

Realizing a Continuous Set of Two-Qubit Gates Parameterized by an Idle Time

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Continuous gate sets are a key ingredient for near-term quantum algorithms. Here, we demonstrate a hardware-efficient, continuous set of controlled arbitrary-phase (CZ_θ) gates acting on flux-tunable transmon qubits. This implementation is robust to control pulse distortions on time scales longer than the duration of the gate, making it suitable for deep quantum circuits. Our calibration procedure makes it possible to parameterize the continuous gate set with a single control parameter, the idle time between the two rectangular halves of the net-zero control pulse. For calibration and characterization, we develop a leakage measurement based on coherent amplification, and a new cycle design for cross-entropy benchmarking. We demonstrate gate errors of 0.7% and leakage of 4×10^{-4} across the entire gate set. This native gate set has the potential to reduce the depth and improve the performance of near-term quantum algorithms compared to decompositions into CZ_π gates and single-qubit gates. Moreover, we expect the calibration and benchmarking methods to find further possible applications.

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I. INTRODUCTION

Variational quantum algorithms on superconducting qubits are under investigation for applications as diverse as quantum chemistry [1,2], simulations of quantum many-body systems [3,4], quantum phase transitions [5], metrology [6], machine learning [7,8], solving optimization problems [9,10], and foundational aspects of quantum information theory [11,12]. They typically include continuously parameterized unitaries [13–15], which may lead to prohibitive circuit depths if compiled into hardware operations from a minimal universal gate set [16]. Hardware-efficient implementations of such parameterized unitaries, in the form of continuous gate sets, allow one to reduce the depth of near-term, noisy quantum circuits and improve their performance [17–20].

Continuous gate sets have been demonstrated for SWAP-like gates [21,22], XY gates [23], three-qubit unitaries

[24,25], as well as for controlled arbitrary-phase (CZ_θ) gates, which apply a conditional phase θ to the $|11\rangle$ state [3,17,22,26]. While high CZ_θ gate fidelities have been demonstrated with tunable qubits in combination with tunable couplers [22], the latter come at the cost of additional wiring and on-chip elements for the flux control of the couplers as well as calibration overhead. This increased complexity is avoided in architectures with fixed coupling between flux-tunable transmon qubits. Controlled-phase gates [27] or controlled arbitrary-phase gates [3,17] can then be implemented by using flux pulses to realize a direct resonant exchange between the $|11\rangle$ state and the non-computational $|20\rangle$ state. However, these implementations are susceptible to control pulse distortions on time scales longer than the gate duration, affecting subsequent gates in long gate sequences, an effect termed “nonatomicity” [28] or “bleeding” [22].

To provide robustness against these distortions, controlled-phase gates were implemented based on net-zero waveforms with vanishing zero-frequency component [28,29]. In this scheme, the flux pulses are split into two halves with opposite polarities and an idle time between them. During the idle time, the population temporarily brought to the $|20\rangle$ state acquires a phase, which directly contributes to the final conditional phase θ . However, while the possibility of implementing a continuous gate set based on this phase contribution was mentioned in Ref. [29], the calibration procedures in Refs. [28,29] focus

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on a fixed conditional phase $\theta = \pi$ and do not suffice to calibrate a whole continuous gate set.

Here, we extend the net-zero CZ_π gate from Ref. [29] to a continuous set of CZ_θ gates. We introduce a novel calibration procedure, which leads to a simple linear dependence of the conditional phase θ on a single control parameter, the idle time t_{idle} . This is in contrast to previous implementations of CZ_θ gate sets, where tuning through all phases θ required adjusting multiple interdependent parameters [3, 17, 22]. We fine-tune the $|11\rangle \leftrightarrow |20\rangle$ resonance beyond the reach of the existing method from Ref. [29], a requirement to attain low leakage to the $|20\rangle$ state across the whole continuous gate set.

Using cross-entropy benchmarking (XEB) [30–32], we demonstrate that the gate performance is uniform over the whole range of phases θ . We implement the continuous gate set between qubits that are far detuned by over 2 GHz when the gate is off, suppressing residual ZZ interactions to below 5 kHz despite a coupling strength of 11.38(1) MHz when the gate is on. This on/off ratio is more than an order of magnitude higher than reported in implementations of controlled-phase gates with a lower idle detuning between statically coupled qubits [17, 29]. We realize the resonant interaction between far-detuned qubits by applying pulses to both qubits simultaneously as in Refs. [5, 33]. Adjusting the amplitudes of these pulses allows us to freely choose the interaction frequency, for example to both avoid interactions with two-level defect modes [33] and frequency collisions with neighboring qubits [5].

Two methods developed in this work are widely applicable beyond the context of calibrating and characterizing continuous gate sets. The first method is a leakage measurement based on the concept of coherent population amplification [34–36]. This concept was previously applied to gate errors of single-qubit [37–39] and two-qubit gates [40], and leakage of single-qubit gates [41]. Here, we demonstrate a coherent leakage amplification for two-qubit gates, and we derive an analysis procedure that considers that the relevant noncomputational state is used as an auxiliary state during the gate. Our analysis method enables us to obtain a quantitative estimate of the leakage per gate. This method can be beneficial for other applications requiring the calibration of low-leakage two-qubit gates, for example quantum error correction [42–48]. The second method we develop is an XEB cycle design that, as opposed to existing protocols [30, 32, 37, 49, 50], allows us to benchmark individual weakly entangling gates.

Below, we introduce the concept of the continuous set of controlled arbitrary-phase gates. We then describe methods to fine-tune the resonance condition necessary to reach low leakage uniformly for all phases. Finally, we present the calibration procedure and benchmarking results for our implementation.

II. PARAMETERIZATION OF THE CONTINUOUS GATE SET

We work with two neighboring qubits in a six-transmon quantum device; see Appendix A for a summary of the system parameters. We write Q_{high} to refer to the high-frequency qubit, idling at 6.3 GHz in this particular experiment, and Q_{low} to refer to the low-frequency qubit, idling at 4.1 GHz. Both qubits idle at first-order flux-insensitive points, which is a requirement for benefiting from the robustness to control pulse distortions provided by net-zero pulses. In cases where this choice of idle point is not feasible, for example due to the presence of two-level defect modes [31], the gate scheme with two resonant interactions and an idle time between them could still be implemented using control waveforms that do not consist of net-zero pulses.

The two qubits can be tuned in frequency with individual flux lines, realized as coplanar waveguides shorted near the superconducting quantum interference device (SQUID) of each qubit. Gaussian-filtered, piecewise constant net-zero flux pulses generated by an arbitrary waveform generator [Fig. 1(a)] translate, via the flux-frequency characteristic of the transmons [Fig. 1(b)], into a time dependence of all the energy levels [Fig. 1(d)]. We describe the measurement setup in Appendix A, and details of the flux waveform generation in Appendix B. To perform a two-qubit gate, we bring the $|11\rangle$ state into resonance with the $|20\rangle$ state [Fig. 1(d)], where the first number in the state label refers to the energy level of Q_{high} and the second one corresponds to Q_{low} . The time dependence of the detuning $\Delta(t) = \omega_{20} - \omega_{11}$ [Fig. 1(c)] shows two plateaus at the minimal detuning $\Delta_{\text{min}} \approx 0$, during which the resonant interaction is switched on, separated by a time interval at maximum detuning, where the qubits are back at their idle frequencies.

We express the Hamiltonian of the coupled system in a frame rotating at the qubit frequencies. This amounts to canceling the single-qubit dynamic phases accumulated by the frequency excursions shown in Fig. 1(d). When performing the experiment, this cancelation is implemented as a redefinition of the qubit frames; see Appendix C. In the two-dimensional subspace spanned by $|11\rangle$ and $|20\rangle$, the Hamiltonian reduces to

$$H(t) = \begin{pmatrix} 0 & g_2/2 \\ g_2/2 & \Delta(t) \end{pmatrix}, \quad (1)$$

where g_2 is the coupling rate in the two-excitation subspace, which is approximately independent of $\Delta(t)$, as shown in Appendix A. During each half of the resonant interaction [the two plateaus in Fig. 1(c)], the $|11\rangle$ population is diabatically swapped to the $|20\rangle$ state, and then swapped back to $|11\rangle$. During the idle time t_{idle} while $\Delta(t) \gg g_2$, the $|20\rangle$ state acquires a phase $\theta_{\text{idle}} = \int_t \Delta(t) dt$ [shaded area in Fig. 1(c)].

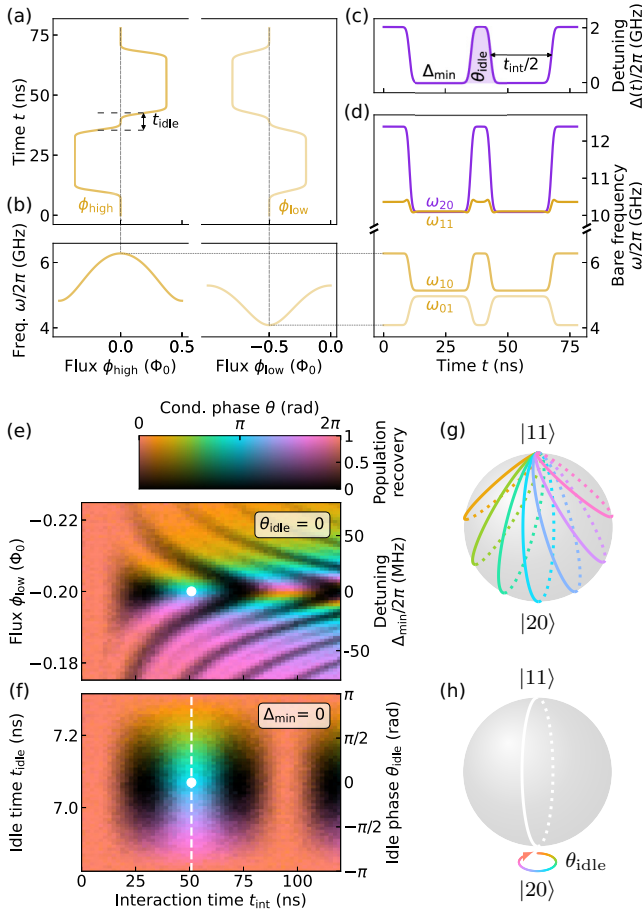


FIG. 1. Concept of the gate set. (a) Control pulses, as a function of time (vertical axis), in units of magnetic flux at the SQUID loop of each transmon. (b) Qubit transition frequencies ω_{10} and ω_{01} of the two transmons as a function of flux. (c),(d) Frequencies of the relevant two-transmon states (d) and detuning $\Delta(t) = \omega_{20} - \omega_{11}$ (c) over time due to the flux pulses in (a) and the conversion to frequencies in (b). The shaded area in (c) indicates the phase θ_{idle} acquired by the $|20\rangle$ state during the idle time t_{idle} . (e),(f) Measured population (brightness) and conditional phase (hue) of the $|11\rangle$ state at the end of the gate; see the measurement techniques in the main text. Shown are two cuts through the three-dimensional parameter space ($\Delta_{\text{min}}, \theta_{\text{idle}}, t_{\text{int}}$), at $\theta_{\text{idle}} = 0$ (e) and $\Delta_{\text{min}} = 0$ (f). The white point indicates a CZ_{π} gate and the dashed line indicates the continuous gate set. (g) Bloch sphere illustrating state trajectories for several Δ_{min} yielding $\theta \in \{\pi/4, \pi/2, \dots, 7\pi/4\}$, for $\theta_{\text{idle}} = 0$ as in (e) (idealized illustration assuming pulses with zero rise time). (h) Illustration of the implementation of the continuous gate set by means of a variable idle phase as in (f); see the main text.

To visualize the parameter space, we measure the population and phase of the $|11\rangle$ state after the two-qubit interaction as a function of the pulse parameters. Keeping the flux amplitude ϕ_{high} of Q_{high} fixed, we sweep the interaction time t_{int} , together with the flux amplitude ϕ_{low} , which sets the minimum detuning Δ_{min} [Fig. 1(e)], or with the idle time t_{idle} , which sets the idle phase θ_{idle} [Fig. 1(f)].

