

Three-Qubit Encoding in Ytterbium-171 Atoms for Simulating 1+1D Quantum Chromodynamics

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Simulating nuclear matter described by quantum chromodynamics using quantum computers is notoriously inefficient because of the assortment of quark degrees of freedom such as matter/antimatter, flavor, color, and spin. Here, we propose to address this resource efficiency challenge by encoding three qubits within individual ytterbium-171 atoms of a neutral atom quantum processor. The three qubits are encoded in three distinct sectors: an electronic “clock” transition, the spin-1/2 nucleus, and the lowest two motional states in one radial direction of the harmonic trapping potential. We develop a family of composite sideband pulses and demonstrate a universal gate set and readout protocol for this three-qubit system. We then apply it to single-flavor quantum chromodynamics in 1+1D axial gauge for which the three qubits directly represent the occupancy of quarks in the three colors. We show that two atoms are sufficient to simulate both vacuum persistence oscillations and a screened hadron-number transition. We consider resource requirements and connections to error detection/correction. Our work is a step toward resource-efficient digital simulation of nuclear matter and opens new opportunities for versatile qubit encoding in neutral atom quantum processors.

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I. INTRODUCTION

In the era of noisy intermediate-scale quantum (NISQ) devices [1], resource efficiency is crucial for utilizing quantum hardware in the short term. As many quantum hardware platforms begin to hit scaling barriers, it is increasingly important to optimize the use of resources such as qubit count, connectivity, and coherence time. A paradigmatic example of this need is the digital quantum simulation of nuclear physics phenomena [2–5]. Simulations of quark matter must contend with several degrees of freedom including flavor, color, matter/antimatter, and spin; and they must also address the underlying fermionic statistics, which require the use of notoriously inefficient fermion-to-qubit mapping schemes such as the Jordan-Wigner transformation [6–8]. Additionally, gauge bosons that mediate interactions between quarks add the challenge of encoding bosonic state spaces that could be large [9].

A natural solution to this challenge is to use larger code spaces in the underlying quantum information carriers of a given hardware platform [4,5]. Specifically, instead of quantum bits (qubits) with a $d = 2$ code space, larger- d quantum spaces called qudits could be employed to pack more information into the same physical entity. In superconducting circuits, this could be accomplished with bosonic encodings [9–11] or with higher-lying states in anharmonic systems like transmons [12,13]. In trapped ions and neutral atoms, the most natural approach to increasing the code space is to use more internal Zeeman states. Indeed, large code spaces are actively being pursued with nuclear and electronic spins, often combined with long-lived metastable states [14–21]. Motional modes of the atoms’ harmonic trapping potential offer another option [9,22–28]. Code spaces of $d = 3$ and $d = 5$ have recently been exploited to efficiently represent gauge bosons in quantum electrodynamics [29].

However, a major challenge for qudits is the complexity of their universal gate sets, the development of protocols for benchmarking and tomography, and the transpilation of algorithms designed for qubit-based hardware. Although gauge bosons may naturally be mapped to a qudit system [29], quark matter, for instance, shares certain behavior with qubits since fermionic statistics forces binary occupancy. A partial solution is a hybrid qubit-qudit processor in which matter particles are represented with qubits while gauge fields are represented with qudits [29]. However, in

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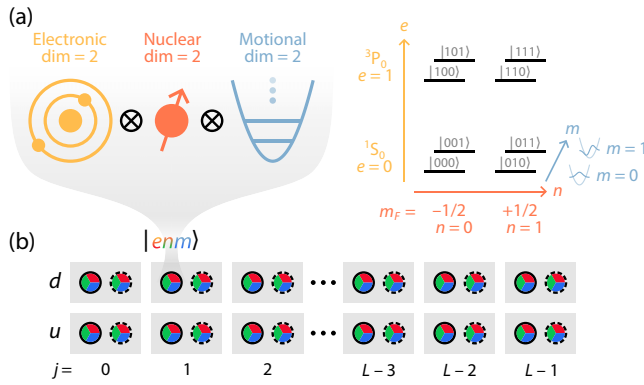


FIG. 1. Overview of the platform. (a) We encode individual qubits in each of the electronic 1S_0 – 3P_0 (yellow, “ e ”), nuclear Zeeman (orange, “ n ”), and two lowest-energy states of the motional (blue, “ m ”) degrees of freedom. The tensor product of these three two-dimensional Hilbert spaces forms the computational “quoct” space. (b) We consider a chain of quocts to encode the possible states of a single flavor of quark for the purpose of a lattice quantum chromodynamics simulation in 1+1D. Here we show the arrangement for a simulation of two quark flavors up (u) and down (d) with two parallel chains. At each site, two adjacent quocts are used to represent the presence or absence of a quark (solid circle) or antiquark (dashed circle) in a simulated space of three colors. The e , n , and m qubits of each quoct encode the presence of a red, green, and blue (anti-)quark at each lattice site. Additional flavors of quarks may be added as additional parallel chains.

general, it remains difficult to fully exploit the potential advantage of qudits and to demonstrate their viability as an alternative to qubit-based processors.

Here, we propose, via results derived from classical simulation, an approach that offers the advantages of both qubits and qudits. Building on previous work [30] with a $d = 4$ “ququart,” we focus on a $d = 8$ “quoct” encoded in neutral ytterbium-171 atoms in optical tweezers, “qubitizing” this space into three distinct qubits that arise from distinct degrees of freedom in the atom [30,31]. Specifically, we employ an electronic qubit, $|e\rangle$, based on the optical “clock” transition; a nuclear-spin qubit, $|n\rangle$, based on the spin-1/2 nucleus; and a motional qubit, $|m\rangle$, based on the lowest two harmonic oscillator states of the atom along a radial direction of the tweezer trap (see Fig. 1). We develop a family of composite sideband pulses that isolate the lowest two motional states. We present a qubit-based universal gate set for intraquoct and interquoct operations that include high-fidelity intraquoct CZ, SWAP, and CCZ gates and interquoct CCZ gates.

Further, we showcase the versatility of our three-qubit encoding platform and its utility by developing circuits to simulate single-flavor quantum chromodynamics (QCD) in 1+1D. By choosing the axial gauge, we obviate the need for gauge bosons in one spatial dimension, enabling a

direct mapping of the occupancy of the three quark colors onto our three qubits (see Fig. 1). We use a row of atoms to represent quarks and another row to represent antiquarks, and we construct Trotterized circuits that, with only a single site (two atoms), faithfully simulate vacuum persistence and a screened hadron-number transition. Our work lays the foundation for other multiqubit encodings and circuit compilations, and it takes a step toward unlocking the potential of quantum hardware for resource-efficient simulation of large-scale nuclear matter.

II. THE QUOCT ARCHITECTURE

Our three-qubit quoct architecture with ^{171}Yb was inspired by, and is an extension of, our recently proposed two-qubit “ququart” architecture based on the electronic optical “clock” transition and the nuclear-spin-1/2 degree of freedom [30]. One of the main new ingredients here is the addition of the third qubit, which is encoded within the harmonic oscillator states of the atom’s motion within the optical trapping potential. Given the recent progress on cooling optically trapped atoms to their motional ground state with high probability [24,32–34], encoding quantum information in motional degrees of freedom has become an active new field for neutral atom arrays [9,24–28]. The motional degree of freedom enjoys long coherence times since it is decoupled from nearly all electromagnetic noise. Indeed a coherence time of $T_2^* \approx 100$ ms has been observed for atoms in tweezers [24], which is primarily limited by intensity noise that can be straightforwardly removed.

We choose to employ a motional qubit in this work instead of, e.g., using a larger nuclear-spin manifold because we want three pseudo-independent degrees of freedom in which to encode our three qubits. Hence, the motional sector offers an opportunity to add a qubit without sacrificing the established capabilities with our two-qubit system of electronic and nuclear degrees of freedom [30], and we note that it may be possible to add two more motional qubits by leveraging motion in the other two dimensions within the three-dimensional optical potential. The three motional degrees of freedom are sufficiently separate due to the intentional or accidental nonisotropic nature of the optical trap. The mode is selected by momentum conservation; since we couple to the motional states via the momentum of the photon that drives the electronic transition, the alignment of the laser beam with respect to the optical trap determines the projection onto the motional eigenbasis. We assume that the laser couples to a single radial motional mode [24].

For the remainder of Sec. II, we will denote three-qubit quoct states in the form $|enm\rangle$ and two-qubit ququart states in the form $|en\rangle$ (where $e, n, m \in \{0, 1\}$). Further, bipartite states and gates with an explicit tensor product will refer

to the quoct or ququart states (or gates thereon) of two adjacent atoms.

A. Intraquoct gates and motion-selective operations

As an extension of the ququart architecture explored in previous work [30], the quoct space inherits a significant portion of its supported operations from that architecture with identical implementations. All operations involving the “clock” transition (e qubit) and the nuclear-spin-1/2 degree of freedom (n qubit) are subsumed under the quoct gate set. For the purposes of this work (see Fig. 9), we specifically inherit the following operations: phase rotation gates to arbitrary angles on the e and n qubits ($Z_e(\alpha)$, $Z_n(\alpha)$) via spin-selective light shifts applied to pairs of ququart states; bit-flip and Hadamard gates on the e qubit (X_e , H_e) via π -polarized pulses on the clock transition; bit-flip and Hadamard gates on the n qubit (X_n , H_n) via simultaneous stimulated Raman pulses within the $e = 0$ and $e = 1$ subspaces mediated by states in the 3P_1 and 3D_1 manifolds, respectively; e - n swap gate ($\text{SWAP}_{e,n}$) via a σ^+ -polarized pulse on the $|01\rangle \leftrightarrow |10\rangle$ clock transition. Here, we also introduce a new ququart operation, which is a π -pulse on the e qubit conditioned on $n = 1$ (CNOT_e^n). This, as well as X_e , X_n , and $\text{SWAP}_{e,n}$ are based on a CORPSE (Compensation for Off-Resonance with a Pulse SEquence)-based [35] motion-preserving pulse (MPP) [36] on their respective transitions, while H_e and H_n use the detuned pulses explored in Ref. [30]. Notably, MPPs only implement a π rotation; for rotations through general angles α about the X -axis of a given transition ($X_{\{e,n\}}(\alpha)$), we resort to conjugation of the appropriate $Z_{\{e,n\}}(\alpha)$ gate with $H_{\{e,n\}}$.

As with ququarts, operations within the quoct space are dependent on inherited compensatory light shifts required to maintain suitable control over the constituent atomic states. Namely, ququart operations require a passive, “background” light shift applied to one of the ququart states to equalize the nuclear Zeeman splittings in the $e = 0$ and $e = 1$ subspaces in order to allow the n -preserving clock transitions to be driven simultaneously with a monochromatic pulse. Secondly, the CNOT_n^e , CNOT_e^n , and $\text{SWAP}_{e,n}$ gates both require compensatory light shifts to correct phases accrued by pairs of ququart states during each operation. In the first case, phase is accrued by the $|10\rangle$ and $|11\rangle$ states due to the operation itself; in the latter two cases, phases are accrued by $|00\rangle$ and $|10\rangle$, and $|00\rangle$ and $|11\rangle$ due to the proximity of the clock transition’s wavelength to those of other transitions accessible from the $e = 0$ subspace, primarily to 3P_1 states. In all cases, accrued phases are corrected with additional light shifts applied to individual ququart states using various other nearby transitions, e.g., those from $e = 0$ states to 3P_1 or from $e = 1$ states to 3D_1 [37].

When extending the computational space to include motional modes, care must be taken to ensure that no ququart operations disturb information encoded in the additional degree of freedom. As specified above, we restrict to only the bottom two motional states and, by construction, MPP-based operations preserve the motional ground state with high fidelity. We find through simulation that this naturally extends to superpositions of the targeted motional states as well, up to a small relative phase ($\lesssim 1^\circ$) between them and a global phase on all states undergoing dynamics due to the MPP. Up to this global phase, we find that MPPs operate on general quoct states with fidelity $\gtrsim 0.999$. Similarly, the Hadamard gate implementation designed in Ref. [30] preserves superpositions of motional states up to a $\approx \pi$ relative phase within the motional sector with fidelity $\gtrsim 0.999$. We examine MPPs and the Hadamard gates in more detail in Appendix A. Any erroneous phases can be corrected after the MPP operation using either selectively applied light shifts within the ququart space or a Z_m gate on the motional qubit (described below). On the other hand, the inherited phase operations (Z_e and Z_n , as well as state-selective light shifts) can all be made entirely agnostic to motion by using light fields that are locally flat in intensity over the length scale of the targeted motional states.

Now we turn to operations that involve the motional states within the quoct space. A key observation to make when working in this space is that although motional states constitute robust quantum memories due to their insensitivity to their environments, transitions between motional states can only be driven indirectly, e.g., via some atomic transition through which a photon can change the atom’s momentum. To achieve control over the population and phase of m -qubit states, we work with the clock transition in the resolved-sideband regime where the Rabi frequency Ω is small compared to the frequency spacing between motional states ω . This allows transitions between specific adjacent pairs of motional states to be individually addressed at the cost of extending gate times due to the reduced Rabi frequency. Here, we fix $\omega = 2\pi \times 100$ kHz as a reasonably large trap frequency attainable for scalable optical tweezer arrays.

The first gate operation we implement is a phase rotation to arbitrary angles on the m qubit ($Z_m(\alpha)$). Operating in the frame corotating with the ground motional state, Z_m can be implemented simply by adiabatically ramping the depth of the tweezer trap up and down such that the phase accrued by the $m = 1$ states is α . We further construct pulse sequences wherein the red sideband on the clock transition is driven to implement an e - m swap gate ($\text{SWAP}_{e,m}$), e - m CZ gate ($\text{CZ}_{e,m}$), and e - n - m doubly controlled Z gate (CCZ). Additionally, an n - m swap gate ($\text{SWAP}_{n,m}$) can be implemented similarly to $\text{SWAP}_{e,m}$ by detuning one of each pair of beams used to drive stimulated Raman transitions within the $e = 0$ and $e = 1$ subspaces

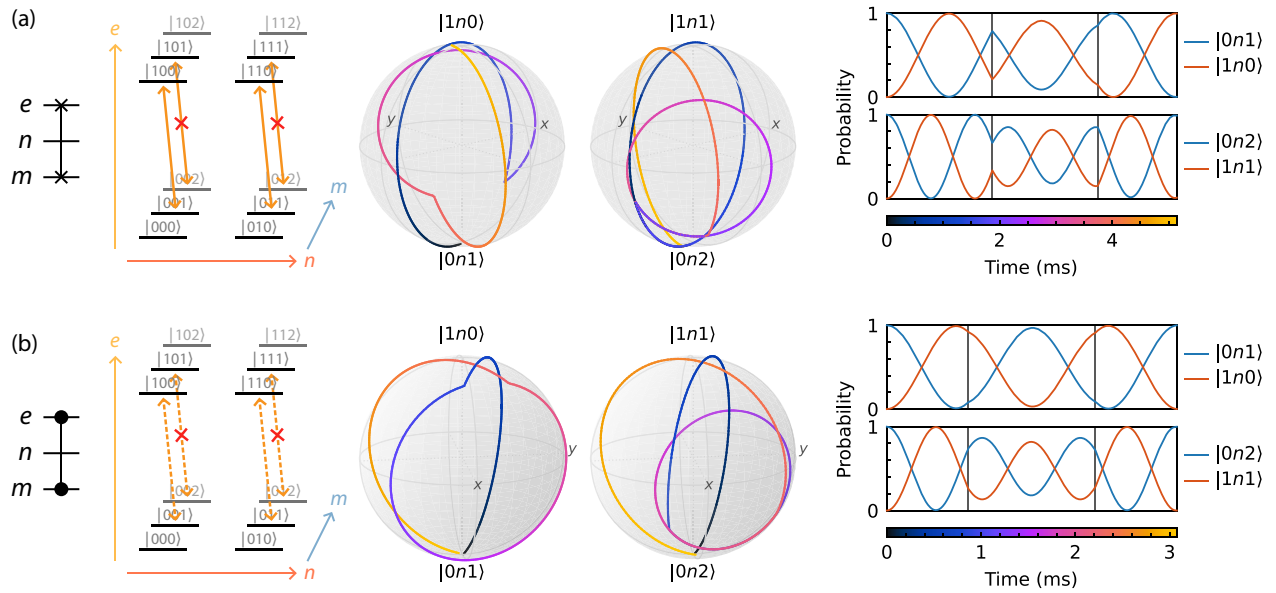


FIG. 2. Motion-selective intraquoct gate sequences. Pulse sequences showing implementations of the e - m SWAP (a) and e - m controlled Z (b) gates with fidelities $\mathcal{F}_{\text{SWAP}_{e,m}} \approx 0.9925$ and $\mathcal{F}_{\text{CZ}_{e,m}} \approx 0.9929$, respectively. For both gates, driven couplings between the relevant quoct states and next-lowest ($m = 2$) motional states are shown, and couplings for which the pulse sequence performs a 2π rotation are marked with a red X. Dashed arrows indicate closed state trajectories through which a phase rotation is performed, rather than population transfer. Simulated Bloch sphere trajectories between the relevant coupled pairs of states and probabilities thereof as functions of time are shown in the middle and right panels. All trajectories start at the south pole of the sphere in black and progress toward yellow on the spheres. The kinks in the trajectories, occurring at times indicated by the vertical lines in the right panels, show the divisions between pulses of varied phase and equal Rabi frequency as described in the main text. The pulse sequences are agnostic to the n qubit because the driving light pulse is π -polarized.

to a red sideband. Each pulse sequence comprises three separate pulses of identical Rabi frequencies and detunings resonant with said sideband but different durations and overall phases, such that the quoct state is rotated through an angle of π on the appropriate transitions, and 2π on corresponding transitions to the set of next-lowest motional states (i.e., $m = 2$). In Figs. 2 and 3, we show the evolution of quoct states through these pulse sequences, projected onto two-dimensional subspaces in which qualitatively distinct dynamics occur. Pulse sequences for each gate are found up to single Z_e , Z_n , and Z_m gates by means of

a gradient ascent-based optimization of the simulated gate fidelity.

