

Performance of Practical Quantum Oblivious Key Distribution

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Motivated by the applications of secure multi-party computation as a privacy-protecting data analysis tool, and identifying oblivious transfer as one of its main practical enablers, we propose a practical realization of randomized quantum oblivious transfer. By using only symmetric cryptography primitives to implement commitments, we construct computationally secure randomized oblivious transfer without the need for public-key cryptography or assumptions imposing limitations on the adversarial devices. We show that the protocol is secure under an indistinguishability-based notion of security and demonstrate an experimental implementation to test its real-world performance. Its security and performance are then compared to both quantum and classical alternatives, showing potential advantages over existing solutions based on the noisy storage model and public-key cryptography.

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I. INTRODUCTION

A. State of the art

Cryptography is a critical tool for data privacy, a task deeply rooted in the functioning of today's digitalized world. Whether it is in terms of secure communication over the Internet or secure data access through authentication, finding ways of protecting sensitive data is of utmost importance. The one-time pad encryption scheme allows communication with perfect secrecy [1], at the cost of requiring the exchange of single-use secret (random) keys of the size of the communicated messages. Distribution of secret keys, therefore, is considered one of the most important tasks in cryptography. Modern cryptography relies

heavily on conjectures about the computational hardness of certain mathematical problems to design solutions for the key distribution problem. However, as quantum computers threaten to make most of the currently used cryptography techniques obsolete [2], better solutions for data protection are needed. This transition towards quantum-resistant solutions becomes particularly crucial when it comes to protecting data associated with the government, finance and health sectors, being already susceptible to *intercept-now-decrypt-later* attacks. Cryptography solutions secure in a post-quantum world, where large-scale quantum computers will be commercially available, have been explored in two directions. Classical cryptography based solutions, also referred as post-quantum cryptography [3–5], involve using a family of mathematical problems that are conjectured to be resilient to quantum computing attacks. On the other hand, quantum cryptography-based solutions [6] using the laws of quantum mechanics can offer information-theoretic security, depending on the physical properties of quantum systems rather than computational hardness assumptions. Quantum key distribution (QKD) [7] is the most well studied and developed of these

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quantum solutions, while other works beyond QKD have been proposed [8].

It is noteworthy that secure communication is not the only cryptographic task where the private data of end users may be exposed to an adversary. Cryptography beyond secure communication and key distribution includes zero-knowledge proofs, secret sharing, contract signing, bit commitment (BC), e-voting, secure data mining, etc. [9]. A huge class of such problems can be cast as multi-party computation (MPC), where distrustful parties can benefit from a joint collaborative computation on their private inputs. It requires parties' individual inputs to remain hidden from each other during the computation, among other security guarantees such as correctness, fairness, etc. [10]. Secure MPC is a powerful cryptographic tool with a vast range of applications as it allows collaborative work with private data. Generic MPC protocols work by expressing the function to evaluate as an arithmetic or Boolean circuit and then securely evaluating the individual gates. These protocols are based on one of two main fundamental primitives [11–14]: *oblivious transfer* (OT) and *homomorphic encryption*, the former of which is the focus of this work.

A 1-out-of-2 OT [15], is the task of sending two messages, such that the receiver can choose only one message to receive, while the sender remains oblivious to this choice. The original protocol, now called all-or-nothing OT, was proposed by Rabin in 1981 [16], where a single message is sent and the receiver obtains it with 1/2 probability. The two flavors of OT were later shown to be equivalent [17]. Notably, it has been shown that it is possible to implement secure MPC using only OT as a building block [18,19]. Relevant to our work is a variation of OT called random oblivious transfer (ROT), which is similar to 1-out-of-2 OT, except that both the sent messages and the receiver's choice are randomly chosen during the execution of the protocol. This can be seen as analogous to the key distribution task, in which both parties receive a random message (the key) as output. By appropriately encrypting messages using the outputs of a ROT protocol as a shared resource, it is possible to efficiently perform 1-out-of-2 OT. As an important consequence, parties expecting to engage in MPC in the future can execute many instances of ROT in advance and save the respective outputs as keys to be later used as a resource to perform fast OTs during an MPC protocol [20]. Because of this, we can think of ROT as a basic primitive for secure MPC.

In the context of quantum cryptography, OT is remarkable because, unlike classically, there exists a reduction from OT to commitment schemes [21]. This result is somewhat undermined by the existence of several theorems regarding the impossibility of unconditionally secure commitments both in classical [22] and quantum [23,24] cryptography, and it was further proven impossible in the more general abstract cryptography framework [25]. These results, in turn, imply that unconditionally secure OT itself

is impossible. In light of this, approaches with different technological or physical constraints on the adversarial power have been proposed. Practical solutions based on hardware limitations, such as bounded and noisy storage [26–29], have the disadvantage that the performance of such protocols decreases as technology improves.

Computationally secure classical protocols have also been proposed [30–33], which work under the assumptions of post-quantum public-key cryptography. Alternatively, we can take advantage of quantum reduction from OT to commitments by implementing commitment schemes using (nontrapdoor) one-way functions (OWFs), such as Hash functions [34] and pseudorandom generators [35], which allows us to construct OT from symmetric cryptography primitives. The existence of general OWFs is a weaker assumption than public-key cryptography [36,37], which requires the existence of the more restrictive *trapdoor* OWFs. This difference is significant, as the latter are defined over mathematically rich structures, such as elliptic curves and lattices, and the computational hardness of the associated problems is less understood than that of their private-key counterparts. For an in-depth study of the relation between OT and OWFs see Ref. [38].

Having established that there is a theoretical merit in using computationally secure quantum protocols to implement secure MPC, it is also important to understand how practical quantum protocols compare with currently used classical solutions in security, computational and communication complexity, and practical speed in current setups. This work focuses in studying the performance of a practical quantum ROT protocol and its potential advantages compared to currently used classical solutions for OT during MPC.

The idea of using quantum conjugate coding and commitments for oblivious transfer was originally proposed by Crépeau and Kilian [17] and then refined by Bennett *et al.*, in Ref. [21] with the BBKS92 protocol (shown in Fig. 1). This construction has been extensively studied from the point of view of its theoretical security [38–43]. However, while practical security analyses and experimental implementations have been made for quantum OT in the noisy storage model [28,29], there are no works analyzing the quantum resource requirements and the resulting performance of implementing the BBKS92 protocol using existing computationally secure commitment schemes based on OWFs. Such analyses are needed to demonstrate secure experimental implementations, and provide an important step in bringing quantum OT to real-world usage.

Motivated by practical considerations, we consider Naor-style statistically binding and computationally hiding commitments, as these are well understood and efficient to implement (note that stronger commitments can be considered, such as the quantum-based commitments studied in Refs. [38,42], however, implementing those requires significantly more computational and quantum resources).

BBCS92 Quantum OT protocol**Parties:** The sender Alice and the receiver Bob.

1. Alice prepares N entangled states of the form $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and, for each state prepared, sends one of the qubits to Bob.
2. Alice randomly chooses a measurement bases string $\theta^A \in \{+, \times\}^N$ and, for each $i = 1, \dots, N$ measures her share of the i -th entangled state in the θ_i basis to obtain outcome x_i^A and the outcome string $x^A = (x_1^A, \dots, x_N^A)$.
3. Bob uses the same process to obtain the measurement bases and outcome strings θ^B and x^B , respectively.
4. For each i , Bob commits (θ_i^B, x_i^B) to Alice.
5. Alice chooses randomly a set of indices $T \subset \{1, \dots, N\}$ of some fixed size and sends T to Bob.
6. For each $j \in T$, Bob opens the commitments associated to (θ_j^B, x_j^B) .
7. Alice checks that $x_j^A = x_j^B$ whenever $\theta_j^A = \theta_j^B$ within the test set. If the test fails Alice aborts the protocol, otherwise she sends the string θ^A to Bob.
8. Bob separates the remaining indices in two sets: I_0 - the indices where Bob's measurement bases match Alice's, and I_1 - the set of indices where their bases do not match. Then, he samples randomly c and sends the ordered pair $(I_c, I_{\bar{c}})$ to Alice.
9. Alice defines the strings $\mathbf{x}_c^A, \mathbf{x}_{\bar{c}}^A$ using the indices in the respective sets $(I_c, I_{\bar{c}})$. Then, she samples randomly a function f from a universal hash family, sends f to Bob and outputs $m_c = f(\mathbf{x}_0)$ and $m_1 = f(\mathbf{x}_{\bar{c}})$ to Bob.
10. Similarly, Bob defines the string $\mathbf{x}^B$ from the set I_0 and outputs $m_c = f(\mathbf{x}^B)$ and c .

FIG. 1. Quantum oblivious transfer protocol based on commitments.

The contributions of this work can be summarized as follows:

We introduce the definition for a quantum ROT protocol, satisfying a strong indistinguishability-based security notion equivalent to the one presented in Ref. [44], which generalizes the security of classical ROT protocols. We present a protocol that realizes said quantum ROT based on the BBCS92 construction. The protocol uses a weakly interactive string commitment scheme, which is statistically binding and computationally hiding, and can be implemented in practice using current QKD setups.

We present a formal finite-key security proof of the proposed protocol accounting for noisy quantum channels assuming only the existence of quantum-secure OWFs,

together with security bounds as functions of the protocol's parameters. We also present calculations for the maximum usable channel error, as well as for the key rate as a function of the number of shared signals per instance of the protocol. Additionally, we study the composability properties of said protocol. In particular, we show that there is a family of weakly interactive commitments, which, when used in the quantum OT protocol, result in universally composable quantum OT in the classical access random oracle model. We experimentally demonstrate our protocol using current technology with a setup based on polarization-entangled photons. We also present a security analysis, which accounts for potential implementation-specific attacks and how they can be circumvented using an appropriate reporting strategy. Finally, we compare our performance results with the performance of current ROT solutions and point out the advantages and disadvantages of using quantum ROT in the context of MPC.

II. QUANTUM RANDOM OBLIVIOUS TRANSFER

In this work, the concept of indistinguishability will be often used to compare the state of systems in a “real” run of the protocol versus another “ideal” desired state. These relations are defined over families of quantum states parametrized by the security parameter of the respective protocol. Hence, indistinguishability relations are statements on the asymptotic behavior of the protocol as the security parameter is increased. For formal definitions of both statistical and computational indistinguishability see Appendix A.

When talking about two indistinguishable families $\{\rho_1^{(k)}\}$ and $\{\rho_2^{(k)}\}$, if the parameter k is implicit, we will just refer to them as ρ_1 and ρ_2 and use the following notation to denote indistinguishability:

$$\rho_1 \approx \rho_2 \quad \text{for statistically indistinguishable;}$$

$$\rho_1 \approx^{(c)} \rho_2 \quad \text{for computationally indistinguishable.}$$

Additionally, in this work we consider protocols that can abort if certain conditions are satisfied. Mathematically, it is useful to consider the state of the aborted protocol as the zero operator. This means that events that trigger the protocol to abort are described as *trace-decreasing* operations, and hence, the operator representing the associated system at the end of the protocol is, in general, not normalized. The probability of the protocol finishing successfully is given then by the trace of the final state of the output registers. Note that the above definitions of indistinguishability can be naturally extended to non-normalized operators since the outcomes of a quantum program can always be represented by the outcomes of a positive operator-valued measure (POVM) $\{F_i\}$, whose probabilities are given by $\text{Tr}[F_i \rho]$, which is a well-defined quantity even for non-normalized ρ .

Definition 1 (Quantum random oblivious transfer). An n -bit quantum random oblivious transfer with security parameter k is a protocol, without external inputs, between two parties S (the sender) and R (the receiver), which,

upon finishing, outputs the joint quantum state ρ_{M_0, M_1, C, M_C} satisfying the following:

- (1) (Correctness) The final state of the outputs when the protocol is run with both honest parties satisfies

$$\rho_{M_0, M_1, C, M_C} \approx \frac{p_{\text{succ}}}{2^{(2n+1)}} \sum_{\substack{m_0, m_1 \in \{0,1\}^n \\ c \in \{0,1\}}} (|m_0\rangle\langle m_0|_{M_0} |m_1\rangle\langle m_1|_{M_1} |c\rangle\langle c|_C |m_c\rangle\langle m_c|_{M_C}), \quad (1)$$

where $p_{\text{succ}} = \text{Tr}[\rho_{M_0, M_1, C, M_C}]$ is the probability of the protocol finishing successfully.

- (2) (Security against dishonest sender) Let H_S be the Hilbert space associated to all of the sender's memory registers. For the final state after running the protocol with an honest receiver it holds that

$$\rho_{S, C} \approx \rho_S \otimes \mathbb{U}_C. \quad (2)$$

- (3) (Security against dishonest receiver) Let H_R be the Hilbert space associated to all of the receiver's memory registers. For the final state after running the protocol with an honest sender, there exists a binary probability distribution given by (p_0, p_1) such that

$$\rho_{R, M_0, M_1} \approx \sum_b (p_b \rho_{R, M_b}^b \otimes \mathbb{U}_{M_b}). \quad (3)$$

The above properties define statistical security for each feature of the ROT protocol. If any of them holds for the case of a dishonest party being limited to efficient quantum operations and the notion of computational indistinguishability $\approx^{(c)}$ instead, we say that the ROT protocol is computationally secure in the respective sense.

We expect the outputs m_0, m_1, c to be uniformly distributed and the receiver always obtaining the correct corresponding m_c . The first property is typically called *correctness* and it states that, when both parties follow the protocol, the probability of it not aborting *and* having incorrect outputs is negligible in the security parameter.

The probability p_{succ} of the protocol finishing appears explicitly in this expression as the success of quantum protocols often depends on external conditions, most notably the noise in the quantum communication channels. For any specific value of p_{succ} and any $\varepsilon^r \leq 1 - p_{\text{succ}}$ we say that, under the associated external conditions, the protocol is $\varepsilon^{(r)}$ *robust*.

The second property, called *security against dishonest sender*, states that regardless of how much the sender deviates from the protocol, their final quantum state (which includes all the information accessible to them) is uncorrelated to the uniformly distributed value of the receiver's choice bit c . Analogously, the third property, called *security against dishonest receiver*, states that even for a receiver running an arbitrary program, by the end of the protocol there is always at least one of the two strings m_0, m_1 that is completely unknown to them (denoted by m_b).

A. Additional schemes

In this section, we define the subroutines used inside of our main protocol. We start by defining a weakly interactive commitment scheme, which gets its name from the fact that the verifier publishes a single random message at the start, which defines the operations that the committer performs.

Definition 2 (String commitment scheme). Let $k, n \in \mathbb{N}$. A weakly interactive n -bit string commitment scheme with security parameter k is a family of efficient (in n , as well as in k) programs `com`, `open`, `ver`

$$\begin{aligned} \text{com} &: \{0, 1\}^n \times \{0, 1\}^{n_s(k)} \times \{0, 1\}^{n_r(k)} \rightarrow \{0, 1\}^{n_c(k)}; \\ \text{open} &: \{0, 1\}^n \times \{0, 1\}^{n_s(k)} \rightarrow \{0, 1\}^{n_o(k)}; \\ \text{ver} &: \{0, 1\}^{n_c(k)} \times \{0, 1\}^{n_o(k)} \times \{0, 1\}^{n_r(k)} \rightarrow \{0, 1\}^n \cup \{\perp\}, \end{aligned} \quad (4)$$

such that

- (1) (correctness) $\text{ver}(\text{com}(m, s, r), \text{open}(m, s), r) = m$ for all $m \in \{0, 1\}^n$, $s \in \{0, 1\}^{n_s}$, and $r \in \{0, 1\}^{n_r}$.
- (2) (hiding property) For all $m_1, m_2 \in \{0, 1\}^n$ and $r \in \{0, 1\}^{n_r}$ the distributions for $\text{com}(m_1, s_1, r)$ and $\text{com}(m_2, s_2, r)$ are computationally (or statistically) indistinguishable in k whenever s_1, s_2 are sampled uniformly.
- (3) (binding property) For uniformly sampled r , the probability $\varepsilon_{\text{bind}}(k)$ that there exists a tuple $(\text{com}, \text{open}_1, \text{open}_2)$ such that $\text{ver}(\text{com}, \text{open}_1, r) \neq \perp$ and

$$\text{ver}(\text{com}, \text{open}_1, r) \neq \text{ver}(\text{com}, \text{open}_2, r), \quad (5)$$

is negligible in k .

Weakly interactive string commitment schemes can be implemented using common cryptographic primitives like hash functions or pseudorandom generators. Most notably, Naor's commitment protocol [35] provides a black-box construction of weakly interactive commitments from OWFs.

Definition 3 (Verifiable information reconciliation scheme). Let $\mathcal{C} \subseteq \{0, 1\}^n \times \{0, 1\}^n$. A verifiable one-way information reconciliation (IR) scheme with security parameter k and leak ℓ for $\mathcal{C}$ is a pair of efficient programs (syn, dec) with

$$\begin{aligned} \text{syn} : \{0, 1\}^n &\rightarrow \{0, 1\}^\ell, \\ \text{dec} : \{0, 1\}^\ell \times \{0, 1\}^n &\rightarrow \{0, 1\}^n \cup \{\perp\}, \end{aligned} \quad (6)$$

such that,

- (1) (correctness) Whenever $(x, y) \in \mathcal{C}$ it holds that $\text{dec}(\text{syn}(x), y) = x$ except with negligible probability in k .
- (2) (verifiability) For any $(x, y) \in \{0, 1\}^n \times \{0, 1\}^n$ it holds that either $\text{dec}(\text{syn}(x), y) = x$ or $\text{dec}(\text{syn}(x), y) = \perp$, except with negligible probability $\varepsilon_{\text{IR}}(k)$.

Due to Shannon's noisy-channel coding theorem, the size of the leak ℓ for any IR scheme over a discrete memoryless channel is lower bounded by $h(p)$, where p represents the bit-error probability, and $h(\cdot)$ denotes the binary entropy function. For concrete IR schemes, we can usually describe their efficiency using the ratio between the scheme's leak and the theoretical optimal: $f = \ell/h(p)$.

III. THE PROTOCOL

In this section we present the protocol π_{QROT} for an n -bit quantum ROT based on the primitives described in the previous section and the use of quantum communication. The protocol's main security parameter is N_0 , which

corresponds to the number of quantum signals sent during the quantum phase. Additionally, it has two secondary security parameters k, k' , which define the security of the underlying commitment and IR schemes, respectively.

In order to facilitate the finite-key security analysis, the description of π_{QROT} features two statistical tolerance parameters, denoted as δ_1, δ_2 . The role of δ_1 is to account for the error in the estimation of the qubit error rate (QBER), while the role of δ_2 is to account for the small variations in the frequency of outcomes of 50/50 events. These parameters can be ignored (set to zero) when considering very large values of N_0 .

In the following description of the protocol we use the common conjugate coding notation used in BB84-based protocols. The bit values 0, 1 denote the computational and Hadamard bases for qubit Hilbert spaces, respectively. For added clarity, we use the superscripts A and B to respectively denote Alice and Bob. Additionally, we use variable x to denote measurement outcomes and θ to denote measurement bases [e.g., the pair (θ_i^A, x_i^A) denotes that Alice measured her i th subsystem in the θ_i^A basis and obtained x_i^A as the outcome]. We use $|\Phi^+\rangle$ to denote the Bell state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Finally, we will use the relative (or normalized) Hamming weight function $r_H : \{0, 1\}^n \rightarrow [0, 1]$ defined for any $x = (x_1, \dots, x_n)$ as

$$r_H(x) = \frac{1}{n} \sum_{i=1}^n x_i. \quad (7)$$

Parameters:

- (1) Parameter estimation sample ratio $0 < \alpha < 1$.
- (2) Statistical tolerance parameters δ_1, δ_2 .
- (3) Maximum qubit error rate $0 \leq p_{\text{max}} \leq 1/2$.
- (4) Coincidence block size $N_0 \in \mathbb{N}$, test set size $N_{\text{test}} = \alpha N_0$, minimum check set size $N_{\text{check}} = (\frac{1}{2} - \delta_2)\alpha N_0$, and raw string block size $N_{\text{raw}} = (\frac{1}{2} - \delta_2)(1 - \alpha)N_0$.
- (5) Weakly interactive 2-bit string commitment scheme $(\text{com}, \text{open}, \text{ver})$, which is computationally hiding and statistically binding, with security parameter $k \in \mathbb{N}$ and associated string lengths n_s, n_r, n_c, n_o .
- (6) Verifiable one-way information reconciliation scheme (syn, dec) on the set $\mathcal{C} = \{(x, y) \in \{0, 1\}^{N_{\text{raw}}} \times \{0, 1\}^{N_{\text{raw}}} : r_H(x \oplus y) < p_{\text{max}} + \delta_1\}$, with security parameter $k' \in \mathbb{N}$ and leak $\ell = f \cdot h(p_{\text{max}} + \delta_1)$.
- (7) Universal hash family $\mathbf{F} = \{f_i : \{0, 1\}^{N_{\text{raw}}} \rightarrow \{0, 1\}^n\}_i$.

Parties: The sender Alice and the receiver Bob.

Protocol steps:

Quantum phase

- (1) Alice generates the state $\bigotimes_{i=1}^{N_0} |\Phi^+\rangle_i$ and sends one qubit of each entangled pair to Bob through a

(potentially noisy) quantum channel. Then she samples the string $\theta^A \in \{0, 1\}^{N_0}$ and for each $i \in I = \{1, \dots, N_0\}$ performs a measurement in the basis θ_i^A on her qubit of $|\Phi^+\rangle_i$ to obtain the outcome string x^A .

- (2) Bob samples the string $\theta^B \in \{0, 1\}^{N_0}$ and for each $i \in I$ performs a measurement in the basis θ_i^B on his qubit of $|\Phi^+\rangle_i$ to obtain the outcome string x^B .

Commit and open phase

- (3) Alice uniformly samples the string $r \in \{0, 1\}^{n_r}$ and sends it to Bob.
 (4) For each $i \in I$, Bob samples a random string $s_i \in \{0, 1\}^{n_s}$, computes

$$(\text{com}_i, \text{open}_i) = (\text{com}((\theta_i^B, x_i^B), s_i, r), \text{open}((\theta_i^B, x_i^B), s_i)), \quad (8)$$

and sends the string $\text{com} = (\text{com}_i)$ to Alice.

- (5) Alice randomly chooses a subset test $I_t \subset I$ of size αN_0 and sends I_t to Bob.
 (6) For each $j \in I_t$, Bob sends open_j to Alice.
 (7) For each $j \in I_t$, Alice checks that $\text{ver}(\text{com}_j, \text{open}_j, r) \neq \perp$. If so, she sets $(\tilde{\theta}_j^B, \tilde{x}_j^B) = \text{ver}(\text{com}_j, \text{open}_j, r)$. Then, Alice computes the set $I_s = \{j \in I_t | \theta_j^A = \tilde{\theta}_j^B\}$ and the quantity

$$p = r_H(x_{I_s}^A \oplus \tilde{x}_{I_s}^B), \quad (9)$$

and checks that $|I_s| \geq N_{\text{check}}$ and $p \leq p_{\text{max}}$. If any of the checks fail Alice aborts the protocol.

String separation phase

- (8) Alice sends $\theta_{I_t}^A$ to Bob.