We measure the population recovery (shown as brightness), by applying the flux pulses to the qubits prepared in the $|11\rangle$ state and performing averaged three-level read-out [51]; see the detailed method in Appendix C. The brightest points in Fig. 1(f) correspond to quantum logic gates, that is, unitaries retaining the state within the qubit subspace. All darker areas correspond to unitaries with imperfect population recovery, that is, leakage to $|20\rangle$. We measure the conditional phase (shown as hue) with the Ramsey-type experiment described in Appendix C.

The conditional phase can be intuitively understood as a nonadiabatic generalization of the Berry phase (Aharonov-Anandan phase [52]), resulting from the geometrical trajectory on the $|11\rangle$ – $|20\rangle$ Bloch sphere. Geometrical phases have previously been explored to implement quantum gates in nuclear magnetic resonance qubits [53], spin qubits [54], trapped ions [55], and in superconducting qubits for single-qubit gates [56], controlled-phase gates in continuous-variable systems [57], and microwave-driven controlled-phase gates [58]. See Appendix D for a geometrical description of the conditional phase in our gate scheme.

If we fix the idle phase $\theta_{\text{idle}} = 0$, as in Fig. 1(e), we can define a continuous gate set along the curve of full population recovery passing through the CZ_{π} gate (white dot). In this case, the conditional phase θ corresponds to the solid angle enclosed by the trajectory on the Bloch sphere, as shown for a few choices of the detuning Δ_{min} in Fig. 1(g) and discussed in detail in Appendix D. However, since the interaction time t_{int} needs to be adjusted for any chosen detuning Δ_{min} such that the trajectory ends in the $|11\rangle$ state, a continuous gate set defined in this way is parameterized by two interdependent parameters that need to be adjusted simultaneously. If, instead of fixing θ_{idle} , we fix the detuning $\Delta_{\text{min}} = 0$ as in Fig. 1(f), the points with full population recovery form a pattern of straight lines, at either constant interaction time t_{int} (vertical lines) or constant idle phase θ_{idle} (horizontal lines). Along the lines $\theta_{\text{idle}} = \pm\pi$, the population recovery is maximized, but the conditional phase is zero, which can be understood as letting the two resonant interactions cancel each other; see Appendix D. By contrast, along the dashed white line of constant interaction time, any conditional phase can be achieved, making it possible to realize a continuous gate set by adjusting a single parameter.

As detailed in Appendix D, the conditional phase θ of this continuous gate set can be expressed as $\theta = \pi - \theta_{\text{idle}}$. The first term is a constant geometrical phase set by the trajectory on the Bloch sphere, which is independent of θ_{idle} , as visible in Fig. 1(h). The second term is a variable dynamic phase set by the integral of the detuning, $\theta_{\text{idle}} = \int_t \Delta(t) dt$. This integral depends linearly on the idle time t_{idle} , provided that t_{idle} is long compared to the rise time of the pulses such that $\Delta(t)$ forms a plateau [Fig. 1(c)]. This means that the conditional phase can be controlled linearly

with the idle time t_{idle} , while keeping all other parameters fixed. In practice, we vary t_{idle} with subpicosecond precision by adjusting the amplitudes at the smooth pulse edges (Appendix B).

III. ACCURATELY ESTABLISHING RESONANCE BETWEEN $|11\rangle$ AND $|20\rangle$

In order to implement the continuous gate set by means of the single parameter t_{idle} without adjusting any other parameters, we need a fixed choice of detuning Δ_{min} and interaction time t_{int} for which the population recovery is maximal, independently of t_{idle} . This independence relies on performing a complete population transfer to $|20\rangle$ [Fig. 1(h)], which requires setting the detuning to $\Delta_{\text{min}} = 0$ with high precision. Appendix E provides a quantitative simulation of the increase in leakage caused by a nonzero Δ_{min} if the interaction time t_{int} is kept fixed while sweeping the idle time t_{idle} .

To visualize the relevant part of the parameter space, we show the population recovery and the conditional phase for a fixed idle phase $\theta_{\text{idle}} = \pi$ in Fig. 2(a), which yields a qualitatively different pattern than in the case $\theta_{\text{idle}} = 0$ in Fig. 1(e). The central point indicated by the middle of the white arrow is the CZ_0 gate, which is an element of the continuous gate set. At this point, the population recovery (shown as brightness) is insensitive to the interaction time t_{int} , but depends quadratically on Δ_{min} ; see Appendix E. This is in contrast to the CZ_π gate at $\theta_{\text{idle}} = 0$ [white dot in Fig. 1(e)], where the population recovery depends quadratically on t_{int} , but is only a fourth-order function of Δ_{min} to leading order. Thus, whereas fixing $\theta_{\text{idle}} = 0$ is adequate to resolve variations in t_{int} , choosing the idle time such that $\theta_{\text{idle}} = \pi$ makes it possible to fine-tune Δ_{min} with a higher sensitivity than in the case $\theta_{\text{idle}} = 0$. By enabling us to establish the $|11\rangle$ – $|20\rangle$ resonance more accurately, a calibration at $\theta_{\text{idle}} = \pi$ therefore reduces the overall leakage across the continuous gate set.

Even with this higher sensitivity, resolving small variations in Δ_{min} is ultimately limited by the readout fidelity and the statistical errors due to finite sampling when measuring the population leaked to $|20\rangle$. To overcome these limitations, we coherently accumulate leakage by repeating the gate N times, before measuring the population $P_{Q_{\text{high}},2}$ of the second excited state of Q_{high} ; see the pulse sequence in Fig. 2(b). Between successive gates, the leaked population acquires a phase because it is detuned from $|11\rangle$ by $\Delta_{\text{max}}/2\pi \approx 2$ GHz. This results in an interference condition that is periodic in the gate separation t_{sep} with period $2\pi/\Delta_{\text{max}} \approx 0.5$ ns, as seen along the vertical direction in Fig. 2(c). In addition, we sweep the pulse amplitude ϕ_{low} (horizontal axis), which determines the detuning Δ_{min} during the interaction. For each value of ϕ_{low} , we then take the maximum of $P_{Q_{\text{high}},2}$ over t_{sep} ; see Fig. 2(d). This yields a

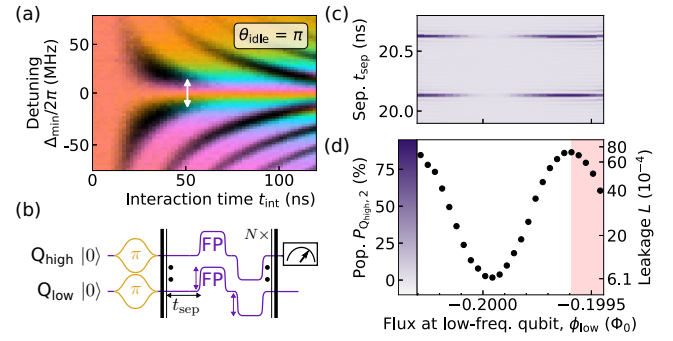


FIG. 2. Characterization of the resonance condition. (a) Population recovery and conditional phase (color scheme as in Fig. 1) as a function of the detuning Δ_{min} (controlled by the pulse amplitude ϕ_{low}) and interaction time t_{int} . For these measurements, θ_{idle} is set to π , resulting in maximum population sensitivity to Δ_{min} (illustrated by the white arrow). (b) Pulse sequence for coherent amplification of leakage, by repeating N times the flux pulses (FP) implementing the gate. Repetition is denoted by vertical bars with dots, and t_{sep} is the time between centers of the falling and rising edges of successive gates. (c) Second excited state population, $P_{Q_{\text{high}},2}$, of Q_{high} , color coded according to the color scale in panel (d), measured after $N = 16$ gates as a function of flux amplitude ϕ_{low} and gate separation t_{sep} [purple and black arrows in (b)]. A line cut for a fixed ϕ_{low} is included in Appendix F. (d) Maximum of $P_{Q_{\text{high}},2}$ (scale on the left vertical axis), taken over all gate separations t_{sep} , at each flux amplitude ϕ_{low} . The approximate conversion to the leakage L of a single gate, shown on the right axis, is given in Appendix F (valid outside the red shaded area).

high-contrast proxy for the leakage of one gate, which we use to calibrate the continuous gate set in Sec. IV below.

On the right vertical axis of Fig. 2(d), we estimate the leakage L per gate corresponding to the observed $|2\rangle$ -state population, using the model that we derive in Appendix F. This conversion is not valid in the red shaded area, where the leakage per gate becomes so high that the coherently accumulated leakage corresponds to a rotation by more than π on the $|11\rangle$ – $|20\rangle$ Bloch sphere, so that the final $|2\rangle$ -state population starts to decrease.

IV. GATE SET CALIBRATION AND BENCHMARKING

Based on the concepts introduced in the two previous sections, we calibrate the continuous set of CZ_θ gates, as summarized here and detailed in Appendix C. We perform measurements of the population recovery and conditional phase, as introduced in Sec. II. In these measurements, we successively sweep the pulse amplitude ϕ_{low} , the pulse length t_{int} , and the idle time t_{idle} , to obtain an approximate calibration of a CZ_θ gate with $\theta = 0$ (i.e., $\theta_{\text{idle}} = \pi$). For this choice of θ , the leakage is maximally sensitive to the detuning Δ_{min} ; see Sec. III. We fine-tune Δ_{min} by updating ϕ_{low} to the value that minimizes the amplified

leakage after $N = 16$ gates; see Fig. 2(d). Then, we modify the idle time t_{idle} to obtain $\theta = \pi$, so that the leakage becomes sensitive to the interaction time t_{int} . We calibrate t_{int} by measuring the leakage after a single gate. This yields a total gate duration of 78.0 ns including buffers; see Appendix C.

Having fixed ϕ_{low} and t_{int} , we measure the conditional phase θ for 12 uniformly spaced choices of the idle time t_{idle} ; see Fig. 3(a). We fit a linear model, which is shown as a black line in Fig. 3(a), and we find fit residuals below $\pm 1^\circ$, as shown in gray in Fig. 3(a). This confirms that we have calibrated a continuous gate set that behaves in close agreement with the linear model introduced in Sec. II. We discuss potential causes for the residuals in Appendix C. The slope of the fitted line is given by the maximal detuning during the idle time, $\Delta_{\text{max}}/2\pi \approx 2$ GHz. With this choice of Δ_{max} , the idle time t_{idle} only needs to vary continuously over a range of 0.5 ns to cover the whole range of conditional phases.

In Appendix C, we describe the calibration procedure of the continuous gate set in more detail, including calibrating a virtual Z rotation [59] to correct the rotating frame by compensating the dynamic phases acquired due to the frequency excursions of each qubit. This yields an effective evolution only in the $|11\rangle$ – $|20\rangle$ subspace; see Eq. (1). Importantly, changing the idle time t_{idle} does not affect the time integral of the frequency excursions [Fig. 1(c)], so this virtual Z rotation only needs to be calibrated once for the entire gate set.

We characterize the continuous gate set using XEB [30–32] with a modified cycle design, overcoming limitations of existing benchmarking methods [23,25,31,37,40]; see Appendix G for a summary of these methods. By comparing the measured and simulated output distributions of random quantum circuits, XEB estimates the error per quantum gate, based on the assumption that the distribution over circuits forms a 2-design, meaning that they generate states approximating aspects of the uniform distribution in the Hilbert space [30]. The original XEB implementation [30,31] fulfilled this condition with a random mixing unitary U_{mix} composed of single-qubit gates, followed by the entangling two-qubit gate under test. However, when followed by a weakly entangling CZ_θ gate with θ close to 0, a single-qubit mixing unitary does not fulfill the randomization assumption for a two-qubit Hilbert space. Therefore, to benchmark the whole continuous gate set, we append the gate under test to an entangling mixing unitary U_{mix} composed of a CZ_π gate and random single-qubit gates $\{X_{\pi/2}, Y_{\pi/2}, Z_{\pi/4}\}$, as shown in Fig. 3(b). In addition, we perform a reference measurement where the XEB cycle consists only of the mixing unitary. We then extract the error of the gate under test from the results of both measurements; see Appendix G. Note that U_{mix} fully explores the Hilbert space, ensuring that the reference measurement captures possibly correlated errors during the single-qubit gates.

