

Upper Bounds on Broadband Absorption and Application to Solar Cells

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We address the question of the optimal broadband absorption of waves in an open dissipative system and its key implications for solar cells. We develop a general framework for absorption induced by multiple overlapping resonances, based on quasinormal modes and radiative and nonradiative decay rates. We demonstrate that the upper bounds on broadband absorption in a slab of thickness d take the simple form $A = 1 - \exp(-F\alpha d)$, where α is the absorption coefficient and F is the path enhancement factor. We apply these results to sunlight absorption in photovoltaics and answer the long-standing debate on the best light-trapping strategy in solar cells. For angle-independent absorption enhanced by scattering, we derive the isotropic upper bound $F = 4n^2$ (where n is the refractive index), extending the well-known Lambertian (or Yablonovitch) limit beyond the ray-optics and weak-absorption regimes. For angle-restricted illumination, we show that F can be further increased up to $8\pi n^2/\sqrt{3}$ using multiresonant absorption induced by periodical patterning. These results have a general scope in the field of wave physics and open new opportunities to maximize absorption, detection, and attenuation of electromagnetic or mechanical waves in ultrathin devices.

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I. INTRODUCTION

Absorption of waves is an elemental process in physics. Maximizing absorption over a broad spectral range is crucial in many applications, such as solar-energy harvesting, optical telecommunications, radar stealth, or attenuation of acoustic and seismic waves.

In optics, the black body is an ideal picture of a perfect broadband absorber, but no actual object absorbs all incident light. In pictorial art, the tiny amount of light reflected by *black* surfaces has inspired the vast artwork of Pierre Soulages, named *Outrenoir* (beyond black). Actually, close to perfect absorption of waves can be achieved with bulk and strongly absorbing materials. In thin and weakly absorbing materials, absorption can be enhanced by leveraging scattering or resonance phenomena, but it remains constrained by the absorption coefficient α and the volume of the absorbing medium. The best strategy

to optimize absorption is a fundamental but still open question that impacts many areas of physics.

Energy conservation and the causality principle have been used to derive theoretical limits of the response of lossy systems in specific cases, leading, for instance, to upper bounds on absorption and scattering cross sections [1–3] and on the bandwidth-thickness absorption limit [4–9], or to constraints on the (local) density of states [10–12]. However, they do not provide general models, guidelines and limits for absorption optimization over a broad spectral range.

In solar cells, Lambertian scattering is commonly used as a reference model for light trapping [13,14] but, contrary to common belief, it is not a fundamental limit [15–17]. An absorption upper bound has been derived by Miñano [15], yet it is restricted to ray-optics and isotropic scattering.

In the following, we first present the historical approaches to optimizing absorption and their limitations. Then, we develop a comprehensive theoretical model for absorption induced by multiple overlapping resonances in an open dissipative system. We derive analytical expressions for upper bounds on broadband absorption, with a wide domain of validity in wave physics. We apply these results to sunlight absorption in photovoltaics and

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we discuss their implications. We show that our theoretical framework embraces and extends the whole scope of previous theories. It reconciles results from the literature with theoretical models and it provides a powerful tool to determine the optimal light-trapping strategy for solar cells, beyond Lambertian scattering.

II. HISTORICAL APPROACHES TO THE ABSORPTION PROBLEM

In optics, absorption models are of particular importance for sunlight harvesting in solar cells [18]. The theoretical efficiency limit of an ideal solar cell, derived by Shockley and Queisser [19], is calculated by assuming perfect absorption for photon energy above the semiconductor band gap E_g and no absorption below. However, this simplistic approximation does not take into account the material properties (refractive index n and absorption coefficient α) and finite thickness d of actual solar cells [20]. As a starting point, assuming no reflection at the front and back interfaces leads to single-pass absorption, expressed by the Beer-Lambert law: $A = 1 - \exp(-\alpha d)$. The combination of a back mirror and surface patterning can be used to reflect, scatter, and trap light within the cell, resulting in an enhancement of the optical path by a factor F . What are the optimal light-trapping strategies and the resulting absorption upper bounds?

Previous light-trapping theories have been developed for ray-optics and isotropic illumination (see Sec. S.1 of the Supplemental Material [21]). They are based on a statistical approach that considers the distribution of propagation directions in the absorber, and the probabilities for light to be trapped by total internal reflection or to escape in free space. It results in an average optical path Fd . Assuming weak absorption, Yablonovitch has found the well-known maximum enhancement factor $F = 4n^2$ in a seminal paper [13] and further extended to restricted acceptance angles θ , as $F = 4n^2 / \sin^2 \theta$ [22]. He has also proposed an absorption model for Lambertian scattering induced by surface texturation: $A_l = \alpha d / (\alpha d + 1/F)$ [13, 14]. This expression, often misleadingly named the Lambertian or Yablonovitch limit, is not a fundamental limit. It has been shown recently that it can be overcome with, e.g., carefully designed regularly patterned absorbers [17,23]. An upper bound on absorption has been derived by Miñano [15] with the same factor F : $A = 1 - \exp(-F\alpha d)$. This expression is valid for both weak and strong absorption but it has been obtained in the ray-optics regime.

