

Entanglement-Assisted Multiparameter Estimation with a Solid-State Quantum Sensor

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Quantum multiparameter estimation promises to extend quantum advantage to the simultaneous high-precision measurements of multiple physical quantities. However, realizing this capability in practical quantum sensors under realistic conditions remains challenging due to intrinsic system imperfections. Here, we experimentally demonstrate multiparameter estimation using a nitrogen-vacancy (NV) center in diamond, a widely adopted solid-state quantum sensor. Leveraging electronic-nuclear spin entanglement and optimized Bell-state measurement at room temperature, we simultaneously estimate the amplitude, detuning, and phase of a microwave drive from a single measurement sequence. Despite practical constraints, our results achieve linear sensitivity scaling for all parameters with respect to interrogation time. This work bridges the gap between foundational quantum estimation theory and real-world quantum sensing, opening pathways toward enhanced multiparameter quantum sensors suitable for diverse scientific and technological applications.

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I. INTRODUCTION

Quantum sensors have found a broad spectrum of applications ranging from biological measurements to investigations of condensed-matter phenomena [1–4]. While the majority of these applications have focused on the estimation of a single parameter, there is a growing interest in quantum multiparameter estimation [5–26], reflecting both the fundamental challenges linked to noncommutativity

and the practical need to fully characterize a complex system or environment by simultaneously measuring multiple physical parameters. By harnessing quantum resources such as entanglement, quantum multiparameter estimation can enable the simultaneous determination of multiple parameters without incurring significant trade-offs in precision. This capability is particularly powerful in real-world applications, where reducing the number of sensors or measurement protocols translates into savings in time and resources. Despite proof-of-principle experiments conducted using photons [16–25] and superconducting qubits [26], a systematic study of the impact of practical experimental limitations is still lacking. In particular, achieving quantum multiparameter estimation in realistic settings—using widely adopted quantum sensors and operating under practical experimental constraints—is crucial for translating theoretical advantages of quantum metrology into tangible benefits for sensing technologies.

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In this work, we demonstrate quantum multiparameter estimation using a solid-state quantum sensor, specifically the nitrogen-vacancy (NV) center in diamond [Fig. 1(a)]. Previous sensing studies using NV centers have demonstrated measurements of DC and AC magnetic-field vector components, as well as the amplitude, frequency, and phase of AC magnetic fields [27–35]. NV centers have also been used to detect electric fields, temperature, and pressure [36–40]. However, all of these demonstrations relied on combinations of separate measurement sequences designed for single-parameter estimation. Consequently, none were based on quantum multiparameter estimation, where multiple parameters are simultaneously extracted from a single measurement sequence by analyzing the resulting probability distribution. In our implementation, we introduce a three-parameter sensing protocol leveraging Bell states within a two-qubit system composed of a sensor qubit and an ancilla qubit [Fig. 1(b)]. This configuration serves as a paradigmatic model for quantum multiparameter estimation, where the sensor qubit interacts directly with the vector field $\vec{B}$, while the ancilla

qubit remains decoupled from it. Such multiparameter estimation schemes have garnered significant recent interest [5,15,24,26]. In previous experimental efforts employing photons [24] and superconducting qubits [26], the interactions with target fields were simulated through decomposed gate sequences. In contrast, our experiment directly estimates three physically meaningful parameters from a real microwave field—its amplitude, frequency, and phase.

II. SYSTEM MODEL AND SENSING FRAMEWORK

A. Hamiltonian of the NV center

We employ the electronic-nuclear spin system of the ^{14}NV center [41,42]. Its Hamiltonian is given by $\tilde{H}_{SI} = DS_z^2 + \vec{B}(t) \cdot (\gamma_e \vec{S} + \gamma_n \vec{I}) + QI_z^2 + AS_z I_z$, where $D = (2\pi) \times 2.87$ GHz is the zero-field splitting, $Q = -(2\pi) \times 4.95$ MHz is the nuclear quadrupole moment, $\gamma_e \approx (2\pi) \times 2.8$ MHz/G, $\gamma_n \approx -(2\pi) \times 0.31$ kHz/G denote the gyromagnetic ratios of the electronic and

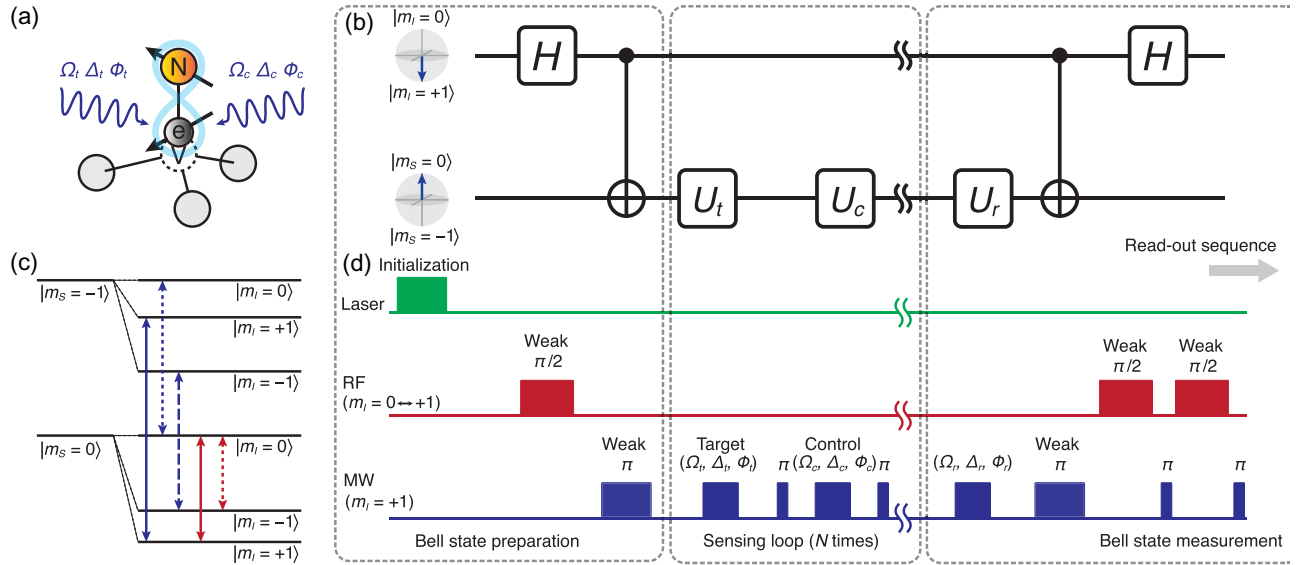


FIG. 1. Principles of multiparameter estimation based on a single NV center in diamond. (a) A single NV center is used to estimate the three target parameters of the microwave field. The electronic spin is used as a sensor qubit, while the nitrogen nuclear spin serves as an ancilla qubit. Entanglement between the two qubits and quantum control via microwave fields are used to enhance the sensitivity. (b) Quantum circuit diagram of the experimental demonstration. It consists of three parts: Bell-state preparation, sensing loop, and Bell-state measurement. (c) Relevant energy levels of the NV center. (d) Optical, radio-frequency (RF), and microwave (MW) pulse sequence used for implementing the quantum circuit shown in (b). The RF field drives the transition between $m_I = 0$ and $m_I = +1$ in the $m_S = 0$ manifold. The microwave field drives the transition between $m_S = 0$ and $m_S = -1$ in the $m_I = +1$ manifold. To prepare the Bell state, the system is first initialized to the $m_S = 0, m_I = +1$ state with a long laser pulse. A $\pi/2$ pulse is then applied to the nuclear spin, followed by a selective (weak) π pulse on the electronic spin. The system is subjected to the target microwave field and a control microwave field, which is flanked by two π pulses to decouple the interaction between the sensor and ancilla qubits. This sensing loop is repeated N times. Subsequently, a microwave pulse for the rotation of the measurement basis, a selective microwave π pulse and a nonselective RF $\pi/2$ pulse are applied. To implement a nonselective RF $\pi/2$ gate on the nuclear spin we embed a nonselective microwave π pulse into two consecutive RF $\pi/2$ pulses. Although omitted from the figure, a selective microwave π pulse is applied at the end to switch the basis.

