

Observation of Non-Hermitian Topology in Cold Rydberg Quantum Gases

Jun Zhang,^{*} Ya-Jun Wang,^{*} Shi-Yao Shao,^{*} Bang Liu, Li-Hua Zhang, Zheng-Yuan Zhang, Xin Liu, Chao Yu, Qing Li, Han-Chao Chen, Yu Ma, Tian-Yu Han, Qi-Feng Wang, Jia-Dou Nan[✉], Yi-Ming Yin, Dong-Yang Zhu, Qiao-Qiao Fang, Dong-Sheng Ding^{✉,†} and Bao-Sen Shi[✉]

Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei, Anhui 230026, China

and Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, Anhui 230026, China



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The pursuit of topological phenomena in non-Hermitian systems has unveiled new physics beyond the conventional Hermitian paradigm, yet their realization in interacting many-body platforms remains a critical challenge. Exploring this interplay is essential to understand how strong interactions and dissipation collectively shape topological phases in open quantum systems. Here, we experimentally demonstrate dynamical spectral topology in a dissipative Rydberg atomic gas and characterize parameter-dependent winding numbers, which quantify a geometric winding of the spectral topology. By increasing the interaction strength, the system evolves from Hermitian to non-Hermitian regime, accompanying emergence of trajectory loop in the complex energy plane. As the scanning time is varied, the spectral topology becomes twisted in the complex energy plane, forming enclosed sub-loops characterized by opposite winding numbers. Furthermore, by changing the scanning direction, we observe the differentiated spectral loops, revealing a signature of scan-direction-dependent nonreciprocity. This work establishes cold Rydberg gases as a versatile platform for exploring the rich interplay between non-Hermitian topology, strong interactions, and dissipative quantum dynamics.

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I. INTRODUCTION

The phase in topology reveals the inherent space property that is preserved under continuous deformations, such as stretching and bending. Topological phase transitions refer to changes in the state of matter that are characterized by alterations in the topological properties of the system, rather than conventional symmetry-breaking mechanisms [1–7]. Studying topological phases and their transitions involves exploring materials' features with distinct global geometric features and investigating how these phases change under different conditions [8–11]. These transitions can occur between different topological phases, where the properties of the system remain invariant under continuous transformations but differ in their topological invariants [12–17], such as the presence of edge states [18–24], topological insulators [25–27] and superconductors [28,29] in quantum

systems. In addition, the topological phase transitions are also investigated in non-equilibrium systems [30,31], non-Hermitian systems [32–36], and dynamical systems [37–40], in which dynamical behaviors and interactions enrich topological properties.

Due to the unique properties of Rydberg atoms [41–43], they not only enhance the complexity of the system's behavior but also provide a platform for investigating topological states and their phase transitions, for example, topological band structure [19] and topological order in the Kagome lattice [44], and other exotic topological features [45–48]. The characteristic of long-range interactions makes the laser-driven Rydberg atomic system displaying rich non-equilibrium physics and nonlinear dynamics [49–66]. The interactions in Rydberg atoms induce non-Hermitian properties [67] and hysteresis loops characterized by asymmetrical responses to varying probe intensities [68]. The precise control over Rydberg atom excitation facilitates the creation of effective non-Hermitian Hamiltonians, which exhibit rich dynamical evolution. This capability provides a powerful platform for investigating the emergence of spectral topology and dynamical characteristics of the spectral.

In this work, we experimentally demonstrate non-Hermitian spectral topology in a dissipative cold

^{*}J. Z., Y.-J. W., and S.-Y. S contributed equally to this work.

[†]Contact author: dds@ustc.edu.cn

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Rydberg atomic gas. Distinct from previous studies on hysteresis trajectories [68], we construct an expanded two-dimensional parameter space to map the effective complex energy spectrum. The emergence of a distinct spectral loop in the complex energy plane reveals a transition from a Hermitian to a non-Hermitian regime. Through measurements of the parameter-dependent complex energy spectra, we extract the corresponding topological winding numbers. Remarkably, we find that the spectral topology itself is not static but dynamical. By tuning the parameter scanning time, the spectral loop can be “twisted”, leading to the emergence of sub-loops characterized by an inversion of the winding number. Furthermore, by reversing the parameter scan direction, we observe a differentiation in the resulting spectral loops. This scan-direction-dependent nonreciprocity in the measurement trajectory provides non-trivial behavior in the non-Hermitian many-body system. The reported results show that the strongly interacting cold Rydberg gases can be regarded as a versatile platform for exploring non-Hermitian topological physics.

II. RESULTS

A. Physical model

To investigate the energy spectrum topological properties, we consider an interacting three-level Rydberg atomic system as illustrated in Fig. 1(a). The system consists of

three atomic state manifolds: the ground state $|g\rangle$, the intermediate excited state $|e\rangle$, and the Rydberg state $|r\rangle$. The probe field with Rabi frequency (detuning) Ω_p (Δ_p) drives the transition $|g\rangle \leftrightarrow |e\rangle$. The coupling field, with Rabi frequency Ω_c and detuning Δ_c , couples the transition $|e\rangle \leftrightarrow |r\rangle$. The spontaneous decay rates for the states $|e\rangle$ and $|r\rangle$ are γ_1 and γ_2 , respectively. The experimental setup is depicted in Fig. 1(b).

To explore the topological properties of the system, we first analyze its Hamiltonian. The effective single-particle Hamiltonian in the interaction picture and under the rotating-wave approximation can be written as

$$H_{\text{eff}} = \sum_j \left[\frac{\Omega_p}{2} |e\rangle\langle g|_j + \frac{\Omega_c}{2} |r\rangle\langle e|_j + \text{H.c.} \right] - \sum_j [\Delta_p |e\rangle\langle e|_j + (\Delta_r + i\gamma_{\text{eff}}) |r\rangle\langle r|_j], \quad (1)$$

where $\Delta_r = \Delta_p + \Delta_c$ represents two-photon detuning. The interactions between atoms in the state $|r\rangle$ influence the many-body quantum dynamics. Here, the interactions induce an additional dissipation on the state $|r\rangle$, resulting in a dissipation term $i\gamma_{\text{eff}}$.

Under the mean-field approximation (the interaction between Rydberg atoms is equivalent to the effect of a single atom in the collective field of surrounding atoms), $\gamma_{\text{eff}} = \gamma_2 + V\rho_{rr}$ exhibits a linear dependence on the