In wave physics, open dissipative systems can be rigorously described in the framework of non-Hermitian physics [24]. In a one-port system (no transmission of incident waves), perfect absorption can be achieved at a given resonant frequency in the well-known critical-coupling regime [25]. It has been extended to two-port systems as the so-called coherent perfect absorption [26],

and more recently to reflectionless multiport systems with the scattering-matrix formalism [27–29].

In contrast to previous approaches, to address the general question of broadband absorption of lossy systems, we need to consider multiple strongly overlapping resonances.

III. GENERAL MULTIRESONANT MODEL FOR ABSORPTION

Here, we present a general framework based on the analysis of the response of a multiresonant system in the frequency domain, described in detail in Secs. S.2, S.3, and S.4 of the Supplemental Material [21]. We consider a slab of absorber with a back mirror, excited by a plane wave, as an illustrative example of a multiresonant open dissipative system [Fig. 1(a)]. Interactions of incident waves with the system can be described by a set of resonant modes, the quasinormal modes (QNMs) [30,31], characterized by their complex frequencies $\omega_m^p = \omega_m + j\gamma$, where the decay rate γ represents the total losses.

A. The single-mode response

We first consider the simple case of a single mode coupled to one radiative channel. It is common to use a phenomenological description of the system dynamics, the temporal coupled-mode theory [32,33], to distinguish two different origins of losses: internal losses due to absorption are proportional to the nonradiative decay rate γ_{nr} , and external losses due to outcoupling are proportional to the radiative decay rate γ_r . For a single resonance m , the absorption spectrum $A_m(\omega)$ is a simple Lorentzian function [33]:

$$A_m(\omega) = \frac{4\gamma_r\gamma_{nr}}{(\omega - \omega_m)^2 + (\gamma_r + \gamma_{nr})^2}. \quad (1)$$

Considering a narrow spectral range relative to the resonance bandwidth $\Delta\omega = 2\gamma = 2(\gamma_r + \gamma_{nr})$, absorption is maximized at the critical-coupling condition $\gamma_r = \gamma_{nr}$, with perfect absorption at the resonance frequency $A_m(\omega_m) = 1$.

The reflection of this system can be written as a pole-zero decomposition:

$$R_m(\omega) = 1 - A_m(\omega) = \left| \frac{\omega - \omega_m^z}{\omega - \omega_m^p} \right|^2, \quad (2)$$

with

$$\omega_m^p = \omega_m + j(\gamma_r + \gamma_{nr}), \quad (3)$$

$$\omega_m^z = \omega_m + j(\gamma_{nr} - \gamma_r). \quad (4)$$

B. From sparse to overlapping resonances

Broadband absorption requires multiple resonances, the spectral distribution, width, and amplitude of which

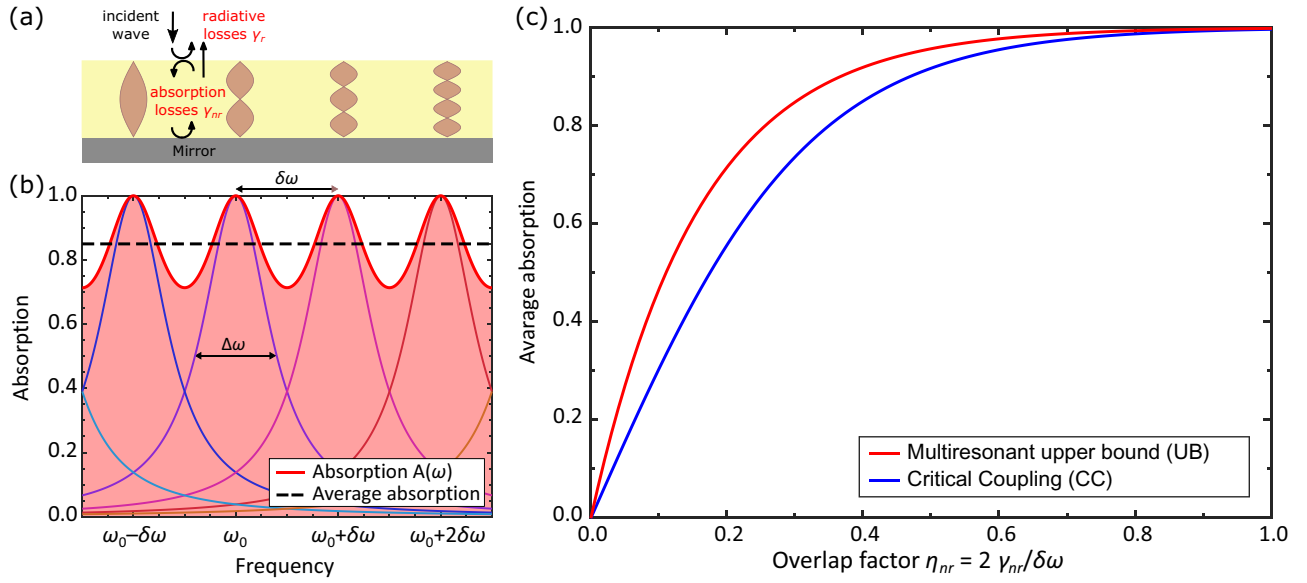


FIG. 1. The general model for multi-resonant absorption. (a) A sketch of an open system absorbing incident waves through multiple resonances and coupled through a single radiative channel. (b) Multiple evenly distributed and independent overlapping resonances (colored Lorentzian curves) in the critical-coupling regime, along with their total absorption envelope (red line and area) and the average absorption (dashed black line). (c) The average absorption as a function of the nonradiative overlap factor η_{nr} in the critical-coupling regime (radiative and nonradiative losses are equal) in blue [Eq. (12)] and the upper bound in red [Eq. (13)].

depend on several parameters. On the one hand, the non-radiative decay rate is defined by the internal properties of the system, i.e., the absorption coefficient and thickness, and cannot be changed. On the other, the spectral response of the system can be tailored by optimizing its boundary conditions, which impact the spectral position and the radiative coupling of modes. For example, a front-side patterning allows tuning the density of resonant modes, and an antireflection coating their free-space coupling [17,34,35].

