

Predicting Liquid Properties and Behavior via Droplet Pinch-off and Machine Learning

Jingtao Wang^{1,*}, Qiwei Chen^{2,*}, C. Ricardo Constante-Amores², Denise Gorse³,
Alfonso Arturo Castrejón-Pita^{4,†} and José Rafael Castrejón-Pita^{1,‡}

¹*Department of Mechanical Engineering, University College London,
Torrington Place, London, WC1E 7JE, United Kingdom*

²*Mechanical Science and Engineering, University of Illinois Urbana-Champaign,
Urbana, Illinois 61801, USA*

³*Department of Computer Science, University College London, London, WC1E 6EA, United Kingdom*

⁴*Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, United Kingdom*



(Received 15 December 2025; accepted 8 June 2026; published 29 July 2026)

Here, we demonstrate that the time-evolving interface during droplet formation—and, in particular, the morphology near pinch-off—encodes sufficient physical information for machine-learning (ML) models to accurately infer key fluid properties, including viscosity and surface tension. A dataset was constructed from high-speed imaging of droplet formation in both dripping and jetting regimes. The frame closest to breakup was selected and the contour of the droplet extracted and recorded, along with the corresponding flow conditions and fluid properties. Experiments were conducted using Newtonian fluids, such as silicon oils, aqueous solutions of ethanol and glycerin, and methanol, under controlled conditions, spanning $0.001 < \text{Re} < 200$ and $0.01 < \text{Oh} < 20$, providing 840 samples. Supervised regression models were trained to predict fluid properties from these inputs. The models accurately infer surface tension and viscosity across a wide range of fluids using the single high-speed snapshot. Moreover, apart from predicting fluid properties from a snapshot, the proposed framework can carry out the inverse problem and predict droplet shape at breakup based on fluid properties and flow conditions. Unsupervised clustering of the learned representations reveals distinct regions in the Re – Oh and Bo – Oh parameter spaces, indicating that the latent space captures meaningful physical structure and provides insight into droplet dynamics. These results establish a data-driven alternative to conventional measurement techniques, reducing experimental complexity and evaluation time while enabling integration into automated systems.

DOI: [10.1103/v3p1-lmzl](https://doi.org/10.1103/v3p1-lmzl)

Subject Areas: Fluid Dynamics

I. INTRODUCTION

The quantification of liquid properties—such as surface tension, viscosity, and density—is an essential quality-control measure in fluid dispensing processes such as inkjet printing [1,2] and spray coating [3,4]. However, some conventional methods for determining fluid properties rely on specialized and complex instrumentation, which often limits their use within small enterprises and complicates automation. Among various existing techniques, the pendant drop method has become a widely adopted approach to determine the surface tension of a liquid [5]. In this method, a droplet is suspended from the tip of a capillary, or needle,

with the droplet contour recorded by optical imaging. The shape of the pendant droplet, and thus its contour, is controlled by a balance between gravity and interfacial forces [6]. Once the droplet boundary is recorded, numerical solutions of the Young-Laplace equation, which accounts for the pressure difference across the liquid interface, the surface curvature, and surface tension [7], are fitted to determine the surface tension σ [8]. This method is valued for its simplicity, small liquid volume requirements, and compatibility with conventional optical imaging systems, making it suitable for both research and industrial environments [9]. In practice, millimeter-scale droplets are commonly employed in this method [10–12]. Other experimental techniques used to characterize surface tension include the Wilhelmy plate, the Du Noüy ring, and the widely used bubble pressure tensiometry. In the Wilhelmy plate method, the surface tension of a liquid is determined by measuring the force exerted by the liquid on a thin fully wetted plate in contact with its surface [13]. Alternatively, and in a similar fashion, the Du Noüy ring method works by lifting a thin metallic ring from a liquid surface while recording the force required to detach it [13].

*These authors contributed equally to this work.

†Contact author: alfonso.castrejon-pita@wadham.ox.ac.uk

‡Contact author: r.pita@ucl.ac.uk

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

These two methods are simple, but their accuracy depends on the skills of the operator and on the use of materials that ensure the complete wetting of the plate or ring. On the other hand, bubble pressure tensiometers are reliable systems that obtain surface tension by measuring the pressure required to produce a bubble at the tip of a capillary submerged in the tested liquid [14]. In this method, the bubble production rate is adjustable, making the technique particularly effective for studying time-dependent interfacial phenomena, such as surfactant diffusion and adsorption [15]. However, bubble pressure tensiometers are usually expensive and less suitable for highly viscous fluids [16].

Viscosity is another key property that controls various aspects of fluid mechanics on the capillary scale, including jetting [17], spray dynamics [18], and liquid breakup [19]. In industrial environments, viscosity is often measured by vibrational viscometers and rheometers. Vibrational methods work by quantifying the viscous damping of shear acoustic waves through the liquid. These viscometers are fast and easy to operate, and they provide reliable results for liquids in which viscosity remains constant regardless of shear; these liquids are commonly referred to as “Newtonian fluids.” A disadvantage of vibrometers is that they require large volumes of liquid, often in the 200 ml range. Liquids with shear-dependent viscosity, such as shear thinning or thickening, and viscoelastics require capillary and/or rotational rheometers [20]. These rheometers determine viscosity by applying a controlled shear stress, measuring the resulting flow response of the fluid. Unfortunately, rheometers are often both expensive and unsuitable for the fast-shear rates found in sprays and inkjet printing [21,22].

While effective, the traditional techniques described above form a segmented toolkit that lacks integration and is often incompatible with automation. In this work, we present an integrated, automation-friendly methodology, utilizing both supervised and unsupervised machine-learning techniques that can determine the surface tension or the viscosity, or both, of Newtonian fluids from a single snapshot of a dripping drop. In our methods, the flow rate, the nozzle size, the density, and, if available, the viscosity or the surface tension are used as additional input.

In more detail, high-resolution and high-speed imaging experiments were conducted in this work to capture the last moments before the breakup of dripping droplets. From these images, droplet contours were extracted and used as key input features to supervised-learning models. Various models are then trained to predict physical parameters, such as viscosity and surface tension, directly from the image data. In addition, models are trained to predict the breakup shape of liquids given their material properties and flow rate. Unsupervised clustering algorithms are additionally employed to discover latent structures within the dataset, enabling the identification of distinct droplet behaviors, or

quality modes, without requiring predefined labels. ML methods require sufficient and representative datasets, along with well-defined input features and target outputs; here, the training is carried out directly from experimental data, taken under a large variety of conditions, ensuring that the learned models capture the intrinsic complexity of wide laboratory conditions. This dual (supervised plus unsupervised) approach not only offers a fast and automated alternative to conventional rheometry and tensiometry but also reveals interpretable morphological patterns linked to fluid properties and experimental conditions. Ultimately, our integration of physics-informed imaging (based on our hypothesis) and machine learning offers a scalable and generalizable pathway for data-driven fluid diagnostics. Importantly, as it will be discussed later, our approach relies on the finite-time morphology observed during experimentally accessible pinch-off dynamics, rather than on the ideal asymptotic similarity solutions that arise close to the singularity.

