

Collective and Nonlinear Structure of Wind-Power Correlations

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We describe the correlation structure of wind-power fluctuations in a farm of 80 turbines, sampled over 5 years. We report the presence of universal, collective, and nonlinear correlations, responsible for the excess persistency and intermittency of farm-aggregated power output. A first cross-correlation analysis of turbine production reveals a dynamical scaling transition (*à la* Family-Vicszek) from local decoherence to large-scale turbulence-driven scaling, and responsible for the geographical smoothing effect, previously reported beyond farm scale [M. M. Bandi, Spectrum of wind-power fluctuations, Phys. Rev. Lett. **118**, 028301 (2017)]. A second bivariate analysis shows the long-range correlation of non-Gaussian features, responsible for their amplification in total farm output. These findings provide important insights into wind energy, the variability of which directly results from atmospheric turbulent forcing. By characterizing the multivariate and intermittent structure of fluctuations, our results advance the current understanding of wind-power variability and provide valuable priors for grid management and storage optimization, thereby supporting improved integration of wind energy into modern power systems.

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I. INTRODUCTION

Renewables occupy a growing share of the global energy portfolio. The main challenge of their exploitation is their variability and poor controllability, direct consequences of the fluctuating nature of resources such as wind [1–3] and solar radiation [4–6]. As a result, renewables constitute a source of risk—both local and global—on infrastructure integrity [7–11], adaptability to demand [12], and market pricing regulation [13,14].

To ensure the safe and efficient integration of these sustainable energy sources, a deep understanding of generation-side fluctuations [15–17] becomes essential. For wind energy in particular, turbine power output $P(t)$ fluctuations mirror the both scale-invariant and intermittent nature of atmospheric wind turbulence [18–31], roughly ranging from turbine reaction time ($\tau_R \sim 10$ s) to day-night injection cycles ($T \sim 24$ h).

Beyond single-turbine production, the larger-scale impact of a wind farm on the energy grid is evaluated by its total output production ($P_{\text{tot}} = \sum_i P_i$). Direct

farm-level data analysis has shown the emergence of a smoothing effect under geographical averaging, at farm level [22] or beyond [18], and its translation into steeper spectral decay. This apparent advantage—more persistent fluctuations might intuitively suggesting improved short-term predictability—is rapidly mitigated by the statistical properties of both individual and aggregated signals. In particular, spatially averaged power outputs retain and even amplify the non-Gaussian features observed at the single-turbine level (see distributions in Ref. [32]), the heavy-tailed nature of which can give rise to potentially damaging extreme events. Understanding these robust statistics across scales is thus a key requirement in the global and sustainable integration of wind energy.

This paper reports an analysis of the statistical structure of wind-power production in a wind farm of 80 turbines, that spans over 20 km. To do so, the wind speed $v_i(t) = \|\mathbf{v}_i\|$ and wind power $P_i(t)$ of each individual turbine were sampled every 10 min—following International Energy Commission standard 61400-12 [33]—for over 5 years. The turbines, of rotor-hub height 80 m and 90 m and diameter 116 m are organized in rows with an average separation of 365 m, and positioned on slightly elevated rolling crest lines of maximum height difference $\Delta = 150$ m. Each rows are facing the direction of the most prevailing wind directions from the south-southwest origin, and are separated by a minimum distance of approximately 1 km, beyond the near wake of turbines [34–37].

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Technical information of the wind farm considered is reported in Sec. I of the Supplemental Material [38]. In Sec. II, a first discussion on the turbulent nature of atmospheric winds (II A) motivates the description of turbine statistics through increments (II B) and multifractal analysis (II C). In Sec. III, these characterizations are extended at farm scale to understand the persistence of extreme fluctuations over farm aggregation (III A): a cross-correlation analysis reveals the coherent structure of wind-induced turbine fluctuations, and its impact over aggregated production (III B). Ultimately, we describe the spatial persistency of extreme events, among and beyond the farm (III C).

II. FROM ATMOSPHERIC TURBULENCE TO WIND-TURBINE STATISTICS

A. Atmospheric wind turbulence

1. Increment statistics

Turbulent flows, commonly characterized through their velocity, pressure, or temperature fields, are known to exhibit multiscaling properties [39–44]. Formally, for a multiscaling field $X(\mathbf{r}, t)$, the increments $\delta_{\ell, \tau} X = X(\mathbf{r} + \ell, t + \tau) - X(\mathbf{r}, t)$ exhibit anomalously scaling absolute moments, namely, the structure functions

$$S_q(\ell, \tau) = \langle |\delta_{\ell, \tau} X|^q \rangle.$$

In particular, temporal increments satisfy

$$S_q^X(\tau) = \langle |\delta_\tau X|^q \rangle \propto \tau^{\zeta_q}, \quad \tau_\eta \ll \tau \ll T, \quad (1)$$

where the scaling-exponent spectrum ζ_q characterizes the self-similar properties of $X(t)$ from the energy-injection timescale T down to the viscous microscale τ_η . Here, the generalized *Hurst* (or roughness) exponent spectrum $H_q = \zeta_q/q$ quantifies the temporal persistence of fluctuations, from typical events ($q \ll 2$) to extreme ones ($q \gg 2$). While a monofractal process with $H_q = H$ exhibits scale-invariant jump statistics, as in fractional Brownian motion, a multifractal process with $H_q \neq H$ displays continuously evolving statistics across scales [45,46]. For example, a decreasing spectrum H_q corresponds to the presence of increasingly heavy-tailed distributions at small scales, a property commonly referred as *small-scale intermittency*.

2. Inertial range of atmospheric turbulence

In atmospheric turbulence, the low kinematic viscosity of air implies velocity fluctuations extending down to very short timescales, of the order of the Kolmogorov timescale $\tau_\eta \sim 10 \mu\text{s}$. Here, however, we focus on the large-scale forcing time T , beyond which fluctuations decorrelate ($S_q(\tau \gg T) \rightarrow \text{const}$), and contrast two physically distinct interpretations for its origin.