In the case of sparse resonances, total absorption can be approximated as a sum of Lorentzian functions [36]. In contrast, optimizing broadband absorption requires considering multiple overlapping resonances, as illustrated in Fig. 1(b). This raises new challenges that are not addressed by previous theoretical frameworks.

C. The multi-resonant absorption model

The reflection coefficient of a plane wave on the system, $r(\omega)$, can be decomposed using the Weierstrass factorization valid for meromorphic functions [37,38]:

$$r(\omega) = \mathbb{A} \cdot \exp(j\omega\mathbb{B}) \cdot \prod_m \frac{\omega - \omega_m^z}{\omega - \omega_m^p}, \quad (5)$$

where $\mathbb{A}$ and $\mathbb{B}$ are two complex numbers, and ω_m^p and ω_m^z are the poles and zeros of the system, respectively.

Our goal is to maximize absorption, which justifies the series of simplifications summarized below and detailed in

Secs. S.3 and S.4 of the Supplemental Material [21]. First, we show that in most common situations (sufficiently high quality factors, strong radiative coupling, weak absorption, sparse resonances, etc.), $|\mathbb{A}| \rightarrow 1$ and $\mathbb{B} \rightarrow 0$, and the Weierstrass factorization [Eq. (5)], leads to a simple pole-zero decomposition for $R(\omega) = |r(\omega)|^2$:

$$R(\omega) = \prod_m \left| \frac{\omega - \omega_m^z}{\omega - \omega_m^p} \right|^2. \quad (6)$$

Second, the optimization of absorption imposes that the poles and zeros are regularly distributed over the spectral range of interest and have the same real part, so that they can be expressed in the form of Eqs. (3) and (4).

This expansion is consistent with the previous description of isolated resonances [Eq. (2)] and can be described phenomenologically as a series of independent resonances m : the total probability of reflection is then expressed as the product of the independent probabilities of reflection of each mode, $R(\omega) = \prod_m R_m(\omega)$.

The requirement of evenly distributed resonances in the frequency domain is key to deriving simple analytical expressions of the average absorption. We construct an infinite set of resonance modes around a frequency ω_0 : $\omega_m = \omega_0 + m\delta\omega$, where m is an integer and $\delta\omega = 1/\rho$, with ρ being the spectral density of modes in the system [see Fig. 1(b)]. This results in a periodical function for

$R(\omega)$, with period $\delta\omega$:

$$R(\omega) = \prod_{m=-\infty}^{+\infty} \frac{1 + \left(\frac{(\gamma_r - \gamma_{nr})/\delta\omega}{\omega/\delta\omega - m} \right)^2}{1 + \left(\frac{(\gamma_r + \gamma_{nr})/\delta\omega}{\omega/\delta\omega - m} \right)^2}, \quad (7)$$

which can be simplified as

$$R(\omega) = \frac{\sin^2\left(\frac{\pi\omega}{\delta\omega}\right) + \sinh^2\left(\frac{\pi(\gamma_r - \gamma_{nr})}{\delta\omega}\right)}{\sin^2\left(\frac{\pi\omega}{\delta\omega}\right) + \sinh^2\left(\frac{\pi(\gamma_r + \gamma_{nr})}{\delta\omega}\right)}. \quad (8)$$

Next, we define the *radiative* and *nonradiative overlap factors*, respectively:

$$\eta_r = 2\gamma_r/\delta\omega, \quad (9)$$

$$\eta_{nr} = 2\gamma_{nr}/\delta\omega. \quad (10)$$

The total overlap factor $\eta = \eta_r + \eta_{nr} = \Delta\omega/\delta\omega$ quantifies the degree of overlap between neighboring resonances. By integration of Eq. (8) over a period, we obtain a simple expression for the average absorption:

$$\langle A \rangle = \frac{2 \sinh(\pi \eta_r) \sinh(\pi \eta_{nr})}{\sinh(\pi (\eta_r + \eta_{nr}))}. \quad (11)$$

Equation (11) is a key result of this work. The average absorption of a general multiresonant system is a simple function of the dimensionless radiative and nonradiative overlap factors. While η_{nr} depends on the material properties of the absorber, η_r can be engineered by tailoring the coupling between the modes within the system and free space. Hence, η_r can be considered as an adjustable parameter to investigate reference models and upper bounds for absorption.