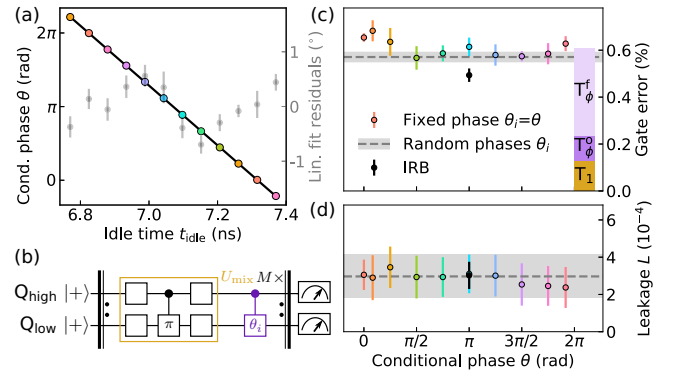


FIG. 3. Gate set calibration and benchmarking. (a) Conditional phase calibration as a function of t_{idle} (colored as in Fig. 1), measured using the method detailed in Appendix C, and linear fit (black line) with residuals (gray). (b) XEB quantum circuit, consisting of M successive cycles. Each cycle i comprises a CZ_{θ_i} gate with phase θ_i , and a mixing unitary composed of randomly chosen single-qubit gates (squares) and a CZ_π gate. (c) Gate error from XEB measurements for 1750 random circuits of up to $M = 256$ cycles, for a fixed phase $\theta_i = \theta$ across all cycles (colored as in Fig. 1), and for a random phase θ_i in each cycle (dashed gray line). The result of interleaved randomized benchmarking (IRB) is shown in black. The colored bars near the right axis of the panel indicate the decomposition of the error estimated from the master equation simulation in Appendix H: dephasing due to flux noise (T_ϕ^f), dephasing due to other sources (T_ϕ^o), and relaxation (T_1). (d) Leakage extracted from XEB and IRB measurements [legend as in (c)].

We extract gate errors by executing 1750 random circuits consisting of up to $M = 256$ cycles; see Fig. 3(b). By using a fixed conditional phase $\theta_i = \theta$ in all cycles i , we characterize individual CZ_θ gates for $\theta \in \{0, \pi/4, \dots, 7\pi/4\}$, as well as for weakly entangling gates with $\theta \in \{\pm\pi/12\}$; see the colored dots in Fig. 3(c). In addition, to estimate the average error over the whole continuous gate set, we perform XEB with a different random phase $\theta_i \in [0, 2\pi)$ in each cycle; see the dashed line in Fig. 3(c). Each individual gate yields a similar error, with a mean of 0.61%, in good agreement with the average measurement at 0.57(2)%, and a maximal deviation from the mean at 0.68(4)% for $\theta = \pi/12$. For the Clifford gate CZ_π , we also perform interleaved randomized benchmarking (IRB) [60,61]; see the black dot in Fig. 3(c). The gate error of 0.49(3)% reported by IRB is in reasonable agreement with the XEB results.

We estimate the purity of the elements of the continuous gate set using speckle purity benchmarking [31]. We obtain incoherent errors that are identical to the total gate errors within error bars, indicating that control errors do not contribute significantly to the total gate error. This is consistent with a master equation simulation presented in Appendix H, from which we estimate the contributions

of relaxation and dephasing to the total gate error; see Fig. 3(c).

The measured gate errors are similar to the infidelity of 0.7% reported for $\theta = \pi$ in the continuous set of CZ_θ gates in Ref. [3], but we note that this gate set was limited to the interval $\theta/\text{rad} \in [0.7, 4.0]$. Moreover, the gate errors reported above are lower than the median error of 2.7% reported for gates from the XY family in Ref. [23] and of 1.7% for a CZ_π gate within the CZ_θ gate set in Ref. [17]. At the expense of the additional overhead of an implementation with tunable couplers, a lower average infidelity of 0.2% was achieved for a continuous set of CZ_θ gates in Ref. [22]. For discrete CZ_π gates implemented with fixed coupling based on a similar gate concept as presented here, a similar infidelity as in our experiment was reported for the best gate in Ref. [33] (0.6%) and for the best gate in Ref. [29] (0.5%).

For a possible explanation of the variations of the gate error as a function of θ observed in Fig. 3(c), consider that the qubits typically cross two-level defects [62–64] during their frequency excursions. Interference effects resulting from these crossings [65] depend on the idle time t_{idle} and are not taken into account in the calibration.

Finally, we extract leakage from XEB [66,67] and IRB [67,68], as shown in Fig. 3(d). The observed values all lie between 2×10^{-4} and 4×10^{-4} , indicating uniformly low leakage over the entire gate set. This is slightly lower than the lowest leakage values reported for gates from the continuous gate sets presented in Refs. [3,17,22,26], which lie between 5×10^{-4} and 15×10^{-4} . A similar amount of leakage was also found for the discrete CZ gates implemented in Refs. [29,33].

V. CONCLUSION

By extending the net-zero CZ_π gate used in previous work [5,29,33], we have designed a continuous set of CZ_θ gates in a fixed-coupling architecture. We have shown how fine-tuning the $|11\rangle\text{--}|20\rangle$ resonance yields a parameter space with a simple structure, where the conditional phase is the sum of a constant geometrical phase and a controllable dynamic phase. As a result, the conditional phase has a linear dependence on a single control parameter, the idle time t_{idle} . In addition, the gate set achieves low leakage and high fidelity independently of t_{idle} . In contrast, other schemes such as those in Refs. [3,17,22] must keep track of several interrelated parameters to define a continuous gate set.

In order to calibrate the $|11\rangle\text{--}|20\rangle$ resonance with sufficient precision, we have introduced a coherent leakage amplification method, which allows quantifying leakage of two-qubit gates below the readout limit. We expect that this method can contribute to both the calibration and benchmarking procedures in broader contexts where low-leakage gates are a key requirement, including two-qubit gates for

quantum error correction [42–48]. We have also introduced a modified XEB cycle design, capable of benchmarking weakly entangling gates by interleaving them with an entangling mixing unitary.

We have implemented and characterized the continuous gate set in an experiment using two qubits with fixed coupling on a six-transmon quantum device, demonstrating state-of-the-art fidelities and leakage across the entire gate set. As the benchmarking results indicate that incoherent errors are dominant in our implementation, we expect that the gate errors can be further reduced in experiments with improved coherence times, e.g., in devices with lower flux noise.

To further improve the gate fidelity while retaining the simple linear dependence of the conditional phase on a single control parameter, future research could investigate the application of the gate scheme in an architecture that combines tunable qubits with tunable couplers [22,41]. Given the typically smaller idle detuning in such architectures, a larger range of idle times would be required, e.g., of the order of 5 ns in the case of a detuning of the order of 200 MHz [41], instead of the range of 0.5 ns reported in Fig. 3(a). The increased idle time could be compensated for by a shorter interaction time resulting from the larger coupling strength of the tunable coupler.

The continuous gate set can be employed, for example, in variational quantum algorithms for quantum machine learning [69–71]. In such applications, we expect that future experiments can demonstrate performance gains compared to implementations based on a single type of two-qubit gate (e.g., only the CZ_π gate); see the related study in Ref. [17].

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C.H. and C.S. conceived the experiments, C.S. and K.H. performed the measurements, and C.S. analyzed the data. C.S., K.H., and A.R. performed numerical simulations, and

C.S., S.L., and N.L. contributed to the measurement software. A.R. and C.S. designed the device, and D.C.Z. and A.F. fabricated it. C.H. and A.W. supervised the work. C.S. and C.H. wrote the manuscript with input from all coauthors.

DATA AVAILABILITY

The data that support the findings of this article are openly available [72].

APPENDIX A: EXPERIMENTAL SETUP AND QUANTUM DEVICE

We performed the experiment on a pair of transmon qubits, on a superconducting quantum device hosting two chains of three qubits; see the micrograph in Fig. 4. We have fabricated the device according to the methods specified in Ref. [64] and summarized below. We have used a high-resistivity intrinsic silicon substrate, with a niobium film as the base metallization layer. The circuit elements are defined using photolithography and reactive-ion etching. The different regions of the ground plane are electrically connected using aluminum/titanium/aluminum trilayer air-bridges. We have deposited the aluminum/aluminum-oxide/aluminum Josephson junctions using double-angle shadow evaporation, with features patterned by electron-beam lithography. We have exposed and deposited aluminum bandages [73] to ensure contact between the leads of the Josephson junctions and the base metallization. The device is mounted in a magnetically shielded sample holder, and operated below 20 mK in a dilution refrigerator; see the setup schematic in Fig. 5.

The two qubits used in this work are read out via dedicated readout resonators and Purcell filters, coupled to a common feedline with frequency multiplexing [74]. The readout signal is amplified by a traveling-wave parametric amplifier (TWPA) [75], followed by a cryogenic high-electron mobility transistor (HEMT) amplifier, and two low-noise amplifiers at room temperature. We initialize the qubits in their ground states using a method based on real-time feedback, by repeatedly performing three-level readout followed by single-qubit drive pulses ($|2\rangle \leftrightarrow |1\rangle$ and $|1\rangle \leftrightarrow |0\rangle$) conditioned on the readout results [76,77].

We implement single-qubit gates by applying microwave pulses via dedicated drive lines, which are capacitively coupled to the qubits. The pulses are generated by an arbitrary waveform generator and up-converted by in-phase and quadrature (IQ) mixers. Single-qubit X and Y gates are realized as derivative removal by adiabatic gates (DRAG) [38,78]. Single-qubit Z gates are implemented as so-called virtual gates by adjusting the phase of subsequent X and Y pulses [59].

The transition frequency of each qubit is controlled by the current flowing in an inductively coupled flux line,

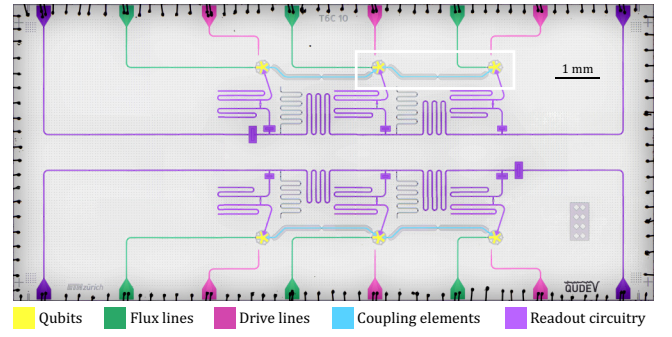


FIG. 4. False-color micrograph of the device used in a preliminary version of this experiment; the device used in the final experiment is nominally identical. Each transmon qubit consists of a capacitive island (yellow) connected to the ground plane via a SQUID, and capacitively coupled to a drive line (pink) and the readout circuitry (purple). Fast flux control is provided by individual flux lines (green), which are inductively coupled to the SQUID of each qubit. The capacitive qubit-qubit coupling is implemented with fixed-frequency coupling resonators (light blue). The measurements have been performed on the pair of qubits indicated by a white frame.

which sets the magnetic flux threading the SQUID loop of the qubit. The idling transition frequency is set by a constant current from an isolated voltage source. The frequency excursions implementing the two-qubit gates are realized by fast flux pulses, generated by an arbitrary waveform generator. The pulses are combined with the bias current via a custom-made bias tee. See Appendix B for more details on flux pulse control. We fit a coupled transmon-resonator model to the transition frequency of each qubit as a function of the flux threading its SQUID loop. These models are represented in Fig. 1(b), and used in the experiments to determine the conversion between flux pulse amplitudes and qubit frequencies.