- (9) Bob constructs the set I_0 by randomly selecting N_{raw} indices $i \in I_t$ for which $\theta_i^A = \theta_i^B$. Similarly, he constructs I_1 by randomly selecting N_{raw} indices $i \in I_t$ for which $\theta_i^A \neq \theta_i^B$. He then samples a random bit c and sends the ordered pair $(I_c, I_{\bar{c}})$ to Alice. If Bob is not able to construct I_0 or I_1 , he aborts the protocol.

Post-processing phase

- (10) Alice computes the strings $(\text{syn}(x_{I_c}^A), \text{syn}(x_{I_{\bar{c}}}^A))$ and sends the result to Bob.
 (11) Bob computes $\text{dec}(x_{I_0}^B, \text{syn}(x_{I_0}^A)) = y^B$. If $y^B = \perp$ Bob aborts the protocol.
 (12) Alice randomly samples $f \in \mathbf{F}$, computes $m_0^A = f(x_{I_c}^A)$ and $m_1^A = f(x_{I_{\bar{c}}}^A)$, sends the description of f to Bob and outputs (m_0^A, m_1^A) .
 (13) Bob computes $m^B = f(y^B)$ and outputs (m^B, c) .

A. Security and performance of the main protocol

We start by stating the main theorem regarding security of the proposed π_{QROT} protocol.

Theorem 1 (Security of π_{QROT}). The protocol π_{QROT} is a statistically correct, computationally secure against dishonest sender, and statistically secure against dishonest receiver n -bit ROT protocol.

A high-level proof of Theorem 1, including the derivation of the security bounds from Lemmas 1 and 2 can be found in Sec. IV and further details can be found in Appendix B. The security of π_{QROT} is given by its main security parameter N_0 , as well as the security parameters of the underlying commitment and IR schemes k and k' , respectively. These values can be computed for the statistical security features of the protocol and are given by the following lemmas:

Lemma 1 (Correctness). The outputs of π_{QROT} when run by honest sender and receiver satisfy

$$\rho_{M_0, M_1, C, M_C} \approx_{\varepsilon} \frac{P_{\text{succ}}}{2^{(2n+1)}} \sum_{\substack{m_0, m_1 \in \{0, 1\}^n \\ c \in \{0, 1\}}} (|m_0\rangle\langle m_0|_{M_0} |m_1\rangle\langle m_1|_{M_1} |c\rangle\langle c|_C |m_c\rangle\langle m_c|_{M_C}), \quad (10)$$

with

$$\varepsilon = 2^{-\frac{1}{2}(N_{\text{raw}} - n)} + 2\varepsilon_{\text{IR}}(k'), \quad (11)$$

where ε_{IR} is a negligible function given by the security of the underlying IR scheme.

Lemma 2 (Security against dishonest receiver). For the final state after running the protocol of π_{QROT} with an honest sender, there exists a binary probability distribution given by (p_0, p_1) such that

$$\rho_{R, M_0, M_1} \approx_{\varepsilon'} \sum_b (p_b \rho_{R, M_b}^b \otimes \mathbf{U}_{M_b}), \quad (12)$$

with

$$\begin{aligned} \varepsilon' = & \sqrt{2} \left(e^{-\frac{1}{2}(1-\alpha)^2 N_{\text{test}} \delta_1^2} + e^{-\frac{1}{2} N_{\text{check}} \delta_1^2} \right)^{\frac{1}{2}} \\ & + e^{-D_{\text{KL}} \left(\frac{1}{2} - \delta_2 \middle| \frac{1}{2} \right) (1-\alpha) N_0} + \varepsilon_{\text{bind}}(k) \\ & + \frac{1}{2} \cdot 2^{\frac{1}{2} \left(n - N_{\text{raw}} \left(\frac{1}{2} - \frac{2\delta_2}{1-2\delta_2} - h \left(\frac{p_{\text{max}} + \delta_1}{\frac{1}{2} - \delta_2} \right) - f \cdot h(p_{\text{max}} + \delta_1) \right) \right)} \end{aligned} \quad (13)$$

where $\mathcal{H}_R$ denotes the Hilbert space associated to all of the receiver's memory registers and $\varepsilon_{\text{bind}}$ is a negligible function given by the security of the underlying commitment scheme.

We can use these results to find the minimum requirements, both in terms of channel losses and number of shared entangled qubits, necessary to securely realize ROT for a given security level. We focus on the quantity

$$\varepsilon_{\text{max}} = \varepsilon + \varepsilon'. \quad (14)$$

For the purposes of this analysis, we assume that the commitment and IR schemes, as well as their security parameters k, k' , are appropriately chosen to satisfy the desired security level and we focus on the dependence of ε_{max} on the channel error rate, characterized by the parameter p_{max} , and the number of quantum signals N_0 . We are also interested in a quantity known as the secret key rate R_{key} . For given values of N_0 , α , δ_1 , δ_2 , p_{max} , and ε_{max} , let n_{max} be the largest number for which the associated n_{max} -bit ROT has at least security ε_{max} , then

$$R_{\text{key}} = \frac{n_{\text{max}}}{N_0}, \quad (15)$$

represents the ratio in which the original measurements of the shared qubits “transform” into the oblivious key. In Fig. 2 we can see the behavior of R_{key} as p_{max} increases. Note that, similarly to the case of quantum key distribution, there is a critical error p_{crit} after which R_{key} becomes negative and no secure key can be generated. The value of p_{crit} is upper bounded by ≈ 0.028 , which is achieved when we set $\alpha, \delta_1, \delta_2 \rightarrow 0$, and $N_0 \rightarrow \infty$.

Another important aspect to analyze is the relation between R_{key} and N_0 , which is shown in Fig. 3. Fixing the $\alpha, \delta_1, \delta_2, p_{\text{max}}$, there is a clearly marked phase transition-like behavior in which, for each ε_{max} , there is a critical value of $N_0 = N_{\text{crit}}$ before which $R_{\text{key}} = 0$, and after which it quickly reaches its maximum value. This result comes from the fact that the parameter estimation requires relatively big sample sizes to reach high confidence. It shows that, even for small n , there is a minimum amount of entangled qubits needed to be shared. In some cases, for

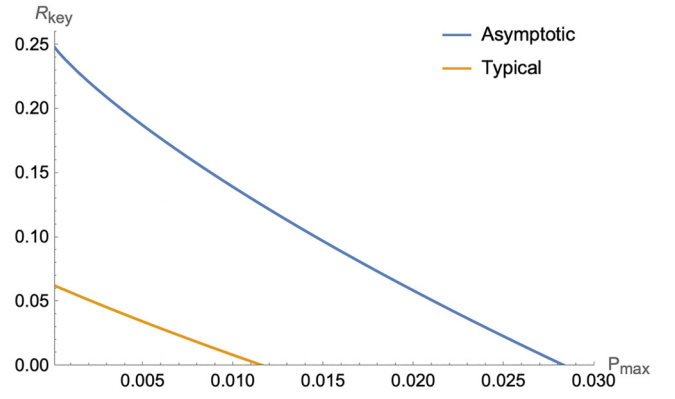


FIG. 2. Maximum key rate output n/N_0 vs error rate p_{max} . The blue line represents the upper bound for the key rate, when $N_0 \rightarrow \infty$, $\alpha, \delta_1, \delta_2$ are taken to be 0 and $f = 1$. The orange line represents a more typical case with $\alpha = 0.35$, $\delta_1 = 0.01$, $\delta_2 = 0.025$, and $f = 1.2$.

instance, generating a 1-bit oblivious key or a 128-bit one may have similar costs in terms of quantum communication. Because the use of resources of the protocol scales with N_0 , the parameters $\alpha, \delta_1, \delta_2$ should be chosen such that N_{crit} is the smallest. Figure 4 exemplifies the dependency of N_{crit} on ε_{max} .

B. Experimental implementation performance

An experiment was implemented to test the performance of the π_{QROT} protocol with contemporary technology. Data was acquired using a picosecond pulsed photon source in a Sagnac configuration [45], producing wavelength degenerate, polarization-entangled photons at 1550 nm. In this setup, entangled photons were produced via spontaneous parametric down-conversion (SPDC) by applying a laser pump beam into a 30-mm-long periodically poled potassium titanyl phosphate (ppKTP) crystal. The photon pairs

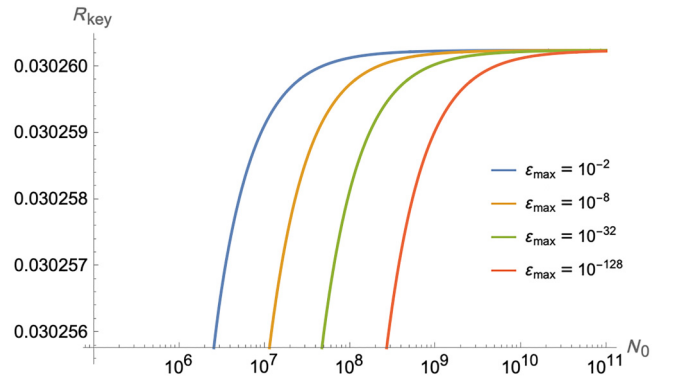


FIG. 3. Maximum key rate behavior as a function of N_0 for different security levels. Parameter values used are $\alpha = 0.35$; $\delta_1 = 9.20 \times 10^{-3}$; $\delta_2 = 3.00 \times 10^{-3}$; $p_{\text{max}} = 0.01$; $f = 1.2$.

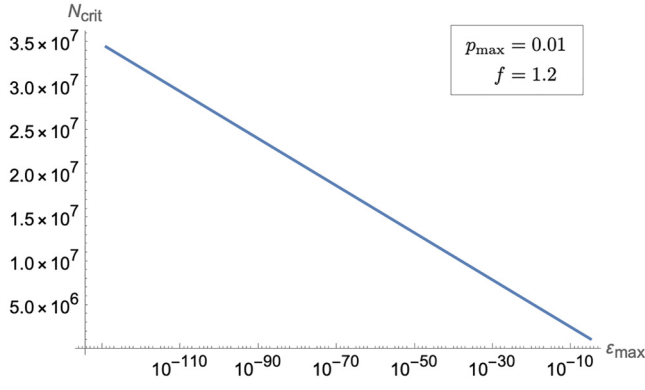


FIG. 4. Critical value N_{crit} of the number of shared qubits needed to obtain positive key rates as a function of the security level. The values of N_{crit} were computed using the parameters $\alpha, \delta_1, \delta_2$ that minimize the value of N_{crit} for each ϵ_{max} .

were split using a half-wave plate (HWP) and a polarizing beam splitter (PBS), and then sent to each party where they are detected using superconducting nanowire single-photon detectors.

To test the OT speed of this implementation, different values for the power P of the laser pump were tested, as well as the use of multiplexing. As the P increases, the amount of coincidences detected per second R_c increases, but the fidelity of the produced entangled pairs decreases, resulting in larger values for qubit error rate, which is represented by the protocol parameter p_{max} . The number of maximum potential OT instances per second is computed as

$$R_{\text{OT}} = \frac{R_c}{N_{\text{crit}}}, \quad (16)$$

where N_{crit} is computed using the optimal values of $\alpha, \delta_1, \delta_2$ for the respective error rate p_{max} and undetected multi-photon rate p_{multi} associated to P , assuming perfectly efficient information reconciliation, $f = 1$ (see Sec. V for the details on the implementation and its security). As seen in Figure 5, for this implementation, the additional coincidence rate gained by increasing P is not enough to compensate for the induced increased error. This result is not immediately obvious, as N_{crit} does not depend explicitly on p_{max} . The decrease in performance comes from the fact that increasing p_{max} limits the values that δ_1 can have while maintaining positive key rates. This restriction on the values of δ_1 ultimately results in an increase in N_{crit} and therefore, a reduction on R_{OT} .

Table I shows an example of the performance of the protocol in a real-world implementation using the data from the experimental setup. For the commit and open phase, the weakly interactive string commitment protocol introduced in Ref. [35] was implemented using the BLAKE3 hash function algorithm as a one-way function. For the

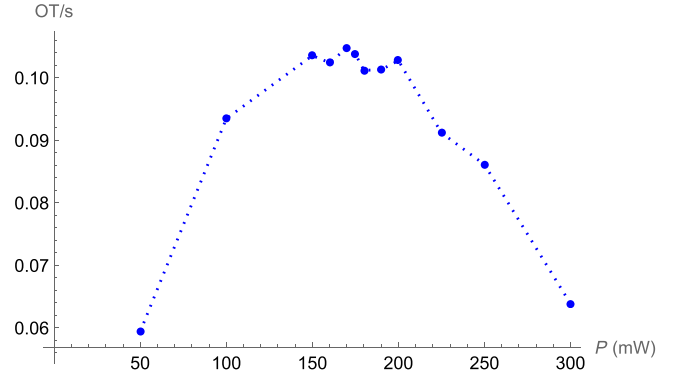


FIG. 5. Maximum potential ROT rates as a function of the pump power P for $\epsilon_{\text{max}} = 10^{-7}$. We see that the best performance is obtained at a laser pump power of $P = 170$ mW, corresponding to a coincidence rate close to 2.45 kHz. The uncertainty on the power measurement (x axis) along with the error bars resulting from the Poissonian noise on the coincidence counts (used to calculate y values) are negligible with respect to the current plot scale.

post-processing phase, a low-density parity-check (LDPC) code was used for IR, and random binary matrices were used to implement the universal hash family for privacy amplification. We evaluated the performance by the number of 128-bit ROT instances able to be completed per second. (It is worth noting that, using a Mac mini M1 2020 16 GB computer, the post-processing throughput was enough to handle all the data from the experiment, the bottleneck being the quantum signal generation rate.) Detector measurement data is publicly available in Ref. [46].

IV. SECURITY ANALYSIS

In this section, we prove the main security result, which relates the overall security of the protocol as a function of

TABLE I. Table of protocols parameters and the resulting performance. The values of N_0 and δ_1 and the laser pump power were optimized to yield the highest ROT rate for an LDPC code with efficiency $f = 1.61$.

Parameter	Symbol	Value
Message size (bits)	n	128
Security level	ϵ_{max}	1.91×10^{-8}
Cost in quantum signals	N_0	5.86×10^6
Max allowed QBER	p_{max}	1.14%
Testing set ratio	α	0.35
Statistical parameter 1	δ_1	9.00×10^{-3}
Statistical parameter 2	δ_2	3×10^{-3}
IR verifiability security	ϵ_{IR}	2^{-32}
Commitment binding security	ϵ_{bind}	2^{-32}
Efficiency of IR	f	1.64
Max allowed multi-photon rate	p_{multi}	3.67×10^{-3}
ROT rate		0.023 ROT/s

its parameters N_0, α, δ_1 , and δ_2 in Theorem 1. For clarity of presentation, we have compacted some of the properties into lemmas, for which detailed proofs can be found in Appendix B. Definitions and properties of entropic quantities can be found in Appendix A

A. Correctness

In order to prove correctness we need to show that either the protocol either finishes with Alice outputting uniformly distributed messages m_0, m_1 and Bob outputting a uniformly random bit c and the corresponding message m_c , or it aborts, except with negligible probability.

Recall that we model the aborted state of the protocol as the zero operator. This way, whenever we have a mixture of states, some of which trigger aborting and some that do not, the abort operation removes the events that trigger it from the mixture, effectively reducing its trace by the probability of aborting. There are three instances where the protocol can abort: first during step (7) if the estimated qubit error rate is larger than $p_{\max}$; the second one is during step (9) if Bob does not have enough (mis)matching bases to construct the sets I_0, I_1 ; and finally during step (11) if the IR verification fails. The probability of aborting in steps (7) and (11) depends on the particular transformation that the states undergo when being sent from Alice's

to Bob's laboratory, about which we make no assumptions. We can group these three abort events and denote by p_{abort} the probability of the protocol aborting by the end of step (11). The state at this point can be written as $(1 - p_{\text{abort}})\rho^\top$, where $\rho^\top$ represents the normalized state conditioned that the protocol has not aborted by this point. As Lemma 3 states, the verifiability property of the information reconciliation scheme guarantees that the states that "survive" past Step (11) have the property that Bob's corrected string y^B is the same as Alice's outcome string $x_{I_0}^A$, which is uniformly distributed.

Lemma 3. Let $X_{I_0}^A, X_{I_1}^A, C, Y^B$ denote the systems holding the information of the respective values $x_{I_0}^A, x_{I_1}^A, c$, and y^B of π_{QROT} . Denote by $\rho^\top$ the parties' joint state at the end of step (11) conditioned that Bob constructed the sets (I_0, I_1) during step (9) and the protocol has not aborted. Assume both parties follow the steps of the protocol, then

$$\rho_{X_{I_0}^A, X_{I_1}^A, C, Y^B}^\top \approx_{\varepsilon_{\text{IR}}(k')} \tilde{\rho}_{X_{I_0}^A, X_{I_1}^A, C, Y^B}^\top, \quad (17)$$

where $\varepsilon_{\text{IR}}(k')$ is a negligible function given by the security of the underlying information reconciliation scheme, k' its associated security parameter, and

$$\tilde{\rho}_{X_{I_0}^A, X_{I_1}^A, C, Y^B}^\top = \frac{1}{2^{(2N_{\text{raw}}+1)}} \sum_{x_{I_0}^A, x_{I_1}^A} |x_{I_0}\rangle\langle x_{I_0}|_{X_{I_0}^A} |x_{I_1}\rangle\langle x_{I_1}|_{X_{I_1}^A} |x_{I_0}\rangle\langle x_{I_0}|_{Y^B} |c\rangle\langle c|_C. \quad (18)$$

During step (12) universal hashing is used in both $x_{I_0}^A$ and $x_{I_1}^A$ to obtain m_c and $m_{\bar{c}}$. Because Eq. (18) describes a state for which the $X_{I_0}^A, X_{I_1}^A$, and C subsystems are independent and uniformly distributed, it follows from Lemma 3 that

$$\begin{aligned} H_{\min}^{\varepsilon_{\text{IR}}(k')} (X_{I_0}^A Y^B | X_{I_1}^A C)_{\rho^\top} &= H_{\min}^{\varepsilon_{\text{IR}}(k')} (X_{I_1}^A | X_{I_0}^A Y^B C)_{\rho^\top} \\ &= N_{\text{raw}}. \end{aligned} \quad (19)$$

Finally, using the quantum leftover hash Lemma A4 twice (once for m_0 and m_1) with the corresponding entropy terms given by Eq. (19), together with Lemma A1(1), we conclude that the state $\rho_{M_0, M_1, C, M_C}^{(\text{out})}$ of the output systems after the post-processing phase satisfies (substituting $p_{\text{succ}} = 1 - p_{\text{abort}}$)

$$\rho_{M_0, M_1, C, M_C}^{(\text{out})} \approx_{\varepsilon} \frac{p_{\text{succ}}}{2^{(2n+1)}} \sum_{\substack{m_0, m_1 \in \{0,1\}^n \\ c \in \{0,1\}}} |m_0\rangle\langle m_0|_{M_0} |m_1\rangle\langle m_1|_{M_1} |c\rangle\langle c|_C |m_c\rangle\langle m_c|_{M_C}, \quad (20)$$

with

$$\varepsilon \leq 2^{-\frac{1}{2}(N_{\text{raw}}-n)} + 2\varepsilon_{\text{IR}}(k'). \quad (21)$$

B. Security against dishonest sender

For this scenario we show that, in the case of an honest Bob and Alice running an arbitrary program, the resulting state after the protocol successfully finishes satisfies Eq. (2). In other words, independently of what quantum state Alice shares at the beginning of the protocol and which operations she performs on her systems, her final state is independent of the value of c . We assume that Alice's laboratory consists of everything outside Bob's. In particular, this means that she controls the environment, which includes the transmission channels. We also assume that Alice is limited to performing efficient computations.

Let A be the system consisting of all of Alice's laboratory after step (1) of the protocol, that is, A contains her part of the shared system and every other ancillary system she may have access, but does not contain any system from Bob's laboratory, including Bob's part of the system shared in step (1). During the execution of the protocol, Alice receives external information from Bob exactly three times: the commitment information shared during step (4), the opening information open_{I_t} for the commitments associated to the test set I_t in step (6), and the information of the pair of sets $(J_0, J_1) = (I_c, I_{\bar{c}})$ during step (9). Let $\text{COM} = (\text{COM}_i)_{i=1}^{N_0}$ and $\text{OPEN} = (\text{OPEN}_i)_{i=1}^{N_0}$ be the respective systems used by Bob to store the information of the strings $\text{com} = (\text{com}_i)_{i=1}^{N_0}$ and $\text{open} = (\text{open}_i)_{i=1}^{N_0}$, and let SEP be the system holding the string separation information (J_0, J_1) . We want to show that, by the end of the protocol, the state of the system $A, \text{COM}, \text{OPEN}_J, \text{SEP}, C$ satisfies

$$\rho_{A, \text{COM}, \text{OPEN}_{\bar{I}_t}, \text{SEP}, C} \approx^{(c)} \rho_{A, \text{COM}, \text{OPEN}_{\bar{I}_t}, \text{SEP}} \otimes \mathbf{U}_C. \quad (22)$$

To guarantee that Alice will not be able to obtain information about the value of c during the string separation phase, it is necessary to show that Alice does not have access to the information of Bob's bases choices θ_{I_0, J_1}^B from the commitments sent by Bob during step (4) of the protocol. As shown by Lemma 4, the shared state of the parties after the commitment information is sent is computationally indistinguishable from a state where Alice's information is independent of $\theta_{I_t}^B$.

Lemma 4. Assuming Bob follows the protocol, for any $J \subseteq I$, the state of the system $A, \text{COM}, \text{OPEN}_J, \Theta_J^B$ after step (4) satisfies

$$\rho_{A, \text{COM}, \text{OPEN}_J, \Theta_J^B} \approx^{(c)} \rho_{A, \text{COM}, \text{OPEN}_J} \otimes \mathbf{U}_{\Theta_J^B}. \quad (23)$$

At step (8) of the protocol, Alice sends Bob the system $\Theta_{I_t}^A$ intended to have the information of her measurement bases. Bob then is able to determine the indices for which $\theta_{I_t}^A$ and $\theta_{I_t}^B$ coincide. With this information, he randomly selects sets $I_0, I_1 \in \bar{I}_t$ of size N_{raw} for which all indices

are associated with matching (for I_0) or nonmatching (for I_1) bases. Then he computes $(J_0, J_1) = (I_c, I_{\bar{c}})$, by flipping the order if the pair (I_0, I_1) depending on the value of c . Clearly, (J_0, J_1) depend on both $\theta_{I_t}^B$ and c , but as Lemma 5 states, any correlation between (J_0, J_1) , c , and Alice's information disappears if one does not have access to $\theta_{I_t}^B$.

Lemma 5. Denote by A' the system representing Alice's laboratory at the start of step (9). Let $\mathcal{E}^{(I_t)} : \mathcal{D}(\mathcal{H}_{A', \Theta_{I_t}^A, \Theta_{I_t}^B, C}) \rightarrow \mathcal{D}(\mathcal{H}_{A', \Theta_{I_t}^A, \Theta_{I_t}^B, C, \text{SEP}})$ be the quantum operation used by Bob to compute the string separation information (J_0, J_1) during step (9) of the protocol. The resulting state after applying $\mathcal{E}^{(I_t)}$ to a product state of the form

$$\mathcal{E}^{(I_t)}(\rho_{A', \Theta_{I_t}^A} \otimes \mathbf{U}_{\Theta_{I_t}^B} \otimes \mathbf{U}_C) = \sigma_{A', \Theta_{I_t}^A, \Theta_{I_t}^B, C, \text{SEP}} \quad (24)$$

satisfies

$$\text{Tr}_{\Theta_{I_t}^A, \Theta_{I_t}^B} \left[\sigma_{A', \Theta_{I_t}^A, \Theta_{I_t}^B, C, \text{SEP}} \right] = \sigma_{A'} \otimes \sigma_{\text{SEP}} \otimes \mathbf{U}_C. \quad (25)$$

A proof of both Lemmas 4 and 5 can be found in Appendix B 3. By setting $J = I_t$, Lemma 4 guarantees that Alice's system's state after the opening information has been sent is computationally indistinguishable from one that is completely uncorrelated with Bob's measurement basis information in $\bar{I}_t$.

