, including iPEPS updates, finite-PEPS full update and belief propagation.

Future directions:

- (1) *Long-range interactions.* Long-range interacting systems, such as Rydberg-atom arrays, have recently attracted considerable attention [101]. Incorporating long-range interactions into tVMC is conceptually straightforward, since the action of long-range couplings only enters through local energy estimators. While new ideas would be desirable to address the increased computational cost from long-range off-diagonal couplings, models with local off-diagonal terms and long-range diagonal couplings are already within the reach of the method, as considered in Ref. [102].
- (2) *Periodic boundary conditions.* Extending PEPS-tVMC to periodic boundary conditions is possible in principle, but the cost of amplitude evaluation of a boundaryless PEPS single layer is substantially higher. One may find inspiration in the large body of work in the tensor network renormalization of two-dimensional classical partition functions [103–106], or the more recent developments of belief propagation [57–59].
- (3) *Finite-temperature calculation.* PEPS-tVMC should be useful for imaginary-time dynamics as well. Two possible routes are the minimally entangled typical thermal state approach [107,108] and thermofield-double purification [109,110].
- (4) *Adiabatic dynamics.* PEPS-tVMC should also be well suited for adiabatic dynamics, which is particularly relevant to how current quantum computers and simulators prepare many-body ground states [111]. In this setting, the system follows (near) ground states of a time-dependent Hamiltonian, which are expected to obey an area law.
- (5) *Spectral functions.* Single-particle spectral functions and density response functions are directly connected to experiments. Once local support collapse is resolved, PEPS-tVMC could become a practical dynamical probe of two-dimensional strongly correlated systems.
- (6) *Single precision.* The simulations in this work use double precision. However, modern GPUs provide much higher throughput for single precision. It is

therefore important to push, as much as possible, PEPS-tVMC to be performed in lower precision.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [95], embargo periods may apply.

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