The coupling between neighboring qubits is mediated by a fixed-frequency resonator (light blue in Fig. 4). Both qubits ($Q_{\text{high}}, Q_{\text{low}}$) are capacitively coupled to the resonator, with coupling rates g_{high} and g_{low} , respectively. The resonator is designed to have a resonance frequency of 24 GHz, such that the frequency differences Δ_{high} and Δ_{low} between the resonator and the transmons are in the dispersive regime, $\Delta \gg g$. In this regime, and with the resonator initially in the vacuum state, the effect of the resonator reduces to an effective capacitive coupling [79]

$$g_1 = 2J_1 = g_{\text{high}}g_{\text{low}} \left(\frac{1}{\Delta_{\text{high}}} + \frac{1}{\Delta_{\text{low}}} \right), \quad (\text{A1})$$

where g_1 is the frequency splitting of the coupling Hamiltonian in the one-excitation subspace, and J_1 the corresponding matrix element. The coupling rate between $|20\rangle$ and $|11\rangle$ (two-excitation subspace) is given by $g_2 = 2J_2 =$

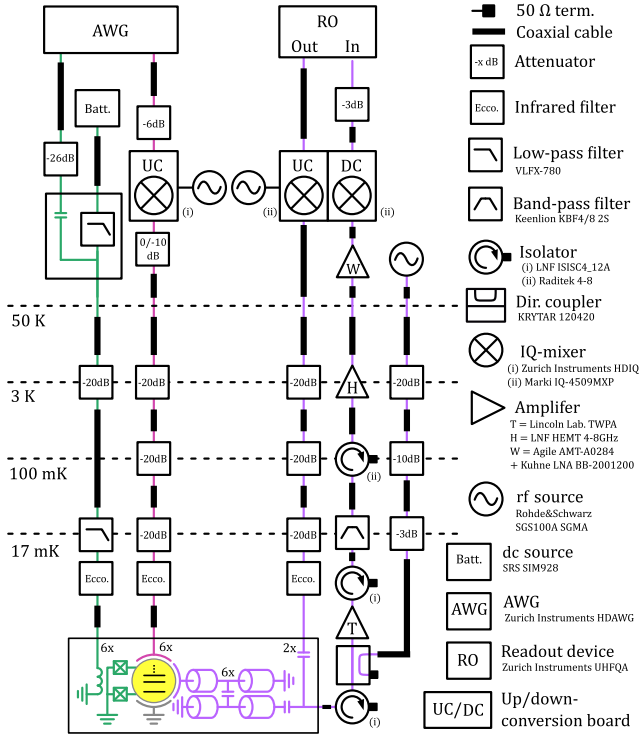


FIG. 5. Simplified schematic of the experimental setup, including the quantum device. Coupling elements between transmons are not represented. See the text for details.

$\sqrt{2}g_1$. This coupling g_2 is the dominating relevant term in the Hamiltonian only when $|20\rangle$ and $|11\rangle$ have similar energies, within a range of approximately g_2 or less. In this small range of qubit frequencies, the detunings Δ_{high} and Δ_{low} are close to constant, and g_2 can therefore be considered constant in the regime relevant for a two-qubit gate.

Table I shows a summary of the properties of the quantum device. The qubit-qubit coupling is specified at the interaction frequency of the gate, and extracted from a fit to the exchange pattern in Fig. 9 below; see Appendix C for a description of the measurement. The residual couplings Δ_{ZZ} are extracted from Ramsey experiments with the other qubit either prepared in the ground or excited state. The error bars are standard error estimates calculated from the variance of the fit residuals. For the parameter σ describing the effective Gaussian filtering of flux control waveforms, see Appendix B.

APPENDIX B: FLUX WAVEFORM GENERATION

To implement two-qubit gates, we apply flux pulses via inductively coupled flux lines, as described in Appendix A. Here, we summarize the methods that we apply to characterize and compensate pulse distortions, we comment on effects due to nonzero rise times of the flux pulses, and we discuss the time resolution for adjusting the idle time θ_{idle} .

TABLE I. Qubit parameters and properties. See the text for details.

Name	Q_{high}	Q_{low}
Qubit idle frequency $\omega/2\pi$ (GHz)	6.278	4.083
Qubit anharmonicity $\alpha/2\pi$ (GHz)	-0.167	-0.191
Readout frequency $\omega_{\text{RO}}/2\pi$ (GHz)	7.423	7.215
Three-state readout error ϵ_{RO} (%)	1.49	1.36
Residual population P_{res} (%)	0.02	0.43
Single-qubit RB error ϵ_{1Q} (%)	0.07	0.05
Lifetime T_1 (μs)	32.6(1)	88.0(4)
Lifetime of the $ 2\rangle$ state, $T_1^{(2)}$ (μs)	14.7(3)	30.3(7)
Ramsey decay time T_2^* (μs)	24(1)	47(1)
Echo decay time T_2^E (μs)	29(1)	50(1)
Echo time at the interaction frequency, $T_{2,\text{int}}^E$ (μs)	8.2(1)	8.5(1)
Qubit-qubit coupling $J_2/2\pi$ (MHz)	11.38(1)	
Residual coupling $\Delta_{ZZ}/2\pi$ (kHz)	1.3(7)	4.4(7)
Waveform smoothing σ (ns)	1.02	1.35

Because of filters in the flux control lines and unwanted effects such as parasitic capacitances, the pulses are distorted before reaching the SQUID loops of the qubits. We characterize long-time distortions above 60 ns with the spectroscopic methods described in Refs. [33,80] and short-time distortions with the cryoscope method from Ref. [81], as detailed in Ref. [82]. We then apply digital predistortion filters to compensate the distortions, such that the combination of digital preprocessing and physical flux line behaves in good approximation as a Gaussian filter. The resulting limited bandwidth makes the two-qubit gates adiabatic with respect to the transmon transition frequencies.

To characterize the width σ of this effective Gaussian filtering, we perform dynamic phase measurements, as described in Appendix C, while sweeping the duration and the amplitude of a rectangular flux pulse on a single qubit. To this dataset, we fit a numerical model that expresses the dynamic phase of the qubit as the time integral of the detuning from the idle frequency during the Gaussian-filtered flux pulse. We consider the parameter σ of the Gaussian filter as the fit parameter, yielding the values shown in Table I. The simulations rely on the flux-frequency characteristic of the transmons obtained in Appendix A and shown in Fig. 1(b).

To study the effect of Gaussian filtering on the population recovery after a two-qubit gate, we show simulation results in Fig. 6. We integrate the Schrödinger equation including all energy levels of the two-transmon system up to two excitations, with a time-dependent Hamiltonian defined by the Gaussian-filtered pulse shapes, and assuming a fixed qubit-qubit coupling (as defined in Appendix A). These simulations do not take into account residual crosstalk between the flux lines, which can influence the exact flux pulse amplitude required to reach the

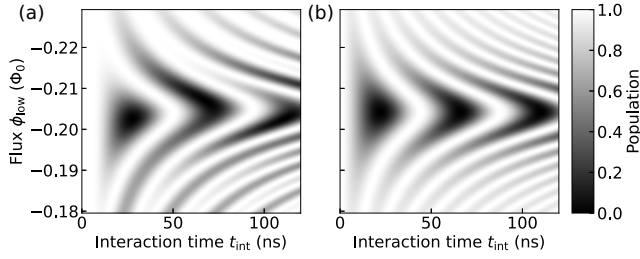


FIG. 6. Simulated population recovery after one gate for (a) Gaussian-filtered net-zero waveforms (see the experimentally measured values of σ in Table I) and (b) unfiltered waveforms ($\sigma = 0$ ns).

resonance condition in the experiment [see the difference between the vertical axis of Fig. 6(a) and the left vertical axis of Fig. 1(e)]. We observe that Gaussian-filtered net-zero waveforms yield an irregular spacing in the curves of population recovery in Fig. 6(a), which is in qualitative agreement with the experimental data in Fig. 1(e). This effect disappears when performing the simulation with piecewise-constant waveforms without applying the Gaussian filter; see Fig. 6(b).

Setting the conditional phase θ of a CZ_θ gate by varying the idle time t_{idle} in the middle of the waveforms, as described in Sec. II, requires one to adjust t_{idle} with a resolution below the sampling period $T_S = 0.42$ ns of the AWG. We first apply a Gaussian filter of width $\sigma = 0.5$ ns, then adjust the amplitudes of the samples at the pulse edges to account for appropriate time shifts as illustrated in Fig. 7, and then apply all other digital filtering. A similar concept was used in, e.g., Ref. [83]. To obtain an order-of-magnitude estimate of the time precision that can be reached, we approximate a rising edge as a line with constant slope within a rise time of $t_{\text{rise}} \approx 2\sigma = 1$ ns, where $\sigma \geq T_S$. With a maximum amplitude $A_{\text{max}} = 1$ V and an amplitude precision $\delta_A = 0.1$ mV (from the datasheet of the AWG), we expect a timing precision $\delta_t \approx t_{\text{rise}} \delta_A / A_{\text{max}} \approx 1 \text{ ns} \times 0.1 \text{ mV} / 1 \text{ V} = 0.1$ ps.

When calibrating the continuous gate set, we observe residuals below $\pm 1^\circ$ in the conditional phase [Fig. 3(a)], which correspond to gate errors below 5×10^{-5} , as discussed in Appendix H. Reaching deviations from the linear model of less than $\pm 1^\circ$ requires a timing resolution of at least $(1^\circ/360^\circ)/\Delta_{\text{max}} = 1.3$ ps for a maximal detuning of $\Delta_{\text{max}} = 2.0$ GHz. Since the precision δ_t estimated above is much finer than 1.3 ps, we conclude that gate errors from limited timing precision are significantly smaller than 5×10^{-5} and not limiting the overall gate fidelity.

APPENDIX C: CALIBRATION PROCEDURE

Implementing the continuous set of CZ_θ gates requires calibrating two basic operations: the half-waveforms

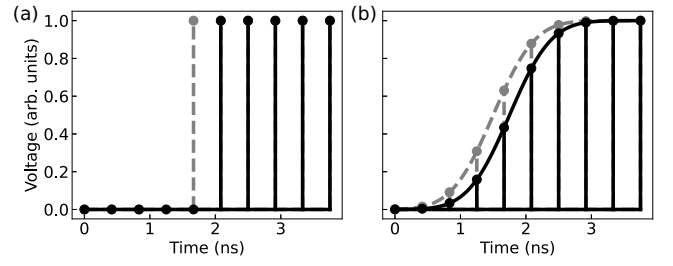


FIG. 7. Concept of shifting (solid black) a given waveform (dashed gray) by means of adjusting the amplitude of the points during the pulse edges. The waveform samples are shown on a $1/(2.4 \text{ GHz})$ grid, as produced by the AWG employed in the experiment. (a) Shifting a waveform with a sharp rising edge, by shifting all samples by a sampling period. (b) Shifting a Gaussian-filtered waveform ($\sigma = 0.5$ ns) by half the sampling period, by adjusting the amplitudes of the samples during the smooth rising edge.

(responsible for the population exchange), and the idle time between them (responsible for the idle phase).

We initialize all the two-qubit gate parameters by estimating them from the flux dependence of the transition frequencies obtained in Appendix A and shown in Fig. 1(b). Applying flux pulses to both qubits gives the degree of freedom of choosing the frequency at which the qubits interact during the gate [5]. When selecting this interaction frequency, we avoid collisions with two-level defect modes by monitoring the population loss of the excited state of each qubit during a flux pulse of variable amplitude [84,85]. We initialize the interaction time t_{int} based on the coupling strength from the device design. In all measurements, we add 10-ns buffers before and after the flux pulses, to prevent possible residual control pulse distortions influencing the next gate.

We start by calibrating a rectangular pulse for each qubit, to perform a full exchange to $|20\rangle$. We use the pulse sequence shown in Fig. 8(a) to measure the populations after this waveform (dark purple). When performing three-level readout, we extract the state populations from the measured signal quadratures by orthogonal projection on calibration points [51]. We measure the excited state populations $P_{Q_{\text{high}},1}$ and $P_{Q_{\text{high}},2}$ of Q_{high} as a proxy for the population of $|11\rangle$ and the leakage to $|20\rangle$. The resulting characteristic pattern of population exchange is shown in Fig. 9, and can be measured at low resolution at the beginning of the calibration to check the initial parameters. To obtain a full population exchange, we calibrate the duration and amplitude of the rectangular pulses by iterating one-dimensional sweeps (white arrows) and maximizing quadratic fits of the leakage.