nuclear spins, $A = -(2\pi) \times 2.16$ MHz is the hyperfine coupling constant, $\vec{S}$ and $\vec{I}$ are the spin-1 operators of the electronic and nuclear spins, respectively. The total magnetic field $\vec{B}(t)$ includes a static component $B_z^0 = 357$ G along the NV axis and AC control fields and the target field. We define two qubits within the subspace spanned by the four levels $\{|m_S, m_I\rangle = |0, +1\rangle, |0, 0\rangle, |-1, +1\rangle, |-1, 0\rangle\}$, while all six levels shown in Fig. 1(c) are used during the Bell-state measurement. At room temperature and under this moderate static field, the readout is restricted to ensemble-averaged fluorescence detection of the electronic spin (i.e., no single-shot readout). We employ microwave fields to control the electronic spin and radio-frequency (RF) fields to drive the nuclear spin; by adjusting the amplitude, frequency, and phase of these fields, we implement the requisite gate operations [Fig. 1(d)]. To analyze the system's dynamics, we move to the rotating frame defined by $H_0 = (D - \gamma_e B_z^0 - A/2)S_z + (Q + \gamma_n B_z^0)I_z$. Expressing the spin-1 operators in terms of effective spin-1/2 operators yields the two-qubit interaction Hamiltonian, $H_{\text{int}} = A(-\sigma_z^e + \sigma_z^n - \sigma_z^e \sigma_z^n)/4$.

B. Three-parameter sensing

To demonstrate multiparameter sensing, we set out to estimate three parameters $\theta = (\Omega_t, \Delta_t, \Phi_t)$ associated with a near-resonant microwave field coupled to the electronic spin: $\Omega_t \cos((\Omega_{\text{mw}}^0 + \Delta_t)T + \Phi_t)\sigma_x^e$, with $\Omega_{\text{mw}}^0 = D - \gamma_e B_z^0 - A/2$. Here, we assume that the target field can be switched on and off at will in order to enable the sequential control described below. This assumption is particularly relevant to condensed-matter experiments that probe the immediate response of materials to externally applied fields, such as spin-wave excitations [43–45]. While the field is time dependent in the frame set by H_0 , we can express it in an appropriate interaction picture as $H(\Omega, \Delta, \Phi) = \vec{B} \cdot \vec{\sigma} = \frac{\Delta}{2}\sigma_z^e + (\Omega/2)(\cos(\Phi)\sigma_x^e - \sin(\Phi)\sigma_y^e)$, where Ω is the Rabi frequency, Δ is the detuning, and Φ is the phase of the microwave. The system evolution under the target field is then described by $U_t = \exp(+i\Delta_t\sigma_z^e T/2) \exp(-i(H(\Omega_t, \Delta_t, \Phi_t) + H_{\text{int}})T)$. Unlike the ideal unitary evolution acting only on the electronic spin, the presence of hyperfine interaction in H_{int} induces more complex two-qubit dynamics, which we address in a subsequent section.

The route to achieving high-sensitivity multiparameter sensing includes (i) preparing an optimal initial state, (ii) choosing an optimal measurement, and (iii) tailoring the time evolution to optimally extract all three parameters. In the ideal case to estimate the vector field $\vec{B}$, it is known from the quantum Fisher information (QFI) arguments that the Bell states and the Bell state measurement are optimal probe state and measurement, respectively, and that one can achieve linear scaling for all three parameters in the interrogation time T for small fields $|\vec{B}| \rightarrow 0$, $\delta B_i \propto 1/T$

for $i \in \{x, y, z\}$ [15,24]. Achieving such scaling is nontrivial, both in the context of multiparameter estimation and even in single-parameter estimation when the Hamiltonian is not in the multiplicative form of the parameter $H(x) = x \text{ G}$ [46]. Moreover, in realistic solid-state quantum sensors based on spin systems such as the one employed in our experiment, several nonidealities arise: (i) the electronic and nuclear spins are always coupled via the hyperfine term $(A/4)\sigma_z^e \sigma_z^n$; (ii) only ensemble-averaged fluorescence from the electronic spin is detected, precluding single-shot two-qubit readout; and (iii) measurements are ultimately photon-shot-noise limited. These significant differences from an ideal system make it a highly nontrivial task to find optimal sensing strategies for practical implementation. In the following, we carefully analyze these effects and develop optimal control and Bell-state measurement protocols to recover near-ideal performance.

C. Sequential control scheme

To achieve linear sensitivity scaling for all three parameters, we build upon the sequential control scheme introduced in Ref. [15]. By applying the control gate $U_c \approx U_t^\dagger$ immediately after the target gate U_t and repeating this sequence N times, an effective bias field can be introduced, allowing detection in regions where the target magnetic field is small, which is equivalent to the condition $|\vec{B}| \rightarrow 0$ in the ideal vector field model.

To implement $U_t^\dagger$, we need to invert the sign of the total Hamiltonian $H_{\text{tot}} = H(\Omega_t, \Delta_t, \Phi_t) + H_{\text{int}}$. Although the first term can be easily inverted by simply adjusting (Ω, Δ, Φ) , the always-on ancilla coupling H_{int} cannot simply be switched off. The effect of the σ_z^n term in H_{int} can be easily canceled by changing the phase of the second $\pi/2$ pulse applied to the nuclear spin [47]. Furthermore, we address the terms $\sigma_z^e \sigma_z^n$ and σ_z^e by introducing a dynamical decoupling approach. The idea is to apply spin-flip π pulses to the electronic spin to effectively reverse the sign of the interaction term $\sigma_z^e \sigma_z^n$, leveraging the identity $\sigma_x \sigma_z \sigma_x = -\sigma_z$. For simplicity, we first consider the near-resonant case $\Delta_t \approx 0$. Setting $\Omega_c = \Omega_t$, $\Delta_c = \Delta_t$, and $\Phi_c = \pi - \Phi_t$ ensures $U_\pi e^{-iH_c T} U_\pi = U_t^\dagger$, where $U_\pi = \sigma_x$ and $H_c = H(\Omega_c, \Delta_c, \Phi_c) + H_{\text{int}}$. For nonzero detuning $\Delta_t \neq 0$, an additional frame-change factor $\exp(+i\Delta_t\sigma_z^e T/2)$ emerges. To cancel this extra phase, we further refine the control scheme by setting the phase of the first electronic spin π -pulse as $2\pi\Delta_t T$, thereby achieving optimal control even under more general target field conditions.