II. BACKGROUND

A. Liquid breakup

For over a century, the breakup of liquid jets and filaments has been at the forefront of fluid mechanics, given its relevance to a myriad of processes ranging from lava ejection and fragmentation to inkjet printing [23,24]. This phenomenon, while seemingly simple, involves complex dynamics where a liquid thins, or necks, rapidly until pinch-off, evolving through dramatic interfacial changes, all within a few microseconds [25]. Indeed, the point of breakup is a finite-time singularity, and its description requires both scaling arguments and high-resolution experiments and modeling to resolve features that arise in its vicinity. A comprehensive theoretical framework has classified the routes to breakup into various canonical regimes depending on the relative importance of inertia, capillarity, and viscosity [25]. In the inviscid regime, where inertia balances surface tension, the thinning of a liquid neck follows the power law $h_{\min} \sim (t_b - t)^{2/3}$, where $h_{\min}$ is the minimum neck radius, t_b is the breakup time, and t is time. Interestingly, liquid breakup under this regime shows self-similarity where, regardless of the flow conditions, the liquid interface near pinch-off converges toward a unique shape, offering a powerful criterion for identifying inviscid liquids [26,27]. This regime is often observed in low-viscosity fluids such as water or ethanol, when surrounded by a negligible ambient environment [28,29]. Previous studies have attempted to infer material properties from pinch-off dynamics by fitting the similarity law $R_{\min} \sim A(t_b - t)^{2/3}$, using a theoretical prefactor $A \approx 0.717$. However, further experiments have demonstrated that this approach can lead to significant errors in the inferred surface tension as experimental observations often occur outside the self-similarity universal regime [30]. Subsequent work showed that the prefactor exhibits a slow

temporal drift and depends on system parameters in this preasymptotic regime [31]. We also note that convergence to the self-similar regime near the capillary singularity is slow and involves multiple transient regimes [31,32], and it is rarely achieved in experiments. As noted by Ref. [33], different regions may exhibit distinct force balances; thus, the route to breakup is parameter dependent, and the morphology reflects this history. Our approach takes advantage of the time history of the dynamics, which remains imprinted in the interface topology up to pinch-off. In summary, asymptotic similarity eliminates independent parameter information in the strict infinite-time limit and only around the pinch-off region. Real finite-time experiments retain sufficient discriminatory structure to allow the separate inference of σ and μ . Our approach makes use of the interface morphology rather than predicted prefactors.

In most other liquids, both viscosity and inertia play a dominant role. In fact, when viscous forces dominate, the thinning is characterized by a linear scaling, i.e., $h_{\min} = 0.0709(\sigma/\mu)(t_b - t)$, where σ is surface tension and μ is the viscosity [34]. In contrast, in cases where all three forces—i.e., capillary, viscous, and inertial—play a role in the dynamics, the viscous-inertial regime leads to the universal scaling $h_{\min} = 0.0304(\sigma/\mu)(t_b - t)$ [35]. A further fourth regime exists where the surrounding media viscosity, often another liquid, dictates the thinning and pinch-off dynamics [36]. It is worth noting that, on the route to breakup, the neck dynamics transition between these regimes based on the local importance of inertia, viscosity, and surface tension effects [19]. The thinning regimes, and their transitions, are parametrized by two dimensionless parameters, the Reynolds ($\text{Re} = [(4\rho Q)/(\pi\mu D)]$) and the Ohnesorge ($\text{Oh} = (\mu/\sqrt{\rho\sigma D})$) numbers, where Q is the volumetric flow rate, ρ is the liquid density, and D is a characteristic length (often the nozzle size or the neck diameter). In fact, the transitional behavior has been found to consist of multiple intermediate transient regimes, which are highly sensitive to the liquid and flow properties. In other words, in their route to breakup, two liquids of similar characteristics can have a very different thinning transitional behavior [19]. Thus, the profile of a liquid neck near, or at, breakup is a repository of fluid dynamics information, reflecting the transitional competition of fluid properties and the flow at the neck. Complementary works have demonstrated that the breakup of liquid filaments and the transition from dripping to jetting are also ruled by the Reynolds and Ohnesorge numbers and by the filament length-to-diameter aspect ratio [37,38]. These findings suggest that the route to breakup is unique for each combination of flow characteristics and liquid properties and thus governed by their complex interplay. Motivated by this idea, we have assembled a large database of snapshots of droplets undergoing breakup (through dripping) to train ML models for liquid-property (and interface) inference.

B. Machine learning

Machine learning is a branch of computer modeling that learns patterns from large amounts of data and makes predictions, or decisions, without prior knowledge beyond what it has learned from these data. In recent years, ML has been applied across various physics topics, including fluid dynamics [39–41], droplet physics [42–46], soft matter [47,48], and materials science [49]. These studies highlight the potential of ML as a tool to complement traditional experimental and theoretical approaches in science. Specifically, supervised learning trains models on labeled datasets (data that have been manually classified or characterized), mapping inputs to known outputs, which is particularly effective for prediction tasks. In contrast, unsupervised learning operates on unlabeled data to identify hidden patterns or underlying structures, i.e., clusters. Both strategies offer valuable means of extracting knowledge from complex experimental observations. In addition, recent advances in computer vision [50] provide powerful tools for analyzing image-based experimental data [51], which are highly relevant for the study of droplet dynamics and pinch-off processes where high-speed imaging is often used in conjunction with image analysis to extract information from images [19].

C. ML applications to droplet dynamics

ML methods have been applied to various systems involving droplet formation; examples of these include direct inkjet writing [52], droplet impact, spreading and bouncing on nanostructure substrates [53], spreading on various materials [54], and droplet splashing [55]. Considering first the forward-prediction paradigm, in which ML models predict droplet behavior from known material and operating conditions, Kim *et al.* [56] trained supervised regressors to predict the jetting behavior of viscoelastic inks from rheological and drive parameters, with later work [57] using reinforcement learning to optimize actuation waveforms, dynamically improving jet stability without an explicit physical model. Additionally, Chen *et al.* [43] employed supervised learning to predict the optimum printable biomaterial formulations for direct ink writing, and Li *et al.* [45] developed a multiparameter droplet optimization model including flow and material parameters. Turning to image-based studies, Kratz and Kierfeld [44] introduced an image-based ML approach for pendant-drop tensiometry, demonstrating that the contour of a static droplet can be used to infer surface tension with the method restricted to steady pendant configurations. Pierzyna *et al.* [55] developed a data-driven model for the splashing thresholds, and Au-Yeung *et al.* [53] developed a method to predict the maximum spreading and impact morphology of droplets on nanostructured surfaces.

In summary, previous works have predominantly used ML to control, or classify, droplet behavior given known physical inputs. In contrast, the present work uses

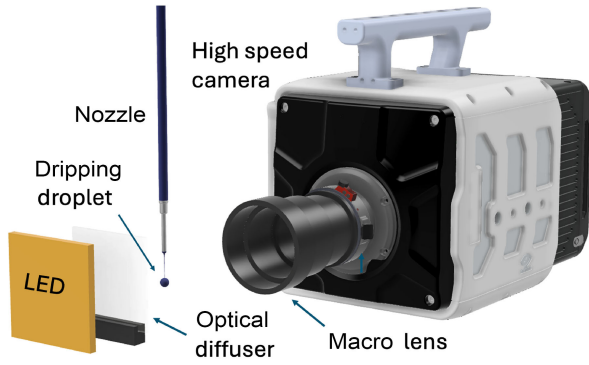


FIG. 1. Experimental setup used to capture the necking and breakup dynamics during droplet dripping. The system is a simple shadowgraph, where visualization is carried out through a macro lens and a diffused light source.

supervised learning to tackle the inverse problem of inferring fluid properties directly from droplet morphology. In addition, we use unsupervised learning to gain insight into the underlying physics of this process.

III. EXPERIMENTAL METHODOLOGY

In our experiments, dripping droplets were produced by pushing liquid, at a controlled flow rate, through long cylindrical nozzles. The flow rate was controlled by a KDS Gemini 88 Plus Dual Rate syringe pump, through Hamilton glass syringes (models 1750, 1001, and 1005; with nominal volumes of 0.5, 1.0, and 5.0 ml, respectively). Polyurethane tubing, of 6.0 mm diameter, connected the syringes to the nozzles by luer adaptors. The following flow rates were

used: 100, 250, 500, 750, 1000, 1250, 1500, 1750, and 2000 $\mu\text{l}/\text{min}$. The actual maximum flow rate per liquid was limited by the fluid viscosity; e.g., the maximum flow rate achieved for pure glycerine was $Q = 1000 \mu\text{l}/\text{min}$. Six different nozzles were used in our experiments, with internal diameters of 0.10, 0.25, 0.83, 1.37, 1.79, and 2.38 mm. Prior to data acquisition, the liquid delivery system was carefully primed to minimize air entrapment. During experiments, the pump imposed a constant volumetric flow rate, producing a pendant drop that grew under gravity, underwent necking, and pinched off. The complete process, from necking to detachment, was recorded for subsequent frame-by-frame (visual) analysis to identify the frame nearest breakup.