Additionally, by recalling that the value of c is sampled independently of any of the considered systems, we know that the state $\rho_{A', \Theta_{I_t}^A, \Theta_{I_t}^B, C}$ before (J_0, J_1) is computed has the required product form and, from Lemma 5, we conclude that the state of all of Alice's system at this point is computationally indistinguishable from a state uncorrelated with C . Let $\mathcal{E}$ be the operation Alice performs in her system from here to the end the protocol. By using Lemma A2 (4) and grouping all of Alice's systems into S , we obtain the desired result:

$$\rho_{S, C} \approx^{(c)} \rho_S \otimes \mathbf{U}_C. \quad (26)$$

C. Security against dishonest receiver

We consider now the scenario in which Alice runs the protocol honestly and Bob runs an arbitrary program. For this analysis, note that Alice trusts her quantum state preparation and detection. We want to show that the state after finishing the protocol successfully satisfies Eq. (3). This means that the state at the end of the protocol can be described as a mixture of states where Bob's system is uncorrelated with at least one of the two strings outputted by Alice. Similarly to the dishonest sender's case, we assume that Bob's laboratory consists of everything

outside Alice's, which means that he controls the communication channels and the environment. However, we do not assume that Bob is restricted to efficient computations.

The values of Alice's output strings depend on several quantities: Alice's measurement outcomes, the choice of the I_t, J_0, J_1 subsets, and the choice of hashing function f during the post-processing phase of the protocol. From all of these, the only ones that are not made explicitly public during the protocol's execution are Alice's measurement outcomes. Instead, partial information of these outcomes is revealed at different steps of the protocol. Let $x_{J_0}^A, x_{J_1}^A$ be the sub-strings of measurement outcomes used to compute Alice's outputs m_0, m_1 , respectively, and let R denote Bob's system at the end of the protocol (which includes all the systems that Alice sent during the execution of the protocol). In order to prove security we need to show that the joint state of the system $X_{J_0}^A, X_{J_1}^A, R$ can be written as a mixture of states ρ^b (with $b \in \{0, 1\}$) such that the conditional min-entropy $H_{\min}^\varepsilon(X_{J_b}^A | R)_{\rho^b}$ is high enough, so that we can use the leftover hash Lemma A4 to guarantee that the outcome of the universal hashing $m_b = f(x_{J_b}^A)$ is uncorrelated with R .

At the start of the protocol the parties share a completely correlated entangled system. If the parties make measurements as intended, their outcomes will be only partially correlated, but if Bob was able to postpone his measurement until after Alice's reveals her measurement bases,

Bob could potentially obtain the whole information of x^A by measuring in the appropriate basis on his system. To prevent this, Bob is required to commit his measurement bases and results to Alice before knowing which set is going to be tested. Then a statistical test is performed in step (7) to estimate the correlation of Alice's measurement outcomes with the ones that Bob committed. As Lemma 6 states, any state passing the aforementioned test is such that, regardless of how Bob defines the sets (J_0, J_1) during the string separation phase, there is a minimum of uncertainty that he has with respect to Alice's measurement outcomes. Recall that, when Alice is honest, the overall state of the protocol before step (8) will be a partially classical state, which could be written as a mixture over all of Alice's classical information. Let $\vec{\tau} = (x_{I_t}^A, \theta^A, r, \text{com}, I_t, I_s, \text{open}_{I_t})$ denote the *transcript of the protocol* up to step (8), and let $\rho_{X^A B}(\vec{\tau}, J_0, J_1)$ be the joint state of Alice's measurement outcomes and Bob's laboratory conditioned to $\vec{\tau}, J_0, J_1$.

Lemma 6. Assuming Alice follows the protocol, let T, SEP, B denote the systems of the protocols transcript, the strings J_0, J_1 , and Bob's laboratory at the end of step (9) of the protocol, and let $\rho_{T, \text{SEP}, X^A, B}$ be the state of the joint system at that point. There exists a state $\tilde{\rho}_{T, \text{SEP}, X^A, B}$, which is classical in T and SEP such as the following:

(1) The conditioned states $\tilde{\rho}_{X^A B}(\vec{\tau}, J_0, J_1)$ satisfy

$$H_{\min}(X_{J_0}^A | X_{J_1}^A B)_{\tilde{\rho}(\vec{\tau}, J_0, J_1)} + H_{\min}(X_{J_1}^A | X_{J_0}^A B)_{\tilde{\rho}(\vec{\tau}, J_0, J_1)} \geq 2N_{\text{raw}} \left(\frac{1}{2} - \delta_2 - h \left(\frac{p_{\max} + \delta_1}{\frac{1}{2} - \delta_2} \right) \right), \quad (27)$$

(2) $\rho_{T, \text{SEP}, X^A, B} \approx_\varepsilon \tilde{\rho}_{T, \text{SEP}, X^A, B}$, with

$$\varepsilon = \left(2 \left(e^{-\frac{1}{2}\alpha(1-\alpha)^2 N_0 \delta_1^2} + e^{-\frac{1}{2}(\frac{1}{2}-\delta_2)\alpha N_0 \delta_1^2} \right) \right)^{\frac{1}{2}} + e^{-D_{\text{KL}}(\frac{1}{2}-\delta_2 | \frac{1}{2}) (1-\alpha) N_0} + \varepsilon_{\text{bind}}(k), \quad (28)$$

where $h(\cdot)$ and $D_{\text{KL}}(\cdot | \cdot)$ denote the binary entropy and the binary relative entropy functions, respectively, and $\varepsilon_{\text{bind}}(k)$ is a negligible function given by the binding property of the commitment scheme.

To reach the desired result, we will first show that a state $\tilde{\rho}_{T, \text{SEP}, X^A, B}$ satisfying Lemma 6(1) also satisfies a tighter version Lemma 2, and then use Lemma 6(2) to attain the bound for the real protocol's outcome. Since $\tilde{\rho}_{T, \text{SEP}, X^A, B}$ is classical in both T and SEP we can write the state of the joint system of Alice's measurement outcomes and Bob's

laboratory as a mixture over all the possible transcripts at that point, that is

$$\tilde{\rho}_{X^A B} = \sum_{\vec{\tau}} P(\vec{\tau}, J_0, J_1) \tilde{\rho}_{X^A B}(\vec{\tau}, J_0, J_1), \quad (29)$$

where $P(\vec{\tau}, J_0, J_1)$ defines a probability distribution, which is dependent on Bob's behavior during the previous steps. We can now separate the $\tilde{\rho}_{X^A B}(\vec{\tau}, J_0, J_1)$ in two categories depending on which of the $x_{J_0}^A, x_{J_1}^A$ is the least correlated with Bob's system. Consider the function $b(\vec{\tau}, J_0, J_1)$ to be equal to 0 if $H_{\min}^\varepsilon(X_{J_0}^A | X_{J_1}^A B)_{\rho(\vec{\tau}, J_0, J_1)} \geq H_{\min}^\varepsilon(X_{J_1}^A | X_{J_0}^A B)_{\rho(\vec{\tau}, J_0, J_1)}$, and equal to 1 otherwise. By regrouping the terms from Eq. (29) for which the value of b is the same, we can rewrite the joint state as

$$\tilde{\rho}_{X^A B} = \sum_{b \in \{0,1\}} P_b \tilde{\rho}_{X^A B}^b, \quad (30)$$

where, from Lemma 6 and recalling that, as Lemma A3 (5) states, the min-entropy of a mixture is lower bounded by that of the term with the least min-entropy, we know that

$$H_{\min}(X_{J_b}^A | X_{J_{\bar{b}}}^A B)_{\tilde{\rho}^b} \geq N_{\text{raw}} \left(\frac{1}{2} - \frac{2\delta_2}{1-2\delta_2} - h\left(\frac{p_{\max} + \delta_1}{\frac{1}{2} - \delta_2}\right) \right). \quad (31)$$

At step (10), Alice shares with Bob the syndromes $S_0 = \text{syn}(x_{J_0}^A)$ and $S_1 = \text{syn}(x_{J_1}^A)$. Since these syndromes are completely determined by the respective sub-strings $x_{J_i}^A$, we know that

$$\begin{aligned} H_{\min}(X_{J_b}^A | S_b S_{\bar{b}} B) &\geq H_{\min}(X_{J_b}^A | S_b X_{J_{\bar{b}}}^A B) \\ &\geq H_{\min}(X_{J_b}^A | X_{J_{\bar{b}}}^A B) - H_{\max}(S_b) \\ &\geq N_{\text{raw}} \left(\frac{1}{2} - \frac{2\delta_2}{1-2\delta_2} - h\left(\frac{p_{\max} + \delta_1}{\frac{1}{2} - \delta_2}\right) - f \cdot h(p_{\max} + \delta_1) \right), \end{aligned} \quad (32)$$

where the second inequality follows from Lemma A3 (3) and (4), and the max-entropy term $H_{\max}(S_b)$ is upper bounded by the size in bits of the syndrome $\ell = N_{\text{raw}}(f \cdot h(p_{\max} + \delta_1))$. Now we can apply Eq. (32) to Lemma A4, which states that, for the outcomes M_0, M_1 of the universal hashing by Alice in step (12) and Bob's final system R it holds that

$$\tilde{\rho}_{R, M_0, M_1}^b \approx_{\varepsilon'} \tilde{\rho}_{R, M_{\bar{b}}}^b \otimes \mathbf{U}_{M_b}, \quad (33)$$

with

$$\varepsilon' = \frac{1}{2} \cdot 2^{\frac{1}{2} \left(n - N_{\text{raw}} \left(\frac{1}{2} - \frac{2\delta_2}{1-2\delta_2} - h\left(\frac{p_{\max} + \delta_1}{\frac{1}{2} - \delta_2}\right) - f \cdot h(p_{\max} + \delta_1) \right) \right)}. \quad (34)$$

Finally, by applying Lemma A1 (3) and (4) to Eqs. (30) and (33), and then Lemma A1 (1) to Eq. (28) we get the desired result:

$$\rho_{R, M_0, M_1} \approx_{\varepsilon} \sum_b P_b \rho_{R, M_{\bar{b}}}^b \otimes \mathbf{U}_{M_b}, \quad (35)$$

with

$$\begin{aligned} \varepsilon &= \sqrt{2} \left(e^{-\frac{1}{2}(1-\alpha)^2 N_{\text{test}} \delta_1^2} + e^{-\frac{1}{2} N_{\text{check}} \delta_1^2} \right)^{\frac{1}{2}} + e^{-D_{\text{KL}}\left(\frac{1}{2} - \delta_2 \middle| \frac{1}{2}\right) (1-\alpha) N_0} + \varepsilon_{\text{bind}}(k) \\ &\quad + \frac{1}{2} \cdot 2^{\frac{1}{2} \left(n - N_{\text{raw}} \left(\frac{1}{2} - \frac{2\delta_2}{1-2\delta_2} - h\left(\frac{p_{\max} + \delta_1}{\frac{1}{2} - \delta_2}\right) - f \cdot h(p_{\max} + \delta_1) \right) \right)}. \end{aligned} \quad (36)$$

D. Composability considerations

Since OT protocols are mainly used as a subroutine of larger applications it is important to understand the

composability properties of π_{ROT} . In general, this is done through simulation-based composability frameworks. As mentioned in Sec. I, this protocol is based on the BBCS92

construction, which has been proven secure in the quantum universal composability (UC) framework by Unruh [41] assuming access to an ideal commitment functionality. This means that we can understand the composability properties of π_{ROT} by understanding the respective properties of the underlying weakly interactive commitment protocol.

It is well known that UC commitments are impossible to realize in the plain model [22,25]. Because of this, protocols are often analyzed within a hybrid model, where the parties have access to some base external functionality. We show in Appendix IVD that there exists a family of commitment schemes that are both weakly interactive and UC-secure in the *classical access* random oracle model (ROM) [47]. This, in tandem with the aforementioned reduction of OT to commitments, results in the following theorem:

Theorem 2. There exists a family of weakly interactive commitment schemes in relation to which π_{ROT} is UC-secure in the classical access ROM.

In relation to Theorem 2, we want to emphasize that, even though limiting the access to the random oracle to be classical may seem at first strong in the context of a quantum protocol (where the parties are required access to some quantum capabilities), it has little impact in the resulting security of larger MPC protocols for which the security is analyzed in the classical setting.

Finally, we would like to stress the merits of Definition 1 by itself. In particular, this definition was studied in Refs. [48] and [44] and stated to ensure security when the protocol is executed sequentially. Furthermore, the indistinguishability properties stated in Definition 1 provide a very strong security guarantee and, because the protocol does not have external inputs and the indistinguishability relations include arbitrary external systems, these properties will still hold in any environment, which makes it relatively straightforward to analyze as part of bigger applications.

V. EXPERIMENTAL IMPLEMENTATION

A. Description of the setup

A schematic representation of the experimental setup can be seen in Fig. 6. Spontaneous parametric down-conversion (SPDC), attributed to Alice, is used to create polarization entangled photon pairs in the state $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$, which are coupled into optical fiber. One photon is sent through a 50:50 fiber beam splitter, probabilistically routing it to one of two polarization projection stages. There, a quarter-wave plate (QWP), a half-wave plate (HWP), and a polarizing beam splitter (PBS) are used to project the photons state onto the linear (H/V) or diagonal ($+/-$) basis, respectively. All photons

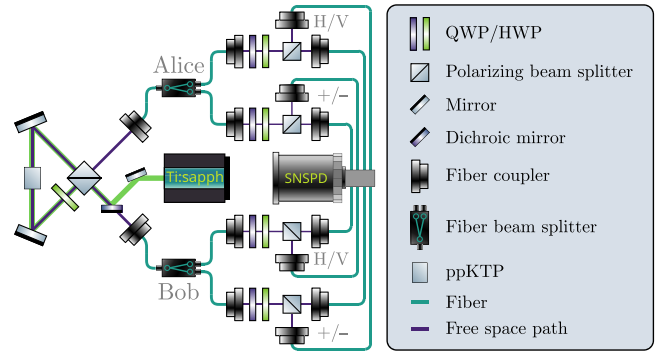


FIG. 6. Experimental setup. Polarization-entangled photon pairs are created using spontaneous parametric down-conversion. Alice's and Bob's photons are individually fiber coupled and each sent to 50:50 fiber beam splitters, which probabilistically route them to free-space polarization projection stages—one projecting onto the linear, and one onto the diagonal basis each for Bob and Alice.

are sent to superconducting nanowire single photon detectors (SNSPDs) and their arrival time is recorded using a time tagging module (TTM). The second photon of the state $|\Psi^+\rangle$, attributed to Bob, travels through an equivalent probabilistic projection setup.

Entangled photon pairs are generated using collinear type-II SPDC in a periodically poled KTiOPO_4 crystal with a poling period of $46.2\ \mu\text{m}$ inside of a Sagnac interferometer. The pump light is produced by a pulsed Ti:sapphire laser (Coherent Mira 900HP) with a pulse width of 2.93 ps and a central wavelength of $\lambda_p = 773\ \text{nm}$, creating degenerate single-photon pairs at $\lambda_s = \lambda_i = 1546\ \text{nm}$. The laser's inherent pulse repetition rate of 76 MHz is doubled twice to 304 MHz using a passive temporal multiplexing scheme [49]. More precisely, for n simultaneously emitted pairs and k multiplexing stages, each doubling the repetition rate, higher-order pair production events are attenuated by a factor of $1/(2^k)^{n-1}$. In our experiment, $k = 2$, so this scheme reduces the probability of emitting a double pair ($n = 2$) by a factor of 4 compared to a source relying on the pump's inherent repetition rate, while the single-pair emission probability remains constant. Finally, about 100 m of single-mode fiber separate the experimental setup from the 1-K cryostat housing the SNSPDs with a detection efficiency of around 95% and a dark-count rate of around 300 Hz.

We note that our entanglement-based implementation presents two main technological advantages over prepare-and-measure configurations:

- (1) It circumvents the need for a certified quantum random number generator or for classical pseudo-randomness that may compromise the security of the quantum phase: instead of feeding random (or

pseudorandom) sequences into the active polarization modulator of a prepare-and-measure scheme, the choice of BB84 state is performed in a passive and uniformly random way by the beam splitters present in both Alice and Bob's measurement setups (also known as "remote state preparation").

- (2) In free space, it avoids the need for active polarization modulation, which imposes a strict upper limit on the protocol's repetition rate governed by the bandwidth of the Pockels cell and its high-voltage amplifier, typically achieving a few hundred kHz to a few hundred MHz [50]. By generating entangled photons that are passively projected onto one of the four BB84 states instead, our OT rate is not limited by any active prepare-and-measure encoding routine, but only by our picosecond-pulsed pump rate of around 300 MHz. With other SPDC sources reaching the GHz [51] to tens of GHz regimes [52], our passive state-preparation routine can perform even better.

B. Practical protocol

The protocol is identical to that from π_{ROT} as described in Sec. III, with the following amendments:

- (1) The parties agree on an additional parameter p_{multi} —the accepted ratio of multi-photon events.
- (2) During the quantum phase of the protocol, Alice may observe detection patterns that are incompatible with the emission of a single photon pair. Instead of sharing N_0 states in step (1), she continues sharing states until, after agreeing on coincidence time-tags with Bob, the parties obtain N_0 coincidences associated with single-photon events on Alice's side. Let N_{tot} be the number of coincidences obtained at this point and $N_{\text{multi}} = N_{\text{tot}} - N_0$. Alice computes the value

$$p'_{\text{multi}} = \frac{N_{\text{multi}}}{3N_{\text{tot}}}, \quad (37)$$

and aborts the protocol if $p'_{\text{multi}} \geq p_{\text{multi}}$.

- (3) Similarly to Alice, Bob may also observe multi-click patterns. While reporting its detection events he uses the following rules:
 - (a) 1 click: assign the correct measured bit value and report a successful round.
 - (b) 2 clicks from the same basis: assign a random bit value to the measurement result and report a successful round.
 - (c) any other click pattern: report an unsuccessful round.

C. Practical security

Any photonic implementation of quantum cryptography presents experimental imperfections, which can be exploited by dishonest parties to enhance their cheating probability and violate ideal security assumptions. Important examples of such imperfections include multi-photon noise, lossy and noisy quantum channels, non-unit detection efficiency and detector dark counts.

Dishonest sender. In our experiment, threshold detectors cannot resolve the incident photon number, and unexpected click patterns can occur. For example, several of the four detectors may simultaneously click for a given round, which leads to an inconclusive measurement outcome that has to be back-reported by the honest receiver. This in turn allows a dishonest sender to gain a significant amount of information about the receiver's measurement basis choice. Adopting the reporting strategy presented above makes the protocol secure against this type of attack. For a complete analysis of both the attack and its countermeasures, see Ref. [53].

Dishonest receiver. Due to Poisson statistics in the SPDC process, emission of double pairs can occur for a given round. When the two photons kept by the sender are projected onto the same state (i.e., only a single click is recorded in the four detectors), the two photons sent to Bob have the same polarization. In this case, a dishonest receiver can split the two photons and measure one in each basis. Assuming four detectors with equal efficiencies (which can be guaranteed in practice by appropriate attenuation the higher efficiency ones), and using the fact that for an SPDC source, whenever multiple pairs are produced, there is no correlation among them, we know that the number of undetected multi-photon events is approximately $\frac{1}{3}$ of the number of detected ones. We can then estimate the probability p'_{multi} of an accepted coincidence to be associated with a multi-photon event with Eq. (37).

Note that the statistical check performed by Alice in the second step of the amended protocol (Sec. VB) ensures security under the assumption that there is no coherence in the photon-number basis. This is the case in our implementation, since SPDC produces states of the form $\sum_{n=0}^{\infty} \sqrt{c_n} |n\rangle_1 |n\rangle_2$ in the number basis $\{|n\rangle\}$ [54], leaving the individual subsystems in incoherent mixtures of the form $\sum_{n=0}^{\infty} c_n |n\rangle \langle n|$.

To account for the leakage caused by undetected multi-photon emissions to our OT rate expression, we effectively grant Bob an amount of information about Alice's measurement outcomes equal to the number of indices in $I_{\bar{t}_i}$ associated to multi-photon events, upper bounded by $p_{\text{multi}}(1 - \alpha)N_0$ for large N_0 . Subtracting this leak to the total entropy expression in Eq. (32) leads to a version of Lemma 2 for security against dishonest receiver corrected for the experimental implementation, which differs from the theoretical version by replacing Eq. (13) with

$$\begin{aligned} \varepsilon'_{\text{exp}} = & \sqrt{2} \left(e^{-\frac{1}{2}(1-\alpha)^2 N_{\text{test}} \delta_1^2} + e^{-\frac{1}{2} N_{\text{check}} \delta_1^2} \right)^{\frac{1}{2}} + e^{-D_{\text{KL}}\left(\frac{1}{2} - \delta_2 \middle| \frac{1}{2}\right)(1-\alpha)N_0} + \varepsilon_{\text{bind}}(k) \\ & + \frac{1}{2} \cdot 2^{\frac{1}{2}} \left(n - N_{\text{raw}} \left(\frac{1}{2} - \frac{2\delta_2}{1-2\delta_2} - h\left(\frac{p_{\text{max}} + \delta_1}{\frac{1}{2} - \delta_2}\right) - f \cdot h(p_{\text{max}} + \delta_1) - \frac{p_{\text{multi}}}{\frac{1}{2} - \delta_2} \right) \right), \end{aligned} \quad (38)$$

VI. DISCUSSION

Using Naor's protocol [35] in conjunction with a linear time OWF (such as a hash function from the SHA3 or BLAKE family), it is possible to implement the required 2-bit commitment in linear time in k . On the other hand, using an LDPC code with soft-decision decoding and hash-based verification, one can implement an IR scheme which is linear in both the block size N_{raw} (and therefore N_0) and k' . Finally, by taking the universal hash set $\mathbf{F}$ to be the set of Toeplitz matrices of size $N_{\text{raw}} \times n$, and using the FFT algorithm for matrix-vector multiplication, the computation of the output strings can be done in time $O(N_{\text{raw}} \log(N_{\text{raw}}))$. Considering that the protocol requires N_0 commitments and all the remaining computations of random subsets and checks can be done in linear time in N_0 , the total protocol running time is $O(N_0(k + k' + \log(N_0)))$.

Regarding the practicality of implementing π_{QROT} , the protocol is designed to be compatible with BB84-based QKD setups, both from the physical layer up to the post-processing, only requiring the addition of the commitment scheme. The most important difference to note is that π_{QROT} has significantly lower tolerance for qubit error rate (QBER). While most common QKD protocols can produce keys through QBERs above 10%, this protocol is limited to a maximum of 2.8%. This comparatively reduces the distances at which the protocol can be successful. However, it is important to note that, as opposed to key distribution between trusting parties, there are legitimate use-cases for OT at short range. While being in proximity to each other can help two trusting parties isolate themselves from a third-party eavesdropper, mistrusting parties do not gain anything (security wise) from being in the same place while attempting to do MPC, making the protocol useful regardless of the distance between the users.