Given our extensive use of clock transition sidebands, the Rabi frequencies Ω at which our pulse sequences are driven play a critical role in determining attainable gate fidelities. In Fig. 4, we plot expected optimum gate fidelities as functions of Ω and highlight chosen values of Ω for each gate (see Appendix C for details). At our chosen Rabi frequencies, we expect to reach gate fidelities of $\mathcal{F}_{\text{SWAP}_{e,m}} \approx 0.9925$, $\mathcal{F}_{\text{CZ}_{e,m}} \approx 0.9929$, $\mathcal{F}_{\text{CCZ}} \approx 0.9814$, and $\mathcal{F}_{\text{SHELVE}_{e,m}} \approx 0.9907$; complete sets of parameters for

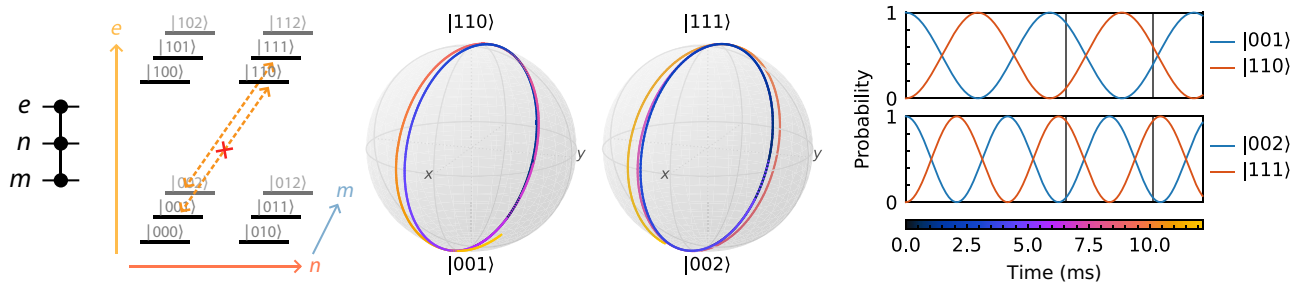


FIG. 3. Intraquoct CCZ gate sequence. The pulse sequence for the e - n - m doubly controlled Z gate with fidelity $\mathcal{F}_{\text{CCZ}} \approx 0.9814$. As in Fig. 2, driven couplings between the relevant quoct states and the next-lowest motional states are shown, with those on which a 2π rotation are performed marked with a red X. Here, pulse sequences depend on the n qubit because the driving light pulse is circularly polarized.

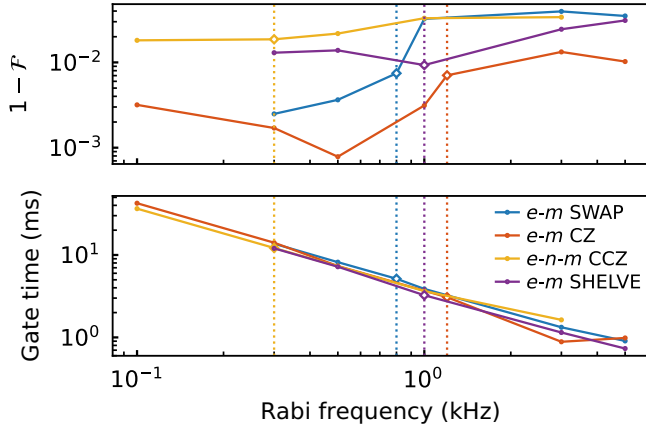


FIG. 4. Motion-selective gate fidelities and execution times. Gate infidelities (top) and times (bottom) are shown as functions of Rabi frequency for each motion-selective gate implemented in this paper. Each point represents the maximum-fidelity condition for each gate that is discoverable via gradient ascent (see Appendix C) for a particular Rabi frequency, with trap frequency fixed to $\omega = 2\pi \times 100$ kHz. The vertical dotted lines and diamond points show conditions selected for minimal gate time up to a ≈ 0.005 gate infidelity tolerance. In the case of the e - n - m CCZ gate, this tolerance was not achievable so the point at maximal gate fidelity with larger Rabi frequency was chosen.

each gate are summarized in Table II. In general, there is a correlation between gate fidelity and gate time. Hence, the fidelity of the CCZ could potentially be increased to the level of the others by slowing it down. Given this gate time/fidelity trade-off, the optimal condition may depend on the algorithm being implemented. As seen below, the CCZ gate is needed only for the mass and chemical potential terms and is thus used relatively infrequently.

B. Interquoct gates

Gates involving the states of two nearby atoms are commonly implemented through Rydberg blockade [38–43]. In ququarts [30], single-photon transitions were driven from states in the $e = 1$ manifold to $6s\,n\,s^3S_1 \, |m_F = \pm 3/2\rangle$ (effective principal quantum number $n^* \approx 55$) Rydberg states to implement a two-qubit CZ gate between the e qubits of two nearby ququarts as well as a four-qubit CCCZ gate involving the e and n qubits of both ququarts. Crucially, the CZ and CCCZ gates were physically identical, up to only which Rydberg transitions were driven: While the CCCZ gate was realized by driving the $|11\rangle \leftrightarrow n^*3S_1 \, |m_F = +3/2\rangle$ transition in both ququarts, the CZ gate used a bichromatic pulse to simultaneously drive $|10\rangle \leftrightarrow n^*3S_1 \, |m_F = -3/2\rangle$ as well, thus making the CCCZ gate agnostic to the n qubits of both ququarts.

Here, we note that since we work with atoms at minimal temperatures—such that spatial wave functions will hardly change over the time scale of the Rydberg pulses—neither interatom gate identified above is sensitive to motion [24]

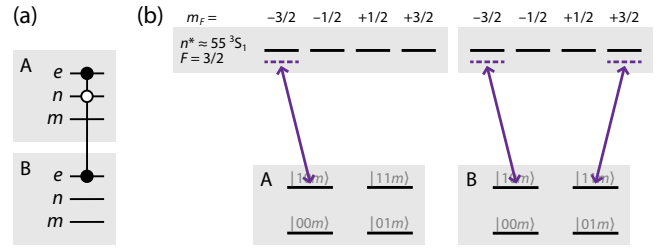


FIG. 5. Interquoct Rydberg-based $\bar{C}\bar{C}\bar{Z}$ gate. (a) The circuit symbol for a CCZ gate on the e and n qubits of one atom and only the e qubit of the other, with the n qubit “anticonnrolled”. (b) The driven Rydberg couplings to implement this gate. By driving both stretched transitions with a bichromatic pulse in quoct B, the gate can be made agnostic to quoct B’s n qubit.

and, hence, the interquoct CZ may be inherited without modification. However, our simulation requires the use of a three-qubit “ $\bar{C}\bar{C}\bar{Z}$ ” gate where one qubit is “anticonnrolled,” i.e., such that the π phase shift is applied only when that qubit is in $|0\rangle$, rather than $|1\rangle$. This gate can be implemented natively as a gate involving the e and n qubits of one quoct and only the e qubit of the other by applying the CCCZ gate pulse to the former with the drive on the $|11m\rangle \leftrightarrow n^*3S_1 \, |m_F = +3/2\rangle$ transition replaced with one on $|10m\rangle \leftrightarrow n^*3S_1 \, |m_F = -3/2\rangle$, and the bichromatic CZ gate pulse to the latter, as shown in Fig. 5. This results in a $\bar{C}\bar{C}\bar{Z}$ gate that flips the phase of the $|10m\rangle \otimes |1nm\rangle$ two-quoct states and acts as the identity for all others. Note that this native implementation of the $\bar{C}\bar{C}\bar{Z}$ gate requires the ability to address individual atoms with the relevant Rydberg pulses, however; if only global Rydberg pulses are available, this gate can alternatively be implemented by conjugating the two-quoct gate $I \otimes X_n$ with $(X_n \otimes I)CCCZ$.

C. Readout protocol

Readout of the quoct space is performed using a multiround procedure, wherein multiple measurements are performed within subsets of the full computation space. As for ququarts, photons are scattered indistinguishably from all $e = 0$ states with additional operations between each round, yielding a single classical bit per round corresponding to the e qubit at the time of measurement: If photons are detected, the readout is said to have yielded a “bright” outcome ($e = 0$), and “dark” ($e = 1$) otherwise. Since we have implementations of both $\text{SWAP}_{e,n}$ and $\text{SWAP}_{e,m}$ available and each round of photon scatter effectively measures the e qubit by mapping $e = 0$ (1) to a “bright” (“dark”) signal on a detector, full readout of the entire space should, in principle, be achievable in three rounds by merely swapping the n and m qubits into the e -qubit position between rounds. However, the photon scattering process couples incoherently to the motional degree of freedom, scrambling information stored in the

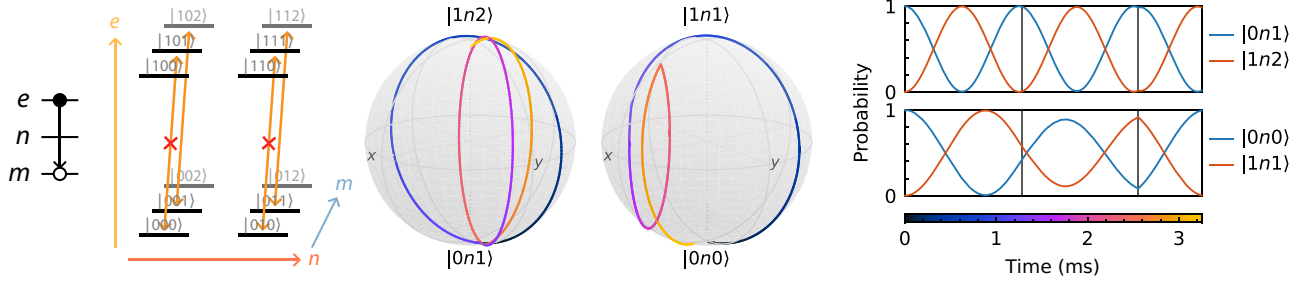


FIG. 6. Motional-selective shelving sequence for readout. Pulse sequence implementing our e - m shelving operation, where we take $|0n1\rangle \leftrightarrow |1n2\rangle$ while performing a simultaneous 2π rotation on $|0n0\rangle \leftrightarrow |1n1\rangle$ with fidelity $\mathcal{F}_{\text{SHELVE}_{e,m}} \approx 0.9907$. As in Fig. 2, we show simulated Bloch sphere trajectories and flattened probability time traces. We denote this operation by the circuit diagram symbol on the left for convenient use in Fig. 7.

m qubit across a wide range of motional states but crucially leaving the n qubit intact. Given some N rounds of readout, we require a procedure capable of mapping classical bitstrings of length N (the outcomes of each round of readout) surjectively onto the space of quoct basis states. Although motional scrambling prevents the use of naive swapping of the n and m qubits into and out of the e qubit, we find that complete readout is possible through the use of an “ e - m shelving” operation constructed to swap a subset of basis states outside the computational space. Specifically, we use the blue sideband on the clock transition to transfer population from the $|0n1\rangle$ states to $|1n2\rangle$ as shown in Fig. 6, and construct a pulse sequence to do so such that the accompanying $|0n0\rangle \leftrightarrow |1n1\rangle$ transition is driven through a net 2π rotation angle. As with the gates discussed in Sec. II A, this operation is implemented using another three-part pulse sequence, the exact parameters of which were found using a gradient ascent optimization of the gate fidelity. In the context of readout and motional scrambling, the e - m shelving operation can be seen as a way to move some information out of the subspace in which the scrambling occurs, such that no two states are scrambled together.

To then construct a full readout protocol, we employ a genetic algorithm to search the space of quoct circuits comprising a number of readout operations with any intraquoct gates in between. Specifically, our algorithm minimizes the number of readout and intraquoct gate operations needed to construct a protocol that performs the requisite surjective mapping from length- N bitstrings to quoct basis states (see Appendix D for details) and we show one such protocol in Fig. 7. This protocol is not unique; we find several others that are equivalent in terms of the number of rounds of readout and total gate count. However, we find none that satisfy the bitstring surjectivity constraint using fewer than four rounds of readout, and none that injectively map initial quoct states to final, output states after the protocol ends. These can be seen as artifacts due to, respectively, the shelving operation reducing the amount of information measured by each readout, and motional

scrambling eventually mixing subsets of states together through multiple rounds of readout.

In Fig. 7(b), we also identify an alternative measurement operation where photons are scattered from only the $|00m\rangle$ states. This operation was implemented experimentally in Refs. [44,45] and allows for direct measurement of the n qubit within the $e = 0$ subspace. Here we note that, provided the ability to do real-time feedback during the execution of a circuit, it can be used to short-circuit full quoct readout in the case that an $e = 0$ result is detected from any of the e -qubit readouts: If photons are scattered during an e -qubit readout after shelving the $|0n1\rangle$ states,

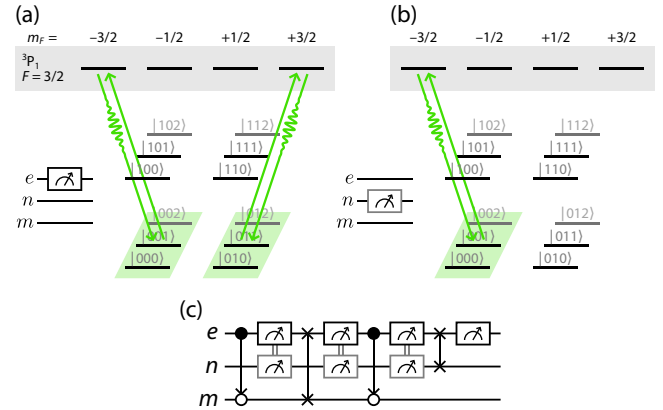


FIG. 7. Measurement protocol. (a) A single round of e -qubit readout. The two stretched $^1S_0 \leftrightarrow ^3P_1$ are driven simultaneously, scattering photons that are indistinguishably detected to effectively measure exactly the e qubit. The n and m qubits are only affected if $e = 0$. (b) An alternative measurement process, where only the stretched transition to $^3P_1 m_F = -3/2$ is driven, amounting to partial readout of the n qubit, only within the $e = 0$ subspace. This measures $|00m\rangle$ or NOT $|00m\rangle$. (c) The circuit diagram for full readout of the quoct computational space. Each full readout requires a maximum of four rounds of e -qubit measurement, but can be short-circuited by a partial readout of the n qubit as in (b) following a “bright” result from the e -qubit readout as indicated by the classically controlled, gray-boxed n -qubit measurements. See Appendix D for more information.

then the quoct collapses into a mixture of the $|0n0\rangle$ states. Hence, a single n -qubit readout using the alternative readout operation described above will then be enough to fully identify a single quoct basis state. In some cases, the readout circuit shown in Fig. 7(c) can be reduced to only two rounds: If the first e -qubit readout yields $e = 0$, an n -qubit readout can be performed immediately after and the entire procedure subsequently terminated. See Appendix D for a more thorough analysis of the possible readout results.