A first estimation assumes that turbulent structures are generated by the vertical shear of the mean wind, with

representative speed $v_0 = \langle v \rangle \sim 8 \text{ m s}^{-1}$, across the height of the atmospheric boundary layer, of characteristic height $L \sim 1 \text{ km}$. Assuming the turbulence to be locally produced by mean flow gradients, one can compute a characteristic timescale $T \approx L/v_0 \sim 125 \text{ s}$. However, this timescale lies well below the upper correlation ranges reported in atmospheric measurements [18,19].

Instead, a second sweeping-based interpretation associates the decorrelation time with the diurnal cycle, $T \simeq 24 \text{ h}$, as directly observed in wind-speed records [18]. In this picture, turbulent structures are advected horizontally by the largest energy-containing motions at a root-mean-square velocity $u_{\text{rms}} = \sqrt{\langle \delta_\tau v^2 \rangle} \sim 5 \text{ m s}^{-1}$ [47,48]. The associated sweeping length, $L = u_{\text{rms}} T \approx 400\text{--}500 \text{ km}$, defines a mesoscale range consistent with meteorological measurements [18,27,28]. This large-scale sweeping picture is further supported by the observation of spatial and temporal Kolmogorov scaling ($\zeta_3 = 1$) [39,40], across various altitudes [18,27,28,49].

3. Space-time structure

A joint space-time description of turbulent statistics provides a powerful experimental framework for characterizing correlations over the full spatiotemporal domain. Taking cue from prior turbulence experiments and numerical studies [47,50], it has been shown that, for isotropic turbulence under linear advection, iso- S_q contours in space-time could be convincingly approximated by elliptical shapes (see, however, Ref. [47] for convective corrections). This leads to an effective parametrization that incorporates both advection and sweeping effects,

$$S_q(\ell, \tau) \propto ((\ell - \mathbf{v}_0 \tau)^2 + u_{\text{rms}}^2 \tau^2)^{\zeta_q/2}, \quad (2)$$

and which interpolates between the Galilean-invariant Taylor frozen-turbulence limit ($v_0 \gg u_{\text{rms}}$) and the continual renewal of turbulent structures described by the Kraichnan-Tennekes random-sweeping hypothesis ($v_0 \ll u_{\text{rms}}$) [48,51].

While the spatial scaling of upper-atmospheric winds is well known to follow Kolmogorov's K41 $k^{-5/3}$ law [49], the ABL is expected to exhibit different and nontrivial spatial features. First, the turbulent organization of the ABL [52] undergoes strong diurnal modulation, ranging from daytime buoyancy-driven convection to nighttime stable stratification at night. Second, viscous effects induced by obstacles—such as topographic variations [53]—introduce local retardation and disrupt the steadiness of wind flows. Third, advection may remain observable between turbine rows in the downwind direction (south-southwest in the data set analyzed here). However, taking an inter-row distance of 1 km and representative wind speed $v_0 = 8 \text{ m s}^{-1}$ yields an advection time $\tau_{\text{advection}} = 1000 \text{ m}/8 \text{ m s}^{-1} = 125 \text{ s}$ well below our sampling resolution $\Delta t = 10 \text{ min}$. For

larger separations, these causal structures should be further dampened by interactions with terrain and other obstacles. We anticipate both effects of Eq. (2) to mix into a single effective sweeping contribution of leading order

$$S_q(\ell, \tau) \propto (\tau^2 + \tau_\ell^2)^{\zeta_q/2}, \quad (3)$$

where the *coherence time* τ_ℓ sets the characteristic sweeping time over distance ℓ below which fluctuations should decorrelate ($S_q(\tau \ll \tau_\ell) \rightarrow \text{const}$) and beyond which multiscaling should be retrieved. Furthermore, we expect interactions with local topographic roughness to affect smaller turbulent structures particularly strongly. This conjecture should manifest itself through a sublinear growth of τ_ℓ with ℓ , will be examined below.

B. Extreme power fluctuations

Wind turbines convert a nearly constant fraction of the incoming wind power $P_{\text{wind}}(t)$ into electrical power $P(t)$. The wind power, associated with the transport at speed v of a cylindrical volume of air carrying kinetic energy $E_c \propto v^2$, leads to the well-established cubic scaling $P \propto v^3$ [20]. Figure 1(a) illustrates the close correspondence between the temporal fluctuations of $v(t)$ and $P(t)$ for a representative turbine in the farm. Deviations from the cubic law arise outside the operational wind-speed range of the turbines, bounded by the cut-in (lower) and rated (upper) speeds [20], beyond which the electrical power saturates. The overall turbine response curves $P = f(v)$ for the two turbine models considered in this study are reported in Sec. I of the Supplemental Material [38].

Similar to standard turbulence analysis, we describe the statistical structure of wind speed and wind power by computing their increments $\delta_\tau v$ and $\delta_\tau P$. We see in Fig. 1(a) that such a finite difference renders the signal stationary and it now evolves around a clearly defined average. The envelopes of the increments, $(|\delta_\tau v|, |\delta_\tau P|)$, themselves fluctuate over larger correlation scales, displaying alternating phases of weak local activity periods and intermittent bursts [46,54]. These stationary yet intermittent variations give rise to time-aggregated non-Gaussian jump statistics, as illustrated in Figs. 1(b) and 1(c), where normalized probability density functions (see caption for standardization rule) exhibit increasingly heavy tails at small τ , reflecting small-scale intermittency. For $\delta_\tau v$, the jump statistics progressively converge toward a Gaussian distribution as τ increases. In contrast, for $\delta_\tau P$, the turbine response function $P = f(v)$ imposes a hard cutoff, confining the power to the interval $[0, P_{\text{sat}}]$. This constraint leads to a condensation of probability near $\delta_\tau P = \pm P_{\text{sat}}$ for the largest increments, and near $\delta_\tau P = 0$ during shutdown or rated operating regimes. As shown in Sec. II of the Supplemental Material [38], defining a corrected power signal