We then construct a CZ_θ gate by concatenating two such rectangular pulses with opposite signs and otherwise identical pulse parameters [Fig. 1(a)]. We allow a variable idle time t_{idle} between the two rectangular pulses. At this

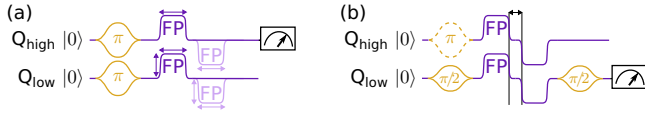


FIG. 8. Calibration measurements. (a) Pulse sequence to estimate the state populations, by reading out (black box) the high-frequency qubit after applying half-waveforms (dark purple) or full waveforms (shaded) of flux pulses (FP) to an initial $|11\rangle$ state prepared by π pulses (yellow). Horizontal arrows indicate a sweep of the interaction time and vertical arrows indicate a sweep of the flux at the low-frequency qubit. (b) Pulse sequence to estimate the conditional phase of $|11\rangle$, extracted from a Ramsey-type measurement on the low-frequency qubit, with the high-frequency qubit prepared in $|1\rangle$ (dashed π pulse) or $|0\rangle$, and sweeping the phase of the final $\pi/2$ pulse.

point, we can directly obtain an approximate calibration of the continuous gate set, by using the Ramsey-type pulse sequence shown in Fig. 8(b) to measure the conditional phase as a function of t_{idle} . The calibration presented up to this point routinely achieved gate errors below 1% on the considered pair of qubits. In order to reproducibly reach the low residuals shown in Fig. 3(a) as well as low leakage, we now introduce additional fine-tuning steps.

We fine-tune the parameters, Δ_{min} and t_{int} , around the CZ_0 and CZ_π gates, respectively, where leakage becomes a quadratic function of the considered parameter; see the Taylor expansions in Appendix E. We first set a value of t_{idle} that yields the conditional phase $\theta = 0$, and fine-tune Δ_{min} by choosing the flux pulse amplitude ϕ_{low} that minimizes the measured accumulated leakage after 16 gates [Fig. 2(d)]. We then set t_{idle} to realize a CZ_π gate, and fine-tune t_{int} by minimizing the measured leakage after one gate.

Having fine-tuned the pulse parameters ϕ_{low} and t_{int} , we characterize the conditional phase θ as a function of t_{idle} , as shown in Fig. 3(a). To implement any CZ_θ gate, we then pick t_{idle} from this dataset, either using a linear fit, or an interpolation for robustness to deviations from the linear model. These deviations are discussed below. In the datasets shown in this paper, we use a cubic spline interpolation. When applying gates in a quantum circuit, we keep their duration fixed by adjusting the buffers around the flux pulses to compensate for the variable idle time between the two half-waveforms. The total gate duration is fixed to $t_{\text{int}} + t_{\text{idle}} + 2t_{\text{buffer}} = 78.0$ ns, where $t_{\text{int}} = 50.9$ ns and $t_{\text{buffer}} \in [9.9 \text{ ns}, 10.2 \text{ ns}]$ for $t_{\text{idle}} \in [6.8 \text{ ns}, 7.3 \text{ ns}]$.

At the end of the calibration procedure, we characterize the single-qubit dynamic phases on $|10\rangle$ and $|01\rangle$, using a Ramsey-type measurement [27]. The dynamic phases depend on the shapes of the half-waveforms, but not on the variable idle time t_{idle} between them, so that a single calibration for the entire gate set is sufficient. We confirm this experimentally by performing the measurements as a function of t_{idle} shown in Fig. 10. The dynamic phases

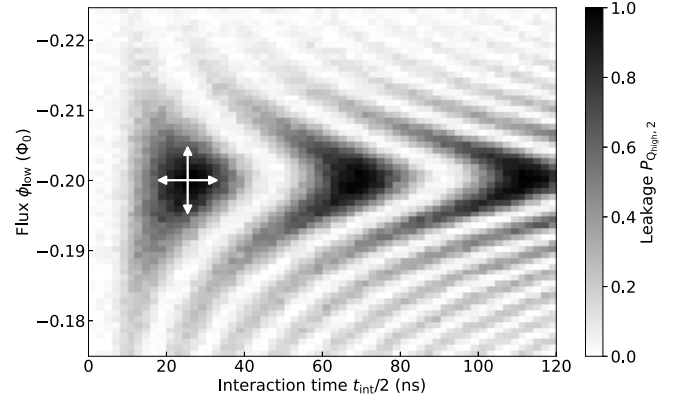


FIG. 9. Measured population recovery after a rectangular half-waveform, as a function of the interaction time t_{int} and the flux ϕ_{low} at the low-frequency qubit during the interaction. White arrows around the point of maximal $|20\rangle$ population illustrate the sweeps shown in Fig. 8(a).

are constant, with residual variations below $\pm 1^\circ$. This is a significant simplification compared to the continuous gate set presented in Ref. [17], where the dynamic phases strongly varied for different values of the conditional phase θ . We cancel the dynamic phases using so-called virtual Z gates [59], i.e., by a redefinition of the rotating frame. An alternative would be to perform the cancelation using additional low-amplitude flux pulses, as done in Ref. [29].

For a possible explanation of the residuals in the conditional phase, we discuss potential effects from residual distortions of the flux control waveforms. After calibrating predistortion filters as described in Appendix B, we expect residual distortions that lead to frequency errors of the order of 1 MHz [82]. These distortions are strongly suppressed during the idle time, where the qubit frequencies are first-order insensitive to flux variations. However, when sweeping the idle time t_{idle} , distortions happening at a time of approximately t_{idle} after the falling edge of the first half of the control waveform are swept through the rising edge of the second half of the waveform. The accumulated idle phase θ_{idle} is sensitive to the exact qubit frequencies during the rising edge. For example, a frequency error of 1 MHz compared to the intended waveform over a duration of 1 ns would correspond to an error of $2\pi \times 1 \text{ MHz} \times 1 \text{ ns} = 0.36^\circ$ in θ_{idle} . Residual flux distortions could thus explain fit residuals in the conditional phase of the order of magnitude observed in Fig. 3(a), where the residuals have a root mean square of 0.37° .

The dynamic phases can be influenced by flux pulse distortions in a similar way. We note that other physical mechanisms can contribute to the deviations of the conditional phase and dynamic phases from their respective models. For instance, interactions with two-level defect modes as well as off-resonant interactions between other transmon states are not included in the model, but can result in residual imperfections of the state populations and

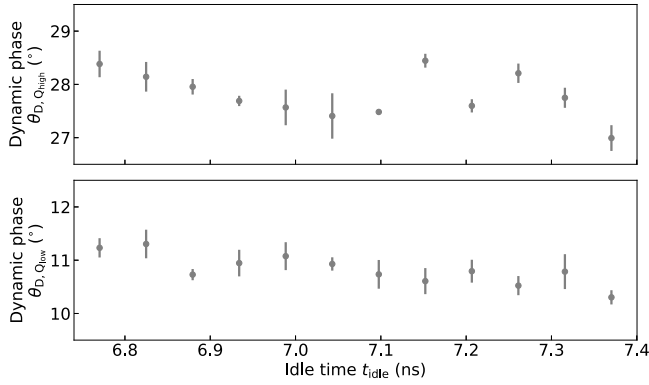


FIG. 10. Dynamic phase measured as a function of the idle time t_{idle} .

phases during the gate. The residuals observed using the above calibration procedure amount to coherent gate errors close to 5×10^{-5} , as detailed in Appendix H. As this value is two orders of magnitude below the observed total gate error, we defer a more detailed study to future work.

Finally, we note that alternative parameterizations of the idle phase are possible. For example, one can use a nonzero flux amplitude during the idle time, changing the integral of the detuning during this time and therefore the conditional phase. Because of the nonlinear flux-frequency relation of the transmons and the finite slope of the pulse edges, this, however, does not yield a linear relation between the control parameter (flux amplitude) and the conditional phase. Moreover, due to Gaussian filtering of the flux control pulses, changing the amplitude during the idle time has the side effect of distorting the edges of the main half-pulses, effectively changing their duration, the interaction time t_{int} . Since this turns the interaction time into a function of the parameter that controls the conditional phase, the continuous gate set cannot be controlled with a single parameter in this parameterization. A variant of this approach was pursued in Ref. [29] by smoothly adjusting the amplitude of a single point at the edge of each half-waveform. When not only calibrating a single CZ_π gate, as in Ref. [29], but an entire continuous gate set, this variant would have the same drawbacks as described above for the approach of adjusting the amplitude during the idle time.

APPENDIX D: GEOMETRICAL PHASE

We express the conditional phase θ of a controlled arbitrary-phase gate

$$\text{CZ}_\theta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\theta} \end{pmatrix} \quad (\text{D1})$$

as a function of the state trajectory on the $|11\rangle$ – $|20\rangle$ Bloch sphere. We consider the implementation based on net-zero pulses presented in the main text and summarized in Fig. 1. The time evolution during the net-zero pulses can be seen as equivalent to a Mach-Zehnder interferometer with two unbalanced beam splitters R and an arbitrary phase shifter Z [28]. The presence of the phase shifter makes it challenging to gain intuition about the conditional phase. We thus instead interpret the state evolution in a geometric picture, where the phase at the end of a cyclic trajectory is given by the Aharonov-Anandan phase [52]:

$$\theta = \underbrace{\oint \langle \Psi | \frac{d}{dt} | \Psi \rangle dt}_{\theta_G} - \underbrace{\oint \langle \Psi | H | \Psi \rangle dt}_{\theta_D}. \quad (\text{D2})$$

In the special case of an adiabatic evolution where the state stays aligned with the axis of rotation at all times, θ would be fully determined by the geometrical phase (Berry phase), $\theta_G = -S/2$, where S is the solid angle enclosed by the trajectory on the Bloch sphere [86]. For a nonadiabatic evolution, Eq. (D2) also includes the dynamic phase contribution θ_D from the instantaneous energy of the state.

From Eq. (1), we can rewrite the time-dependent Hamiltonian restricted to the two relevant states as

$$H(t) = \begin{pmatrix} 0 & g_2/2 \\ g_2/2 & \Delta(t) \end{pmatrix} = \Delta(t) \frac{\sigma_e}{2} + g_2 \frac{\sigma_x}{2}, \quad (\text{D3})$$

where I , σ_z , σ_x are the identity and Pauli matrices acting on the $|11\rangle$ – $|20\rangle$ subspace, and we have defined $\sigma_e = I - \sigma_z$. This yields an instantaneous axis of rotation on the Bloch sphere $\vec{\Omega} = (g_2, 0, -\Delta(t))$. Assuming that $\Delta(t)$ is a piecewise-constant, symmetric function of time suitable to reach full population recovery, a net-zero pulse sequence implements the unitary

$$U = R_\lambda^\vec{r} Z_{\theta_D} R_\lambda^\vec{r}, \quad (\text{D4})$$

where the rotation generated by H is denoted as Z when $\Delta(t) \gg g_2$ (rotation axis pointing downwards), and $R_\lambda^\vec{r}$ otherwise (rotation by an angle λ around an arbitrary axis $\vec{r}$). Since we have defined a reference energy $\langle 11 | H(t) | 11 \rangle = 0$ by canceling the single-qubit dynamic phases (see the main text), and since full population recovery implies that both of the R -type rotations pass through the $|11\rangle$ state, $\langle H \rangle = 0$ is a constant of motion during the two R -type rotations. As a result, the entire contribution from the dynamic phase θ_D is contained in the Z_{θ_D} rotation.

