

Sequential and Programmable Squeezing Gates for Optical Non-Gaussian Input States

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Quantum computing has been pursued with various hardware platforms and an optical system is one of the most reasonable choices for large-scale computation. In the optical continuous-variable computation scheme, the incorporation of Gaussian gates and a highly nonclassical non-Gaussian state enables universal quantum computation. Although basic technologies for Gaussian gates and non-Gaussian state generation have long been developed, these building blocks have not yet been integrated in a scalable fashion. Here, we integrate them to develop a scalable and programmable optical quantum computing platform that can sequentially perform an essential Gaussian gate, the squeezing gate, on a non-Gaussian input state. The key enablers are a loop-based optical circuit with dynamical and programmable controllability and its time synchronization with the probabilistic non-Gaussian state generation. We verify the deterministic, programmable, and repeatable quantum gates on a typical non-Gaussian state by implementing up to three-step gates. The gates implemented are of such high quality that strong evidence of the nonclassicalities of the states, the negativities of the Wigner functions, are preserved even after multistep gates. This platform is compatible with other non-Gaussian states and can in principle realize large-scale universal quantum computing by incorporating other existing processing technologies.

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I. INTRODUCTION

While various physical systems have emerged for quantum computing [1–3], an optical system employing the continuous-variable (CV) scheme has been attracting much interest due to its strength in scalability [4]. The optical CV scheme, which utilizes the quadrature amplitude of the light field for quantum computing, has offered deterministic quantum gates [5–7], with its practical advantage lying in the unconditionally prepared ancillary squeezed vacuum and the highly efficient homodyne measurement. A promising idea to integrate the deterministic gates to build a scalable quantum computing platform is to adopt the time-domain multiplexing approach unique

to an optical system. In this approach, the quantum information is encoded in sequential optical pulses on a single (or a few) path(s) and the same optical components are repeatedly used to process them at different times. This approach, in fact, has recently led to the development of several platforms able to perform programmable multistep quantum gates on a single- or multimode input state [8–11].

However, these scalable platforms are currently limited to performing calculations within the Gaussian realm, which can be efficiently simulated by a classical computer [12]. This limitation implies a lack of the ability to perform universal quantum computation. The missing piece for achieving universality is a non-Gaussian quantum state. When a specific non-Gaussian ancillary state known as a cubic phase state is injected, Gaussian platforms, which consist of optical Gaussian gates and homodyne measurements, enable universal computation including a non-Gaussian gate [13]. Furthermore, fault tolerance is also attainable on these platforms by injecting non-Gaussian qubit states, specifically Gottesman-Kitaev-Preskill (GKP)

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qubits [14,15]. Nevertheless, the realization of scalable Gaussian platforms compatible with non-Gaussian states had long been obstructed by technical challenges arising from the probabilistic generations of such states: non-Gaussian states thus far had only undergone single-step gates without electrical programmability in a non-scalable fashion [16,17]. Only recently, time synchronization of a dynamical optical circuit with a probabilistic non-Gaussian source has been reported in a study of a scalable platform [18].

In this work, as a crucial step toward simultaneously achieving scalability and universality, we develop an optical quantum computing platform that can repeatedly and programmably perform a fundamental Gaussian gate, the squeezing gate, on a non-Gaussian input state. This platform in principle provides arbitrarily many steps of squeezing gates with a single setup and the gates are electrically programmable step by step. This is achieved by incorporating two major components. One is a scalable and programmable loop-based processor [10]. The other is a time-domain-multiplexed light-source unit that prepares a train of optical pulses consisting of the non-Gaussian input state (the Schrödinger cat state) and the ancillary states for the gate, which is sent to the processor. Time synchronizing these two components with the probabilistic non-Gaussian state generation enables performance of the gate known as the measurement-induced squeezing [19] on that state. This is an essential gate that composes a universal gate set and also serves as the foundation for various other measurement-induced gates. We verify the deterministic, programmable, and repeatable squeezing gates by implementing various single- or multistep gates. In addition, the high qualities of the gates are supported by the remaining negativities of the Wigner functions, which are widely recognized criteria for the nonclassicalities of the quantum states, even after two-step gates without any loss correction. The squeezing gate can be combined with other easier-to-implement gates to enable any single-mode or multimode Gaussian gate, suggesting a versatile test bed for quantum computing with a non-Gaussian input as a near-term application of our platform. Moreover, since a non-Gaussian gate, which completes the universal gate set, can be realized through a similar measurement-based gate protocol with an alternative non-Gaussian ancilla [10,13], our work will lead to fully universal quantum computing.

II. RESULTS

A. Our platform for a multistep squeezing gate

Before describing our platform, we begin by briefly introducing the measurement-induced squeezing gate [19] in Fig. 1(a), which illustrates a typical schematic of a multistep gate. The single-step $\hat{x}$ squeezing is composed of the following steps, where $\hat{x}$ and $\hat{p}$ represent the orthogonal quadrature operators of the light field: (1) an arbitrary

input state is coupled to an ancillary $\hat{x}$ -squeezed vacuum at a beam splitter with reflectivity R and transmissivity T ($R + T = 1$); (2) the $\hat{p}$ quadrature of one of the resultant modes is measured by the homodyne detector (HD); (3) the measurement outcome m is fed forward to displace the $\hat{p}$ quadrature of the other mode by gm to cancel out the antisqueezing component of the ancillary state, where $g = \sqrt{T/R}$ is called the feedforward gain. In the ideal limit of infinitely squeezed ancilla, this gate transforms the quadrature operators of the input state as $\hat{x} \mapsto \sqrt{R}\hat{x}$ and $\hat{p} \mapsto \hat{p}/\sqrt{R}$ in the Heisenberg picture. Note that this input-output relation changes to $\hat{x} \mapsto \sqrt{T}\hat{x}$ and $\hat{p} \mapsto \hat{p}/\sqrt{T}$ with $g = -\sqrt{R/T}$ only for the first step in Fig. 1(a), due to the inverse positioning of the input and the ancilla. These transformations correspond to the squeezing gates and the degree of squeezing depends only on the beam-splitter reflectivity R . It is worth noting that the quadrature being squeezed, $\hat{x}$ in the above example, can be controlled by adjusting the phase of the ancilla with respect to the input and, accordingly, changing the two quadratures being measured or displaced.