III. LATTICE QCD SIMULATION IN 1+1D

A. Lattice Hamiltonians to qubitized hardware

1. Kogut-Susskind Hamiltonian in the axial gauge

We start with a review of the Kogut-Susskind (KS) Hamiltonian. The KS Hamiltonian for a three-color fermion $\psi = (\psi_r, \psi_g, \psi_b)^T$ in $1 + 1$ spacetime dimensions with one flavor takes the form [4, 46–48]

$$\hat{H} = \frac{1}{2a} \sum_{n=0}^{2L-2} \left(\hat{\psi}_n^\dagger \hat{U}_n \hat{\psi}_{n+1} + \text{h.c.} \right) + m \sum_{n=0}^{2L-1} (-1)^n \hat{\psi}_n^\dagger \hat{\psi}_n + \mu \sum_{n=0}^{2L-1} \hat{\psi}_n^\dagger \hat{\psi}_n + \frac{ag^2}{2} \sum_{n=0}^{2L-2} \hat{E}_n^2, \quad (1)$$

where a is the lattice spacing, and m, μ, g are the bare mass, baryon chemical potential, and coupling strength, respectively. The fermions are staggered so that the L two-component spinor degrees of freedom are split into single-component fermions which live on the even and odd sites of a $2L$ -site lattice (see Fig. 1). In this staggered model, the quarks and antiquarks are collinear and separated by a . On even sites, excitations are understood as quarks $\hat{\psi}_n = (\hat{c}_r, \hat{c}_g, \hat{c}_b)^T$, and on odd sites as antiquarks $\hat{\psi}_n = (\hat{c}_{\bar{r}}, \hat{c}_{\bar{g}}, \hat{c}_{\bar{b}})^T$. In a multiflavor model, the quarks could also transform under a nontrivial flavor symmetry in a straightforward way by introducing a new index, or augmenting an existing one as described in [4]. $\hat{U}_n = \exp(-iag\hat{A}_n)$ is the Wilson line which transports $SU(3)$ color charge between site n and $n + 1$, and lives on the link. In the continuum limit $A_\mu \in \mathfrak{su}(3)$ is the gauge field which generates quantum chromodynamics akin to how the $A_\mu \in [0, 2\pi)$

in QED generates the $U(1)$ gauge group. $\hat{E}_n^2$ is the $SU(3)$ Casimir which measures color-electric flux and also lives on the link.

For spatial dimensions $d > 1$, the KS Hamiltonian contains a color-magnetic term which introduces dynamics into the gauge field. In its absence, there are no independent gauge degrees of freedom. Fixing the axial gauge $A_x = 0$ and solving the Gauss law constraint for A_t , the Hamiltonian becomes

$$\hat{H} = \frac{1}{2a} \sum_{n=0}^{2L-2} \left(\hat{\psi}_n^\dagger \hat{\psi}_{n+1} + \text{h.c.} \right) + m \sum_{n=0}^{2L-1} (-1)^n \hat{\psi}_n^\dagger \hat{\psi}_n + \mu \sum_{n=0}^{2L-1} \hat{\psi}_n^\dagger \hat{\psi}_n + \frac{ag^2}{2} \sum_{n=0}^{2L-1} \sum_{a=1}^8 \left(\sum_{m=1}^n \hat{Q}_m^a \right)^2, \quad (2)$$

where $\hat{Q}_n^a = \hat{\psi}_n^\dagger T^a \hat{\psi}_n$ represents the color charge at site n which preserves Gauss' law. Here we work with open boundary conditions (OBCs) and thus ignore topological effects. The continuum limit is discussed in Appendix F.

Observe that we have exchanged our local bosonic degrees of freedom $\hat{E}_n^2$ with nonlocal charge-pairing terms like $\hat{Q}_m^a \hat{Q}_n^a$ which decompose into fermionic degrees of freedom. Now that there are only fermionic degrees of freedom in our theory, we can introduce a finite-dimensional Hilbert space at each site. For the even sites we start with the unoccupied state $|0\rangle$ and then raise by $\hat{\psi}_n^\dagger = (\hat{c}_r^\dagger, \hat{c}_g^\dagger, \hat{c}_b^\dagger)$, yielding $8 = 2^3$ possible states. As fermionic fields, the $\hat{c}_c$ satisfy the anticommutation relations $\{\hat{c}_c, \hat{c}_{c'}^\dagger\} = \delta_{c,c'}$ and $\{\hat{c}_c, \hat{c}_{c'}\} = \{\hat{c}_c^\dagger, \hat{c}_{c'}^\dagger\} = 0$. Therefore a fixed antisymmetric ordering of the states must be chosen. For example labeling the quarks on the *same* site q_1 and q_2 , we define for the diquark states

$$|br\rangle := |q_1^b q_2^r\rangle - |q_1^r q_2^b\rangle = -(|q_1^r q_2^b\rangle - |q_1^b q_2^r\rangle) = -|rb\rangle. \quad (3)$$

Since there is a unique antisymmetric state, we do not need to create superpositions of states once we encode in the quantum computer; we can label the states as combinations of color excitations and then interpret them as the correct antisymmetric wavefunction. This gives us the quoct or “qu8it” whose formalism was first developed in [48] and is reviewed in Appendix E,

$$\{| \text{quoct} \rangle\} = \{ |0\rangle, \hat{c}_r^\dagger |0\rangle, \hat{c}_g^\dagger |0\rangle, \hat{c}_b^\dagger |0\rangle, \hat{c}_g^\dagger \hat{c}_b^\dagger |0\rangle, \hat{c}_r^\dagger \hat{c}_b^\dagger |0\rangle, \hat{c}_r^\dagger \hat{c}_g^\dagger |0\rangle, \hat{c}_r^\dagger \hat{c}_g^\dagger \hat{c}_b^\dagger |0\rangle, \\ |0\rangle, |r\rangle, |g\rangle, |b\rangle, |gb\rangle, |rb\rangle, |rg\rangle, |rgb\rangle \}. \quad (4)$$

2. Translating quoct operators to hardware gateset

Given operators $\hat{O}$ on the lattice, we can represent them as matrices $\tilde{O} = [\hat{O}]_{\text{quoct}}$ in the quoct Hilbert space. The main new work here is translating operators on the abstract quoct basis into circuit gates acting on the electronic, nuclear, and motional degrees of freedom $[\hat{O}]_{\text{ennm}}$ and then exploring NISQ and near-future observables.

For $L = 1$ with one quark and one antiquark site, the quoct matrix representation for the Hamiltonian is

$$\tilde{H} = \frac{1}{2a} \sum_c ((\tilde{c}_c^\dagger \tilde{P}) \otimes \tilde{c}_c^\dagger - (\tilde{c}_c \tilde{P}) \otimes \tilde{c}_c) + \frac{ag^2}{2} \tilde{Q}^2 \otimes \tilde{I} + m(\tilde{M} \otimes \tilde{I} + \tilde{I} \otimes \tilde{M}) + \mu(\tilde{M} \otimes \tilde{I} - \tilde{I} \otimes \tilde{M}). \quad (5)$$

There is flexibility when encoding this representation as excitations in the $|enm\rangle$ basis. The simplest assignment maps the quark colors directly so that red, green, and blue excitations correspond directly to the electronic, nuclear, and motional degrees of freedom, respectively. Under this encoding, the basis states are reordered to follow the usual computational basis $|000\rangle, |001\rangle, \dots, |111\rangle$. This simple assignment has the benefit of clarifying how the circuit should be structured. The circuit is broken into four parts for each term in the Hamiltonian and is Trotterized to form the full circuit.

Kinetic term: For the kinetic Hamiltonian, we can decompose the creation operators into intraquoct gates. For example looking at Eq. (E2) we read off

$$\tilde{c}_r^\dagger = \frac{1}{2}(X - iY)_r \otimes CZ_{\bar{g}b} \quad (6)$$

where $CZ_{\bar{g}b}$ introduces a phase when $|b\rangle$ is occupied but $|g\rangle$ is not. Likewise $\tilde{P} = Z_r Z_g Z_b$. Thus for the red part of the kinetic term,

$$\begin{aligned} (\tilde{c}_r^\dagger \tilde{P})_0 \tilde{c}_{r,1}^\dagger - (\tilde{c}_r \tilde{P})_0 \tilde{c}_{r,1} &= \frac{1}{2}(X_0 X_1 - Y_0 Y_1)_r (CZ)_{\bar{g}b,0} CZ_{\bar{g}b,1} \\ &= \frac{1}{2}(\tilde{X}_0^r \tilde{X}_1^r - \tilde{Y}_0^r \tilde{Y}_1^r), \end{aligned} \quad (7)$$

where

$$\tilde{X}_0^r = X_r CZ_{g\bar{b}} = \begin{array}{c} r \text{---} \boxed{X} \text{---} \\ | \\ g \text{---} \bullet \text{---} \\ | \\ b \text{---} \circ \text{---} \end{array} \quad (8)$$

$\tilde{X}_1^r = X_r CZ_{\bar{g}b}$, and $\tilde{Y}_0^r$ and $\tilde{Y}_1^r$ have Y gates replace X . We then find these circuit representations for the green and blue terms of the kinetic Hamiltonian in a similar manner. Since $\tilde{X}_i^c$ and $\tilde{Y}_i^c$ are unitary, hermitian, and exchange states, we can circuitize the associated time-evolution unitary using the identity

$$x \text{---} \boxed{\exp(-\frac{i\alpha}{2} A \otimes B)} \text{---} = \begin{array}{c} q \text{---} \bullet \text{---} \boxed{A(\alpha)} \text{---} \bullet \text{---} \\ | \\ \bar{q} \text{---} \boxed{B} \text{---} \text{---} \boxed{B} \text{---} \end{array} \quad (9)$$

where the controlled- B quoct operator represents a gate which is B when q is a certain family of states, and the identity $\tilde{I}$ otherwise, and $A(\alpha) = \exp(-\frac{i\alpha}{2} A)$ is the

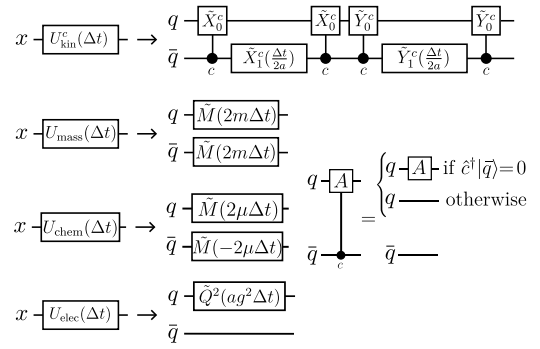


FIG. 8. Trotter decomposition for the $L = 1$ Hamiltonian. For a given quoct operator A , we define $A(\alpha) := \exp(-\frac{i\alpha}{2} A)$ for the unitary operator describing its exponentiation. We also make use of a generalized controlled operator CA which controls on a family of states.

$SU(8)$ generalization of the usual rotations. This identity is proven in Appendix G 1. Thus the kinetic term

$$\tilde{H}_{\text{kin}} = \frac{1}{4a} \sum_c \tilde{X}_0^c \tilde{X}_1^c - \tilde{Y}_0^c \tilde{Y}_1^c \quad (10)$$

exponentiates to the interquoct gate $U_{\text{kin}}^c(\Delta t)$ shown in Fig. 8. Since we have chosen a basis for the quoct in terms of (r, g, b) and (e, n, m) , we can write down the circuit for the controlled $C\tilde{X}_0^c$ and rotating $\tilde{X}_1^c(\alpha)$. This is demonstrated in Fig. 9. The $C\tilde{X}_0^c$ is straightforward since it has a single red control (which controls half of the states). $\tilde{X}_1^c(\alpha)$ can be understood as a simple $X(\alpha)$ rotation with an extra phase produced by the $C\bar{C}Z$ gate. Observe that when transpiling to the optimal gateset for the hardware, native SWAP gates are often used to shuttle data between the motional and electronic/nuclear sectors. Thus while the targets of the interquoct $C\bar{C}Z$ gate change in the color framework, they are all implemented by the same $C\bar{C}Z$ gate acting on the ququart $|en\rangle \otimes |en\rangle$ subspace of states shown in Fig. 5. More details for the intraquoct and interquoct gates are found in Secs. II A and II B.

Mass and Chemical Potential: The mass and chemical potential are always single intraquoct gates as seen in Fig. 9. Using the simple (r, g, b) to (e, n, m) mapping, we have that $\tilde{M} = 3/2 - \frac{1}{2}Z_r - \frac{1}{2}Z_g - \frac{1}{2}Z_b$. The constant term is irrelevant, and so $\tilde{M}$ exponentiates to the product of rotation gates $Z(\alpha)$.

Electric term: As noted for $L = 1$, the electric term is a single intraquoct gate. The charge operator has the simple form

$$\frac{ag^2}{2} \tilde{Q}^2 = \frac{ag^2}{6} (3 - Z_r Z_g - Z_g Z_b - Z_r Z_b), \quad (11)$$

which transpiles to the CX and RZ circuit in Fig. 9 in the usual way for Pauli strings.

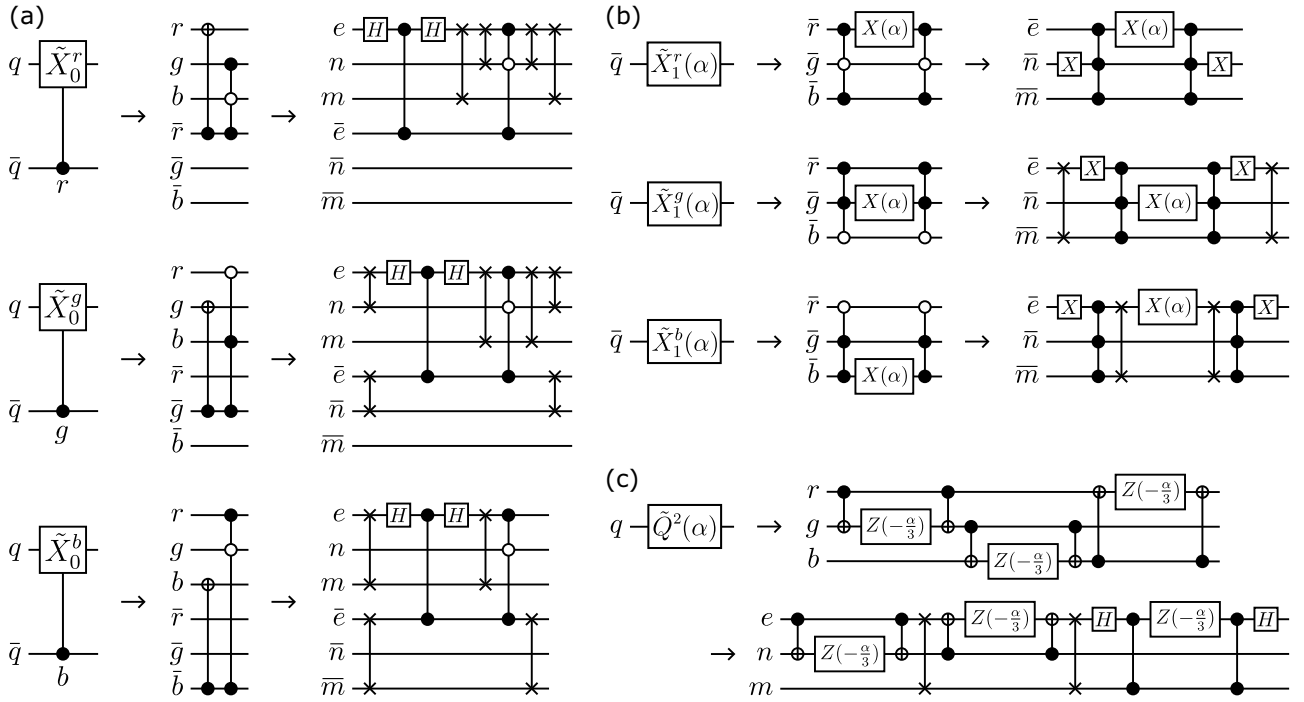


FIG. 9. Circuit compilation for the $L = 1$ Hamiltonian. Each circuit is first understood as acting on quots, then in terms of operators acting on each color sector, and lastly transpiled into the desired gateset.

The full circuit with some simplifications can then be found in Appendix G 1. Note that while the direct $(r, g, b) \rightarrow (e, n, m)$ mapping makes developing the circuit simple, it is not necessary.