As a special case, the circular trajectories with $\theta_{\text{idle}} = 0$ [as in Figs. 1(e) and 1(g), reproduced in Fig. 11(a)] only consist of two R -type rotations, so that θ_D vanishes. We thus compute the conditional phase from the solid angle S

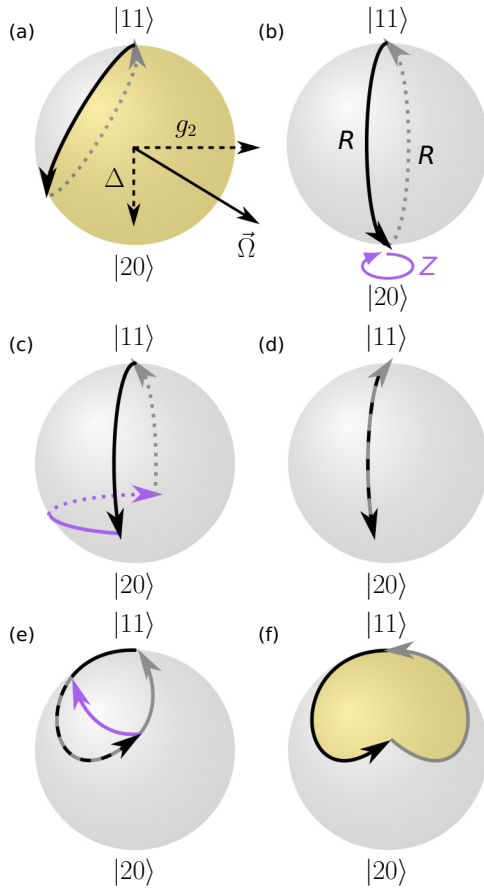


FIG. 11. State trajectories and geometrical phase during a net-zero pulse. The solid angle (yellow) is equivalent to the conditional phase in the absence of a Z rotation (purple); see the text for details. We use the convention that a positive phase increase of $|20\rangle$ corresponds to a counterclockwise rotation when seen from the top of the Bloch sphere. Accordingly, a Z rotation rotates clockwise for a detuning $\Delta > 0$ pointing downward. The time evolution is represented for (a) a pulse with $\theta_{\text{idle}} = 0$ as in Fig. 1(g), (b) a CZ_0 gate from the continuous gate set, (c) a pulse implementing a short CZ_0 gate, (d) the unitary equivalent to (c) after commuting out the Z rotation as explained in the text, yielding a vanishing solid angle, (e) a pulse with freely chosen parameters, and (f) the unitary equivalent to (e) after commuting out the Z rotation.

enclosed by a circle on the Bloch sphere:

$$\begin{aligned} \theta &= \theta_G = -\frac{1}{2}S = -\frac{1}{2}2\pi(1 - \langle \vec{r}, \vec{z} \rangle) \\ &= -\pi \left(1 + \frac{\Delta}{\sqrt{g_2^2 + \Delta^2}} \right). \end{aligned} \quad (\text{D5})$$

This is equivalent to the conditional phase given in Ref. [17] for a gate implemented with a rectangular flux pulse.

As a second special case, we consider the continuous gate set [dashed line in Fig. 1(f) and trajectory in Fig. 1(h), reproduced in Fig. 11(b)]. In this case, each R-type rotation

with $\Delta = 0$ performs a full population exchange, yielding $\theta_G = \pi$. The Z-type rotation happens while the state is $|20\rangle$, yielding no contribution to the trajectory on the Bloch sphere (no contribution to θ_G) and giving a dynamic phase that is equal to the negative time integral of the detuning; see Eq. (D2) combined with Eq. (D3) for a $|20\rangle$ state. By defining θ_{idle} as the (positive) time integral of the detuning, we obtain the formula used in the main text:

$$\theta = \theta_G + \theta_D = \pi - \theta_{\text{idle}}. \quad (\text{D6})$$

We now consider the special case corresponding to the CZ_0 gates at the top and bottom of Fig. 1(f). An example trajectory with $\theta_D = \pi$ and $\Delta = 0$ is shown in Fig. 11(c). The geometrical phase is

$$\theta_G = \int_0^{-\pi} \int_0^\lambda \left(-\frac{1}{2} \right) dS = \pi \sin^2 \frac{\lambda}{2}. \quad (\text{D7})$$

During the $Z_{-\pi}$ rotation (in purple) of duration $t_Z = \pi/\Delta$, we obtain

$$\begin{aligned} \theta_D &= - \int_0^{t_Z} \langle H \rangle dt \\ &= - \int_0^{\pi/\Delta} \Delta \sin^2 \frac{\lambda}{2} dt \\ &= -\pi \sin^2 \frac{\lambda}{2} \\ &= -\theta_G. \end{aligned} \quad (\text{D8})$$

This yields $\theta = 0$, resulting in a CZ_0 gate independently of the value of the rotation angle λ .

We can find an alternative description where the conditional phase is directly reflected by the geometrical phase. Commuting the Z rotation through the second R gives

$$U = R_\lambda^{\vec{r}} Z_{\theta_D} R_\lambda^{\vec{r}} = Z_{\theta_D} R_{\lambda'}^{\vec{r}'} R_\lambda^{\vec{r}}, \quad (\text{D9})$$

where the new rotation axis $\vec{r}'$ differs from $\vec{r}$ by the phase θ_D . If the time evolution ends in the $|11\rangle$ state, which is not affected by the pure phase shift on $|20\rangle$, Eq. (D9) can be simplified as

$$U \equiv R_{\lambda'}^{\vec{r}'} R_\lambda^{\vec{r}}. \quad (\text{D10})$$

The resulting equivalent operation is only composed of R-type rotations, which have a vanishing dynamic phase, as explained above. For the special case considered here, the trajectory under this equivalent operation is shown in Fig. 11(d), where $\vec{r}' = -\vec{r}$ yields a vanishing solid angle. The $Z_{-\pi}$ rotation therefore mirrors the time evolution of the second R-type rotation, such that the two R-type rotations cancel each other. This produces a CZ_0 gate with full

population recovery and vanishing conditional phase for any angle λ of the R rotation.

Using the transformation in Eq. (D10), the conditional phase can be interpreted as a geometrical phase for any gate described by Eq. (D4), including the more general cases visible in Figs. 12 and 14 in Appendix E. Figures 11(e)–11(f) provide an example of a generic trajectory, where the solid angle shown in yellow reflects the conditional phase after the gate.

APPENDIX E: SENSITIVITY OF THE POPULATION RECOVERY

In the neighborhood of the optimal parameters $\Delta_{\min}$ and t_{int} , the sensitivity of the population recovery to these parameters depends on the conditional phase θ . We study this effect analytically for small deviations around the optimal parameters.

In the two-dimensional subspace spanned by $|11\rangle$ and $|20\rangle$, assuming piecewise-constant net-zero waveforms, the ideal time evolution is given by three successive unitaries, as already studied graphically in Appendix D: a partial Rabi rotation R during the time $t_{\text{int}}/2$, a phase rotation $Z_{-\theta_{\text{idle}}}$, and a second, identical R rotation.

We now derive the corresponding unitaries explicitly, by rewriting the Hamiltonian (D3) as

$$H(t) = \begin{pmatrix} 0 & g_2/2 \\ g_2/2 & \Delta(t) \end{pmatrix} = \frac{\Delta(t)}{2}I + \frac{\Omega(t)}{2} \left(-\frac{\Delta(t)}{\Omega(t)}\sigma_z + \frac{g_2}{\Omega(t)}\sigma_x \right), \quad (\text{E1})$$

where $\Omega(t) = \sqrt{g_2^2 + \Delta^2(t)}$ is the effective rotation frequency under $H(t)$. Fixing $\Delta(t) = \Delta_{\min}$ and $\Omega = \sqrt{g_2^2 + \Delta_{\min}^2}$ yields the R rotation by exponentiating the Hamiltonian using Eq. (4.8) of Ref. [87]:

$$R = e^{-iHt_{\text{int}}/2} = e^{-i(\Delta_{\min}/2)(t_{\text{int}}/2)} \times \left[\cos\left(\frac{\Omega}{2} \frac{t_{\text{int}}}{2}\right)I - i \sin\left(\frac{\Omega}{2} \frac{t_{\text{int}}}{2}\right) \left(-\frac{\Delta_{\min}}{\Omega}\sigma_z + \frac{g_2}{\Omega}\sigma_x \right) \right]. \quad (\text{E2})$$

Fixing instead $\Delta(t) \gg g_2$ yields the phase rotation

$$Z_{-\theta_{\text{idle}}} = |11\rangle\langle 11| + e^{-i\theta_{\text{idle}}} |20\rangle\langle 20|. \quad (\text{E3})$$

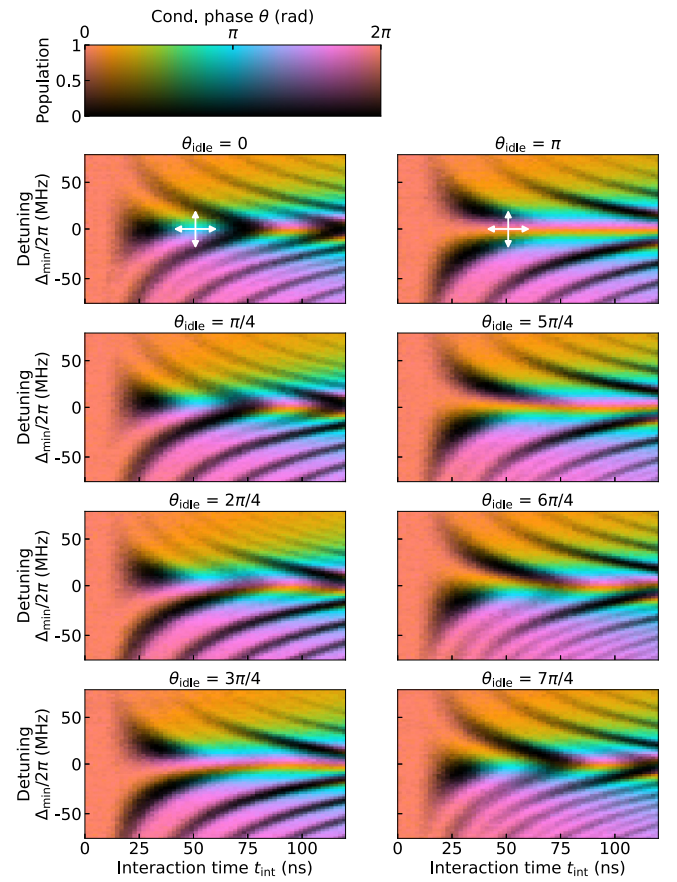


FIG. 12. Measured $|11\rangle$ population and conditional phase after one gate for different values of the idle phase θ_{idle} . The white arrows indicate the one-dimensional cuts shown in Fig. 13 and approximated in Eq. (E5).

The final probability amplitude of the $|11\rangle$ state is then the matrix element

$$c_{|11\rangle} = \langle 11| RZ_{-\theta_{\text{idle}}} R |11\rangle = e^{-i(\Delta_{\min}/2)t_{\text{int}}} \times \left[\xi + (1 - \xi) \cos\left(\frac{\Omega}{2} t_{\text{int}}\right) + i \frac{\Delta_{\min}}{\Omega} \sin\left(\frac{\Omega}{2} t_{\text{int}}\right) \right] \quad (\text{E4})$$

with $\xi = (1 - e^{-i\theta_{\text{idle}}})g_2^2/2\Omega^2$ accounting for the phase rotation in the middle of the evolution. This matrix element is a function of three controllable parameters: $\Delta_{\min}$, t_{int} , and θ_{idle} . In Fig. 12, we show measurement data for the population recovery $P_{|11\rangle} = |c_{|11\rangle}|^2$ (encoded as brightness) and the conditional phase $\arg c_{|11\rangle}$ (color), as a function of $\Delta_{\min}$ and t_{int} , for eight values of θ_{idle} . The cases $\theta_{\text{idle}} = 0$ and $\theta_{\text{idle}} = \pi$ correspond to Figs. 1(e) and 2(a), respectively. The other panels illustrate the continuous transition between these two extreme cases.