In Fig. 1(b), we show a conceptual diagram of our platform, which is markedly more scalable while having exactly the same functionality as the circuit in Fig. 1(a). In this architecture, the optical pulses are arranged in the time domain and a single dynamical processing unit is repeatedly used to sequentially process them in a resource-efficient manner. This platform is composed of three parts: the time-domain-multiplexed light source, the loop-based optical circuit, and the HD. Their individual properties or functionalities are as follows. The light source plays the role of preparing the optical pulses arranged in the time domain: the constituent optical switch sequentially transfers either a probabilistically generated non-Gaussian input state or a deterministically generated ancillary state to the loop-based circuit with a time separation of τ . The loop-based circuit has an optical delay line with a round-trip time of τ , which makes the incoming two adjacent optical pulses coincide at a variable beam splitter (VBS) for interference. At the downstream part of the loop, the HD with a variable measurement basis $\phi(t)$ is placed to measure $\hat{x} \cos \phi(t) + \hat{p} \sin \phi(t)$ of the incident beam. All the dynamical parameters—the switching status, the VBS reflectivity $R(t)$, and the homodyne measurement basis $\phi(t)$ —are electrically programmable. Notably, as a platform for a non-Gaussian input, these parameters are integrally controlled in synchronization with an electric signal that heralds the generation of the non-Gaussian state. A typical timing chart is shown in Fig. 1(c).

This platform performs an n -step squeezing gate in the following way. When a non-Gaussian input state is generated, the optical switch sends it to the loop part with neighboring ancillas, forming a pulse train in the order in Fig. 1(b). In the loop part, the reflectivity of the VBS is first set to zero [$R_0 = 0$ in Fig. 1(a)], which depicts an

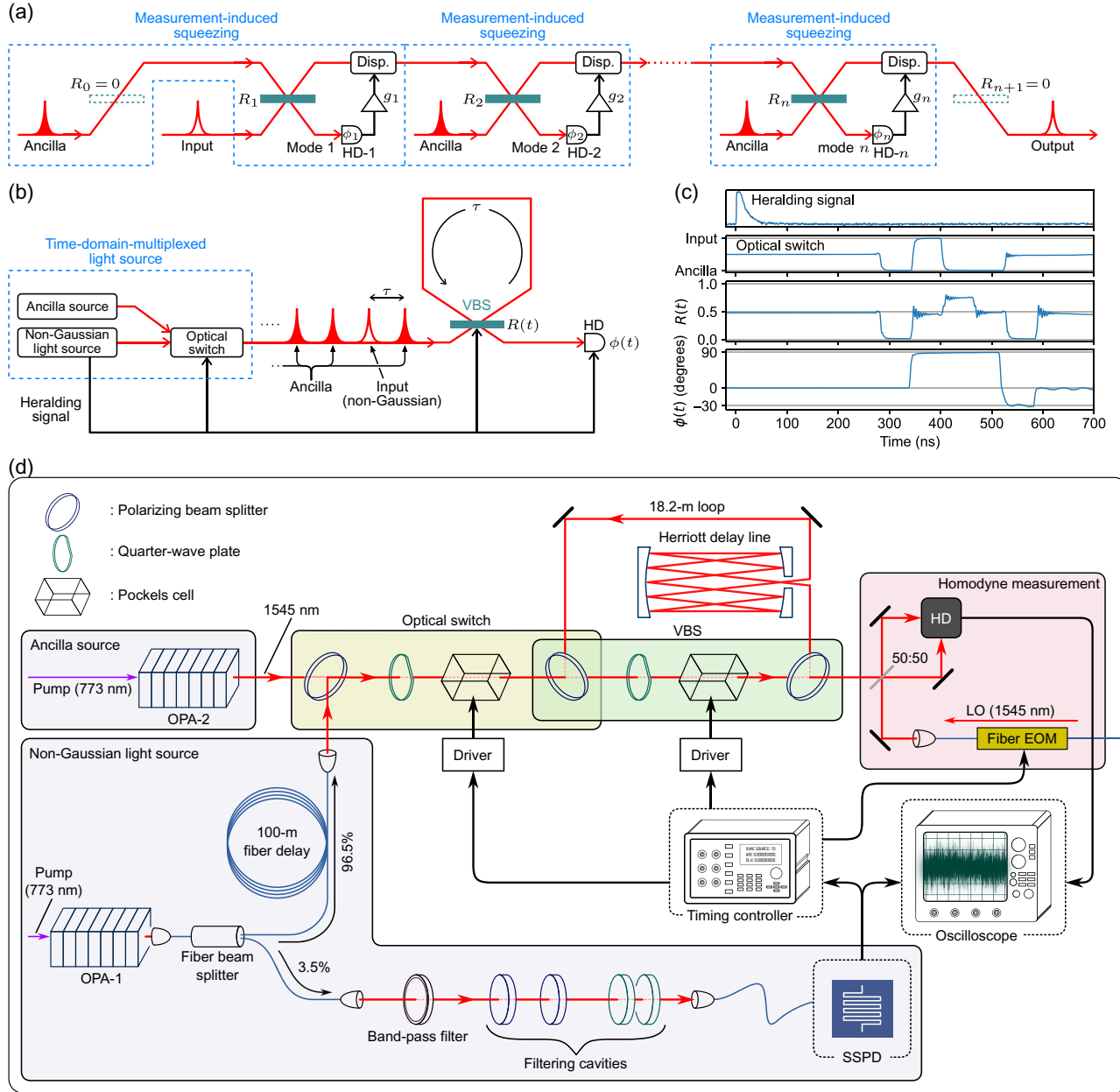


FIG. 1. A conceptual diagram of our platform and the experimental setup. (a) The typical implementation of a multistep measurement-induced squeezing based on path encoding. Disp., displacement operation; HD, homodyne detector. (b) A conceptual diagram of our platform. VBS, variable beam splitter. (c) The timing chart of each component for the third gate in Table II. Zero on the horizontal axis corresponds to the rising of the heralding signal. (d) The experimental setup. EOM, electro-optic modulator; LO, local oscillator; OPA, optical parametric amplifier; SSPD, superconducting nanostrip single-photon detector.

equivalent circuit of our platform] for the loop to take in the ancilla. To avoid the fragile non-Gaussian state suffering excess loss in the loop, the ancilla should be injected into the loop in advance for the first step out of the n steps. The VBS reflectivity then sequentially takes n values ($R_1, \dots, R_n$) to act as n different beam splitters. At the same time, the HD continuously measures one of the VBS outputs, with its basis $\phi(t)$ dynamically controlled for the desired gate. This continuous measurement can be

split into multiple measurements (HD-1, ..., HD- n) on different time bins. Finally, the VBS is again made transparent ($R_{n+1} = 0$) to let the output state exit the loop and the HD measures it for state characterization. The full outcome of the homodyne measurement is stored in a classical computer, on which all the displacement operations are performed by numerical postprocessing. Owing to the dynamical controllability and the loop-based structure enabling sequential processing, our platform can perform

TABLE I. The working conditions and fidelities for the single-step squeezing gates. “ $\hat{x}$ -sq.” (“ $\hat{p}$ -sq.”) in the “Ancilla” column refers to the $\hat{x}$ -squeezed vacuum ($\hat{p}$ -squeezed vacuum). R_1 , ϕ_1 , and g_1 represent the VBS reflectivity, the measurement angle, and the feedforward gain, respectively. These symbols can be found in Fig. 1(a), which depicts an equivalent circuit of our platform. The definitions of the fidelities, F_{real} , F_{ideal} , and $F_{\text{ideal}}^{(\text{theory})}$, are given in the text. The center values and the error ranges are estimated as averages and standard errors when splitting the whole 36 000 data into five independent subsets, respectively. The same is the case for Table II.