D. Multiresonant critical coupling

We propose the *multiresonant critical-coupling* (CC) regime as a first reference model. We consider that the CC condition $\eta_r = \eta_{nr}$ is fulfilled for every resonance, as illustrated in Fig. 1(b), leading to the average absorption

$$\langle A \rangle_{cc} = \tanh(\pi \eta_{nr}). \quad (12)$$

E. Absorption upper bound

Although the CC regime results in perfect absorption at each resonance frequency, it does not provide the highest average absorption. Indeed, for a given η_{nr} , $\langle A \rangle$ is an increasing function of η_r [Eq. (11)]. The maximal multiresonant absorption is reached as $\eta_r \rightarrow \infty$ (the overcoupling

regime):

$$\langle A \rangle_{ub} = 1 - e^{-2\pi \eta_{nr}}. \quad (13)$$

Equation (13) gives the general *upper bound on broadband absorption* in a dissipative system coupled to free space through a single radiative channel.

Equations (12) and (13) are simple yet effective formulas. Multiresonant absorption limits are expressed as a function of a single parameter, the nonradiative overlap factor $\eta_{nr} = 2\gamma_{nr}\rho$, which accounts for the intrinsic properties of the system. The CC model and the upper bound are compared in Fig. 1(c), where the absorption is plotted as a function of the nonradiative overlap factor η_{nr} . The average absorption approaches unity for both models when η_{nr} is close to 1 or above. The difference becomes significant as η_{nr} decreases, and the ratio between the upper bound and the CC absorption tends to a factor of 2 in the weak-absorption regime ($\eta_{nr} \rightarrow 0$).

F. Path enhancement factor

Assuming that the resonant system is a slab of linear, homogeneous, and isotropic material with thickness d , the attenuation (or absorption) coefficient α of a propagating wave can be linked to its energy decay time $2\gamma_{nr}$ and group velocity v_g through $2\gamma_{nr} = \alpha v_g$, and the nonradiative overlap factor is expressed as $\eta_{nr} = \alpha v_g \rho$.

The absorption upper bound can be written as a function of an effective path length Fd , where F is defined as the wave-path enhancement factor:

$$\langle A \rangle_{ub} = 1 - e^{-F\alpha d}, \quad (14)$$

$$F = \frac{2\pi \eta_{nr}}{\alpha d} = 2\pi v_g \rho / d. \quad (15)$$

This expression allows for a direct comparison with the usual Beer-Lambert law obtained for single-pass absorption, $F = 1$. It is worth noting that F is a useful figure of merit only for spectrally averaged absorption enhancement, and holds no physical meaning at a given frequency (e.g., $F \rightarrow +\infty$ at the resonance frequency in the case of critical coupling).

G. Isotropic absorption induced by scattering

The above derivation assumes a single radiative channel, which is appropriate to describe a planar slab textured with a subwavelength periodical pattern, i.e., with no diffraction in free space. We are now interested in describing isotropic absorption induced by nonperiodic patterning, i.e., for a system with internal modes isotropically coupled to all directions in free space.

We start by assuming a pattern with a large period inducing N diffracted waves. The total reflection $R(\omega)$ of an

incident plane wave is the sum of reflections toward the N different diffracted orders:

$$R(\omega) = \sum_{q=1}^N |r_q(\omega)|^2. \quad (16)$$

Isotropic absorption means that the reflectivity $R(\omega)$ is independent of the incidence angle. As a consequence, the reflection coefficients $r_q(\omega)$ should be identical and have the same pole-zero decomposition, and the theoretical framework developed for a single radiative channel ($N = 1$) can be extended to the case of N identical radiative channels (see Sec. S.4 of the Supplemental Material Sec. [21]). The previous derivation of the absorption upper bound holds, and Eqs. (14) and (15) take the same form by replacing η_{nr} with η_{nr}/N .

Nonperiodic structures that induce omnidirectional scattering in free space can be modeled by periodical patterns with infinitely large periods ($N \rightarrow +\infty$) and the requirement of isotropic absorption translates into uniform diffraction efficiencies. This allows us to use the previous model for absorption upper bounds.

In other words, the upper bound on broadband absorption given by Eqs. (14) and (15) applies to isotropic absorption.

It is important to emphasize that, *a priori*, there is no unique solution to achieve isotropic absorption. Various nonperiodic patternings can lead to various angular distributions of wave scattering in the absorber that all comply with the constraint of isotropic absorption. In solar cells, in the weak-absorption regime, the Lambertian light-trapping model, which involves scattering at the slab interfaces, results in isotropic absorption of light. However, as will be discussed later, Lambertian scattering is not the optimal scheme when absorption becomes significant.

H. Main assumptions

The assumptions used in the above derivation are detailed in Sec. S.3 of the Supplemental Material [21]. The main requirement is a slow variation of the overlap factors over the spectral ranges relative to the resonance spacing $\delta\omega$ and the resonance width $\Delta\omega$. The average absorption is not significantly affected by other assumptions. For instance, replacing the infinite distribution of evenly spaced modes by a finite number of resonances or by a nonuniform distribution has little impact on $\langle A \rangle$.

There is an apparent contradiction between the overcoupling regime of the upper bound and the limited losses required to apply the Lorentzian approximation. In practice, we show that radiative overlap factors as low as 0.5 are sufficient to reach 95% of the upper-bound absorption. We also note that the closed-form expressions of Eqs. (11), (12), and (13) are intrinsically bounded between 0 and 1,

ensuring that absorption tends to 1 for large internal losses, even beyond the validity of the Lorentzian approximation.