We now study the behavior of the population recovery in the neighborhood of the continuous gate set. Below,

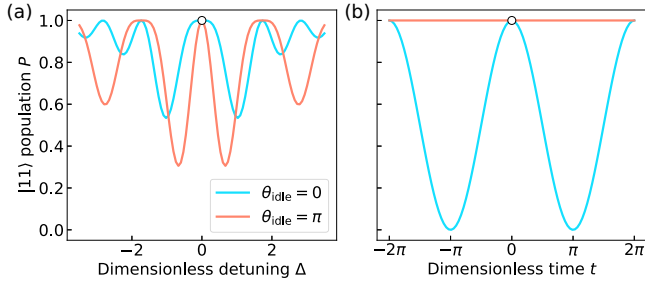


FIG. 13. Simulated $|11\rangle$ population after one gate, using Eq. (E4). The population is shown as a function of the dimensionless parameters $\Delta = \Delta_{\min}/g_2$ and $t = t_{\text{int}}g_2 - 2\pi$, around the CZ_π gate ($\theta_{\text{idle}} = 0$) and the CZ_0 gate ($\theta_{\text{idle}} = \pi$). The continuous gate set, corresponding to both parameters set to 0, is indicated as a white dot.

we use dimensionless parameters $\Delta = \Delta_{\min}/g_2$ and $t = t_{\text{int}}g_2 - 2\pi$. Note the 2π offset in t , such that the ideal continuous gate set corresponds to unitaries with both $\Delta = 0$ and $t = 0$. Figure 13 shows the population recovery $P_{|11\rangle} = |c_{|11\rangle}|^2$, with $c_{|11\rangle}$ calculated from Eq. (E4), as a function of either parameter, both at $\theta_{\text{idle}} = 0$ and $\theta_{\text{idle}} = \pi$.

Taylor expanding $P_{|11\rangle}(\Delta, t)$ around 0 yields, after a few lines of algebra,

$$P_{|11\rangle}(\Delta, 0) = \begin{cases} 1 - \frac{\pi^2 \Delta^4}{4} + O(\Delta^6) & \text{if } \theta_{\text{idle}} = 0, \\ 1 - 4\Delta^2 + O(\Delta^4) & \text{if } \theta_{\text{idle}} = \pi, \end{cases} \quad (\text{E5})$$

$$P_{|11\rangle}(0, t) = \begin{cases} 1 - \frac{t^2}{4} + O(t^4) & \text{if } \theta_{\text{idle}} = 0, \\ 1 & \text{if } \theta_{\text{idle}} = \pi. \end{cases}$$

These four cases are indicated by the white arrows in Fig. 12. The population recovery is only fourth-order sensitive to the detuning Δ at $\theta_{\text{idle}} = 0$. Intuitively, around the CZ_π gate (see Fig. 12 at $\theta_{\text{idle}} = 0$), small changes of Δ are tangent to the curve of maximal population recovery, and therefore have very little effect on the population. On the other hand, the population recovery is second-order sensitive to Δ at $\theta_{\text{idle}} = \pi$. This is why we fine-tune the detuning at $\theta_{\text{idle}} = \pi$, as discussed in the main text. Similarly, we fine-tune the interaction time t_{int} at $\theta_{\text{idle}} = 0$, noting that $\theta_{\text{idle}} = \pi$ would yield no sensitivity to t_{int} at all. This insensitivity can be seen from Eq. (E5) and can be interpreted as the cancellation of the two resonant interactions discussed for the CZ_0 gate in Appendix D.

We now quantitatively study the effect of an imperfect calibration of the detuning Δ , since fine-tuning the detuning is a requirement to reach the region of the parameter space shown in Fig. 1(f), where the effects of the interaction time and idle time are decoupled. Figure 14 shows an example of population recovery and conditional phase at a nonzero detuning, yielding a skewed landscape compared to Fig. 1(f). The unrealistically high value of the detuning,

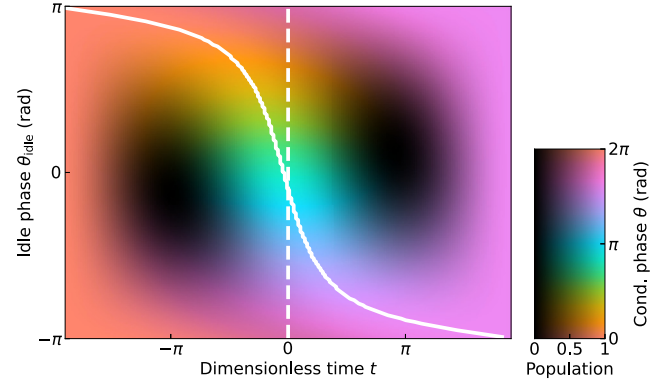


FIG. 14. Simulated $|11\rangle$ population and conditional phase after one gate for an exaggeratedly high value of the dimensionless detuning, $\Delta = 0.2$, as a function of the idle phase θ_{idle} and the dimensionless time t . The curve of maximal population recovery is shown as a solid white line. The dashed white line indicates the curve of maximal population recovery if Δ was instead equal to 0.

$\Delta = 0.2$, is chosen for the purpose of illustration. The solid white curve indicates the points of maximal population recovery, evaluated numerically. This would correspond to a continuous gate set parameterized by both the idle phase and the interaction time. For decreasing $|\Delta|$, this curve approaches the vertical dashed line at $t = 0$, parameterized by the idle phase only. However, for a nonzero $|\Delta|$, a continuous gate set parameterized only by the idle phase cannot reach zero leakage for all gates.

We now study such a continuous gate set parameterized by θ_{idle} only. We assume for concreteness that $t = 0$, such that the population recovery will be lowest at $\theta_{\text{idle}} = \pm\pi$ (the extremities of the dashed white line in Fig. 14). At this point, the leakage is $1 - P_{|11\rangle}(\Delta, 0) \approx 4\Delta^2$ due to Eq. (E5). The spacing of $33 \mu\Phi_0$ between points used in the calibration in Fig. 2(d) corresponds to a detuning of 4×10^{-3} and would, therefore, allow leakage below $4\Delta^2 \approx 7 \times 10^{-5}$ in the absence of all other imperfections.

APPENDIX F: LEAKAGE AMPLIFICATION

To infer the leakage per gate in the calibration procedure of the continuous gate set, we use the fact that the leakage after a series of N gates displays interference fringes as a function of the gate separation. We first derive this behavior analytically in a simplified model by assuming a unitary process, and then quantitatively in simulations by fitting a master equation model to the measured data. Our numerical analysis is valid for a two-qubit gate that uses a noncomputational state as an auxiliary state, considering that dephasing of the superposition between the $|11\rangle$ and $|20\rangle$ states contributes to incoherent leakage. We do not rely on small-angle approximations, so the analysis is applicable for leakage varying by orders of magnitude during the gate calibration.

In an error amplification measurement where the errors to be characterized add up incoherently, for example in randomized benchmarking [60,61,68,88], the resulting amplified error scales linearly with the number of gates. If, on the other hand, the errors add coherently, the resulting interferences must be constructive in order to observe an amplified error. In the case of single-qubit gates, constructive interference can be achieved by adjusting the phase of each microwave control pulse to follow the rotating frame of the qubit [59]. Another possibility is to adjust the temporal separation in a sequence of gates to cancel the dynamic phase accumulated by the leaked population, as demonstrated in Ref. [41], and in experiments conducted in parallel to our work [89,90]. As the degree of freedom of adjusting the phase of control pulses does not exist for the two-qubit gate implementation in this paper, we adjust the gate separation.

We start by deriving the leakage after N gates analytically, by assuming fully coherent leakage. We restrict the description to the subspace spanned by $|11\rangle$ and $|20\rangle$. We model an entire gate as the unitary

$$U = Y_\lambda Z_\beta = \begin{pmatrix} \cos(\lambda/2) & -\sin(\lambda/2) \\ \sin(\lambda/2) & \cos(\lambda/2) \end{pmatrix} \begin{pmatrix} e^{-i\beta/2} & \\ & e^{i\beta/2} \end{pmatrix} \quad (\text{F1})$$

parameterized by λ and β . Compared to a fully generic unitary $e^{i\alpha} Z_{\beta_1} Y_\lambda Z_{\beta_2}$ (Theorem 4.1 of Ref. [87]), we have dropped the global phase and have absorbed a Z rotation into the (arbitrary) choice of the phase reference for the $|20\rangle$ state. In the parameterization in Eq. (F1), λ encompasses the contributions of any coherent rotation errors and β represents the phase acquired due to the separation t_{sep} between successive gates. Ideally, we have $\lambda = 0$ for a calibrated gate and $\beta = 0$ to obtain constructive interferences.

Expressing U^N using Eq. (5) from Ref. [34] yields

$$\langle 20 | U^N | 11 \rangle = \langle 20 | U | 11 \rangle \frac{\sin(N \arccos \text{Re}[\langle 11 | U | 11 \rangle])}{\sin(\arccos \text{Re}[\langle 11 | U | 11 \rangle])}, \quad (\text{F2})$$

and taking the square modulus gives the total leakage after N gates

$$L_N = \underbrace{\sin^2\left(\frac{\lambda}{2}\right)}_{L_1} \frac{\sin^2\{N \arccos[\cos(\lambda/2) \cos(\beta/2)]\}}{\sin^2\{\arccos[\cos(\lambda/2) \cos(\beta/2)]\}}. \quad (\text{F3})$$

The interference pattern described by Eq. (F3) is plotted in Fig. 15 as a function of λ and β .

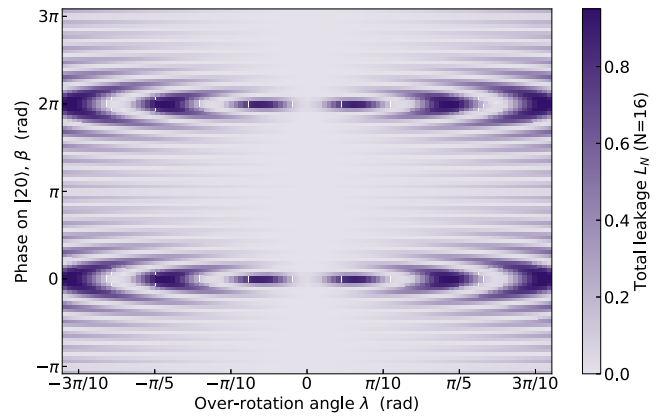


FIG. 15. Calculated interference pattern of the coherent leakage after $N = 16$ gates using the simplified decoherence-free model in Eq. (F3). The leakage is shown as a function of the over-rotation angle λ , which sets the strength of the leakage, and of the phase acquired by the population in the $|20\rangle$ state between two gates, which sets the interference condition.

Equation (F3) can be rewritten to make the dependence on the leakage of a single gate L_1 explicit:

$$L_N = L_1 \underbrace{\frac{\sin^2(N \arccos\{\cos[\arcsin(\sqrt{L_1})] \cos(\beta/2)\})}{\sin^2(\arccos\{\cos[\arcsin(\sqrt{L_1})] \cos(\beta/2)\})}}_{f(L_1, \beta)}. \quad (\text{F4})$$

In the special case of small leakage L_1 per gate [see the range around $\lambda = 0$ in Fig. 15 and the experimental data in Fig. 2(c)], we can approximate Eq. (F4) as its first-order Taylor expansion in L_1 :

$$L_N = L_1 f(0, \beta) + O(L_1^2) \approx L_1 \frac{\sin^2(N\beta/2)}{\sin^2(\beta/2)} = L_1 \frac{1 - \cos(N\beta)}{1 - \cos(\beta)}. \quad (\text{F5})$$

The resulting expression can be interpreted as the interference pattern of N waves of equal amplitude L_1 , each shifted by an integer multiple of the phase β accumulated after each gate. This explains the interference fringes visible as a function of β along a vertical cut in Fig. 2(c) or Fig. 15; see an example in Fig. 16.