III. STATE PREPARATION AND MEASUREMENT

A. Bell-state measurement scheme

In conventional single-parameter sensing, each measurement sequence yields only one outcome, limiting the

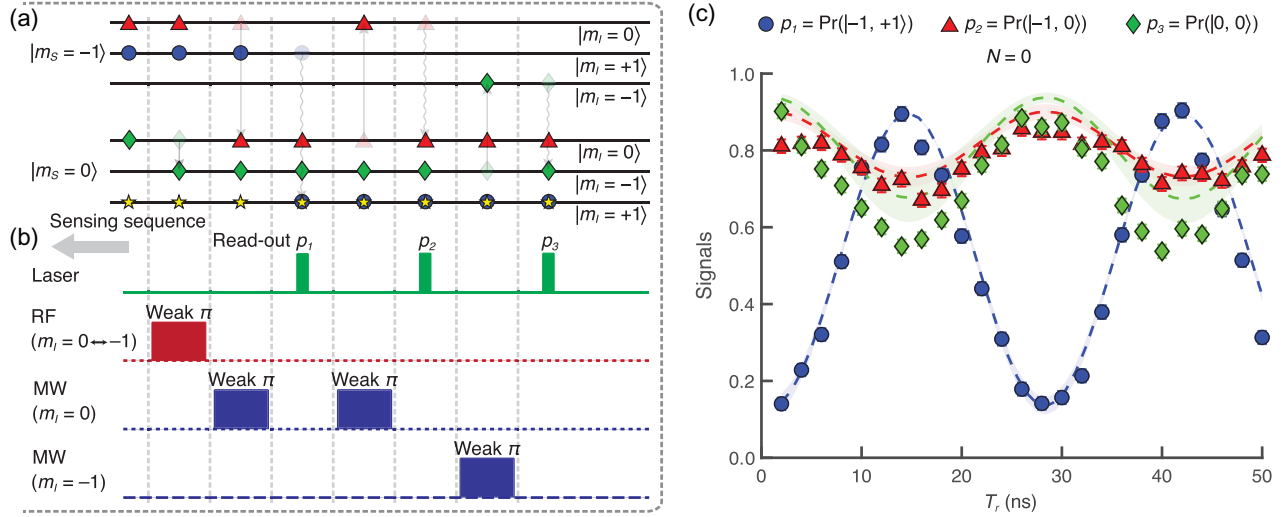


FIG. 2. Readout sequence and experimental demonstration of Bell-state measurement. (a) Energy-level diagram illustrating the readout sequence. After applying the disentangling gates shown in Fig. 1, the Bell-state measurement reduces to measuring the population of the $|-1, +1\rangle$ (p_1 , blue circle), $|-1, 0\rangle$ (p_2 , red triangle), and $|0, 0\rangle$ (p_3 , green diamond) states. (b) Optical, radio-frequency (RF), and microwave (MW) pulse sequence used for the readout shown in (a). Initially, an RF π pulse between the $m_I = 0$ and $m_I = -1$ states shelves population p_3 into the unused $|m_S = 0, m_I = -1\rangle$ state. Next, a selective (weak) microwave π pulse conditioned on $m_I = 0$ transfers population p_2 to the $|0, 0\rangle$ state. A short laser pulse measures the electronic spin, yielding p_1 without affecting the nuclear spin. Although this measurement erases the distinction between p_1 and p_4 , the shelving enables extraction of both p_2 and p_3 , which are sequentially mapped onto the electronic spin for optical readout. (c) Example of the Bell-state measurement as a function of the pulse duration for the rotation gate T_r for fixed amplitude $\Omega_r = (2\pi) \times 21$ MHz, detuning $\Delta_r = (2\pi) \times 30$ MHz, and phase $\Phi_r = 30^\circ$. We plot the normalized signals without any sensing loop ($N = 0$). To cancel the frame-change factor $\exp(+i\Delta_r\sigma_z T/2)$, we set the phase of the following π pulse as $2\pi\Delta_r T_r$. The dashed lines represent a model that accounts for state preparation and measurement (SPAM) errors due to nonideal behavior of nuclear spin states under laser irradiation, while the shaded regions represent the 95% confidence intervals derived from parameter uncertainties (P_e, η, γ, ζ). Error bars represent the standard deviation of the signal determined through error propagation from the photoluminescence intensity. Each displayed point is a result of $n = 4 \times 10^6$ averages of sequences.

extracted information to a single parameter. By entangling the sensor with an ancilla qubit, we effectively expand the measurement basis to two qubits, enabling simultaneous estimation of multiple parameters. A Bell-state measurement on this entangled system, then, provides all three target parameters from a single measurement sequence, thus realizing genuine quantum multiparameter sensing.

Bell-state measurements of the electronic-nuclear spin system associated with the NV center have been demonstrated in conditions that are not always compatible with the most general sensing tasks, in particular at cryogenic temperatures or under high magnetic fields [48–50]. Here we achieve averaged readout of the Bell basis at room temperature and under a low static DC field by utilizing the three-level system of the nitrogen-14 nuclear spin (see Fig. 2). In addition to rotating the Bell state to the computational basis using CNOT and Hadamard gates, the key insight is that we can extract information about both the electronic and nuclear spin states by performing three electronic spin readouts, while the nonqubit level of the nuclear spin acts as a (classical) memory. Specifically, we transfer the $m_S = 0, m_I = 0$ state to the $m_S = 0, m_I = -1$ state, which lies outside the qubit subspace, effectively “shelving” the state. The sequential measurements

illustrated in Figs. 2(a) and 2(b) allow determining the three populations $p_i, i \in \{1, 2, 3\}$.

The readout sequence yields three signals, as shown in Fig. 2(c), which presents the measurement results obtained by rotating the Bell basis without performing the sensing loop, i.e., $N = 0$. The dashed lines indicate a simple model that accounts for state preparation and measurement (SPAM) errors due to the nonideal behavior of nuclear spin states under laser irradiation, as we describe next.

B. Modeling SPAM errors

As illustrated in Fig. 1, the Bell state is prepared by applying a $\pi/2$ pulse ($\exp(+i\pi\sigma_y/4)$) on the nuclear spin and a selective π pulse on the electronic spin. With the $20\mu\text{s}$ laser pulse for initialization, the nuclear spin achieves only partial polarization, quantified by $P = 0.89$ [47]. This results in a mixed initial state $\rho = P|0, +1\rangle\langle 0, +1| + (1 - P)|0, 0\rangle\langle 0, 0|$, which, after the entangling gates, evolves into a classical mixture of Bell states: $\rho = P|\Phi_+\rangle\langle\Phi_+| + (1 - P)|\Phi_-\rangle\langle\Phi_-|$, where $|\Phi_\pm\rangle = (|0, 0\rangle \pm |-1, +1\rangle)/\sqrt{2}$. When $P < 1$, the signal amplitude decreases, degrading the sensitivity. In addition, we account for a small population of $|m_S = 0, m_I = -1\rangle$

with $P_{-1} = 0.039$, which manifests as offsets in signals p_1 and p_2 [47].

