

# Fast Gates for Bit-Flip Protected Superconducting Qubits

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Superconducting qubits offer an unprecedentedly high degree of flexibility in terms of circuit encoding and parameter choices. However, in designing the qubit parameters, one typically faces the conflicting goals of long coherence times and simple control capabilities. Both are determined by the wavefunction overlap of the qubit basis states and the corresponding matrix elements. Here, we address this problem by introducing a qubit architecture with real-time tunable bit-flip protection. In the first, the “heavy” regime, the energy relaxation time can be on the order of hours for fluxons located in two near-degenerate ground states, as recently demonstrated in Ref. [Hassani *et al.*, Nat. Commun. **14** (2023)]. The second, “light” regime, facilitates high-fidelity control on nanosecond timescales without the need for microwave signals. We propose two different tuning mechanisms of the qubit potential and show that base-band flux-pulses of around 10 ns are sufficient to realize a universal set of high-fidelity single- and two-qubit gates. This realizes a hardware-efficient and strongly noise-biased qubit platform with analogies to cat qubits with the potential for efficient quantum error correction. We expect that the introduced concept of fast and real-time wavefunction control can also be applied to other hardware-protected qubit designs.

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## I. INTRODUCTION

Superconducting qubits have significantly contributed toward the development of a universal quantum computer [1]. The most prominent one is the transmon, given the simple fabrication and protection against charge noise-induced dephasing [2]. Despite the significant results that have been demonstrated with transmon qubits, the low anharmonicity, as well as relaxation due to dielectric loss and material defects, limit their performance. Therefore, different types of superconducting qubits such as the fluxonium qubit—an rf-SQUID type circuit with enhanced anharmonicity—have been developed [3]. Recently, high-fidelity single-qubit and two-qubit gates have been investigated theoretically [4–8] and implemented experimentally [9–17] and the potential as a scalable architecture demonstrated [18,19].

Fluxonium qubits have higher characteristic impedance compared to flux qubits and can be realized in two main

regimes: coined “heavy” and “light” [20,21]. In the heavy regime, transitions between the two lowest fluxon states are exponentially suppressed with the energy barrier separating them. This intrinsic protection, however, complicates active control of the qubit. In the light regime on the other hand, the matrix elements are substantially larger and the reduced energy dispersion leads to a smaller flux-noise sensitivity. This regime—with the Blochonium being the most extreme realization—is more suitable for control at the cost of lower bit-flip protection [21,22]. A special case of an “ultra-heavy” fluxonium is the inductively shunted transmon (IST) qubit—an intermediate-scale impedance rf-SQUID with a large capacitance and a large inductance. In this case, a relaxation time between the two fluxon states in the range of hours has been observed close to the half-flux point [23]. However, due to the vanishingly small matrix elements, coherent control of the qubit subspace has not been experimentally demonstrated.

In this work, we propose to realize gates between the exponentially bit-flip-protected fluxon states by real-time tuning the degree of protection: between the heavy regime with long  $T_1$  times for information storage and the light regime that allows fast, high-fidelity gates due to the increased matrix elements. Moreover, we show that the mere act of tuning the qubit into the light regime for an optimized duration implements a single- or two-qubit gate of choice, up to local phase rotations. With this approach, the advantages of both regimes are combined in a single circuit.

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Experimentally, fast flux control has already been used for the implementation of high-fidelity single-qubit gates [13,24], by the means of non-adiabatic Landau-Zener transitions [25]. In contrast to ramping the external flux bias, we propose tuning the Josephson energy [26,27] or alternatively the effective capacitance of the circuit. This approach is conceptually related to the single-qubit gates proposed for the oscillator-stabilized flux qubit [28,29], flux qubits [30], and Kerr cat qubits [31].

Intrinsically protected qubits hold the promise to substantially reduce the massive hardware overhead necessary for quantum error correction. Although conceptually fully protected qubits have been recently implemented, such as the  $0 - \pi$  qubit [32] or the grid-state qubit [33], coherent control in the protected regime remains very challenging. In the case of strongly unequal decoherence channels, optimized error-correction codes can be applied [34–36]. Being tailored to the underlying error structure, they require substantially fewer resources, for instance, in the case of erasure qubits [37] or noise-biased qubits [38]. Similarly to the noise-biased cat qubits [39], for which the control parameter of the noise bias is the average photon number, the equivalent tuning knob in our proposed circuits is the effective Josephson energy. Considering realistic parameters, our estimated noise bias (phase-flip versus bit-flip errors) is around  $10^5$ , which is fairly similar to recent experimental results for dissipative cat qubits of  $10^5$ – $10^6$  [40].

## II. TUNABLE CIRCUITS

We propose two different circuits that allow fast gate operations between the two fluxon states with disjoint support in the  $\varphi$ -coordinate. In Fig. 1(a), the first circuit

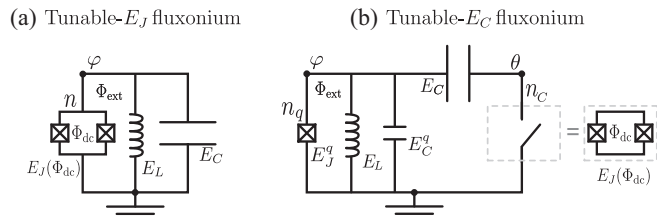


FIG. 1. Tunable circuits. (a) Tunable- $E_J$  fluxonium: The Josephson energy  $E_J$  of the dc-SQUID is tuned through the flux  $\Phi_{dc}$ .  $E_L$  being the inductive energy,  $E_C$  the capacitive energy,  $n$  the excess charge on the island,  $\varphi$  the phase variable and  $\Phi_{ext}$  is the effective flux in the rf-SQUID. (b) Tunable- $E_C$  fluxonium:  $E_J^q$  represents the Josephson energy,  $E_L$  is the inductive energy, and  $E_C^q$  is the capacitive energy of the qubit in the light regime.  $E_C$  is the capacitive energy of an additional large shunt capacitance connected to a dc-SQUID with a tunable  $E_J$  value, acting as a “switch.”  $n_q$  is the excess charge on the rf-SQUID island,  $n_c$  the charge on the dc-SQUID island, and  $\varphi$  and  $\theta$  are the corresponding phases in the two loops.

depicts the tunable- $E_J$  fluxonium, for which the potential landscape of the qubit is tuned. Compared to an ordinary fluxonium, the single Josephson Junction is replaced by a dc-SQUID [41,42], allowing to alter the effective value of the Josephson energy  $E_J$ , by tuning its external magnetic flux  $\Phi_{dc}$ . The “heavy” regime ( $E_J/E_C \gg 1$ ) is attained for a high value of  $E_J$ , i.e. the external magnetic flux in the dc-SQUID being  $\Phi_{dc}/\Phi_0 = 0$ , where  $\Phi_0$  is the magnetic flux quantum. Oppositely for  $\Phi_{dc}/\Phi_0 \approx 0.5$ , the circuit is in the “light” regime ( $E_J/E_C \sim 10$ ). The circuit Hamiltonian can be written as

$$\hat{H}_{E_J} = E_C(2\hat{n})^2 + \frac{E_L}{2}(\hat{\varphi} - \varphi_{ext})^2 - E_J(\varphi_{dc}) \cos(\hat{\varphi}), \quad (1)$$

with  $\varphi_{dc} = 2\pi\Phi_{dc}/\Phi_0$  being the flux inside the dc-SQUID and  $E_J(\varphi_{dc}) = E_{J,max}|\cos(\frac{\varphi_{dc}}{2})|$ , thus forming two first-order insensitive working points at zero and half flux. For clarity, a possible asymmetry of the Josephson junctions in the dc-SQUID is omitted here. In the presence of a finite asymmetry, the Hamiltonian can be rewritten in a similar form, up to an additional  $\Phi_{dc}$ -dependent correction to the reduced external flux  $\varphi_{ext} = 2\pi\Phi_{ext}/\Phi_0$ , that can be adjusted for, as shown in the full derivation in Appendix C. In this case,  $E_J/h$  can not fully be tuned to zero, but a finite value, which we account for by considering a range of 1–12 GHz.

In the case of the tunable- $E_C$  fluxonium, shown in Fig. 1(b), the  $\varphi$ -dependent part of the potential is unperturbed and instead the effective mass is tuned to increase the wavefunction overlap of the fluxon states. Since tuning a capacitance itself on a nanosecond timescale is difficult, we propose to effectively connect and disconnect a large capacitance to the fluxonium circuit. This “switch” is similarly realized through a dc-SQUID, being equivalent to a short for a low inductance around  $\Phi_{dc}/\Phi_0 = 0$  and equivalent to an open for  $\Phi_{dc}/\Phi_0 = 0.5$ . The Hamiltonian for this circuit can be written as

$$\begin{aligned} \hat{H}_{E_C} = & E_C^q(2\hat{n}_q)^2 + \frac{E_L}{2}(\hat{\varphi} - \varphi_{ext})^2 - E_J^q \cos \hat{\varphi} \\ & + 4(E_C^q + E_C)(\hat{n}_C - n_q)^2 - E_J(\varphi_{dc}) \cos \hat{\theta} \\ & - 8E_C^q \hat{n}_C \hat{n}_q. \end{aligned} \quad (2)$$

In addition to the fluxonium phase  $\hat{\varphi}$ , another phase over the dc-SQUID  $\hat{\theta}$  appears. This gives a two-dimensional potential as displayed in Fig. 2(c), with the cross section over  $\varphi$  remaining an ordinary fluxonium potential, but in the  $\theta$  coordinate the wavefunctions are delocalized in the light regime. From Eq. (2), this circuit can be equivalently interpreted as a fluxonium capacitively coupled to a transmon with  $E_C^t = E_C^q + E_C$ , with  $\hat{\theta}$  representing the phase of this effective transmon mode.