Gate		Working condition				Experimental fidelities		Theoretical fidelity
r	Squeezed quadrature	Ancilla	R_1	ϕ_1 (degrees)	g_1	F_{real}	F_{ideal}	$F_{\text{ideal}}^{(\text{theory})}$
0.26	$\hat{x}$	$\hat{x}$ -sq.	0.40	90	−0.82	0.976(3)	0.956(2)	0.961
0.46	$\hat{x}$	$\hat{x}$ -sq.	0.60	90	−1.22	0.956(4)	0.907(3)	0.907
−0.26	$\hat{p}$	$\hat{p}$ -sq.	0.40	0	−0.82	0.9783(9)	0.959(3)	0.961
−0.46	$\hat{p}$	$\hat{p}$ -sq.	0.60	0	−1.22	0.979(3)	0.904(4)	0.911

arbitrarily many steps of squeezing gates with a constant number of hardware components, offering scalability.

The physical setup is depicted in Fig. 1(d). Any non-Gaussian input state is acceptable as long as the heralding signal arises and we choose the Schrödinger cat state for the demonstration. For state preparation with the photon-subtraction scheme [20,21], a squeezed vacuum is produced by an optical parametric amplifier (OPA) and subsequently injected into a fiber beam splitter (FBS), one output of which leads to a superconducting nanostrip single-photon detector (SSPD). Photon detection on the SSPD generates the Schrödinger cat on the other output of the FBS, in a specific wave packet defined by filtering optics before the SSPD. The ancillary squeezed vacuum is continuously produced from another OPA. Except for these light sources, the whole optical system is constructed in free space, including the loop-based circuit, most of which is folded in the mechanically stable Herriott-type delay line [22]. The output state of the gate is fully characterized by the homodyne tomography [23] with neither postselection nor loss correction. The timing controller synchronizes the entire system with the photon-detection signal from the SSPD on a nanosecond time scale. The detail of the setup is described in Appendix A.

B. Single-step gate

For the certification of the programmable and deterministic gates of our platform, we implement single-step squeezing gates with various squeezing parameters. A squeezing gate is characterized by an operator $\hat{S}(r)$ that transforms quadrature operators as $\hat{S}^\dagger \hat{x} \hat{S}(r) = e^{-r} \hat{x}$ and $\hat{S}^\dagger \hat{p} \hat{S}(r) = e^r \hat{p}$. We call r a squeezing parameter and we choose $r = \pm 0.26, \pm 0.46$ for the demonstration. The working condition of the system for each gate is listed in Table I.

The experimental results are summarized in Fig. 2 and Table I. We have first measured the time-domain-multiplexed input Schrödinger cat and the ancillary squeezed vacuum by setting the VBS maximally

reflecting, before implementing the squeezing gates. The reconstructed Wigner functions of the input cats for the squeezing gates with positive and negative squeezing parameters are shown in Figs. 2(a) and 2(d), respectively, which are identical in principle. The squeezing levels of the ancillas have been measured to be approximately −4 dB (for more precise figures, see Appendix C). We then present the output Wigner functions of the squeezing gates in Figs. 2(b), 2(c), 2(e), and 2(f). Alongside each experimental outcome, the theoretical outcomes derived from the realistic or ideal model are shown with the corresponding label attached. The realistic model involves the finitely squeezed ancilla and the optical loss, whereas the ideal model has the infinitely squeezed ancilla and no optical loss (the detail is described in Appendix C). Graphically, all the experimental outcomes agree well with the realistic predictions.

For quantitative evaluation, we employ three different figures of merit: quadrature variances representing the degree of squeezing, Wigner negativity representing the quality of the gate, and fidelities comprehensively representing the circuit performance. We first present the quadrature variances of the output states in Figs. 2(g) and 2(h). They are normalized by those of the corresponding input state: in the specific form, $\Delta^2 q$ ($q = x, p$) is defined as $(\langle \hat{q}_{\text{out}}^2 \rangle - \langle \hat{q}_{\text{out}} \rangle^2) / (\langle \hat{q}_{\text{in}}^2 \rangle - \langle \hat{q}_{\text{in}} \rangle^2)$, where $\langle \cdots \rangle$ denotes the mean value and the subscripts “out” and “in” refer to the output and the input states, respectively. For four different target squeezing parameters, the experimentally obtained variances have little deviation from the realistic predictions, indicating that our platform performs squeezing gates properly, in a programmable manner. Next, we show the Wigner negativities of the output states, where the negativity is defined as the value of $W(0, 0)$ for the output Wigner function $W(x, p)$, below the square panels in Fig. 2. This value, in our case, represents the quality of the gate. This is because, in general, the negative $W(0, 0)$ signifies the nonclassicality of the state and is degraded through the gate process, whereas the ideal squeezing keeps it constant. All the experimental results show clear