For the measurement error, we employ a simple rate model (instead of the more complex Lindblad master equation) that captures the effects of nuclear spin population transfer (“leakage”) occurring during short laser pulses used for electron spin readout. Let the populations prior to the readout sequence be denoted as $p_1 = \Pr(|-1, +1\rangle)$, $p_2 = \Pr(|-1, 0\rangle)$, $p_3 = \Pr(|0, 0\rangle)$, and $p_4 = \Pr(|0, +1\rangle)$. The first readout pulse measures p_1 , which is unaffected by leakage. However, after this measurement, population transfer between the nuclear spin states with $m_s = 0$ occurs, such that $\Pr(|0, 0\rangle) = (1 - \zeta)p_2 + \eta p_3$ and $\Pr(|0, -1\rangle) = (1 - \gamma - \eta)p_3$. Consequently, the population measured by the next short laser pulse is $(1 - \zeta)p_1 + \eta p_3$. During this interval, the state $\Pr(|0, -1\rangle)$ further evolves to $\Pr(|0, -1\rangle) = (1 - 2\gamma - 2\eta)p_3$, which is then read out by the final laser pulse. The parameters $\zeta = 0.20$, $\gamma = 0.16$, and $\eta = 0.030$ are determined experimentally [47]. In addition, the model accounts for incomplete electron spin initialization, $P_e = 93\%$ of the maximum polarization due to the short (400 ns) laser pulses. The remaining population in the $m_s = -1$ state introduces a minor correction to subsequent measurements.

This parameter-free model accurately captures the experimental data, particularly the signal amplitudes, even when neglecting additional imperfections such as pulse errors. As will be discussed later, the signal amplitude plays a critical role in determining the sensor’s overall sensitivity.

IV. SENSITIVITY ANALYSIS

In theoretical studies of quantum multiparameter estimation, sensitivity is typically quantified using the quantum Fisher information matrix (QFIM) [11], whereas experimental quantum sensing often relies on error-propagation methods [2] that naturally incorporate classical readout noise. Here, we generalize this experimentally motivated framework from the single-parameter to the multiparameter regime by extending from the binomial to the multinomial distribution. This approach explicitly accounts for classical readout imperfections—an often overlooked aspect in theoretical analyses—and establishes a comprehensive and practical framework for quantitative multiparameter sensitivity analysis, offering realistic guidance for experimental implementations.

A. Quantum noise

Quantum projection noise is one of the most fundamental sources of error in quantum sensing. In a projective measurement, rather than directly obtaining probabilities p_i , one of K possible outputs is observed according to the multinoulli (or categorical) distribution. To accurately estimate p_i , the experiment is repeated n times, and the number

of occurrences for each output is collected into a histogram. The estimator for p_i is given by $\hat{p}_i = n_i/n$, where n_i denotes the number of times outcome i is obtained, following a multinomial distribution. The total number of measurements is given by $n = T_{\text{tot}}/(T + T_m)$, where T_{tot} is the total available measurement time, T is the measurement duration, and T_m is the additional time required for sensor initialization, manipulation, and readout [2]. The covariance matrix of p_i is therefore determined by that of the multinomial distribution,

$$[\Sigma_{\hat{p}, \text{quantum}}]_{i,j} = \begin{cases} p_i(1 - p_i)/n, & \text{if } i = j, \\ -p_i p_j / n, & \text{if } i \neq j, \end{cases} \quad (1)$$

which indicates that projective measurements add noise on the order of $1/\sqrt{n}$. The covariance matrix for the estimate of the target parameters is obtained via error propagation,

$$\Sigma_{\hat{\theta}} = J^{-1} \Sigma_{\hat{p}} (J^{-1})^T, \quad (2)$$

where the Jacobian matrix J is defined as $J_{i,j} = \partial p_i / \partial \theta_j$. The square roots of the diagonal elements of $\Sigma_{\hat{\theta}}$ yield the standard deviations of the estimates, providing a quantitative measure of sensitivity. For the ideal Bell-state measurement, with the probabilities given in Appendix A, Eq. (A1), for the ideal vector field model, we recover that $\Sigma_{\hat{\theta}}$ coincides with the QFIM.

B. Classical noise

In many scenarios, the projection noise is dominated by the classical noise introduced during the readout of the sensor. Then, the estimate of p_i can be obtained from the average of the physical readout value y_i ,

$$\hat{p}_i = \frac{\bar{y}_{p_i} - y_{p_i=0}}{y_{p_i=1} - y_{p_i=0}},$$

where $y_{p_i=0}$ and $y_{p_i=1}$ denote the average physical readout values corresponding to $p_i = 0$ and $p_i = 1$, respectively. In our experiments, where photon-shot noise dominates, the variance in the estimate of p_i due to readout noise can be approximated as $\sigma_{y_{p_i}} \approx \sqrt{\bar{y}_{p_i}}$. The total covariance matrix of $\hat{p}$ is then

$$\Sigma_{\hat{p}} = \Sigma_{\hat{p}, \text{quantum}} + \text{diag}(\sigma_{\hat{p}_1, \text{readout}}^2, \dots, \sigma_{\hat{p}_K, \text{readout}}^2), \quad (3)$$

where $\sigma_{\hat{p}_i, \text{readout}}^2 = \sigma_{y_i}^2 / (y_{p_i=1} - y_{p_i=0})^2$. Since in our experiments all $\sigma_{\hat{p}_i, \text{readout}}^2$ are approximately equal and dominate the quantum contribution, we can approximate the total covariance as $\Sigma_{\hat{p}} \approx \sigma^2 I$, from which we experimentally extract the parameter covariance matrix Σ_{θ} .

V. RESULTS

We quantify the sensor's performance by extracting the parameter covariance matrix $\Sigma_{\hat{\theta}}$ from the measurement signals, that is, by evaluating the measurement covariance $\Sigma_{\hat{p}}$ and the Jacobian J . This procedure extends what typically done in single-parameter estimation (see e.g., Ref. [27]) where the signal variance and its slope when varying the parameter are measured to obtain the sensitivity. Even in the multiparameter case, for small variations in θ , the diagonal elements of $\Sigma_{\hat{\theta}}$ yield the sensitivities. Given the shot-noise dominated diagonal covariance matrix described above, optimal sensitivity occurs when the measurement signal slope is steepest.

In a standard Ramsey experiment for estimating the parameter B_z in the Hamiltonian $H = (B_z/2)\sigma_z$, the steepest-slope condition can be realized either by introducing a bias field or by adjusting the measurement basis. This flexibility arises because the ultimate sensitivity bound, given by the quantum Fisher information (QFI) as T^2 , is independent of B_z . This optimal sensitivity is achieved by initializing the system in an equal superposition state, such as $|+\rangle$, and performing measurements within the equatorial

plane. Unfortunately, for multiparameter sensing applying a bias field is no longer a feasible strategy since $|\vec{B}| \neq 0$ generally spoils linear sensitivity scaling, as previously discussed. Instead, optimizing the measurement basis becomes the primary viable strategy. Specifically, we introduce a rotation gate $U_r = \exp(-i(\pi/3\sqrt{3})(\sigma_x^e + \sigma_y^e + \sigma_z^e))$ prior to the disentangling gates. This rotation ensures uniform output probabilities $p_i = 1/4$ when starting from the initial state $|\Phi_+\rangle$, which is analogous to the optimal operating point $p = 1/2$ used in Ramsey experiments under classical noise. This strategy is close to the optimal basis rotation that provides a slight improvement over the uniform probability case, highlighting the intricate effects of classical noise in multiparameter estimation [47].