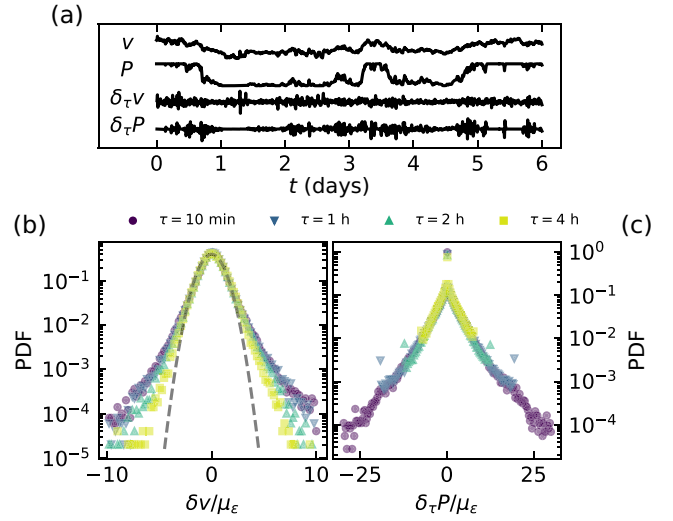


FIG. 1. The jump statistics of the wind speed v and the wind power P . (a) The weekly evolution of the wind speed v , the power output P , and their jumps $\delta_\tau v$ and $\delta_\tau P$ ($\tau = 10$ min). (b),(c) The probability density functions (PDFs) of jumps $(\delta_\tau v, \delta_\tau P)$ for $\tau = 10$ min, 1 h, 2 h, 4 h. To avoid the bias introduced by cutoff effects (especially for P), the core of the PDFs has been collapsed by rescaling jumps by the ratio between empirical and Gaussian low order ($\epsilon \ll 1$) moments: $\mu_\epsilon = [(\langle |\delta_\tau X|^\epsilon \rangle \sqrt{\pi} 2^{-\epsilon/2} / \Gamma((1+\epsilon)/2))]^{1/\epsilon}$.

P_c , obtained by removing these saturating regimes, is sufficient to retrieve the statistics of v^3 , except for extreme fat-tail events that cannot be reproduced by such filtering.

C. Multifractal analysis

Following seminal contributions to turbulence [39,40,55] and more recent analytical developments [45,46,56,57], it is now well established that the non-Gaussian statistics of turbulent flows can be efficiently described within the multifractal formalism. In the following, we investigate the scaling properties of our wind-farm data set.

1. From velocity to speed and power

We expect the intermittent non-Gaussian statistics of v and P to originate from the multifractal nature of the underlying turbulent wind velocity field $\mathbf{v}$ [40], as encoded in the scaling properties of its increments. To first order, the increments of speed and power can be approximated as

$$\delta_\tau v = \delta_\tau \mathbf{v} \cdot \mathbf{e}_v, \quad \delta_\tau P = v^2 (\delta_\tau \mathbf{v} \cdot \mathbf{e}_v) = v^2 \delta_\tau v, \quad (4)$$

where $\mathbf{e}_v$ denotes the unit vector in the direction of the mean wind velocity $\mathbf{v}$. In both expressions, velocity increments are effectively projected onto the dominant flow direction. While identical scaling properties are therefore expected for $\mathbf{v}$ and v , the multiplicative factor v^2 in the power increment introduces additional variability, leading

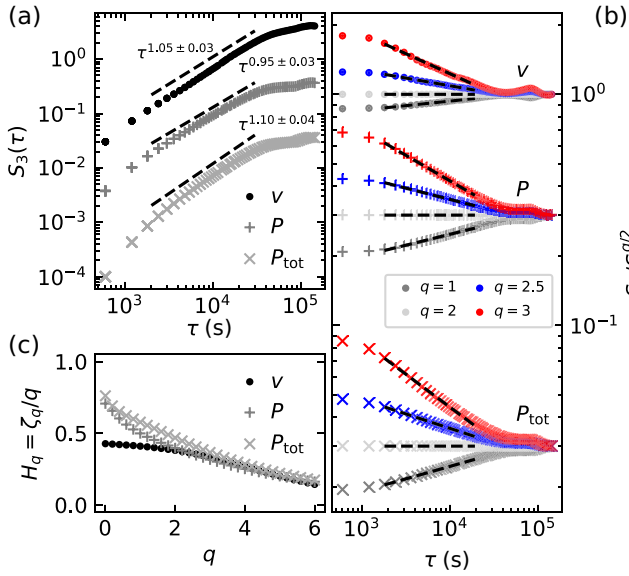


FIG. 2. The intermittency analysis of the wind speed v ($\circ$), the single-turbine power output P ($+$), and the total farm production $P_{\text{tot}} = \sum_i P_i$ ($\times$). (a) $S_3(\tau)$ versus τ . (b) $S_q(\tau)$ rescaled by $(S_2(\tau))^{q/2}$ for $q = 1, 2, 2.5, 3$, with fits for $30 \text{ min} \leq \tau \leq 5 \text{ h}$ shown as dotted black lines. (c) The scaling-exponent spectrum $H_q = \zeta_q/q$.

to potential deviations in the statistics of P , as discussed below.

For real wind turbines, however, the electrical power is bounded by the rated value P_{sat} . In this regime, the effective scaling of power increments, $\delta_\tau P \sim P^{2/3} \delta_\tau v$, is expected to closely follow that of the wind speed, yielding

$$S_q^v \sim S_q^P \sim S_q^P \sim \tau^{\zeta_q}.$$

The limiting case of ideal turbines, for which $P_{\text{sat}} \rightarrow +\infty$, will be discussed separately.