B. Physical Observables

1. Vacuum persistence

A simple observable is vacuum persistence $|\langle \psi(0) | \psi(t) \rangle|^2 = |\langle 0 | e^{-iHt} | 0 \rangle|^2$, where the system is initialized in the unoccupied state and allowed to time evolve [4]. For finite ag the unoccupied state is not an eigenstate, and so the system will evolve nontrivially. Trotter error occurs as the kinetic term does not commute with the other terms (which are all mutually commuting). Using first-order Trotterization the error to lowest order in t is proportional to t^2/D where D is the number of Trotter steps. In particular for the $L = 1$ Hamiltonian we can quantify the Trotter error bound as

$$\left\| \frac{t^2}{2D} [\tilde{H}_{\text{kin}}, \tilde{H}_{\text{elec}} + \tilde{H}_{\text{mass}} + \tilde{H}_{\text{chem}}] |0\rangle \right\|^2 = \frac{\sqrt{3}t^2(3m + ag^2)}{6aD} \quad (12)$$

to lowest order in t . However this lowest-order bound can significantly overestimate the error. As seen in Fig. 10, the vacuum persistence of the three Trotter step circuit matches the exact diagonalization result very well up to

$t = 3$. At this time, the fidelity $\mathcal{F} = |\langle \psi | \tilde{\psi} \rangle|^2$ between the exact state ψ and the three Trotter step state $\tilde{\psi}$ is 0.83, which is much smaller than the naive Trotter error analysis would suggest. While one Trotter step is clearly insufficient as the pair creation terms of the kinetic term take the state away from the vacuum, we can see that three Trotter

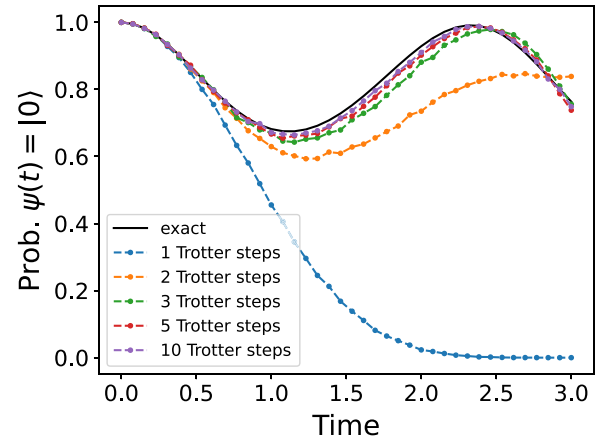


FIG. 10. Vacuum persistence for $L = 1$. Here we plot $|\langle 0 | \psi(t) \rangle|^2$ or the probability that $\psi(t) = |0\rangle$. Each curve has a fixed number of Trotter steps so Trotter error grows for larger t . The fully unoccupied state oscillates into color-neutral multi-particle states for finite g . In this plot $a = m = g = 1$, $\mu = 0$. Convergence to the exact diagonalization (ED) result is seen for higher Trotter step density.

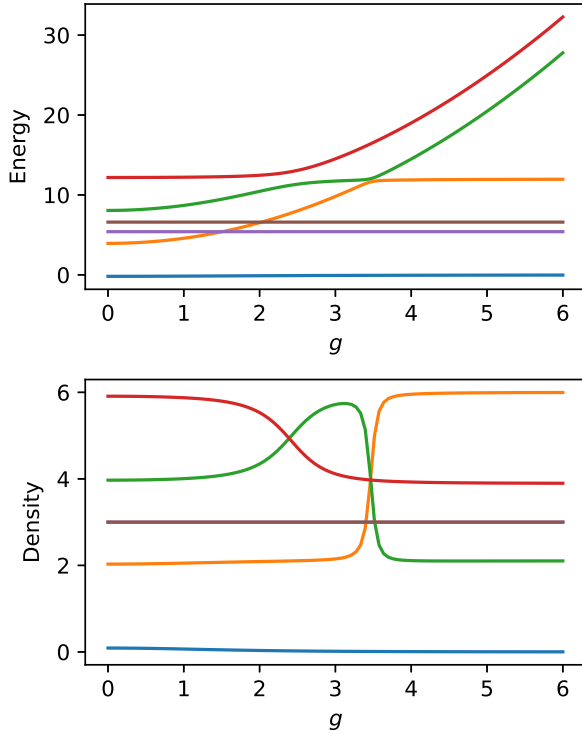


FIG. 11. Spectrum and expectation value of the fermion number density from exact diagonalization for $L = 1$. Here we restrict to color-singlet states and fix $a = 1, m = 2, \mu = 0.2$, varying the coupling g . At $g = 0$, the first excited state is a meson realized as a quark on the $n = 0$ site, an antiquark at $n = 1$, and a color-electric flux-tube between them. At large g this state transitions to a baryon rgb and antibaryon $\bar{r}\bar{g}\bar{b}$ with no flux between. At $g = 0$ and large g , the six states of the spectrum correspond to the ground state, the single-baryon and antibaryon state (split by the chemical potential μ), and linear combinations of the meson $q\bar{q}$, two-meson $qq\bar{q}\bar{q}$, and the two-baryon state. Note that the transition of the single meson state (in orange) is accompanied by a small gap between it and the next excited state.

steps is sufficient to capture the oscillatory behavior of the system at least until three time units.

2. Adiabatic hadron-number transition

In the $g \rightarrow \infty$ limit and ignoring color nonsinglet states that may appear from the finite size of the lattice, we expect only the baryon states to have finite energy since in the lattice they have no color-electric flux. However at low g , the single meson state has lower energy since it has less mass. From ED in Fig. 11 we can see in the spectrum that this is a transition from a quark-antiquark state connected by a color flux-tube to a bound baryon state, as noted in [4]. This transition, which is state mixing at $L = 1$, is the avatar of string breaking in larger simulations. We can follow the transition of this excited state by adiabatically increasing or decreasing g .

For adiabatic evolution, we have an initial time-independent Hamiltonian H_i with an eigenstate that we

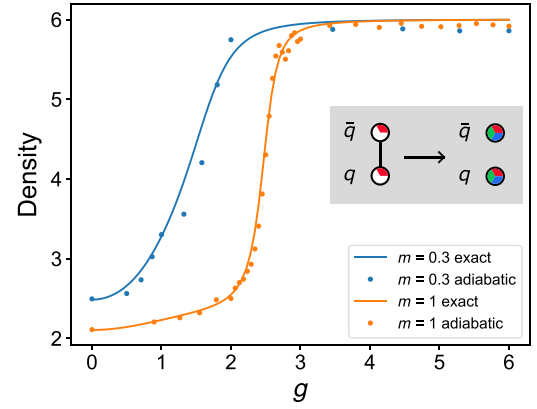


FIG. 12. Adiabatic evolution of the single meson state at weak coupling to the two-baryon state at strong coupling. Here $L = a = 1, \mu = 0$. The evolution is broken into three Trotter “speeds” depending on the spectral gap. In particular the transition in density from the single meson to the two-baryon states is accompanied by a small gap between it and the next excited state, requiring more adiabatic time and thus more Trotter steps. Circuits are measured at unit intervals in T . For $am = 1$ in orange, the maximum circuit depth is 250 Trotter steps. For $am = 0.3$ in blue the maximum circuit depth is 60 Trotter steps.

know well, $|\psi_i\rangle$, and we have a target Hamiltonian H_f with less well-known eigenstates $|\psi_f\rangle$. Consider the time-dependent Hamiltonian

$$H(t) = \left(1 - \frac{t}{T}\right) H_i + \frac{t}{T} H_f. \quad (13)$$

Then $H(0) = H_i$ and $H(T) = H_f$. We can transport our initial state by time evolution as $|\psi_i(t)\rangle = U(t) |\psi_i(0)\rangle$, where

$$U(T) = \mathcal{T} e^{-i \int_0^T H(t) dt} \approx \mathcal{T} \prod_{j=0}^D e^{-i H(j \Delta t) \Delta t} \approx \underbrace{e^{-i H(T-\Delta t) \Delta t} \dots e^{-i H(\Delta t) \Delta t} e^{-i H(0) \Delta t}}_D \quad (14)$$

and where $\Delta t = T/D \ll 1$ and D records the depth of the circuit. Assuming that T is long enough, then the adiabatic theorem tells us $|\psi_i(T)\rangle \cong |\psi_f\rangle$ up to a phase [49].

We can rewrite $H(t)$ for our particular Hamiltonian,

$$H_g = H_K + H_M + g^2 H_E. \quad (15)$$

Then

$$U(T) = \mathcal{T} \prod_{j=0}^D e^{-i H_{g_{\text{eff}}} \left(\frac{j}{D}\right) \Delta t}, \quad (16)$$

where $g_{\text{eff}}^2 = g_i^2 + (j/D)(g_f^2 - g_i^2)$ and where Δt is small enough to approximate the time-ordered integration and allow for Trotterization.

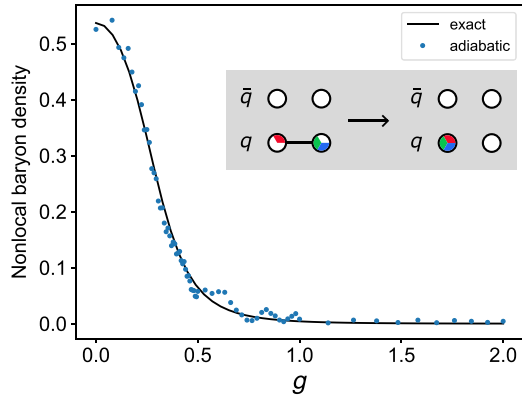


FIG. 13. Adiabatic evolution of the density of nonlocal baryons for $L = 2, m = 2, \mu = 0.1$. For $L = 2$ the baryon can be extended on to two lattice sites, e.g., $|r\rangle$ on site 0 and $|gb\rangle$ on site 2. As g approaches unity, the baryon is localized to only one site. The maximum circuit depth is 500 Trotter steps.

For $g = 0$ the eigenstates of the Hamiltonian are exactly solvable [4] for arbitrary m, a, μ , and so the system can be initialized at this point and then in principle evolved to any intermediate strength g even for systems that are not tractable on classical computers. An example of this is shown in Fig. 12. For larger m the gap between excited states shrinks at the transition, and the $g = 0$ and $g \rightarrow \infty$ states partition better into integer densities. For $L = 1$, the single-baryon states are split by nonzero μ , but cannot evolve as g changes, making their dynamics fairly trivial. For $\mu > m$, the lowest-energy state is the single antibaryon as expected.

3. Baryon size

As L increases, the spectrum of physical states becomes richer. Baryon number n_B is a symmetry of the Hamiltonian, and for $L = 1$ the single-baryon and antibaryon states are fixed as g evolves since there is only one state for each sector. For $L = 2$ and finite g , there are 20 states in the $n_B = 1$ sector which allows for the low-energy state pictured in Fig. 13 to evolve as g is increased. Along with single-baryon states on one site, the $n_B = 1$ eigenstates also have support on states with two quarks on one site and one quark on the other site, with a color-electric flux-tube connecting them. Thus we have baryons which are “nonlocal”. The density of these states can be measured using the following operator:

$$(2 \text{ particles})_0 \otimes (1 \text{ particle})_2 + (1 \text{ particle})_0 \otimes (2 \text{ particles})_2, \quad (17)$$

which projects out these nonlocal states.

Similar to the hadron-number changing observable, increasing g means the flux-tube becomes energetically expensive. However, for this low-energy state, rather than

breaking the flux-tube by producing color-neutralizing partners, the baryon eliminates the flux-tube by huddling all the quarks on one site. Thus the total number density remains constant, but the local number density changes. These nonlocal baryons can be thought of as having a finite size relative to the lattice.

C. Resource Estimation and $L > 1$

For $L = 1$ the basic circuit for one Trotter step is shown in Fig. 17. Based on the MPP ququart operations with fidelities $\gtrsim 0.999$ and the intraquoct gates with fidelities shown in Table II, the overall fidelity of one Trotter step is 0.691. Averaging the gate times to ~ 1 ms, then each Trotter step takes ≈ 100 ms. Since coherence times are ~ 1 s, we can run $\lesssim 10$ Trotter steps within coherence times at a total fidelity of 0.025. However as noted, the three Trotter step vacuum persistence circuit matches the ED results closely and would have a larger fidelity of 0.331. Thus while the fidelity makes adiabatic evolution for observables like hadron number and baryon locality challenging, it is clear that simple observables like vacuum persistence are achievable in the short-term.

It should be noted that the low number of interquoct gates is particular to $L = 1$, as the electric term of the Hamiltonian is confined to one quoct as a $\tilde{Q}^2$ term. For $L > 1$ the electric term is nonlocal and generates $(2L - 1)$ choose 2 pairs of the type $\tilde{Q} \otimes \tilde{Q}$ and $\tilde{Q} \otimes \tilde{Q}$. Thus while the other terms of the Hamiltonian scale linearly with L , the electric term grows quadratically.

Moreover, the nonlocal charge terms do not have simple qubit or quoct circuit representations like the other terms. One way to represent them is to decompose the Hamiltonian term via Pauli string decomposition, and then diagonalize mutually commuting families to cut down on circuit complexity as done in [4]. It is simple to then write circuits which exponentiate these Pauli strings composed of Z and I .

We can also examine these terms and try to write quoct operators directly. For example $\hat{Q}^1$ exchanges red and green quarks so it is built around a $\text{SWAP}_{e,n}$ gate. Unlike $\tilde{X}_i^c$ and $\tilde{Y}_i^c$, however,

$$\begin{aligned} \tilde{Q}^1 = & |100\rangle \langle 010| + |010\rangle \langle 100| - |101\rangle \langle 011| \\ & - |011\rangle \langle 101| \end{aligned} \quad (18)$$

is not unitary which makes its circuit representation once exponentiated less trivial. We can devise operations in terms of quocts and ququarts

$$\exp(i\alpha\tilde{Q}^1) = g \begin{array}{c} \text{---} r \text{---} \\ \text{---} \times \text{---} (\alpha) \text{---} \\ \text{---} b \text{---} \\ \text{---} Z \text{---} \end{array} \begin{array}{c} \text{---} \bullet \text{---} \\ \text{---} \bullet \text{---} \\ \text{---} \bullet \text{---} \\ \text{---} Z(-\alpha) \text{---} Z(-\alpha) \end{array} = g \begin{array}{c} \text{---} r \text{---} \\ \text{---} \times \text{---} (\alpha) \text{---} \\ \text{---} b \text{---} \\ \text{---} R(-\alpha) \text{---} R(-\alpha) \end{array} \quad (19)$$

TABLE I. Gate depth of zero-correlation circuits for $L \geq 1$. A listing of the different gates required for larger circuits, separating the motion-preserving operations from the motion-selective ones.

System size	Single	Ququart	SWAP _{$e,n/m$}	CCZ _{enm}	CZ _{e,m}
$L = 1$	53	31	26	6	2
$L = 2$	153	93	78	18	6
$N = 2L - 1$	$\sim 50N$	$\sim 30N$	$\sim 25N$	$6N$	$2N$

where the boxed three-qubit unitary represents a “rotated” SWAP _{e,n} Z _{m} gate,

$$(\text{SWAP}_{e,n} Z_m)(\alpha) = \cos\left(\frac{\alpha}{2}\right) I_{64} - i \sin\left(\frac{\alpha}{2}\right) \text{SWAP}_{e,n} Z_m, \quad (20)$$

where the boxed two-qubit unitary represents just the rotated SWAP _{e,n} , P_1 is the phase shift gate, and $P_0 = XP_1X$. If possible, these operations leverage the quoct architecture more efficiently.

Unfortunately, both methods generate color nonsinglet states due to the noncommutativity of the charge operators $[\hat{Q}_m^a, \hat{Q}_n^b] = i\delta_{mn}f^{abc}\hat{Q}_m^c$, where f^{abc} is the $SU(3)$ structure constant. Thus Trotter error will mix physical and non-physical states. One solution is to artificially increase the energy of nonsinglet states by introducing a term proportional to the sum of the charge over the lattice squared $(\sum_{n=0}^{2L-1} \hat{Q}_n^a)^2$ as done in Refs. [4,48]. Then Gauss’ law is reestablished by exponentially suppressing the nonphysical states. However this introduces $(2L \text{ choose } 2)$ more of these charge-pairing terms. A quoct operator that exactly exponentiates the charge-coupling terms would resolve this.

Recently Farrell *et al.* showed that confinement in the Schwinger model introduces a correlation length ξ which can serve as a cutoff for the size of the charge coupling terms once a correct basis for the charges was chosen [50]. Thus in the strong-coupling regime we can make a lower-bound estimate for the gate depth for larger systems L by ignoring charge coupling (i.e., setting $\xi < a$) as seen in Table I. In the weak-coupling regime, however, we will need new gates to couple sites separated by less than a few correlation lengths.

These charge-coupling terms are also a necessary component of multiflavor models [4]. As the different flavors lie on the same staggered sites, truly flavorful physics requires interquoct charge-coupling terms. Although the flavor and site charge-coupling operators couple different degrees of freedom, they can be represented by the same nonlocal operator in the quoct basis $\hat{Q}_i^a \hat{Q}_j^a$, where i, j can be a multi-index for flavor and lattice site. Thus in particular

we only need two interquoct gates QQ and $Q\bar{Q}$ that transform two quocts $|qq'\rangle$ by $\sum_{a=1}^8 \tilde{Q}^a \otimes \tilde{Q}^a$ and $\sum_{a=1}^8 \tilde{Q}^a \otimes \tilde{\bar{Q}}^a$, respectively, as defined in Appendix E. Indeed, for future work which has finite-correlation length and multiple flavors, a quoct-native charge-coupling operator is a vital next step.