Drop formation and pinch-off were recorded using the back-lit, side-view, high-speed imaging system illustrated in Fig. 1. The setup consists of a high-speed camera (Phantom, TMX-5010), an off-the-shelf macro lens (Tamron macro), a vertical nozzle for gravity-driven dripping, a 500-watt LED light, and an optical diffuser. The camera-lens axis was aligned orthogonally to the nozzle to obtain the side view. The diffuser is used to produce a uniform background and high-contrast images; some examples are seen in Fig. 2. Images were acquired at a spatial resolution of 1280×800 pixels, at a frame rate of $50\,000 \text{ frames s}^{-1}$ and at an exposure time of $19 \mu\text{s}$. The optical resolution is in the range of 112 to 116 pixels/mm. This configuration provides sufficient temporal and spatial resolution to easily identify and capture the entire necking and pinch-off dynamics for all the conditions explored in this work.

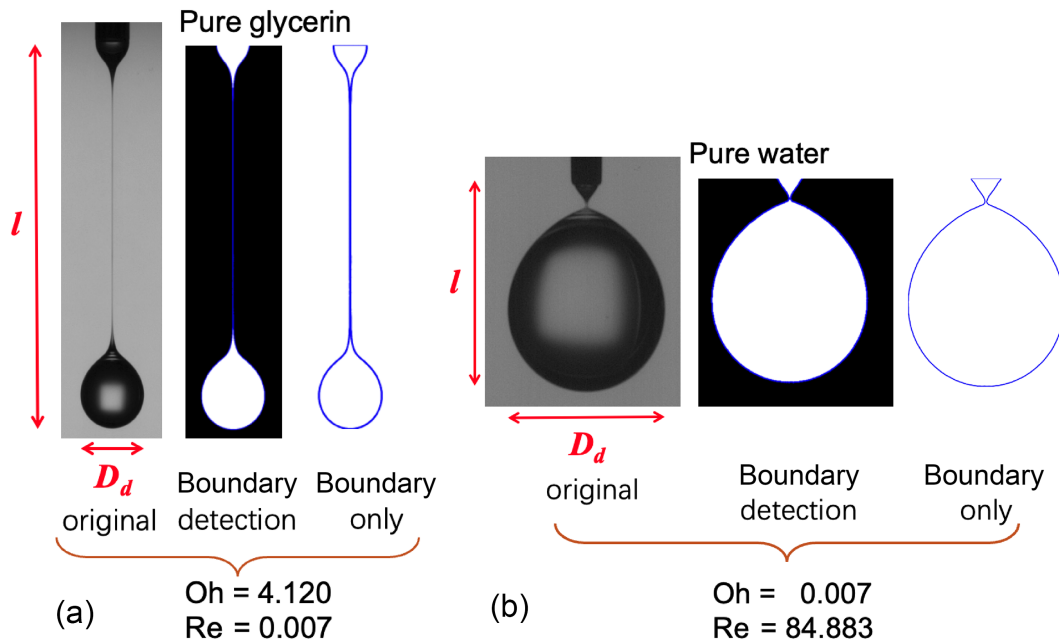


FIG. 2. Examples of the image-processing pipeline. The left panel shows the droplet at breakup, the middle panel shows the binary segmentation, and the right panel shows the extracted interface silhouette.

TABLE I. Physical properties of the tested liquids at $T = 20.0 \pm 1.0^\circ\text{C}$. Values of viscosity μ , density ρ , and surface tension σ are used in this work.

Liquid	μ (mPa s)	ρ (kg/m ³)	σ (mN/m)
Water	1.0	1000	72.0
90% water / 10% glycerin	1.5	1005	71.5
70% water / 30% glycerin	2.8	1033	69.2
50% water / 50% glycerin	6.8	1082	67.4
40% water / 60% glycerin	12.1	1115	66.6
30% water / 70% glycerin	32.9	1156	65.8
20% water / 80% glycerin	56.5	1187	64.8
10% water / 90% glycerin	225.1	1206	63.8
5% water / 95% glycerin	505.3	1230	63.3
Glycerin	1350.0	1248	62.8
90% water / 10% ethanol	1.4	951	50.4
80% water / 20% ethanol	1.9	938	38.5
70% water / 30% ethanol	2.5	927	34.2
50% water / 50% ethanol	2.5	886	28.6
10% water / 90% ethanol	1.6	811	23.9
30% water / 70% ethanol	2.3	827	26.1
5% water / 95% ethanol	1.5	790	22.8
Ethanol	1.2	789	22.2
Methanol	0.6	784	22.3
Silicon oil (2 cSt)	1.8	900	17.7
Silicon oil (5 cSt)	4.6	916	19.7
Silicon oil (50 cSt)	48.6	960	20.8
Silicon oil (350 cSt)	348.0	969	21.1
Silicon oil (1000 cSt)	996.4	970	21.2

A. Liquid materials and characterization

All the liquids used in our experiments are well-known Newtonians and are listed in Table I. They consist of aqueous solutions of glycerin and ethanol, pure methanol, and silicon oils. Deionized water was sourced locally, from a IQ 7000 Milli-Q purification system (at 18.2 M Ω .cm and

2.3 ppb TOC). (Pure) ethanol (TWL00000010683) and methanol (TWL0000001064) were obtained from Fisher Scientific, and glycerin was sourced from UK ONYX Ingredients (99.9% pure). Silicone oils were acquired from Sigma Aldrich; these are 5 cSt (317667), 50 cSt (378356), and 1000 cSt (378399). The 2 cSt and 350 cSt silicone oils were obtained from Azeils (PMX-200).

The physical properties—i.e., density, viscosity, and surface tension—of the test liquids were characterized at (working) room temperature of $(21.0 \pm 1.0^\circ\text{C})$. Density was determined gravimetrically by weighing a known volume of liquid using an analytical balance. Viscosity was measured on a rotational rheometer (Anton Paar MCR 102e) equipped with a cone-plate geometry at a constant shear rate of 50 s^{-1} . Each sample was measured 20 times with the mean value taken as the working viscosity; the typical error in the viscosity measurement was $\delta\mu_{\text{true}} = \pm 0.10 \text{ mPa s}$. For liquids of low and moderate viscosity ($\mu \lesssim 60 \text{ mPa s}$), the surface tension was evaluated via the maximum bubble pressure method using a SITA pro line T15 tensiometer at bubble lifetimes from 1.0 s to 15.0 s. Within this range, surface tension was found to be independent of surface age, with a standard error of $\delta\sigma_{\text{true}} = \pm 0.2 \text{ mN/m}$. As noted in the Introduction, bubble-pressure tensiometers are known to provide unreliable readings for highly viscous liquids due to viscous and hydrodynamic effects [16,58]. Accordingly, for glycerin and water mixtures of a concentration greater than or equal to 80 wt% glycerin, and for 350 cSt and 1000 cSt silicone oils, surface tension values were taken from the manufacturers' data sheets. The resulting liquid property data are presented in Table I; we note that viscosity values span more than 3 orders of magnitude while surface tension triples in value. The domain of our data, parametrized in terms of Re and Oh, can be found in Fig. 3(a). As can be