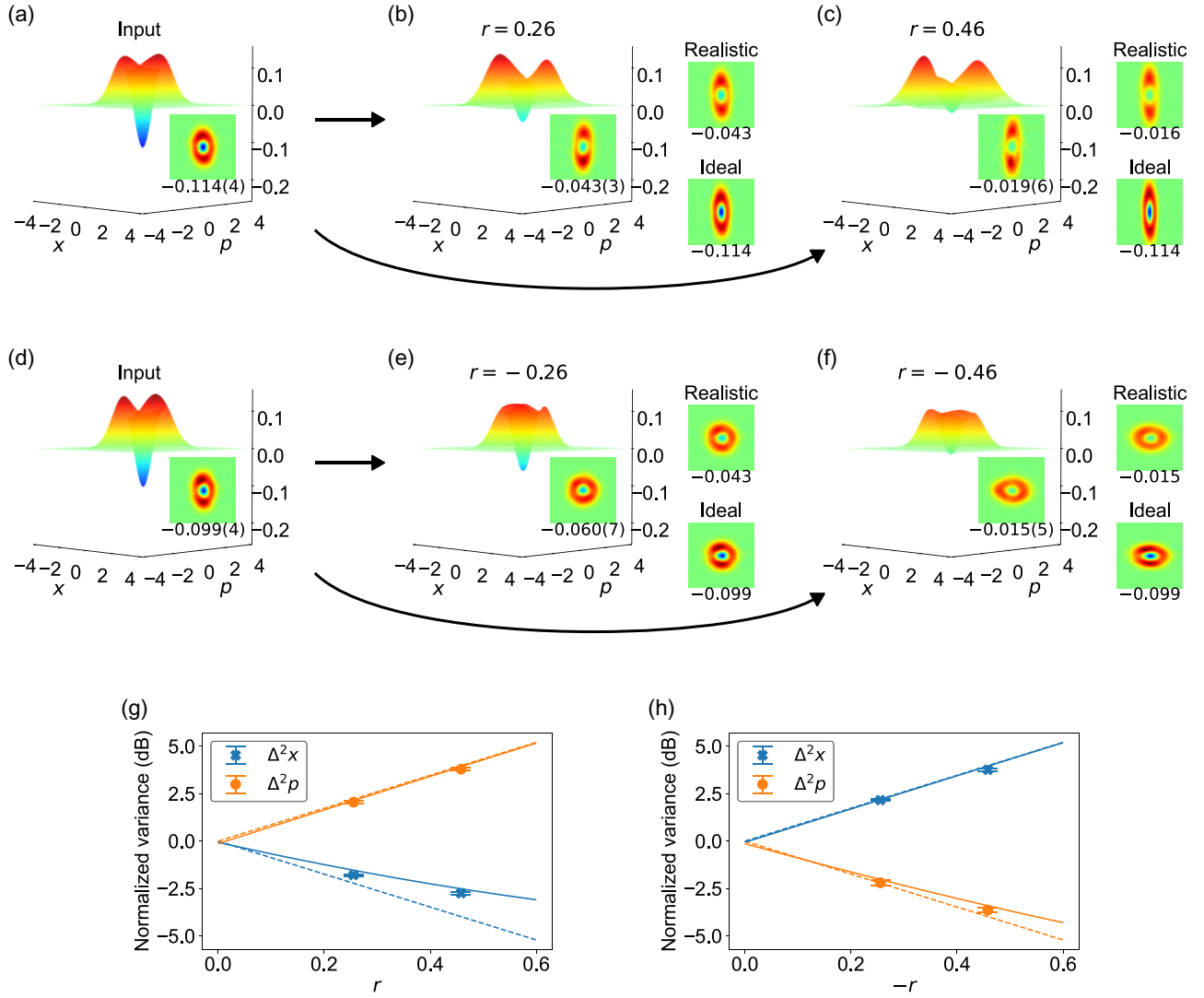


FIG. 2. State characterizations for the single-step squeezing gates. (a),(d) The Wigner function of the input Schrödinger cat state for the squeezing, with positive and negative squeezing parameters, respectively (in principle, they are identical). The convention $\hbar = 1$ is adopted. (b),(c),(e),(f) The Wigner function of the experimentally obtained output state and the theoretically derived states. The horizontal and the vertical directions of the inset square panel correspond to x and p , respectively. The figure below each inset square panel represents the value of $W(0,0)$. For the description of the theoretical predictions, see the text. (g),(h) The normalized variances as a function of a target squeezing parameter. The solid and the dashed lines are derived from the realistic and the ideal models, respectively. For the definitions of the normalized variances, see the text. For all items, the presented center values and the error ranges are estimated as averages and standard errors when splitting the whole 36 000 data into five independent subsets, respectively. The same is the case for Fig. 3.

negative values, revealing the high qualities of the gates of our platform. Finally, as comprehensive indicators, we present two fidelities, F_{real} and F_{ideal} , in Table I. F_{real} (F_{ideal}) is the fidelity [24] between the experimental output and the theoretical output derived from the realistic (ideal) model described above. We also include the theoretical fidelity $F_{\text{ideal}}^{(\text{theory})}$ in the same table for reference. This value, obtained numerically, represents the fidelity between the theoretical outputs derived from the realistic and the ideal models, predicting F_{ideal} . For all gates implemented, both F_{ideal} and F_{real} are reasonably high. Nonunity values of F_{real}

can be attributed to deviations between the simple theoretical model and the actual experimental situation, such as the spatial-mode mismatch at the VBS between the incoming and the circulated beams.

As a whole, Fig. 2 and Table I graphically and quantitatively demonstrate that our platform deterministically performs high-quality squeezing gates on the non-Gaussian cat state in a programmable manner. We have also evaluated each gate with a different input state, the vacuum field, and confirmed that the experimental output is again consistent with the theoretical prediction. The results

TABLE II. The working conditions and fidelities for the multistep squeezing gates. All items are listed in the same manner as in Table I. The symbols R_i , ϕ_i , and g_i ($i = 1, 2, 3$) can be found in Fig. 1(a).

Gate	Squeezed quadrature	Ancillas	Working condition		Experimental fidelities		Theoretical fidelity $F_{\text{ideal}}^{(\text{theory})}$
			$(R_1, R_2, \dots)$	$(\phi_1, \phi_2, \dots)$ (degrees)	$(g_1, g_2, \dots)$	F_{real} F_{ideal}	
(0.33)	$\hat{x}$	$\hat{x}$ -sq.	(0.48)	(90)	(-0.96)	0.968(2) 0.939(4)	0.944
$(0.33, 0.14)$	$\hat{x}$	$\hat{x}$ -sq.	$(0.48, 0.75)$	$(90, 90)$	$(-0.96, 0.58)$	0.958(4) 0.876(4)	0.905
$(0.33, 0.14, 0.37)$	$\hat{x}$	$\hat{x}$ -sq.	$(0.48, 0.75, 0.48)$	$(90, 90, 90)$	$(-0.96, 0.58, 1.04)$	0.930(3) 0.753(6)	0.793
(-0.33)	$\hat{p}$	$\hat{p}$ -sq.	(0.48)	(0)	(-0.96)	0.981(2) 0.948(5)	0.945
$(-0.33, -0.14)$	$\hat{p}$	$\hat{p}$ -sq.	$(0.48, 0.75)$	$(0, 0)$	$(-0.96, 0.58)$	0.978(3) 0.888(5)	0.910
$(-0.33, -0.14, -0.37)$	$\hat{p}$	$\hat{p}$ -sq.	$(0.48, 0.75, 0.48)$	$(0, 0, 0)$	$(-0.96, 0.58, 1.04)$	0.971(2) 0.770(7)	0.821

are shown in Table S1 and Fig. S1 of the Supplemental Material [25].

C. Multistep gate

For a demonstration of the ability of our platform to perform multistep gates, we implement up to three-step squeezings. The squeezing parameters are chosen to be $(r_1, r_2, r_3) = \pm(0.33, 0.14, 0.37)$, where r_i refers to the one for the i th step. The working conditions are summarized in Table II.

In Fig. 3 and Table II, we present the step-by-step results similarly to Fig. 2 and Table I. The output Wigner functions in Figs. 3(b)–3(d) and 3(f)–3(h), along with the quadrature variances in Figs. 3(i) and 3(j), show that, both graphically and quantitatively, the experimental results agree well with the theoretical predictions. Furthermore, the two fidelities, including F_{ideal} , in Table II remain at a relatively high level over the multiple steps. These results collectively demonstrate that our platform reliably performs multistep squeezings while programmably changing the squeezing parameter. The preserved Wigner negativities for up to two steps highlight the qualities of the gates. In Figs. 3(k) and 3(l), we show the evolution of the negativity as a function of the number of gates, along with theoretical lines derived from the realistic and ideal models, as well as models under improving system parameters. A detailed description of these additional models can be found in Appendix D 1, in the context of mitigating optical losses. It should be emphasized here that the available number of steps in our architecture is not restricted to three but, in principle, is infinite.