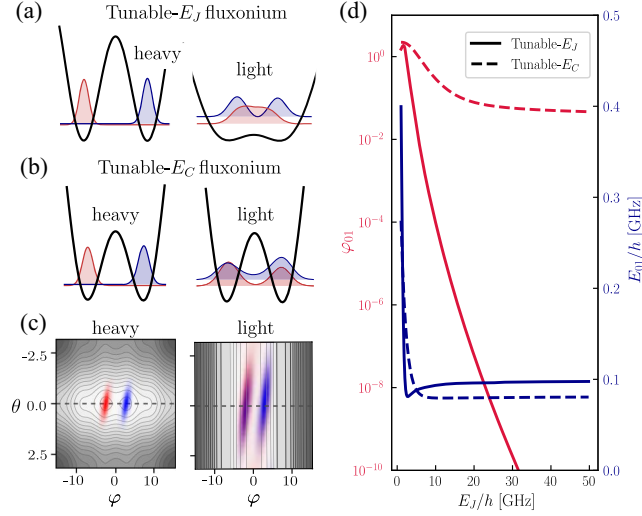


FIG. 2. Qubit properties at flux working point  $\Phi_{\text{ext}}/\Phi_0 = 0.495$ . (a) Potential and absolute value of wavefunctions of the first two fluxon levels (red and blue) of the tunable- $E_J$  fluxonium with  $E_C/h = 0.25$  GHz,  $E_L/h = 0.5$  GHz,  $E_J/h = 1$  GHz in the light regime and 12 GHz in the heavy regime. (b)  $\theta = 0$  cross section of the 2D-potential and wavefunctions of the tunable- $E_C$  fluxonium with  $E_C/h = 0.15$  GHz,  $E_C^q/h = 1$  GHz,  $E_L/h = 0.5$  GHz,  $E_J^q/h = 3$  GHz and  $E_J/h$  of the dc-SQUID ranging from 1 to 50 GHz. (c) 2D-potential (gray scale and contour lines) and wavefunctions of the tunable- $E_C$  fluxonium. Dashed lines indicate the cross section  $\theta = 0$  shown in (b). (d) Phase matrix elements  $\varphi_{01}$  (red) and transition frequencies  $E_{01}/h$  (blue) between the two lowest fluxon states  $|0\rangle$  and  $|1\rangle$  for the tunable- $E_J$  (solid) and tunable- $E_C$  (dashed) circuit as a function of  $E_J$  of the dc-SQUIDs.

Figure 2(d) shows the transition energy  $E_{01}$  and phase matrix element  $\varphi_{01} = |\langle 0|\hat{\phi}|1\rangle|$  between the two lowest energy fluxon states  $|0\rangle$  and  $|1\rangle$  as a function of the tuning parameter  $E_J(\varphi_{\text{dc}})$  of the dc-SQUID for both proposed qubits. The phase matrix element  $\varphi_{01}$  is proportional to the charge matrix element  $\langle 0|\hat{\phi}|1\rangle = \frac{8E_C}{\omega} \langle 0|\hat{n}|1\rangle$  [14] and an indicator for the degree of protection from energy relaxation in the fluxon basis. The energy splitting of the qubit states  $E_{01}$  is quite similar for both qubits as a function of  $E_J(\varphi_{\text{dc}})$ ; however,  $\varphi_{01}$  is substantially stronger suppressed for the tunable- $E_J$  fluxonium. Given the stronger protection and simpler circuit fabrication, in the following sections we focus predominantly on the tunable- $E_J$  fluxonium.

### III. SINGLE-QUBIT GATES

Typically, superconducting qubits are controlled by microwave pulses, whereas in this proposal, all required gates can be performed solely with base-band flux pulses without IQ-modulation. Physically, the qubits are tuned to the light regime by applying a flux pulse to the

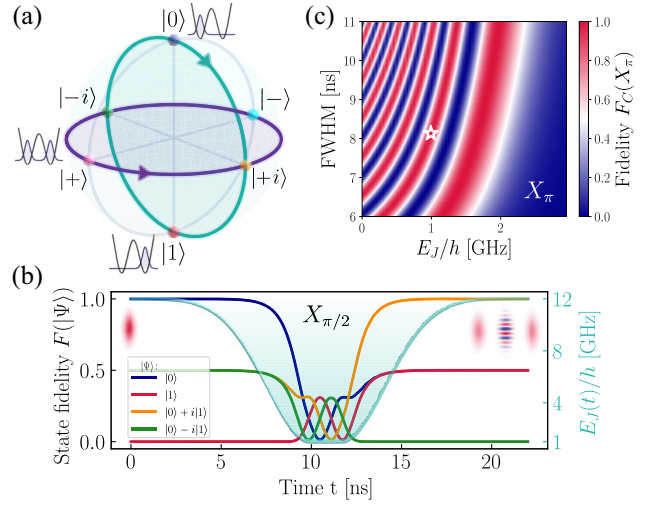


FIG. 3. Flux-pulse control of the tunable- $E_J$  fluxonium. (a) Bloch sphere representation of the qubit subspace with  $|0\rangle, |1\rangle$  being the protected fluxon states. The purple curve depicts a rotation around the  $\sigma_z$  axis, performed via detuning from the external flux point  $\Phi_{\text{ext}}/\Phi_0 = 0.50$ . The rotation around the  $\sigma_x$  axis is visualized by the turquoise curve, implemented by tuning  $\Phi_{\text{dc}}$ . (b) State fidelity  $F(|\Psi\rangle)$  of the initial state  $|0\rangle$  visualized over time throughout a  $X_{\pi/2}$  gate. The optimized, flat-top Gaussian flux pulse (turquoise curve) that varies  $E_J/h$  from 12 GHz to 1 GHz is indicated by the secondary axis. Colored cardinal points on the Bloch sphere correspond to the individual labeling of the curves. The initial and final states are depicted by their Wigner function. (c) Closed-system gate fidelity  $F_C(U_T = X_\pi)$  as a function of the FWHM and  $E_J/h$ , with the flat-top length  $l_{\text{flat}}$  of the pulse being varied between 0–5 ns. The white asterisk indicates the optimized parameter set.

dc-SQUID that acts as a switch for the tunable- $E_C$  fluxonium and lowers the potential barrier for the tunable- $E_J$  fluxonium.

At the flux point  $\Phi_{\text{ext}}/\Phi_0 = 0.5$ , the potential of the tunable- $E_J$  fluxonium is well approximated by a double-well potential and the lowest eigenstates are the symmetric and anti-symmetric superpositions of the fluxon states localized in individual wells. We use the convention for which those are identified as  $|+\rangle, |-\rangle$  and define the computational basis  $|0\rangle, |1\rangle$  as the left and right fluxon states, being visualized on the Bloch sphere in Fig. 3(a) (up to an exponentially small correction, being detailed in Appendix A). By modifying the potential landscape, the protection is lifted and the energy splitting is enlarged temporarily. In this choice of basis, a flux pulse lowering the potential barrier realizes a continuous rotation around the  $\sigma_x$  axis of the Bloch sphere with respect to the eigenstates of the idle Hamiltonian.

The turquoise curve on the Bloch sphere displays this  $X$ -rotation, and the angle is set by the area of a flat-top Gaussian flux-pulse [as depicted in Fig. 3(b)]. Physically,

the coherent quantum phase tunneling (CQPT) rate is tuned from around 10 Hz in the heavy to 1 GHz in the light regime during the pulse (Appendix A). For universal qubit control, an additional, continuous rotation around the  $\sigma_Z$  axis can be performed, by tuning  $\Phi_{\text{ext}}$  away from the half flux point for a finite duration, which breaks the symmetry of the potential and lifts the energy splitting of the fluxon states temporarily. This continuous Z-rotation on the equator of the Bloch sphere is depicted through the purple curve, and the gate characterization and achievable fidelities are detailed in Appendix A.