2. Kolmogorov scaling

Figure 2(a) shows the third-order structure functions $S_3(\tau)$ of both v and P for a representative turbine, which display clear power-law scaling up to $\tau \sim 12 \text{ h}$, in accordance with the diurnal forcing timescale. A fit performed over approximately one decade, $30 \text{ min} \leq \tau \leq 5 \text{ h}$ —the smallest scales being typically affected by windowing procedures—yields $S_3(\tau) \propto \tau^{\zeta_3}$ with $\zeta_3 \simeq 1$. We show in Sec. III of the Supplemental Material [38] that this scaling behavior is observed for all turbines in the farm and is consistent with previous measurements reported in Ref. [18], down to the turbine response time $\tau_R \approx 30 \text{ s}$. It corresponds to the Kolmogorov (K41) [39] prediction for a direct turbulent cascade, as has been similarly reported in Dutch and Corsican meteorological observations [27,28].

This result indicates that, within a real wind-farm configuration, the temporal autocorrelation properties of the

wind-speed and power fluctuations are essentially identical across turbines, although this does not imply their statistical independence. This behavior contrasts with results from idealized experiments and numerical simulations, where regularly designed farms with periodic turbine placement, linearly advected turbulence, and identical rotor heights could imprint modification on individual spectral behavior [58–61]. Here, the subjection of wind turbines to real meteorological and topographical conditions suppresses such effects.

3. Multifractal analysis

The full scaling spectrum ζ_q is obtained by combining two independent measures: ζ_3 , extracted from the fit shown in Fig. 2(a), and the deviations $[\zeta_q - q\zeta_2/2]$, inferred from the ratios $S_q/S_2^{q/2}$ displayed in Fig. 2(b). That last observable extends the classical flatness ($q = 4$) commonly used in turbulence studies into a *generalized flatness* measure. As mentioned in previous works [62], fitting ratios of structure functions effectively enlarge the inertial range and improve the robustness of scaling exponents. Consistent results are also obtained using an extended self-similarity (ESS) analysis [32,63]; however, without enforcing the constraint $\zeta_3 = 1$. The resulting generalized Hurst exponent spectrum, $H_q = \zeta_q/q$, is reported in Fig. 2(c) and its behavior is discussed below.

First, in regard to previously established works [18], the same exponents $H_2 \approx 0.4$ have been retrieved, in full correspondence with other energy-spectrum estimates [18, 19]. This allows us to discuss the relative position of exponents at lower and higher order. For $q < 2$, the moments primarily describe the scale invariance of small-amplitude fluctuations, at the core of increment distributions. Turbine shutdown and saturated regimes produce persistent extended periods of low-amplitude jumps $\delta_\tau P \approx 0$, resulting in larger roughness exponents H_q for P compared to v . For $q > 2$, the moments are expected to describe increasingly intense fluctuations associated with the tails of the distribution. However, the mean-reverting statistics of v induce an effect analogous to the bounded structure of P , leading to similar exponents.

4. Extension to ideal turbines

The scaling properties of the measured power P are strongly influenced by the response function of turbines. Therefore, we extend the analysis to ideal turbines with infinite rated power ($P_{\text{sat}} \rightarrow +\infty$) by directly examining v^3 and the corresponding ideal farm output $\sum_i v_i^3$. Figure 3(a) reveals a clear scaling behavior for v^3 , with a roughness exponent $H_3 \simeq 0.25$, smaller than the Kolmogorov predictions ($H_3 = \zeta_3/3 \simeq 1/3$) and identically observed for P_c (not shown here).

We attribute this effect to the autoregressive properties of the wind speed, reported in Ref. [20] and which

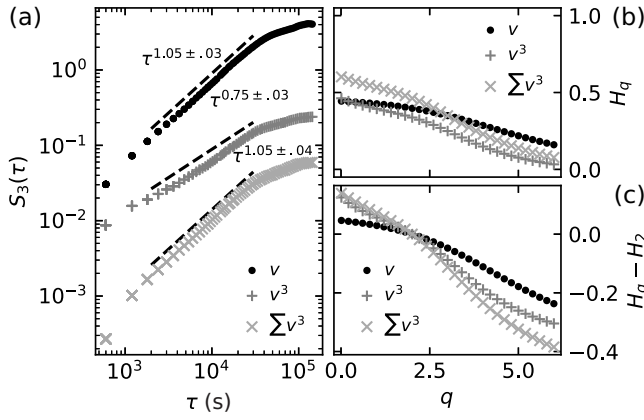


FIG. 3. The intermittency analysis of the wind speed v ($\circ$), v^3 ($+$), and $\sum v^3$ ($\times$). (a) The structure functions S_3 versus τ . (b) The generalized Hurst exponents $H_q = \zeta_q/q$ versus q , fitted as in Fig. 2 (c) $H_q - H_2$ versus q .

results in positive correlations between the wind amplitude v and the increment amplitude $|\delta_\tau v|$, i.e., strong mean winds yield stronger wind fluctuations. As a result, the increments of the exponentiated velocity should scale as a higher-order power law of wind-speed increments:

$$\delta_\tau(v^m) \approx v^{m-1} \delta_\tau v \sim (\delta_\tau v)^{\alpha_m},$$

with $\alpha_m > 1$, increasing with m . In consequence, wind-power increments ($m = 3$ for ideal turbines) will be shifted, $q \rightarrow \alpha_m q$, and will yield the decrease in H_2 reported for v^3 in Fig. 3(b). We have found, by collapsing the exponent spectra of v^2 and v^3 onto v , that $\alpha_2 \approx 1.3$ and $\alpha_3 \approx 1.5$.

III. CORRELATION STRUCTURE OF WIND-FARM PRODUCTION

A. Statistics of aggregated turbines

From the standpoint of wind-farm operators, the total farm production $P_{\text{tot}} = \sum_i P_i$ is an observable of interest for power transmission and grid design. Although one would expect the summation of production over turbines to yield smoother Gaussian statistics and fewer extreme events, we will see below that this intuition does not hold. A preliminary result may be that the daily deviation to mean ratio only weakly decays from P_i to P_{tot} (see Table I) and remains significantly larger than the discount predicted by the central limit theorem (CLT) $N^{-1/2} \sim 0.1$. We expect such behavior to result from the presence of temporal and spatial correlations among and between turbines. That intuition is confirmed in Fig. 4, where a qualitative comparison of single- and farm-aggregated turbine output statistics evidences strong similarities, which we now investigate quantitatively.