IV. UPPER BOUNDS ON SUNLIGHT ABSORPTION

A. Multiresonant absorption

In the following, we apply this multiresonant theoretical framework to optics, and we address the important question of the upper bounds on sunlight absorption in a solar cell modeled by a slab of semiconductor (see details in Sec. S.5 of the Supplemental Material [21]). A perfect back-reflector is used to avoid transmission through the back side of the absorber. Multiple well-defined resonances can be induced by a two-dimensional (2D) regular pattern at the front or back side, leading to diffraction effects and guided-modes resonances in the absorber [39–41] [see Fig. 2(a)]. The spectral density of modes depends on the period p . Their coupling to free space, γ_r , can be controlled by tuning the shape of the subwavelength patterning, and by using an antireflection coating. Since most optical modes are well confined in the absorber, the group velocity can be expressed as $v_g = c/n$, and $F = 2\pi c\rho/nd$.

Sunlight incident on the absorber can be decomposed as an ensemble of independent plane waves linearly polarized along two orthogonal directions [42]. In the most general case, polarization rotation can be induced by the resonant system, such that two radiative channels should be considered. However, if the 2D pattern has a plane of symmetry perpendicular to the slab, then the two polarization states parallel and orthogonal to the mirror plane do not couple for normal incidence [43]. The total density of modes 2ρ is then divided in two halves, each of which is coupled to a single radiative channel. For each of the two polarization states, the previous derivations [Eqs. (12), (13), (14), and (15)] apply.

Optimizing sunlight absorption requires maximizing the density of modes ρ in the system while avoiding diffraction in free space. Considering the reciprocal space of the diffraction pattern, the problem is equivalent to the optimization of circle packing in the plane. The optimal lattice is hexagonal and the optical-path enhancement factor is

$$F = 2\sqrt{3}\pi n^2 \left(\frac{p}{\lambda}\right)^2, \quad (17)$$

where p is the period of the lattice. The upper bound is achieved for $p \rightarrow 2\lambda/\sqrt{3}$:

$$F_{ub} = \frac{8}{\sqrt{3}}\pi n^2. \quad (18)$$

The plane of symmetry simplifies the design and requires less computationally intensive simulations. It also enables the use of the multiresonant critical-coupling model valid only with a single radiative channel. This symmetry can be broken by design or with oblique incidence.

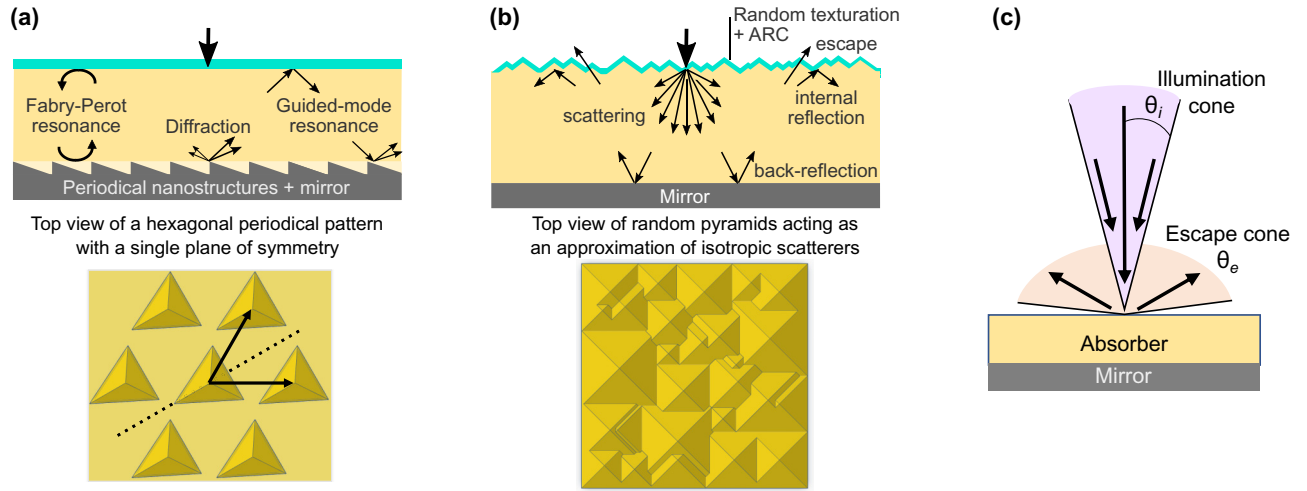


FIG. 2. Light-trapping structures based on periodic and random patterning, and free-space coupling schemes. (a),(b) Sketches of the two reference light-trapping strategies: (a) Multiresonant absorption induced by a regular pattern, here an hexagonal lattice of tetrahedra on the back side; (b) isotropic scattering induced by random texturation, here pyramids of random size and position on the front side, as obtained by chemical etching of silicon wafers and typically used in solar cells. (c) Illumination and escape cones constrain the free-space coupling.

In this case, every mode (density 2ρ) is coupled to both radiative channels and we find the same path enhancement factor and upper bound. On the contrary, higher orders of symmetry would result in mode degeneracies and lower average absorption. As a result, the optimal pattern has no more than one plane of symmetry [43].