Comparisons between classical and quantum protocols can be difficult because physical and technological assumptions, such as access to quantum communication or noisy quantum storage, do not straightforwardly compare with computational hardness assumptions. Furthermore, there is no natural way of quantitatively comparing statistical versus computational security. We can, however, contrast the (dis)advantages of using a computationally secure quantum OT protocol as compared to both fully classical computationally secure protocols, as well as statistically secure quantum ones.

Classical OT protocols based on asymmetric cryptography comprise the overwhelming majority of current real-world implementations of OT. The obvious main advantage of quantum OT is the weaker computational hardness assumption (OWF vs asymmetric cryptography), while the main advantage of current post-quantum classical OT implementations is speed. As shown in Fig. 5, the presented experimental setup is able to produce up to 0.10 OT/s, which pales in comparison to contemporary classical protocols, such as Refs. [30–33], that can achieve upwards of 10^5 OT/s (not including latency between parties) with current off-the-shelf hardware (for more details, see Ref. [33]). This difference can be mitigated by the use of OT extension algorithms, as the difference in speed would only matter during the generation of the base OTs. Note that in this case one should use a OT extension that matches the computational assumption of this work, such as Ref. [55].

Quantum protocols, both discrete variable (DV) [27] and continuous variable (CV) [29], have been shown to achieve statistically secure OT in the quantum noisy-storage model (QNS). Their experimental implementations show comparable values of quantum communication cost in terms of shared signals: 10^8 (no memory encoding assumption), and 10^5 (Gaussian encoding) for CV, and 10^7 for DV. As shown in Fig. 4, our protocol requires 10^6 quantum signals when matching their security ($\varepsilon = 10^{-7}$), which improves upon the alternatives when no additional assumption on the memory encoding of the adversary is made. Less straightforward to compare is the strength of the assumptions of noisy storage and OWFs. We note that the existence of OWFs is an assumption that permeates modern cryptography, from block cipher encryption and message authentication up to public-key cryptography protocols [56], which makes π_{QROT} more suited to be introduced in current cipher suites than protocols with alternative assumptions. In particular, as noted above, OWFs are required for OT extension algorithms. A summary of comparisons between the different approaches can be found in Table II.

Regarding potential improvements and further work, we can identify two main directions to build upon this work: performance and security. Regarding performance, we note that the dominant term in the expression for ε_{max} is the one associated with the significance of the parameter estimation [the first term in Eq. (13)]. This translates into the

TABLE II. Comparison of our work with other approaches for OT. N denotes the respective security parameter. Acronyms for assumptions are as follows: OWF, one-way functions; QNS, quantum noisy storage; DDH, decisional Diffie-Hellmann; RLWE, ring learning with errors; SLS, space-like separation enforced. Protocols marked with superscript a do not have a reference experimental implementation at the time of writing.

Protocol	Type	Assumption	Quantum Cost	Security
This work	Quantum discrete variable	OWF	$\mathcal{O}(N)$	Indistinguishability UC ROM
GLSV21 [38] ^a	Quantum discrete variable	OWF	$\text{Poly}(N)$	Stand-alone plain model
S10 [27,48]	Quantum discrete variable	QNS	$\mathcal{O}(N)$	Indistinguishability
FGSPSW18 [29]	Quantum continuous variable	QNS	$\mathcal{O}(N)$	Indistinguishability
MR19 [30]	Classical	DDH	—	Stand-alone ROM
BFGMMS21 [33]	Classical	RLWE	—	UC ROM
P16 [57] ^a	Quantum and relativistic discrete variable	SLS	$\mathcal{O}(N)$	Other

relatively large values of N_0 needed to achieve adequate security, which was the bottleneck in the performance of our implementation. One way to reduce the number of signals needed per OT is to modify the protocol to perform many concurrent ROTs in a single run. This would mean performing one single estimation, albeit of a larger sample, that would work for many OTs in such a way that the required number of signals per ROT is decreased. On the topic of increasing security two main directions come to mind. First, we can consider the constructions of *collapsing* hash functions proposed in Refs. [58,59] to implement statistically hiding, computationally collapse binding commitments, which in turn allow for OT protocols that feature forward security (the OT remains secure even if the underlying hash function can be attacked after the commit and open phase of the protocol). The second direction would be a deeper exploration of the composable security of the protocol in the ROM. This can come from generalizing Theorem 2 for any weakly interactive commitments (currently the proof applies only to the LRV25 construction), or applying the techniques developed in Ref. [60] to prove UC security of commitments in the quantum ROM to remove the adversary’s limitation of classical access to the oracle. From the practical implementation perspective, it seems natural to integrate quantum OT into both QKD setups for a unified physical layer capable of providing secure communication and computation powered by OT extension and MPC algorithms, bringing the benefits of quantum OT closer to real world usage.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [46].

APPENDIX A: PRELIMINARIES

1. Quantum computational efficiency and distinguishability

We model the quantum capabilities of parties through programs running on quantum computers, for which we adopt a model based on deterministic-control quantum Turing machines [61]. For the purposes of the following definitions, a quantum computer is a device that has a classical interface and a quantum part, which contains the quantum memory registers available to the party. The classical interface has the capabilities of a classical computer augmented with the ability to perform a predefined universal set of quantum operators on the quantum memory registers and perform measurements in the canonical (computational) basis. Given a specified type of quantum computer, a quantum program is a classical description of a set of instructions to be run by the computer, including the quantum operations and measurements to be executed in the quantum part, as well as any classical computation. Quantum programs can be compared with probabilistic classical programs as they both have natural numbers as inputs and outputs. When a quantum computer runs the

program T with input $x \in \mathbb{N}$, we assume that the quantum part of the computer starts with some predefined initial state, performs a sequence of operations on its quantum registers, and upon halting, it outputs $T(x) \in \mathbb{N}$ on its classical interface by reading the appropriate registers associated with the program's output. Each execution of a quantum program is then associated to a quantum operation, which is the result of all the operations performed on the quantum part during the execution of the program.

Definition A1 (Computational efficiency). Let T be a quantum program. We say that T is computationally efficient (or polynomial time) if there exists a polynomial P such that the running time of $T(x)$ is $O(P(x))$.

Definition A2 (Distinguishing advantage). Let X_1, X_2 be two random variables with values in $\mathbb{N}$. For any quantum program T , the *distinguishing advantage* of X_1, X_2 using T is defined as

$$\text{Adv}_T(X_1, X_2) = |\Pr[T(X_1) = 1] - \Pr[T(X_2) = 1]|. \quad (\text{A1})$$

Analogously, let $\rho_1, \rho_2 \in \mathcal{D}(\mathcal{H})$. For any quantum program T , the *distinguishing advantage* of ρ_1, ρ_2 using T is defined as

$$\text{Adv}_T(\rho_1, \rho_2) = |\Pr[T^{(\rho_1)} = 1] - \Pr[T^{(\rho_2)} = 1]|, \quad (\text{A2})$$

where $T^{(\rho)}$ denotes the classical output of the program starting with the quantum state ρ and zero classical input.

Definition A3 (Indistinguishability-finite). Let $\rho_1, \rho_2 \in \mathcal{D}(\mathcal{H})$ and $\varepsilon \geq 0$. We say that ρ_1 and ρ_2 are ε -indistinguishable, denoted by $\rho_1 \approx_\varepsilon \rho_2$, whenever

$$\text{Adv}_T(\rho_1, \rho_2) \leq \varepsilon, \quad \text{for all quantum programs } T. \quad (\text{A3})$$

ε -indistinguishability for random variables is defined analogously.

As the following proposition states, to show that two states are ε -indistinguishable, it is enough to upper bound their trace distance D . (For more detail on the relationship of these quantities, see Refs. [62,63].)

Proposition A1. For any pair of quantum states $\rho_1, \rho_2 \in \mathcal{D}(\mathcal{H})$ it holds that

$$\rho_1 \approx_{D(\rho_1, \rho_2)} \rho_2. \quad (\text{A4})$$

Definition A4 (Indistinguishability-asymptotical). Let $\{\rho_1^{(k)} \in \mathcal{D}(\mathcal{H}_k)\}$ and $\{\rho_2^{(k)} \in \mathcal{D}(\mathcal{H}_k)\}$ be two families of density operators. We say that the two families are *statistically indistinguishable* if there exists a negligible function $\varepsilon(k) \geq 0$ such that

$$\rho_1^{(k)} \approx_{\varepsilon(k)} \rho_2^{(k)} \quad \text{for all } k \in \mathbb{N}. \quad (\text{A5})$$

Furthermore, we say the two families are *computationally indistinguishable* if for every efficient quantum program T ,

there exists a negligible function $\varepsilon_T(k) \geq 0$ such that

$$\text{Adv}_T(\rho_1^{(k)}, \rho_2^{(k)}) \leq \varepsilon_T(k) \quad \text{for all } k \in \mathbb{N}. \quad (\text{A6})$$

Statistical and computational indistinguishability for random variables is defined analogously.

Recall from Sec. II that, when the parameter k is implicit, we may omit the explicit dependence on k and use $\approx$ and $\approx^{(c)}$ for statistical and computational indistinguishability, respectively. We now turn our attention to the properties of indistinguishable states. It is worth noting that computational indistinguishability is only meaningful in terms of information security when the adversary is assumed to have limited computational capabilities. It is important then to define the type of quantum operations such an adversary can perform.

Definition A5 (Efficient quantum operation). We say that a family $\{\mathcal{E}^{(k)}\}_{k=1}^\infty$ of quantum operations is efficient if there exists an efficient quantum program T such that, for each k , $\mathcal{E}^{(k)}$ is the associated operation applied to the quantum part of the machine while running T on input k .

The following properties are straightforward to prove from Definitions A3 and A4 and the properties of trace distance:

Lemma A1 (Properties of indistinguishable states I). Let $\rho_1, \rho_2, \rho_3 \in \mathcal{D}(\mathcal{H})$:

- (1) $\rho_1 \approx_\varepsilon \rho_2 \wedge \rho_2 \approx_{\varepsilon'} \rho_3 \Rightarrow \rho_1 \approx_{\varepsilon+\varepsilon'} \rho_3$.
- (2) $\rho_1 \approx_\varepsilon \rho_2 \wedge \sigma_1 \approx_{\varepsilon'} \sigma_2 \Rightarrow \rho_1 \otimes \sigma_1 \approx_{\varepsilon+\varepsilon'} \rho_2 \otimes \sigma_2$.
- (3) Let $x \in \mathcal{X}$. For any probability distribution P_x , assume that $(\forall x \in \mathcal{X}) \rho_1^x \approx_{\varepsilon^x} \rho_2^x$. Then

$$\sum_{x \in \mathcal{X}} P_x \rho_1^x \approx_{\varepsilon^{\max}} \sum_{x \in \mathcal{X}} P_x \rho_2^x \quad \text{where } \varepsilon^{\max} = \max_{x \in \mathcal{X}} \{\varepsilon^x\}.$$

- (4) $\rho_1 \approx_\varepsilon \rho_2 \Rightarrow \mathcal{E}(\rho_1) \approx_\varepsilon \mathcal{E}(\rho_2)$, for any completely positive, trace non-increasing map $\mathcal{E}$.

Lemma A2 (Properties of indistinguishable states II). Let $\{\rho_1(k)\}, \{\rho_2(k)\}, \{\rho_3(k)\}$ be families of density operators parameterized by $k = 1, 2, \dots$. The following statements hold for asymptotic computational indistinguishability:

- (1) $\rho_1 \approx^{(c)} \rho_2 \wedge \rho_2 \approx^{(c)} \rho_3 \Rightarrow \rho_1 \approx^{(c)} \rho_3$.
- (2) $\rho_1 \approx^{(c)} \rho_2 \wedge \sigma_1 \approx^{(c)} \sigma_2 \Rightarrow \rho_1 \otimes \sigma_1 \approx^{(c)} \rho_2 \otimes \sigma_2$.
- (3) Let $x \in \mathcal{X}$. For any probability distribution P_x , assume that $(\forall x \in \mathcal{X}) \rho_1^x \approx^{(c)} \rho_2^x$. Then

$$\sum_{x \in \mathcal{X}} P_x \rho_1^x \approx^{(c)} \sum_{x \in \mathcal{X}} P_x \rho_2^x.$$

- (4) $\rho_1 \approx^{(c)} \rho_2 \Rightarrow \mathcal{E}(\rho_1) \approx^{(c)} \mathcal{E}(\rho_2)$, where $\{\mathcal{E}^{(k)}\}$ is an efficient family of quantum operations acting on the respective $\rho_i(k)$.

2. Entropic quantities

We start off by defining a useful pair of quantities for measuring information in quantum systems: the max-entropy and the conditional min-entropy. The max-entropy is a measure of the number of possible different outcomes that can result from measuring a quantum state, whereas the conditional min-entropy is a way of measuring the information that a party can infer from a quantum system given access to another correlated quantum system. This measure will be useful to bound the distance between states based on their internal correlations.

Definition A6 (Max-entropy). Let $\rho \in \mathcal{D}(\mathcal{H})$. The max-entropy of ρ is defined as

$$H_{\max}(\rho) = \log(\dim(\text{supp}(\rho))), \quad (\text{A7})$$

where $\text{supp}(\rho)$ denotes the support subspace of ρ and $\dim$ denotes its dimension.

Definition A7 (Min-entropy and conditional min-entropy). Let $\rho \in \mathcal{D}(\mathcal{H})$ and $\lambda_{\max}(\rho)$ denote the maximum eigenvalue of ρ . The min-entropy of ρ is defined as

$$H_{\min}(\rho) = -\log(\lambda_{\max}(\rho)). \quad (\text{A8})$$

Let $\rho_{AB} \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_B)$ and $\sigma_B \in \mathcal{D}(\mathcal{H}_B)$. The conditional min-entropy of ρ_{AB} given σ_B is defined as

$$H_{\min}(\rho_{AB} | \sigma_B) = -\log(\lambda_{\sigma_B}), \quad (\text{A9})$$

where λ_{σ_B} is the minimum real number such that $\lambda_{\sigma_B}(\mathbb{I}_A \otimes \sigma_B) - \rho_{AB}$ is non-negative. The conditional min-entropy of ρ_{AB} given $\mathcal{H}_B$ is defined as

$$H_{\min}(A | B)_\rho = \sup_{\sigma_B \in \mathcal{D}(\mathcal{H}_B)} H_{\min}(\rho_{AB} | \sigma_B). \quad (\text{A10})$$

Furthermore, let $\varepsilon > 0$. The ε -smooth conditional min-entropy is defined as

$$H_{\min}^\varepsilon(A | B)_\rho = \sup_{\rho'_{AB} \in \mathcal{B}^\varepsilon(\rho_{AB})} H_{\min}(A | B)_{\rho'}, \quad (\text{A11})$$

where $\mathcal{B}^\varepsilon(\rho_{AB}) = \{\rho'_{AB} : D(\rho_{AB}, \rho'_{AB}) < \varepsilon\}$.

The smooth conditional min-entropy is in general hard to compute. Because of this, it is useful to have some tools to bound it for states that have some specific forms. In our case we are interested in states that are partially *classical*.

Definition A8 (Partially classical states). A quantum state described by the density operator $\rho_{AB} \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_B)$ is classical in $\mathcal{H}_A$ (or classical in A) if it can be written in the form

$$\rho_{AB} = \sum_x \lambda_x |x\rangle\langle x|_A \otimes \rho_B^x, \quad (\text{A12})$$

where the set $\{|x\rangle\}_x$ is an orthonormal basis for $\mathcal{H}_A$. A multipartite state is said to be classical if it is classical in all its parts.

When dealing with partially classical states as shown in Eq. (A12), we will refer to the operators ρ_B^x as the state of the system B conditioned to x .

Lemma A3 (Properties of min- and max-entropy). Let $\varepsilon, \varepsilon' \geq 0$:

- (1) $H_{\min}(\rho_A \otimes \rho_B | \rho_B) = -\log(\lambda_{\max}(\rho_A))$.
- (2) $H_{\min}^{\varepsilon+\varepsilon'}(AA' | BB')_{\rho \otimes \rho'} \geq H_{\min}^\varepsilon(A | B)_\rho + H_{\min}^{\varepsilon'}(A' | B')_{\rho'}$.
- (3) $H_{\min}^\varepsilon(A | BC)_\rho \leq H_{\min}^\varepsilon(A | B)_\rho$.
- (4) $H_{\min}^\varepsilon(AB | C)_\rho \leq H_{\min}^\varepsilon(A | BC)_\rho + H_{\max}(\rho_B)$.
- (5) $H_{\min}^\varepsilon(A | B)_\rho \geq \inf_x \{H_{\min}^\varepsilon(\rho_A^x)\}$, whenever the state ρ_{AB} is classical on B .

We use universal hashing to implement randomness extraction in the final steps of the protocol. The proof both Lemmas A3 and A4 can be found in Ref. [64].

Definition A9 (Universal hashing). A set of functions $\mathbf{F} = \{f_i : \{0, 1\}^m \rightarrow \{0, 1\}^n\}$ is a universal hash family if, for all $x, y \in \{0, 1\}^m$, such that $x \neq y$, and i chosen uniformly at random, we have

$$\Pr[f_i(x) = f_i(y)] \leq \frac{1}{2^n}. \quad (\text{A13})$$

Lemma A4 (Quantum leftover hash). Let $\mathbf{F} = \{f_i : \{0, 1\}^n \rightarrow \{0, 1\}^\ell\}$ be a universal hash family, let $\mathcal{H}_A, \mathcal{H}_B, \mathcal{H}_F, \mathcal{H}_E$ be Hilbert spaces such that $\{|x\rangle\}_{x \in \{0, 1\}^n}, \{|f_i\rangle\}_{f_i \in \mathbf{F}}$, and $\{|e\rangle\}_{e \in \{0, 1\}^\ell}$ are orthonormal bases for $\mathcal{H}_A, \mathcal{H}_F$, and $\mathcal{H}_E$ respectively. Then for any $\varepsilon \geq 0$ and any state of the form

$$\rho_{ABFE} = \frac{1}{|\mathbf{F}|} \sum_{\substack{x \in \{0, 1\}^n \\ f_i \in \mathbf{F}}} (\lambda(x) |f_i\rangle\langle f_i|_F |f_i(x)\rangle\langle f_i(x)|_E \otimes |x\rangle\langle x|_A \rho_B^x), \quad (\text{A14})$$

it holds that

$$\rho_{EBF} \approx_{\varepsilon'} \mathbf{U}_E \otimes \rho_{BF}, \quad (\text{A15})$$

with

$$\varepsilon' = \varepsilon + \frac{1}{2} \cdot 2^{-\frac{1}{2}(H_{\min}^\varepsilon(A|B)_\rho - \ell)}. \quad (\text{A16})$$

APPENDIX B: DETAILED PROOF OF THEOREM 1

1. Supporting lemmas

One of the main features to analyze for the security against a dishonest receiver is the potential information that he can learn about the sender's strings given the quantum state that remains with him after the commit and open phase. In order to talk about the security of the protocol independently of the specific cheating strategy that may be used by the dishonest parties (or possible effects that the environment can have in the shared quantum state), we want to understand the properties that a quantum state that passes Alice's test at step (7) can have. We do this through the following lemma, a version of which was originally proven in Ref. [40]. Here, we provide a more self-contained statement and make explicit the trace distance bound.

Lemma B1. Let $\varepsilon > 0$, $I = \{1, \dots, N\}$, and $\rho_{T,\hat{X},X,E}$ be a density operator of the form

$$\begin{aligned} \rho_{T,\hat{X},X,E} &= \sum_{\substack{I_1, I_2 \\ \in \mathcal{P}(I) \setminus \{\emptyset\}}} q(I_1, I_2) |I_1, I_2\rangle\langle I_1, I_2|_T \\ &\quad \otimes |\hat{x}\rangle\langle \hat{x}|_{\hat{X}} \otimes |\psi\rangle\langle \psi|_{X,E} \\ |\psi\rangle_{X,E} &= \sum_{x \in \{0,1\}^N} \beta^x |x\rangle_X |\phi^x\rangle_E, \end{aligned} \quad (\text{B1})$$

where $\dim(\mathcal{H}_{\hat{X}}) = \dim(\mathcal{H}_X) = 2^N$ and $\mathcal{P}(I)$ denotes the set of all subsets of I . For each I_1, I_2 define the set

$$B_{I_1, I_2} = \{x \in \{0,1\}^N : |r_H(x_{I_1} \oplus \hat{x}_{I_1}) - r_H(x_{I_2} \oplus \hat{x}_{I_2})| < \varepsilon\}. \quad (\text{B2})$$

Additionally, let $\mathcal{Q}(N, \varepsilon)$ be a function such that, whenever the subsets I_1, I_2 are sampled according to q , it holds that

$$\Pr[|r_H(x_{I_1} \oplus \hat{x}_{I_1}) - r_H(x_{I_2} \oplus \hat{x}_{I_2})| > \varepsilon] \leq \mathcal{Q}(N, \varepsilon) \quad (\text{B3})$$

independently of x . There exists a state $\tilde{\rho}_{T,\hat{X},X,E}$ of the form

$$\begin{aligned} \tilde{\rho}_{T,\hat{X},X,E} &= \sum_{\substack{I_1, I_2 \\ \in \mathcal{P}(I) \setminus \{\emptyset\}}} q(I_1, I_2) |I_1, I_2\rangle\langle I_1, I_2|_T \\ &\quad \otimes |\hat{x}\rangle\langle \hat{x}|_{\hat{X}} \otimes |\psi_{I_1, I_2}\rangle\langle \psi_{I_1, I_2}|_{X,E} \\ |\psi_{I_1, I_2}\rangle &= \sum_{x \in B_{I_1, I_2}} \tilde{\beta}_{I_1, I_2}^x |x\rangle_X |\phi_{I_1, I_2}^x\rangle_E, \end{aligned} \quad (\text{B4})$$

such that

$$D(\rho_{T,\hat{X},X,E}, \tilde{\rho}_{T,\hat{X},X,E}) \leq \mathcal{Q}(N, \varepsilon)^{\frac{1}{2}}. \quad (\text{B5})$$

Proof. First, we choose an adequate definition for the $\tilde{\beta}_{I_1, I_2}^x$ and then show that, under that choice, the bound in Eq. (B5) holds. Note that we can write the state

$$\begin{aligned} |\psi\rangle_{X,E} &= \sum_{x \in \{0,1\}^N} \beta_x |x\rangle_X |\phi^x\rangle_E \\ &= \underbrace{\left(\sum_{x \in B_{I_1, I_2}} |\beta_x|^2 \right)^{\frac{1}{2}}}_{\lambda_{I_1, I_2}} \underbrace{\frac{\sum_{x \in B_{I_1, I_2}} \beta_x |x\rangle_X |\phi^x\rangle_E}{\left(\sum_{x \in B_{I_1, I_2}} |\beta_x|^2 \right)^{\frac{1}{2}}}}_{|\psi_{I_1, I_2}\rangle_{X,E}} + \underbrace{\left(\sum_{x \notin B_{I_1, I_2}} |\beta_x|^2 \right)^{\frac{1}{2}}}_{\lambda_{I_1, I_2}^\perp} \underbrace{\frac{\sum_{x \notin B_{I_1, I_2}} \beta_x |x\rangle_X |\phi^x\rangle_E}{\left(\sum_{x \notin B_{I_1, I_2}} |\beta_x|^2 \right)^{\frac{1}{2}}}}_{|\psi_{I_1, I_2}^\perp\rangle_{X,E}}. \end{aligned} \quad (\text{B6})$$

The trace distance between the pure states $|\psi\rangle$ and $|\psi_{I_1, I_2}\rangle$ is given by $\sqrt{1 - |\langle \psi | \psi_{I_1, I_2} \rangle|^2} = \lambda_{I_1, I_2}^\perp$, hence the trace distance between the complete joint states is given by

$$\begin{aligned} D(\rho_{T,\hat{X},X,E}, \tilde{\rho}_{T,\hat{X},X,E})^2 &\leq \left(\sum_{I_1, I_2 \in \mathcal{T}(N, \alpha)} q D(|\psi\rangle\langle \psi|_{X,E}, |\psi_{I_1, I_2}\rangle\langle \psi_{I_1, I_2}|_{X,E}) \right)^2 \\ &= \left(\sum_{I_1, I_2 \in \mathcal{T}(N, \alpha)} q \lambda_{I_1, I_2}^\perp \right)^2 \\ &\leq \sum_{I_1, I_2 \in \mathcal{T}(N, \alpha)} q \lambda_{I_1, I_2}^{\perp 2}, \end{aligned} \quad (\text{B7})$$

where Jensen's inequality was used in the last step. We proceed now to bound the right side of Eq. (B7). For that purpose consider the function

$$\xi(I_1, I_2, x) = \begin{cases} 0, & \text{if } x \in B_{I_1, I_2}, \\ 1, & \text{otherwise} \end{cases}, \quad (\text{B8})$$

so that

$$\begin{aligned} & \sum_{I_1, I_2} q(I_1, I_2) \xi(I_1, I_2, x) \\ &= \Pr[|r_H(\hat{x} \oplus x|_{I_1}) - r_H(\hat{x} \oplus x|_{I_2})| > \varepsilon] \\ &= \mathcal{Q}(N, \varepsilon). \end{aligned} \quad (\text{B9})$$

Hence,

$$\begin{aligned} & \|\rho_{T, \hat{X}, X, E} - \tilde{\rho}_{T, \hat{X}, X, E}\|^2 \\ & \leq \sum_{I_1, I_2} q(I_1, I_2) \lambda_{I_1, I_2}^{\perp 2} \\ &= \sum_{I_1, I_2} q(I_1, I_2) \sum_{x \notin B_{I_1, I_2}} |\beta_x|^2 \\ &= \sum_{I_1, I_2} q(I_1, I_2) \sum_{x \in \{0,1\}^N} \xi(I_1, I_2, x) |\beta_x|^2 \\ &= \sum_{x \in \{0,1\}^N} |\beta_x|^2 \sum_{I_1, I_2} q(I_1, I_2) \xi(I, x) \\ & \leq \sum_{x \in \{0,1\}^N} |\beta_x|^2 \mathcal{Q}(N, \varepsilon) = \mathcal{Q}(N, \varepsilon), \end{aligned} \quad (\text{B10})$$

as required. $\blacksquare$

In order to use the above result in the context of the π_{QROT} protocol, we need to find an appropriate function $\mathcal{Q}(N, \varepsilon)$ that satisfies Eq. (B3) for the case when the $\hat{x}, x$ are the respective measurement outcomes of Alice and Bob when measuring in the same basis. We do this through the following lemma based on the Hoeffding inequality for sampling without replacement.