To extract the experimentally determined Jacobian matrix, we sweep the target parameters and measure the resulting signal variations, as illustrated in Fig. 3. Each element of the Jacobian is a partial derivative evaluated at the operating point; to obtain it experimentally we sweep one parameter at a time around the optimal point while keeping the other two fixed at the control field. Figure 4 presents both measured and simulated sensitivities as a function of $N = T/\tau$. Here, $\tau = 30$ ns is chosen to satisfy

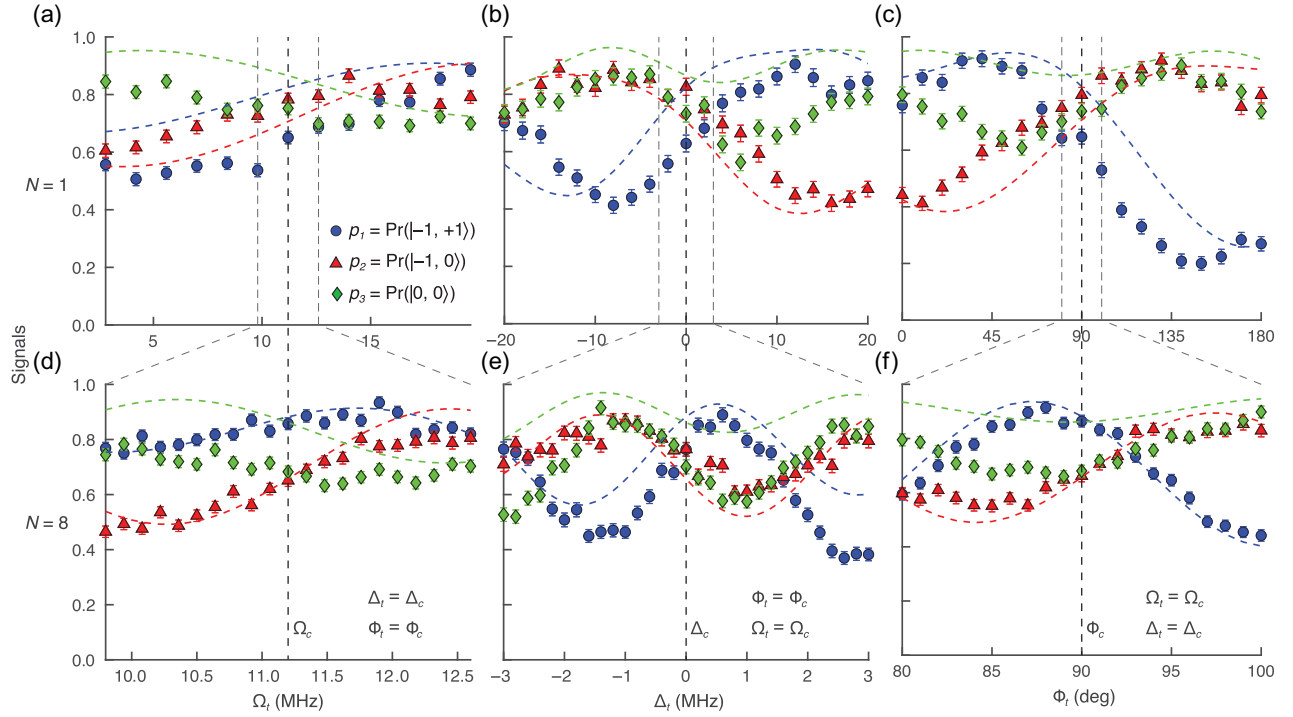


FIG. 3. Characterization of the sensing protocol performance. (a)–(f) Examples of measured signals as functions of the amplitude Ω_t (a) and (d) detuning Δ_t (b) and (e) and phase Φ_t (c) and (f) of the target microwave field for two different numbers of repetitions, $N = 1$ (a)–(c) and $N = 8$ (d)–(f). Here, the control field parameters were set as follows: amplitude $\Omega_c = (2\pi) \times 11.2$ MHz, detuning $\Delta_c = 0$ MHz, and phase $\Phi_c = 90^\circ$. The sensitivity is limited by the uncertainty in signal measurement $\sigma \approx 0.02$, with the maximum slope point achieved at the parameters of the control field. At this point, the sensor is most sensitive to changes in the target parameters. Compared to the case of $N = 1$, the signal is significantly more sensitive to variations in the target parameters when $N = 8$. The dashed lines represent the model that accounts for SPAM errors due to laser irradiation, as shown in Fig. 2(c). Error bars represent the standard deviation of the signal. Each displayed point is a result of $n = 3 \times 10^6$ averages of sequences.

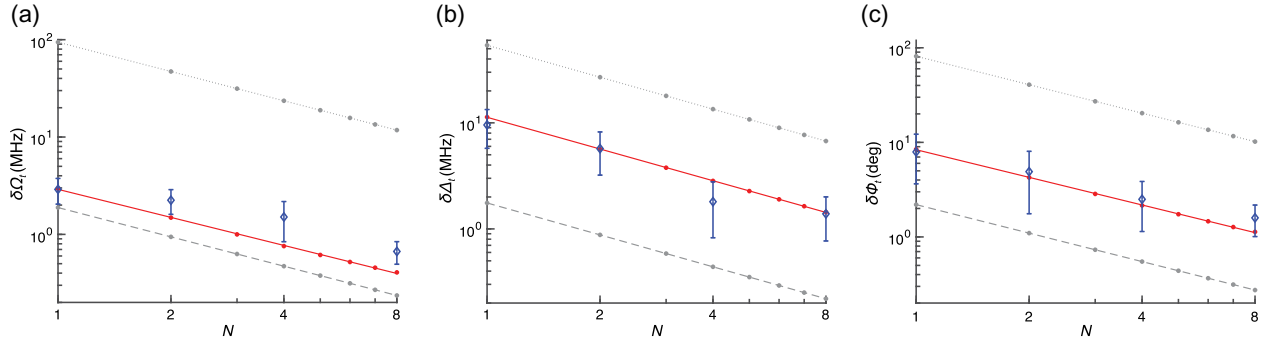


FIG. 4. Characterization of sensitivity. (a)–(c) The minimum measurable amplitude (a), detuning (b), and phase (c) of the target microwave field as functions of number of repetitions N . The uncertainties in the estimates of the target parameters, calculated from error propagation, are plotted for both the simulation (red) and the experiment (blue) at the point of maximum sensitivity, i.e., around the parameters of the control field shown in Fig. 3. Error bars on the experimental data indicate the standard error of the uncertainty, obtained via error propagation from the standard error of the fitted slopes. A fit to the simulation (red line) shows that the sensitivity improves as the number of repetitions N , which is proportional to the interrogation time T , demonstrating the linear scaling for all three parameters. Gray dashed and dotted lines represent photon-shot noise-limited sensitivities for the ideal model with full and low ancilla polarization ($P = 1$ and $P = 0.51$), respectively. The divergence near $P = 0.5$ highlights the breakdown of simultaneous three-parameter estimation with a single qubit.