The multiresonant upper bound given by Eq. (18) is derived for normal-incidence light ($\theta_i = 0$) and no angular restriction for photon escape (no diffraction allowed in the whole hemisphere above the absorber, $\theta_e = \pi/2$). It can be generalized by imposing the single-channel condition to an illumination cone with half-angle θ_i and an escape cone restricted to a half-angle $\theta_e \geq \theta_i$ [Fig. 2(c)]:

$$F_{ub}(\theta_i, \theta_e) = \frac{F_{ub}}{[\sin \theta_i + \sin \theta_e]^2}. \quad (19)$$

The path enhancement factors apply to both the critical-coupling reference model and the upper bound. The formulas are summarized in Table I.

B. Isotropic absorption

As an alternative to multiresonant absorption, nonperiodic structures can be used to induce scattering effects and trap sunlight in the absorber [Fig. 2(b)]. Oblique propagation and total internal reflection lead to light-path enhancement that has been widely studied in the ray-optics regime. Our multiresonant theoretical framework provides a way to generalize previous results: scatterers are modeled as large-period structures that allow free-space coupling to a large number N of identical channels with the same coupling rate γ_r [36]. The light-path enhancement is then (see

Sec. S.6 of the Supplemental Material [21])

$$F = \frac{2\pi c \rho}{nd N}, \quad (20)$$

and in the limit of large periods ($p \gg \lambda$), both ρ and N scale as p^2 , leading to

$$F_s = 4n^2. \quad (21)$$

Assuming restricted illumination and escape cones with the same half-angle $\theta_e = \theta_i$, the maximum path enhancement factor for isotropic absorption becomes

$$F_s(\theta_i) = \frac{F_s}{\sin^2 \theta_i}. \quad (22)$$

Equations (14), (21), and (22) (see also Table I) recover the well-known $4n^2$ factor and the upper bound previously derived for isotropic absorption in the ray-optics regime [13,15,22]. We show that they are also consistent with previous results obtained for wave optics in the weak-absorption regime [36,40] (see Sec. S.7 of the Supplemental Material [21]) and that the “thermodynamic upper bound” derived in Ref. [44] does not hold (see Sec. S.8 of the Supplemental Material [21]). Here, these seminal results are generalized to a wide domain of validity that includes both thick and ultrathin solar cells in the weak- and strong-absorption regimes.

C. Optimal light-trapping strategy

In Fig. 3, we compare the absorption enhancement achieved by multiresonant absorption and isotropic

TABLE I. A summary of the reference absorption models and upper bounds for light trapping in solar cells using multiresonant absorption and isotropic absorption induced by scattering.

	Multiresonant absorption	Isotropic absorption
Reference models	Critical coupling: $\langle A \rangle_{cc} = \tanh\left(\frac{F\alpha d}{2}\right)$	Lambertian scattering: $A_l = \frac{\alpha d}{\alpha d + 1/F}$
Upper bound	$\langle A \rangle_{ub} = 1 - e^{-F\alpha d}$	
Maximum path enhancement factor F	$F = \frac{8}{\sqrt{3}}\pi n^2$	$F = 4n^2$
Restricted cones for illumination (θ_i) and escape (θ_e)	$F(\theta_i, \theta_e) = \frac{8}{\sqrt{3}} \frac{\pi n^2}{[\sin \theta_i + \sin \theta_e]^2}$	$F(\theta_i = \theta_e) = \frac{4n^2}{\sin^2 \theta_e}$

absorption as a function of the half-angle of the illumination cone θ_i , assuming no restriction for photon escape ($\theta_e = \pi/2$). At normal incidence, multiresonant absorption is the most efficient strategy by far, and its enhancement factor F stays beyond the $4n^2$ limit of isotropic absorption for illumination half-angles up to 64.8° . For wider angles and in the case of angular restriction on photon escape with $\theta_e = \theta_i$, isotropic absorption leads to higher F factors, with a gain over multiresonant absorption limited to a maximum of $2\sqrt{3}/\pi \approx 1.10$.

As an illustration, concentration photovoltaics is currently limited to factors up to about 1000 suns,

corresponding to illumination cones close to normal incidence with an half-angle of 8.5° , well below the value of 64.8° .

V. APPLICATION TO SILICON PHOTOVOLTAICS

In this section, we apply the absorption models to silicon solar cells and we compare the different light-trapping strategies.

Reducing the thickness of Si wafers with respect to the current industry standards (about $150 \mu\text{m}$), with no performance degradation, is an important pathway for material, energy, and cost savings [45,46]. In the following, we assume normal incidence and no restriction on the escape cone. The refractive index of Si is taken from Ref. [47]. The model assumptions are satisfied over the whole range of thicknesses and wavelengths investigated (see Sec. S.9 of the Supplemental Material [21]).

As an illustration, we first consider a $1\text{-}\mu\text{m}$ -thick silicon layer, about 2 orders of magnitude thinner than commercial solar cells. The absorption spectra plotted in Fig. 4(a) show the large gap between the two main light-trapping strategies: light scattering and multiresonant absorption. Lambertian scattering, the reference light-trapping scheme for state-of-the-art solar cells, provides a huge improvement over single-pass absorption. However, it remains far from the isotropic upper bound. Critical-coupling absorption is close to the multiresonant upper bound and exhibits a significant gain over light-scattering strategies.