Similarly to the case of single-step gates, we have applied the same gates to the vacuum field and presented the results in Table S2 and Fig. S2 of the Supplemental Material [25], which again proves the validity of the gates.

III. DISCUSSION AND CONCLUSIONS

In conclusion, we have developed an optical quantum computing platform that can repeatedly and programmably perform a measurement-induced squeezing gate on a non-Gaussian state. We have verified the deterministic, programmable, and repeatable nature of the gates. Our platform can deal with not only the Schrödinger cat state but also any other non-Gaussian states as long as the heralding signal arises, including the cubic phase state [26] and the GKP state [27]. In addition to the squeezing gate on input non-Gaussian states, the current platform has the potential to acquire another functionality when a specific non-Gaussian state, instead of the cat, is injected as an ancilla. Notably, the ancillary cubic phase state, together with the already reported nonlinear feedforward system [28], enables the implementation of the cubic phase gate [10]. Further adding a phase shifter inside the loop evolves the platform to a single-mode universal one, since

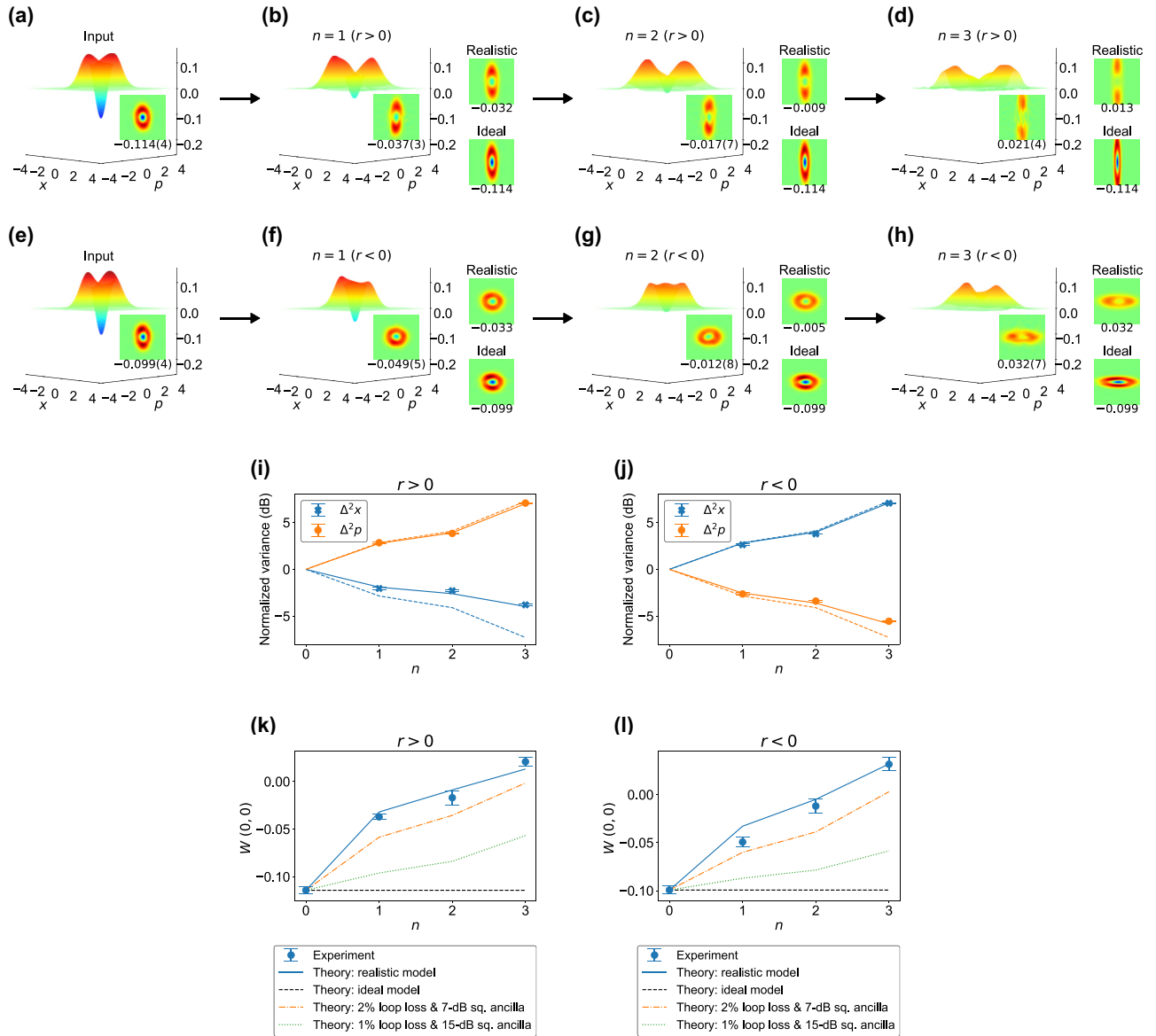


FIG. 3. State characterizations for the multistep squeezing gates. (a),(e) The Wigner function of the input Schrödinger cat state for the squeezing, with positive and negative squeezing parameters, respectively. These are the same figures as in Fig. 2. The convention $\hbar = 1$ is adopted. (b),(c),(d),(f),(g),(h) The Wigner function of the experimentally obtained output state and the theoretically derived states. n denotes the number of steps. (i),(j) The normalized variances as a function of the number of steps. (k),(l) The $W(0,0)$ values of the output $W(x,p)$ as a function of the number of steps. The solid blue and dashed black lines are drawn using the realistic and ideal models described in the text, respectively. The dash-dot orange line (dotted green line) is obtained with a model with 2% (1%) loop loss and an ancilla of 7-dB (15-dB) squeezed vacuum, where “sq.” in the legend denotes “squeezed.” Parts (a)–(j) are shown in the same manner as in Fig. 2, except that (i) and (j) are shown as functions of n .

an arbitrary single-mode gate is composed of displacement, squeezing, phase shift, and the cubic phase gate [29]. The main technical challenge, deterministic processing of the non-Gaussian state in the time domain, has been addressed in this work; the remaining task is the integration of these technologies. Furthermore, our platform can be extended for multimode processing [30] and eventually leads to fault-tolerant computing, which only necessitates Gaussian platforms once the proper non-Gaussian qubit

states are supplied [14,15]. Therefore, our work represents a fundamental milestone toward the realization of scalable, universal, and fault-tolerant quantum computing.