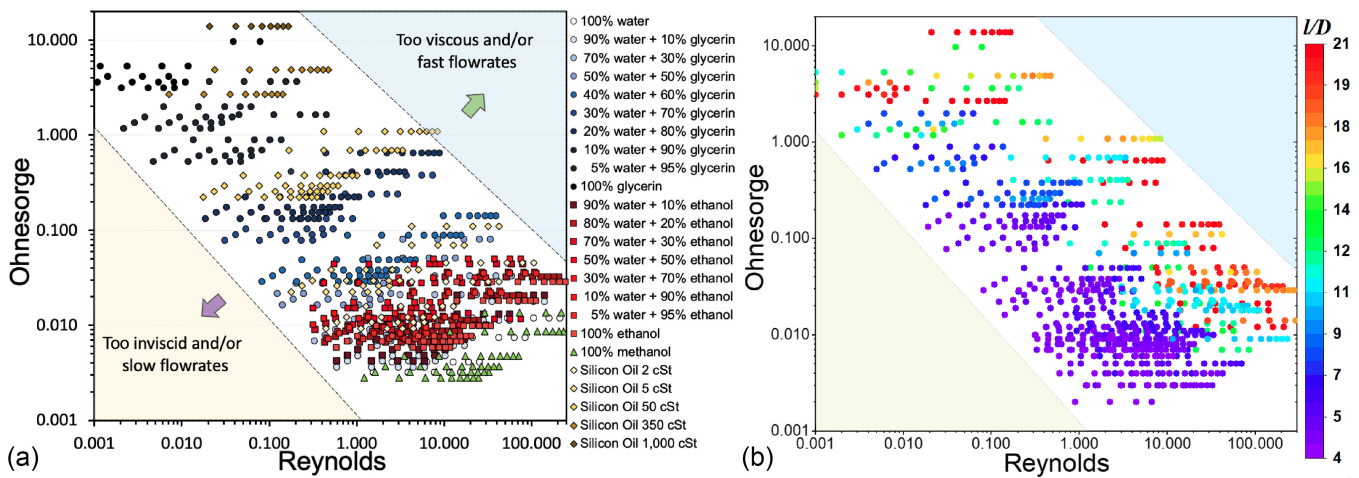


FIG. 3. Data domains in terms of the Reynolds and Ohnesorge numbers. (a) Parametric space of fluid properties and flow characteristics in terms of the liquid type and formulation; most liquids correspond to aqueous solutions of ethanol and glycerin. (b) Parametric space in terms of the drop-length-to-nozzle inner diameter ratio (l/D); as seen, the ratio is not a simple function of Oh and Re as many conditions lead to the same aspect ratio.

seen there, our conditions cover 4 orders of magnitude in the Ohnesorge number and five in Reynolds, and include simple dripping and jetting as defined in previous works [37].

B. Image analysis

In our method, high-speed images were analyzed to identify the frame at which full breakup occurred. The preceding frame (within 20 μs , as limited by the frame rate) was then selected, recorded, and analyzed. This process was repeated for each experiment. We carried out 840 experiments with varying conditions, each producing an image (snapshot) showing the breakup of the droplet. Each image was processed by a custom MATLAB code to obtain the droplet profile and its contour. The code works by first subtracting a prerecorded background (image without a droplet, just the background) from the droplet snapshot to enhance the interfacial contrast. The code then employs a global threshold function (Otsu's method) to obtain a binary image [59], followed by steps to remove image artifacts and holes. The largest connected region in the binary image is then identified as the droplet, and its exterior boundary is extracted as the raw contour. Finally, the boundary is smoothed and resampled uniformly. The needle outer diameter was used as a size reference (pixels to micrometers) to determine lengths, such as the droplet diameter D_d at pinch-off and the drop length l [the axial distance from the nozzle exit plane to the lowest point of the drop (apex) along the centerline], for each experiment. Figure 2 illustrates some of the steps of the image analysis and the droplet shape at the point of breakup for (a) pure glycerin and (b) water. These boundary-only silhouettes constitute the sole image inputs for our computational methods.

IV. MACHINE-LEARNING METHODOLOGY

Supervised and unsupervised machine-learning techniques were employed for the data-driven prediction of liquid properties and droplet shapes. The supervised methods aim to either predict fluid properties based on physical inputs and latent shape features or to reconstruct droplet shapes directly from physical parameters. In contrast, the unsupervised method focuses on identifying intrinsic groupings, i.e., clusters, of droplet characteristics based on experimental conditions and shape features.

A. Extracting low-dimensional representation of boundary-only image with autoencoder

A common foundation across both approaches is the use of a convolutional neural network autoencoder (CNN-AE) [50] to extract low-dimensional representations of the droplet snapshots, in the form of a latent vector $\mathbf{z}$. The structure of the autoencoder can be found in Table II in Appendix A. The primary motivation for employing a CNN

is its ability to filter and denoise raw image data while reducing the input dimension. These features are then used as model inputs in the supervised-learning tasks or as clustering coordinates in the unsupervised-learning analyses. Our dataset is constructed from the droplet boundary-only images previously described. From the dataset of 840 samples, we randomly selected 600 samples for training, 120 samples for validation, and 120 samples for testing the autoencoder and the other methods. At each step, these groups of samples remained isolated from each other to avoid data leakage. The network was trained using the Adam optimizer [60] with a learning rate of 2.5×10^{-4} . An early stopping criterion was applied: If the validation loss did not decrease for 30 consecutive epochs, training was terminated. Under these conditions, the training converged after 140 epochs with a batch size of 32. Loss curves for the autoencoder are provided in Fig. 11 in Appendix A.

The model uses binary cross-entropy (BCE) loss, as the output is normalized. Mathematically, the autoencoder consists of an encoder E_θ and a decoder D_ϕ , parametrized by θ and ϕ , respectively. Given an input image x , the BCE reconstruction loss is defined as

$$\mathcal{L}_{\text{BCE}}(x, D_\phi(E_\theta(x))) = -\frac{1}{N} \sum_{i=1}^N [x_i \log(D_\phi(E_\theta(x)))_i + (1 - x_i) \log(1 - D_\phi(E_\theta(x)))_i],$$

where x is the ground-truth image, $D_\phi(E_\theta(x))$ is the reconstructed image (both with pixel values in $[0, 1]$), and N is the total number of pixels. This loss encourages the model to accurately predict pixel intensities in the reconstructed image [61].

The validation set was then used to evaluate the reconstruction performance of the autoencoder. Figure 4(a) shows the relative reconstruction error as a function of the latent dimension. Our results indicate that the relative error decreases rapidly as the latent dimension d (the length of vector $\mathbf{z}$) increases and plateaus around $d = 14$, indicating that adding more dimensions does not improve accuracy. Consequently, in our methods, we set the autoencoder to produce a vector $\mathbf{z}$ of size $d = 14$. We note that we have found a low-dimensional representation with a dimension in $\mathcal{O}(10^1)$, starting from an input dimension of $\mathcal{O}(10^5)$; thus, a reduction of 4 orders of magnitude is achieved. A representative reconstruction example is found in Fig. 4(b), contrasting the original contour and the reconstruction (with $d = 14$), and demonstrating that the autoencoder effectively captures the overall droplet geometry.

B. Predicting liquid properties and droplet shape with supervised learning

In this section, we describe the supervised models used to infer either one or two fluid properties (viscosity and/or

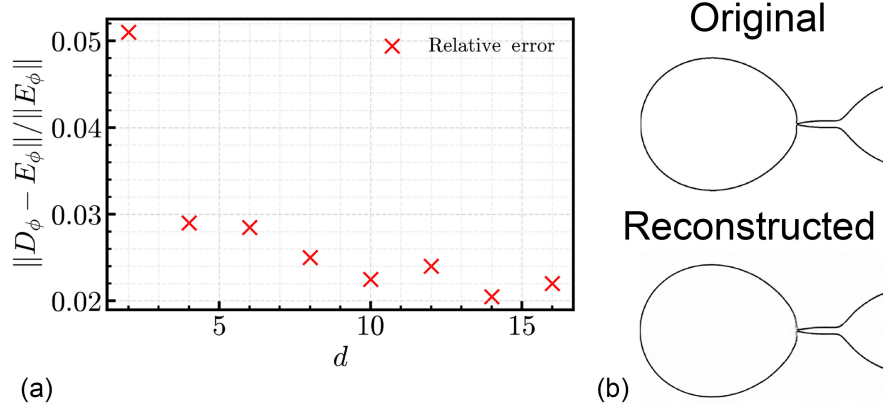


FIG. 4. Autoencoder error analysis. (a) Relative reconstruction error $\|D_\phi - E_\phi\|/\|E_\phi\|$ on the validation set versus the latent dimension d of the shape vector ($z \in \mathbb{R}^d$). As seen, the minimum error is found at $d = 14$, which is the value used in our models. (b) Qualitative comparison of a representative boundary: ground truth (original) vs autoencoder output (reconstructed). Here, $Re = 7.943$ and $Oh = 0.004$ (90% water + 10% glycerin).

surface tension) from images or to predict the droplet shape from a set of known physical parameters. All supervised-learning models were implemented using two types of algorithms: multilayer perceptron (MLP) and gradient boosting decision trees (XGBoost) [62], which differ in their learning paradigms. The four XGBoost tasks, or models, considered are summarized in Fig. 13, and details of the MLP model are found in Appendix B. Neural networks learn smooth continuous mappings, while XGBoost captures nonlinear feature interactions through ensembles of decision trees—allowing a complementary comparison between deep and tree-based methods. Models 1–3 combine the autoencoder latent vector $\mathbf{z}$ with physical parameters (nozzle diameter D , flow rate Q , density ρ , viscosity μ , and surface tension σ) to predict fluid properties. Model 4 performs the inverse mapping, predicting droplet shape from the same physical parameters, thereby establishing a bidirectional link between geometry and material behavior.