The short-circuit current J_{sc} represents the figure of merit for the overall absorption in a solar cell under one-sun illumination. Assuming that each photon absorbed in the semiconductor contributes to one electron-hole pair, J_{sc} is calculated by integrating the absorption spectrum weighted by the spectral density of photon flux (AM1.5G spectral irradiance) between 300 nm and 1200 nm. The results are plotted as a function of the slab thickness in Fig. 4(b). They confirm that there is plenty of room for improvement beyond the Lambertian light-trapping strategy, with a tenfold reduction in thickness enabled by approaching to the multiresonant upper bound.

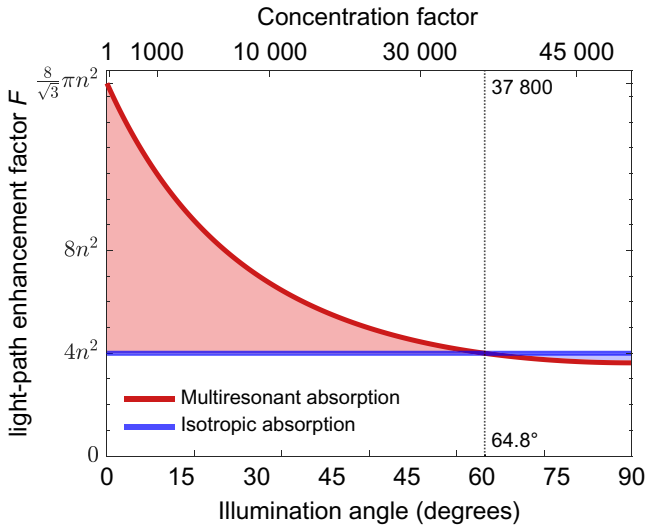


FIG. 3. Upper bounds on light absorption with multiple resonances and isotropic absorption. The maximum light-path enhancement factor F as a function of the half-angle of the illumination cone for multiresonant absorption with no diffraction in free space and for isotropic absorption. Photon escape is considered here without restriction (full hemisphere, $\theta_e = \pi/2$). The half-angle θ_i can be translated into concentration factors $C = \sin^2(\theta_i)/\sin^2(\theta_{1\text{sun}})$, where $\theta_{1\text{sun}} = 0.27^\circ$ is the half-angle of the sun as seen from the earth.

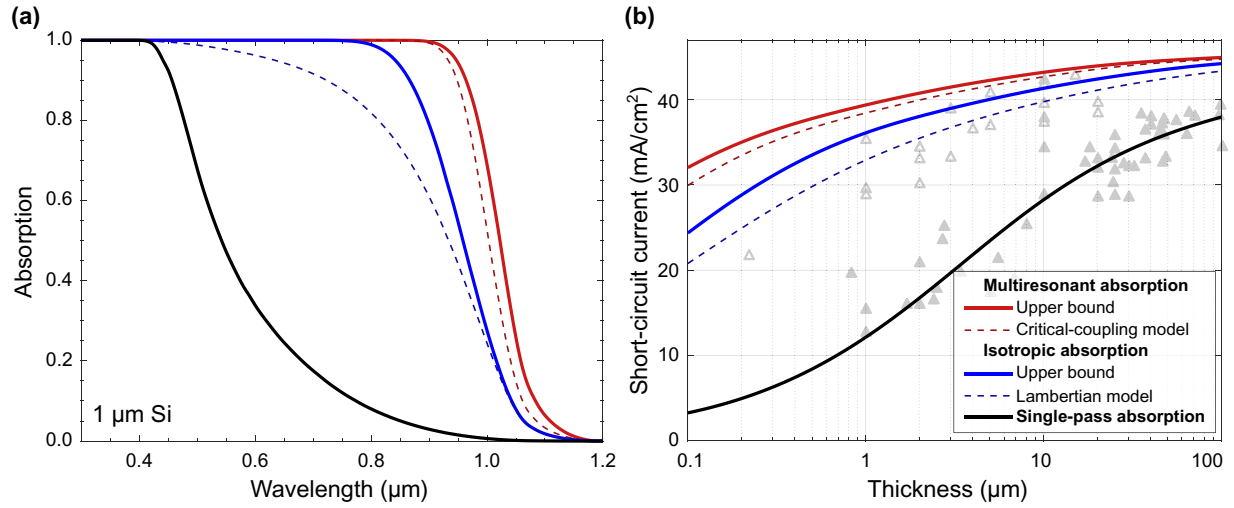


FIG. 4. Upper bounds on light-trapping in silicon solar cells. A comparison of the different light-trapping models for silicon solar cells modeled as a simple textured slab with a back mirror. (a) Absorption spectra for a 1-μm-thick silicon slab. (b) The short-circuit current density J_{sc} calculated for the reference models and upper bounds for one-sun illumination. The gray symbols show results of short-circuit current measurements (filled triangles) and simulation (open triangles) taken from the literature (see Ref. [17]).

The derivation of reference models and upper bounds provides guidelines for the design of ultrathin solar cells [17]. In practice, the spectral position and free-space coupling of resonant modes are determined by the shape, size, and arrangement of nanostructures. We have shown that the best multiresonant absorption scheme requires a pattern with a hexagonal arrangement, no more than one plane of symmetry, and a maximum interspacing of $2\lambda_0/\sqrt{3}$. If the pattern is placed at the back side, diffraction in free space may only occur after a double pass in the absorber and for $\lambda < \lambda_0$. Hence, λ_0 defines the long-wavelength region in which light trapping is needed.