Definition B1. Given a set I and an integer $n \leq |I|$, define the set $\mathcal{T}(n, I)$ as the set of all subsets of I with size n .

Lemma B2 (Hoeffding's inequalities). Let $x \in \{0, 1\}^N$, $\delta > 0$, $I = \{1, \dots, N\}$ and $0 < \alpha < \frac{1}{2}$ such that $\alpha N \in \mathbb{N}$.

- (a) (Inequality for sampling without replacement comparing the sampled subset with the whole set) For $I_t \in \mathcal{T}(\alpha N, I)$ sampled uniformly, it holds that

$$\Pr[|r_H(x|_{I_t}) - r_H(x)| > \delta] \leq 2e^{-2\alpha N \delta^2}. \quad (\text{B11})$$

- (b) (Inequality for sampling without replacement comparing the sampled subset with its complement)

$$\Pr[|r_H(x|_{I_t}) - r_H(x|_{\bar{I}_t})| > \delta] \leq 2e^{-2\alpha(1-\alpha)^2 N \delta^2}. \quad (\text{B12})$$

- (c) (Inequality for sampling without replacement comparing the sampled subset with its complement and ignoring part of the sample) Let $n \in \{n_0, \dots, \alpha N\}$ be sampled according to some distribution $P(n)$. For $I_s \in \mathcal{T}(n, I_t)$ sampled uniformly, it holds that

$$\begin{aligned} & \Pr[|r_H(x|_{I_s}) - r_H(x|_{\bar{I}_t})| > \delta] \\ & \leq 2 \left(e^{-\frac{1}{2}\alpha(1-\alpha)^2 N \delta^2} + e^{-\frac{1}{2}n_0 \delta^2} \right). \end{aligned} \quad (\text{B13})$$

Proof.

- (a) This is the original Hoeffding inequality for sampling without replacement, the proof of which can be found in Ref. [65].
 (b) Note that we can write $r_H(x) = \alpha r_H(x|_{I_t}) + (1 - \alpha) r_H(x|_{\bar{I}_t})$. Substituting $r_H(x)$ in Eq. (B11) we get

$$\begin{aligned} & \Pr[|r_H(x|_{I_t}) - \alpha r_H(x|_{I_t}) + (1 - \alpha) r_H(x|_{\bar{I}_t})| > \delta'] \\ &= \Pr[|r_H(x|_{I_t}) - r_H(x|_{\bar{I}_t})| > \delta' / (1 - \alpha)] \end{aligned} \quad (\text{B14})$$

$$\leq 2e^{-2\alpha N \delta'^2}. \quad (\text{B15})$$

The result is obtained by taking $\delta' = (1 - \alpha)\delta$

- (c) Let us consider first the case where n is fixed. From the triangle inequality we know that

$$|r_H(x|_{I_{s(n)}}) - r_H(x|_{\bar{I}_t})| > \delta \Rightarrow |r_H(x|_{I_{s(n)}}) - r_H(x|_{I_t})| + |r_H(x|_{I_t}) - r_H(x|_{\bar{I}_t})| > \delta \quad (\text{B16})$$

$$\Rightarrow |r_H(x|_{I_{s(n)}}) - r_H(x|_{I_t})| > \delta/2 \vee |r_H(x|_{I_t}) - r_H(x|_{\bar{I}_t})| > \delta/2, \quad (\text{B17})$$

and hence, by the union bound

$$\Pr[|r_H(x|_{I_{s(n)}}) - r_H(x|_{\bar{I}_t})| > \delta] \leq \Pr[|r_H(x|_{I_{s(n)}}) - r_H(x|_{I_t})| + |r_H(x|_{I_t}) - r_H(x|_{\bar{I}_t})| > \delta] \quad (\text{B18})$$

$$\leq \Pr[|r_H(x|_{I_{s(n)}}) - r_H(x|_{I_t})| > \delta/2] + \Pr[|r_H(x|_{I_t}) - r_H(x|_{\bar{I}_t})| > \delta/2] \quad (\text{B19})$$

$$\leq 2e^{-\frac{1}{2}n\delta^2} + 2e^{-\frac{1}{2}\alpha(1-\alpha)^2 N \delta^2}, \quad (\text{B20})$$

where the last expression comes from applying the (b) and (a) inequalities to the first and second terms of Eq. (B19) respectively. Using this, we can consider the case in which n is not fixed, but instead follows a probability distribution $P(n)$ such that $P(n) = 0$ for $n < n_0$. For this case

$$\begin{aligned} & \Pr[|r_H(x|_{I_s}) - r_H(x|_{\bar{I}_t})| > \delta] \\ &= \sum_n P(n) \Pr[|r_H(x|_{I_s(n)}) - r_H(x|_{\bar{I}_t})| > \delta] \end{aligned} \quad (\text{B21})$$

$$\leq \sum_n P(n) 2 \left(e^{-\frac{1}{2}\alpha(1-\alpha)^2 N \delta^2} + e^{-\frac{1}{2}n \delta^2} \right) \quad (\text{B22})$$

$$\leq 2 \sum_n P(n) \left(e^{-\frac{1}{2}\alpha(1-\alpha)^2 N \delta^2} + e^{-\frac{1}{2}n_0 \delta^2} \right) \quad (\text{B23})$$

$$= 2 \left(e^{-\frac{1}{2}\alpha(1-\alpha)^2 N \delta^2} + e^{-\frac{1}{2}n_0 \delta^2} \right). \quad (\text{B24})$$

■

The following lemma helps us bound the conditional min-entropy of a partially measured pure state by comparing it with the one of an appropriately chosen, partially measured mixed state. A proof of this result can be found in Ref. [66].

Lemma B3 (Entropy bound for post-measurement states). Let $\mathcal{H}_A$ and $\mathcal{H}_E$ be Hilbert spaces and $\{|x\rangle\}_{x \in \mathcal{X}}$, $\{|y\rangle\}_{y \in \mathcal{Y}}$ be orthonormal bases for $\mathcal{H}_A$. Let $J \subseteq \mathcal{X}$, define the states

$$\rho_{AE} = |\phi\rangle\langle\phi|_{AE} \quad \text{with } |\phi\rangle_{AE} = \sum_{x \in J} \beta_x |x\rangle_A |\phi^x\rangle_E, \quad (\text{B25})$$

$$\rho_{AE}^{\text{mix}} = \sum_{x \in J} |\beta_x|^2 |x\rangle\langle x|_A \otimes |\phi^x\rangle\langle\phi^x|_E. \quad (\text{B26})$$

Denote by σ_{YE} and σ_{YE}^{mix} the states resulting from measuring the subsystem A of ρ_{AE} and ρ_{AE}^{mix} respectively in the basis $\{|y\rangle\}_{y \in \mathcal{Y}}$, storing the result in the system Y , and then tracing out the A subsystem; then it holds that

$$H_{\min}(Y | E)_\sigma \geq H_{\min}(Y | E)_{\sigma^{\text{mix}}} - \log |J|. \quad (\text{B27})$$

2. Proof of Lemma 3

Here we present a proof of Lemma 3 used in the protocol's correctness analysis in Sec. IV.

Lemma B4. Let $X_{I_0}^A, X_{I_1}^A, C, Y^B$ denote the systems holding the information of the respective values $x_{I_0}^A, x_{I_1}^A, c$, and y^B of π_{QROT} . Denote by $\rho^\top$ the parties' joint state at the

end of step (11) conditioned that Bob constructed the sets (I_0, I_1) during step (9) and the protocol has not aborted. Assume both parties follow the steps of the protocol, then

$$\rho_{X_{I_0}^A, X_{I_1}^A, C, Y^B}^\top \approx_{\varepsilon_{\text{IR}}(k')} \tilde{\rho}_{X_{I_0}^A, X_{I_1}^A, C, Y^B}^\top, \quad (\text{B28})$$

where $\varepsilon_{\text{IR}}(k')$ is a negligible function given by the security of the underlying information reconciliation scheme, k' its associated security parameter, and

$$\begin{aligned} & \tilde{\rho}_{X_{I_0}^A, X_{I_1}^A, C, Y^B}^\top \\ &= \frac{1}{2^{(2N_{\text{raw}}+1)}} \sum_{x_{I_0}^A, x_{I_1}^A} |x_{I_0}\rangle\langle x_{I_0}|_{X_{I_0}^A} |x_{I_1}\rangle\langle x_{I_1}|_{X_{I_1}^A} |x_{I_0}\rangle\langle x_{I_0}|_{Y^B} |c\rangle\langle c|_C. \end{aligned} \quad (\text{B29})$$

Proof. Note that, because the state shared by Alice at step (1) of the protocol is a tensor product of maximally entangled states, the state of Alice's part is a product of maximally mixed states. This means that, regardless of the measurement bases θ^A , the outcome of her measurements x^A is always uniform in $\{0, 1\}^{N_0}$. Let $\rho_{\Theta^A, \Theta^B, X^A, X^B}^{(2)}$ be the state of the parties' respective measurement bases and outcomes at the end of step (2) of the protocol, we can write

$$\begin{aligned} & \rho_{\Theta^A, \Theta^B, X^A, X^B}^{(2)} \\ &= \frac{1}{2^{2N_0}} \sum_{\theta^A, \theta^B} |\theta^A, \theta^B\rangle\langle\theta^A, \theta^B|_{\Theta^A \Theta^B} \frac{1}{2^{N_0}} \sum_{x^A} |x^A\rangle\langle x^A|_{X^A} \\ &\otimes \sum_{x^B} P(x^B | x^A, \theta^A, \theta^B) |x^B\rangle\langle x^B|_{X^B}, \end{aligned} \quad (\text{B30})$$

where $P(x^B | x^A, \theta^A, \theta^B)$ denotes the probabilities of Bob's outcomes given each parties measurement bases and Alice's measurement outcomes, which in turn depends on the effect of the transmission channel when the state was shared from Alice's laboratory. All operations will be classical from this point onwards. To arrive to Eq. (B29), we first need to show that the abort operations within the protocol do not bias the distribution of possible values of x_{I_0} and x_{I_1} , and then use the verifiability property of the IR scheme to ensure that $y^B = x_{I_0}$ with high probability.

We will consider now the two abort instructions at steps (7) and (9) as a single quantum operation $\mathcal{E}$ that maps the state to the zero operator if any of the two abort conditions is satisfied and applies the identity map otherwise. Let us first separate the values of $\theta^{A/B}$ and x^B that "survive" the abort operation. For any given values of x^A and I_t , define the sets

$$J_{I_t, x^A}^{(1)} = \{(\theta^A, \theta^B) : w_H(\overline{\theta_t^A \oplus \theta_t^B}) \geq N_{\text{check}} \wedge N_{\text{raw}} \leq w_H(\theta_t^A \oplus \theta_t^B) \leq (1 - \alpha)N_0 - N_{\text{raw}}\} \quad (\text{B31})$$

$$J_{I_t, x^A, \theta^A, \theta^B}^{(2)} = \{x^B : r_H(x_{I_s}^A \oplus x_{I_s}^B) \leq p_{\text{max}}\}, \quad (\text{B32})$$

where $w_H(\cdot)$ denotes the Hamming weight function. Let $\mathcal{T} = \mathcal{T}(\alpha N_0, I)$ be the set of all subsets of $I = \{1, \dots, N_0\}$ of size αN_0 , and denote by S the system where Bob holds the information of the sets I_0 and I_1 . The joint state of the systems $C, X^{A/B}, \Theta^{A/B}, S$ at the end of step (10) of the protocol is

$$\begin{aligned} \rho_{C, X^A, X^B, \Theta^A, \Theta^B, S}^{(10)} &= \frac{1}{2} \sum_c |c\rangle\langle c|_C \frac{1}{2^{N_0}} \sum_{x^A} |x^A\rangle\langle x^A|_{X^A} \frac{1}{|\mathcal{T}| \cdot 2^{2N_0}} \sum_{I_t} \sum_{\substack{(\theta^A, \theta^B) \\ \in J^{(1)}}} |\theta^A, \theta^B\rangle\langle \theta^A, \theta^B|_{\Theta^A, \Theta^B} \\ &\otimes \sum_{x^B \in J^{(2)}} P(x^B | x^A, \theta^A, \theta^B) |x^B\rangle\langle x^B|_{X^B} \sum_{I_0, I_1} P(I_0, I_1 | I_t, \theta^A, \theta^B) |I_0, I_1\rangle\langle I_0, I_1|_S, \end{aligned} \quad (\text{B33})$$

where the conditional distribution $P(I_0, I_1 | I_t, \theta^A, \theta^B)$ notably does not depend on x^A or c . Tracing out the $\Theta^{A/B}$ systems and rearranging terms we get

$$\begin{aligned} \rho_{S, C, X^A, X^B}^{(10)} &= \sum_{I_0, I_1} P(I_0, I_1) |I_0, I_1\rangle\langle I_0, I_1|_S \\ &\otimes \underbrace{\frac{1}{2^{N_0+1}} \sum_{x^A, c} |x^A, c\rangle\langle x^A, c|_{X^A, C} \sum_{x^B} P(x^B | I_0, I_1, x^A) |x^B\rangle\langle x^B|_{X^B}}_{\text{conditioned state } \rho_{C, X^A, X^B}^{(10)}(I_0, I_1)}, \end{aligned} \quad (\text{B34})$$

where

$$P(I_0, I_1) = \frac{1}{|\mathcal{T}| \cdot 2^{2N_0}} \sum_{I_t} \sum_{\substack{(\theta^A, \theta^B) \\ \in J^{(1)}}} P(I_0, I_1 | I_t, \theta^A, \theta^B), \quad (\text{B35})$$

and

$$P(x^B | I_0, I_1, x^A) = \frac{\sum_{I_t} \sum_{\substack{(\theta^A, \theta^B) \\ \in J^{(1)}}} P(x^B | x^A, \theta^A, \theta^B) P(I_0, I_1 | I_t, \theta^A, \theta^B)}{\sum_{I_t} \sum_{\substack{(\theta^A, \theta^B) \\ \in J^{(1)}}} P(I_0, I_1 | I_t, \theta^A, \theta^B)}. \quad (\text{B36})$$

Now that we have a form for the conditioned state as pointed out in Eq. (B34), we can move to the action of step (11), where Bob computes $y^B = \text{dec}(\text{syn}(x_{I_0}^A, x_{I_1}^B))$. The resulting state of the systems $C, X_{I_0}^A, X_{I_1}^A, Y^B$ is then given by

$$\rho_{C, X_{I_0}^A, X_{I_1}^A, Y^B}^{(11)}(I_0, I_1) = \frac{1}{2^{2N_{\text{raw}}+1}} \sum_{\substack{x_{I_0}^A, x_{I_1}^A \\ c}} |x_{I_0}^A, x_{I_1}^A\rangle\langle x_{I_0}^A, x_{I_1}^A|_{X_{I_0}^A, X_{I_1}^A} |c\rangle\langle c|_C \otimes \left(P_{\text{correct}} |x_{I_0}^A\rangle\langle x_{I_0}^A|_{Y^B} + P_{\perp} |\perp\rangle\langle \perp|_{Y^B} + P_{\text{error}} \sigma_{Y^B} \right), \quad (\text{B37})$$

for some coefficients $P_{\text{correct}}, P_{\perp}, P_{\text{error}}$, and state σ orthogonal to both $|x_{I_0}^A\rangle\langle x_{I_0}^A|$ and $|\perp\rangle\langle \perp|$. By applying the verifiability property of the IR scheme with security parameter k' (where $P_{\text{error}} = \varepsilon_{\text{IR}}(k')$), and conditioning the resulting state to not having aborted, we get the desired result

$$\rho_{X_{I_0}^A, X_{I_1}^A, Y^B, C}^{(11), \top}(I_0, I_1) \approx_{\varepsilon_{\text{IR}}(k')} \frac{1}{2^{2N_{\text{raw}}+1}} \sum_{\substack{x_{I_0}^A, x_{I_1}^A \\ c}} |x_{I_0}^A, x_{I_1}^A\rangle\langle x_{I_0}^A, x_{I_1}^A|_{X_{I_0}^A, X_{I_1}^A} |x_{I_0}^A\rangle\langle x_{I_0}^A|_{Y^B} |c\rangle\langle c|_C. \quad (\text{B38})$$

■

3. Proof of Lemma 4

Here we present a proof of Lemma 4 introduced in Sec. IV B regarding the hiding property of the commitment in the context of π_{QROT} .

Lemma B5. Assuming Bob follows the protocol, for any $J \subseteq I$, the state of the system $A, \text{COM}, \text{OPEN}_J, \Theta_J^B$ after step (4) satisfies

$$\rho_{A, \text{COM}, \text{OPEN}_J, \Theta_J^B} \approx^{(c)} \rho_{A, \text{COM}, \text{OPEN}_J} \otimes \mathbf{U}_{\Theta_J^B}, \quad (\text{B39})$$

where

$$\mathbf{U}_{\Theta_J^B} = \frac{1}{2^{N_0 - |J|}} \sum_{\theta_J^B} |\theta_J^B\rangle\langle\theta_J^B|_{\Theta_J^B}, \quad (\text{B40})$$

denotes the uniform distribution over all possible values of θ_J^B .