In the special case of constructive interference ($\beta = 0$), the denominator in Eq. (F4) cancels with the prefactor L_1 . For small leakage L_1 , plugging in the Taylor expansion $L_1 = 1 - P_{|11\rangle} \approx 4\Delta^2 = 4\Delta_{\text{min}}^2/g_2^2$ from Appendix E yields

$$L_N \approx \sin^2\left(2N \frac{\Delta_{\text{min}}}{g_2}\right). \quad (\text{F6})$$

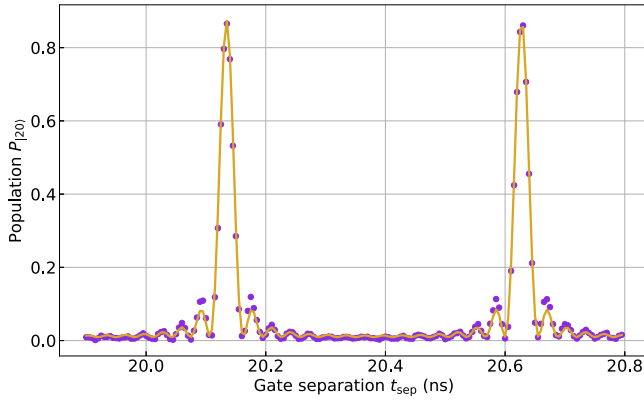


FIG. 16. Amplified leakage (dots) along a one-dimensional cut in the dataset shown in Fig. 2(c), at the point of highest contrast near $\phi_{\text{low}} = -0.1996 \Phi_0$, and fit (line) using the model in Eq. (F7).

With a first-order approximation of Δ_{min} as a function of the pulse parameter ϕ_{low} , this explains the sinusoidal shape of the data shown in Fig. 2(d) as a function of ϕ_{low} .

We now study the total leakage quantitatively, by computing

$$L_N = \text{Tr}[|20\rangle\langle 20| U^N (|11\rangle\langle 11|)] \quad (\text{F7})$$

numerically in a master equation simulation [91]. We model the time evolution in an effective three-dimensional Hilbert space including the $|11\rangle$ and $|20\rangle$ states and an auxiliary state representing all lower-energy states. We take decoherence into account by including Lindblad terms describing decay from $|11\rangle$ to the lower-energy states with rate $1/T_1^{(11)}$, decay from $|20\rangle$ to the lower-energy states with rate $1/T_1^{(20)}$, and dephasing between $|11\rangle$ and $|20\rangle$ with rate $1/T_\phi$. We then truncate the Hilbert space and the operators to the $|11\rangle$ – $|20\rangle$ subspace.

We model the time evolution assuming piecewise-constant, net-zero waveforms. To approximate the increased sensitivity to flux noise during the frequency excursions [33] while keeping a limited number of parameters, we only apply the dephasing term during the interaction time, whereas we apply the relaxation terms during the entire time evolution. From the experimentally determined relaxation times in Table I we obtain $T_1^{(11)} = (1/T_1^{(10)} + 1/T_1^{(01)})^{-1} = 23.8 \mu\text{s}$ and $T_1^{(20)} = 14.7 \mu\text{s}$. This is an approximation, which neglects the frequency dependence of the relaxation times. Plugging in measured values for the dephasing time would require accounting for the strong frequency dependence of the sensitivity to flux noise and for the echo effect coming from the net-zero pulse [28]. We thus take the simpler approach of only considering an effective dephasing time T_ϕ , which we fit to the data. As further fit parameters, we choose the detuning

Δ_{min} during the interaction time (responsible for coherent errors), and a time offset t_{offset} to account for the rise times of the control pulses by expressing β in Eq. (F1) as $\beta = (\omega_{20} - \omega_{11})(t_{\text{sep}} - t_{\text{offset}})$ with the gate separation t_{sep} .

In order to extract the leakage of a single gate L_1 from the total leakage L_N , we fit the model in Eq. (F7) to a one-dimensional subset of the dataset shown in Figs. 2(c)–2(d), as a function of the gate separation t_{sep} , for a fixed value of the flux amplitude near $\phi_{\text{low}} = -0.1996 \Phi_0$ yielding the highest contrast. The fit as a function of the gate separation is shown in Fig. 16. We obtain the fitted value $T_\phi = 18(2) \mu\text{s}$. For comparison, from a model approximating the noise power spectral density as white, using Eqs. (19) and (A3) of Ref. [92], this effective dephasing time would correspond to an incoherent leakage $L_1 \approx t_{\text{int}}/4T_\phi = 6.9 \times 10^{-4}$. This is of the same order of magnitude as the leakage measured in Fig. 3(d) for a calibrated gate.

Keeping T_ϕ and t_{offset} fixed, we now reuse the fitted model for the rest of the dataset, as follows. We compute both L_N and L_1 as a function of Δ_{min} , and interpolate numerically $L_N = f(L_1)$. We use f to compute the right-hand vertical axis in Fig. 2(d). The area that is not shaded in red indicates the range where this conversion can be done in a unique way.

APPENDIX G: CROSS-ENTROPY BENCHMARKING

We use XEB, introduced in Refs. [30,31], to estimate the fidelity of a gate or the whole continuous set of gates. XEB does not require a set of gates forming a group, as needed in interleaved randomized benchmarking [37,40,60,61], thus providing more flexibility in the choice of gates to be characterized. Unlike state tomography [93], XEB evaluates random circuits containing many gates and of different lengths, thereby canceling the contribution of state preparation and measurement errors, and allowing one to capture errors that only arise in long gate sequences such as non-Markovian effects due to long-time control pulse distortions. XEB has been applied to benchmark entangling two-qubit gates, for example in Ref. [31]. Alternatively, a continuous gate set was characterized using randomized benchmarking in Refs. [23,25], by benchmarking the Clifford unitary $CZ_\theta CZ_{\pi-\theta}$, although this method does not distinguish the individual errors of each gate of the pair of gates.

An XEB experiment consists in measuring the statistics of observing each n -qubit basis state, for random circuits with a varying number M of gate cycles. An unbiased estimate of the fidelity per cycle can be computed under the assumptions of low error rates and chaotic state evolution [30–32,94]. In this context, chaotic evolution means that a few cycles are sufficient to form a 2-design, that is, the distribution over random circuits resembles the uniform

distribution up to second-order statistical moments. This is the reason for adding the mixing unitary U_{mix} in the cycle design shown in Fig. 3(b). From the output of the randomized circuits, one then estimates the XEB depolarization fidelity [22,30], which is a normalized function of the cross-entropy between the ideal (simulated) and measured probability distributions. The XEB fidelity decreases exponentially with the number of cycles M [31,67]:

$$F_{\text{XEB}} = A \left(1 - \frac{D}{D-1} \epsilon_a \right)^M + B. \quad (\text{G1})$$

Here, ϵ_a is the average error per cycle, D is the size of the Hilbert space ($D = 4$ in our case), and A, B represent contributions of state preparation and measurement errors. The average cycle error ϵ_a can be extracted from an exponential fit of F_{XEB} as a function of M .

In an XEB experiment, we run 250 different random circuits consisting of U_{mix} and the gate under test [Fig. 3(b)] for each number of cycles $M \in \{4, 8, 16, 32, 64, 128, 256\}$, each repeated 2048 times. We perform three-level single-shot readout to detect leakage in the final readout; see Fig. 3(d). After excluding the 3.6% of the experimental runs where leakage was detected in the final readout, we renormalize the remaining data such that the populations sum to one in the computational subspace, as in Ref. [38]. We fit the exponential decay of the average XEB depolarization fidelity and extract the average cycle error. In addition, we perform a reference experiment benchmarking only U_{mix} . We finally extract the error ϵ_{CZ} of a gate under test by writing, similarly to the product over cycles in Eq. (G1), the product of fidelities within a cycle $1 - \epsilon_{\text{tot}} \approx (1 - \epsilon_{\text{CZ}})(1 - \epsilon_{U_{\text{mix}}})$, where ϵ_{tot} and $\epsilon_{U_{\text{mix}}}$ are the average cycle errors for the measurements with and without the gate under test, respectively.

APPENDIX H: ERROR BUDGET

We estimate the error budget of a CZ_θ gate using a Lindblad master equation describing the four computational states as in Ref. [33]. This approach neglects potential differences between the decoherence of the $|20\rangle$ and $|11\rangle$ states. We have verified numerically that modeling the decoherence of the second excited state as in Ref. [28] leads to similar results.

We include three decoherence terms: dephasing due to flux noise (T_ϕ^f), dephasing due to other sources (T_ϕ^v), and relaxation (T_1). To account for the inherent echo effect of the net-zero gate [28], we describe dephasing based on the echo decay times T_2^E of the qubits. To this end, we fit the model

$$\frac{1}{T_\phi} = \frac{1}{T_2^E} - \frac{1}{2T_1} = a\omega' + b, \quad (\text{H1})$$

where a and b are fit parameters, and ω' is the derivative of qubit frequency with respect to flux. We use ten measurements, distributed over a week, of T_2^E and T_1 at the idle frequency, as well as 16 measurements at equally spaced flux bias points in an interval corresponding to approximately 150 MHz around the interaction frequency. By evaluating the fitted model from Eq. (H1) at the flux applied during the interaction and at the idle flux, we obtain the relevant T_ϕ^f and T_ϕ^v . The corresponding values of T_2^E as well as the relaxation times T_1 are reported in Table I.

We approximate the time evolution during a gate as piecewise constant. We include five steps, with Lindbladian $\mathcal{L}_i$ and duration δt_i , corresponding to the two halves of the flux pulse, the idle time between them, and the initial and final buffers. We then compute the time evolution using

$$\mathcal{E} = \prod_{i=1}^5 e^{\mathcal{L}_i \delta t_i}, \quad (\text{H2})$$

where we use the coherence times at the interaction point in steps $i = 2$ and $i = 4$, and the coherence times at the idle frequencies otherwise.

We convert $\mathcal{E}$ into a χ matrix [95] to obtain the average gate fidelity

$$F = \frac{\chi_{0,0} D + 1}{D + 1}, \quad (\text{H3})$$

where $\chi_{0,0}$ is the first diagonal element of χ and $D = 4$ is the Hilbert space dimension; see Eqs. (2) and (6) of Ref. [95]. We obtain the contribution of each error source by setting the other sources to zero in the master equation, yielding the following error budget: 0.37% from dephasing due to flux noise, 0.10% from dephasing due to other sources, and 0.13% from relaxation. This amounts to a total of 0.61% incoherent gate errors, which is consistent with the measured gate errors shown in Fig. 3(c).

In the following, to estimate the coherent gate error resulting from a miscalibration of the conditional phase θ , we derive the average gate fidelity using the method in Ref. [96]. We first compute the entanglement fidelity $F_e = |\langle \Psi | U_{\text{err}} \otimes I | \Psi \rangle|^2$, where $|\Psi\rangle$ is a maximally entangled state. As the error unitary $U_{\text{err}} = \text{diag}[1, 1, 1, e^{i\theta_{\text{err}}}]$ is diagonal, we directly obtain

$$F_e = \frac{|1 + 1 + 1 + e^{i\theta_{\text{err}}}|^2}{16} = \frac{10 + 6 \cos(\theta_{\text{err}})}{16}, \quad (\text{H4})$$

yielding the gate fidelity [96,97]

$$F = \frac{F_e D + 1}{D + 1} = \frac{7 + 3 \cos(\theta_{\text{err}})}{10} \quad (\text{H5})$$

with $D = 4$. We note that the gate error is, to leading order, a quadratic function of the error θ_{err} in the conditional

phase. For θ_{err} below 1° as in Fig. 3(a), we obtain a gate error of less than 5×10^{-5} , two orders of magnitude lower than the incoherent error.

For a dynamic phase error, we repeat the calculation with the error unitary $U_{\text{err}} = \text{diag}[1, e^{i\theta_{\text{err}}}, 1, e^{i\theta_{\text{err}}}]$, yielding

$$F = \frac{3 + 2 \cos(\theta_{\text{err}})}{5}. \quad (\text{H6})$$

A residual imperfection of a dynamic phase by $\theta_{\text{err}} = 1^\circ$ would therefore correspond to a gate error of 6×10^{-5} .

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