Experimental and numerical results taken from the literature are also shown in Fig. 4(b) [17]. Importantly, several data points lie above the Lambertian model and the isotropic upper bound. It is interesting to point out that these results have been obtained with a periodical texture that complies with most of the criteria required for optimal multiresonant absorption: the pattern is made of slanted cones with a single plane of symmetry, and a period of $0.85 \mu\text{m}$ [35]. Therefore, our theoretical framework reconciles the results found in the literature with reference models and upper bounds, and sets the upper limits for each light-trapping scheme. Still, intensive numerical work and efficient optimization strategies are needed to account for the vast geometrical parameter space and reach absorption close to the upper bound with realistic designs for ultrathin solar cells [17].

The fundamental efficiency limit of a single-junction silicon solar cell is the result of a trade-off between photon absorption and intrinsic charge-carrier recombination. Assuming Lambertian scattering, an optimal thickness of $100 \mu\text{m}$ leads to a theoretical efficiency of 29.4%, with

$J_{sc} = 43.4 \text{ mA/cm}^2$ [48]. Actually, the same short-circuit current is obtained with thicknesses of only $12 \mu\text{m}$ for the multiresonant upper bound, and $42 \mu\text{m}$ with isotropic absorption induced by scattering. Such thickness reduction results in decreased Auger recombination and a predicted voltage gain of more than 20 mV [48]. Hence, the maximum theoretical efficiency of silicon solar cells should be reassessed and is expected to be above 30% for thicknesses of a few tens of μm .

In this section, our theoretical models have been compared to the experimental and numerical results published in the literature for thin and ultrathin silicon solar cells. The upper bounds on absorption have a wide domain of validity and can be applied to other materials, including direct-band-gap semiconductors. The example of GaAs is studied in Sec. S.10 of the Supplemental Material [21].

VI. DISCUSSION

Reference absorption models and the upper bounds derived in this paper are summarized in Table I. Overall, they provide an answer to the long-debated question of the best light-trapping strategy for solar cells [34,49,50].

A. Approaching the absorption upper bound

While the theoretical upper bound is derived under the idealized condition $\eta_r \rightarrow \infty$, our analysis shows that 99% of the absorption upper bound can be achieved with η_r values below unity (see Sec. S.3.4 of the Supplemental Material [21]). This confirms that the $\eta_r \rightarrow \infty$ requirement is a mathematical idealization, not a practical constraint. Realistic systems can approach the upper bound by, e.g., using the multiresonant critical-coupling regime $\eta_r = \eta_{mr}$.

B. Beyond the absorption upper bound

It is interesting to note that optical absorption beyond the multiresonant upper bound can be achieved in specific cases. First, the density of modes used in the above derivation assumes bulk materials. Absorption can be further enhanced by increasing the local density of states close to subwavelength structures such as plasmonic particles [51–54]. Such effects are limited to very thin absorbers or inhomogeneous nanostructured media. Second, we can break the requirement of a single radiative channel used to derive the multiresonant upper bound. Larger periods lead to an increase of the density of modes, but also of free-space losses due to diffraction. Sweet spots can be found for normal incidence illumination by canceling the coupling to diffracted waves in free-space, although this will occur at the expense of the acceptance angle [55].

C. On the role of disorder

The absorption models provide guidelines to design optimal light-trapping structures. Our theoretical framework has been applied to perfectly periodical patterning for multiresonant absorption and to random nonperiodic scattering structures for isotropic absorption. However, significant efforts still need to be made to optimize numerically the shape and size of patterning to get close to the upper bounds. For instance, this raises the challenge of designing texturing that meets the requirement of large γ_r over a broad spectral range.

For nonperiodic patterning, the angular distribution of light scattering in the absorber depends on both the degree of disorder and the shape of structures. The optimal scattering distribution to approach the isotropic upper bound remains an open question. While Lambertian scattering is the emblematic example of isotropic distribution of scattering within the absorber, it yields optical absorption significantly below the isotropic upper bound.

The physics of correlated disordered media have emerged recently and open new routes to control finely both free-space coupling and the direction of light propagation in absorbers [56]. In particular, hyperuniform structures offer a promising route to favor diffraction at high angles in the absorber [57–60]. The extension of our theoretical framework to the different classes of correlated disordered patterning is an intriguing research direction, and we can speculate the existence of scattering structures with a controlled disorder that would result in isotropic absorption and help to approach the isotropic upper bounds.

D. CONCLUSION

In conclusion, we have proposed a general theoretical framework for multiresonant absorption in an open dissipative system. Analytical expressions for the absorption upper bounds have been derived for a slab with either

random or periodical patterning and applied to light trapping in solar cells. Our results show the route toward high-efficiency ultrathin solar cells, enabling manufacturing with lower material usage and carbon footprint, and the design of flexible devices [17]. Beyond the realm of optics and photovoltaics, our general formalism can be applied to any kind of electromagnetic and mechanical wave, and should guide the development of ultrathin broadband detectors and absorbers for radar waves [61], compact acoustic sinks [62,63], the attenuation of seismic waves [64,65], or energy harvesting of ocean water waves [5].

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DATA AVAILABILITY

The data that support the findings of this paper are not publicly available. The data are available from the authors upon reasonable request.

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