In addition, the same train, validation, and test split as used for the autoencoder was used here, with the same strict separation of these sets, in order to prevent data leakage. Each model was trained using the Adam optimizer with an initial learning rate of 10^{-3} and the mean-squared-error (MSE) loss function,

$$\mathcal{L}_{\text{MSE}} = \frac{1}{N} \sum_{i=1}^N \|\hat{y}_i - y_i\|_2^2,$$

where N denotes the number of samples in a batch. A batch size of 32 was employed, and a StepLR scheduler was applied with a step size of 100 epochs and a decay factor of 0.8. Training was terminated if the validation loss did not improve for 100 consecutive epochs, and the model weights corresponding to the lowest validation loss were restored for testing. For targets with a large dynamic range (e.g., viscosity), the outputs were log-transformed during

training to improve numerical stability and then mapped back to the physical scale for evaluation.

The performance of the models was assessed by the coefficient of determination R^2 , defined as

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2},$$

where y_i denotes the true (experimentally obtained) values, $\hat{y}_i$ the (ML-obtained) predicted values, and $\bar{y}$ the mean of the true values.

C. Uncovering natural groupings in droplet behavior with unsupervised learning

In addition, we have employed unsupervised learning to directly uncover naturally occurring group behavior in droplet dynamics from the observed shapes and imposed flow conditions, without requiring prior knowledge of the underlying fluid properties.

This work is motivated by experimental scenarios in which viscosity or surface tension is not readily measurable, yet droplet morphology and control parameters are accessible, e.g., in systems where liquid volumes are too small. The autoencoder latent vector $\mathbf{z} \in \mathbb{R}^{14}$ was combined with two directly controlled experimental parameters, nozzle inner diameter and flow rate, for clustering. Notably, we deliberately excluded any information related to the fluid properties, as these quantities are often difficult to measure in practice. We applied both k -means and Gaussian mixture models (GMM) with cluster numbers K , whose algorithm settings are provided in Appendix C. To identify optimal settings, we systematically tested $K \in \{2, 7\}$. The choice of the upper bound $K = 7$ reflects a practical consideration: While larger values of K might further partition the data, excessively fine-grained clustering may produce groups that lack clear physical interpretation in the context of droplet dynamics. After the

evaluation of K discussed in Appendix D, we adopt $K = 5$ in this work.

V. EXPERIMENTAL OBSERVATIONS

Figure 5 shows example snapshots one frame prior to breakup. The images are organized by nozzle diameter D and the liquid composition. As can be seen, water (a near inviscid fluid) displays a characteristic sharp neck that previous works have confirmed takes a 18-degree angle. In contrast, the 50% mixture forms a longer, more gradually evolving neck, which indicates viscosity is playing a role in the dynamics. Interestingly, regardless of having very similar Reynolds numbers, pure water and 50% glycerin (with $D = 0.25$ mm) have visibly different shapes, with the glycerin solution having a larger bulb. Pure glycerin shows a slender, quasicylindrical long filament that pinches off at the middle, consistent with a viscous-dominated dynamics. The Reynolds numbers listed beneath the panels increase as D decreases; e.g., for water, $Re = 0.77 \rightarrow 2.56 \rightarrow 6.79$, and for glycerin, $Re = 0.001 \rightarrow 0.023 \rightarrow 0.080$. As can be seen, increasing viscosity (decreasing Re) shifts the minimum neck radius upstream and increases the breakup length-to-nozzle diameter aspect ratio.

The parameter map in Fig. 3(a) identifies different liquids within the Re - Oh plane. As expected, methanol is found at the lowest Oh values due to its low viscosity. In contrast, aqueous ethanols are found in the low- Oh and moderate- to high- Re regions due to their low viscosities and surface tensions. Glycerin-water mixtures occupy intermediate Oh values across a wide range of Re , with pure glycerin and high-viscosity silicone oils situated at high Oh and low Re . The accessible domain of our setup is

limited in the low left by very low viscosity and very slow flow rates, and in the upper right by the maximum pumping pressure, which limits the flow rates. We argue that our domain of experimental conditions is sufficiently extensive, as it includes very low viscosity liquids (methanol and water), intermediate viscosities (solutions), very high viscous fluids (silicon oil and glycerin), very low surface tension (silicon oils), and high surface tension liquids (water).

Figure 3(b) provides an alternative perspective by color-mapping the breakup length-to-nozzle diameter ratio L/D across the Re - Oh space. As noted in the Introduction, the L/D ratio has been used to determine whether a liquid filament will break up or not and to distinguish dripping from jetting conditions [37,38]. From our results [the color redundancy in Fig. 3(b)], various Re and Oh conditions lead to the same dimensionless breakup length; it is hence clear that the aspect ratio alone is insufficient to predict liquid properties or a flow characteristic.

Figure 6 shows the interface shapes of three breaking-up droplets, all of which exhibit the characteristic shallow angle and surface overturning, suggesting its route to self-similarity at the singularity for the near-inviscid case. Here, at $Oh = 0.004$, these characteristics are observed only near the location of the pinch-off as shown by Ref. [26], but importantly, these features do not extend to the whole fluid interface. Away from the capillary singularity region, differences in the relevant dimensionless ratios lead to distinct global droplet geometries and temporal evolutions. As seen in Fig. 6, the interface near the (differently sized) nozzles and the droplet sizes and shapes are very different. These larger-scale features provide the information used by our machine-learning models to distinguish between different fluids and

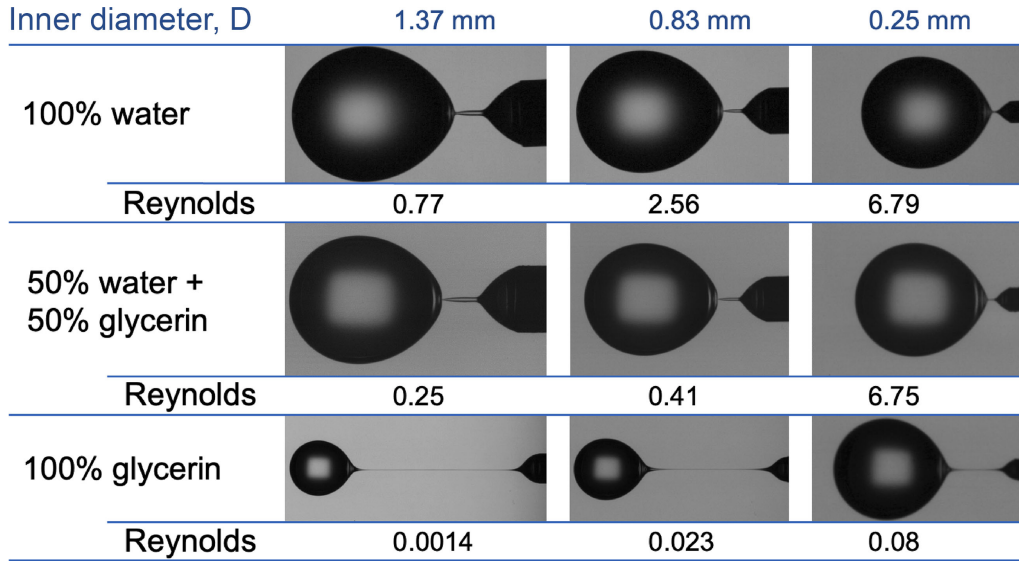


FIG. 5. Examples of breaking-up droplets at various conditions of Reynolds numbers for different liquids. As expected, pure water shows self-similarity near the pinch-off point, and pure glycerin presents long filaments that breakup from the middle. Snapshots of pure glycerin are not to scale.