TABLE I. The statistics of the wind speed v , the individual turbine power P , and the total farm power P_{tot} . Here $(-)$ indicates most extreme values among turbines, and $(\pm)$ error bars from the curves fitted in Fig. 3(a).

X	v	P	P_{tot}
$\langle X \rangle$	8.0–9.0 m s ⁻¹	1.09–1.36 MW	100.2 MW
$\sqrt{\langle \delta_\tau X^2 \rangle}$	4.4–5.3 m s ⁻¹	1.12–1.28 MW	91.6 MW
$\sqrt{\langle \delta_\tau X^2 \rangle} / \langle X \rangle$	0.54–0.64	0.90–1.04	0.93
ζ_3	1.05 ± 0.03	0.95 ± 0.03	1.10 ± 0.04

As shown in Fig. 2(a), the aggregated farm production P_{tot} reproduces the scale-invariant behavior observed for individual turbine power P , with, however, modified scaling exponents. Timescales shorter than the sweeping time across the farm, $\tau_D = D/u_{\text{rms}} \sim 2$ h, are expected to be affected by spatial aggregation. In practice, the relaxation toward decorrelation and the presence of retarded structures extend these modifications up to 2–3 τ_D ; for a direct comparison between the scaling behavior of P and P_{tot} , see Sec. IV of the Supplemental Material [38]. The resulting generalized Hurst exponents H_q , shown in Fig. 2(c), are systematically larger for P_{tot} at low orders ($q < 2$), while they coincide with those of P for higher orders ($q > 2$). This, respectively, indicates the emergence of a more spread distribution core, as well as the conservation of non-Gaussian tail properties over aggregation. Note that the increase of H_2 indicates the emergence of more persistent time correlations.

While the second-order exponent H_2 primarily characterizes the correlation structure of the time series—persistent (antipersistent) dynamics corresponding to $H_2 > 1/2$ ($H_2 < 1/2$)—the scale dependence of standardized jump statistics is more naturally captured by the relative roughness exponent $H_q - H_2$, directly fitted from generalized flatness functions. In the idealized limit $P_{\text{sat}} = +\infty$, Fig. 3(c) shows that low-order relative exponents remain essentially unchanged, whereas high-order ones decay sharply: the core of the increment distributions is preserved, while

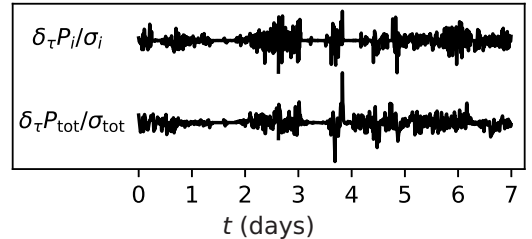


FIG. 4. The wind-power increments $\delta_\tau P$ ($\tau = 10$ min) of a single turbine (top) and of the total farm (bottom), sampled over a week. Each time series has been normalized by its empirical deviation.

extreme events, unbounded here, constructively add up to extend the tail of aggregated production.

B. Cross-structure analysis

The multifractal analysis introduced above confirms the presence of cross-correlations among turbines, which we now directly describe. As shown previously and further described in Sec. I of the Supplemental Material [38], the statistics of P have been found to be similar to v^3 within the operating regime $P \propto v^3$. To describe the wind-field structure dictating turbine production, we then directly use wind speeds (v_i), standardized by their deviations and mean values. We describe pairwise correlations by using cross-structure functions:

$$S_q^{ij}(\tau) = \langle |v_i(t + \tau) - v_j(t)|^q \rangle.$$

We focus on the second-order case $q = 2$ to retrieve the a direct equivalence with cross-spectrum analysis $\mathcal{F}[v_i]\mathcal{F}[v_j]$ [64]. Note that replacing v by v^3 , or cleaned turbine output P_c , yields identical conclusions in the following.

1. Cross-structure analysis

Figure 5(upper inset) shows $S_2^{ij}(\tau)$ for four turbines separated by distances $\ell_{ij} - \ell_{ij} = 0$ corresponding to $i = j$. Two scaling regimes, postulated in Eq. (3), appear: (i) for low τ , S_2^{ij} flattens as turbulence decorrelates at short timescales; (ii) beyond this short timescale, the signal coherence [64] is retrieved as an ℓ_{ij} -dependent transition toward scale invariance, $S_2^v \propto \tau^{\zeta_2}$.

This asymptotic behavior being identical to the effective sweeping predicted in Eq. (3), we have decided in Fig. 5(lower inset) to fit and display, for all (i, j) , an *effective coherence time* parameter τ_{ij} defined from

$$S_2^{ij}(\tau) \propto \left(\tau^2 + \tau_{ij}^2 \right)^{\zeta_2/2}. \quad (5)$$

We have found τ_{ij} to evolve from a few minutes to the previously estimated sweeping time over the farm $\tau_D = 2$ h. Estimation of the sweeping speed $u_{ij} = \ell_{ij} / \tau_{ij}$ for neighboring turbines ($u_{ij} = 500 \text{ m}/300 \text{ s} = 1.3 \text{ m s}^{-1}$) indicates that the decoherence regime over short distances is dictated by slower more retarded sweeping dynamics. This result is confirmed in Fig. 5(lower inset), where a power-law fit $\tau_{ij} \propto \ell_{ij}^\beta$ $\beta = 0.60 \pm 0.15$ evidences the sublinear growth of sweeping times over separation ℓ_{ij} .

2. Influence of ABL regimes

The scattering of measured τ_ℓ has multiple causes: wake patterns, rough topography, and seasonal changes. In particular, we know the wind-speed correlation structure to be strongly affected by day-night modulations of the ABL.