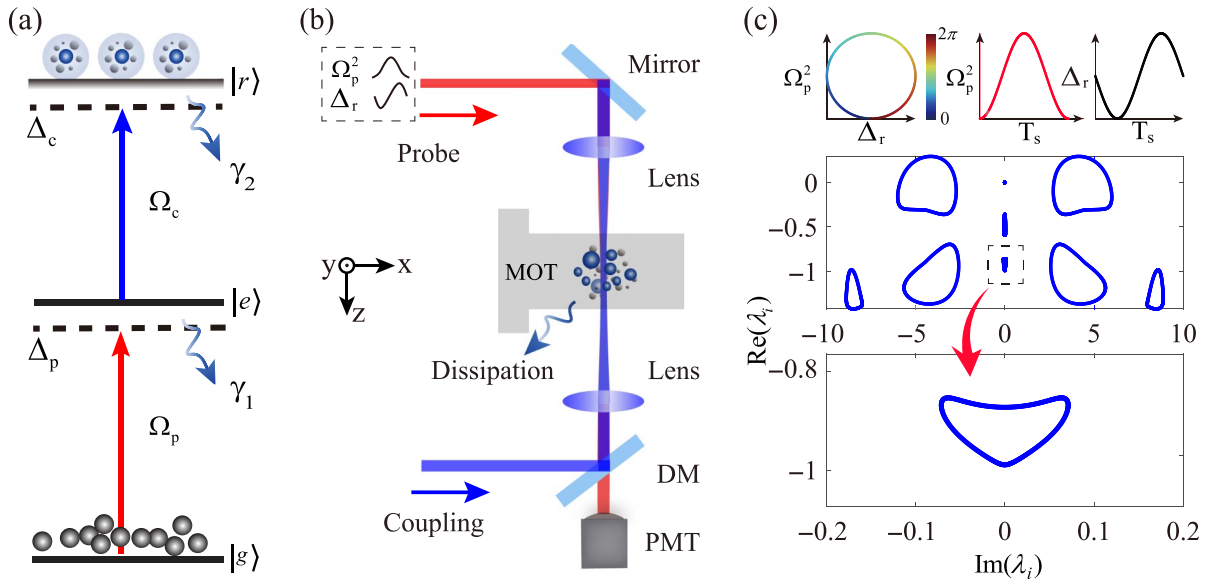


FIG. 1. Experimental diagram and energy spectral topology. (a) Energy level diagrams. Probe and coupling fields excite atoms from the ground state $|g\rangle$ to the Rydberg state $|r\rangle$. (b) Schematic diagram of the experimental setup. The probe beam is incident opposite to the coupling beam and focused in cold ^{85}Rb atoms. MOT: magneto-optical trap, PMT: photomultiplier tube, DM: dichroic mirror. (c) Numerical simulations of Liouvillian eigenspectrum varying with parameter $\text{Arg}(Z)$ on the complex energy plane, where $Z = \Delta_r + i\Omega_p^2$. The parameters are $\{\Omega_c, \Delta_c, \gamma_{\text{eff}}, \gamma_1, \gamma_2\} = \{4, 0, 2, 0.8, 1.2\}$, $\Omega_p^2 \in [0, 6]$, and $\Delta_r \in [-3, 3]$. The enlarged view represents the eigenvalue λ_{rr} of eigenstate ρ_{rr} . The upper panel shows the diagram of changing trends of parameters Ω_p^2 and Δ_r .

population of $|r\rangle$ state ρ_{rr} [68]. The Lindblad master equation including the associated dissipative terms can be written as

$$\dot{\rho} = -i[H_{\text{eff}}, \rho] + \sum_j \left(L_j \rho L_j^\dagger - \frac{1}{2} \{L_j^\dagger L_j, \rho\} \right) = \mathcal{L}\rho, \quad (2)$$

where $\mathcal{L}$ denotes the Liouvillian superoperator, and the dissipative processes are described by the Lindblad jump operators $L_1 = \sqrt{\gamma_1}|g\rangle\langle e|$ and $L_2 = \sqrt{\gamma_2}|e\rangle\langle r|$. To fully investigate the emergence of the system's non-Hermitian topological property, we perform a full spectral decomposition by solving the Liouvillian eigenvalue equation $\mathcal{L}\hat{\rho}_i = \lambda_i \hat{\rho}_i$, λ_i and $\hat{\rho}_i$ are the eigenvalues and eigenstates of the Liouvillian, respectively. The complex eigenspectrum of the Liouvillian superoperator is shown in Fig. 1(c). The enlarged view represents the eigenvalue λ_{rr} , which governs the dynamical evolution of the density matrix element ρ_{rr} , and its trajectory reveals a “point-gap” topology in the complex plane.

We then characterize the spectral topology for our system [10], where the Liouvillian space $\mathcal{L}(Z)$ depends on the complex parameter $Z = \Delta_r + i\Omega_p^2$. Similar to the general non-Hermitian case, the eigenvalues $\lambda(Z) \in \mathbb{C}$ lie in the complex energy plane. As Z varies along a closed contour C_Z in the parameter space, the C_Z forms a loop that may enclose a finite area in the complex plane. This loop can wind around a reference energy point λ , giving rise to a nontrivial spectral winding number $\mathcal{W}_\lambda$, which is defined as [10,69,70]:

$$\mathcal{W}_\lambda = \frac{1}{2\pi i} \oint_{C_Z} d\vec{Z} \cdot \nabla_Z \ln \det (\mathcal{L}(Z) - \lambda I). \quad (3)$$

It counts the number of times the spectrum winds around the reference eigenvalues λ and I is an identity matrix of the same dimension as $\mathcal{L}$. Here, λ denotes a reference point inside the closed spectral trajectory, ensuring the well-defined $\mathcal{W}_\lambda$ [4,10]. For a single clockwise (CW) encirclement, the winding number yields $\mathcal{W}_\lambda = -1$, while a single counterclockwise (CCW) one gives $\mathcal{W}_\lambda = 1$. This quantized winding number serves as a geometric metric to characterize the evolution of spectral trajectories.

B. Spectral topology on the complex plane

Figure 2(a) shows a numerical simulation of the complex energy eigenvalue λ_{rr} as a function of the dissipation γ_{eff} . As the interaction γ_{eff} increases, the eigenvalue trajectory evolves into distinct loops on the complex plane. To quantitatively probe the topological features, we employ electromagnetically induced transparency (EIT) in a cold atomic ensemble of ^{85}Rb atoms. Using an

acousto-optic modulator, we periodically sweep the probe field intensity and detuning Δ_r with controlled period T_s , tracing closed contours in the (Ω_p^2, Δ_r) parameter space. The measured results are displayed in Figs. 2(b)–2(e), where the trajectory of the eigenenergy is projected onto the parameter space spanned by the frequency ω and transmission loss ξ (see the definition in Appendix B).

The trajectory is traced by varying the argument of the complex parameter $Z = \Delta_r + i\Omega_p^2$. In this framework, we set the frequency $\omega = \Delta_r$, where the two-photon detuning Δ_r represents the energy mismatch between the driving lasers and the Rydberg state. This energy mismatch determines the frequency of the coherent evolution, corresponding to the imaginary parts of the Liouvillian eigenvalues. The observable loss ξ reflects the decay rate of the Rydberg states, which effectively characterizes the real parts of the Liouvillian eigenvalues [71–73]. The combination of ω and ξ effectively reconstructs the complete complex-plane structure of the Liouvillian spectrum qualitatively. Although it does not strictly track specific instantaneous eigenvalues, it efficiently captures the trends of the Liouvillian spectrum.

It can be seen that the system exhibits “point-gap” spectral topology with a winding number $\mathcal{W}_\lambda = 1$, and the reference point is chosen inside the closed spectral trajectory. This quantized metric serves as a robust signature of the spectral topology, maintaining invariant under continuous parameter variations even when the spectral trajectory undergoes geometric distortions, as evidenced in Figs. 2(c)–2(e).

At small Ω_p^2 , dissipation is weak and the system exhibits quasi-Hermitian behavior: the transmission loss remains nearly constant and the spectral path lies close to the real axis, as shown in Fig. 2(b). In this regime, the system is dominated by intrinsic single-particle decay, which provides a near-zero loss ξ . However, as Ω_p^2 increases, Rydberg many-body interactions induce significant dissipation. This enhanced Rydberg decay rate is directly reflected in the observable loss ξ , shifting the dynamics into the non-Hermitian regime. Similar to the theory, the spectral trajectory becomes larger as the interaction increases. Thus, the trajectory expands significantly, highlighting the role of interaction-induced dissipation.