In the numerical simulations, the following realistic parameters were used  $E_C/h = 0.25$  GHz,  $E_L/h = 0.5$  GHz, and  $E_J/h$  ranging from 1 to 12 GHz. Therefore, in the idle mode the circuit is situated in-between a heavy fluxonium [13] and the IST-qubit [23] limit, and for the short duration of the single- and two-qubit gates between a typical fluxonium and a flux qubit. The flat-top Gaussian pulse is characterized by its full width at half maximum (FWHM), with a flat-top length  $l_{\text{flat}}$  and an amplitude in terms of  $E_J/h$ , as shown in Fig. 3(b) for an optimized  $X_{\pi/2}$  gate. We extract the gate fidelity by reconstruction of the Pauli process matrix  $\chi_R$  [43] with respect to the one for the ideal gate  $\chi$  using

$$F = \frac{2\text{Tr}(\chi^T \chi_R) + 1}{3}. \quad (3)$$

In the closed-system case, the infidelity results in  $1 - F_C(X_\pi) = 3 \times 10^{-6}$  for an  $X_\pi$  gate with the optimal parameters being FWHM = 8 ns,  $l_{\text{flat}} = 2$  ns and  $E_J/h = 0.963$  GHz. The Hilbert space of the fluxonium in the simulation of the tunable- $E_J$  qubit is truncated to 100 Fock states. In the open-system case, the time evolution is simulated in the presence of noise, that is modeled using the expected decoherence rates presented in sec. V, and the system is truncated to the qubit subspace. For the specified optimal parameters, we find an infidelity of  $1 - F_O(X_\pi) = 5 \times 10^{-4}$ . The infidelity is fully limited by the dephasing in the beginning and end of the pulse since the error acquired during idling in the heavy regime for the same duration is even higher  $1 - F_O(I) = 7 \times 10^{-4}$ . Figure 3(c) depicts the  $X_\pi$ -gate fidelity  $F_C$  as a function of the FWHM and  $E_J$  with the optimized gate parameters being indicated by an asterisk. Multiple maxima highlight that for a given pulse length, an optimal pulse amplitude in terms of  $E_J$  can always be found to maximize the gate fidelity.

In the heavy regime, we expect a significant reduction of  $T_1$  times approaching the half-flux bias point due to hybridization of higher excited states that dominate bit-flip times at finite temperature in the heavy regime, as discussed in Sec. V. For this reason, we demonstrate that the proposed flux-gates can also be performed slightly detuned from the half-flux point in principle. If we do not

introduce a new rotating frame, the Bloch vector rotates around the  $\sigma_z$  axis in the idle-mode, due to the finite energy level splitting between the fluxon states. Choosing the working point  $\Phi_{\text{ext}}/\Phi_0 = 0.495$ , we simulate the Hadamard gate ( $\pi$ -rotation around the axis  $\sigma_H = (\sigma_x + \sigma_z)/\sqrt{2}$ ) with a flat-top Gaussian pulse [similar to Fig. 3(b)] with FWHM = 7.5 ns,  $l_{\text{flat}} = 1.5$  ns and  $E_J/h = 0.9872$  GHz, resulting in the closed-system gate-infidelity of  $1 - F_C(H) = 2 \times 10^{-5}$ . The open-system gate-infidelity results in  $1 - F_O(H) = 6 \times 10^{-4}$ . The concrete error budget as well as the optimized pulse parameters and simulated fidelities for the continuous Z-rotation are given in Appendix A.

A similar optimization can be performed for the tunable- $E_C$  fluxonium, for which we find a closed system infidelity of  $1 - F_C = 1 \times 10^{-3}$  for the Hadamard gate at the same bias flux working point. This result was obtained using a coarser optimization routine due to the substantially larger Hilbert space involved. While we believe that higher fidelities could be achieved in principle, we expect the achievable optimum to be lower compared to the tunable- $E_J$  qubit, in part due to less localized basis states in the light regime, for the experimental parameters considered. Moreover, the open-system fidelity is expected to be further degraded by charge noise due to the additional qubit island in this circuit.

In general, the introduced method of tuning the wavefunction overlap can in principle always be complemented by additional microwave pulses in the light regime to further push the gate fidelity toward unity. This would be at the expense of additional required rf-control capabilities.

## IV. TWO-QUBIT GATES

For the implementation of a two-qubit gate, a simple capacitive coupling between the two qubits suffices, since the interaction is tunable through the individual dc-SQUIDS, as visualized in Fig. 4(a) for the tunable- $E_J$  fluxonium. The coupling capacitance is chosen, such that the two-qubit subspaces can be considered decoupled when they are operated in the heavy regime due to the vanishingly small respective charge matrix element. Similarly to the single-qubit gates, tuning both qubits to the light regime leads to a hybridization of the qubit states and allows fast, high-fidelity two-qubit gates. For both qubits, the energy values are chosen close to the single-qubit case, but slightly distinct. The parameters of the first qubit are  $E_{C1}/h = 0.30$  GHz,  $E_{L1}/h = 0.52$  GHz and of the second  $E_{C2}/h = 0.30$  GHz,  $E_{L2}/h = 0.50$  GHz. The tunability of the respective  $E_{Ji}/h$  value is considered in the range of 1–12 GHz, identical to the single-qubit case. The joint Hamiltonian of the capacitively coupled qubits is given by

$$\hat{H}_{12} = \hat{H}_1 + \hat{H}_2 + 8 \frac{E_{C1} E_{C2}}{E_{C1} + E_{C2} + E_{C_g}} \hat{n}_1 \hat{n}_2, \quad (4)$$

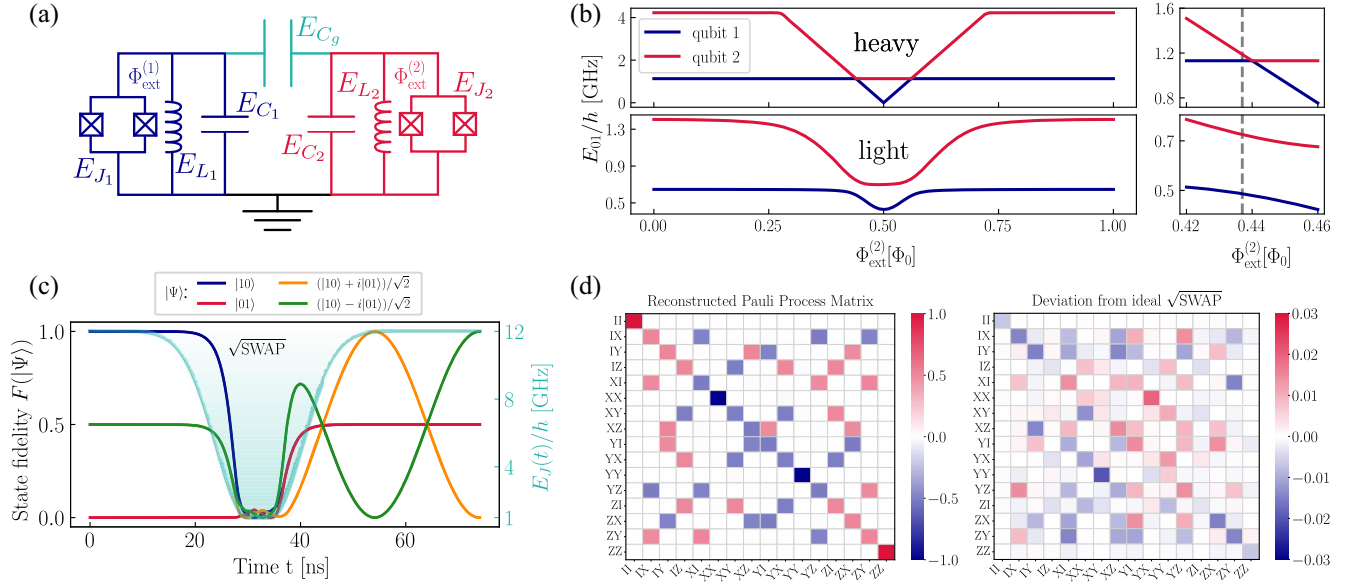


FIG. 4. Two-qubit gate scheme. (a) Electrical circuit diagram for two capacitively coupled tunable- $E_J$  qubits. (b) Energy spectrum for the qubits operated in the heavy (top), and light (bottom) regime. The abscissa represents the external flux of the second qubit  $\Phi_{\text{ext}}^{(2)}$  with  $\Phi_{\text{ext}}^{(1)} = 0.44 \Phi_0$  being fixed. In the insets in the right column, the gray dashed line depicts the working point value for the  $\sqrt{\text{SWAP}}$  gate. (c) Fidelity of the state evolution throughout the flux-pulse of the initial state  $|10\rangle$ . Oscillations in the  $\{|10\rangle, |01\rangle\}$  manifold are due to the energy splitting between the two states in this frame. The flat-top Gaussian pulse shape applied to both qubits is displayed in turquoise in terms of  $E_J/h$ . (d) Left: Reconstructed Pauli process matrix for the simulated optimized  $\sqrt{\text{SWAP}}$  gate. Right: Deviation with respect to the ideal target  $\sqrt{\text{SWAP}}$  gate.

with  $\hat{H}_i$  being the respective single-qubit Hamiltonian in Eq. (1) and  $E_{C_g}/h = 6.67$  GHz is the energy of the coupling capacitor.