$\tau\sqrt{\Omega_t^2 + \Delta_t^2} < \pi/2$, ensuring operation within the linear regime ($\sin x \approx x$) for each repetition unit N . The red solid lines are fits to $\delta\hat{\theta}_j \propto N^{-m}$ for $\theta_j \in \{\Omega_t, \Delta_t, \Phi_t\}$ for the simulation data (red circles), where $m \approx 1$, which shows linear scaling with respect to the interrogation time. The experimental data, shown as blue diamonds, agree with these simulations. As seen in Fig. 3, increasing N makes the signals more sensitive to small variations in the target parameters, thereby illustrating why the sensitivity improves with $N \propto T$. The linear enhancement with N discussed here is valid when dephasing effects remain negligible. A detailed decoherence-limited analysis is provided in Appendix B.

Additionally, we plot sensitivities corresponding to two cases of the ideal vector field model: the photon-shot noise limit and the case of low ancilla polarization. The gray dashed line represents the sensitivity in the ideal scenario with covariance matrix $\Sigma_{\hat{p}} = \sigma^2 \mathbf{I}$, defining the photon-shot noise limit. In general, the photon-shot noise-limited sensitivity of NV sensors is typically about two orders of magnitude worse than the quantum projection limit, which, in our case, saturates the bound given by the QFIM. The gray dotted line illustrates the photon-shot noise-limited sensitivity assuming the ancilla's initial polarization is $P = 0.51$, to approximate the scenario where the ancilla does not aid the sensing task. Indeed, as the polarization P approaches 0.5, the sensitivity diverges, and at exactly $P = 0.5$, the Jacobian matrix becomes singular. This singularity corresponds to the fundamental limitation that three parameters cannot be simultaneously estimated with a single qubit.

In addition to the aforementioned SPAM errors, several factors contribute to the degradation of sensitivity in our simulation model and actual experiments compared to the

photon-shot noise limit of the ideal model. These include pulse errors in sequential control, state preparation and measurement, finite pulse length effects of π pulses, and the presence of the frame-change factor.

VI. CONCLUSION AND OUTLOOK

Our demonstration achieved linear sensitivity scaling with respect to the interrogation time T for three parameters, despite various experimental imperfections. In the ideal case of estimating a static vector field $\vec{B}$ with a sensor-ancilla system, the optimal sequential control scheme simultaneously achieves optimal precision for all three parameters, offering a threefold sensitivity improvement over separate single-parameter estimations with a single sensor [15]. It is worth noting that the target parameters in our case originate from a time-dependent AC field in the laboratory frame, specifically, its amplitude, frequency, and phase. By applying a suitable rotating frame transformation and sequential controls, we effectively map the problem onto a time-independent Hamiltonian case, and achieve the scaling $\delta\hat{\theta}_j \propto 1/T$ for all parameters $\theta_j \in \{\Omega_t, \Delta_t, \Phi_t\}$. However, this does not necessarily correspond to the highest possible precision for each parameter, since the optimal scaling for estimating the frequency of an AC field can reach a quadratic scaling behavior [25, 51, 52]. In future work, we aim to investigate whether it is possible to simultaneously attain the highest precision for the parameters of the AC magnetic field (Ω, Δ, Φ) under realistic experimental conditions, and to explore potential trade-offs between these parameters.

Additionally, when considering the total measurement time T_{tot} as a resource, the time required for the Bell-state

preparation and the Bell-state measurement, T_m , constitutes an overhead compared to the initialization and readout of single-parameter estimation schemes which usually require much simpler operations. However, if we suppose both the sensor and the ancilla enable single-shot readout, and the readout time dominates the whole measurement time ($T_{\text{tot}} \approx T_m$), the Bell-state measurement—requiring only two readouts—might offer an advantage over performing three separate single-parameter estimations.

NV quantum sensors offer exceptional spatial resolution and versatility, making them particularly well-suited for multiparameter estimation in contexts where simultaneous measurement of multiple physical quantities at a single localized point is required. In such scenarios, the use of multiple bulky sensors is often impractical, further highlighting the advantages of NV-based approaches. In our demonstration, we focused on the simultaneous estimation of three parameters—the amplitude, detuning, and phase of a microwave drive. While microwave measurements have direct applications such as probing spin waves [43–45], in principle, our scheme can be extended to estimate any set of three parameters encoded in the Hamiltonian. Future research directions include the development of pulse sequences and Hamiltonian engineering techniques to simultaneously estimate multiple physical quantities accessible to the NV center, such as vector components of static magnetic and electric fields, temperature, and pressure.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [53].

APPENDIX A: SENSITIVITY IN VECTOR FIELD SENSING

1. Mixed probe state

As shown in the previous section, when using a maximally entangled state as the probe and performing a projective measurement on the Bell basis, the estimation can reach the local precision limit. However, experimental limitations might prevent the preparation of a perfect initial state. In this section, we consider the scenario where the measurement remains a Bell-basis projection, but the probe state is mixed. In particular, we assume an attempt to prepare an entangled state using Hadamard and CNOT gates as in the previous section, but starting from a state where the ancilla is mixed: $\rho_0 = |0\rangle\langle 0| \otimes (\mathbf{I} + \mathcal{P}\sigma_z)/2$ instead of $|00\rangle$. This models the experimental limitation in polarizing the nuclear spin and broadly highlights the role of the ancilla qubit in achieving optimal metrological performance.

To evaluate the metrological bounds and compare the QFIM and the classical Fisher information matrix (FIM), we use scalar quantum and classical Fisher information metrics under two choices of weight matrices: $W = \mathbf{I}$ and $W_a = \text{diag}[1, 1/B^2, 1/(B^2 \sin^2 \alpha)]$. The latter corresponds to the mean squared error in estimating the components of the magnetic-field vector.

In the limit $B \rightarrow 0$ (achievable via control and shown to be optimal in the pure-state case), the scalar quantum Fisher information (QFI) becomes

$$\text{Tr}[\text{QFIM}W_a] = T^2(2 + \mathcal{P}^2).$$

Notably, if we assume no ancilla is used, the scalar QFI reduces to

$$\text{Tr}[\text{QFIM}_1W_a] = 2T^2$$

for a single qubit with an initial state $|+\rangle$. We may express the QFI in terms of the concurrence $\mathcal{C}$, which quantifies entanglement. For our prepared state, we have $\mathcal{C} = |\mathcal{P}|$. Then, the scalar QFI becomes $\text{Tr}[\text{QFIM}W_a] = T^2(2 + \mathcal{C}^2)$, highlighting that any gain over the ancilla-free scenario is due to the presence of entanglement.