The resulting transmission trajectories reveal loop structures that are directly tied to the spectral winding number. The absence of such loops in the weak-dissipation limit $V\rho_{rr} \rightarrow 0$, where $V \propto C_6/R^6$ denotes the van der Waals interaction between Rydberg atoms, depending on the van der Waals coefficient C_6 and the interatomic distance R . This absence underscores the essential role of strong interactions in breaking Hermitian constraints and enabling topology driven by dissipation. These results illustrate a continuous transition mediated by interaction strength, where the system evolves from a Hermitian-like

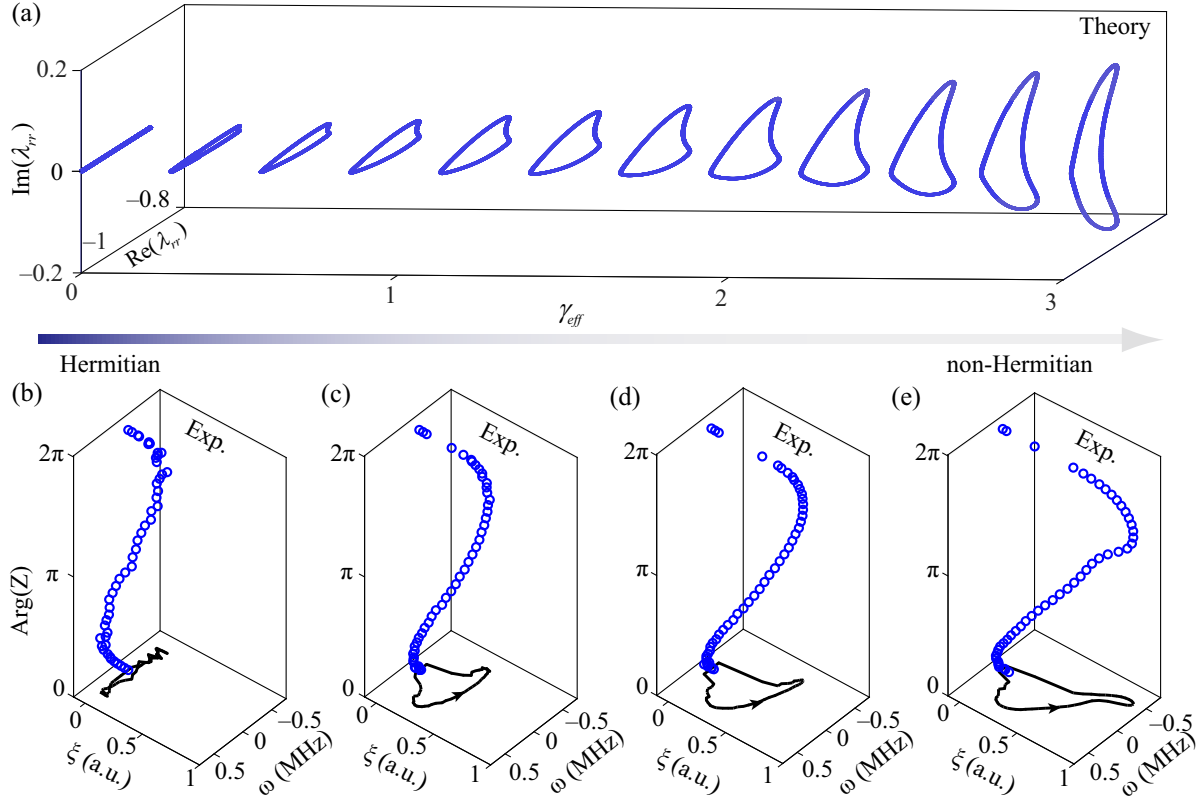


FIG. 2. Non-Hermitian energy spectral topology. (a) Numerical simulation of the energy eigenvalue λ_{rr} of Rydberg steady state versus γ_{eff} on the complex plane, where the parameters are $\{\Omega_c, \Delta_c, \gamma_1, \gamma_2\} = \{4, 0, 0.8, 1.2\}$. The Liouvillian eigenspectrum forms a loop in the complex plane. (b)–(e) The evolution trajectory of Rydberg eigenstate in the complex energy plane is strongly correlated with the variation of probe transmission and the energy level detuning Δ_r in the experiment varies with the parameter $\text{Arg}(Z)$, where $Z = \Delta_r + i\Omega_p^2$ in the scanning time of $T_s = 18 \mu\text{s}$. Panels (b)–(e) display the spectral features for $(\Omega_p/2\pi)^2 \in [0, 1] \text{ MHz}^2$, $(\Omega_p/2\pi)^2 \in [0, 4] \text{ MHz}^2$, $(\Omega_p/2\pi)^2 \in [0, 10] \text{ MHz}^2$, and $(\Omega_p/2\pi)^2 \in [0, 20] \text{ MHz}^2$, respectively.

regime with trivial spectral lines into a non-Hermitian phase with non-zero spectral winding.

C. Twisted topology

In our system, the direction of parameter scanning governs a fundamental transition between dominant quantum processes: from absorption to interaction-driven dissipation. This is vividly captured in Fig. 3(a), where the imaginary part of the coherence, $\text{Im}(\rho_{\text{eg}})$, exhibits a twisted hysteresis between the up-scan and down-scan processes of the probe field Ω_p . The crossing of the transmission spectra reveals a dynamical intersection point: at lower Ω_p , absorption dominates, as evidenced by the larger $\text{Im}(\rho_{\text{eg}})$ during the increasing scan compared to the decreasing scan. In contrast, at higher Ω_p strong Rydberg interactions induce enhanced dissipation, reversing the hysteresis so that the decreasing scan now yields greater $\text{Im}(\rho_{\text{eg}})$. This crossover reflects the competition between radiative absorption and the non-Hermitian dissipation arising from Rydberg many-body interactions. Thus, the resulting topological winding number in the

regimes of absorption-dominated and dissipation-dominated has different signs, as illustrated in Fig. 3(b).

The dynamical nature of this competition is further controlled by the scanning time, which governs the system's dynamical response, memory, and non-adiabaticity. Additionally, T_s determines the accumulation rate of Rydberg atoms, thereby affecting the collective dissipation and non-Hermitian effects. As shown in Figs. 3(c)–3(h), the projection of the spectral trajectories in the (ξ, ω) plane evolves dramatically with the scan period T_s , reflecting the characteristics of a non-static dynamical spectral topology. When T_s is long (e.g., $19 \mu\text{s}$), the system responds adiabatically, and the transmission loop exhibits a well-defined winding structure characteristic of a certain topological phase. As T_s shortens, the increasing non-adiabaticity alters the interplay between absorption and dissipation, leading to growing distortion of the loop. Eventually, at sufficiently short periods (e.g., $1 \mu\text{s}$), the loop becomes twisted and even self-intersecting. This self-intersection reshapes the spectral topology, forming enclosed sub-loops characterized by opposite winding numbers $\mathcal{W}_\lambda$, which signify opposite spectral evolution directions.