Since new features of the theory are present in the simulation as L increases, such as the finite size of the baryon in Fig. 13, it is also necessary to work on developing circuits that scale reasonably for larger L . Furthermore, larger-scale simulations will be important for scattering simulations that are out of reach for classical simulation [51,52].

IV. OUTLOOK

We have demonstrated a hardware-efficient approach to digital quantum simulation of nuclear physics with atomic qudits in which the higher-dimensional space within each atom is “qubitized.” We have demonstrated a universal gate set and have shown that two atoms (a quark site and an antiquark site) are sufficient to simulate oscillations between vacuum and quark-antiquark pairs and to simulate hadron number changing as the coupling strength is varied. We believe that our multiqubit encoding scheme will contribute to a broad range of applications within quantum computing and digital quantum simulation. Finally, we believe that our use of motional states and our family of composite pulses to isolate the lowest two motional states will aid the development of hybrid digital/continuous variable and bosonic encodings for neutral atoms [9,25–28].

Although the resource requirements for the near-term realizations identified above are daunting, we highlight opportunities to mitigate or even correct errors. These opportunities are enabled by a “physics-inspired” code-sign for error detection/correction protocols that respect the underlying symmetries of the Hamiltonian. Specifically, for $L = 0$ with the constraint of zero total baryon number, our system exhibits a matter-antimatter symmetry. Hence, the occupancy of a quark of any color guarantees the occupancy of an antiquark of the same color. Accordingly, a minimal strategy would be to post-select on the 8/64 physically meaningful states: $|rbg\bar{r}\bar{b}\bar{g}\rangle = \{|000000\rangle, |100100\rangle, |010010\rangle, |001001\rangle, |110110\rangle, |101101\rangle, |011011\rangle, |111111\rangle\}$. While still useful, this approach drastically reduces the success rate and does not identify self-correcting errors that occur during the circuit. Thus, we propose to add color-by-color parity checks after each Trotter step. This could be accomplished with an ancilla qubit for each color, or potentially with an ancilla quoct that contains three ancilla qubits. This approach would keep a record of matter-antimatter parity errors during the circuit, which may enable classical decoding protocols to salvage those operations. Additionally, it may

be possible to correct such errors with a feedforward operation that flips one of the two bits. However, since the energy of a $|00\rangle$ (for a $|c\bar{c}\rangle$ state) may be significantly different than that of $|11\rangle$, depending on g , not knowing which of the two to flip could lead to a substantial injection or depletion of energy.

Nevertheless, we anticipate that “physics-inspired” error detection/correction protocols that are codesigned with our multiqubit-encoded hardware will open the door to robust and resource-efficient simulation of nuclear physics phenomena, and we leave this study for future work. Given the advantages of using qudits to encode the many degrees of freedom of nuclear matter, we anticipate that optimal error detection/correction protocols should be based on qudit schemes rather than the canonical picture of error correcting qubit codes. Examples might include bosonic codes such as the Gottesman-Kitaev-Preskill (GKP) code [53] or closely related angular momentum-based codes [54,55]. It will be interesting to interpret the code words of such codes as robust states of quark matter. Our “qubitization” of qudits and qubit-to-color mapping may aid in the code-sign of error correcting codes inspired by the underlying nuclear physics. Alternatively, it could be interesting to fully embrace the quoct space without qubitizing, potentially offering a more direct translation to bosonic and qudit error correcting codes.

Indeed, our “qubitization” and qubit-to-color mapping is one of many possible methods to implement the KS Hamiltonian. Our large gateset suggests that it could be possible to directly operate in the quoct space, which could more readily enable extensions of our model, such as the use of Weyl gauge instead of the axial gauge. For the specific model considered here (1+1D QCD in axial gauge with $L = 1$), we do not believe that any other encoding scheme is more resource efficient, but we anticipate that the resource cost differential between different encoding schemes will begin to deviate as L increases, as we include a multiflavor model, and as we go to higher dimensions.

An obvious extension of 1+1D QCD is 2+1D and 3+1D QCD. However, several elements conspire to make these models much more challenging. On the matter side, fermionic exchange becomes more cumbersome using the Jordan-Wigner transform since the system must be made quasi-1D [9]. On the gauge-boson side, the additional dimension introduces a new “plaquette” term to the Hamiltonian. Although in certain models for Z_2 and $U(1)$ this term is a four-link operation [9,29], previous work and others’ suggest that a reduced-electric basis for $SU(3)$ requires a control structure entangling at a minimum twelve links [56,57]. Thus the extension of lattice QCD from one dimension to two and three remains an active area of research, and will require new gates and new insights. Nevertheless, our approach offers an intuitive and compelling route to efficiently study small-scale nuclear physics on near-term quantum hardware.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: ANALYSIS OF MPPs

Here, we analyze the effects of MPPs [36] in the context of the larger quoct computational space. MPPs are a specific use of the Compensation for Off-Resonance with a Pulse SEquence (CORPSE) scheme [35] where two CORPSEs, each implementing a net 90° rotation on the relevant transition, are concatenated to perform a total rotation of 180° . The authors of Ref. [36] found that this six-part composite pulse sequence also preserves the motional ground state and used it as a way to mitigate unwanted atomic heating when driving the clock transition: Even when the Rabi frequency Ω at which the clock transition is driven is much larger than the trap frequency ω , the MPP evolves the atomic state through such a trajectory that an atom beginning in its motional ground state will be left with negligible overlap with higher motional states. We find in simulation that the MPP sequence additionally preserves the $m = 1$ and $m = 2$ states, which allows it to be used in the full quoct space for more general-purpose operations. We also find that this behavior is maintained even at larger trap frequencies than those originally studied in Ref. [36]. Whereas Lis *et al.* work at a trap frequency of $2\pi \times 10$ kHz, we fix to $\omega = 2\pi \times 100$ kHz so that larger Rabi frequencies may be used for the motion-selective operations discussed in Appendix C. In Fig. 14, we plot the change in average motional quantum and associated π -pulse fidelity for MPPs applied to states starting in an initial motional state of $m_0 \in \{0, 1, 2\}$ as a function of Ω , and identify an optimum Rabi frequency $\Omega_0 \approx 2\pi \times 810$ kHz. Ω_0 is approximately equal in proportion to that in Ref. [36], where the authors found an optimum at $\approx 2\pi \times 80$ kHz for a trap frequency of $2\pi \times 10$ kHz. Fixing the MPP Rabi frequency to Ω_0 , we find that MPPs preserve superpositions of motional states, up to a negligible phase shift ($\lesssim 1^\circ$) within the superposition, and a global shift of $\approx -88^\circ$ applied to all states involved the transition (i.e., that

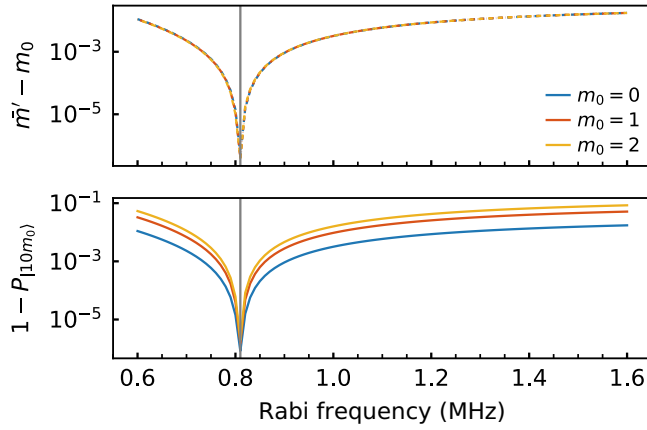


FIG. 14. Preservation of motional quantum numbers with motion-preserving pulses. Starting in the $|00m_0\rangle$ quoct state, we simulate a $X_e(\pi)$ using a motion-preserving pulse and plot the change in average motional quantum (top) and the π -pulse infidelity (bottom) as a function of the Rabi frequency Ω used for the gate for a trap frequency of $\omega = 2\pi \times 100$ kHz. The change in motional quantum is nearly identical for all m_0 , and we identify $\Omega_0 \approx 2\pi \times 810$ kHz as the optimum Rabi frequency for both quantities. We note that Ω_0 agrees proportionally to that in Ref. [36], which found an optimum at $\approx 2\pi \times 80$ kHz for $\omega = 2\pi \times 10$ kHz.

undergo dynamics due to the pulse). We note additionally that population transfer between the $e = 0$ and $e = 1$ subspaces can alternatively be achieved using a three-photon process (rather than driving the clock transition directly) [58,59] to reduce the requisite optical power for a ≈ 810 kHz Rabi frequency.

We similarly analyze the ququart-space pulse implementing the Hadamard gate from Ref. [30] in the context of varied initial motional states and superpositions thereof. Unlike for MPPs, the Hadamard pulse protocol does not strictly require a certain total nutation angle: For a given Rabi frequency, the detuning of the pulse is chosen such that population inversion occurs between the single computational basis states coupled by the transition and the appropriate X-basis states thereof, while the gate time can be freely chosen such that the final state at the end of the pulse has greatest overlap with the expected X-basis state. As such, we again simulate the change in average motional quantum and overall pulse fidelity for initial motional quantum $m_0 \in \{0, 1\}$ ($m_0 = 2$ is excluded because it is only relevant to measurement), but now freely vary the gate time in addition to the Rabi frequency. The results are shown in Fig. 15. We find optimal Rabi frequency $\Omega_0 \approx 2\pi \times 350$ kHz and gate time $T_0 \approx 5.1$ μ s for both $m_0 = 0$ and $m_0 = 1$. For this condition, we find that both the change in average motional quantum and the pulse infidelity are $\lesssim 10^{-3}$, with a phase shift of $\approx 175^\circ$ applied to all $m = 1$ states.

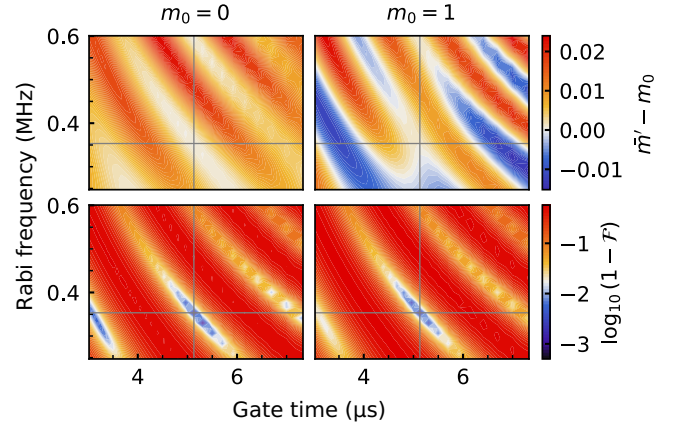


FIG. 15. Preservation of motional quantum numbers with Hadamard pulses. The change in average motional quantum (top) and overall pulse infidelity (bottom) for initial motional state $m_0 = 0$ (left) and $m_0 = 1$ (right) as functions of Rabi frequency and gate time. The vertical and horizontal lines show values for both parameters that minimize the pulse fidelities, coinciding at the same values for both initial motional quanta.

APPENDIX B: MOTIONAL QUBIT FIDELITY IN THE PRESENCE RYDBERG GATE OPERATIONS

We additionally analyze the effect of Rydberg gate operations on the motional qubit. Specifically, we consider a standard “release-and-recapture”-style gate whereby trapped atoms are initially released from optical tweezer confinement while the gate protocol is performed, and then recaptured. The purpose of this is to avoid complications arising from the large electric dipole matrix elements associated with Rydberg states, which can significantly affect the trappability of those states otherwise. However, these large matrix elements also allow for significantly larger Rabi frequencies ($\sim$ MHz) to be used for the gate, typically giving negligible effects on atomic motion for most practical applications.

Here, we consider two main effects on atomic motion with respect to our chosen Rydberg gate protocol [30,43], which has a gate time of $\tau \approx 43$ ns for a typical Rabi frequency of $\Omega = 5$ MHz. The first is momentum transfer from the ≈ 302 nm light used to drive the protocol. Given the scale of Ω , the atomic dynamics during the gate pulses can be taken as highly coherent, from which we may conclude that any incoming photons absorbed by an atom are then emitted back into the laser mode soon thereafter, resulting in no net transfer of momentum to the atom while it is in free space. Additionally, we estimate any shifts to the overall position of the wavefunction occur at the level of ≈ 500 pm, which is nearly a factor of 100 smaller than the scale of the atomic wavefunctions. Thus, we can safely neglect this effect.

The second effect is the spread of the wavefunction that occurs as the atom is allowed to move freely outside

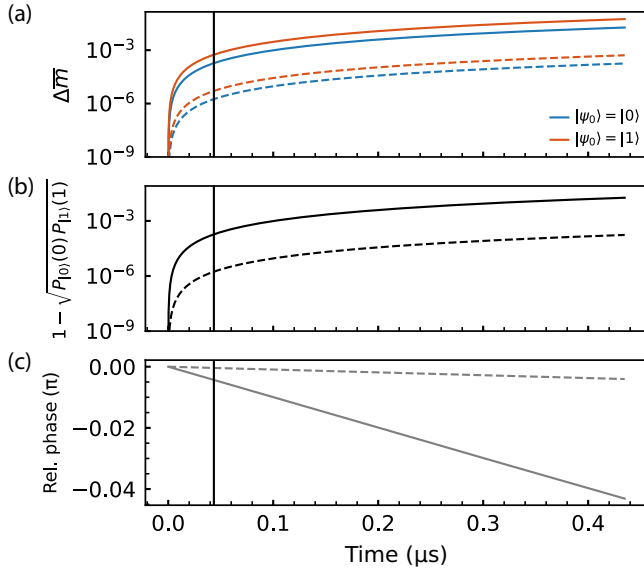


FIG. 16. Preservation of the m qubit during a Rydberg gate protocol. Assuming a release-and-recapture-style Rydberg gate as in Refs. [30,43], we simulate the evolution of motional qubit states when quenched into free space, for initial state $|m = 0\rangle$ (blue) and $|m = 1\rangle$ (red) as well as initial trap frequencies $\omega = 100$ kHz (solid) and $\omega = 10$ kHz (dashed). We show (a) the change in average motional quantum, (b) the geometric average of the overlap probabilities of the evolved states with their initial states, and (c) the accrued relative phase between the two motional qubit states. The black vertical lines show the ideal gate time for the protocol for a typical Rabi frequency of 5 MHz.

its trap. For this, we simulate the evolution of the radial wavefunctions in free space as a function of the gate time and construct a fidelity measure for the motional qubit as a whole. We consider a ^{171}Yb atom initially trapped in a $1\text{ }\mu\text{m}$ -diameter optical tweezer. In independent simulations, the atom is initialized in the $m = 0$ or $m = 1$ motional qubit modes, quenched into free space, and then allowed to evolve freely over the gate time. These simulations are performed for an initial tweezer depth corresponding to the $\omega = 100$ kHz trap frequency of the normal working condition assumed in the main text as well as a shallower $\omega = 10$ kHz condition, as it is typical in a practical setting to first ramp the depth of the tweezer down by a factor of ≈ 100 before releasing atoms for the gate.

In each simulation, we compute the change in average motional quantum associated with each initial state, the relative phase accrued between the two, and an averaged state-preservation fidelity $\mathcal{F}_{\text{mot}} = \sqrt{P_{|0\rangle}(0)P_{|1\rangle}(1)}$, where $P_{|m_0\rangle}(m')$ is the overlap probability of the wavefunction evolved from initial state $|m_0\rangle$ with the m' th motional eigenstate in the appropriate-depth tweezer. The results, shown in Fig. 16, show preservation of the motional qubit at the $\gtrsim 0.999$ level, with accrued relative phase $< 0.005\pi$.

APPENDIX C: PULSE SEQUENCE OPTIMIZATION

The pulse sequences implementing intraquoc two- and three-qubit gates presented in the main text [see Figs. 2 and 6] were found by means of a gradient ascent optimizer applied to simulated multilevel dynamics under applied laser pulses. While some approaches to the general problem of pulse optimization may optimize a time-dependent phase to implement a particular gate [60], we choose to optimize a technically simpler sequence of only three concatenated pulses. Each pulse is on resonance with the relevant sideband of the clock transition, and our optimization scheme varies only the nutation and phase angles for each pulse. This can be seen as a simplified instance of the approach taken in Ref. [61], which expands the computational space to include higher motional modes as a general qudit.