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T.Y. and S.T. designed the whole system. T.Y. and D.O. contributed to the development and debugging of the experimental setup, the data acquisition and analysis, and the numerical simulation. T.K. and T.U. supplied the OPA modules. S.M., H.T., M.Y., and F.C. supplied the SSPD. S.T. conceived, planned, and supervised the project. T.Y. wrote the manuscript, with assistance from D.O., S.T., and other coauthors.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

APPENDIX A: EXPERIMENTAL SETUP

The experimental setup, which is an extended version of the one found in our previous work [18], is illustrated in Fig. 1(d). For more information on the common part, see Ref. [18].

We use a continuous-wave laser with a wavelength of 1545 nm. Frequency-doubled beams pump two OPAs [31,32] to produce squeezed vacua, from one of which a photon is subtracted to obtain the non-Gaussian Schrödinger cat state. A band-pass filter and two filtering cavities before the SSPD define the temporal wave packet $f(t)$ of the cat state as

$$f(t) \propto (e^{\gamma_1(t-t_0)} - e^{\gamma_2(t-t_0)}) \Theta(t_0 - t), \quad (\text{A1})$$

where γ_1 and γ_2 are the bandwidths (half width at half maximum) of the cavities and t_0 is the photon-detection timing on the SSPD. Θ denotes the Heaviside step function. The configuration of the VBS is the same as those in our previous works [10,11,18,33]: a Pockels cell and a quarter-wave plate (QWP) sandwiched between two polarizing beam splitters. Another set of these optics forms the optical switch, where the QWP makes the two incident beams interfere when no voltage is applied to the Pockels cell. This is intended to make it possible to sense and stabilize the phase difference between them. The structure of the loop-shaped optical path and the configuration of the homodyne-measurement part are the same as those in Ref. [18].

A typical timing chart of the dynamical components is shown in Fig. 1(c). All the components have a rise and fall time of approximately 10 ns, in some cases followed by transient oscillations of approximately 20 ns around the target steady values. The loop-shaped circuit is 18.2 m long

and has a corresponding round-trip time (τ) of 60.8 ns, which determines the interval between the neighboring optical pulses to be 60.8 ns. Consequently, each component is controlled every 60.8 ns.

The phase conditions of the entire optical system are stabilized using auxiliary classical beams. Injecting the classical beams enables us to obtain the error signals and stabilize the system by using the feedback controls. During the main measurement, the auxiliary beams are chopped and the whole system is kept in the stabilized state. The phase of the ancillary squeezed vacuum with respect to that of the input cat state is changed by inverting the corresponding feedback polarity.

APPENDIX B: DATA ANALYSIS

We experimentally determine the three parameters γ_1 , γ_2 , and t_0 in Eq. (A1), representing the mode function of the cat state. As described in Sec. IIB, we acquire a time series of homodyne data containing the generated cat state before performing the squeezing gates. From the time series, the three parameters are optimized to maximize the quadrature variance, based on the core idea in Ref. [34]. The determined values for γ_1 and γ_2 are $2\pi \times 29.8$ MHz and $2\pi \times 95.6$ MHz, respectively.

Each temporal mode in the time-domain-multiplexed processing is defined as $f(t - (i-1)\tau)$ ($i = 1, 2, \dots$), where $f(t)$ denotes the function of Eq. (A1) with the above-determined parameters. The mode index i corresponds to that shown in Fig. 1(a). The raw voltage data from the oscilloscope are converted to the quadrature amplitude by normalizing them by those of the vacuum field obtained immediately before the main measurement.

To fully characterize the gate outcome, we repeat the gate and the measurement for the measurement basis from 90° to -75° at 15° intervals. 3000 independent data are acquired for each basis. The Wigner function is reconstructed from the 3000×12 quadratures by the maximum-likelihood method [23].

APPENDIX C: THEORETICAL MODELS

To derive the theoretical-output Wigner functions shown in Figs. 2 and 3, we employ two different models: the realistic model with a finite squeezing level of the ancillas and optical loss, and the ideal model with an infinite squeezing level and no optical loss.

The realistic model emulates the experiment as follows: (1) the input cat state is coupled to the finitely squeezed vacuum at the VBS; (2) one output of the VBS is immediately measured at the HD with unity efficiency; (3) the other output is subjected to the propagation efficiency of 0.96 in the loop and followed by the measurement at the HD with unity efficiency. For a multistep gate, the whole process is repeated with the input state replaced

with the previous output. For the emulation of squeezing gates with positive (negative) squeezing parameters, we use the experimentally obtained Wigner function in Fig. 2(a) [Fig. 2(d)] for the initial cat state and the ancillary squeezed vacua are numerically reproduced by applying an optical loss of 22 % (27 %) to the pure squeezed vacua with a squeezing level of -6.8 dB (-7.0 dB). The amount of loss and the pure squeezing level have been estimated from the measurement data simultaneously obtained with those of the input cat. Note that in this model, the first-step squeezing introduces another 4 % loss on the ancilla, since its implementation obliges the state to pass through the loop to interfere with the input. The effect of nonunity efficiency of the homodyne measurement is assumed to be reflected on the measured input cat and ancilla. The model does not include other experimental imperfections such as the nonideal phase difference between the cat and the ancilla and the spatial-mode mismatch at the VBS between the incoming and the circulated beams.

The ideal model has infinitely squeezed ancilla and no optical loss, deriving the output Wigner function by simply rescaling the experimentally obtained input Wigner function by e^{-r} and e^r along the x and p axis, respectively, with the target squeezing parameter r .

We also use these models to calculate the output-quadrature variances from the experimentally measured input variances.

APPENDIX D: CURRENT LIMITATIONS AND FUTURE PROSPECTS

1. Optical loss

The intrinsic optical losses in the current system are as follows. Based on a preliminary measurement of the squeezing spectrum, we estimate the optical loss between OPA-1 and the HD in Fig. 1(d) to be approximately 30 %, comprising approximately 12 % internal loss in the OPA module [31], approximately 8 % in-fiber propagation loss, approximately 3 % free-space propagation loss, and other unidentified losses. Similarly, the loss between OPA-2 and the HD is estimated to be approximately 20 %, including approximately 12 % (or less, due to the absence of fiber coupling at the output) internal loss in the module and approximately 7 % free-space propagation loss. The round-trip loss in the loop is measured at 4 % and the readout loss at the HD is approximately 3 %.

During the squeezing-gate process, the input cat state suffers propagation loss in the loop, which degrades the gate quality, as indicated by the reduction in the negativity of the Wigner function and the fidelity F_{ideal} . In addition, the losses on the ancillary squeezed vacuum constrain its squeezing level to approximately -4 dB, introducing excess noise with the corresponding variance into one of the quadratures of the gate output state [see Eq. (2) in Ref.

[19]]. This effect also contributes to the degradation of the gate performance.

Let us now discuss the strategy to mitigate these losses in future experiments. For the light source, the loss of the OPA module could be reduced by improving the fabrication technique of the waveguide nonlinear crystals or by eliminating extra optical components within the module [31,32]. Regarding the measurement system, the use of an OPA followed by a broadband HD could enable low-loss and high-bandwidth measurements [35]. Such broadband measurements would reduce the optical pulse length. This reduction would eventually shorten the loop and consequently reduce the optical loss in it, since the minimal loop length is limited by the optical pulse length as well as the electric switching time of the VBS.