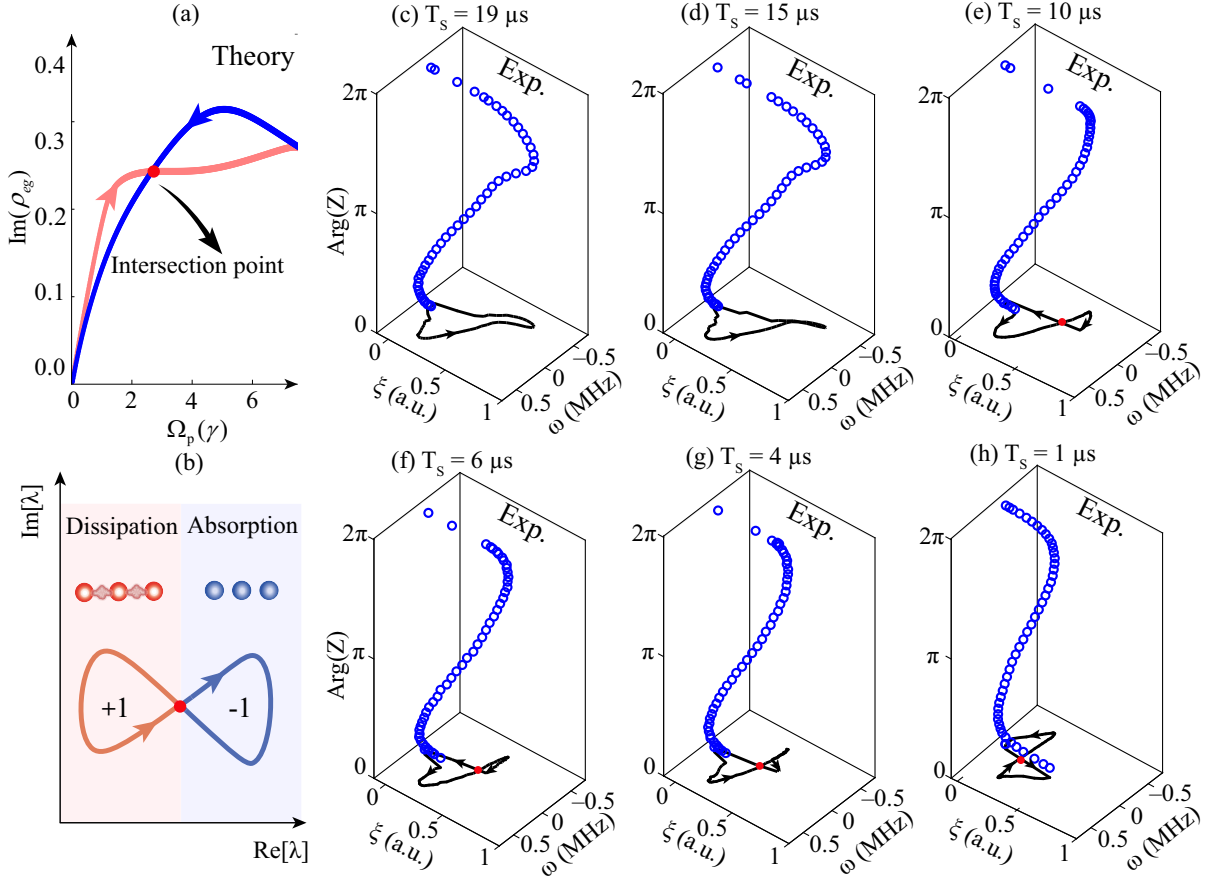


FIG. 3. Twisted topology. (a) The simulated $\text{Im}(\rho_{eg})$ during the up-scan and down-scan processes of Ω_p^2 through Lindblad master equation. (b) Schematic diagram of the different winding numbers corresponding to the absorption- and dissipation-dominated spectra of the system. (c)–(h) The spectral trajectory of the transmitted signal and the energy level detuning Δ_r in the experiment varies with the parameter $\text{Arg}(Z)$. The scanning times are as follows: $T_s = 19 \mu\text{s}$ in (c), $T_s = 15 \mu\text{s}$ in (d), $T_s = 10 \mu\text{s}$ in (e), $T_s = 6 \mu\text{s}$ in (f), $T_s = 4 \mu\text{s}$ in (g) and $T_s = 1 \mu\text{s}$ in (h). The red dots indicate the intersection points of the complex energy spectrum.

This evolution underscores the role of the scanning time as a control parameter for topology: it regulates whether the system can follow the instantaneous eigenstates or is driven into a non-equilibrium regime where dissipation and coherent dynamics compete. The resulting spectral loops are not mere geometric constructs but reflect the accumulation of non-adiabatic phase effects and the non-reciprocal flow of energy and coherence in an open quantum system. Thus, by varying $\text{Arg}(Z)$ and T_s , we directly manipulate the topological structure of the system's response, demonstrating the rich interplay between non-Hermitian topology, many-body interactions, and quantum dynamics out of equilibrium.

D. Scan-direction-dependent nonreciprocity

Figure 4 demonstrates the emergence of reverse spectral topology controlled by the scanning direction of $\text{Arg}(Z)$. As illustrated in Fig. 4(a), two distinct scanning paths are implemented in the complex parameter space defined by Ω_p^2 and Δ_r : a CCW and a CW

trajectory. These directions are not equivalent due to the dissipative, non-conservative nature of the system, which breaks time-reversal symmetry at the level of the effective non-Hermitian Hamiltonian.

Figures 4(b) and 4(c) show the experimentally measured spectral responses under these two scanning directions, in which the topological trajectory in the complex energy plane and its projection (black curve) are mapped. The trajectory in Fig. 4(b), corresponding to CW scanning, exhibits a loop structure with characteristic winding, while the reversed scan in Fig. 4(c) results in a distinct, mirror-like path, highlighting the directional sensitivity of the spectral topology. This demonstrates that the spectral topology is not a static property but is intrinsically tied to the direction of parameter variation, revealing the dynamical and geometry-dependent nature of non-Hermitian energy spectra.

Figure 4(d) directly compares the projections of these spectral trajectories. The clear separation between the two curves underscores the path-dependence of the system's response, revealing a signature of

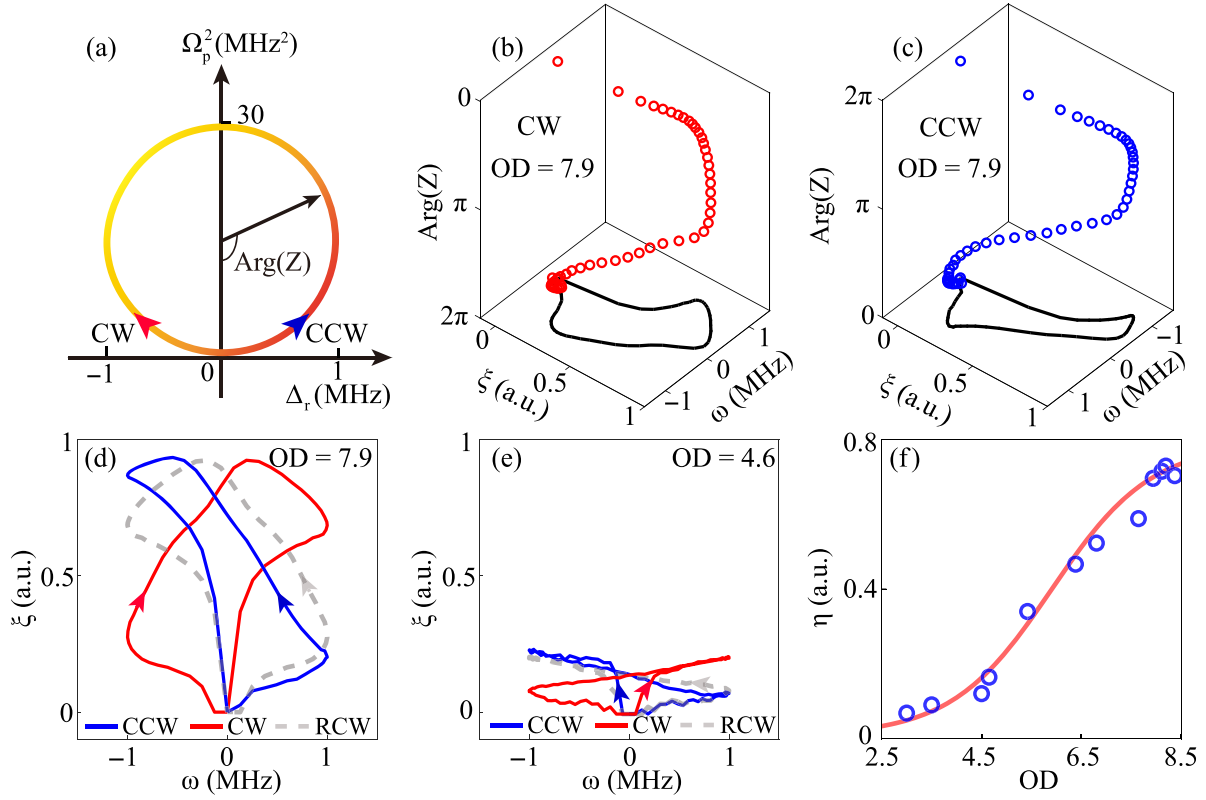


FIG. 4. Scan-direction-dependent nonreciprocity. (a) The clockwise (CW) and counterclockwise (CCW) scan paths of the parameters Ω_p^2 and Δ_r . (b) The spectral topological trajectory and projection (the black line) under the CW scanning path. (c) The spectral topological trajectory and projection (the black line) under the CCW scanning path. (d),(e) The projections of spectra under two different scanning paths on the parameter plane, at the condition of optical depth $OD = 7.9$ (d) and $OD = 4.6$ (e). The gray dash curve denotes the mirror curve of the projected topological trajectory under the CW scanning. (f) The nonreciprocity parameter η as a function of OD , and the red curve is a function fit $y = L/[1 + e^{-k(x-x_0)}]$ ($L = 0.78$, $k = 1.0$, and $x_0 = 5.9$).

scan-direction-dependent nonreciprocity. This behavior arises from the combination of strong Rydberg-mediated dissipation and the adiabatic (or non-adiabatic) driving of system parameters, which together induce this nonreciprocity in the measurement trajectory. These results provide a direct visualization of how dissipation and interactions collaborate to produce directional-sensitive topological phases in open quantum systems.