Our scheme is based on the observation that the effective Rabi frequency at which a particular sideband transition will be driven (by a pulse of fixed power and detuning) varies with the motional quantum numbers of the states involved. This allows the motional degree of freedom to be controlled such that undesired leakage out of the computational space is prevented via sequences that perform a net- 2π rotation on erroneous transitions. More generally, there is a trade-off in the number of pulses used in a sequence between the fidelity and duration of the overall operation: A larger number of pulses results in greater control over state trajectories, but longer gate times as well. Since we work in the resolved-sideband regime, the gate time associated with additional pulses in a sequence can be costly. In this regime, Rabi frequencies Ω are constrained to be much less than the trap frequency ω . This introduces another factor in the trade-off between fidelity and gate time, where a larger Rabi frequency will reduce not only the gate time but motional selectivity as well. It follows that increasing ω will then also allow Ω to be increased, but we find an upper limit to this effect in practice. We plot the fidelity and gate time as functions of Ω for a realistic limiting trap frequency $\omega = 2\pi \times 100$ kHz in Fig. 4. Here, we fix to sequences of three pulses and suitable Rabi frequencies to maintain gate fidelities of $\gtrsim 0.99$, with $\sim$ ms gate times.

We simulate the quoc space with motional quantum numbers up to $m = 2$, assuming equalization of nuclear splittings in the ground and clock manifolds via a suitable set of background light shifts [30]. When simulating gates that only involve a subset of the encoded qubits, however, we reduce the size of the simulated Hilbert space to only the relevant degrees of freedom. Given a target unitary matrix U_0 describing a gate operation to implement, we construct an objective function $\mathcal{F}$ from the gate fidelity comparing U_0 to an effective unitary U' computed for pulse angles $\{\theta_k\}, \{\varphi_k\}$ ($k \in \{0, 1, 2\}$) by initializing in

TABLE II. Optimized pulse sequence angles for motion-selective operations. Full listing of the nutation (θ_i), phase (φ_i), and Z gate (α, β, γ) angles for each gate implementation described in Appendix C, with expected gate fidelity $\mathcal{F}$ and gate time τ in milliseconds.

Gate	θ_0	θ_1	θ_2	φ_0	φ_1	φ	α	β	γ	$\mathcal{F}$	τ
SWAP _{<i>e,m</i>}	1.498 π	1.498 π	1.123 π	1.908 π	1.196 π	1.583 π	1.338 π	—	1.415 π	0.9925	5.147
CZ _{<i>e,m</i>}	1.036 π	1.620 π	1.036 π	0.138 π	0.417 π	0.695 π	1.299 π	—	0.377 π	0.9929	3.076
CCZ	1.972 π	1.066 π	0.613 π	0.103 π	0.122 π	0.150 π	1.966 π	1.966 π	1.986 π	0.9814	12.17
SHELVE _{<i>e,m</i>}	1.277 π	1.277 π	0.694 π	0.398 π	0.616 π	1.845 π	0.811 π	—	0.503 π	0.9907	3.248

each available basis state, simulating the pulse sequence, and taking the final states. The full objective function is then

$$\mathcal{F} = \max_{\alpha, \beta, \gamma} \frac{1}{N} \left| \text{Tr} \left(U_0^\dagger \mathcal{Z}(\alpha, \beta, \gamma) U^\dagger \right) \right|^2 \quad (\text{C1})$$

and gradient ascent proceeds on $\{\theta_k\}, \{\varphi_k\}$ from there. Since the full simulated Hilbert space is larger than the computational space, we restrict the trace in the above to only computational states (but extend to include the $|1n2\rangle$ states in the case of the SHELVE_{*e,m*} sequence), adjusting the normalization constant N such that the maximum value attainable by $\mathcal{F}$ is 1. We additionally insert $\mathcal{Z}(\alpha, \beta, \gamma) \equiv Z_m(\gamma)Z_n(\beta)Z_e(\alpha)$ describing a set of phase rotations on each qubit—i.e., maximization of $\mathcal{F}$ results in a pulse sequence that implements U_0 up to single-qubit phase rotations. We list the pulse angles $\{\theta_k\}, \{\varphi_k\}$, phase correction angles α, β, γ , and expected fidelities for chosen Rabi frequencies for each pulse sequence gate in Table II.

APPENDIX D: QUOCT READOUT PROTOCOL

1. Construction and optimization

We construct the quoct measurement protocol (see Fig. 7) using a genetic algorithm to find a minimal set of operations to map computational states into the set of bitstrings describing bright/dark atomic readout results. The state readout process is inherited from previous work on the “ququart” space [30], where photons are scattered identically from all $|0nm\rangle$ states, but with extra considerations regarding motion. Namely, the process of scattering photons will couple to the motion degree of freedom and cause some degree of scrambling within it to any states that are bright with respect to the scattering transition. When this occurs, multiple quoct basis states can be mixed with each other and rendered indistinguishable, even through later rounds of measurement.

We account for this by using the *e-m* shelving operation in Fig. 6 to swap half of the states within the subspace affected by motional scrambling to a designated set of states outside the computational space that is immune. Since no scrambling occurs along the *e*- and *n*-qubit degrees of freedom, this means the states that are not shelved are scrambled independently of all other quoct

states (and each other), such that no two states are scrambled together and all quoct states remain distinguishable.

To implement genetic optimization on measurement protocols, we model a population of protocols undergoing selection and crossover where an individual protocol is a series of short circuits punctuated by a round of photon scattering as described above. We sample an initial population of 100 protocols, where individual protocols are generated by drawing a number of readout rounds from a Poisson distribution with mean 1.5, with an extra condition that the number be at least 1. For each round, a number of gates is drawn from another Poisson distribution with mean 2, and each gate is drawn uniformly from the quoct-native gateset.

The initial population is evolved for one million generations; at each generation, the 50 least-fit protocols are discarded, and 40 new protocols are generated via crossover. Crossover between protocols is implemented as pairwise random selection between the pre-readout circuits of the parent protocols. That is, the k th pre-readout circuit of the offspring is randomly selected from the k th circuits of the parents with equal probability; if one parent comprises fewer circuits than the other, the remaining circuits from the other are individually either passed on or discarded. Following crossover, mutations are applied individually to all protocols whereby the number of circuits in a given protocol is increased by 1 with probability 0.3 (with the new circuit generated following the initial sampling procedure), or decreased by 1 with probability 0.3. For each circuit in the protocol, the number of gates is increased or decreased by 1, each with probability 0.3. Finally, each gate is resampled from the native gateset with probability 0.5. After mutation, 10 new protocols are generated via the initial sampling procedure to maintain population size.

Fitness is evaluated by simulating the protocol for each computational basis state and recording the resulting bitstrings of bright/dark outcomes. A composite fitness score is constructed from a boolean value b_{dups} that is 0 if the bitstrings are unique for all basis states (1 otherwise), combined with a positive integer f

$$f = (1 + n_{\text{rounds}})(1 + n_{\text{gates}})(1 + n_{\text{dups}}) \quad (\text{D1})$$

where n_{rounds} is the number of rounds of readout, n_{gates} is the total number of gates, and n_{dups} is the number of

duplicate output bitstrings. Protocols for which b_{dups} is 0 are coarsely considered fitter than those otherwise, and f is used for fine-scale comparisons within these groups.

At the end of each generation, any protocols with $b_{\text{dups}} = 0$ are copied to a separate population not acted upon by the algorithm. After evolution ends, all such protocols with minimal f are collected. In general, multiple distinct protocols can satisfy the desired conditions, and the particular protocol presented in the main text is arbitrarily selected from this set. However, the optimal protocols our algorithm converges to share distinct characteristics, which we briefly discuss here. Namely, we find no protocols that uniquely map quoct states to bitstrings of length fewer than four. Further, we find no protocols that leave each initial computational basis state in a unique final state—that is, in addition to mapping quoct basis states to bitstrings, each protocol also constitutes a map within the quoct state space, and we find no suitable protocols for which this map is injective. Both of these characteristics may be seen as manifestations of motional scrambling. This additionally highlights the importance of the e - m shelving operation as one that can move information out of the readout subspace—without disturbing information stored elsewhere in the computational space—such that the preserved state information can be extracted through subsequent rounds of readout. Indeed, we find no solutions when it is removed from the gate set, and no solutions that do not use it once before the first measurement.

Given a chosen measurement protocol [that shown in Fig. 7(c), for example], analysis of the association between length- N bitstrings and basis states reveals the possibility for further optimization. Specifically, we consider the sets of input basis states to the protocol that map to the same leading substrings of length $M < N$. We find that any substring with at least one bright readout result is mapped to a set of at most two basis states and, further, that the two basis states differ by only their n qubit. This is due to the nature of the shelving operation in combination with the e -qubit measurement as shown in Fig. 7(a). For instance, after the initial shelving operation and assumed bright first e -qubit measurement in Fig. 7(c), the quoct state is collapsed to a mixed state containing only information initially encoded in the $|0n0\rangle$ states. Given the ability to perform real-time feedback on measurement outcomes, this allows the protocol to be short-circuited by a (partial) n -qubit measurement as shown in Fig. 7(b), which will measure out the remaining information in the state with just one operation. If, on the other hand, an e -qubit measurement is dark, no motional scrambling occurs and the protocol can proceed as shown until the next bright outcome, after which a short-circuiting n -qubit measurement can be applied in the same manner. Thus, in some cases, full quoct readout can be performed in as few as two rounds of either e - or n -qubit readout. Below, we document the minimal bitstrings needed to identify each quoct basis

state using the protocol in Fig. 7(c), in which bright and dark outcomes yielded by an e -qubit readout are denoted as B and D , while those by a short-circuiting n -qubit readout are B' and D' ,

$$\begin{aligned} |000\rangle &: BB' & |010\rangle &: BD', \\ |100\rangle &: DBB' & |110\rangle &: DBD', \\ |001\rangle &: DDBB' & |011\rangle &: DDBD', \\ |101\rangle &: DDDDB & |111\rangle &: DDDD. \end{aligned} \quad (\text{D2})$$

2. Expected readout fidelity

Analysis of the fidelity with which full quoct readout can be performed is based on a calculation of the probability with which the readout protocol above will produce strings of measurement results satisfying the signatures identified in Eq. (D2). As in Ref. [30], we construct directed acyclic graphs (DAGs) to describe the evolution of all initial quoct states through the protocol under the effects of realistic imperfections. Major sources of error in the context of neutral atom experiments include the fidelity $\mathcal{F}_{\text{det}}$ with which bright or dark measurement results can be discriminated and faithfully represent the underlying atomic state, as well as the probabilities P_{surv} and P_{flip} with which atoms survive (i.e., are not ejected from their traps) or have their nuclear qubit flipped within the $e = 0$ manifold by the e - or n -qubit imaging processes. Here, we also include the fidelities $\mathcal{F}_{\text{SHELVE}_{e,m}}$, $\mathcal{F}_{\text{SWAP}_{e,m}}$, and $\mathcal{F}_{\text{SWAP}_{e,n}}$ of the gates involved in the readout protocol.

The readout fidelities presented here are of the form

$$\mathcal{F}_q = P(\text{sig}(q)|q), \quad (\text{D3})$$

where $\text{sig}(q)$ denotes the measurement result signature of the quoct state q , as in Eq. (D2)—that is, we define the readout fidelity on a per-quoct state basis as the probability that our readout protocol produces the correct measurement results for a given input quoct state. The DAGs used to calculate these fidelities are constructed using atomic basis states up to $m = 2$ motional quantum and including the null “empty-tweezer state” resulting from atom loss (possibly with associated measurement results, such as being in the $|000\rangle$ state with associated B result from a particular round of e -qubit readout) as vertices. Transition probabilities involving the aforementioned parameters are edge weights, and hence the probability of a particular measurement string is found by calculating the product of all edge weights along paths satisfying a certain signature and summing the results. Thus substituting realistic values of $\mathcal{F}_{\text{det}} = P_{\text{surv}} = 1 - P_{\text{flip}} = \mathcal{F}_{\text{SWAP}_{e,n}} = 0.999$ as well as values for $\mathcal{F}_{\text{SHELVE}_{e,m}}$ and $\mathcal{F}_{\text{SWAP}_{e,m}}$ from Table II,

we calculate the readout fidelities

$$\begin{aligned} \mathcal{F}_{|000\rangle} &= 0.987 & \mathcal{F}_{|010\rangle} &= 0.988, \\ \mathcal{F}_{|100\rangle} &= 0.988 & \mathcal{F}_{|110\rangle} &= 0.989, \\ \mathcal{F}_{|001\rangle} &= 0.976 & \mathcal{F}_{|011\rangle} &= 0.977, \\ \mathcal{F}_{|101\rangle} &= 0.969 & \mathcal{F}_{|111\rangle} &= 0.970. \end{aligned} \quad (\text{D4})$$

We note that, broadly, the readout fidelities for states with longer signatures are slightly lower due to the extra operations required to fully interrogate the state. Additionally, the fidelities for states whose signatures terminate with a D or D' result are slightly higher than those of an equal length with B or B' because the empty-tweezer state is an attractor in the protocol, making dark measurement results slightly more likely.

APPENDIX E: REVIEW OF THE QUOCT BASIS FOR THE KS HAMILTONIAN

The ground state for the Dirac field is the Fermi sea with all the negative energy states occupied. The $\hat{\psi}_n = \hat{c}_{\bar{c},n}$ operator on odd sites then produces antiparticles by lowering the occupied state. We follow [48] and make the particle-hole transformation $\hat{c}_{\bar{c}} \rightarrow \hat{c}_c^\dagger$ so that quoct excited states correspond to the creation of antiquarks. The KS Hamiltonian can then be written in the basis of the quoct states. This amounts to a representation of operators $\hat{O}$ as matrices $\tilde{O} = [\hat{O}]_{\text{quoct}}$.

Kinetic term: The kinetic Hamiltonian is

$$\hat{H}_{\text{kin}} = \frac{1}{2} \sum_c \sum_{n=0}^{r,g,b \ 2L-2} (-1)^n \left(\hat{c}_{c,n}^\dagger \hat{c}_{c,n+1}^\dagger + \text{h.c.} \right). \quad (\text{E1})$$

To construct the matrix representations of the ladder operators (denoted $\tilde{c}_c$ or $[\hat{c}_c]_{\text{quoct}}$) in the single-site Hilbert space of each quoct, we examine how each operator acts on the quoct basis,

$$\begin{aligned} \tilde{c}_r &= |0\rangle \langle r| + |g\rangle \langle rg| - |b\rangle \langle rb| + |gb\rangle \langle rgb|, \\ \tilde{c}_g &= |0\rangle \langle g| - |r\rangle \langle rg| + |b\rangle \langle gb| + |rb\rangle \langle rgb|, \\ \tilde{c}_b &= |0\rangle \langle b| + |r\rangle \langle rb| - |g\rangle \langle gb| + |rg\rangle \langle rgb|. \end{aligned} \quad (\text{E2})$$

Observe that the minus signs come from the choice of fermion ordering for the basis and satisfy anticommutation relations on the single site [62]. As in the Jordan-Wigner transformation, we need to include a phase dependent on the number of occupied fermions in order to satisfy anticommutativity on different sites. On each site, define $(-1)^{n_q}$ which measures the number of fermions present; in the quoct basis, we define the matrix $\tilde{P} = \text{diag}(1, -1, -1, -1, 1, 1, 1, -1)$ on each site. Thus the operator $\hat{c}_{c,n}^\dagger$ gets mapped to the quoct string

$\tilde{P} \otimes \cdots \otimes \tilde{P} \otimes \tilde{c}_c^\dagger$, where the site label on the operator determines the size of the string. Thus

$$[\hat{c}_{c,n}^\dagger \hat{c}_{c,n+1}^\dagger]_{\text{quoct}} = \tilde{I} \cdots \tilde{I} \otimes (\tilde{c}_c^\dagger \tilde{P}) \otimes \tilde{c}_c^\dagger \otimes \tilde{I} \cdots \tilde{I}. \quad (\text{E3})$$

Then the matrix form of the kinetic Hamiltonian is (suppressing single-site identity factors),

$$\tilde{H}_{\text{kin}} = \frac{1}{2} \sum_c \sum_{n=0}^{r,g,b \ 2L-2} (-1)^n \left[(\tilde{c}_c^\dagger \tilde{P})_n \tilde{c}_{c,n+1}^\dagger - (\tilde{c}_c \tilde{P})_n \tilde{c}_{c,n+1} \right]. \quad (\text{E4})$$

Mass and Chemical Potential: The mass term is

$$\hat{H}_{\text{mass}} = m \sum_c \sum_{n=0}^{r,g,b \ L-1} \hat{c}_{c,n}^\dagger \hat{c}_{c,n} \quad (\text{E5})$$

and in the quoct basis

$$[\hat{c}_c^\dagger \hat{c}_c]_{\text{quoct}} = \tilde{M} = \text{diag}(0, 1, 1, 1, 2, 2, 2, 3). \quad (\text{E6})$$