The trace of the QFIM with the identity weight matrix, $\text{Tr}[\text{QFIM}]$, shows a more intricate angular dependence, even in the $B \rightarrow 0$ limit,

$$\text{Tr}[\text{QFIM}] = T^2[1 - (1 - \mathcal{P}^2) \sin^2(\alpha) \cos^2(\beta)],$$

while for the single-qubit case without ancilla,

$$\text{Tr}[\text{QFIM}_1] = T^2[1 - \sin^2(\alpha) \cos^2(\beta)].$$

This shows that for $\mathcal{P} \neq \pm 1$, the QFI depends on the angles, reflecting no capability to simultaneously estimate all three parameters without an ancilla. Importantly, in all these cases, even with a mixed ancilla, the QFI retains the $\propto T^2$ scaling. However the optimal scaling $\propto 3T^2$ is only achieved for $\mathcal{P} = \pm 1$.

Lastly, we compare the quantum and classical Fisher information. While they may coincide for specific parameter values, in general, the trace of FIM is substantially smaller than the quantum counterpart. This indicates that the Bell-state measurement is no longer optimal when the initial state is mixed.

2. Classical readout error: Averaged readout

Here, we focus on the scenario where readout error dominates, a situation that is particularly relevant to our experiment. We consider the task of measuring a vectorial field

$\vec{B} = B\{\cos \alpha, \sin \alpha \cos \beta, \sin \alpha \sin \beta\}$ which corresponds to our target microwave field in the rotating frame. In this case, the average of the measurement is still proportional to the ideal (projective measurement) probabilities,

$$\begin{aligned} p_1 &= \cos^2(BT), \\ p_2 &= \sin^2(BT) \cos^2(\alpha), \\ p_3 &= \sin^2(BT) \sin^2(\alpha) \cos^2(\beta), \\ p_4 &= \sin^2(BT) \sin^2(\alpha) \sin^2(\beta). \end{aligned} \quad (\text{A1})$$

but the noise is assumed to be isotropic and uncorrelated, so that $\Sigma_{\hat{p}} = \sigma^2 \mathbf{I}$.

Using error propagation, we obtain $\Sigma_{\vec{B}} = J^{-1} \Sigma_{\hat{p}} (J^{-1})^T$.

Explicitly, this gives

$$\Sigma_{\vec{B}} = \begin{pmatrix} \frac{\sigma^2 \csc^2(2BT)}{T^2} & \frac{\sigma^2 \cot(\alpha) \csc^3(BT) \sec(BT)}{4T} & \frac{\sigma^2 \cot(\beta) \csc^3(BT) \csc^2(\alpha) \sec(BT)}{4T} \\ \frac{\sigma^2 \cot(\alpha) \csc^3(BT) \sec(BT)}{4T} & \frac{\sigma^2}{32} (11 + 4 \cos(2\alpha) + \cos(4\alpha)) \csc^4(BT) \csc^2(\alpha) \sec^2(\alpha) & \frac{\sigma^2}{8} (3 + \cos(2\alpha)) \cot(\beta) \csc^4(BT) \csc^3(\alpha) \sec(\alpha) \\ \frac{\sigma^2 \cot(\beta) \csc^3(BT) \csc^2(\alpha) \sec(BT)}{4T} & \frac{\sigma^2}{8} (3 + \cos(2\alpha)) \cot(\beta) \csc^4(BT) \csc^3(\alpha) \sec(\alpha) & \frac{\sigma^2}{16} (7 + 4 \cos(2\beta) + \cos(4\beta)) \csc^4(BT) \csc^4(\alpha) \csc^2(\beta) \sec^2(\beta) \end{pmatrix}$$

The corresponding figure of merit becomes

$$\begin{aligned} \delta \hat{B}_x^2 + \delta \hat{B}_y^2 + \delta \hat{B}_z^2 &= \delta \hat{B}^2 + B^2 \delta \hat{\alpha}^2 + B^2 \sin^2(\alpha) \delta \hat{\beta}^2 \\ &= \frac{\sigma^2 \csc^2(BT)}{4T^2} [\sec^2(BT) + B^2 T^2 \csc^2(BT) (\csc^2(\alpha) (3 \csc^2(\beta) + \sec^2(\beta)) + \tan^2(\alpha))]. \end{aligned}$$

The result again diverges at $B = 0$, and the sensitivity is the worst.

3. Rotation of Bell measurement basis

Interestingly, the loss of sensitivity at $B = 0$ due to classical noise can be recovered by rotating the measurement basis. In particular, we apply the unitary transformation: $U = \exp(-i(\pi/3\sqrt{3})(\sigma_x + \sigma_y + \sigma_z))$, which redefines the measurement basis.

In this rotated basis, the outcome probabilities become

$$\begin{aligned} p_1 &= \frac{1}{4} [\cos(BT) + \sin(BT) (\cos(\alpha) \\ &\quad + \sin(\alpha) (\cos(\beta) + \sin(\beta)))^2, \\ p_2 &= \frac{1}{4} [\cos(BT) - \sin(BT) (\cos(\alpha) \\ &\quad + \sin(\alpha) (\cos(\beta) - \sin(\beta)))^2, \end{aligned}$$

$$\begin{aligned}
p_3 &= \frac{1}{4} [\cos(BT) + \sin(BT) (\cos(\alpha) \\
&\quad - \sin(\alpha) (\cos(\beta) + \sin(\beta)))]^2, \\
p_4 &= \frac{1}{4} [\cos(BT) - \sin(BT) (\cos(\alpha) \\
&\quad + \sin(\alpha) (-\cos(\beta) + \sin(\beta)))]^2.
\end{aligned}$$

At $B = 0$, all four probabilities become equal: $p_1 = p_2 = p_3 = p_4 = 1/4$. With these probabilities, one finds again

$$\begin{aligned}
\Sigma_{\hat{B}} &= J^{-1} \Sigma_{\hat{p}} (J^{-1})^\top \\
&= \frac{1}{4n} \begin{pmatrix} \frac{1}{T^2} & 0 & 0 \\ 0 & \csc^2(BT) & 0 \\ 0 & 0 & \csc^2(BT) \csc^2(\alpha) \end{pmatrix} \quad (\text{A2})
\end{aligned}$$

for the noiseless case.

We perform error propagation for both the single-shot and averaged readout cases to obtain the covariance matrix $\Sigma_{\hat{B}}$. In general, concise expressions for the covariance matrix and the figure of merit, $\delta\hat{B}_x^2 + \delta\hat{B}_y^2 + \delta\hat{B}_z^2$ are not available. Numerical simulations [47] of the figure of merit as a function of B and T show that the optimal point at $B = 0$ is recovered.