To quantify the nonreciprocity of this scan-direction-dependent parameter winding, we define the nonreciprocity parameter

$$\eta = \oint_{C_Z} d\vec{Z} |O_{RCW}(Z) - O_{CCW}(Z)|. \quad (4)$$

Here, $O_{RCW}(Z)$ and $O_{CCW}(Z)$ denote the measured observables along the mirror-symmetric clockwise (RCW) and CCW scanning paths at the complex parameter Z . The parameter η quantifies the relative asymmetry between the system's responses under these two scanning directions. The non-zero value indicates a signature of scan-direction-dependent nonreciprocity, as it measures

the system's asymmetry under the reversal of the parameter scan direction. In the experiment, we evaluate an effective nonreciprocity parameter by taking the observable $O(Z)$ as the probe transmission loss ξ .

To quantitatively investigate this nonreciprocity, we vary the optical density (OD) from 3.0 to 8.3. As the OD increases, the separation between the RCW and CCW projected trajectories becomes more distinct. This trend arises because a higher OD involves more atoms in the interaction volume, which enhances the effective Rydberg-mediated interaction-induced dissipation γ_{eff} in the system. The increased dissipation further reinforces the non-Hermitian topological character, resulting in a more clearly defined spectral winding loop, as clearly seen at the two cases of $OD = 7.9$ in Fig. 4(d) and $OD = 4.6$ in Fig. 4(e). This direct relationship is conclusively demonstrated in Fig. 4(f), where the nonreciprocity parameter η increases with OD .

III. CONCLUSION

In this work, we have demonstrated an experiment to study non-Hermitian topological physics in the presence of

strong interactions and dissipation in cold Rydberg atomic gases. Through precise manipulation of system parameters, such as probe intensity, two-photon detuning, and scanning time, we have shown the emergence of spectral loops in the complex energy plane, indicating a transition from Hermitian to non-Hermitian regimes. The observed twisting and self-intersection of these loops under varying scanning times reveal a dynamical topological reconfiguration of the spectral trajectory into interconnected sub-loops featuring opposite winding numbers. Additionally, the asymmetric spectral responses under CW and CCW parameter scans provide direct evidence of the scan-direction-dependent nonreciprocity, an intrinsic characteristic of non-Hermitian dynamical topology. These findings underscore the rich interplay between dissipation, interactions, and topology in open quantum systems, and establish cold Rydberg gases as a platform for studying dynamical topological phenomena beyond the Hermitian framework.

The long-range interactions between Rydberg atoms play a vital role, as they not only enhance many-body correlations but also mediate the dissipative processes that are essential for realizing non-Hermitian topological states. This results in non-static topological properties that evolve dynamically with the scanning process. The interplay between long-range interactions and dissipation enriches the topological landscape, enabling the emergence of phases that are otherwise inaccessible in short-range interacting systems. The observed topology is not merely a static property of the Hamiltonian but emerges from the interplay between coherent dynamics, dissipation, and the measurement process itself, revealing the non-equilibrium nature of topology in open quantum systems. This work opens new avenues for exploring non-equilibrium topological phases and exotic non-Hermitian phenomena, such as interaction-induced exceptional points [68,74,75] and many-body skin effects [76,77], as well as their potential applications in quantum simulation and information processing.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [78].

APPENDIX A: DETAILS OF THE EXPERIMENTAL SETUP

Our investigation into non-Hermitian spectral topology utilizes a cold ensemble of ^{85}Rb atoms confined in a three-dimensional magneto-optic trap. Through optical pumping, the atoms are prepared in the hyperfine ground state $|g\rangle = |5S_{1/2}, F=3\rangle$. To ensure a stable magnetic environment and minimize dephasing, the MOT is enclosed within a double-layer magnetic shield, which suppresses the residual magnetic field to below 10 mGauss. This configuration can avoid the dephasing from the Earth's magnetic field. The atoms are excited to the Rydberg state $|r\rangle = |40D_{5/2}\rangle$ via a two-photon transition. A probe beam ($\omega_p \approx 20 \mu\text{m}$) drives the $|g\rangle \rightarrow |e\rangle = |5P_{3/2}, F=4\rangle$ transition, while a counter-propagating coupling beam ($\omega_c \approx 30 \mu\text{m}$) drives the $|e\rangle \rightarrow |r\rangle$ transition. To ensure the high coherence and frequency stability required for EIT, probe and coupling lasers are actively locked using the Pound-Drever-Hall method. The system's response is characterized by measuring the probe beam's transmission through the atomic ensemble using a photomultiplier tube.

In the experiment, a sine wave and a cosine wave generated by the signal generator (RIGOL DG4062) were applied to the acousto-optic modulator (AOM). Specifically, a cosine wave in the form of $\cos(2\pi t/T_s + \theta_1) + C$ was applied for amplitude modulation on the AOM to control Ω_p^2 , while a cosine wave in the form of $\sin(2\pi t/T_s + \theta_2) + D$ was utilized for frequency modulation of the AOM to control Δ_p . This configuration generates the variable $\text{Arg}(Z)$ in the parameter space, where $Z = \Delta_r + i\Omega_p^2$, as illustrated in Fig. 1(c). Here, T_s denotes the scanning period, θ_1 and θ_2 are the initial phases, C and D are constant offsets. The initial phases θ_1 and θ_2 , determine the rotation direction of the parameter $\text{Arg}(Z)$: a CW rotation corresponds to $\theta_1 = \theta_2$, while a CCW rotation is achieved when $|\theta_2 - \theta_1| = \pi$.

APPENDIX B: LINDBLAD MASTER EQUATION

We simulate transmission and absorption in the Rydberg atomic dissipation system by solving the Lindblad master equation, which is written as

$$\dot{\rho}_{gg} = i\frac{\Omega_p}{2}(\rho_{eg} - \rho_{ge}) + \gamma_1\rho_{ee}, \quad (\text{B1})$$

$$\dot{\rho}_{ee} = i\frac{\Omega_p}{2}(\rho_{ge} - \rho_{eg}) + i\frac{\Omega_c}{2}(\rho_{re} - \rho_{er}) - \gamma_1\rho_{ee} + \gamma_2\rho_{rr}, \quad (\text{B2})$$

$$\dot{\rho}_{rr} = i\frac{\Omega_c}{2}(\rho_{er} - \rho_{re}) - \gamma_2\rho_{rr}, \quad (\text{B3})$$

$$\dot{\rho}_{re} = i\frac{\Omega_c}{2}(\rho_{ee} - \rho_{rr}) - i\frac{\Omega_p}{2}\rho_{rg} + \left(-i\Delta_c - \frac{\gamma_2}{2} - \frac{\gamma_1}{2}\right)\rho_{re}, \quad (\text{B4})$$

$$\dot{\rho}_{gr} = i\frac{\Omega_p}{2}\rho_{er} - i\frac{\Omega_c}{2}\rho_{ge} + \left(i(\Delta_p + \Delta_c) - \frac{\gamma_2}{2}\right)\rho_{gr}, \quad (\text{B5})$$

$$\dot{\rho}_{eg} = i\frac{\Omega_p}{2}(\rho_{gg} - \rho_{ee}) + i\frac{\Omega_c}{2}\rho_{rg} + \left(-i\Delta_p - \frac{\gamma_1}{2}\right)\rho_{eg}, \quad (\text{B6})$$

The remaining equations are given by $\rho_{ge} = \rho_{eg}^*$, $\rho_{rg} = \rho_{gr}^*$, and $\rho_{er} = \rho_{re}^*$. In the mean-field approximation ($\gamma_2 \rightarrow \gamma_{\text{eff}} = \gamma + V\rho_{rr}$), we replace the time-dependent population $\rho_{rr}(t)$ by the term $\dot{\rho}_{rr}(t)$ according to Eq. (B1): $\rho_{rr}(t) = i\Omega_c/2\Gamma_2(\rho_{er}(t) - \rho_{re}(t)) - \dot{\rho}_{rr}(t)/\Gamma_2$. Thus, the slope of Rydberg population $\dot{\rho}_{rr}(t)$ plays a crucial role in influencing the transient behavior of $\rho_{rr}(t)$. By solving the time-dependent Lindblad master equation outlined above, we obtained the matrix element ρ_{eg} . The

theoretical results are presented in Fig. 3(a), where we present the dynamic evolution curve of the system as the Rabi frequency Ω_p varies, with a detailed derivation of the dynamical hysteresis mechanism provided in Ref. [68].