The chemical potential will have an extra sign factor $(-1)^n$ that computes the difference in occupation numbers on even (fermionic) and odd (antiferromionic) sites,

$$\hat{H}_{\text{chem}} = \mu \sum_c \sum_{n=0}^{r,g,b \ L-1} (-1)^n \hat{c}_{c,n}^\dagger \hat{c}_{c,n},$$

$$\tilde{H}_{\text{mass}} + \tilde{H}_{\text{chem}} = \sum_{n=0}^{L-1} (m + (-1)^n \mu) \underbrace{\tilde{I} \cdots \tilde{I} \otimes \tilde{M} \otimes \tilde{I} \cdots \tilde{I}}_n. \quad (\text{E7})$$

Electric term: We can write the charge operator on even sites in terms of the creation and annihilation operators as discussed in Appendix F,

$$\hat{Q}^a = \sum_{c=r,g,b} \sum_{c'=r,g,b} \hat{c}_c^\dagger T_{c,c'}^a \hat{c}_{c'}. \quad (\text{E8})$$

From this it is clear that $|0\rangle$ is annihilated and single quark states transform by T^a . What about the three diquark and one triquark states? From an algebraic perspective, $|0\rangle$ is a color singlet and transforms under the trivial, one-dimensional representation (1) and the color-charge generator is 0. Likewise the single quark states $|c\rangle$ transform under the three-dimensional fundamental representation (3) so the color-charge generator is T^a . The diquark states come from the tensor decomposition of $3 \otimes 3 = 6 \oplus \bar{3}$. Since our quark states have to be antisymmetric, the diquark states must then transform under the three-dimensional antifundamental representation ($\bar{3}$) and their

color-charge generator is $\tilde{T}^a := -(T^a)^*$. Lastly the triquark states come from $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$ and only the $\mathbf{1}$ is totally antisymmetric so their color-charge generator is 0. This can also be computed directly by applying $\hat{Q}^a$ to a color state and using the fact that T^a is hermitian and traceless. Thus from computing how $\hat{Q}^a$ acts on quoct states, we can write the matrix representation $\tilde{Q}^a$ in the quoct basis

$$[\hat{Q}^a]_{\text{quoct}} = \tilde{Q}^a = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & T^a & 0 & 0 \\ 0 & 0 & \tilde{T}^a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (\text{E9})$$

On odd sites, the charge operator has the following form:

$$\hat{Q}^a = \sum_{c=r,g,b} \sum_{c'=r,g,b} \hat{c}_{\tilde{c}}^\dagger T_{\tilde{c},\tilde{c}'}^a \hat{c}_{\tilde{c}'}. \quad (\text{E10})$$

So on odd sites we have a different matrix representation for the charge operator

$$[\hat{Q}^a]_{\text{quoct}} = \tilde{\tilde{Q}}^a = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \tilde{T}^a & 0 & 0 \\ 0 & 0 & T^a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (\text{E11})$$

Since the charge operators on different sites commute, the electric term of the Hamiltonian can be written

$$\begin{aligned} \hat{H}_{\text{elec}} &= \frac{ag^2}{2} \sum_{n=0}^{2L-2} \left(\sum_m^n \hat{Q}_m^2 + 2 \sum_{a=1}^8 \sum_{m < m'}^n \hat{Q}_m^a \hat{Q}_{m'}^a \right) \\ &= \frac{ag^2}{2} \sum_{n=0}^{2L-2} (2L-1-n) \hat{Q}_n^2 \\ &\quad + ag^2 \sum_{a=1}^8 \sum_{n=0}^{2L-2} \sum_{m > n}^{2L-2} (2L-1-m) \hat{Q}_n^a \hat{Q}_m^a \end{aligned} \quad (\text{E12})$$

On both even and odd sites, the operator $\hat{Q}_m^2 = \sum_a \hat{Q}_m^a \hat{Q}_m^a$ has a simple site-local representation

$$\tilde{Q}^2 = (\tilde{Q}^a)^2 = (\tilde{\tilde{Q}}^a)^2 = \frac{4}{3} \text{diag}(0, 1, 1, 1, 1, 1, 1, 0). \quad (\text{E13})$$

The charge-pairing terms $\hat{Q}_m^a \hat{Q}_{m'}^a$ will be discussed more in Sec. III B 3 but a useful fact is that $\tilde{Q}^a \otimes \tilde{\tilde{Q}}^a = \tilde{\tilde{Q}}^a \otimes \tilde{Q}^a$ and $\tilde{Q}^a \otimes \tilde{Q}^a = \tilde{\tilde{Q}}^a \otimes \tilde{\tilde{Q}}^a$. Thus we can rewrite the quoct

representation of $\hat{H}_{\text{elec}}$ (suppressing identities),

$$\begin{aligned} \tilde{H}_{\text{elec}} &= \frac{ag^2}{2} \sum_{n=0}^{2L-2} (2L-1-n) \tilde{Q}_n^2 \\ &\quad + ag^2 \sum_{a=1}^8 \sum_{m=0}^{2L-2} \sum_{m' > m}^{2L-2} (2L-1-m') \tilde{Q}_m^a \tilde{Q}_{m'}^a \\ &\quad + ag^2 \sum_{a=1}^8 \sum_{k=0}^{2L-2} \sum_{k' > k}^{2L-2} (2L-1-k') \tilde{Q}_k^a \tilde{\tilde{Q}}_{k'}^a, \end{aligned} \quad (\text{E14})$$

where the second sum is over m, m' both even or both odd, and the third sum is over k, k' where either k is even and k' is odd, or vice versa.

APPENDIX F: AXIAL GAUGE IN THE CONTINUUM TO KS

Consider the locally gauge-invariant $SU(3)$ Lagrangian density [63],

$$\mathcal{L} = -\frac{1}{4g^2} F^{\mu\nu a} F_{\mu\nu}^a + \bar{\psi}_i (i \not{D}_{ij} - m \delta_{ij}) \psi_j, \quad (\text{F1})$$

where

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c \quad (\text{F2})$$

describes the color field strength indexed in the adjoint representation by a and

$$D_\mu \psi_i = \partial_\mu \psi_i - i A_\mu^a T_{ij}^a \psi_j \quad (\text{F3})$$

is the covariant derivative indexed in the fundamental representation by i . We find the canonical momenta the usual way,

$$\begin{aligned} \Pi^{\mu a} &= \frac{\delta \mathcal{L}}{\delta \partial_0 A_\mu^a} = \frac{1}{g^2} F^{\mu 0 a}, \\ \pi_i &= i \psi_i^\dagger. \end{aligned} \quad (\text{F4})$$

and note the primary constraint $\Pi_0^a = 0$. This means that A_0^a is nondynamical. We can then compute the Hamiltonian density the usual way [64],

$$\mathcal{H} = \Pi^{\mu a} \partial_0 A_\mu^a + \frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu a} + \pi \partial_0 \psi - \bar{\psi} (i \not{D} - m) \psi. \quad (\text{F5})$$

We compute the gauge part first. Using

$$-g^2 \Pi_\mu^a = F_{0\mu}^a = \partial_0 A_\mu^a - \partial_\mu A_0^a + f^{abc} A_0^b A_\mu^c \quad (\text{F6})$$

and taking the signature $(+, -, -, -)$, we have

$$-\partial_0 A_k^a = g^2 \Pi_k^a + \partial_k A_0^a + f^{abc} A_0^b A_k^c \quad (\text{F7})$$

and

$$\frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu a} = -\frac{g^2}{2} \Pi_k^a \Pi^{ka} + \frac{1}{4g^2} F_{kl}^a F^{kla}. \quad (\text{F8})$$

Thus the pure gauge-field Hamiltonian density is

$$\begin{aligned} \mathcal{H}_A \Pi^{\mu a} \partial_0 A_\mu^a + \frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu a} \\ = \frac{g^2}{2} \Pi_k^a \Pi^{ka} + \partial_k A_0^a \Pi^{ka} + f^{abc} \Pi^{ka} A_0^b A_k^c + \frac{1}{4g^2} F_{kl}^a F^{kla}. \end{aligned} \quad (\text{F9})$$

For the matter Hamiltonian density, we obtain

$$\mathcal{H}_M = -i\bar{\psi} \gamma^k \partial_k \psi - \bar{\psi} A^a T^a \psi + m\bar{\psi} \psi. \quad (\text{F10})$$

Now we can simplify our expression by restricting to $1+1$ dimensions and fixing an axial gauge $A_1 = 0$,

$$\begin{aligned} \mathcal{H} = \frac{g^2}{2} \Pi^a \Pi^a + \partial_x A_0^a \Pi^a - i\psi^\dagger \gamma^0 \gamma^x \partial_x \psi - A_0^a \psi^\dagger T^a \psi \\ + m\psi^\dagger \gamma^0 \psi, \end{aligned} \quad (\text{F11})$$

where $\Pi^a := \Pi^{1,a}$. Integrating by parts, identifying $-E^a = F^{10a} = \Pi^a$, and identifying the charged matter current $j^{\mu a} = \bar{\psi} \gamma^\mu T^a \psi$, we get

$$\begin{aligned} \mathcal{H} = -i\psi^\dagger \gamma^0 \gamma^x \partial_x \psi + m\psi^\dagger \gamma^0 \psi + \frac{g^2}{2} (E^a)^2 \\ + A_0 (\partial_x E^a - j^{0a}), \end{aligned} \quad (\text{F12})$$

where A_0 takes the form of a Lagrange multiplier enforcing Gauss' law $\partial_x E^a = \psi^\dagger T^a \psi$.

Imposing Gauss' law, taking the gamma matrix representation $\gamma^0 = \sigma_z$, $\gamma^1 = i\sigma_y$, and decomposing the spinor $\psi = (\psi_1, \psi_2)^T$, the Hamiltonian becomes

$$\begin{aligned} H = \int dx \left(-i\psi_1^\dagger \partial_x \psi_2 - i\psi_2^\dagger \partial_x \psi_1 + m\psi_1^\dagger \psi_1 - m\psi_2^\dagger \psi_2 \right. \\ \left. + \frac{g^2}{2} (E^a)^2 \right). \end{aligned} \quad (\text{F13})$$

Discretizing space and staggering the fields so that $\psi_1 \rightarrow \frac{1}{\sqrt{a}} \psi_{2n}$ and $\psi_2 \rightarrow \frac{1}{\sqrt{a}} \psi_{2n+1}$, we recover the KS Hamiltonian,

$$H = \sum_n \psi_n^\dagger \frac{(\psi_{n+1} - \psi_{n-1})}{ia} + m(-1)^n \psi_n^\dagger \psi_n + \frac{ag^2}{2} E_n^2. \quad (\text{F14})$$

We can then use the discretized Gauss' law constraint to express the chromo-electric energy in terms of the integral

of the charge density in the absence of a background field

$$E_n^a - E_{n-1}^a = \psi_n^\dagger T^a \psi_n =: Q_n^a. \quad (\text{F15})$$

With this we recover Eq. (2) for $\mu = 0$. An alternate derivation can be found in [47].

APPENDIX G: CIRCUIT COMPILATION

1. Circuit identity

Let A and B be two-quoct operators that are hermitian, unitary, and choose a basis such that A exchanges states, i.e., given the basis $\{|1\rangle, \dots, |8\rangle\}$, there is a partition into two sets $\{|i_1\rangle, \dots, |i_4\rangle\}$ and $\{|j_1\rangle, \dots, |j_4\rangle\}$ such that $A|i_n\rangle = |j_n\rangle$. Then we can prove the quoct circuit identity in Eq. (9),

$$x \text{---} \boxed{\exp(-\frac{i\alpha}{2} A \otimes B)} \text{---} = \begin{array}{c} q \text{---} \bullet \text{---} \boxed{A(\alpha)} \text{---} \bullet \text{---} \\ | \\ \bar{q} \text{---} \boxed{B} \text{---} \text{---} \boxed{B} \end{array} \quad (\text{G1})$$

where the control is on the four states $|i_n\rangle$ and will match the transitions of A . Since A and B are hermitian and unitary, they are involutory so

$$\exp(i\alpha A \otimes B) = \cos(\alpha) I_{64} + i \sin(\alpha) A \otimes B, \quad (\text{G2})$$

where I_{64} is the 64-dim identity matrix. Recall that

$$A \otimes B = \begin{pmatrix} a_{11}B & \dots & a_{18}B \\ \vdots & \ddots & \vdots \\ a_{81}B & \dots & a_{88}B \end{pmatrix} = a_{kl} B \quad (\text{G3})$$

and since A exchanges states, $a_{kl} = 0$ whenever k and l belong to the same partition.

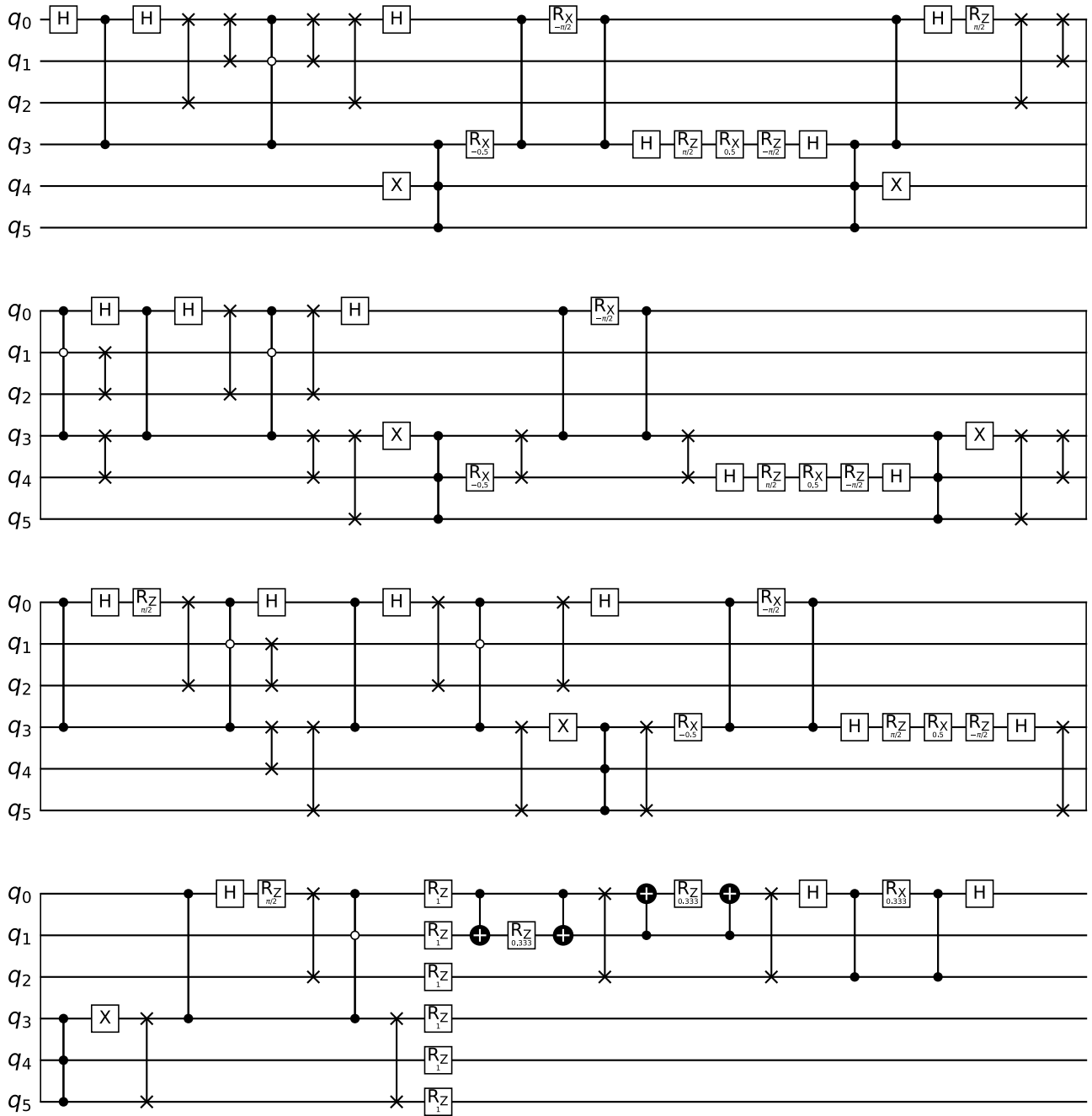
Next, consider the controlled- B operator CB . In a matrix representation CB is a block-diagonal matrix which has a B block when $q = i_n$ and has an I block when $q = j_n$. The $A(\alpha)$ gate along with the empty wire has the matrix representation

$$\exp(i\alpha A) \otimes I_8 = \cos(\alpha) I_{64} + i \sin(\alpha) A \otimes I_8. \quad (\text{G4})$$

Thus we want to show $CB(A \otimes I_8)CB = A \otimes B$. Since CB is block-diagonal, left-multiplication acts on the rowspace and right-multiplication acts on the column space. Observe then that since $B^2 = I_8$,

$$CB(A \otimes I_8)CB = a_{kl} \begin{cases} I_8 & \text{both } k, l \text{ in same partition,} \\ B & k, l \text{ in opposite partitions,} \end{cases} \quad (\text{G5})$$

since we have noted that $a_{kl} = 0$ when k and l are in the same partition, the equality is proven.