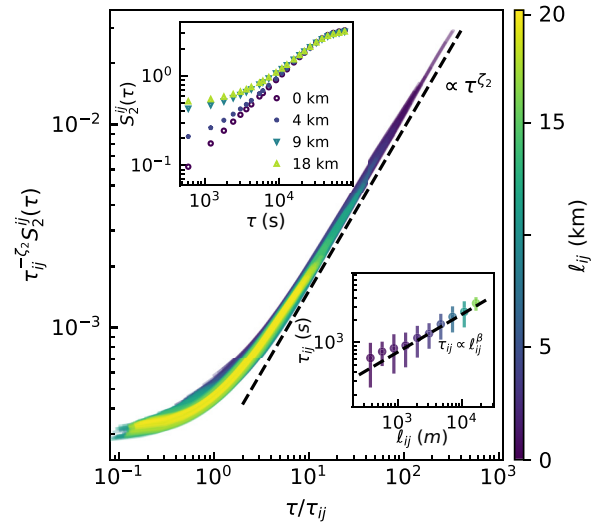


FIG. 5. The cross-structure analysis of wind turbines. The upper inset shows S_2^{ij} versus τ for neighbouring turbines, with the pair turbine distance ℓ_{ij} in the legend. The main figure shows the rescaled S_2^{ij} versus τ , fitted and collapsed using Eq. (5). The scaling asymptote is shown in black, with $\zeta_2 = 0.8$. The lower inset shows the fitted coherence times τ_{ij} as functions of ℓ_{ij} and the fitted power-law scaling shown in black, with $\beta = 0.60 \pm 0.15$.

Indeed, during the day, solar heating creates vertical mixing that brings stronger winds from higher altitudes down to the surface, making winds typically stronger and gustier. At night, the ground cools, stabilizing the atmosphere and suppressing vertical mixing, which leads to calmer winds with horizontally stratified structures. We reproduce the analysis of Fig. 5 by masking (v_i) into day (vertical and/or convective) and night (horizontal and/or sweeping) ABL regimes [52]. We have retrieved $\beta = 0.3 \pm 0.1$ and $\beta = 0.8 \pm 0.1$, corresponding to: (i) convective turbulence, for which τ_{ij} is expected to saturate [47,65] ($\beta = 0$); and (ii) horizontally swept and/or advected turbulence, for which $\tau_{ij} \propto \ell_{ij}$ ($\beta = 1$). We expect higher-frequency sampling over a few days to yield more refined estimates of β .

3. Collapse to master curve

In the main plot of Fig. 5, S_2^{ij} versus τ curves have been rescaled from Eq. (5) using $\zeta_2 = 0.8$. As a result, all of the data set has collapsed onto a single master curve $x \mapsto f(x)$, defined by

$$f(x) \sim \begin{cases} 1, & \text{for } x \ll 1, \\ x^{\zeta_2}, & \text{for } x \gg 1, \end{cases}$$

where $x = \tau/\tau_{ij}$. This form is directly analogous to dynamic scaling, *à la Family-Vicszek*, where space-time correlations collapse onto a universal function. The coherence time τ_{ij} encapsulates a strong statistical dependence

on turbine separation ℓ_{ij} , demonstrating that the transient and coherence regimes are universal across the farm.

4. From correlations to spectral steepening

The increase of the scaling exponents of P_{tot} can now be traced back to the cross-correlation identified previously. For $q = 2$, the structure function of the total power can be written as

$$S_2^{\text{tot}}(\tau) = \sum_i S_2^{ii}(\tau) + \frac{1}{2} \sum_{i \neq j} [S_2^{ij}(\tau) - S_2^{ij}(0)],$$

which, in the limit $N \gg 1$, is dominated by the second sum on the right-hand side, involving $N(N-1) \sim N^2$ contributions. These pairwise contributions can be expanded following

$$S_2^{ij}(\tau) - S_2^{ij}(0) \propto \tau_{ij}^{\zeta_2} \left[f\left(\frac{\tau}{\tau_{ij}}\right) - f(0) \right] \sim \left(\frac{\tau}{\tau_{ij}}\right)^\alpha,$$

where the exponent α controls the scaling properties of f around 0. In particular, the effective sweeping form of Eq. (3) predicts ballistic scaling with $\alpha = 2$, whereas the elliptic approximation of Eq. (2), dominated by advection effects, leads to diffusive scaling with $\alpha = 1$. We note, however, that the scaling range of the individual cross-structure terms $S_2^{ij}(\tau) - S_2^{ij}(0)$ is not sufficiently extended in the present data set to allow for a definitive discrimination between these two regimes.

C. Extreme fluctuations: Excess intermittency and range

1. Copula analysis

While the cross-scaling analysis above accounts for the increase of the roughness exponents H_q under turbine-output aggregation, it does not by itself explain the persistence of non-Gaussian statistics, nor their enhancement, at short timescales ($\tau \rightarrow 0$). This effect is directly observable in Fig. 4, where the relative amplitude of calm to extreme fluctuations appear is visually higher for P_{tot} than for a single turbine.

To address this point, we turn to a multivariate description of power increments and analyze the joint PDFs $\mathbb{P}(\delta_\tau P_i, \delta_\tau P_j)$, focusing on their dependence on the time lag τ and the interturbine separation ℓ_{ij} . As shown in Sec. V of the Supplemental Material [38], the progressive alignment of the iso- $\mathbb{P}$ contours along the diagonal reveals the buildup of correlations between power fluctuations, as τ increases and ℓ_{ij} decreases, corresponding, respectively, to temporal integration and turbine separation.