In Fig. 4(b), the energy spectrum of the two capacitively coupled qubits in the heavy and light regime is displayed versus the external flux  $\Phi_{\text{ext}}^{(2)}$  of qubit 2, while for qubit 1 being fixed to  $\Phi_{\text{ext}}^{(1)} = 0.44 \Phi_0$ . By tuning both qubits to the light regime, the anti-crossing is around 239 MHz for  $\Phi_{\text{ext}}^{(2)} = 0.437 \Phi_0$ . In principle, any value close to  $\Phi_{\text{ext}}^{(1)} = 0.5 \Phi_0$  is a valid working point, since the size of the anti-crossing remains of the order of 250–400 MHz at any flux point. At this working point, a flat-top Gaussian pulse is applied to the individual dc-SQUID loops, as depicted in Fig. 4(c). The flux-pulse is optimized with regard to the  $\sqrt{\text{SWAP}}$  gate, which forms, together with individual qubit control, a universal quantum gate set. The optimal parameters found for the  $\sqrt{\text{SWAP}}$  gate are: a FWHM = 19 ns of the pulse with a flat-top length of  $l_{\text{flat}} = 4$  ns, the external fluxes of both qubits set to  $\Phi_{\text{ext}}^{(1)} = 0.4399 \Phi_0$ ,  $\Phi_{\text{ext}}^{(2)} = 0.4369 \Phi_0$  and the minimum value of  $E_{J_i}/h = 1.0068$  GHz.

For ease of optimization, we kept the pulse parameters of both qubits the same, but individual fine-tuning might improve the gate fidelity even further. We reconstruct the full Pauli process matrix of the  $\sqrt{\text{SWAP}}$  gate in the closed system case, as visualized in Fig. 4(d) and extract an

infidelity of  $1 - F_C = 3 \times 10^{-4}$ . In the numerical simulations, the Hilbert space truncation is set to 50 Fock states. In the open-system case, we find an infidelity  $1 - F_O = 4 \times 10^{-3}$ , which is dominated by dephasing as in the single-qubit case. In comparison, the infidelity for both qubits idling in the heavy regime for the same duration is  $3 \times 10^{-3}$ . The  $\sqrt{\text{SWAP}}$  gate was the choice for the two-qubit gate as it resulted in the highest fidelity, however, other two-qubit gates, i.e.  $i\text{SWAP}$  or  $\sqrt{i\text{SWAP}}$  can be realized by adjusting the flux-pulse parameters. The two-qubit gate simulations for the tunable- $E_C$  qubit are left for future work.

Notably, this proposed tuning of the Josephson energy gives the advantage of realizing high-fidelity two-qubit gates on the nanosecond scale without the need of additional circuit elements as in other architectures, i.e. in the form of tunable couplers [4,16].

## V. EXPECTED COHERENCE PROPERTIES

In this section, we present the coherence properties for the tunable- $E_J$  fluxonium. Since the parameters of the proposed qubits are close to those of the IST-qubit in [23], similarly long relaxation times of the fluxon states are expected, however, not in the range of hours due to the smaller value of  $E_J$ . Typically for fluxonium devices, the coherence times are estimated using Fermi's Golden rule, by computation of the matrix element of a respective noise

operator  $\langle 0|\hat{O}|1\rangle$  and the associated noise spectral density  $S_O(\omega)$ . However, given the very small matrix element of the fluxon states in the heavy regime, direct transitions between the two qubit states are estimated to occur on timescales of years, but do not represent the dominant decay channel. Instead, leakage to higher levels within a well, e.g. due to finite temperature, results in incoherent, multi-level decay processes.

Here, we consider the full energy-level structure of the fluxonium Hamiltonian and compute the transition rates between the respective levels. Given the experimental results for fluxonium devices in [42,44], we estimate the  $T_1$  times to be limited by dielectric loss. The transition rate between two respective levels  $\Gamma_{ij}^{\text{diel}}$  for dielectric loss is computed using

$$\Gamma_{ij}^{\text{diel}} = \frac{\hbar\omega_{ij}^2}{4E_C Q_{\text{cap}}(\omega_{ij})} |\langle i|\hat{\phi}|j\rangle|^2 \frac{\coth\left(\frac{\hbar|\omega_{ij}|}{2k_B T_{\text{eff}}}\right)}{1 + \exp\left(-\frac{\hbar\omega_{ij}}{k_B T_{\text{eff}}}\right)}, \quad (5)$$

where the capacitive quality factor is assumed to be  $Q_{\text{cap}}(\omega_{ij}) = 10^5 \times (\frac{2\pi \times 6 \text{ GHz}}{\omega_{ij}})^{0.7}$  and an effective bath temperature  $T_{\text{eff}}$  of 60 and 70 mK respectively. For the estimates of  $T_1$ , we simulate the time evolution in the eigenstate basis of the fluxonium Hamiltonian under the Lindblad equation with jump operators being defined as  $L_{ij} = \sqrt{\Gamma_{ij}}|j\rangle\langle i|$ , to include all possible decay channels and extract the value of  $T_1$  from the main population decay. Due to the multiple decay channels, the population does not undergo a single exponential decay; however, other quantities can be used for decoding [45,46], which allows extracting a single decay constant.

The values of  $T_1$  are visualized in Fig. 5(a) as a function of the external flux  $\Phi_{\text{ext}}$  for the two temperatures in the heavy and light regime. Around the working points of the single- and two-qubit gates (indicated by disc symbols), the immense difference between the two regimes by a factor of around  $10^5$ – $10^6$  is visible. In the heavy regime, the expected  $T_1$  drops by more than an order of magnitude approaching the half-flux point, which is attributed to increased hybridization of higher-lying states with support in both wells, activating faster transitions. In panel (b),  $T_1$  is plotted versus the flux through the dc-SQUID  $\Phi_{\text{dc}}$  and the effective  $E_J$  value for both the single- and two-qubit gate working points and temperatures. The curves display an exponential scaling with the potential barrier height, which is proportional to the value of  $E_J$ .

Notably, the curves for the two flux points show a distinct behavior. For the working point of the single-qubit gate, the scaling is similar, but it depicts a “staircase” pattern, as observed experimentally in the double-well system of the Kerr-cat qubit [47,48]. This feature is similarly here attributed to the number of energy eigenstate

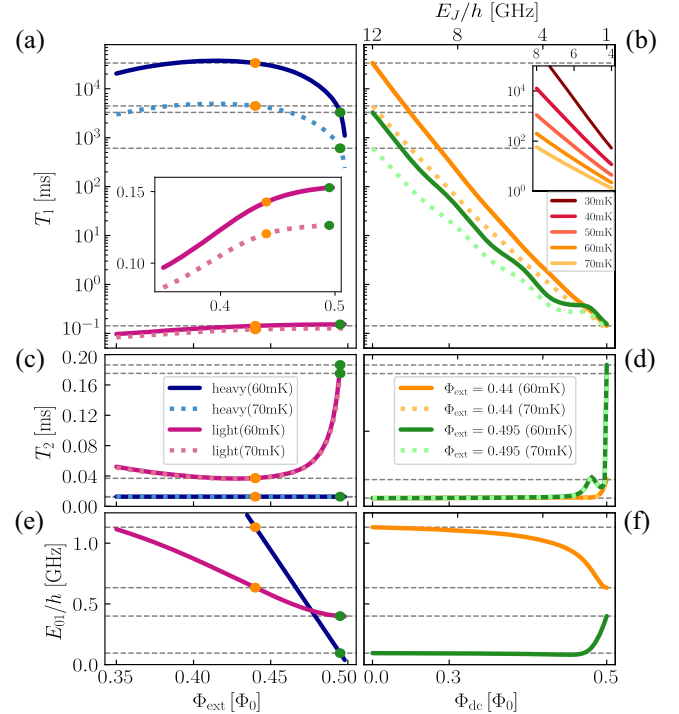


FIG. 5. Coherence properties for the tunable- $E_J$  fluxonium. Left column: Coherence time values  $T_1$  in (a) and  $T_2$  in (c) as well as the transition frequency  $E_{01}/h$  in (e) as a function of  $\Phi_{\text{ext}}$  in the heavy (blue) and light (purple) regime for two different temperatures 60 and 70 mK. The disc symbols indicate the working points of the single-qubit gate (green)  $\Phi_{\text{ext}} = 0.495 \Phi_0$  and the two-qubit gate  $\Phi_{\text{ext}} = 0.44 \Phi_0$  (orange). Right column: Coherence times  $T_1$  in (b) and  $T_2$  in (d) as well as the transition frequency  $E_{01}/h$  in (f) versus  $\Phi_{\text{dc}}$  that tunes the effective value of  $E_J/h$  (top axis) in the range of 1–12 GHz. Curves for the working points of the single- and two-qubit gates are displayed in green and orange, respectively. The inset in panel (b) shows  $T_1$  versus  $E_J/h$  at  $\Phi_{\text{ext}} = 0.44 \Phi_0$  for additional temperatures. The dashed gray lines indicate the vertical position of the working points.

pairs that are located within the wells. This is clearly absent from the flux point  $\Phi_{\text{ext}} = 0.44 \Phi_0$ , for which the potential has a stronger asymmetry and no level-pairing occurs.