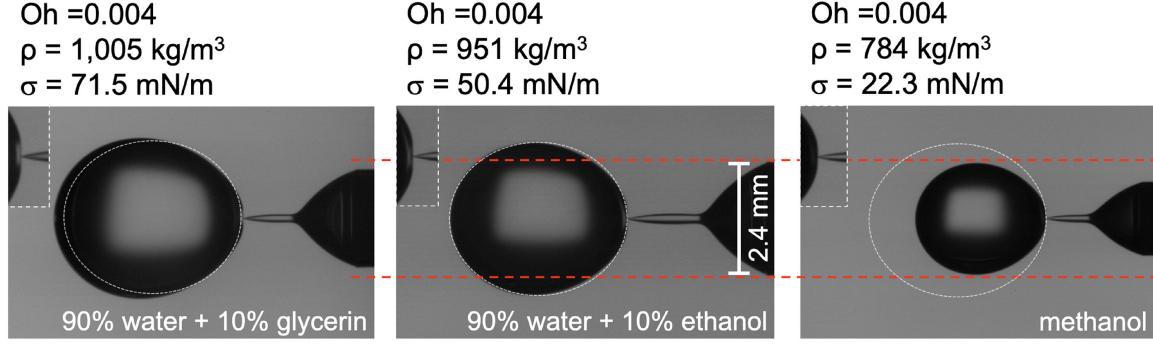


FIG. 6. Breakup in the inertial-capillary regime for three different liquids at $Oh = 0.004$. The interface near the singularity exhibits apparent self-similar behavior, as shown in the insets. However, the shape away from the location of the breakup differs significantly. The red dashed lines indicate the nozzle sizes, while the white dashed line compares the droplet sizes; the water and glycerin solution produces the largest droplet, whereas methanol produces the smallest.

flow conditions, regardless of the apparent self-similar shapes near the point of breakup. We note that, in the experiments seen in Fig. 6, density varies from 1005 to 784 kg/m³, surface tension ranges from 22.2 to 71.5 mN/m, and viscosity is in the range of 0.6 to 1.5 mPa s.

VI. MACHINE-LEARNING RESULTS

A. Supervised-learning results

The four supervised models provide complementary perspectives on the extent to which fluid properties and droplet geometries can be inferred from a combination of physical parameters and learned shape representations. We evaluated the predictive performance of all four

supervised-learning models on the test set, using the coefficient of determination (R^2) between predicted and true values as the main metric.

Model 1, implemented as a MLP, predicts the viscosity from the other liquid properties, the flow characteristics, and the image-derived shape features. Figure 7(a) uses a logarithmic scale and covers more than 3 orders of magnitude; we obtain $R^2 = 0.9428$. In contrast, with the same inputs, gradient boosting decision trees (XGBoost) reach $R^2 = 0.9978$ [Fig. 8(a)], confirming that the viscosity signal is extremely well aligned with the latent geometry. We note that XGBoost is known to handle small datasets well.

Model 2 addresses the complementary task of predicting surface tension from the same inputs as model 1 but with

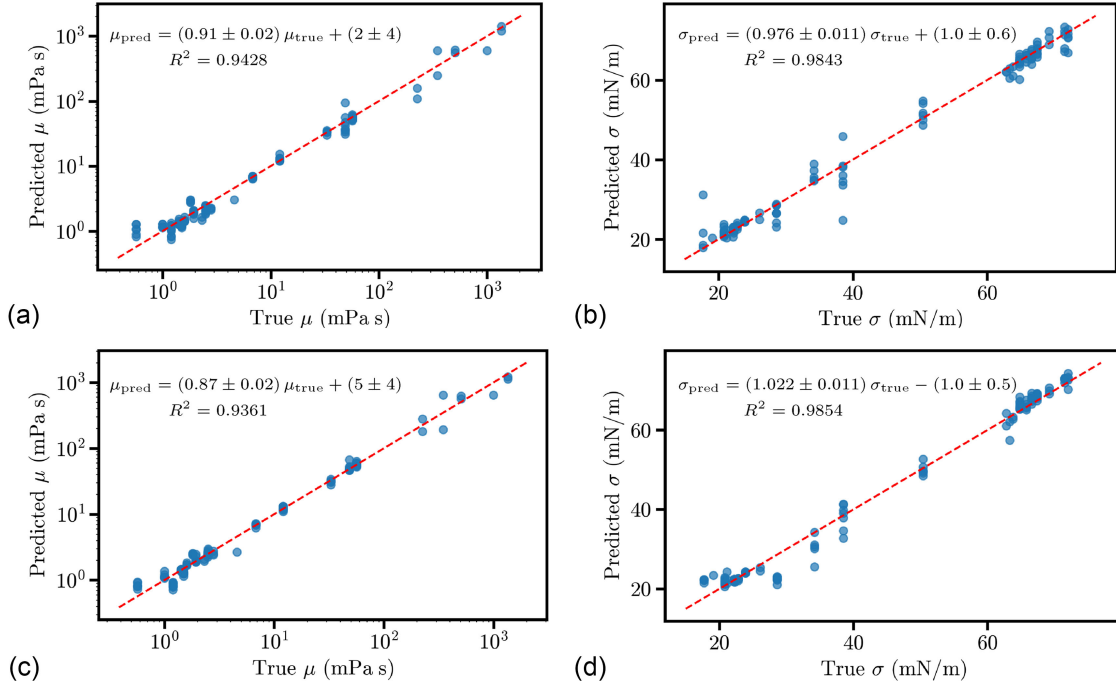


FIG. 7. Predicted vs true values on the test set for models 1–3 (MLP). Model 1 (a) predicts viscosity, model 2 (b) predicts surface tension, and model 3 predicts both viscosity (c) and surface tension (d).

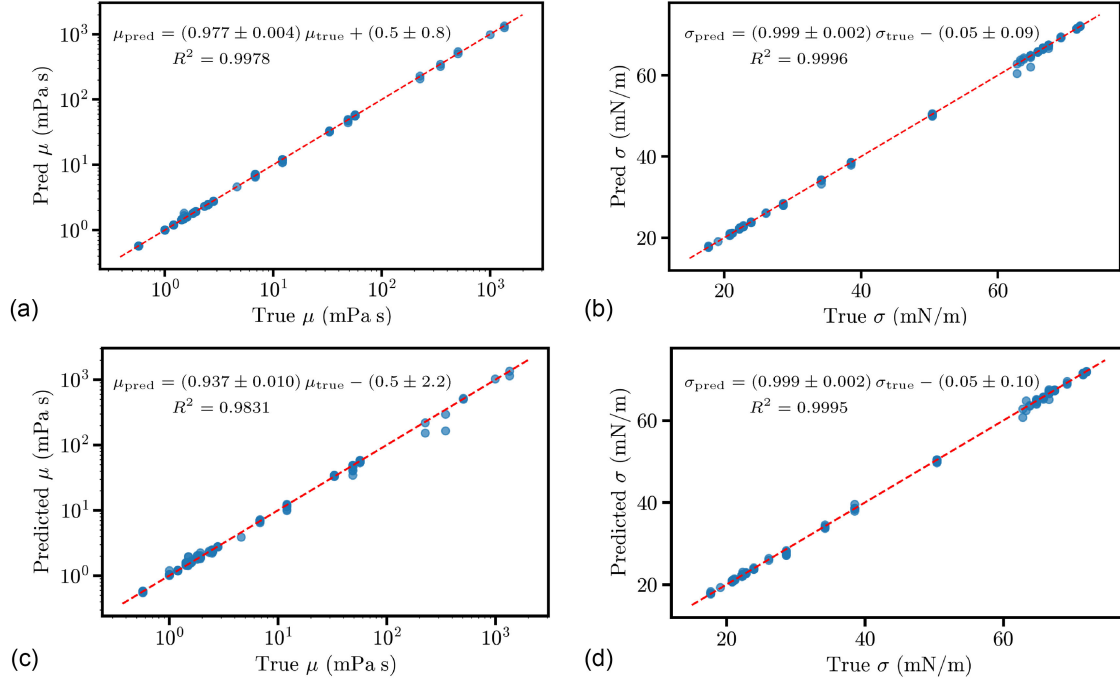


FIG. 8. Predicted vs true values on the test set for models 1–3 (XGBoost). Model 1 (a) predicts viscosity, model 2 (b) predicts surface tension, and model 3 predicts both viscosity (c) and surface tension (d).

surface tension removed from the inputs and viscosity provided instead. The MLP attains $R^2 = 0.9843$ [Fig. 7(b)], while the XGBoost model slightly improves this result to $R^2 = 0.9996$ [Fig. 8(b)], indicating that the latent vector indeed contains geometric markers that are controlled by capillarity.