Proof. We start by describing the general form of the state prepared by Alice at the beginning of the protocol, which is sent to Bob. Because the value of $r \in \{0, 1\}^{n_r}$ sent by Alice in step (3) as part of the commitment scheme is independent of any of Bob's actions, we can consider without loss of generality that it is prepared at the start of the protocol. The state shared at the beginning of the protocol (after Bob receives his qubit shares) has a general form given by

$$\begin{aligned} \rho_{\Phi^B, R, A}^{(0)} &= |\psi^{(0)}\rangle\langle\psi^{(0)}|_{\Phi^B, R, A} \quad |\psi^{(0)}\rangle_{\Phi^B, R, A} \\ &= \sum_{x, r} \alpha^{x, r} |x\rangle_{\Phi^B} |r\rangle_R |\phi^{x, r}\rangle_A, \end{aligned} \quad (\text{B41})$$

where the $|\phi^{x, r}\rangle$ are not necessarily orthogonal. Using the Hadamard operator H , we can define the states

$$|x, \theta\rangle = H^\theta |x\rangle = H^{\otimes \theta_1} \otimes \dots \otimes H^{\otimes \theta_{N_0}} |x\rangle, \quad (\text{B42})$$

and write, for any string of basis choices $\theta \in \{0, 1\}^{N_0}$, the state $|\psi^{(0)}\rangle_{\Phi^B, R, A}$ as

$$\begin{aligned} |\psi^{(0)}\rangle_{\Phi^B, R, A} &= \sum_{x', r} \alpha^{x', r} |\phi^{x', r}\rangle_A |r\rangle_R \sum_x \langle x | H^\theta | x' \rangle |x, \theta\rangle_{\Phi^B} \\ &= \sum_{x, r} \beta^{x, \theta, r} |x, \theta\rangle_{\Phi^B} |r\rangle_R |\phi^{x, \theta, r}\rangle_A, \end{aligned} \quad (\text{B43})$$

with

$$\begin{aligned} \beta^{x, \theta, r} &= \left(\sum_{x'} |\langle x | H^\theta | x' \rangle \alpha^{x', r}|^2 \right)^{\frac{1}{2}} |\phi^{x, \theta, r}\rangle \\ &= (\beta^{x, \theta, r})^{-1} \sum_{x'} \langle x | H^\theta | x' \rangle \alpha^{x', r} |\phi^{x', r}\rangle. \end{aligned} \quad (\text{B44})$$

After uniformly sampling the values of θ^B , Bob proceeds to perform his measurement on his qubit shares. Let $\mathcal{H}_{X^B}$ denote the system where Bob records the outcome string x^B . Additionally, at step (3) Bob receives the value of r , this is a classical message, which we model as Bob receiving the $\mathcal{H}_R$ system and measuring it in the computational basis upon arrival. We can now easily use Eq. (B43) to get the post-measurement state at the end of step (3) after tracing out $\mathcal{H}_{\Phi^B}$, which is given by

$$\rho_{X^B, \Theta^B, R, A}^{(1)} = \frac{1}{2^{N_0}} \sum_{x^B, \theta^B, r} |\beta^{x^B, \theta^B, r}|^2 |\theta^B\rangle\langle\theta^B|_{\Theta^B} |x^B\rangle\langle x^B|_{X^B} |r\rangle\langle r|_R |\phi^{x^B, \theta^B, r}\rangle\langle\phi^{x^B, \theta^B, r}|_A. \quad (\text{B45})$$

Before proceeding with the protocol, it will be useful to state some basic properties of the above state. Note that even though each of the $|\phi^{x^B, \theta^B, r}\rangle$ depends on θ^B , the partial trace

$$\begin{aligned} \text{Tr}_{X^B}[\rho^{(1)}] &= \frac{1}{2^{N_0}} \sum_{\theta^B} |\theta^B\rangle\langle\theta^B|_{\Theta^B} \otimes \sum_{x, r} |\alpha^{x, r}|^2 |r\rangle\langle r|_R |\phi^{x, r}\rangle\langle\phi^{x, r}|_A \end{aligned} \quad (\text{B46})$$

has a product form. Furthermore, because honest Bob measures each of his qubits independently, for any $I' \subseteq I$, the

partial trace

$$\text{Tr}_{X_{I'}^B}[\rho^{(1)}] = \frac{1}{2^{|I'|}} \sum_{\substack{\theta_{I'}^B \\ \in \{0, 1\}^{|I'|}}} |\theta_{I'}^B\rangle\langle\theta_{I'}^B|_{\Theta_{I'}^B} \otimes \rho_{X_{I'}^B, \Theta_{I'}^B, R, A}^{(1)} \quad (\text{B47})$$

also has a product form. As Alice will be able to perform quantum operations on her part of the joint state, it is important to note that the above property holds even after the A subsystem undergoes an arbitrary CPTP transformation independent of $\Theta_{I'}^B$ and $X_{I'}^B$. During step (4) Bob commits his values of θ^B and x^B , for that he samples the values of $s = (s_1, \dots, s_{N_0})$ and computes

$$\begin{aligned}\text{com} &= \left(\text{com}((\theta_1^B, x_1^B), s_1, r), \dots, \text{com}((\theta_{N_0}^B, x_{N_0}^B), s_{N_0}, r) \right), \\ \text{open} &= \left(\text{open}((\theta_1^B, x_1^B), s_1), \dots, \text{open}((\theta_{N_0}^B, x_{N_0}^B), s_{N_0}) \right),\end{aligned}\quad (\text{B48})$$

leading to the state

$$\begin{aligned}\rho^{(2)} &= \frac{1}{2^{N_0}} \sum_{x^B, \theta^B, r} |\beta^{x^B, \theta^B, r}|^2 |\theta^B\rangle\langle\theta^B|_{\Theta^B} |x^B\rangle\langle x^B|_{X^B} |r\rangle\langle r|_R |\phi^{x^B, \theta^B, r}\rangle\langle\phi^{x^B, \theta^B, r}|_A \\ &\quad \bigotimes_{i \in I} \left(\frac{1}{2^{n_s}} \sum_{s_i} |\text{com}((\theta_i^B, x_i^B), s_i, r)\rangle\langle\text{com}((\theta_i^B, x_i^B), s_i, r)|_{\text{COM}_i} \otimes |\text{open}((\theta_i^B, x_i^B), s_i)\rangle\langle\text{open}((\theta_i^B, x_i^B), s_i)|_{\text{OPEN}_i} \right).\end{aligned}\quad (\text{B49})$$

Let $J \subseteq I$, we now want to use the hiding property of the commitment scheme to approximate the state (B49) to one where the values of com and open_J do not provide any information about θ_J^B . First, we proceed to rewrite the expression for the COM_i and OPEN_i subsystems by grouping the individual values of com_i

$$\begin{aligned}&\frac{1}{2^{n_s}} \sum_{s_i} |\text{com}((\theta_i^B, x_i^B), s_i, r)\rangle\langle\text{com}((\theta_i^B, x_i^B), s_i, r)|_{\text{COM}_i} |\text{open}((\theta_i^B, x_i^B), s_i)\rangle\langle\text{open}((\theta_i^B, x_i^B), s_i)|_{\text{OPEN}_i} \\ &= \sum_{\substack{\text{com}_i \\ \in \{0,1\}^{n_c}}} P_{\text{com}}^{\theta_i, x_i, r}(\text{com}_i) |\text{com}_i\rangle\langle\text{com}_i|_{\text{COM}_i} \sum_{\substack{\text{open}_i \\ \in \mathcal{C}_r(\text{com}_i)}} P_{\text{open}}^{\theta_i, x_i, r}(\text{com}_i, \text{open}_i) |\text{open}_i\rangle\langle\text{open}_i|_{\text{OPEN}_i} \\ &= \sigma_{\text{COM}_i, \text{OPEN}_i}^{\theta_i, x_i, r},\end{aligned}\quad (\text{B50})$$

where $P_{\text{com}}^{\theta_i, x_i, r}$ is the respective distribution for com_i for uniformly sampled s_i , which depends on the commitment scheme used, and $\mathcal{C}_r(\text{com}_i)$ is the set of strings open_i that satisfy $\text{ver}(\text{com}_i, \text{open}_i, r) \neq \perp$. Substituting Eq. (B50) into Eq. (B49) and tracing out $\text{OPEN}_{\bar{J}}$ and R results in

$$\rho^{(2)} = \frac{1}{2^{N_0}} \sum_{x^B, \theta^B, r} |\beta^{x^B, \theta^B, r}|^2 |\theta^B\rangle\langle\theta^B|_{\Theta^B} |x^B\rangle\langle x^B|_{X^B} |\phi^{x^B, \theta^B, r}\rangle\langle\phi^{x^B, \theta^B, r}|_A \underbrace{\bigotimes_{i \in J} \sigma_{\text{COM}_i, \text{OPEN}_i}^{\theta_i, x_i, r}}_{\sigma_{\text{COM}_J, \text{OPEN}_J}^{\theta_J, x_J, r}} \underbrace{\bigotimes_{i \in \bar{J}} \sigma_{\text{COM}_i}^{\theta_i, x_i, r}}_{\sigma_{\text{COM}_{\bar{J}}}^{\theta_{\bar{J}}, x_{\bar{J}}, r}}. \quad (\text{B51})$$

The hiding property of the commitment scheme states that for any fixed r , the distributions $P_{\text{com}}^{\theta_i, x_i, r}$ are computationally indistinguishable among themselves. Let $P_{\text{com}}^r = P_{\text{com}}^{0,0,r}$, then

$$\sigma_{\text{COM}_i}^{\theta_i, x_i, r} \approx^{(c)} \tilde{\sigma}_{\text{COM}_i}^r, \quad (\text{B52})$$

with

$$\tilde{\sigma}_{\text{COM}_i}^r = \sum_{\substack{\text{com}_i \\ \in \{0,1\}^{n_c}}} P_{\text{com}}^r(\text{com}_i) |\text{com}_i\rangle\langle\text{com}_i|_{\text{COM}_i}. \quad (\text{B53})$$

Applying Eq. (B52) to the $\bar{J}$ subset in Eq. (B51), and from Lemma A2 (2) and (3) we get that

$$\rho_{\Theta^B, X^B, A, \text{COM}, \text{OPEN}_J}^{(2)} \approx^{(c)} \tilde{\rho}_{\Theta^B, X^B, A, \text{COM}, \text{OPEN}_J}^{(2)}, \quad (\text{B54})$$

where

$$\tilde{\rho}^{(2)} = \frac{1}{2^{N_0}} \sum_{x^B, \theta^B, r} |\beta^{x^B, \theta^B, r}|^2 |\theta^B\rangle\langle\theta^B|_{\Theta^B} |x^B\rangle\langle x^B|_{X^B} |\phi^{x^B, \theta^B, r}\rangle\langle\phi^{x^B, \theta^B, r}|_A \sigma_{\text{COM}_J, \text{OPEN}_J}^{\theta_J, x_J, r} \tilde{\sigma}_{\text{COM}_{\bar{J}}}^r. \quad (\text{B55})$$

Note that, since both $\sigma_{\text{COM}_J, \text{OPEN}_J}^{\theta_J, x_J}$ and $\tilde{\sigma}_{\text{COM}_J}$ are independent of θ_J^B, x_J^B , we can use Eq. (B47) with $I' = \bar{J}$ such that, after tracing the Θ_J^B, X^B subsystem, we obtain the state

$$\tilde{\rho}_{\Theta_J^B, A, \text{COM}, \text{OPEN}_J}^{(2)} = \mathbf{U}_{\Theta_J^B} \otimes \tilde{\rho}_{A, \text{COM}, \text{OPEN}_J}^{(2)}. \quad (\text{B56})$$

Finally, using Lemma A2(1) and (2) we obtain the required result

$$\rho_{\Theta_J^B, A, \text{COM}, \text{OPEN}_J}^{(2)} \approx^{(c)} \mathbf{U}_{\Theta_J^B} \otimes \rho_{A, \text{COM}, \text{OPEN}_J}^{(2)}. \quad (\text{B57})$$

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4. Proof of Lemma 5

In this section we present a proof of Lemma 5, which states that the string separation step of π_{QROT} does not leak any information about the random bit c to the receiver.

Lemma B6. Let $\mathcal{E}^{(I_t)} : \mathcal{H}_{A, \Theta_{I_t}^A, \Theta_{I_t}^B, C} \rightarrow \mathcal{H}_{A, \Theta_{I_t}^A, \Theta_{I_t}^B, C, \text{SEP}}$ be the quantum operation used by Bob to compute the string separation information (J_0, J_1) during step (9) of the protocol. The resulting state after applying $\mathcal{E}^{(I_t)}$ to a product state of the form

$$\mathcal{E}^{(I_t)}(\rho_{A, \Theta_{I_t}^A} \otimes \mathbf{U}_{\Theta_{I_t}^B} \otimes \mathbf{U}_C) = \sigma_{A, \Theta_{I_t}^A, \Theta_{I_t}^B, C, \text{SEP}} \quad (\text{B58})$$

satisfies

$$\text{Tr}_{\Theta_{I_t}^A, \Theta_{I_t}^B} \left[\sigma_{A, \Theta_{I_t}^A, \Theta_{I_t}^B, C, \text{SEP}} \right] = \sigma_A \otimes \sigma_{\text{SEP}} \otimes \mathbf{U}_C. \quad (\text{B59})$$

Proof. Let $\theta^{\text{ch}} = \theta^A \oplus \theta^B$ and, for $b \in \{0, 1\}$, define the sets $S_b = \{i \in \bar{I}_t | \theta_i^{\text{ch}} = b\}$. Bob's operation consists on randomly choosing subsets I_0, I_1 of size N_{raw} , from S_0 and S_1 , respectively, and then computing $J_0 = I_c, J_1 = I_{\bar{c}}$. Denote by $N_1 = (1 - \alpha)N_0$ the size of the working set $\bar{I}_t$, so that $N_{\text{raw}} = (\frac{1}{2} - \delta_2)N_1$. If the number of matching bases in $\bar{I}_t$, given by the Hamming weight $w_H(\theta_{I_t}^{\text{ch}})$, is either smaller than N_{raw} or greater than $N_1 - N_{\text{raw}}$, Bob will not be able to construct either I_0 or I_1 , in which case he sends an abort message to Alice independently of the value of c and Eq. (B59) is satisfied. On the other hand, we will show that whenever $N_{\text{raw}} \leq w_H(\theta_{I_t}^{\text{ch}}) \leq N_1 - N_{\text{raw}}$, the probability of choosing I_0, I_1 is the same for every I_0, I_1 . Define

$$C(I_0, I_1) = \{\theta^{\text{ch}} : \forall i \in I_0 \forall j \in I_1 (\theta_i^{\text{ch}} = 0 \ \& \ \theta_j^{\text{ch}} = 1)\}. \quad (\text{B60})$$

Note that, because for any two pairs $(I_0^1, I_1^1), (I_0^2, I_1^2)$ the elements of $C(I_0^1, I_1^1)$ and $C(I_0^2, I_1^2)$ are related to each other through a permutation of indices, the size of the $C(I_0, I_1)$ is independent of I_0, I_1 . The probability of Bob choosing I_0, I_1 is then given by

$$\begin{aligned} P(I_0, I_1) &= \sum_{\theta_{I_t}^B \in C(I_0, I_1)} P(I_0, I_1 | \theta_{I_t}^B) P(\theta_{I_t}^B) \\ &= \sum_{n=N_{\text{raw}}}^{N_0 - N_{\text{raw}}} \sum_{\substack{w_H(\theta_{I_t}^{\text{ch}}) = n \\ \theta^{\text{ch}} \in C(I_0, I_1)}} \binom{n}{N_{\text{raw}}}^{-1} \binom{N_1 - n}{N_{\text{raw}}}^{-1} P(\theta^{\text{ch}}) \\ &= \sum_{n=N_{\text{raw}}}^{N_0 - N_{\text{raw}}} \sum_{\substack{w_H(\theta_{I_t}^{\text{ch}}) = n \\ \theta^{\text{ch}} \in C(I_0, I_1)}} \binom{n}{N_{\text{raw}}}^{-1} \binom{N_1 - n}{N_{\text{raw}}}^{-1} 2^{-N_1} \\ &= P^{\text{SEP}}, \end{aligned} \quad (\text{B61})$$

where the combinatorial factors come from the fact that, for each θ^{ch} , the I_0, I_1 are chosen uniformly among all available compatible combinations, and the 2^{-N_1} factor comes from the fact that both θ^A and θ^B are sampled independently and θ^B is sampled uniformly [as guaranteed by the product form Eq. (B58)], and the last equality comes from the fact that the number of elements in $C(I_0, I_1)$ is constant, and hence the number of terms in the summation is the same for every (I_0, I_1) . Importantly, note that P^{SEP} is independent of (I_0, I_1) . To obtain Eq. (B59) we start by computing

$$\begin{aligned} \sigma_{\text{SEP}, \Theta_{I_t}^B, C} &= \mathcal{E}^{(I_t, \Theta_{I_t}^A)}(\mathbf{U}_{\Theta_{I_t}^B} \otimes \mathbf{U}_C) \\ &= \frac{1}{2^{N_1}} \sum_{\theta_{I_t}^B} |\theta_{I_t}^B\rangle\langle\theta_{I_t}^B|_{\Theta_{I_t}^B} \otimes \frac{1}{2} \sum_c |c\rangle\langle c|_C \\ &\quad \otimes \sum_{I_0, I_1} P(I_0, I_1 | \theta_{I_t}^B) |I_c, I_{\bar{c}}\rangle\langle I_c, I_{\bar{c}}|_{\text{SEP}} \\ &= \frac{1}{2} \sum_c |c\rangle\langle c|_C \otimes \sum_{I_0, I_1} P(I_0, I_1) |I_c, I_{\bar{c}}\rangle\langle I_c, I_{\bar{c}}|_{\text{SEP}} \\ &\quad \otimes \sum_{\theta_{I_t}^B \in C(I_0, I_1)} P(\theta_{I_t}^B | I_0, I_1) |\theta_{I_t}^B\rangle\langle\theta_{I_t}^B|_{\Theta_{I_t}^B}, \end{aligned} \quad (\text{B62})$$

where the sum in SEP goes over all possible I_0, I_1 given I_t . Tracing out $\Theta_{I_t}^B$ and using Eq. (B61) we obtain

$$\begin{aligned} \text{Tr}_{\Theta_{I_t}^B} \left[\sigma_{\text{SEP}, \Theta_{I_t}^B, C} \right] &= \frac{1}{2} \sum_c |c\rangle\langle c|_C \otimes \sum_{I_0, I_1} P^{\text{SEP}} |I_c, I_{\bar{c}}\rangle\langle I_c, I_{\bar{c}}|_{\text{SEP}} \\ &= \frac{1}{2} \sum_c |c\rangle\langle c|_C \otimes \sum_{I_0, I_1} P^{\text{SEP}} |I_0, I_1\rangle\langle I_0, I_1|_{\text{SEP}} \\ &= \mathbf{U}_{\text{SEP}}^{I_t} \otimes \mathbf{U}_C. \end{aligned} \quad (\text{B63})$$

■

5. Proof of Lemma 6

In this section we present a proof of Lemma 6, introduced in Sec. IV as part of the security analysis against a dishonest receiver. Recall that the transcript of the protocol $\vec{\tau} = (x_{I_t}^A, \theta^A, r, \text{com}, I_t, I_s, \text{open}_{I_t}, r)$ is defined to consist of all classical information (with the exception of her measurement outcomes) that Alice has access up to step (8) of the protocol.

Lemma B7. Assuming Alice follows the protocol, let X^A, B denote the systems of Alice measurement outcomes and Bob's laboratory at the end of step (9) of the protocol, and let $\rho_{X^A, B}$ be the state of the joint system at that point. There exists a state $\tilde{\rho}_{X^A, B}$, such as

- (1) The conditioned states $\tilde{\rho}_{X^A, B}(\vec{\tau}, J_0, J_1)$ satisfy

$$\begin{aligned} & H_{\min}(X_{J_0}^A | X_{J_1}^A B)_{\tilde{\rho}(\vec{\tau}, J_0, J_1)} + H_{\min}(X_{J_1}^A | X_{J_0}^A B)_{\tilde{\rho}(\vec{\tau}, J_0, J_1)} \\ & \geq 2N_{\text{raw}} \left(\frac{1}{2} - \frac{2\delta_2}{1 - 2\delta_2} - h\left(\frac{p_{\max} + \delta_1}{\frac{1}{2} - \delta_2}\right) \right); \end{aligned} \quad (\text{B64})$$

- (2) $\rho_{X^A, B} \approx_{\varepsilon} \tilde{\rho}_{X^A, B}$, with

$$\begin{aligned} \varepsilon = & \left(2 \left(e^{-\frac{1}{2}\alpha(1-\alpha)^2 N_0 \delta_1^2} + e^{-\frac{1}{2}\left(\frac{1}{2}-\delta_2\right)\alpha N_0 \delta_1^2} \right) \right)^{\frac{1}{2}} \\ & + e^{-D_{\text{KL}}\left(\frac{1}{2}-\delta_2 \middle| \frac{1}{2}\right)(1-\alpha)N_0} + \varepsilon_{\text{bind}}(k), \end{aligned} \quad (\text{B65})$$

where $h(\cdot)$ and $D_{\text{KL}}(\cdot | \cdot)$ denote the binary entropy and the binary relative entropy functions, respectively, and $\varepsilon_{\text{bind}}(k)$ is a negligible function given by the binding property of the commitment scheme.

Proof. We proceed by tracking the properties of Alice's and Bob's shared state as the protocol develops in order to bound the conditional min-entropy of Alice's measurement outcomes given the information the Bob gains during the protocol, then we use Lemma A4 to obtain the desired result. Let $\rho_{\Theta^A, T, R}^{\text{rand}}$ denote the quantum state associated to the systems holding Alice's basis choice θ^A , test subset I_t , and the value of r used in the commit and open phase, which we can treat as if they are sampled at the beginning of the protocol since their distribution is fixed, and is given by

$$\begin{aligned} \rho_{\Theta^A, T, R}^{\text{rand}} = & \frac{1}{2^{N_0}} \sum_{\theta^A} |\theta^A\rangle\langle\theta^A|_{\Theta^A} \otimes \frac{1}{|\mathcal{T}(\alpha N_0, I)|} \sum_{I_t} |I_t\rangle\langle I_t|_T \\ & \otimes \frac{1}{2^{n_r}} \sum_r |r\rangle\langle r|_R, \end{aligned} \quad (\text{B66})$$

where $\mathcal{T}(\alpha N_0, I)$ denotes the set of subsets of $I = \{1, 2, \dots, N_0\}$ with αN_0 elements. Let $S_{\text{bind}}(k)$ be the

set of all $r \in \{0, 1\}^{n_r(k)}$ for which there exists a tuple $(\text{com}, \text{open}_1, \text{open}_2)$ such that

$$\perp \neq \text{ver}(\text{com}, \text{open}_1, r) \neq \text{ver}(\text{com}, \text{open}_2, r) \neq \perp. \quad (\text{B67})$$

From the binding property of the commitment scheme we know that there exists a negligible function $\varepsilon_{\text{bind}}(k)$ such that, for a commitment security parameter k it holds that

$$\frac{S_{\text{bind}}(k)}{2^{n_r(k)}} = \varepsilon_{\text{bind}}(k), \quad (\text{B68})$$

and hence the state

$$\begin{aligned} \tilde{\rho}_{\Theta^A, T, R}^{\text{rand}} = & \frac{1}{2^{N_0}} \sum_{\theta^A} |\theta^A\rangle\langle\theta^A|_{\Theta^A} \otimes \frac{1}{|\mathcal{T}(\alpha N_0, I)|} \sum_{I_t} |I_t\rangle\langle I_t|_T \\ & \otimes P_R \sum_{r \in S_{\text{bind}}} |r\rangle\langle r|_R, \end{aligned} \quad (\text{B69})$$

where $P_R = \frac{1}{2^{n_r} - |S_{\text{bind}}|}$, satisfies

$$\rho_{\Theta^A, T, R}^{\text{rand}} \approx_{\varepsilon_{\text{bind}}(k)} \tilde{\rho}_{\Theta^A, T, R}^{\text{rand}}. \quad (\text{B70})$$

In other words, the state of the system holding the value of the variable r is indistinguishable to one where the commitment scheme is perfectly binding (for all com strings, there is at most one open string that passes verification).

Additionally, the state of the shared resource system as after Bob receives his shares at the beginning of the protocol is given by

$$\begin{aligned} \rho^{(0)} = & \tilde{\rho}_{\Theta^A, T, R}^{\text{rand}} \otimes |\psi^{(0)}\rangle\langle\psi^{(0)}| |\psi^{(0)}\rangle \\ = & \frac{1}{\sqrt{2^{N_0}}} \sum_x |x\rangle_{\Phi^A} |x\rangle_{\Phi^B}. \end{aligned} \quad (\text{B71})$$

Since the measurement on Alice subsystem is performed independently from Bob's actions, we can equivalently consider a version of the protocol in which Alice does not measure her side of the shared resource state until it is needed to perform the check at step (7) (for the indices in I_t) and the computation of the syndromes at step (10) (for the remaining indices).