The additional inset displays curves for more temperatures for the two-qubit working point at  $\Phi_{\text{ext}} = 0.44 \Phi_0$ . Using the numerically simulated dependence of the fluxon state lifetime  $T_1 \propto \exp(\gamma E_J/k_B T_{\text{eff}})$  as a function of  $E_J$  and temperature  $T_{\text{eff}}$  as shown in the inset of panel (b), we extract a scaling constant  $\gamma = 1.48 \pm 0.06$ , confirming the exponential dependence with  $E_J/T_{\text{eff}}$  [46].

For a numerical estimate of the dephasing time we use [42]

$$T_\phi = (\ln(2)(\tilde{A}_1^2 + \tilde{A}_2^2 + 2c_{12}\tilde{A}_1\tilde{A}_2))^{-1/2}, \quad (6)$$

TABLE I. Simulated  $T_1$  and  $T_2$  values at the two gate working-points.

| $T_{\text{eff}}$                                         | $T_1^{\text{heavy}}$ [s] | $T_2^{\text{heavy}}$ [ $\mu\text{s}$ ] | $T_1^{\text{light}}$ [ $\mu\text{s}$ ] | $T_2^{\text{light}}$ [ $\mu\text{s}$ ] |
|----------------------------------------------------------|--------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| Single-qubit gate ( $\Phi_{\text{ext}} = 0.495 \Phi_0$ ) |                          |                                        |                                        |                                        |
| 60 mK                                                    | 3.3                      | 12.8                                   | 150                                    | 143                                    |
| 70 mK                                                    | 0.61                     | 12.8                                   | 140                                    | 130                                    |
| Two-qubit gate ( $\Phi_{\text{ext}} = 0.44 \Phi_0$ )     |                          |                                        |                                        |                                        |
| 60 mK                                                    | 32                       | 12.8                                   | 150                                    | 35                                     |
| 70 mK                                                    | 4.5                      | 12.8                                   | 130                                    | 34                                     |

with  $\tilde{A}_i = A_i \frac{\partial \omega_{01}}{\partial \Phi_i}$  and the noise correlation coefficient  $c_{12}$  is assumed to be 0.5. The variables refer to the physical fluxes  $\Phi_1$  and  $\Phi_2$  and are related through  $\Phi_1 = \Phi_{\text{dc}}$  and  $\Phi_2 = \Phi_{\text{ext}} - \Phi_{\text{dc}}/2$  (Appendix C), with realistic flux noise amplitudes  $A_1 = A_2 = 5 \mu\Phi_0$  for regularly sized SQUIDS [42,49]. In the heavy regime, the locally flat region around the sweet-spot  $\Phi_{\text{ext}} = 0.5 \Phi_0$  is very narrow, as detailed in Appendix B. Therefore, to obtain realistic estimates, we report and use in all the single-qubit gate simulations, the  $T_\phi$  value calculated at the flux-noise sensitive point  $\Phi_{\text{ext}} = 0.495 \Phi_0$ .

The panels (c) and (d) show the calculated values of  $T_2 = (\frac{1}{2T_1} + \frac{1}{T_\phi})^{-1}$  versus the external flux  $\Phi_{\text{ext}}$  and  $\Phi_{\text{dc}}$ , respectively. In the heavy regime,  $T_2$  is estimated to be around 12.8  $\mu\text{s}$  for both working points and in the light regime 143  $\mu\text{s}$  for  $\Phi_{\text{ext}} = 0.495 \Phi_0$  and 35  $\mu\text{s}$  for  $\Phi_{\text{ext}} = 0.44 \Phi_0$ . As depicted in (d),  $T_2$  is mainly limited by  $T_\phi$  in the heavy regime and is fairly constant except for small values of  $E_J$ . This is attributed to the vanishing double-well structure and, consequently, the absence of fluxon states approaching small  $E_J$  values. The value of  $T_2$  can be increased by targeting lower realistic values of  $E_L$ , i.e. by a factor of 5 to 62  $\mu\text{s}$  ( $E_{L/h} = 0.10 \text{ GHz}$ ), which is detailed in Appendix B. The panels (e) and (f) in Fig. 5 display the energy splitting between the two fluxon states versus  $\Phi_{\text{ext}}$  and  $\Phi_{\text{dc}}$ .

The comparison of the simulated coherence times for the working points of the single- and two-qubit gate is summarized in Table I.  $T_1$  in the protected regime is estimated to be in the range of seconds and is by a factor  $10^5$ – $10^6$  longer than in the light regime. Therefore, on relevant timescales, we expect to be mainly limited by  $T_2$  and possible relaxation errors during the short duration of the single- and two-qubit gates. Importantly, while an exponential gain in  $T_1$  is seen with increasing  $E_J$ , our simulations indicate that this is not at the cost of reduced  $T_2$  in the high  $E_J$  region. This is in contrast to the phase coherence scaling in bit-flip protected cat qubits [40,47,48,50–52].

## VI. CONCLUSION

In this letter, we propose a new type of superconducting qubit platform, a fluxonium qubit with exponentially

tunable bit-flip protection. The tunability is based on flux control of a dc-SQUID, which varies either the value of  $E_J$  or alternatively  $E_C$  of the circuit. The qubits are operated in two regimes—the heavy regime for information storage, for which  $T_1$  is estimated to show an exponential scaling with the value of  $E_J$  and has even been experimentally demonstrated to reach hours [23]. For gate operations, the protection is lifted for the duration of a few nanoseconds, which is small compared to the estimated coherence times. Since the gate implementation is based solely on flux pulses and no microwave lines are needed except for readout, this approach can be advantageous with regard to scalable architectures, as frequency crowding will not affect the control capabilities.

The main limitation of the proposed qubits is the low  $T_2$  due to a high dephasing rate, which is caused by flux noise. Reducing the flux noise amplitude has been achieved through appropriate surface treatment [53] and due to its low-frequency nature, a variety of active mitigation techniques [54–58] exist. Given the strong noise-bias of the proposed qubits, they can be further concatenated with hardware-efficient error-correcting codes that leverage this bias [36,38,52,59–61]. Although the proposed gates are not bias-preserving, they are operated on a timescale much faster than the decoherence times in the light regime. A very rapid control of  $E_J$ , in the sub-nanosecond regime, might even allow to preserve the noise-bias partially [46]. Moreover, the long  $T_1$  times make them promising candidates as ancilla qubits since the propagation of harmful errors to the storage system can be exponentially suppressed [62–64].

## ACKNOWLEDGMENTS

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## DATA AVAILABILITY

The data supporting this article are available on Zenodo [69].

## APPENDIX A: SYSTEM HAMILTONIAN AND GATES

In this appendix, the universal control of the qubit subspace through flux pulses is detailed. We start by giving the circuit Hamiltonian for the tunable- $E_J$  fluxonium

$$\hat{H}_{E_J} = E_C(2\hat{n})^2 + \frac{E_L}{2}\hat{\varphi}^2 - E_J(\varphi_{\text{dc}})\cos(\hat{\varphi} + \varphi_{\text{ext}}). \quad (\text{A1})$$

In this section the external flux  $\varphi_{\text{ext}}$  is entirely allocated with the Josephson Junction, since the gate mechanisms do not rely on non-adiabatic dynamics and a more intuitive picture can be obtained in this gauge. The treatment for time-dependent flux control in the irrotational gauge is found in Appendix C. In the heavy limit  $E_J/E_C \gg 1$ , the two ground states at the half flux point  $\varphi_{\text{ext}} = \pi$  can be well approximated as

$$\begin{aligned} |g\rangle &\propto \int d\varphi' e^{-\varphi'^2/2\sigma^2} (|\varphi = -\pi + \varphi'\rangle + |\varphi = \pi + \varphi'\rangle) \\ &\propto \int dn' e^{-n'^2\sigma^2/2} \cos(\pi n') |n = n'\rangle \\ |e\rangle &\propto \int d\varphi' e^{-\varphi'^2/2\sigma^2} (|\varphi = -\pi + \varphi'\rangle - |\varphi = \pi + \varphi'\rangle) \\ &\propto \int dn' e^{-n'^2\sigma^2/2} \sin(\pi n') |n = n'\rangle. \end{aligned} \quad (\text{A2})$$

In its  $\varphi$ -representation, the wavefunctions are the symmetric and anti-symmetric superpositions of the two Gaussian fluxon states spaced by  $2\pi$ . In the conjugate quadrature  $n$ , the respective wavefunction is represented by a cosine and sine with frequency  $\pi$  and a Gaussian envelope with variance  $\frac{1}{\sigma^2}$ . The two states have same support in the  $\varphi$ -representation but have an even and odd charge distribution and are thus distinct in parity. Whereas their symmetric superposition states  $\frac{1}{\sqrt{2}}(|g\rangle \pm |e\rangle) \approx \int d\varphi' e^{-\varphi'^2/2\sigma^2} |\varphi = \pm\pi + \varphi'\rangle := |\pm\pi\rangle$  have distant support in the  $\varphi$ -quadrature. Throughout the paper the following convention is used for the logical basis states and its

superposition states

$$\begin{aligned} |+\rangle &= |g\rangle, \quad |0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) = |\pi\rangle + O(e^{-\pi^2/\sigma^2}) \\ |-\rangle &= |e\rangle, \quad |1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) = |-\pi\rangle + O(e^{-\pi^2/\sigma^2}) \end{aligned} \quad (\text{A3})$$

where we have omitted the subscripts. Since  $|+\rangle, |-\rangle$  have an even and odd parity they are exactly orthogonal ( $\langle + | - \rangle = 0$ ), whereas their superpositions have a non-zero overlap ( $\langle 0 | 1 \rangle = O(e^{-\pi^2/\sigma^2})$ ).