To eliminate the bias induced by the marginal distributions of power increments, we construct the associated Gaussian copulas by mapping each marginal onto

a zero-mean unit-variance normally distributed variable, $u_i = \mathcal{F}[\delta_\tau P_i]$. The monotonic transformation $\mathcal{F}$ is found to be approximately identical across turbines. The resulting copulas,

$$\rho(u_i = \mathcal{F}[\delta_\tau P_i], u_j = \mathcal{F}[\delta_\tau P_j]) = \mathbb{P}(\delta_\tau P_i, \delta_\tau P_j), \quad (6)$$

are computed from Gaussian-kernel-density methods [66, 67] and are shown in Figs. 6(a) and 6(b) (see also Sec. V of the Supplemental Material [38]). They reveal a distinctly nonelliptical dependence structure, closely resembling that observed in joint-return statistics in finance [68], the significance of which we now explain. At fixed τ , extreme events remain strongly correlated for all separations ℓ_{ij} , yielding a characteristic cross-shaped joint PDF, while for neighboring turbines their signs align preferentially, as evidenced by the pronounced elongation along the $u_i = u_j$ direction. This joint alignment of signs and persistence of large amplitudes directly drives the excess intermittency, since coincident extreme fluctuations add coherently and dominate the aggregated statistics.

2. Magnitude correlations

To quantitatively measure the space-time range of extreme-events correlations, we compute from our time series a measure of local deviation, the *log-volatility* (or *magnitude*), defined as

$$\omega_\varepsilon = \ln |\delta_\varepsilon X| + \Omega_\varepsilon, \quad (7)$$

where the constant term Ω_ε ensures that $\langle \omega_\tau \rangle = 0$. Interestingly, this measure has been introduced in Ref. [45] and used in Refs. [27,28,46,54] as a complementary description of multifractal statistics. In fact, for a multifractal process of *intermittency coefficient* $\lambda = -\zeta_q''(0)$, it has been shown [27,45] that the covariance of its magnitude should decay following

$$C_{\omega_\varepsilon}(\tau) = \langle \omega_\varepsilon(t) \omega_\varepsilon(t + \tau) \rangle = -\lambda \ln \tau / \mathcal{T}, \quad \tau > \varepsilon, \quad (8)$$

where $\mathcal{T}$ is an upper multifractal range, usually coinciding with upper inertial range T —although see Refs. [46,54] for counterexamples. In Fig. 6(c), the autocovariance of the wind-speed magnitude ω_τ^v computed from Eq. (8) expectedly shows logarithmic decay for $\tau > \varepsilon$, with $\mathcal{T} = T = 24\text{h}$, and $\lambda = 0.04$, identically reported in Ref. [27,28] for hourly sampled ($\varepsilon = 1\text{h}$) Dutch and Corsican wind data. Note that fitting H_q around $q \rightarrow 0$ from our wind-speed data yields the same $\lambda = 0.04$ value. The excess correlations reported for $\tau < \varepsilon$ result from the coarse graining introduced in the volatility computation.

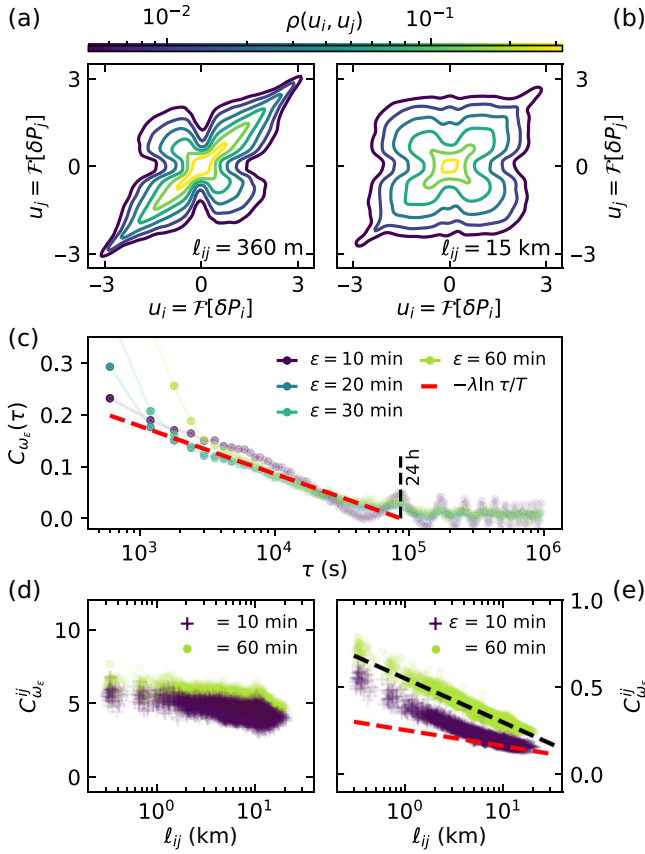


FIG. 6. The copula analysis and spatial correlations. (a),(b) Iso-density lines of the Gaussian copula $\rho(u_i, u_j)$ of power increments [computed from Eq. (6)], for (a) neighboring and (b) distant turbines. (c) The wind-speed autocovariance C_{ω_ε} versus the lag τ for different ε values. The red line is the decaying logarithm of parameters $T = 24$ h, and $\lambda = 0.04$ [28]. (d),(e) The covariances $C_{\omega_\varepsilon}^{ij}$ versus ℓ_{ij} [computed from Eq. (8)] for $\varepsilon = 10$ min, and 60 min, for (d) P and (e) v . The black and red logarithmic curves, respectively, use $\lambda = 0.10$, $\xi = 200$ km and $\lambda = 0.04$, $\xi = 600$ km (similar to Ref. [28]).

We extend our analysis to spatial correlations of the magnitude by defining and computing the pairwise covariance,

$$C_{\omega_\varepsilon}^{ij} = \langle \omega_\varepsilon^i(t) \omega_\varepsilon^j(t) \rangle, \quad (9)$$

for all $80^2 = 1600$ pairs (i, j) of magnitude measures of P and v , using $\varepsilon = 10$ min and 60 min [28]. Figures 6(d) and 6(e) show the decaying trend of these covariances as functions of turbine separation ℓ_{ij} . For ω_P , the correlations have been found to slightly decrease over turbine distance, an effect that we ascribe to the presence of saturating and shutdown regimes.