We now turn our attention to step (4), when Bob computes and sends his commitment strings after receiving the value of r . Denote by B_0 the system containing all of Bob's laboratory at the beginning of the protocol, and let U_1 be the transformation that Bob performs on his system to produce the commitments, which has the general form

$$U_1 |r\rangle_R |x\rangle_{\Phi^B} |0\rangle_{B_0} = \sum_{\text{com}} \alpha^{r, x, \text{com}} |\text{com}\rangle_{\text{COM}} |\phi^{r, x, \text{com}}\rangle_{B_1}, \quad (\text{B72})$$

where $\mathcal{H}_R \otimes \mathcal{H}_{\Phi^B} \otimes \mathcal{H}_{B_0} = \mathcal{H}_{\text{COM}} \otimes \mathcal{H}_{B_1}$, and $\text{com} = (\text{com}_1, \text{com}_2, \dots, \text{com}_{N_0})$ with $\text{com}_i \in \{0, 1\}^{n_c(k)}$. Bob then proceeds to send the COM system to Alice, who measures it in the computational basis. The joint shared state as Bob sends the commitment information is

$$\rho^{(1)} = \frac{1}{2^{N_0} |\mathcal{T}(\alpha N_0, I)|} \sum_{\theta^A} |\theta^A\rangle\langle\theta^A|_{\Theta^A} \sum_{I_t} |I_t\rangle\langle I_t|_T \sum_{r \in \tilde{S}_{\text{bind}}^{\text{com}}} P_{\text{com}}^r |\text{com}\rangle\langle\text{com}|_{\text{COM}} |\eta^{r, \text{com}}\rangle\langle\eta^{r, \text{com}}|_{\Phi^A B_1}, \quad (\text{B73})$$

where

$$P_{\text{com}}^r = \frac{P_R}{2^{N_0}} \sum_x |\alpha^{r, x, \text{com}}|^2$$

$$|\eta^{r, \text{com}}\rangle_{\Phi^A B_1} = \sum_x \underbrace{\sqrt{\frac{P_R}{2^{N_0}}} (P_{\text{com}}^r)^{-\frac{1}{2}} \alpha^{r, x, \text{com}}}_{\beta^{r, x, \text{com}}} |x\rangle_{\Phi^A} |\phi^{r, x, \text{com}}\rangle_{B_1}. \quad (\text{B74})$$

We intend to use Lemma B1 to bound the form of the shared state after the parameter estimation step, and then Lemma B3 to bound the amount of correlation between Alice's measurement outcomes on the system Φ^A and Bob's system. For that, we first need to associate Bob's commitments with their corresponding committed strings x^B and θ^B . For an arbitrary dishonest Bob the strings that Alice received are not guaranteed to be outcomes of the com function and may not have an associated preimage. Consider now the functions $x_i^B(r, \text{com}), \theta_i^B(r, \text{com}) : \{0, 1\}^{n_r} \times \{0, 1\}^{n_c} \rightarrow \{0, 1\}$ defined as follows:

$$x_i^B(r, \text{com}) = \begin{cases} x, & \text{if } \text{com}_i = \text{com}((\theta, x), s, r) \text{ for some } \theta \in \{0, 1\}, s \in \{0, 1\}^{n_s}, \\ 0, & \text{otherwise,} \end{cases} \quad (\text{B75})$$

$$\theta_i^B(r, \text{com}) = \begin{cases} \theta, & \text{if } \text{com}_i = \text{com}((\theta, x), s, r) \text{ for some } x \in \{0, 1\}, s \in \{0, 1\}^{n_s}, \\ 0, & \text{otherwise,} \end{cases} \quad (\text{B76})$$

and denote

$$x^B(r, \text{com}) = (x_i^B(r, \text{com}))_i, \quad \theta^B(r, \text{com}) = (\theta_i^B(r, \text{com}))_i. \quad (\text{B77})$$

We know the above functions are well defined for all $r \in \tilde{S}_{\text{bind}}$ because, by definition of S_{bind} , for each possible value of com_i , there is at most a single opening that passes verification. For any $\theta^B \in \{0, 1\}^{N_0}$ we can write the state $|\eta^{r, \text{com}}\rangle_{\Phi^A B_1}$ in the θ^B basis of Φ^A as

$$|\eta^{r, \text{com}}\rangle_{\Phi^A B_1} = \sum_x \underbrace{\sum_{x'} \beta^{r, x, \text{com}} \langle x, \theta^B | x' \rangle |\phi^{r, x', \text{com}}\rangle_{B_1}}_{\beta^{r, x, \theta^B, \text{com}} |\phi^{r, x, \theta^B, \text{com}}\rangle} |x, \theta^B\rangle_{\Phi^A}. \quad (\text{B78})$$

Recall that $I_s(\theta^A, \theta^B, I_t) = \{i \in I_t : \theta_i^A \oplus \theta_i^B = 0\}$. From Lemma B1 we know that there exists a state

$$\tilde{\rho}^{(1)} = \frac{1}{2^{N_0} |\mathcal{T}(\alpha N_0, I)|} \sum_{r \in \tilde{S}_{\text{bind}}^{\text{com}}} P_{\text{com}}^r |\text{com}\rangle\langle\text{com}|_{\text{COM}} \sum_{\theta^A, I_t} |\theta^A\rangle\langle\theta^A|_{\Theta^A} |I_t\rangle\langle I_t|_T |\eta^{r, \text{com}, I_s, \tilde{I}_t}\rangle\langle\eta^{r, \text{com}, I_s, \tilde{I}_t}|_{\Phi^A B_1}, \quad (\text{B79})$$

where the $|\eta^{r, \text{com}, I_s, \tilde{I}_t}\rangle$ have the form

$$|\eta^{r, \text{com}, I_s, \tilde{I}_t}\rangle_{\Phi^A B_1} = \sum_{x \in B(\theta^B, r, \text{com}, I_s, I_t)} \tilde{\beta}^{x, \theta^B, r, \text{com}, I_s, I_t} |x, \theta^B(r, \text{com})\rangle_{\Phi^A} |\phi^{x, \theta^B, r, \text{com}, I_s, I_t}\rangle_{B_1} \quad (\text{B80})$$

$$B(r, \text{com}, I_s, I_t) = \{x : |r_H(x_{I_s} \oplus x_{I_s}^B(r, \text{com})) - r_H(x_{\tilde{I}_t} \oplus x_{\tilde{I}_t}^B(r, \text{com}))| \leq \delta_1\}, \quad (\text{B81})$$

such that

$$D(\rho^{(1)}, \tilde{\rho}^{(1)}) \leq \sqrt{2} \left(e^{-\frac{1}{2}\alpha(1-\alpha)^2 N_0 \delta_1^2} + e^{-\frac{1}{2}(\frac{1}{2}-\delta_2)\alpha N_0 \delta_1^2} \right)^{\frac{1}{2}}. \quad (\text{B82})$$

We are ready now proceed to step (6) of the protocol, in which Bob sends the string $\text{open}_{I_t} = (\text{open}_i)_{i \in I_t}$, which is expected to contain the opening information for all the commitments $\text{com}_i, i \in I_t$.

$$U_{\text{open}} |\phi^{x,r,\text{com},I_s,I_t}\rangle_{B_1} = \sum_{\text{open}_{I_t}} \alpha^{x,r,\text{com},I_s,I_t,\text{open}_{I_t}} |\phi^{x,r,\text{com},I_s,I_t,\text{open}_{I_t}}\rangle_{B_2} |\text{open}_{I_t}\rangle_{\text{OPEN}_{I_t}}, \quad (\text{B83})$$

where $\mathcal{H}_{B_1} = \mathcal{H}_{B_2} \otimes \mathcal{H}_{\text{OPEN}_{I_t}}$. Such that

$$U_{\text{open}} |\eta^{r,\text{com},I_s,\bar{I}_t}\rangle_{\Phi^A B_1} = \sum_{\text{open}_{I_t}} \sum_{x \in B} \tilde{\beta}^{x,r,\text{com},I_s,I_t} \alpha^{x,r,\text{com},I_s,I_t,\text{open}_{I_t}} |x, \theta^B(r, \text{com})\rangle_{\Phi^A} \otimes |\phi^{x,r,\text{com},I_s,I_t,\text{open}_{I_t}}\rangle_{B_2} |\text{open}_{I_t}\rangle_{\text{OPEN}_{I_t}}. \quad (\text{B84})$$

During step (7), after receiving the opening information and measuring the OPEN system in the computational basis, she aborts the protocol unless $\text{ver}(\text{com}_i, \text{open}_i, r) \neq \perp$ for all $i \in I_t$. Let $H(r, I_t)$ be the set of strings com for which Alice's first check *can* be passed. From the binding property of the commitment scheme, we know that, for any $r \in \bar{S}_{\text{bind}}$, if $\text{com} \in H(r, I_t)$ there is only one $\text{open}'(r, \text{com}, I_t)$ for which $\text{ver}(\text{com}_i, \text{open}'_i, r) \neq \perp$ for all $i \in I_t$. Because the protocol aborts if Alice's test is not passed, the state of the joint system after Alice performs this check is given by (note that from here, by removing the mixture over all opens, we are reducing the overall trace of the system). Effectively, we are keeping only the runs of the protocol that did not abort in the commitment check part of step (7). The amount for which the trace is reduced is given by the sum of the $|\alpha^{x,r,\text{com},I_s,I_t,\text{open}_{I_t}}|^2$ over the values of $\text{open} \neq \text{open}'(\text{com}, I_t)$ or for which $I_s < N_{\text{check}}$:

$$\begin{aligned} \tilde{\rho}^{(2)} &= \frac{1}{2^{N_0} |\mathcal{T}(\alpha N_0, I)|} \sum_{I_t, \theta^A} |I_t\rangle\langle I_t|_T \sum_{\substack{r \in \bar{S}_{\text{bind}} \\ \text{com} \in H(r, I_t)}} P_{\text{com}}^r P_{\text{open}'}^r |\text{com}\rangle\langle \text{com}|_{\text{COM}} |\text{open}'\rangle\langle \text{open}'|_{\text{OPEN}_{I_t}} \\ &\otimes |\theta^A\rangle\langle \theta^A|_{\Theta^A} |\tilde{\eta}^{r,\text{com},I_s,\bar{I}_t}\rangle\langle \tilde{\eta}^{r,\text{com},I_s,\bar{I}_t}|_{\Phi^A B_2}, \end{aligned} \quad (\text{B85})$$

with

$$P_{\text{open}'}^r = \sum_{\substack{x \in \\ B(r,\text{com},I_s,I_t)}} |\tilde{\beta}^{x,r,\text{com},I_s,I_t} \alpha^{x,r,\text{com},I_s,I_t,\text{open}'}|^2$$

and

$$|\tilde{\eta}^{r,\text{com},I_s,\bar{I}_t}\rangle = \sum_{\substack{x \in \\ B(r,\text{com},I_s,I_t)}} \left(\underbrace{(P_{\text{open}'}^r)^{-\frac{1}{2}} \tilde{\beta}^{x,r,\text{com},I_s,I_t} \alpha^{x,r,\text{com},I_s,I_t,\text{open}'}}_{\gamma^{x,r,\text{com},I_s,I_t} |\tilde{\phi}^{x,r,\text{com},I_s,I_t}\rangle} |\phi^{x,r,\text{com},I_s,I_t,\text{open}'}\rangle_{B_2} \otimes |x, \theta^B(r, \text{com})\rangle_{\Phi^A} \right). \quad (\text{B86})$$

Alice then proceeds to measure her part of the state. Let us divide her measurement in two parts: the measurement of the qubits in I_t , and the measurement of the reminder qubits. For the first part, the action of measuring the subsystem $\Phi_{I_t}^A$ in a state $|\tilde{\eta}^{\text{com},I_s,\bar{I}_t}\rangle$ and in the $\theta_{I_t}^A$ basis is

$$\begin{aligned} |\tilde{\eta}^{r,\text{com},I_s,\bar{I}_t}\rangle\langle \tilde{\eta}^{r,\text{com},I_s,\bar{I}_t}| &\rightarrow \sum_{x_{I_t}^A} |x_{I_t}^A, \theta_{I_t}^A\rangle\langle x_{I_t}^A, \theta_{I_t}^A| \left(|\tilde{\eta}^{r,\text{com},I_s,\bar{I}_t}\rangle\langle \tilde{\eta}^{r,\text{com},I_s,\bar{I}_t}| \right) |x_{I_t}^A, \theta_{I_t}^A\rangle\langle x_{I_t}^A, \theta_{I_t}^A| \\ &= \sum_{x_{I_t}^A} P_{x_{I_t}^A}^r |x_{I_t}^A, \theta_{I_t}^A\rangle\langle x_{I_t}^A, \theta_{I_t}^A|_{\Phi_{I_t}^A} |\tilde{\eta}^{r,\text{com},I_s,\bar{I}_t,\theta^A, x_{I_t}^A}\rangle\langle \tilde{\eta}^{r,\text{com},I_s,\bar{I}_t,\theta^A, x_{I_t}^A}|_{\Phi_{I_t}^A B_2}, \end{aligned} \quad (\text{B87})$$

where

$$P_{x_{I_t}^A}^r = \sum_{x \in B} |\langle x_{I_t}, \theta_{I_t}^B(r, \text{com}) | x_{I_t}^A, \theta_{I_t}^A \rangle \gamma^{x, r, \text{com}, I_s, I_t}|^2 \quad (\text{B88})$$

and

$$|\tilde{\eta}^{r, \text{com}, I_s, \bar{I}_t, x_{I_t}^A, \theta^A}\rangle = \sum_{x \in B} (P_{x_{I_t}^A}^r)^{-\frac{1}{2}} \underbrace{\langle x_{I_s}, \theta_{I_s}^B | x_{I_s}^A, \theta_{I_s}^B \rangle}_{\delta(x_{I_s}, x_{I_s}^A)} \langle x_{I_t \setminus I_s}, \theta_{I_t \setminus I_s}^B | x_{I_t \setminus I_s}^A, \bar{\theta}_{I_t \setminus I_s}^B \rangle \gamma^{x, \theta^B, r, \text{com}, I_s, I_t} |\tilde{\phi}^{x, \theta^A, r, \text{com}, I_t}\rangle_{B_2} |x_{\bar{I}_t}, \theta_{\bar{I}_t}^B\rangle_{\Phi_{\bar{I}_t}^A}, \quad (\text{B89})$$

where in the last expression, and going forward, we omit the explicit dependence of both x^B and θ^B on r, com . By defining

$$G(x_{I_s}^A, r, \text{com}) = \{x_{\bar{I}_t} : |r_H(x_{I_s}^A \oplus x_{I_s}^B) - r_H(x_{\bar{I}_t} \oplus x_{\bar{I}_t}^B)| \leq \delta_1\}, \quad (\text{B90})$$

we can rewrite

$$|\tilde{\eta}^{\text{com}, I_s, \bar{I}_t, x_{I_t}^A, \theta^A}\rangle = \sum_{x_{I_t}^A \in G} \sum_{x_{I_t \setminus I_s}} (P_{x_{I_t}^A}^r)^{-\frac{1}{2}} \langle x_{I_t \setminus I_s}, \theta_{I_t \setminus I_s}^B | x_{I_t \setminus I_s}^A, \theta_{I_t \setminus I_s}^B \rangle \gamma^{x_{I_t}, x_{I_t \setminus I_s}^A, r, \text{com}, I_s, I_t} |\tilde{\phi}^{x_{I_t}, x_{I_t \setminus I_s}^A, r, \text{com}, I_t}\rangle_{B_2} \otimes |x_{\bar{I}_t}, \theta_{\bar{I}_t}^B\rangle_{\Phi_{\bar{I}_t}^A}. \quad (\text{B91})$$

$\underbrace{\gamma^{x_{I_t}, x_{I_t \setminus I_s}^A, r, \text{com}, I_s, I_t} |\tilde{\phi}^{x_{I_t}, x_{I_t \setminus I_s}^A, r, \text{com}, I_t}\rangle}_{\gamma^{x_{I_t}, r, \text{com}, I_s, I_t} |\tilde{\phi}^{x_{I_t}, r, \text{com}, I_s, I_t, \theta^A}\rangle}$

After performing the measurement, Alice aborts the protocol whenever $r_H(x_{I_s}^A \oplus x_{I_s}^B(r, \text{com})) > p_{\max}$. The state of the shared system after this check is (tracing out the $T, \text{COM}, \text{OPEN}$ subsystems)

$$\begin{aligned} \tilde{\rho}^{(3)} &= \frac{1}{2^{N_0} |\mathcal{T}(\alpha N_0, I)|} \sum_{I_t} \sum_{\substack{r \in \tilde{\mathcal{S}}_{\text{bind}} \\ \text{com} \in H(r, I_t)}} P_{\text{com}}^r P_{\text{open}'}^r \sum_{\theta^A} |\theta^A\rangle \langle \theta^A|_{\Theta^A} \\ &\otimes \sum_{\substack{x_{I_t}^A \in \\ J_{p_{\max}}}} P_{x_{I_t}^A}^r |x_{I_t}^A, \theta_{I_t}^A\rangle \langle x_{I_t}^A, \theta_{I_t}^A|_{\Phi_{I_t}^A} |\tilde{\eta}^{r, \text{com}, I_s, \bar{I}_t, x_{I_t}^A, \theta^A}\rangle \langle \tilde{\eta}^{r, \text{com}, I_s, \bar{I}_t, x_{I_t}^A, \theta^A}|_{\Phi_{\bar{I}_t}^A B_2}, \end{aligned} \quad (\text{B92})$$

where

$$J_{p_{\max}} = \{x_{I_t}^A : r_H(x_{I_s}^A \oplus x_{I_s}^B) \leq p_{\max}\}. \quad (\text{B93})$$

Before proceeding, it will be useful to approximate the above state to a state where the number of mismatching bases in $\bar{I}_t$ is “high enough.” More precisely, this means approximating $\tilde{\rho}^{(3)}$ to a state for which the sum over θ^A runs explicitly over strings $\theta^A \in K_{\text{com}} = \{\theta^A : w_H(\theta_{\bar{I}_t}^{\text{ch}}) \geq N_{\text{raw}}\}$. Let

$$\begin{aligned} \tilde{\tilde{\rho}}^{(3)} &= \frac{1}{2^{N_0} |\mathcal{T}(\alpha N_0, I)|} \sum_{I_t} \sum_{\substack{r \in \tilde{\mathcal{S}}_{\text{bind}} \\ \text{com} \in H(r, I_t)}} P_{\text{com}}^r P_{\text{open}'}^r a(\text{com}) \sum_{\theta^A \in K_{\text{com}}} |\theta^A\rangle \langle \theta^A|_{\Theta^A} \\ &\otimes \sum_{\substack{x_{I_t}^A \in \\ J_{p_{\max}}}} P_{x_{I_t}^A}^r |x_{I_t}^A, \theta_{I_t}^A\rangle \langle x_{I_t}^A, \theta_{I_t}^A|_{\Phi_{I_t}^A} |\tilde{\eta}^{r, \text{com}, I_s, \bar{I}_t, x_{I_t}^A}\rangle \langle \tilde{\eta}^{r, \text{com}, I_s, \bar{I}_t, x_{I_t}^A}|_{\Phi_{\bar{I}_t}^A B_2}, \end{aligned} \quad (\text{B94})$$

with $a(\text{com}) = 2^{N_0}/|K_{\text{com}}|$. The distance between $\tilde{\rho}^{(3)}$ and $\tilde{\tilde{\rho}}^{(3)}$ is bounded by the probability of a uniformly chosen θ^A not being in K_{com} . Using the Chernoff-Hoeffding bound we get

$$D(\tilde{\rho}^{(3)}, \tilde{\tilde{\rho}}^{(3)}) \leq e^{-D_{\text{KL}}(\frac{1}{2} - \delta_2 | \frac{1}{2})(1-\alpha)N_0}, \quad (\text{B95})$$

where $D_{\text{KL}}(\frac{1}{2} - \delta_2 | \frac{1}{2})$ represents the relative entropy between the binary distributions defined by the respective probabilities $p_1 = \frac{1}{2} - \delta_2$ and $p_2 = \frac{1}{2}$.

During step (8), Alice sends the Θ^A system to Bob, who then computes J_0, J_1 (in the actual protocol, Alice sends only $\Theta_{I_t}^A$, but to simplify the expressions we can assume, without loss of generality, that she sends the whole register Θ^A). To simplify the list of dependencies, denote the transcript of the protocol up until step (8) as $\vec{\tau} = (x_{I_t}^A, \theta^A, r, \text{com}, I_t, I_s, \text{open}_{I_t})$. Keep in mind that, although $\vec{\tau}$ consists of seven quantities, I_s and open_{I_t} are completely defined by the other five. In the remaining of the proof, unless noted otherwise, the sums over $\vec{\tau}$ run over the values of its variables as shown in Eq. (B94). By defining

$$P_{\vec{\tau}} = \frac{P_{\text{com}}^r P_{\text{open}}^r P_{x_{I_t}^A}^r a(\text{com})}{2^{N_0} |\mathcal{T}(\alpha N_0, I)|}, \quad (\text{B96})$$

we can write

$$\tilde{\rho}^{(3)} = \sum_{\vec{\tau}} P_{\vec{\tau}} |\theta^A\rangle \langle \theta^A|_{\Theta^A} |x_{I_t}^A, \theta_{I_t}^A\rangle \langle x_{I_t}^A, \theta_{I_t}^A|_{\Phi_{I_t}^A} |\tilde{\eta}^{r, \text{com}, I_s, \bar{I}_t, x_{I_t}^A}\rangle \langle \tilde{\eta}^{r, \text{com}, I_s, \bar{I}_t, x_{I_t}^A}|_{\Phi_{I_t}^A B_2}. \quad (\text{B97})$$

During step (9), after receiving θ^A , Bob sends the SEP system, containing the (classical) string separation information J_0, J_1 to Alice. By following the same treatment as in steps (4) and (6), let U_{sep} be the operation that Bob performs on the B_2 system to compute the information to be sent to Alice in the SEP system:

$$U_{\text{sep}} |\tilde{\phi}^{x_{I_t}^A, r, \text{com}, I_s, I_t, \theta^A}\rangle_{B_2} |\theta^A\rangle_{\Theta^A} = \sum_{J_0, J_1} \alpha^{x_{I_t}^A, \vec{\tau}, J_0, J_1} |\phi^{x_{I_t}^A, \vec{\tau}, J_0, J_1}\rangle_{B_3} |J_0, J_1\rangle_{\text{SEP}}, \quad (\text{B98})$$

where $\mathcal{H}_{B_2} \otimes \mathcal{H}_{\Theta^A} = \mathcal{H}_{B_3} \otimes \mathcal{H}_{\text{SEP}}$ and the summation over J_0, J_1 goes over all possible values compatible with I_t . The state after step (9) after Alice receives the SEP system and measures in the computational basis is then given by (tracing out SEP)

$$\tilde{\rho}^{(4)} = \sum_{\substack{\vec{\tau} \\ J_0, J_1}} P_{\vec{\tau}, J_0, J_1} |x_{I_t}^A, \theta_{I_t}^A\rangle \langle x_{I_t}^A, \theta_{I_t}^A|_{\Phi_{I_t}^A} |v^{\vec{\tau}, J_0, J_1}\rangle \langle v^{\vec{\tau}, J_0, J_1}|_{\Phi_{I_t}^A B_3}, \quad (\text{B99})$$

where

$$\begin{aligned} P_{\vec{\tau}, J_0, J_1} &= \sum_{x_{I_t}^A \in G} |\tilde{\gamma}^{x_{I_t}^A, r, \text{com}, I_s, I_t} \alpha^{x_{I_t}^A, \vec{\tau}, J_0, J_1}|^2 \\ |v^{\vec{\tau}, J_0, J_1}\rangle_{\Phi_{I_t}^A B_3} &= \sum_{x_{I_t}^A \in G} \underbrace{(P_{\vec{\tau}, J_0, J_1})^{-\frac{1}{2}} \tilde{\gamma}^{x_{I_t}^A, r, \text{com}, I_s, I_t} \alpha^{x_{I_t}^A, \vec{\tau}, J_0, J_1}}_{\beta^{x_{I_t}^A, \vec{\tau}, J_0, J_1}} |x_{I_t}^A, \theta_{I_t}^A\rangle_{\Phi_{I_t}^A} |\phi^{x_{I_t}^A, \vec{\tau}, J_0, J_1}\rangle_{B_3}. \end{aligned} \quad (\text{B100})$$