### 1. Continuous X-rotation

The single-qubit gate mechanisms rely on breaking the inherent symmetries of the system. The Hamiltonian in Eq. (A1) is invariant under a parity transformation  $\hat{P}: \vec{\varphi} \rightarrow -\vec{\varphi}$  at the half flux point  $\varphi_{\text{ext}} = \pi$  for all values of  $E_J$  [46], being set through the flux  $\varphi_{\text{dc}}$ . An intuitive picture can be established by expressing the qubit states in the Fock state basis as visualized in Fig. 6. Importantly, the states  $|+\rangle, |-\rangle$  have a distinct even and odd population structure in the Fock basis. Therefore, by tuning the parameter  $E_J$ , only rotations around the  $\sigma_X$  axis (visualized in turquoise on the Bloch sphere) are allowed, as otherwise a change in parity would be necessary. This parity structure is further visible in the interference pattern of the Wigner functions that are visualized in the individual insets in Fig. 6. To generate a population transfer across parity sectors, thus a continuous rotation around the  $\sigma_Z$  axis, the parity symmetry can be broken by tuning  $\varphi_{\text{ext}}$  away from the half flux point temporarily. This can be understood as introducing a slight asymmetry to the double-well

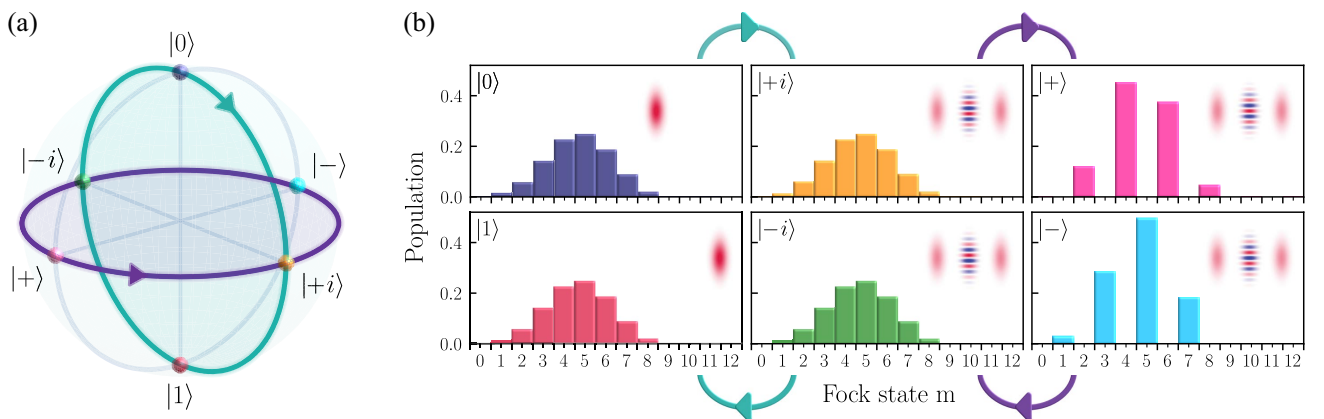


FIG. 6. (a) Bloch sphere representation of the qubit subspace at half-flux. The continuous X- and Z-rotation is depicted through turquoise and purple curves, respectively, and the cardinal states are highlighted in distinct colors. (b) Fock state population distribution of the 6 cardinal states visualized in the corresponding color. Colored arrows highlight the continuous X- and Z-rotation in the respective manifold. The inset in each plot displays the Wigner function of the individual state.

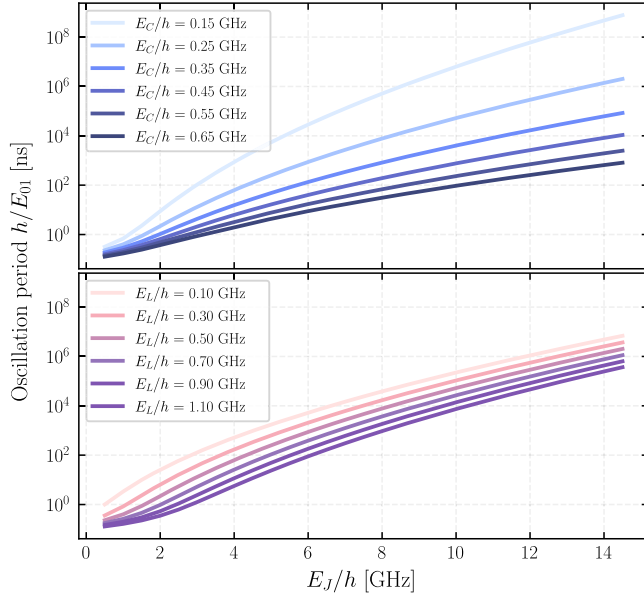


FIG. 7. Rabi oscillation period of the fluxon states versus  $E_J$  for different values of  $E_C$  (top, fixed  $E_L/h = 0.5$ ) and  $E_L$  (bottom, fixed  $E_C/h = 0.25$ ) displayed through distinct colored curves. The oscillation period is directly given by  $T_X = \frac{h}{E_{01}}$ .

potential, resulting in a different phase accumulation of the fluxon states.

The two ground states of the Hamiltonian are not completely degenerate, but have an energy tunnel splitting  $E_{01}$  that is exponentially suppressed with  $E_J$ .  $E_{01}/h$  gives the CQPT rate of the fluxon states and in our basis choice, the frequency of the oscillations around the  $\sigma_X$  axis in the Bloch sphere. The oscillation period  $T_X$  of the two fluxon states is visualized in Fig. 7 as a function of  $E_J$  for different values of  $E_C$  and  $E_L$ , respectively. Through the tunability of  $E_J$  in this circuit,  $T_X$  can be changed from 10–100 milliseconds in the heavy regime to a few nanoseconds in the light regime.

For targeting  $X(\theta)$  gate durations within a few nanoseconds, Fig. 7 identifies possible ideal values of  $E_{J\min}$  and  $E_{J\max}$  for various values of  $E_C$  and  $E_L$ , respectively. This translates into a  $E_{J\min}$  possible of around 1–2 GHz, which agrees well with the color plot in Fig 3(c) in the main text. By choosing even lower values of  $E_{J\min}$ , the  $X(\theta)$  gate can even be realized on sub-nanosecond time-scales in principle, which is however more challenging in experimental implementations. In the heavy regime,  $T_X$  should be larger than 10 ms for achieving small gate infidelities. The exact gate angle  $\theta$  is set by the short duration in the light regime and the ramping time of the pulse.

## 2. Continuous Z-rotation

A rotation around the  $\sigma_z$  axis of the Bloch sphere necessitates to break the parity symmetry, in order to

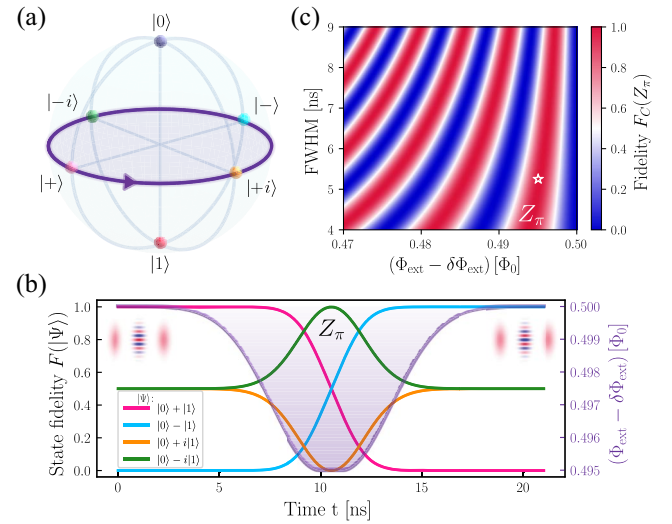


FIG. 8. (a) Bloch sphere representation depicting the Z rotation in purple. (b) State fidelity  $F(|\Psi\rangle)$  plotted versus time  $t$  for the initial state  $|+\rangle$  and target state  $|-\rangle$  (target gate:  $Z_\pi$ ), which are also visualized by their Wigner functions in the insets. The secondary axis displays the amplitude of the pulse  $\Phi_{\text{ext}} - \delta\Phi_{\text{ext}}$  in units of  $\Phi_0$ . (c) 2D closed-system fidelity  $F_C(Z_\pi)$  colorplot for a  $Z_\pi$  gate as a function of the FWHM and the amplitude  $\Phi_{\text{ext}} - \delta\Phi_{\text{ext}}$ . The optimized parameters of the pulse shape in (b) are indicated by a white asterisk.