In Figs. 3(k) and 3(l), we present two theoretical lines illustrating potential improvements in gate quality achieved by reducing the loop loss and enhancing the squeezing level of the ancillary squeezed vacuum. The dash-dot orange line reflects the effect of approximately halving both the loop loss and the ancilla noise from their current amounts described above. The dotted green line depicts an outcome under a quarter of the current loop loss and 15-dB squeezed ancilla noise, which is the highest recorded squeezing level to date [36]. In this latter scenario, if the loss during non-Gaussian state preparation is also reduced to 13 %, as achieved in a similar state-generation experiment yielding the minimum Wigner negativity [37], a squeezing gate with a squeezing parameter of $r = 0.1, 0.2, 0.3, 0.4$ could be iterated up to 18, 10, 7, and 5 times, respectively, while preserving the negative region of the output Wigner function. This level of scalability is realistically anticipated in our system.

2. Clock frequency

Here, we discuss the practical temporal limitations of our architecture. The current system has an optical pulse interval of 60.8 ns or a clock frequency of 16 MHz. This interval is limited by the sum of the optical pulse length of approximately 20 ns and response time of the dynamical components, of approximately 30 ns. The latter is currently limited by the response of the VBS, which includes a rise and fall time of approximately 10 ns and a subsequent oscillation lasting approximately 20 ns [see Fig. 1(c)] and demands a buffer time of a few nanoseconds to compensate for the electric jitter.

The pulse length is currently restricted by the bandwidth of the HD, approximately 200 MHz. This length can be reduced by adopting highly broadband measurements leveraging an OPA, enabling a bandwidth exceeding 10 GHz [35]. The response rate of the VBS can also be improved to several tens of gigahertz by using a waveguide modulator in place of the current bulk Pockels cell.

Therefore, with these enhancements, a clock frequency of 10 GHz or higher is feasible with our architecture.

3. Scalability with probabilistically generated non-Gaussian states

Even if a platform can process non-Gaussian states in a scalable manner, the probabilistic nature of non-Gaussian state generation often limits the overall efficiency of computing protocols involving these states, thereby hindering scalability. However, a methodology has been reported that enables scalable computing with probabilistically generated non-Gaussian states, as long as the generation rate exceeds a certain threshold [38]. The generation rate can be enhanced by broadening the bandwidth of the state, employing a more sophisticated generation scheme [39], or multiplexing the non-Gaussian light source [40,41]. Therefore, our platform has the potential to eventually achieve true scalability.

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- [1] C. D. Bruzewicz, J. Chiaverini, R. McConnell, and J. M. Sage, Trapped-ion quantum computing: Progress and challenges, *Appl. Phys. Rev.* **6**, 021314 (2019).
 - [2] Google Quantum AI, *et al.*, Suppressing quantum errors by scaling a surface code logical qubit, *Nature* **614**, 676 (2023).
 - [3] K. Wintersperger, F. Dommert, T. Ehmer, A. Houshanov, J. Klepsch, W. Maurer, G. Reuber, T. Strohm, M. Yin, and S. Luber, Neutral atom quantum computing hardware: Performance and end-user perspective, *EPJ Quantum Technol.* **10**, 1 (2023).
 - [4] S. Takeda and A. Furusawa, Toward large-scale fault-tolerant universal photonic quantum computing, *APL Photonics* **4**, 060902 (2019).
 - [5] J. Yoshikawa, T. Hayashi, T. Akiyama, N. Takei, A. Huck, U. L. Andersen, and A. Furusawa, Demonstration of deterministic and high fidelity squeezing of quantum information, *Phys. Rev. A* **76**, 060301 (2007).
 - [6] R. Ukai, S. Yokoyama, J. Yoshikawa, P. van Loock, and A. Furusawa, Demonstration of a controlled-phase gate for continuous-variable one-way quantum computation, *Phys. Rev. Lett.* **107**, 250501 (2011).
 - [7] X. Su, S. Hao, X. Deng, L. Ma, M. Wang, X. Jia, C. Xie, and K. Peng, Gate sequence for continuous variable one-way quantum computation, *Nat. Commun.* **4**, 2828 (2013).
 - [8] W. Asavanant, B. Charoensombutamon, S. Yokoyama, T. Ebihara, T. Nakamura, R. N. Alexander, M. Endo, J. Yoshikawa, N. C. Menicucci, H. Yonezawa, and A. Furusawa, Time-domain-multiplexed measurement-based quantum operations with 25-MHz clock frequency, *Phys. Rev. Appl.* **16**, 034005 (2021).
 - [9] M. V. Larsen, X. Guo, C. R. Breum, J. S. Neergaard-Nielsen, and U. L. Andersen, Deterministic multi-mode gates on a scalable photonic quantum computing platform, *Nat. Phys.* **17**, 1018 (2021).
 - [10] Y. Enomoto, K. Yonezu, Y. Mitsuhashi, K. Takase, and S. Takeda, Programmable and sequential Gaussian gates in a loop-based single-mode photonic quantum processor, *Sci. Adv.* **7**, eabj6624 (2021).
 - [11] K. Yonezu, Y. Enomoto, T. Yoshida, and S. Takeda, Time-domain universal linear-optical operations for universal quantum information processing, *Phys. Rev. Lett.* **131**, 040601 (2023).
 - [12] S. D. Bartlett, B. C. Sanders, S. L. Braunstein, and K. Nemoto, Efficient classical simulation of continuous variable quantum information processes, *Phys. Rev. Lett.* **88**, 097904 (2002).
 - [13] K. Miyata, H. Ogawa, P. Marek, R. Filip, H. Yonezawa, J. Yoshikawa, and A. Furusawa, Implementation of a quantum cubic gate by an adaptive non-Gaussian measurement, *Phys. Rev. A* **93**, 022301 (2016).
 - [14] B. Q. Baragiola, G. Pantaleoni, R. N. Alexander, A. Karanjai, and N. C. Menicucci, All-Gaussian universality and fault tolerance with the Gottesman-Kitaev-Preskill code, *Phys. Rev. Lett.* **123**, 200502 (2019).
 - [15] H. Yamasaki, T. Matsuura, and M. Koashi, Cost-reduced all-Gaussian universality with the Gottesman-Kitaev-Preskill code: Resource-theoretic approach to cost analysis, *Phys. Rev. Res.* **2**, 023270 (2020).
 - [16] Y. Miwa, J. Yoshikawa, N. Iwata, M. Endo, P. Marek, R. Filip, P. van Loock, and A. Furusawa, Exploring a new regime for processing optical qubits: Squeezing and unsqueezing single photons, *Phys. Rev. Lett.* **113**, 013601 (2014).
 - [17] M. Wang, M. Zhang, Z. Qin, Q. Zhang, L. Zeng, X. Su, C. Xie, and K. Peng, Experimental preparation and manipulation of squeezed cat states via an all-optical in-line squeezer, *Laser Photonics Rev.* **16**, 2200336 (2022).
 - [18] D. Okuno, T. Yoshida, R. Arita, T. Kashiwazaki, T. Umeki, S. Miki, H. Terai, M. Yabuno, F. China, and S. Takeda, Time-domain programmable beam-splitter operations for an optical phase-sensitive non-Gaussian state, *Phys. Rev. A* **110**, 023706 (2024).
 - [19] R. Filip, P. Marek, and U. L. Andersen, Measurement-induced continuous-variable quantum interactions, *Phys. Rev. A* **71**, 042308 (2005).
 - [20] J. S. Neergaard-Nielsen, B. M. Nielsen, C. Hettich, K. Mølmer, and E. S. Polzik, Generation of a superposition of odd photon number states for quantum information networks, *Phys. Rev. Lett.* **97**, 083604 (2006).
 - [21] K. Wakui, H. Takahashi, A. Furusawa, and M. Sasaki, Photon subtracted squeezed states generated with periodically poled KTiOPO₄, *Opt. Express* **15**, 3568 (2007).
 - [22] D. Herriott, H. Kogelnik, and R. Kompfner, Off-axis paths in spherical mirror interferometers, *Appl. Opt.* **3**, 523 (1964).
 - [23] A. I. Lvovsky, Iterative maximum-likelihood reconstruction in quantum homodyne tomography, *J. Optics B: Quantum Semiclassical Opt.* **6**, S556 (2004).
 - [24] R. Jozsa, Fidelity for mixed quantum states, *J. Mod. Opt.* **41**, 2315 (1994).
 - [25] See the Supplemental Material at <http://link.aps.org/supplemental/10.1103/PRXQuantum.6.010311> for descriptions of the reliable evaluation of the squeezing gates on the vacuum field. The Supplemental Material also includes Ref. [42].