We can now consider Alice's measurement on the $\Phi_{I_t}^A$ system. So far we have tracked the evolution of the joint state in order to describe the relationship between both parties' information. To finalize the proof we only need to keep track of the conditional min-entropy of Alice's outcomes given Bob's part of the joint system. Let $\rho_{X_{I_t}^A, B_3}(\vec{\tau}, J_0, J_1)$ be the resulting (conditioned) state after measuring the system $\Phi_{I_t}^A$ in the $\theta_{I_t}^A$ basis, recording the respective outcomes in the $X_{I_t}^A$ system, and tracing out the $\Phi_{I_t}^A$ subsystem. We can write the state of the joint system after the measurement as

$$\tilde{\rho}^{(5)} = \sum_{\substack{\vec{\tau} \\ J_0, J_1}} P_{\vec{\tau}, J_0, J_1} \tilde{\rho}_{X_{I_t}^A, B_3}(\vec{\tau}, J_0, J_1). \quad (\text{B101})$$

Additionally, for any given J_0, J_1 , denote by J_d the complement of $J_0 \cup J_1$ in $\bar{I}_t$. Following Lemma A3 (3) and (5) we know that for any $b \in \{0, 1\}$

$$\begin{aligned} H_{\min}^e(X_{J_b}^A | X_{J_b}^A B_3)_{\tilde{\rho}^{(5)}} &\geq H_{\min}^e(X_{J_b}^A | X_{J_b}^A X_{J_d}^A B_3)_{\tilde{\rho}^{(5)}} \geq \inf_{\vec{\tau}, J_0, J_1} \left\{ H_{\min}^e(X_{J_b}^A | X_{J_b}^A X_{J_d}^A B_3)_{\tilde{\rho}(\vec{\tau}, J_0, J_1)} \right\} \\ &\geq \inf_{\vec{\tau}, J_0, J_1} \left\{ \inf_{x_{J_{\bar{b}, d}}^A} \left\{ H_{\min}^e(X_{J_b}^A | B_3)_{\tilde{\rho}(x_{J_{\bar{b}, d}}^A, \vec{\tau}, J_0, J_1)} \right\} \right\}. \end{aligned} \quad (\text{B102})$$

We can invoke Lemma B3 to obtain an expression for the above quantity explicitly in terms of the protocol parameters N_0, α, δ_1 , and δ_2 . For that, we must take a small detour to define the associated mixed states $\rho_{\Phi_{J_b}^A B_3}^{\text{mix}}$ and compute their respective post-measurement entropy. First, for $b \in \{0, 1\}$, we compute the reduced states

$$\rho_{\Phi_{J_b}^A B_3}(\vec{\tau}, J_0, J_1) = \text{Tr}_{\Phi_{J_{\bar{b},d}}^A} \left[|v^{\vec{\tau}, J_0, J_1}\rangle \langle v^{\vec{\tau}, J_0, J_1}|_{\Phi_{J_b}^A B_3} \right] = \sum_{x_{J_{\bar{b},d}}} P_{x_{J_{\bar{b},d}}}^{\vec{\tau}, J_0, J_1} |v^{x_{J_{\bar{b},d}}, \vec{\tau}, J_0, J_1}\rangle \langle v^{x_{J_{\bar{b},d}}, \vec{\tau}, J_0, J_1}|_{\Phi_{J_b}^A B_3}, \quad (\text{B103})$$

with

$$\begin{aligned} P_{x_{J_{\bar{b},d}}}^{\vec{\tau}, J_0, J_1} &= \sum_{\substack{x_{J_b} \in \\ B_b(x_{J_{\bar{b},d}})}} \left| \beta^{x_{J_b}, \vec{\tau}, J_0, J_1} \right|^2 \\ |v^{x_{J_{\bar{b},d}}, \vec{\tau}, J_0, J_1}\rangle_{\Phi_{J_b}^A B_3} &= \sum_{\substack{x_{J_b} \in \\ B_b(x_{J_{\bar{b},d}})}} \underbrace{(P_{x_{J_{\bar{b},d}}}^{\vec{\tau}, J_0, J_1})^{-\frac{1}{2}} \beta^{x_{J_b}, \vec{\tau}, J_0, J_1}}_{\lambda^{x_{J_b}, \vec{\tau}, J_0, J_1}} |x_{J_b}, \theta_{J_b}^B\rangle_{\Phi_{J_b}^A} |\phi^{x_{J_b}, \vec{\tau}, J_0, J_1}\rangle_{B_3}, \end{aligned} \quad (\text{B104})$$

and

$$\begin{aligned} B_b(x_{J_{\bar{b},d}}) &= \left\{ x_{J_b} : x_{J_b, \bar{b}, d} \in G(x_{I_s}^A, r, \text{com}) \right\} \\ &= \left\{ x_{J_b} : \left| \left(\frac{1}{2} - \delta_2 \right) r_H(x_{J_b} \oplus x_{J_b}^B) + \left(\frac{1}{2} + \delta_2 \right) r_H(x_{J_{\bar{b},d}} \oplus x_{J_{\bar{b},d}}^B) - r_H(x_{I_s}^A \oplus x_{I_s}^B) \right| \leq \delta_1 \right\}, \end{aligned} \quad (\text{B105})$$

where the explicit dependence of B_b on $x_{I_s}^A, r, \text{com}$ has been omitted for compactness. Note that since $r_H(x_{I_s}^A \oplus x_{I_s}^B) \leq p_{\max}$ the size of $B_b(x_{J_{\bar{b},d}})$ is upper bounded by

$$|B_b(x_{J_{\bar{b},d}})| \leq 2^{h\left(\frac{p_{\max} + \delta_1}{\frac{1}{2} - \delta_2}\right) N_{\text{raw}}}, \quad (\text{B106})$$

where the h stands for the binary entropy function. We can now define

$$\rho_{\Phi_{J_b}^A B_3}^{\text{mix}}(x_{J_{\bar{b},d}}, \vec{\tau}, J_0, J_1) = \sum_{\substack{x_{J_b} \in \\ B_b(x_{J_{\bar{b},d}})}} \left| \lambda^{x_{J_b}, \vec{\tau}, J_0, J_1} \right|^2 |x_{J_b}, \theta_{J_b}^B\rangle \langle x_{J_b}, \theta_{J_b}^B|_{\Phi_{J_b}^A} \otimes |\phi^{x_{J_b}, \vec{\tau}, J_0, J_1}\rangle \langle \phi^{x_{J_b}, \vec{\tau}, J_0, J_1}|_{B_3}. \quad (\text{B107})$$

Measuring the above state in the $\theta_{J_b}^A$ basis, recording the results in X_{J_b} and tracing out $\Phi_{J_b}^A$ leads to

$$\rho_{X_{J_b}^A B_3}^{\text{mix}}(x_{J_{\bar{b},d}}, \vec{\tau}, J_0, J_1) = \sum_{x_{J_b}^A} \sum_{\substack{x_{J_b} \in \\ B_b(x_{J_{\bar{b},d}})}} \left| \lambda^{x_{J_b}, \vec{\tau}, J_0, J_1} \right|^2 |\langle x_{J_b}^A, \theta_{J_b}^A | x_{J_b}, \theta_{J_b}^B \rangle|^2 |x_{J_b}^A\rangle \langle x_{J_b}^A|_{X_{J_b}^A} \otimes |\phi^{x_{J_b}, \vec{\tau}, J_0, J_1}\rangle \langle \phi^{x_{J_b}, \vec{\tau}, J_0, J_1}|_{B_3}, \quad (\text{B108})$$

defining $J_b^{0/1} = \{i \in J_b : \theta_i^{\text{ch}} = 0/1\}$ we can write the factors

$$\left| \langle x_{J_b}^A, \theta_{J_b}^A | x_{J_b}, \theta_{J_b}^B \rangle \right|^2 = \prod_{i \in J_b^0} \underbrace{|\langle x_i^A, \theta_i^A | x_i, \theta_i^B \rangle|^2}_{\delta(x_i^A, x_i)} \prod_{i \in J_b^1} \underbrace{|\langle x_i^A, \theta_i^A | x_i, \theta_i^B \rangle|^2}_{\left| \frac{1}{\sqrt{2}} \right|^2}, \quad (\text{B109})$$

substituting in Eq. (B108) we get

$$\begin{aligned} \rho_{X_{J_b}^A B_3}^{\text{mix}}(x_{J_{b,d}}, \vec{\tau}, J_0, J_1) &= \left(\frac{1}{2}\right)^{w_H(\theta_{J_b}^{\text{ch}})} \sum_{\substack{x_{J_b}^A \\ x_{J_b}^1}} |x_{J_b}^A\rangle\langle x_{J_b}^A|_{X_{J_b}^1} \\ &\otimes \sum_{\substack{x_{J_b} \in \\ B_b(x_{J_{b,d}})}} \left| \lambda^{x_{J_b}, \vec{\tau}, J_0, J_1} \right|^2 |x_{J_b}^0\rangle\langle x_{J_b}^0|_{X_{J_b}^0} |\phi^{x_{J_b}, \vec{\tau}, J_0, J_1}\rangle\langle \phi^{x_{J_b}, \vec{\tau}, J_0, J_1}|_{B_3}, \end{aligned} \quad (\text{B110})$$

which is a product state between the systems $X_{J_b}^A$ and $X_{J_b}^0 B_3$. From Lemma A3 (1) and (2) we know that

$$\begin{aligned} H_{\min}^\varepsilon(X_{J_b}^A | B_3)_{\rho^{\text{mix}}(x_{J_{b,d}}, \vec{\tau}, J_0, J_1)} &\geq H_{\min}^0(X_{J_b}^A | B_3)_{\rho^{\text{mix}}(x_{J_{b,d}}, \vec{\tau}, J_0, J_1)} \\ &\geq H_{\min}^0(X_{J_b}^A | B_3)_{\rho^{\text{mix}}(x_{J_{b,d}}, \vec{\tau}, J_0, J_1)} + H_{\min}^0(X_{J_b}^1)_{\rho^{\text{mix}}(x_{J_{b,d}}, \vec{\tau}, J_0, J_1)} \\ &\geq H_{\min}^0(X_{J_b}^1)_{\rho^{\text{mix}}(x_{J_{b,d}}, \vec{\tau}, J_0, J_1)} \\ &= -\log\left(\left(\frac{1}{2}\right)^{w_H(\theta_{J_b}^{\text{ch}})}\right) = w_H(\theta_{J_b}^{\text{ch}}). \end{aligned} \quad (\text{B111})$$

Application of Lemma B3 together with Eqs. (B111) and (B106) leads to

$$\begin{aligned} H_{\min}^\varepsilon(X_{J_b}^A | B_3)_{\tilde{\rho}(x_{J_{b,d}}, \vec{\tau}, J_0, J_1)} &\geq H_{\min}^\varepsilon(X_{J_b}^A | B_3)_{\rho^{\text{mix}}(x_{J_{b,d}}, \vec{\tau}, J_0, J_1)} - \log(|B_b(x_{J_{b,d}})|) \\ &\geq w_H(\theta_{J_b}^{\text{ch}}) - h\left(\frac{p_{\max} + \delta_1}{\frac{1}{2} - \delta_2}\right) N_{\text{raw}}. \end{aligned} \quad (\text{B112})$$

Note that the above expression depends only on the number of nonmatching bases θ^{ch} associated to the indices in J_b and the parameters of the protocol, which in turn makes the infimum in Eq. (B102) straightforward to compute. We can now add the respective conditional min-entropies for $X_{J_0}^A$ and $X_{J_1}^A$, which results in

$$\begin{aligned} H_{\min}^\varepsilon(X_{J_0}^A | X_{J_1}^A B_3)_{\tilde{\rho}(\vec{\tau}, J_0, J_1)} + H_{\min}^\varepsilon(X_{J_1}^A | X_{J_0}^A B_3)_{\tilde{\rho}(\vec{\tau}, J_0, J_1)} &\geq w_H(\theta_{J_0}^{\text{ch}}) + w_H(\theta_{J_1}^{\text{ch}}) - 2h\left(\frac{p_{\max} + \delta_1}{\frac{1}{2} - \delta_2}\right) N_{\text{raw}} \\ &\geq N_{\text{raw}} - 2\delta_2(1 - \alpha)N_0 - 2h\left(\frac{p_{\max} + \delta_1}{\frac{1}{2} - \delta_2}\right) N_{\text{raw}} \\ &\geq 2N_{\text{raw}} \left(\frac{1}{2} - \frac{2\delta_2}{1 - 2\delta_2} - h\left(\frac{p_{\max} + \delta_1}{\frac{1}{2} - \delta_2}\right) \right). \end{aligned} \quad (\text{B113})$$

The result follows by recalling, from Eqs. (B82) and (B95), that the real state at this point in the protocol has distance from $\tilde{\rho}^{(5)}$ bounded by

$$\varepsilon = \sqrt{2} \left(e^{-\frac{1}{2}\alpha(1-\alpha)^2 N_0 \delta_1^2} + e^{-\frac{1}{2}\left(\frac{1}{2}-\delta_2\right)\alpha N_0 \delta_1^2} \right)^{\frac{1}{2}} + e^{-D_{\text{KL}}(\frac{1}{2}-\delta_2|\frac{1}{2})(1-\alpha)N_0} + \varepsilon_{\text{bind}}(k). \quad (\text{B114})$$

■

APPENDIX C: UC SECURITY IN THE RANDOM ORACLE MODEL

Following the discussion made in Sec. IV D, we prove the composability of a specific family of weakly interactive commitment schemes in the *classical access random oracle model*, which we will refer to as ROM from here onwards. These commitments, originally proposed by Lorünser *et al.* [67], build upon the original Naor bit commitment [35] and efficiently generalize it for arbitrary k -bit string commitments without the need of error-correcting codes. A description of the commitment protocol is shown in Fig. 7, whose correctness, binding, and hiding properties, have been proven in Ref. [67]. Instead, we will thus limit ourselves to prove that the LRV commitment protocol UC-emulates the commitment functionality $\mathcal{F}_{\text{COM}}$ (defined in Fig. 8) when the hash function is modeled as an oracle $\mathcal{F}_{\text{RO}}$ which computes a random function.

Let Π_A and Π_B represent the programs for the verifier and prover, respectively, as shown in Fig. 7. Note that, for simplicity, the external inputs that trigger the start and end of the commit and reveal phases have been omitted from Fig. 7; without loss of generality, we can consider them to take the form of the respective inputs and outputs as shown in the $\mathcal{F}_{\text{COM}}$ functionality. More specifically, the commit phase starts when Π_B receives the input $(\text{commit}, \text{sid}, \mathbf{b})$ and ends when Π_A outputs $(\text{committed}, \text{sid})$, etc. We proceed now to separate the security in two cases, in which the adversary controls Alice or Bob, respectively, as shown in Fig. 9. In order to prove security we must show that for any efficient (i.e., polynomial-time) adversary Adv

with classical access to the oracle there exists a respective simulator $\mathcal{S}$ such that for any environment, which is able to send and receive inputs and outputs through the loose wires in the right and left of the diagrams, the *real-world* and *ideal-world* scenarios are indistinguishable. Denote by H the function that the random oracle computes.

a. Dishonest Bob

We construct the simulator in terms of the following subprograms:

- (1) $\mathcal{F}_{\text{RO}}^*$: The same as $\mathcal{F}_{\text{RO}}$, except that it saves a list L of all the queries that have been made to the internal memory of $\mathcal{S}$.
- (2) Π_A^* : The same as Π_A , except that after receiving c from Adv it runs through the current list L of queries. When it finds an $\mathbf{x}' \in L$ and $\mathbf{b} \in \{0, 1\}^n$ such that

$$\mathbf{c} = G(\mathbf{x}') \oplus \sum_{i=1}^n b_i \cdot \mathbf{r}_i, \quad (\text{C1})$$

it sends $(\text{commit}, \text{sid}, \mathbf{b})$ to $\mathcal{F}_{\text{COM}}$. If no pair $(\mathbf{b}, \mathbf{x}')$ is found, it samples uniformly a value $\mathbf{b}$ and sends $(\text{commit}, \text{sid}, \mathbf{b})$ to $\mathcal{F}_{\text{COM}}$. In the reveal phase, if the check is passed, it sends $(\text{open}, \text{sid})$ to $\mathcal{F}_{\text{COM}}$.

Because of the binding property of the commitment protocol, the simulator may find at most one pair $(\mathbf{b}, \mathbf{x}')$ satisfying Eq. (C1) when looking through the list, except

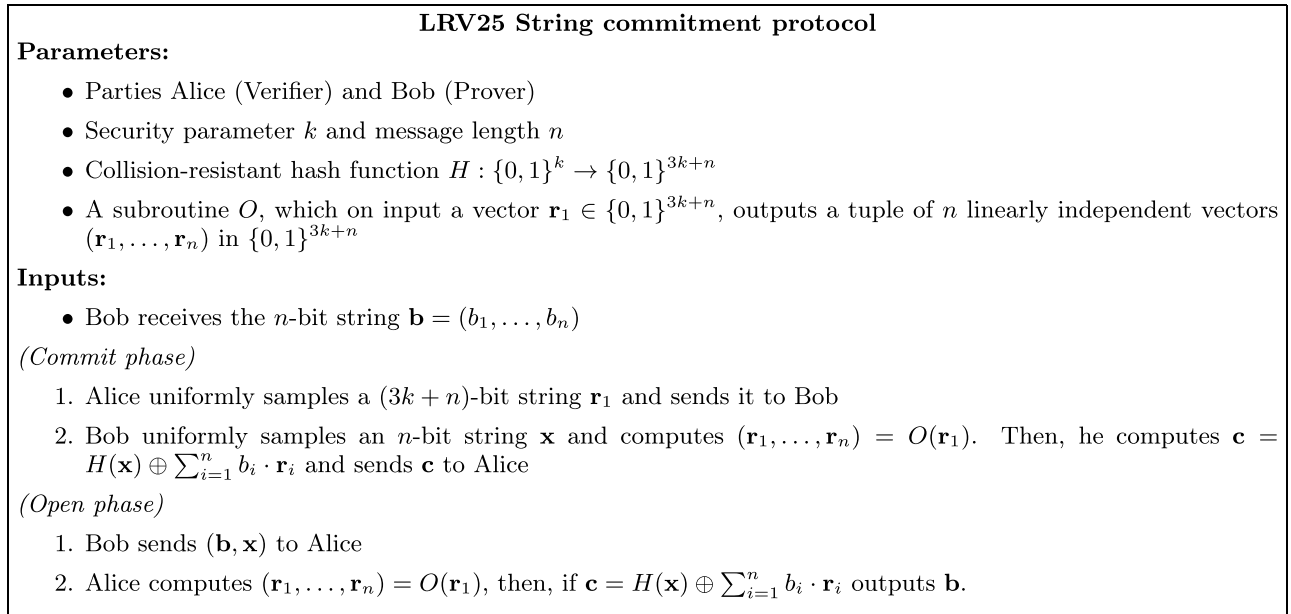


FIG. 7. Weakly interactive string commitment scheme based on hash functions.

Functionality $\mathcal{F}_{\text{COM}}$	
Parameters:	
• Parties Alice (Verifier) and Bob (Prover)	
• Message length n	
1. Upon receiving an input $(\text{commit}, \text{sid}, \mathbf{b})$ from Bob, if no value has previously been committed, output the message $(\text{committed}, \text{sid})$ to Alice	
2. Upon receiving the input $(\text{open}, \text{sid}, \mathbf{b})$ from Bob, if a value $\mathbf{b}$ has previously been committed, output the message $(\text{open}, \text{sid}, \mathbf{b})$ to Alice	

FIG. 8. Commitment ideal functionality.

with negligible probability (this is because the probability of there existing more than one valid openings for a given value of $\mathbf{c}$ is negligible). This allows $\mathcal{S}$ to correctly extract the committed value from $\mathbf{c}$ and commit it to $\mathcal{F}_{\text{COM}}$. Note that in the case no valid opening is found from L , the simulator commits a random value to $\mathcal{F}_{\text{COM}}$. If the adversary is able to provide a valid opening pair $(\mathbf{b}, \mathbf{x})$ in the Reveal phase the two scenarios could be distinguished. However, from the preimage resistance of random oracles, an efficient adversary cannot find a valid opening from a value of $\mathbf{c}$ without having obtained it by querying the oracle, meaning that regardless of $\mathcal{S}$ committing a random value to $\mathcal{F}_{\text{COM}}$, the probability of it being opened is negligible.

b. Dishonest Alice

Similarly, we construct the simulator in terms of the following subprograms:

- (1) $\mathcal{F}_{\text{RO}}^*$: The same as for the dishonest Bob case, except it may be reprogrammed on individual query-output pairs.
- (2) Π_B^* : The same as Π_B , except upon receiving an input of the form $(\text{committed}, \text{sid})$ from $\mathcal{F}_{\text{COM}}$, it samples uniformly the value $\mathbf{c}'$ and sends it to Adv . In the Reveal phase, upon receiving $(\text{open}, \text{sid}, \mathbf{b})$ from $\mathcal{F}_{\text{COM}}$, samples a random $\mathbf{x}'$ not in L , sets $\mathcal{F}_{\text{RO}}^*$

so that

$$G(\mathbf{x}') = \mathbf{c}' \oplus \sum_{i=1}^n b_i \cdot \mathbf{r}_i, \quad (\text{C2})$$

and sends $(\mathbf{b}, \mathbf{x}')$ to Adv .

From the hiding property of the commitment protocol, the value $\mathbf{c}$ received by Alice during the commit phase does not give a significant advantage to an efficient adversary in finding the committed value $\mathbf{b}$ as compared to a random string. Because of this, an efficient adversary cannot distinguish if the randomly sampled $\mathbf{c}'$ corresponds to any possible committed value, except with negligible probability. During the reveal phase, the reprogramming of the oracle according to Eq. (C2) guarantees that have $\mathbf{c}'$ will be consistent with the committed values from $\mathcal{F}_{\text{COM}}$. The only difference between the real and ideal scenarios is the change in the behavior of the oracle. Because the value $\mathbf{c}'$ was sampled uniformly, the associated outcome $G(\mathbf{x}')$ as defined by Eq. (C2) is also uniformly distributed and independent on the rest of the values $G(\mathbf{x} \neq \mathbf{x}')$, resulting in both scenarios being consistent with the oracle computing a random function, and therefore indistinguishable from each other.

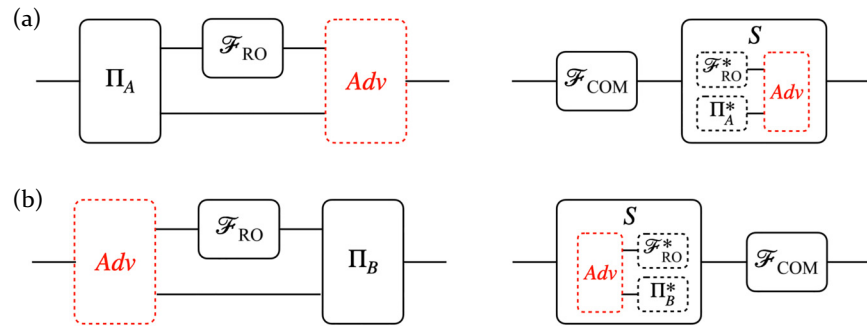


FIG. 9. Box diagrams for the execution of the protocol for (a) dishonest Bob and (b) dishonest Alice. The left sides represent the *real world* protocol interacting with an adversary Adv while the right sides represent the *ideal world* functionality interacting with the respective simulator S .

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