activate oscillations across parity sectors as visualized in Fig. 6. This can be implemented by tuning the external flux  $\Phi_{\text{ext}}/\Phi_0 = 0.50$  of the sweet spot working point temporarily. This allows to induce fast oscillations in the equatorial plane of the Bloch sphere between the states  $|\pm\rangle$  and  $|\pm i\rangle$ , as highlighted in Fig 8(a). In (b) an optimal flat-top Gaussian pulse is displayed in terms of the external flux being tuned to  $\Phi_{\text{ext}}/\Phi_0 = 0.495$  with a length  $l_{\text{flat}} = 1.15$  ns and FWHM = 5 ns. Similarly to the X rotation, the closed system fidelity  $F_C(Z_\pi)$  is plotted as a function of the FWHM and amplitude given by  $\Phi_{\text{ext}} - \delta\Phi_{\text{ext}}$  in Fig. 8(c). The oscillation frequency is proportional to the level splitting, therefore, the larger the deviation from half flux, the faster the  $Z(\beta)$  gate speed, as visualized in the 2D color plot. The infidelities reached for the  $Z(\beta)$ -rotation in the closed system case are below  $10^{-8}$ . In the open-system case, the fidelity is similarly to the other gates limited by the short dephasing time in the heavy regime. Further details on the precise error contributions are provided in the next section.

## 3. Gate-error budget and leakage analysis

The flux-pulse ramp in our numerical simulations is characterized by the width  $\sigma$  of the flat-top Gaussian pulse. In order to quantify the induced error by those nanosecond ramping times, we report the leakage to other higher-lying states in the fluxonium potential. Predominantly, the X-gate and the two-qubit gate ( $\sqrt{\text{SWAP}}$ ) are affected, due

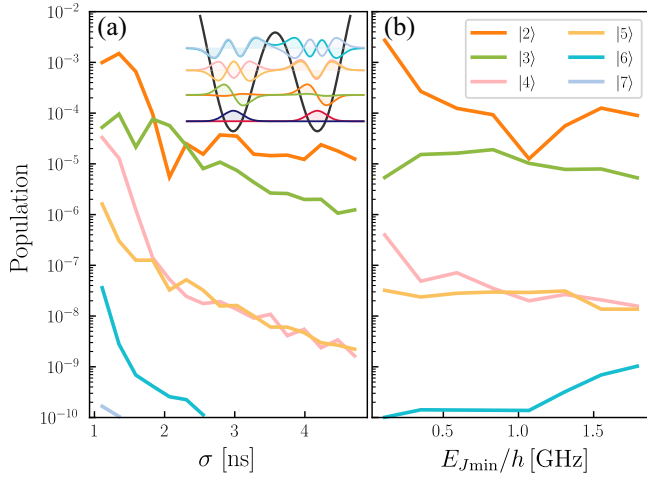


FIG. 9. The population of the higher lying states for a  $X_\pi$  gate is visualized as a function of the width  $\sigma$  of the flat-top Gaussian pulse (a) and the amplitude in terms of  $E_{J\min}$  (b). In (a)  $E_{J\min}/h = 1$  GHz is kept fixed and in (b)  $\sigma = 2.55$  ns. The inset displays the wavefunctions of the fluxonium states, and the color of each state corresponds to the population curves in (a) and (b).

to the wide tuning range of  $\Phi_{dc}$ , compared to the narrow one of  $\Phi_{ext}$  during the  $Z$ -gate. We visualize the leakage error for the  $X$ -gate in Fig. 9 as a function of  $\sigma$  of the flat-top Gaussian pulse (a) and the amplitude in terms of  $E_{J\min}$  (b). For each  $\sigma$  and  $E_{J\min}$ , the optimal flat-top length  $l_{flat}$  is found to minimize the infidelity of a  $X_\pi$  gate. A substantial decrease of the higher-state population is seen for increasing values of  $\sigma$ . The values for our gate simulations in the main text of  $\sigma = 2.55$  ns and  $E_{J\min}/h \approx 1$  GHz are chosen as a compromise between the leakage error and the gate duration.

Further, we report the error budget for the single- and two-qubit gates, introduced in the main text, to quantify the impact of leakage errors and the respective decoherence channels. During the single- and two-qubit gates, the Hamiltonian is modified, and thereby the coherence times  $T_1$  and  $T_2$  change at each incremental time-step. For a simplified analysis, which does not quantitatively impact the results, we only use the values of  $T_1$  and  $T_2$  in the heavy and the light regime, reported in Table I. For half the ramping time, the decoherence times of the heavy and for the other half the ones of the light regime are applied. The error mechanisms considered are dielectric loss and flux noise, as discussed in detail in Sec. V.

In Fig. 10, the error budgets for the  $X_\pi$ ,  $Z_\pi$ , and  $\sqrt{\text{SWAP}}$  gate are visualized. The infidelity contributions due to coherent errors, leakage as well as the finite  $T_1$  and  $T_2$  in the heavy and light regime are computed independently, and their individual impact is displayed. The dominant contribution, as stated in the main text, is due to the short  $T_2$  in the heavy regime (dark blue). This results in the open-system infidelities during idling exceeding those of

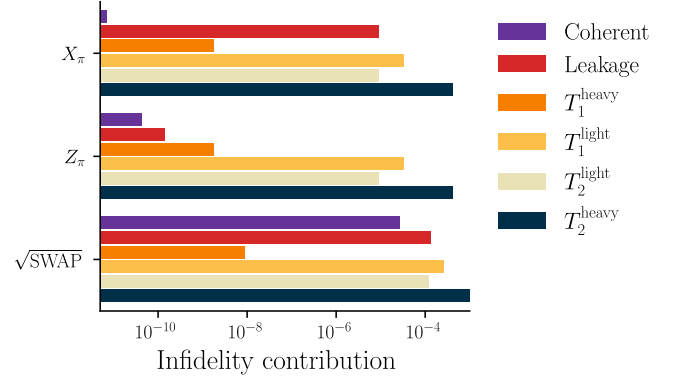


FIG. 10. Individual infidelities plotted on a logarithmic scale for the  $X_\pi$ ,  $Z_\pi$  and  $\sqrt{\text{SWAP}}$  gate. Contributions due to coherent errors, leakage, finite  $T_1$  and  $T_2$  in the heavy and light regime are depicted by individual bars.

the gates for the same duration. To improve visibility of the remaining error contributions, the respective values are displayed on a logarithmic scale, as they are at least an order of magnitude smaller. Figure 10 further highlights the stronger impact of leakage errors (red) during the  $X_\pi$  and  $\sqrt{\text{SWAP}}$  gates. However, they do not represent a dominant error source and can be further reduced by increasing the ramping time, as visualized in Fig. 9.

## APPENDIX B: DEPHASING TIME

We study the expected dephasing times for the tunable- $E_J$  fluxonium with respect to the circuit parameters.

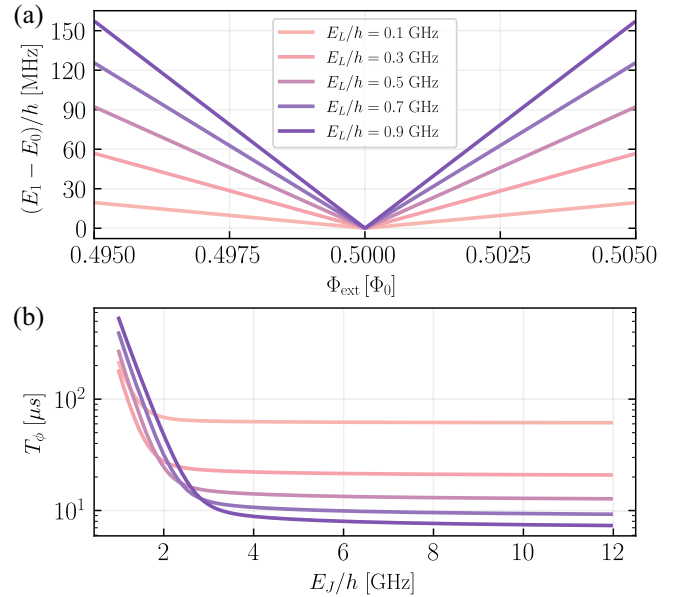


FIG. 11. (a) Transition frequency of the fluxon states in the heavy regime plotted versus  $\Phi_{ext}$  for various values of  $E_L$ . (b) Dephasing time  $T_\phi$  as a function of  $E_J$  for same values of  $E_L$ .