Model 3 is the multitask setting, predicting both viscosity and surface tension together. Our MLP yields $R_\mu^2 = 0.9361$ for viscosity and $R_\sigma^2 = 0.9854$ for surface tension. The XGBoost version of the same task improves the numbers to $R_\mu^2 = 0.9831$ and $R_\sigma^2 = 0.9995$ [Figs. 8(c) and 8(d)]. Despite having to predict two parameters simultaneously, the multitask setting maintains essentially the same fidelity as the single-task models.

Model 4 differs from the others in that it predicts the *shape* (the latent vector) directly from physical parameters, without any image as input. Using the MLP, we previously obtained $R^2 = 0.9145$ on the test set when comparing the predicted latent vector. The XGBoost model reaches $R^2 = 0.9713$ in the normalized latent space. Because the decoder is a nonlinear map from latent coordinates to pixels, part of this advantage is smoothed out after decoding, but the high latent-space R^2 confirms that a large fraction of the image variability can be explained directly from the physical descriptors. Representative predictions are shown in Figs. 9(a) and 9(b), we can see that both MLP and XGBoost models provide good predictions. Both models reproduce the global morphology reliably: The bulb radius, neck position, and overall aspect ratio match the ground truth. From the differences in the figures, it can be

seen that MLP maintained the continuity of the lines better but slightly underestimated the terminal bulbs, while XGBoost underestimated the thickness of the filament neck.

We note that, across all four supervised tasks, both the MLPs and the XGBoost models consistently achieve test set $R^2 > 0.9$, and in the viscosity case, the boosted model is effectively at the noise floor ($R^2 \approx 0.998$). This case compares favorably with prior work [42,46]. Remarkably, the tree-based models (XGBoost) match or even slightly outperform the MLPs on several tasks, which is likely because the physical properties in our dataset, e.g., viscosity and surface tension, take only discrete values, determined by the limited categories of liquids used. While the models presented here interpolate robustly within the experimentally sampled parameter space, the discrete set of fluid properties used in the training data may limit extrapolation beyond this range. While the MLP attempts to learn a smooth, continuous mapping, XGBoost is naturally better suited for capturing piecewise-constant relationships arising from such discrete labels. The performance of all four models shows that once a good latent vector is available, even a relatively shallow ensemble of boosted trees can exploit it extremely well.

At first sight, the ability to infer fluid properties (such as surface tension σ and viscosity μ) from a single snapshot of a breaking droplet appears to contradict self-similarity and universality. In the asymptotic limit approaching pinch-off, the dynamics is governed by self-similar and universal solutions in which only ratios of material parameters enter

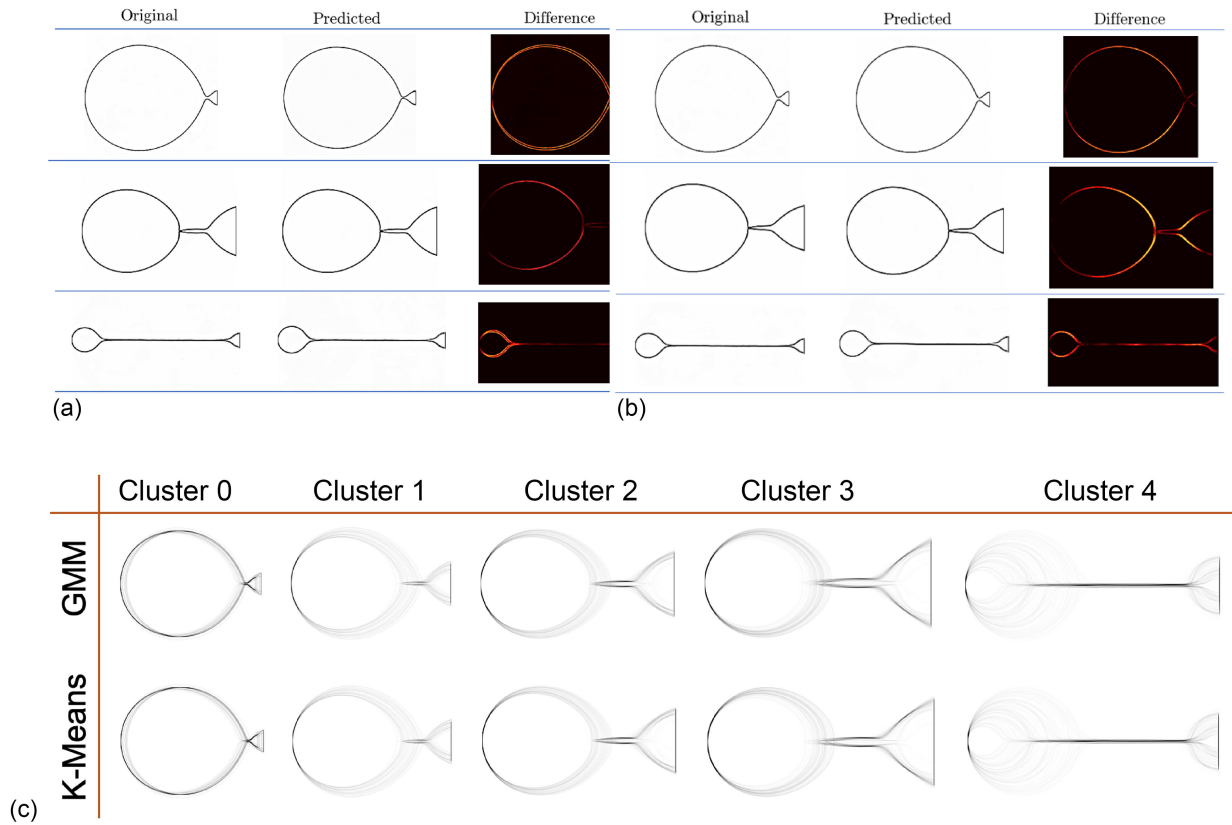


FIG. 9. Model 4 predictions: experimental vs predicted droplet shapes for both (a) MLP and (b) XGBoost variants. (c) Unsupervised-learning results: average droplet shapes discovered by GMM and *K*-means clustering in latent space.

(e.g., σ/ρ for the inertial-capillary regime [63] and σ/μ for the inertia-capillary-viscous regime [64]). In this strict limit, it seems that independent recovery of σ and μ would therefore be impossible. However, a substantial body of experimental and theoretical work has established that it is extremely challenging to achieve the true asymptotic similarity regime during the dynamics of pinch-off. Instead, the approach to singularity is characterized by a complex sequence of transient regimes, slow convergence to similarity, and nonuniversal prefactors. Experiments and simulations have shown that neck thinning does not proceed via a single direct transition between canonical regimes [63–65] but rather through multiple intermediate dynamical states that delay the onset of the final similarity regime [66]. Moreover, similarity solutions may exhibit oscillatory or slowly converging behavior, arising from complex eigenvalue spectra and stability properties [67]. These transient regimes reflect different balances of inertia, viscosity, and capillarity, and their occurrence depends sensitively on the material parameters. As a result, even when the system approaches a nominal similarity regime, the interface observed near pinch-off still bears information about its early dynamics. In addition, deviations from idealized similarity can arise when additional physical effects introduce additional length or time scales, such as heterogeneity or microstructure. These results highlight

that the breakup dynamics is inherently path dependent and that the evolution of the interface (of the whole system, from nozzle to neck to bottom of the drop) cannot, in general, be described by a single universal solution.

Therefore, the (whole) droplet morphology observed at the moment of breakup does not necessarily correspond to the universal similarity solution but rather a finite-time snapshot of a trajectory through multiple dynamical regimes. Crucially, the sequence of regimes, the evolution of prefactors, and the spatial location of regime transitions depend separately on the fluid properties, as well as on flow conditions and geometry. The morphology therefore encodes the route to singularity rather than merely the final asymptotic scaling law. Attempts to infer surface tension by fitting pinch-off data to asymptotic scaling laws with fixed prefactors have been shown to produce significant errors when the system is not fully in the similarity regime [68]. In contrast, the present approach does not assume universality; instead, it leverages the nonuniversal, parameter-dependent morphology present in finite-time breakup dynamics.