- [26] M. Yukawa, K. Miyata, H. Yonezawa, P. Marek, R. Filip, and A. Furusawa, Emulating quantum cubic nonlinearity, *Phys. Rev. A* **88**, 053816 (2013).
- [27] S. Konno, W. Asavanant, F. Hanamura, H. Nagayoshi, K. Fukui, A. Sakaguchi, R. Ide, F. China, M. Yabuno, S. Miki, *et al.*, Logical states for fault-tolerant quantum computation with propagating light, *Science* **383**, 289 (2024).
- [28] A. Sakaguchi, S. Konno, F. Hanamura, W. Asavanant, K. Takase, H. Ogawa, P. Marek, R. Filip, J. Yoshikawa, E. Huntington, and H. Yonezawa, Nonlinear feedforward enabling quantum computation, *Nat. Commun.* **14**, 3817 (2023).
- [29] S. Lloyd and S. L. Braunstein, Quantum computation over continuous variables, *Phys. Rev. Lett.* **82**, 1784 (1999).
- [30] S. Takeda and A. Furusawa, Universal quantum computing with measurement-induced continuous-variable gate sequence in a loop-based architecture, *Phys. Rev. Lett.* **119**, 120504 (2017).
- [31] T. Kashiwazaki, T. Yamashima, N. Takanashi, A. Inoue, T. Umeki, and A. Furusawa, Fabrication of low-loss quasi-single-mode PPLN waveguide and its application to a modularized broadband high-level squeezer, *Appl. Phys. Lett.* **119**, 251104 (2021).
- [32] T. Kashiwazaki, T. Yamashima, K. Enbutsu, T. Kazama, A. Inoue, K. Fukui, M. Endo, T. Umeki, and A. Furusawa, Over-8-dB squeezed light generation by a broadband waveguide optical parametric amplifier toward fault-tolerant ultra-fast quantum computers, *Appl. Phys. Lett.* **122**, 234003 (2023).
- [33] S. Takeda, K. Takase, and A. Furusawa, On-demand photonic entanglement synthesizer, *Sci. Adv.* **5**, eaaw4530 (2019).
- [34] O. Morin, C. Fabre, and J. Laurat, Experimentally accessing the optimal temporal mode of traveling quantum light states, *Phys. Rev. Lett.* **111**, 213602 (2013).
- [35] A. Inoue, T. Kashiwazaki, T. Yamashima, N. Takanashi, T. Kazama, K. Enbutsu, K. Watanabe, T. Umeki, M. Endo, and A. Furusawa, Toward a multi-core ultra-fast optical quantum processor: 43-GHz bandwidth real-time amplitude measurement of 5-dB squeezed light using modularized optical parametric amplifier with 5G technology, *Appl. Phys. Lett.* **122**, 104001 (2023).
- [36] H. Vahlbruch, M. Mehmet, K. Danzmann, and R. Schnabel, Detection of 15 dB squeezed states of light and their application for the absolute calibration of photoelectric quantum efficiency, *Phys. Rev. Lett.* **117**, 110801 (2016).
- [37] A. Kawasaki, K. Takase, T. Nomura, S. Miki, H. Terai, M. Yabuno, F. China, W. Asavanant, M. Endo, J. Yoshikawa, and A. Furusawa, Generation of highly pure single-photon state at telecommunication wavelength, *Opt. Express* **30**, 24831 (2022).
- [38] J. E. Bourassa, R. N. Alexander, M. Vasmer, A. Patil, I. Tzitrin, T. Matsuura, D. Su, B. Q. Baragiola, S. Guha, G. Dauphinais, K. K. Sabapathy, N. C. Menicucci, and I. Dhand, Blueprint for a scalable photonic fault-tolerant quantum computer, *Quantum* **5**, 392 (2021).
- [39] H. Tomoda, A. Machinaga, K. Takase, J. Harada, T. Kashiwazaki, T. Umeki, S. Miki, F. China, M. Yabuno, H. Terai, D. Okuno, and S. Takeda, Boosting the generation rate of squeezed single-photon states by generalized photon subtraction, *Phys. Rev. A* **110**, 033717 (2024).
- [40] M. J. Collins, C. Xiong, I. H. Rey, T. D. Vo, J. He, S. Shahnia, C. Reardon, T. F. Krauss, M. J. Steel, A. S. Clark, and B. J. Eggleton, Integrated spatial multiplexing of heralded single-photon sources, *Nat. Commun.* **4**, 2582 (2013).
- [41] F. Kaneda, B. G. Christensen, J. J. Wong, H. S. Park, K. T. McCusker, and P. G. Kwiat, Time-multiplexed heralded single-photon source, *Optica* **2**, 1010 (2015).
- [42] G. Mauro D'Ariano, M. G. A. Paris, and M. F. Sacchi, Parameter estimation in quantum optics, *Phys. Rev. A* **62**, 023815 (2000).