In these simulations the flux noise amplitudes introduced in the main text are used ( $A_1 = A_2 = 5 \mu\Phi_0$ ). The dephasing time is calculated from Eq. (6) and proportional to the derivative of the transition frequency with respect to flux. Since in the heavy regime, the sweet-spot is very sharp, the dephasing time is computed slightly off the sweet spot  $\Phi_{\text{ext}}/\Phi_0 = 0.495$  for a realistic estimate. In our case, the contribution with respect to the external flux  $\Phi_{\text{ext}}$  dominates, which is plotted in Fig. 11. The transition frequency of the fluxon states is visualized in panel (a) around the sweet spot for different values of  $E_L$  in the heavy regime, clearly highlighting the flattening of the energy levels with decreasing  $E_L$ . In addition, the expected improvement of  $T_\phi$  for lower values of  $E_L$  is shown in panel (b) as a function of  $E_J$ . Notably,  $T_\phi$  stays effectively constant by increasing  $E_J$ , and it is therefore not in conflict with the exponential  $T_1$  protection.

### APPENDIX C: DERIVATION OF THE TUNABLE- $E_J$ HAMILTONIAN

We present the derivation of the time-dependent Hamiltonian for the tunable- $E_J$  fluxonium, in presence of a finite junction asymmetry. We start with the Hamiltonian in the irrotational gauge as proposed in [70] for correct flux allocation for time-dependent dynamics

$$\begin{aligned} \hat{H}_{E_J} = & E_C(2\hat{n})^2 + \frac{E_L}{2} \left( \hat{\phi} - \varphi_1 \frac{C_{J_1}}{C_\Sigma} - \varphi_2 \frac{C_{J_1} + C_{J_2}}{C_\Sigma} \right)^2 \\ & - E_{J_1} \cos \left( \hat{\phi} + \varphi_1 \frac{C_{J_2} + C}{C_\Sigma} + \varphi_2 \frac{C}{C_\Sigma} \right) \\ & - E_{J_2} \cos \left( \hat{\phi} - \varphi_1 \frac{C_{J_1}}{C_\Sigma} + \varphi_2 \frac{C}{C_\Sigma} \right), \end{aligned} \quad (\text{C1})$$

with  $E_{J_i}$  ( $C_{J_i}$ ) the energy (capacitance) of the Josephson junction, capacitance  $C$  of the inductor and total capacitance of the circuit  $C_\Sigma = C + C_{J_1} + C_{J_2}$ .  $\varphi_1$  is the reduced physical flux in the dc-loop and  $\varphi_2$  in the rf-loop. Given the geometry of the circuit, the effective reduced fluxes of the rf-squid and the dc-SQUID are  $\varphi_{\text{ext}} = \varphi_2 + \varphi_1/2$  and  $\varphi_{\text{dc}} = \varphi_1$  and we express the asymmetry of the dc-SQUID as  $E_{J_1/J_2} = E_J(1 \pm d)$  and  $C_{J_1/J_2} = C_J(1 \pm d)$

$$\begin{aligned} \hat{H}_{E_J} = & E_C(2\hat{n})^2 + \frac{E_L}{2} \left( \hat{\phi} - \varphi_{\text{dc}} \frac{d C_J}{C_\Sigma} - \varphi_{\text{ext}} \frac{2 C_J}{C_\Sigma} \right)^2 \\ & - 2E_J \cos \left( \hat{\phi} + \varphi_{\text{ext}} \frac{C}{C_\Sigma} - \varphi_{\text{dc}} \frac{d C_J}{C_\Sigma} \right) \cos \left( \frac{\varphi_{\text{dc}}}{2} \right) \\ & + 2dE_J \sin \left( \hat{\phi} + \varphi_{\text{ext}} \frac{C}{C_\Sigma} - \varphi_{\text{dc}} \frac{d C_J}{C_\Sigma} \right) \sin \left( \frac{\varphi_{\text{dc}}}{2} \right). \end{aligned} \quad (\text{C2})$$

The latter term due to the finite junction asymmetry  $d$  is very harmful to the proposed gate mechanism, since  $\varphi_{\text{dc}}$  is

tuned from 0 to  $\pi$ . Thereby, the potential of the qubit is changed by a quarter of a period over this range, preventing the hybridization of the fluxon states. However, the Hamiltonian can be rewritten [41,42] in a more practical form

$$\begin{aligned} \hat{H}_{E_J} = & E_C(2\hat{n})^2 + \frac{E_L}{2} \left( \hat{\phi} - \varphi_{\text{dc}} \frac{d C_J}{C_\Sigma} - \varphi_{\text{ext}} \frac{2 C_J}{C_\Sigma} \right)^2 \\ & - E_J(\varphi_{\text{dc}}) \cos \left( \hat{\phi} - \varphi_{\text{dc}} \frac{d C_J}{C_\Sigma} + \varphi_{\text{ext}} \frac{C}{C_\Sigma} + \varphi_{\text{corr}} \right), \end{aligned} \quad (\text{C3})$$

with  $E_J(\varphi_{\text{dc}}) = E_{J,\text{max}} |\sqrt{\cos^2(\varphi_{\text{dc}}/2) + d^2 \sin^2(\varphi_{\text{dc}}/2)}|$  and  $\varphi_{\text{corr}} = \text{atan}(d \tan(\varphi_{\text{dc}}/2))$ . The latter contribution  $\varphi_{\text{corr}}$  encapsulates the change of the effective working point for  $\varphi_{\text{dc}}$  being modified.

In Fig. 12 this change in the external flux  $0.5 - \varphi_{\text{corr}}/2\pi$  with respect to the dc-flux is plotted for various values of the asymmetry  $d$ . The visualized potential and wavefunction graphics correspond to the idle point (blue) in the heavy regime ( $\Phi_{\text{ext}}/\Phi_0 = 0.5$ ,  $\Phi_{\text{dc}}/\Phi_0 = 0$ ) and in the light regime ( $\Phi_{\text{ext}}/\Phi_0 = 0.25$ ,  $\Phi_{\text{dc}}/\Phi_0 = 0.5$ ) highlighted in orange. The latter shows the apparent symmetry breaking, which has a drastic effect on the gate mechanisms. In order to preserve the double-well structure and the sweet-spots in the light regime (highlighted in magenta),  $\varphi_{\text{ext}}$  has to be actively transformed to  $\varphi_{\text{ext}} + \varphi_{\text{corr}}$  during the gate implementations. In that manner, the dc-SQUID asymmetry  $d$  can be mitigated and does not pose a challenge in fabrication. Since the gradient of the curves for small asymmetry in Fig. 12 is large approaching  $\Phi_{\text{dc}} = 0.5 \Phi_0$ , higher values of  $d$  are preferable experimentally for more robustness against flux noise.

The Hamiltonian in the main text in Eq. (1) underlies two simplifications that do not alter the proposed gate mechanism. First, since  $\varphi_{\text{corr}}$  can be compensated for,

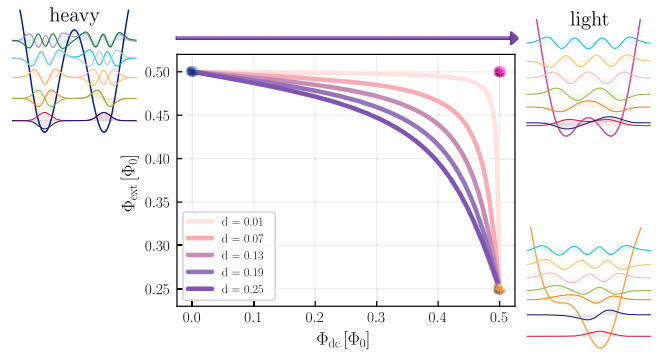


FIG. 12. The external flux  $\Phi_{\text{ext}}$  is visualized versus  $\Phi_{\text{dc}}$  for different values of the Josephson Junction asymmetry  $d$ . The wavefunction plots indicate the idling point in the heavy regime (blue) and during the gate in the light regime, in absence (orange,  $\Phi_{\text{ext}} = 0.25 \Phi_0$ ) and presence (magenta,  $\Phi_{\text{ext}} = 0.50 \Phi_0$ ) of compensation of the term  $\varphi_{\text{corr}}$ .

we absorb it in  $\varphi_{\text{ext}}$  and thus simplifying it to the case of  $d = 0$ , but with a finite minimum value of  $E_J$ . Second, we consider the limit  $C_j \gg C$  to arrive at the Hamiltonian in Eq. (1)

$$\hat{H}_{E_j} = E_C(2\hat{n})^2 + \frac{E_L}{2}(\hat{\varphi} - \varphi_{\text{ext}})^2 - E_J(\varphi_{\text{dc}}) \cos(\hat{\varphi}). \quad (\text{C4})$$

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