In the experiment, we obtain loss of the probe field $\xi = 1 - I/I_0 = 1 - e^{-k\text{Im}[\chi]l}$, where $\chi = 2m|\mu_{eg}|^2\rho_{eg}/[\epsilon_0\Omega_p]$ is polarizability, l is the distance the probe laser beam travels through the cold atomic gases, k is the wave number of the probe light in vacuum, m is the atomic number density, μ_{eg} is transition dipole moment, and I and I_0 represent the intensity of the input and output of the probe light respectively.

Under the weak probe approximation, $\xi \propto \text{Im}[\rho_{eg}]$ [62,79]. To calculate the absorption analytically, we solve the steady-state master equation, which differs from the dynamical hysteresis process. Applying the steady-state condition ($\dot{\rho}_{ij} = 0$), the weak interaction approximation ($V = 0$), and the experimental condition $\Delta_c = 0$ we obtain the relations for ρ_{eg} in terms of ρ_{rr} :

$$\rho_{eg} \simeq \frac{\rho_{rr}}{\Omega_p} \left[\frac{i\gamma_1(\gamma_2 + 2i\Delta_p)}{\gamma_1 + \gamma_2} + \frac{-i\gamma_2(\gamma_1 + \gamma_2)(\gamma_2^2 + 4\Delta_p^2) + \gamma_2(-i\gamma_2 + 2\Delta_p)\Omega_p^2}{(\gamma_1 + \gamma_2)\Omega_c^2} \right. \\ \left. + \frac{i[\gamma_2^2 + \gamma_1(\gamma_2 + 2i\Delta_p) + 4\Delta_p^2 + \Omega_p^2][\gamma_1^2\gamma_2(\gamma_2 - 2i\Delta_p) + \gamma_2^2\Omega_c^2 + \gamma_1(\gamma_2 - 2i\Delta_p)(\gamma_2^2 + \Omega_c^2) + \gamma_1\gamma_2\Omega_p^2]}{\Omega_c^2[(\gamma_1 + \gamma_2)(\gamma_1\gamma_2 + \Omega_c^2) + \gamma_1\Omega_p^2]} \right]. \quad (\text{B7})$$

Under the weak excitation approximation ($\Omega_p \ll \Omega_c$), we neglect higher-order terms in Ω_p , and we obtain

$$\rho_{eg} \simeq \frac{\rho_{rr}}{\Omega_p} \frac{\gamma_1(\gamma_2 + 2i\Delta_p)[(\gamma_2 - 2i\Delta_p)(i\gamma_1 + 2\Delta_p) + i\Omega_c^2][\gamma_2(\gamma_1 + \gamma_2) + \Omega_c^2]}{(\gamma_1 + \gamma_2)\Omega_c^2(\gamma_1\gamma_2 + \Omega_c^2)}. \quad (\text{B8})$$

It is evident that ρ_{ge} is proportional to ρ_{rr} . As evidenced by Fig. 2(a), the system exhibits near-zero transmission loss in the weak-interaction regime, confirming that intrinsic linear decay is negligible. Therefore, the observed macroscopic loss ξ is dominated by interactions between Rydberg atoms, which reflects the real part of the Liouvillian spectrum of ρ_{rr} . The interaction-induced

dissipation γ_{eff} introduces an effective non-Hermiticity into the system, which ultimately manifests as the observed probe transmission loss. Additionally, based on the steady-state solution of the Lindblad master equation, the population of the Rydberg state ρ_{rr} in the weak-probe regime can be analytically expressed as

$$\rho_{rr} \simeq \frac{(\gamma_1 + \gamma_2)\Omega_c^2(\gamma_1\gamma_2 + \Omega_c^2)}{\gamma_1[\gamma_2(\gamma_1 + \gamma_2) + \Omega_c^2][(\gamma_1^2 + 4\Delta_p^2)(\gamma_2^2 + 4\Delta_p^2) + 2(\gamma_1\gamma_2 - 4\Delta_p^2)\Omega_c^2 + \Omega_c^4]}\Omega_p^2 + O(\Omega_p^4). \quad (\text{B9})$$

This result allows us to express the population as $\rho_{rr} \propto \Omega_p^2$. This relationship justifies our construction of the complex parameter $Z = \Delta_r + i\Omega_p^2$, which provides a direct mapping from experimental observables to the Liouvillian spectral topology.