FIG. 17. Fully transpiled circuit for the $L = 1$ Hamiltonian.

2. Full $L = 1$ circuit

Since the circuit can be simplified by combining SWAPs and to get a sense for one Trotter step of the $L = 1$ circuit, we include an example with $\mu = 0$ for completeness.

- [1] J. Preskill, Quantum computing in the NISQ era and beyond, *Quantum* **2**, 79 (2018).

- [2] M. C. Bañuls, R. Blatt, J. Catani, A. Celi, J. I. Cirac, M. Dalmonte, L. Fallani, K. Jansen, M. Lewenstein, S. Montangero, C. A. Muschik, B. Reznik, E. Rico, L. Tagliacozzo, K. Van Acoleyen, F. Verstraete, U.-J. Wiese, M. Wingate, J. Zakrzewski, and P. Zoller, Simulating lattice gauge theories within quantum technologies, *Eur. Phys. J. D* **74**, 165 (2020).
- [3] D. Paulson, L. Dellantonio, J. F. Haase, A. Celi, A. Kan, A. Jena, C. Kokail, R. van Bijnen, K. Jansen, P. Zoller, and C. A. Muschik, Simulating 2D effects in lattice gauge

- theories on a quantum computer, *PRX Quantum* **2**, 030334 (2021).
- [4] R. C. Farrell, I. A. Chernyshev, S. J. M. Powell, N. A. Zemlevskiy, M. Illa, and M. J. Savage, Preparations for quantum simulations of quantum chromodynamics in 1+1 dimensions. I. Axial gauge, *Phys. Rev. D* **107**, 054512 (2023).
 - [5] R. C. Farrell, I. A. Chernyshev, S. J. M. Powell, N. A. Zemlevskiy, M. Illa, and M. J. Savage, Preparations for quantum simulations of quantum chromodynamics in 1+1 dimensions. II. Single-baryon β -decay in real time, *Phys. Rev. D* **107**, 054513 (2023).
 - [6] P. Jordan and E. Wigner, Über das Paulische Äquivalenzverbot, *Z. Phys.* **47**, 631 (1928).
 - [7] S. B. Bravyi and A. Y. Kitaev, Fermionic quantum computation, *Ann. Phys.* **298**, 210 (2002).
 - [8] D. González-Cuadra, D. Bluvstein, M. Kalinowski, R. Kaubruegger, N. Maskara, P. Naldesi, T. V. Zache, A. M. Kaufman, M. D. Lukin, H. Pichler, B. Vermersch, J. Ye, and P. Zoller, Fermionic quantum processing with programmable neutral atom arrays, *Proc. Natl. Acad. Sci. U.S.A.* **120**, e2304294120 (2023).
 - [9] E. Crane, K. C. Smith, T. Tomesh, A. Eickbusch, J. M. Martyn, S. Kühn, L. Funcke, M. A. DeMarco, I. L. Chuang, N. Wiebe, A. Schuckert, and S. M. Girvin, Hybrid oscillator-qubit quantum processors: Simulating fermions, bosons, and gauge fields, [arXiv:2409.03747](https://arxiv.org/abs/2409.03747).
 - [10] B. M. Terhal, J. Conrad, and C. Vuillot, Towards scalable bosonic quantum error correction, *Quantum Sci. Technol.* **5**, 043001 (2020).
 - [11] W. Cai, Y. Ma, W. Wang, C.-L. Zou, and L. Sun, Bosonic quantum error correction codes in superconducting quantum circuits, *Fundam. Res.* **1**, 50 (2021).
 - [12] L. E. Fischer, A. Chiesa, F. Tacchino, D. J. Egger, S. Carretta, and I. Tavernelli, Universal qudit gate synthesis for transmons, *PRX Quantum* **4**, 030327 (2023).
 - [13] Z. Wang, R. W. Parker, E. Champion, and M. S. Blok, High-EJ/EC transmon qudits with up to 12 levels, *Phys. Rev. Appl.* **23**, 034046 (2025).
 - [14] P. J. Low, B. M. White, A. A. Cox, M. L. Day, and C. Senko, Practical trapped-ion protocols for universal qudit-based quantum computing, *Phys. Rev. Res.* **2**, 033128 (2020).
 - [15] S. Omanakuttan, A. Mitra, M. J. Martin, and I. H. Deutsch, Quantum optimal control of ten-level nuclear spin qudits in ^{87}Sr , *Phys. Rev. A* **104**, L060401 (2021).
 - [16] M. Ringbauer, M. Meth, L. Postler, R. Stricker, R. Blatt, P. Schindler, and T. Monz, A universal qudit quantum processor with trapped ions, *Nat. Phys.* **18**, 1053 (2022).
 - [17] P. Hrmo, B. Wilhelm, L. Gerster, M. W. van Mourik, M. Huber, R. Blatt, P. Schindler, T. Monz, and M. Ringbauer, Native qudit entanglement in a trapped ion quantum processor, *Nat. Commun.* **14**, 2242 (2023).
 - [18] S. Omanakuttan, A. Mitra, E. J. Meier, M. J. Martin, and I. H. Deutsch, Qudit entanglers using quantum optimal control, *PRX Quantum* **4**, 040333 (2023).
 - [19] P. J. Low, B. White, and C. Senko, Control and readout of a 13-level trapped ion qudit, *Npj Quantum Inf.* **11**, 85 (2025).
 - [20] S. Omanakuttan, V. Buchemmavari, J. A. Gross, I. H. Deutsch, and M. Marvian, Fault-tolerant quantum computation using large spin cat-codes, *PRX Quantum* **5**, 020355 (2024).
 - [21] I. V. Zalivako, A. S. Nikolaeva, A. S. Borisenko, A. E. Korolkov, P. L. Sidorov, K. P. Galstyan, N. V. Semenin, V. N. Smirnov, M. A. Aksenov, K. M. Makushin, E. O. Kiktenko, A. K. Fedorov, I. A. Semerikov, K. Y. Khabarova, and N. N. Kolachevsky, Towards multiqubit quantum processor based on a $^{171}\text{Yb}^+$ ion string: Realizing basic quantum algorithms, *Quantum Rep.* **7**, 19 (2025).
 - [22] J. D. Jost, J. P. Home, J. M. Amini, D. Hanneke, R. Ozeri, C. Langer, J. J. Bollinger, D. Leibfried, and D. J. Wineland, Entangled mechanical oscillators, *Nature* **459**, 683 (2009).
 - [23] C. Flühmann, T. L. Nguyen, M. Marinelli, V. Negnevitsky, K. Mehta, and J. P. Home, Encoding a qubit in a trapped-ion mechanical oscillator, *Nature* **566**, 513 (2019).
 - [24] A. L. Shaw, P. Scholl, R. Finkelstein, R. B.-S. Tsai, J. Choi, and M. Endres, Erasure cooling, control, and hyperentanglement of motion in optical tweezers, *Science* **388**, 845 (2025).
 - [25] P. T. Grochowski, H. Pichler, C. A. Regal, and O. Romero-Isart, Quantum control of continuous systems via nonharmonic potential modulation, *Quantum* **9**, 1824 (2025).
 - [26] Y. Liu, S. Singh, K. C. Smith, E. Crane, J. M. Martyn, A. Eickbusch, A. Schuckert, R. D. Li, J. Sinanan-Singh, M. B. Soley, T. Tsunoda, I. L. Chuang, N. Wiebe, and S. M. Girvin, Hybrid oscillator-qubit quantum processors: Instruction set architectures, abstract machine models, and applications, [arXiv:2407.10381](https://arxiv.org/abs/2407.10381).
 - [27] Y. Liu, J. M. Martyn, J. Sinanan-Singh, K. C. Smith, S. M. Girvin, and I. L. Chuang, Toward mixed analog-digital quantum signal processing: Quantum AD/DA conversion and the Fourier transform, *IEEE Trans. Signal Process.* **73**, 3641 (2025).
 - [28] L. H. Bohnmann, D. F. Locher, J. Zeiher, and M. Müller, Bosonic quantum error correction with neutral atoms in optical dipole traps, *Phys. Rev. A* **111**, 022432 (2025).
 - [29] M. Meth, J. Zhang, J. F. Haase, C. Edmunds, L. Postler, A. J. Jena, A. Steiner, L. Dellantonio, R. Blatt, P. Zoller, T. Monz, P. Schindler, C. Muschik, and M. Ringbauer, Simulating two-dimensional lattice gauge theories on a qudit quantum computer, *Nat. Phys.* **21**, 570 (2025).
 - [30] Z. Jia, W. Huie, L. Li, W. K. C. Sun, X. Hu, Aakash, H. Kogan, A. Karve, J. Y. Lee, and J. P. Covey, An architecture for two-qubit encoding in neutral ytterbium-171 atoms, *npj Quantum Inf.* **10**, 106 (2024).
 - [31] W. C. Campbell and E. R. Hudson, Polyqubit quantum processing, [arXiv:2210.15484](https://arxiv.org/abs/2210.15484).
 - [32] A. M. Kaufman, B. J. Lester, and C. A. Regal, Cooling a single atom in an optical tweezer to its quantum ground state, *Phys. Rev. X* **2**, 041014 (2012).
 - [33] J. D. Thompson, T. G. Tiecke, A. S. Zibrov, V. Vuletić, and M. D. Lukin, Coherence and Raman sideband cooling of a single atom in an optical tweezer, *Phys. Rev. Lett.* **110**, 133001 (2013).
 - [34] A. Jenkins, J. W. Lis, A. Senoo, W. F. McGrew, and A. M. Kaufman, Ytterbium nuclear-spin qubits in an optical tweezer array, *Phys. Rev. X* **12**, 021027 (2022).

- [35] H. K. Cummins, G. Llewellyn, and J. A. Jones, Tackling systematic errors in quantum logic gates with composite rotations, *Phys. Rev. A* **67**, 042308 (2003).
- [36] J. W. Lis, A. Senoo, W. F. McGrew, F. Rönchen, A. Jenkins, and A. M. Kaufman, Midcircuit operations using the omg architecture in neutral atom arrays, *Phys. Rev. X* **13**, 041035 (2023).
- [37] L. Li, X. Hu, Z. Jia, W. Huie, W. K. C. Sun, Aakash, Y. Dong, N. Hiri-O-Tuppa, and J. P. Covey, Parallelized telecom quantum networking with a ytterbium-171 atom array, *Nat. Phys.* **21**, 1826 (2025).
- [38] L. Isenhower, E. Urban, X. L. Zhang, A. T. Gill, T. Henage, T. A. Johnson, T. G. Walker, and M. Saffman, Demonstration of a neutral atom controlled-NOT quantum gate, *Phys. Rev. Lett.* **104**, 010503 (2010).
- [39] M. Saffman, T. G. Walker, and K. Mølmer, Quantum information with Rydberg atoms, *Rev. Mod. Phys.* **82**, 2313 (2010).
- [40] T. Wilk, A. Gaëtan, C. Evellin, J. Wolters, Y. Miroshnychenko, P. Grangier, and A. Browaeys, Entanglement of two individual neutral atoms using Rydberg blockade, *Phys. Rev. Lett.* **104**, 010502 (2010).
- [41] Y.-Y. Jau, A. M. Hankin, T. Keating, I. H. Deutsch, and G. W. Biedermann, Entangling atomic spins with a Rydberg-dressed spin-flip blockade, *Nat. Phys.* **12**, 71 (2016).
- [42] T. M. Graham, M. Kwon, B. Grinkemeyer, Z. Marra, X. Jiang, M. T. Lichtman, Y. Sun, M. Ebert, and M. Saffman, Rydberg-mediated entanglement in a two-dimensional neutral atom qubit array, *Phys. Rev. Lett.* **123**, 230501 (2019).
- [43] H. Levine, A. Keesling, G. Semeghini, A. Omran, T. T. Wang, S. Ebadi, H. Bernien, M. Greiner, V. Vuletić, H. Pichler, and M. D. Lukin, Parallel implementation of high-fidelity multiqubit gates with neutral atoms, *Phys. Rev. Lett.* **123**, 170503 (2019).
- [44] W. Huie, L. Li, N. Chen, X. Hu, Z. Jia, W. K. C. Sun, and J. P. Covey, Repetitive readout and real-time control of nuclear spin qubits in 171Yb atoms, *PRX Quantum* **4**, 030337 (2023).
- [45] M. A. Norcia *et al.*, Midcircuit qubit measurement and rearrangement in a Yb-171 atomic array, *Phys. Rev. X* **13**, 041034 (2023).
- [46] J. Kogut and L. Susskind, Hamiltonian formulation of Wilson's lattice gauge theories, *Phys. Rev. D* **11**, 395 (1975).
- [47] P. Sala, T. Shi, S. Kühn, M. C. Bañuls, E. Demler, and J. I. Cirac, Variational study of U(1) and SU(2) lattice gauge theories with gaussian states in 1 + 1 dimensions, *Phys. Rev. D* **98**, 034505 (2018).
- [48] M. Illa, C. E. P. Robin, and M. J. Savage, Qu8its for quantum simulations of lattice quantum chromodynamics, *Phys. Rev. D* **110**, 014507 (2024).
- [49] T. Albash and D. A. Lidar, Adiabatic quantum computation, *Rev. Mod. Phys.* **90**, 015002 (2018).
- [50] R. C. Farrell, M. Illa, A. N. Ciavarella, and M. J. Savage, Quantum simulations of hadron dynamics in the Schwinger model using 112 qubits, *Phys. Rev. D* **109**, 114510 (2024).
- [51] S. P. Jordan, K. S. M. Lee, and J. Preskill, Quantum algorithms for quantum field theories, *Science* **336**, 1130 (2012).
- [52] Z. Davoudi, C.-C. Hsieh, and S. V. Kadam, Scattering wave packets of hadrons in gauge theories: Preparation on a quantum computer, *Quantum* **8**, 1520 (2024).
- [53] D. Gottesman, A. Kitaev, and J. Preskill, Encoding a qubit in an oscillator, *Phys. Rev. A* **64**, 012310 (2001).
- [54] V. V. Albert, J. P. Covey, and J. Preskill, Robust encoding of a qubit in a molecule, *Phys. Rev. X* **10**, 031050 (2020).
- [55] S. P. Jain, E. R. Hudson, W. C. Campbell, and V. V. Albert, Absorption-emission codes for atomic and molecular quantum information platforms, *Phys. Rev. Lett.* **133**, 260601 (2024).
- [56] P. Balaji, C. Conefrey-Shinozaki, P. Draper, J. K. Elhaderi, D. Gupta, L. Hidalgo, A. Lytle, and E. Rinaldi, Quantum circuits for SU(3) lattice gauge theory, *Phys. Rev. D* **112**, 054511 (2025).
- [57] A. Ciavarella, N. Klco, and M. J. Savage, Trailhead for quantum simulation of SU(3) Yang-Mills lattice gauge theory in the local multiplet basis, *Phys. Rev. D* **103**, 094501 (2021).
- [58] D. S. Barker, N. C. Pisenti, B. J. Reschovsky, and G. K. Campbell, Three-photon process for producing a degenerate gas of metastable alkaline-earth-metal atoms, *Phys. Rev. A* **93**, 053417 (2016).
- [59] G. Panelli, S. C. Burd, E. J. Porter, and M. Kasevich, Doppler-free three-photon spectroscopy on narrow-line optical transitions, *Phys. Rev. A* **111**, 033112 (2025).
- [60] S. Jandura and G. Pupillo, Time-optimal two- and three-qubit gates for Rydberg atoms, *Quantum* **6**, 712 (2022).
- [61] Y. Liu, J. Sinanan-Singh, M. T. Kearney, G. Mintzer, and I. L. Chuang, Constructing qudits from infinite-dimensional oscillators by coupling to qubits, *Phys. Rev. A* **104**, 032605 (2021).
- [62] If we chose $|br\rangle$ instead of $|rb\rangle$, for example, there would be no minus phase in $\tilde{c}_r$ but two in $\tilde{c}_g$ and $\tilde{c}_b$.
- [63] M. D. Schwartz, *Quantum Field Theory and the Standard Model* (Cambridge University, Cambridge, England, 2014).
- [64] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University, Cambridge, England, 1996).