4. Comparison of multiparameter estimation and repeated single-parameter estimation under averaged readout

We now compare the sensitivity at the optimal point $B = 0$, recovered by the aforementioned basis rotation in the averaged readout scenario, with the sensitivity achievable via single-parameter estimation. For averaged readout, the optimal variance for each component B_i is given by $\sigma^2/T^2 = \sigma_0^2/nT^2$, where σ_0^2 is the variance of each repetition. To estimate three parameters sequentially under a fixed total resource constraint, we assume that each parameter is measured independently and repeated $n/3$ times, yielding a figure of merit,

$$\delta\hat{B}_x^2 + \delta\hat{B}_y^2 + \delta\hat{B}_z^2 = \frac{3\sigma_0^2}{(n/3)T^2} = \frac{9\sigma_0^2}{nT^2}.$$

In contrast, for the multiparameter estimation scenario with $\Sigma_{\hat{p}} = (\sigma_0/\sqrt{n})^2 \mathbf{I}$, the corresponding figure of merit is

$$\begin{aligned}
\delta\hat{B}_x^2 + \delta\hat{B}_y^2 + \delta\hat{B}_z^2 &= \text{tr}(\Sigma_{\hat{B}}) \\
&= (\sigma_0/\sqrt{n})^2 \text{tr}(J^{-1}(J^{-1})^\top).
\end{aligned}$$

The Jacobian J at the point $B_x = B_y = B_z = 0$ can be evaluated analytically as

$$J = \frac{T}{2} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}.$$

Using this Jacobian, the figure of merit for the multiparameter scheme becomes

$$\delta\hat{B}_x^2 + \delta\hat{B}_y^2 + \delta\hat{B}_z^2 = \frac{6\sigma_0^2}{nT^2}.$$

Thus, the multiparameter estimation scheme demonstrates a sensitivity advantage over repeated single-parameter estimation even in the case of averaged readout.

APPENDIX B: DECOHERENCE-LIMITED SENSITIVITY SCALING

We analyze how decoherence ultimately limits the sensitivity scaling with respect to the number of sensing repetitions N , i.e., the interrogation time. In general, the influence of decoherence on optimal probe state, control, and measurement in quantum multiparameter estimation is highly nontrivial. For instance, in the ideal vector sensing model, it is known that in the presence of decoherence, the Bell-state measurement no longer satisfy QFIM = FIM [54]. Nevertheless, we can investigate how the sensitivity scaling is affected by modeling the dynamics of the target and control microwave fields using the Lindblad master equation,

$$\dot{\rho} = -i[H_{\text{tot}}(t), \rho] + \Gamma_e \mathcal{D} \left[\frac{\sigma_z^e}{\sqrt{2}} \right] \rho,$$

where $\mathcal{D}[L]\rho = L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}$, and electronic dephasing is set by $\Gamma_e = 1/T_2^e$.

Figures 5(a) and 5(c) illustrates the sensitivities of the target microwave amplitude, frequency, and phase for $T_2^e = 10 \mu\text{s}$. To better highlight the effects of decoherence, we assume ideal state preparation, readout, and π -pulses. Furthermore, for ease of comparison with conventional methods, the number of repetitions is defined as $n = T_{\text{tot}}/2T$, and the sensitivities are normalized with respect to a unit total time T_{tot} , expressed as η_i for $i \in \{\Omega_t, \Delta_t, \Phi_t\}$. Interestingly, the optimal number of sensing repetitions N that minimizes the sensitivity differs across parameters. As shown in Figs. 5(d)–5(f), the minimum sensitivity $\eta_i^{\min}$ with respect to T_2^e scales as $1/\sqrt{T_2^e}$, consistent with the well-known results of single-parameter estimation [2]. For $T_2^e = 10 \mu\text{s}$, the sensitivities for amplitude and frequency are on the order of $10 \text{ nT}/\sqrt{\text{Hz}}$ and $1 \text{ kHz}/\sqrt{\text{Hz}}$, respectively, where we converted the amplitude unit via $\Omega_t = \gamma_e B_t/\sqrt{2}$. It is worth mentioning that

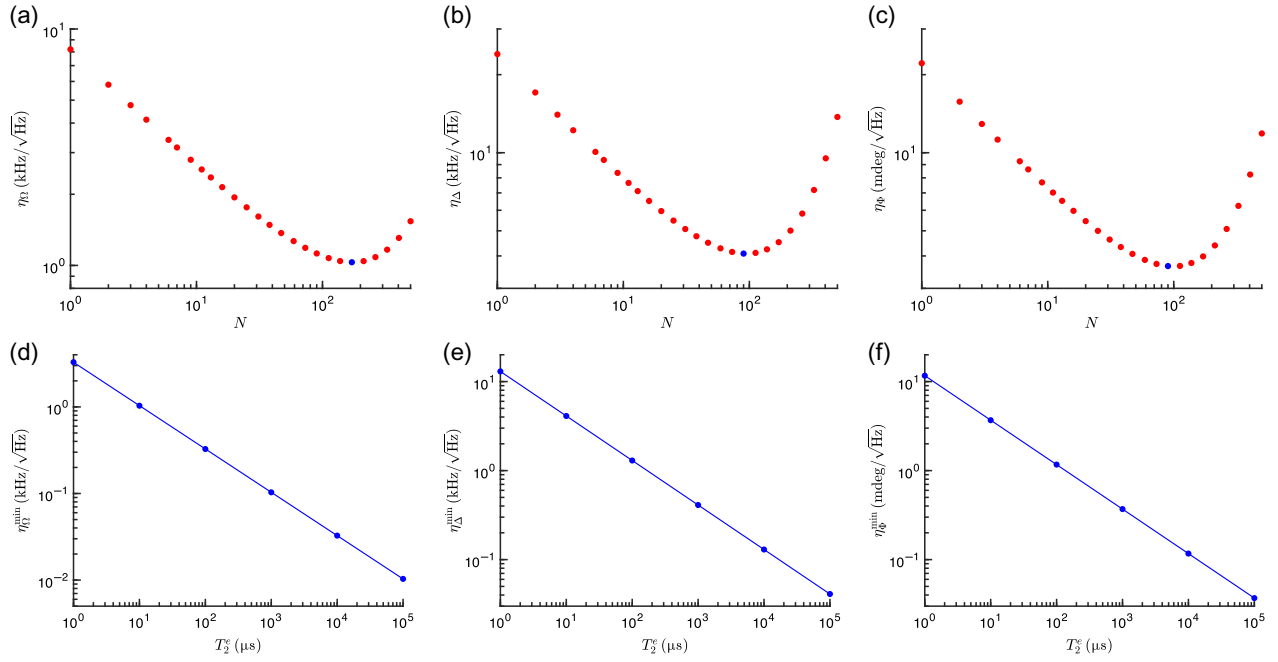


FIG. 5. Decoherence-limited sensitivity scaling. (a)–(c) Sensitivities of the target microwave amplitude, frequency, and phase for $T_2^e = 10 \mu\text{s}$ as a function of the sensing repetitions N . Ideal state preparation, readout, and π -pulses are assumed, and sensitivities are normalized by T_{tot} . The optimal number of repetitions N differs across parameters. (d)–(f) Minimum sensitivities as a function of T_2^e . The sensitivities scale as $1/\sqrt{T_2^e}$, in agreement with the well-established result of single-parameter estimation. The parameters used here match those in Fig. 4 of the main text.

these are comparable to those achievable with conventional Rabi and CPMG protocols [55,56]. However, as discussed in the main text, practical imperfections such as imperfect state preparation and readout, pulse errors, and the temporal overhead associated with the Bell-state measurement degrade the sensitivity, and overcoming these limitations remains an important challenge for future work.

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