For ω_v , Fig. 6(d) shows that the spatial covariance organizes along logarithmically decaying curves as a function

of turbine separation, of the form

$$C_{\omega_\varepsilon}(\ell) = -\lambda \ln \ell / \xi. \quad (10)$$

For a coarse-graining timescale $\varepsilon = 1$ h, a fit [black curve in Fig. 6(d)] yields $\xi = 200 \pm 50$ km $\simeq L/2$, slightly below the forcing scale associated with diurnal variability, and $\lambda = 0.10 \pm 0.01$, significantly larger than the value inferred from the temporal correlations of ω_ε in the $\tau > \varepsilon$ regime.

In reality, to evidence a universal correlation regime for ω_v , the sweeping approximation yields a condition similar to $\tau > \varepsilon$ found for the autocovariance analysis: $\ell > \varepsilon u_{\text{rms}}$, which for $\varepsilon = 60$ min leads to a quasimaximum turbine separation $\ell = 18$ km $\sim D$. As a result, logarithmic fits obtained for larger values of ε will display excess correlations and coarse-graining-dependent fitting parameters.

For $\varepsilon = 10$ min, the sweeping condition reduces to $\ell > 3$ km. In this case, Fig. 6(d) reveals a crossover from excess short-range correlations to a logarithmic decay consistent with the parameters $\lambda = 0.04$ and $\xi = 600$ km (black curve) reported in Ref. [28] for $\ell > \varepsilon$, although the data points around $\ell \sim 20$ km remain systematically above the fit. This value of ξ is consistent with previous estimates of correlation lengths for instantaneous ($\varepsilon \rightarrow 0$) power fluctuations across wind plants in Texas ($L \approx 300$ km) [69], as well as with the estimate $L \approx 400$ km obtained from the sweeping approximation.

Ultimately, in the context of wind power, our results show that amplitude correlations extend over distances comparable to, and well beyond, the physical size of the wind farm. We further identify a clear crossover from excess short-range correlations induced by signal filtering to a surface-layer-boundary turbulent regime, consistent with previously reported statistics [28]. As a consequence, spatial aggregation does not suppress extremes: rather, extreme fluctuations combine coherently, allowing the farm power output to retain and even amplify its non-Gaussian character.

IV. CONCLUSIONS

A. Results

We conclude by summarizing our main findings. We have shown that wind turbines inherit the scale-invariant and non-Gaussian statistics of atmospheric wind-speed fluctuations. At the farm level, however, the aggregated power output departs from single-turbine behavior, by displaying steeper spectral decay and more intermittent statistics. These features have been investigated through a combination of cross-correlation analyses and nonlinear multivariate descriptions, applied consistently to both wind-speed and power signals.

We have demonstrated the existence of robust and universal cross-correlations between turbine outputs. These correlations show a transition from short-time decoherence to turbulence-driven scaling regime, controlled by a coherence time τ_{ij} directly related to turbine pair-distance ℓ_{ij} . In particular, we have found the sublinear relation $\tau_{ij} \sim \ell_{ij}^\beta$ with $\beta < 1$, indicating the stronger retardation of smaller scale structures in the farm.

By using Gaussian copula and autocorrelation analyses, we have further shown that wind-power fluctuations exhibit persistent correlations in both amplitude and sign across turbines. In particular, and consistently with mesoscale boundary layer fluctuations, amplitude correlations extend well beyond the physical size of the farm (approximately 20 km). As a result, extreme events in neighboring turbines align in jump directions, leading to coherent summation of power bursts, thus reinforcing intermittent behavior in the aggregated output.

We have systematically extended our analysis to ideal turbines with unbounded rated power ($P_{\text{sat}} = +\infty$, $P \propto v^3$). In this limit, the scaling of individual turbine production has been found to deviate from the Kolmogorov scaling of the wind speed, a behavior that we attribute to the autoregressive nature of wind-speed fluctuations. Despite the absence of saturation effects, the aggregated power in this idealized setting exhibits the same increase in persistence (jump sign correlations) and intermittency reported for real turbines. This confirms that nonlinear response regimes and turbulence-induced correlations are both independently responsible for the robustness of non-Gaussian features under wind-power aggregation.

B. Perspectives

Our results rely on the scale-invariant properties of atmospheric wind turbulence; we therefore expect the reported findings to remain valid, and to be further refined, at higher temporal resolutions. In particular, a better identification of causal wind structures, boundary-layer variability, and ε -independent magnitude correlations would be beneficial.

For the resolution considered, $\Delta t = 10$ min, the mapping of the wind speed and/or power introduced by the response function $P = f(v)$ suggests practical routes for forecasting intraday power fluctuations. One option would be to enrich multivariate wind-speed models with a spatial prior in order to improve short-time predictions, as explored in Refs. [30,70], where Fokker-Planck-based predictors for wind speed and power have been developed—albeit relying solely on local-in-space measurements. In this context, the long-range magnitude correlations identified here, and previously reported in the literature [28], provide a valuable basis for wind-speed volatility forecasting, in close analogy with methods developed for asset-price dynamics [71–73].

More broadly, the framework developed in this work provides quantitative tools to assess wind-farm sensitivity to extreme events and to evaluate their impact on electrical infrastructure. In a way similar to the phenomenological description of financial markets [74–76], we expect that the continued uncovering and characterization of the non-linear correlation structure of wind-power fluctuations will provide actionable insights for local buffering and control strategies.

While the large-scale transition toward sustainability will rely on coordinated policies and cross-system collaboration [15,77], we expect our results, together with recent advances in renewable variability, to provide quantitative benchmarks to inform and calibrate future policy and operational decisions.

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DATA AVAILABILITY

The data that support the findings of this paper are not publicly available because they are owned by a third party and the terms of use prevent public distribution. The data are available from the authors upon reasonable request.

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