- [1] S.-Y. Xu, Y. Xia, L. Wray, S. Jia, F. Meier, J. Dil, J. Osterwalder, B. Slomski, A. Bansil, H. Lin *et al.*, *Topological phase transition and texture inversion in a tunable topological insulator*, *Science* **332**, 560 (2011).
- [2] M. Caraglio, C. Micheletti, and E. Orlandini, *Stretching response of knotted and unknotted polymer chains*, *Phys. Rev. Lett.* **115**, 188301 (2015).
- [3] K. Kawabata, S. Higashikawa, Z. Gong, Y. Ashida, and M. Ueda, *Topological unification of time-reversal and particle-hole symmetries in non-Hermitian physics*, *Nat. Commun.* **10**, 297 (2019).
- [4] K. Kawabata, K. Shiozaki, M. Ueda, and M. Sato, *Symmetry and topology in non-Hermitian physics*, *Phys. Rev. X* **9**, 041015 (2019).
- [5] W. Ji and X.-G. Wen, *Categorical symmetry and non-invertible anomaly in symmetry-breaking and topological phase transitions*, *Phys. Rev. Res.* **2**, 033417 (2020).
- [6] S. Xia, D. Kaltsas, D. Song, I. Komis, J. Xu, A. Szameit, H. Buljan, K. G. Makris, and Z. Chen, *Nonlinear tuning of pt symmetry and non-Hermitian topological states*, *Science* **372**, 72 (2021).
- [7] H. Qin, Z. Zhang, J. Wang, and R. Fleury, *Topological hysteretic winding for temporal anti-lasing*, *Nat. Commun.* **16**, 6189 (2025).
- [8] S.-Y. Xu, C. Liu, N. Alidoust, M. Neupane, D. Qian, I. Belopolski, J. Denlinger, Y. Wang, H. Lin, L. a. Wray *et al.*, *Observation of a topological crystalline insulator phase and topological phase transition in $Pb_{1-x}Sn_xTe$* , *Nat. Commun.* **3**, 1192 (2012).
- [9] C. Heide, Y. Kobayashi, D. R. Baykusheva, D. Jain, J. A. Sobota, M. Hashimoto, P. S. Kirchmann, S. Oh, T. F. Heinz, D. A. Reis *et al.*, *Probing topological phase transitions using high-harmonic generation*, *Nat. Photonics* **16**, 620 (2022).
- [10] K. Ding, C. Fang, and G. Ma, *Non-Hermitian topology and exceptional-point geometries*, *Nat. Rev. Phys.* **4**, 745 (2022).
- [11] Z. Lin, L. Zhang, X. Long, Y.-A. Fan, Y. Li, K. Tang, J. Li, X. Nie, T. Xin, X.-J. Liu *et al.*, *Experimental quantum simulation of non-Hermitian dynamical topological states using stochastic schrödinger equation*, *npj Quantum Inf.* **8**, 77 (2022).
- [12] S.-L. Zhu, Z.-D. Wang, Y.-H. Chan, and L.-M. Duan, *Topological bose-mott insulators in a one-dimensional optical superlattice*, *Phys. Rev. Lett.* **110**, 075303 (2013).
- [13] S. Mittal, S. Ganeshan, J. Fan, A. Vaezi, and M. Hafezi, *Measurement of topological invariants in a 2D photonic system*, *Nat. Photonics* **10**, 180 (2016).
- [14] J. Y. Lee, J. Ahn, H. Zhou, and A. Vishwanath, *Topological correspondence between Hermitian and non-Hermitian systems: Anomalous dynamics*, *Phys. Rev. Lett.* **123**, 206404 (2019).
- [15] K. Wang, T. Li, L. Xiao, Y. Han, W. Yi, and P. Xue, *Detecting non-bloch topological invariants in quantum dynamics*, *Phys. Rev. Lett.* **127**, 270602 (2021).
- [16] G. Li, L. Wang, R. Ye, Y. Zheng, D.-W. Wang, X.-J. Liu, A. Dutt, L. Yuan, and X. Chen, *Direct extraction of topological Zak phase with the synthetic dimension*, *Light: Sci. Appl.* **12**, 81 (2023).
- [17] X. Liu, J. Wang, R. Mao, H. Hu, S.-Y. Zhu, X. Xu, H. Cai, and D.-W. Wang, *Zak phase induced topological non-reciprocity*, *Phys. Rev. Lett.* **134**, 193602 (2025).
- [18] L. Xiao, X. Zhan, Z. Bian, K. Wang, X. Zhang, X. Wang, J. Li, K. Mochizuki, D. Kim, N. Kawakami *et al.*, *Observation of topological edge states in parity-time-symmetric quantum walks*, *Nat. Phys.* **13**, 1117 (2017).
- [19] S. de Léséleuc, V. Lienhard, P. Scholl, D. Barredo, S. Weber, N. Lang, H. P. Büchler, T. Lahaye, and A. Browaeys, *Observation of a symmetry-protected topological phase of interacting bosons with Rydberg atoms*, *Science* **365**, 775 (2019).
- [20] H. Cai and D.-W. Wang, *Topological phases of quantized light*, *Nat. Sci. Rev.* **8**, nwaa196 (2021).
- [21] M. Yang, H.-Q. Zhang, Y.-W. Liao, Z.-H. Liu, Z.-W. Zhou, X.-X. Zhou, J.-S. Xu, Y.-J. Han, C.-F. Li, and G.-C. Guo, *Topological band structure via twisted photons in a degenerate cavity*, *Nat. Commun.* **13**, 2040 (2022).
- [22] D. Hao, L. Wang, X. Lu, X. Cao, S. Jia, Y. Hu, and Y. Xiao, *Topological atomic spin wave lattices by dissipative couplings*, *Phys. Rev. Lett.* **130**, 153602 (2023).
- [23] M. Yang, Y.-W. Liao, H.-Q. Zhang, Y. Li, Z.-H. Hao, Z.-W. Zhou, X.-W. Luo, J.-S. Xu, C.-F. Li, and G.-C. Guo, *Observing half-integer topological winding numbers in non-Hermitian synthetic lattices*, *Light Sci. Appl.* **14**, 225 (2025).
- [24] Z. Yue, Y.-F. Mao, X. Liang, Z.-X. Hua, P. Ge, Y.-X. Chao, K. Li, C. Jia, M. K. Tey, Y. Xu, and L. You, *Observation of average topological phase in disordered Rydberg atom array*, *arXiv:2505.06286*.
- [25] T. Insulator, *Topological phase transition and texture inversion in a tunable*, *Science* **1201607**, 332 (2011).
- [26] J. Zhang, C.-Z. Chang, P. Tang, Z. Zhang, X. Feng, K. Li, L.-L. Wang, X. Chen, C. Liu, W. Duan *et al.*, *Topology-driven magnetic quantum phase transition in topological insulators*, *Science* **339**, 1582 (2013).
- [27] Q. Lin, T. Li, L. Xiao, K. Wang, W. Yi, and P. Xue, *Observation of non-Hermitian topological Anderson insulator in quantum dynamics*, *Nat. Commun.* **13**, 3229 (2022).
- [28] G. Zhang *et al.*, *Majorana fermions and a topological phase transition in semiconductor-superconductor heterostructures*, *Phys. Rev. Lett.* **105**, 10 (2013).
- [29] P. Zhang, K. Yaji, T. Hashimoto, Y. Ota, T. Kondo, K. Okazaki, Z. Wang, J. Wen, G. D. Gu, H. Ding *et al.*, *Observation of topological superconductivity on the surface of an iron-based superconductor*, *Science* **360**, 182 (2018).
- [30] S. Vajna and Bázs Dóra, *Topological classification of dynamical phase transitions*, *Phys. Rev. B* **91**, 155127 (2015).
- [31] M. Heyl and J. C. Budich, *Dynamical topological quantum phase transitions for mixed states*, *Phys. Rev. B* **96**, 180304 (2017).
- [32] Z. Gong, Y. Ashida, K. Kawabata, K. Takasan, S. Higashikawa, and M. Ueda, *Topological phases of non-Hermitian systems*, *Phys. Rev. X* **8**, 031079 (2018).
- [33] S. Longhi, *Topological phase transition in non-Hermitian quasicrystals*, *Phys. Rev. Lett.* **122**, 237601 (2019).
- [34] Q. Lin, T. Li, L. Xiao, K. Wang, W. Yi, and P. Xue, *Topological phase transitions and mobility edges in*

- non-Hermitian quasicrystals*, *Phys. Rev. Lett.* **129**, 113601 (2022).
- [35] T. Dai, Y. Ao, J. Mao, Y. Yang, Y. Zheng, C. Zhai, Y. Li, J. Yuan, B. Tang, Z. Li *et al.*, *Non-Hermitian topological phase transitions controlled by nonlinearity*, *Nat. Phys.* **20**, 101 (2024).
- [36] Z. Li, L.-W. Wang, X. Wang, Z.-K. Lin, G. Ma, and J.-H. Jiang, *Observation of dynamic non-Hermitian skin effects*, *Nat. Commun.* **15**, 6544 (2024).
- [37] N. Fläschner, D. Vogel, M. Tarnowski, B. S. Rem, D.-S. Lühmann, M. Heyl, J. C. Budich, L. Mathey, K. Sengstock, and C. Weitenberg, *Observation of dynamical vortices after quenches in a system with topology*, *Nat. Phys.* **14**, 265 (2018).
- [38] M. Heyl, *Dynamical quantum phase transitions: A review*, *Rep. Prog. Phys.* **81**, 054001 (2018).
- [39] R. J. Valencia-Tortora, N. Pancotti, M. Fleischhauer, H. Bernien, and J. Marino, *Rydberg platform for nonergodic chiral quantum dynamics*, *Phys. Rev. Lett.* **132**, 223201 (2024).
- [40] H.-X. Xiao, W. Jiang, P. Qian, H. Li, Z. Li, H. Shen, and B. Chen, *Dynamical topological quantum phase transitions in high-order topological systems*, *Phys. Rev. B* **110**, 064306 (2024).
- [41] M. Saffman, T. G. Walker, and K. Mølmer, *Quantum information with Rydberg atoms*, *Rev. Mod. Phys.* **82**, 2313 (2010).
- [42] C. S. Adams, J. D. Pritchard, and J. P. Shaffer, *Rydberg atom quantum technologies*, *J. Phys. B* **53**, 012002 (2019).
- [43] A. Browaeys and T. Lahaye, *Many-body physics with individually controlled Rydberg atoms*, *Nat. Phys.* **16**, 132 (2020).
- [44] R. Samajdar, W. W. Ho, H. Pichler, M. D. Lukin, and S. Sachdev, *Quantum phases of Rydberg atoms on a kagome lattice*, *Proc. Natl. Acad. Sci. U.S.A.* **118**, e2015785118 (2021).
- [45] X. Li and S. D. Sarma, *Exotic topological density waves in cold atomic Rydberg-dressed fermions*, *Nat. Commun.* **6**, 7137 (2015).
- [46] K. Li, J.-H. Wang, Y.-B. Yang, and Y. Xu, *Symmetry-protected topological phases in a Rydberg glass*, *Phys. Rev. Lett.* **127**, 263004 (2021).
- [47] R. Verresen, M. D. Lukin, and A. Vishwanath, *Prediction of toric code topological order from Rydberg blockade*, *Phys. Rev. X* **11**, 031005 (2021).
- [48] S. K. Kanungo, J. D. Whalen, Y. Lu, M. Yuan, S. Dasgupta, F. B. Dunning, K. R. A. Hazzard, and T. C. Killian, *Realizing topological edge states with Rydberg-atom synthetic dimensions*, *Nat. Commun.* **13**, 972 (2022).
- [49] T. E. Lee, H. Häffner, and M. C. Cross, *Collective quantum jumps of Rydberg atoms*, *Phys. Rev. Lett.* **108**, 023602 (2012).
- [50] C. Carr, R. Ritter, C. Wade, C. S. Adams, and K. J. Weatherill, *Nonequilibrium phase transition in a dilute Rydberg ensemble*, *Phys. Rev. Lett.* **111**, 113901 (2013).
- [51] H. Schempp, G. Günter, M. Robert-de Saint-Vincent, C. S. Hofmann, D. Breyel, A. Komnik, D. W. Schönleber, M. Gärttner, J. Evers, S. Whitlock, and M. Weidemüller, *Full counting statistics of laser excited Rydberg aggregates in a one-dimensional geometry*, *Phys. Rev. Lett.* **112**, 013002 (2014).
- [52] M. Marcuzzi, E. Levi, S. Diehl, J. P. Garrahan, and I. Lesanovsky, *Universal nonequilibrium properties of dissipative Rydberg gases*, *Phys. Rev. Lett.* **113**, 210401 (2014).
- [53] I. Lesanovsky and J. P. Garrahan, *Out-of-equilibrium structures in strongly interacting Rydberg gases with dissipation*, *Phys. Rev. A* **90**, 011603 (2014).
- [54] A. Urvoy, F. Ripka, I. Lesanovsky, D. Booth, J. P. Shaffer, T. Pfau, and R. Löw, *Strongly correlated growth of Rydberg aggregates in a vapor cell*, *Phys. Rev. Lett.* **114**, 203002 (2015).
- [55] F. M. Gambetta, F. Carollo, M. Marcuzzi, J. P. Garrahan, and I. Lesanovsky, *Discrete time crystals in the absence of manifest symmetries or disorder in open quantum systems*, *Phys. Rev. Lett.* **122**, 015701 (2019).
- [56] D.-S. Ding, Z.-K. Liu, B.-S. Shi, G.-C. Guo, K. Mølmer, and C. S. Adams, *Enhanced metrology at the critical point of a many-body Rydberg atomic system*, *Nat. Phys.* **18**, 1447 (2022).
- [57] S. Helmrich, A. Arias, G. Lochead, T. M. Wintermantel, M. Buchhold, S. Diehl, and S. Whitlock, *Signatures of self-organized criticality in an ultracold atomic gas*, *Nature (London)* **577**, 481 (2020).
- [58] D.-S. Ding, H. Busche, B.-S. Shi, G.-C. Guo, and C. S. Adams, *Phase diagram of non-equilibrium phase transition in a strongly-interacting Rydberg atom vapour*, *Phys. Rev. X* **10**, 021023 (2020).
- [59] T. M. Wintermantel, Y. Wang, G. Lochead, S. Shevate, G. K. Brennen, and S. Whitlock, *Unitary and nonunitary quantum cellular automata with Rydberg arrays*, *Phys. Rev. Lett.* **124**, 070503 (2020).
- [60] K. Klocke, T. Wintermantel, G. Lochead, S. Whitlock, and M. Buchhold, *Hydrodynamic stabilization of self-organized criticality in a driven Rydberg gas*, *Phys. Rev. Lett.* **126**, 123401 (2021).
- [61] K. Wadenpfuhl and C. S. Adams, *Emergence of synchronization in a driven-dissipative hot Rydberg vapor*, *Phys. Rev. Lett.* **131**, 143002 (2023).
- [62] D. Ding, Z. Bai, Z. Liu, B. Shi, G. Guo, W. Li, and C. S. Adams, *Ergodicity breaking from Rydberg clusters in a driven-dissipative many-body system*, *Sci. Adv.* **10**, ead15893 (2024).
- [63] X. Wu, Z. Wang, F. Yang, R. Gao, C. Liang, M. K. Tey, X. Li, T. Pohl, and L. You, *Dissipative time crystal in a strongly interacting Rydberg gas*, *Nat. Phys.* **20**, 1389 (2024).
- [64] B. Liu, L.-H. Zhang, Q.-F. Wang, Y. Ma, T.-Y. Han, J. Zhang, Z.-Y. Zhang, S.-Y. Shao, Q. Li, H.-C. Chen *et al.*, *Higher-order and fractional discrete time crystals in floquet-driven Rydberg atoms*, *Nat. Commun.* **15**, 9730 (2024).
- [65] B. Liu, L.-H. Zhang, Y. Ma, T.-Y. Han, Q.-F. Wang, J. Zhang, Z.-Y. Zhang, S.-Y. Shao, Q. Li, H.-C. Chen *et al.*, *Quantum synchronization at the critical point of floquet driven Rydberg atoms*, *arXiv:2404.12180*.
- [66] B. Liu, L.-H. Zhang, Y. Ma, Q.-F. Wang, T.-Y. Han, J. Zhang, Z.-Y. Zhang, S.-Y. Shao, Q. Li, H.-C. Chen, G.-C. Guo, D.-S. Ding, and B.-S. Shi, *Bifurcation of time crystals*

- in driven and dissipative Rydberg atomic gas*, *Nat. Commun.* **16**, 1419 (2025).
- [67] P. Delplace, T. Yoshida, and Y. Hatsugai, *Symmetry-protected multifold exceptional points and their topological characterization*, *Phys. Rev. Lett.* **127**, 186602 (2021).
- [68] J. Zhang, E.-Z. Li, Y.-J. Wang, B. Liu, L.-H. Zhang, Z.-Y. Zhang, S.-Y. Shao, Q. Li, H.-C. Chen, Y. Ma *et al.*, *Exceptional point and hysteresis trajectories in cold Rydberg atomic gases*, *Nat. Commun.* **16**, 3511 (2025).
- [69] J. C. Budich and E. J. Bergholtz, *Non-Hermitian topological sensors*, *Phys. Rev. Lett.* **125**, 180403 (2020).
- [70] K. Wang, A. Dutt, K. Y. Yang, C. C. Wojcik, J. Vučković, and S. Fan, *Generating arbitrary topological windings of a non-Hermitian band*, *Science* **371**, 1240 (2021).
- [71] R. W. Boyd, A. L. Gaeta, and E. Giese, *Nonlinear optics*, in *Springer Handbook of Atomic, Molecular, and Optical Physics* (Springer, Cham, 2008), pp. 1097–1110, https://link.springer.com/chapter/10.1007/978-3-030-73893-8_76.
- [72] K. Sun and W. Yi, *Encircling the Liouvillian exceptional points: A brief review*, *AAPPS Bull.* **34**, 22 (2024).
- [73] F. Minganti, A. Biella, N. Bartolo, and C. Ciuti, *Spectral theory of Liouvillians for dissipative phase transitions*, *Phys. Rev. A* **98**, 042118 (2018).
- [74] T. Yoshida and Y. Hatsugai, *Fate of exceptional points under interactions: Reduction of topological classifications*, *Phys. Rev. B* **107**, 075118 (2023).
- [75] M. Kato and T. Yoshida, *Interaction-enabled two- and three-fold exceptional points*, [arXiv:2602.14056](https://arxiv.org/abs/2602.14056).
- [76] S.-B. Zhang, M. M. Denner, Táš Bzdušek, M. A. Sentef, and T. Neupert, *Symmetry breaking and spectral structure of the interacting Hatano-Nelson model*, *Phys. Rev. B* **106**, L121102 (2022).
- [77] K. Kawabata, K. Shiozaki, and S. Ryu, *Many-body topology of non-Hermitian systems*, *Phys. Rev. B* **105**, 165137 (2022).
- [78] [10.5281/zenodo.19677052](https://zenodo.org/record/19677052).
- [79] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, *Electromagnetically induced transparency: Optics in coherent media*, *Rev. Mod. Phys.* **77**, 633 (2005).