Within this framework, machine-learning models act as nonlinear feature extractors that map the high-dimensional geometric information contained in the shape of the interface to the underlying material parameters. The success of this approach therefore does not reflect a violation of

similarity theory but rather the fact that experimentally accessible pinch-off dynamics captures the regime in which independent information about σ , ρ , and μ (and other conditions) is preserved.

B. Unsupervised-learning results

We employed unsupervised clustering on the feature representations extracted by the convolutional neural network to identify intrinsic groupings in the droplet dynamics, based on the droplet images in our validation dataset. From the behavior of the silhouette and DBI metrics across candidate values of K (see Appendix D), we selected $K = 5$ for both K -means and GMM, corresponding to a maximum silhouette score. In the case of the DBI, extending K beyond this value did not result in a large improvement, and, as previously noted, a smaller number of clusters is preferable for insight into the underlying physics.

We computed the average decoded shape of all droplets within a cluster to visualize the typical droplet morphology for that cluster. Figure 9(c) shows these mean shapes for the GMM and K -means partitions. Both methods readily and consistently identify cluster 4 as a bulb attached to a very long filament and cluster 0 as a wide and rounded bulb coming from a nozzle that is small in comparison (with no connecting filament). The other three clusters only differentiate in the relative size of the nozzle

and its connecting filament. Cluster 1 consists of elongated droplets connected to a small nozzle through a short filament, cluster 3 is a droplet connected, through a filament, to a nozzle that is equal in size, and cluster 2 is a droplet connected to a filament and nozzle that are intermediate in size. The close agreement between the average shapes from the two methods confirms that both clustering approaches capture coherent and meaningful droplet shape features.

A central question arising from the clustering is whether the unsupervised method effectively identifies, or differentiates, the underlying regimes and the various other factors influencing the dripping and breakup dynamics. Following previous works [25] that have classified the liquid breakup dynamics in terms of scaling arguments, Figs. 10(b) and 10(d) show the cluster data projected onto the log-log (Re , Oh) plane for GMM and K -means. Importantly, some clusters are readily physically interpretable. Decreasing Oh indicates weakening viscous effects relative to inertial-capillary stresses, while increasing Re reflects stronger inertia at fixed viscosity. Both methods clearly separate cluster 4 and cluster 0. Cluster 4 lies predominantly in the upper-left quadrant (high Oh , low Re), consistent with viscous-damped pinch-off with limited necking and slow recoil, and visually associated with very long filaments. In contrast, cluster 0 concentrates in the lower-right region (low Oh , high Re), characteristic of

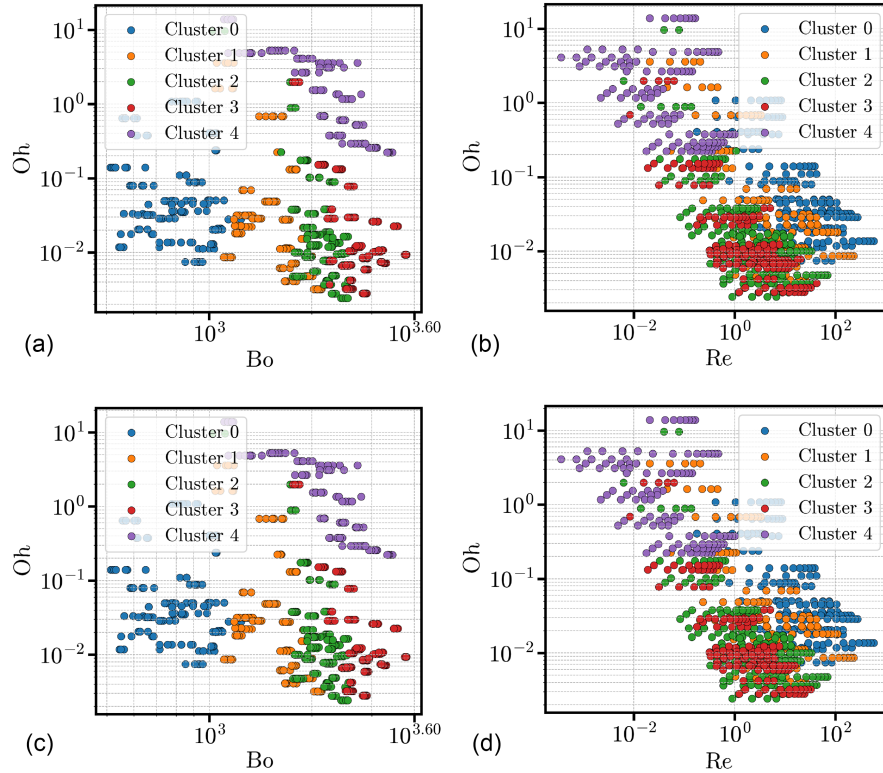


FIG. 10. Clustering of droplet breakup regimes in the Bo - Oh and Re - Oh parameter spaces. Panels (a) and (b) show K -mean clustering, while panels (c) and (d) present the GMM results.

inertial-capillary breakup producing small, nearly spherical droplets with weak viscous damping. Further analyses can be carried out by reflecting on the distribution of liquid formulations among the clusters, as summarized in Tables III and IV in Appendix E. A clear pattern emerges from these analyses: High viscosity fluids, such as glycerin-rich solutions and high cSt silicone oils, are consistently grouped into cluster 4 by both GMM and K -means. These samples correspond to the most elongated droplets, supported by the high ratios L_2/L_1 and the mean shapes shown in Fig. 9(c). In contrast, low-viscosity and low-surface-tension fluids are more frequently assigned to cluster 0. These groups correspond to rounder, more stable droplets with minimal tailing.

Interestingly, clusters 2 and 3 overlap in the low Re and low Oh quadrant, a region reserved for low viscosity liquids dripping at low flow rates. Furthermore, cluster 1 occupies the whole parametric space with no distinct regime, or liquid, or flow property in common. Intermediate formulations such as ethanol-rich fluids are more diffusely distributed across clusters 2–4, suggesting transitional regimes. The clustering results also show strong consistency between GMM and K -means for the same liquids; this alignment reinforces the idea that the latent feature space captures essential morphological cues that reflect fluid properties—even though clustering is entirely unsupervised and image driven.

The lack of a common denominator among data clusters 1–3 suggests that a further factor—apart from the flow rate, the nozzle size, and the liquid characteristics—plays a role in driving the dripping dynamics. Fortunately, dripping is well known to be governed by a third dimensionless parameter, i.e., the Bond number $Bo = \Delta\rho g D_d^2 / \sigma$, where $\Delta\rho$ is the difference between liquid density and air density [25]. We note that, at the point of breakup, the nozzle size is no longer the relevant characteristic length (as the flow is detached), and the droplet diameter D_d is thus used in the Bond number. While D_d is unknown *a priori*, unsupervised clustering does not require any input. Figures 10(a) and 10(c) show the data and clustering in the Bo-Oh space, where all five clusters are well separated, indicating distinct regimes governed by the relative importance of gravity (Bo) and viscosity (Oh). Clusters 0–3 occupy a narrow range of Oh but progressively shift toward larger Bo. This increase in Bo indicates that these clusters primarily reflect the gradual strengthening of gravitational forces relative to capillarity, while viscous effects remain comparatively constant. Cluster 0 corresponds to the capillary-dominated regime, where droplets are nearly spherical and detach quasistatically. The gravitational effect strengthens among clusters 1–3; even though they have comparable viscosity, the increasing gravitation elongates the filaments. In contrast, cluster 4 deviates markedly from this Bo-driven sequence; it spans to much higher Oh values while maintaining moderate Bo, which corresponds to high

viscous effects in which surface tension and gravity are both resisted by strong viscous damping.

As described in Sec. IV C, the clustering was performed on a feature vector that concatenates droplet-shape descriptors with the experimental settings (nozzle diameter and mean flow speed) and does not explicitly include fluid properties. The data separation in Re-Oh and Bo-Oh therefore suggests that these features implicitly encode the inertia-viscosity-gravity balance. Notably, the physical properties of the liquids were not input into the clustering. Thus, the separation in Re-Oh and Bo-Oh space indicates that our method successfully uncovered the underlying physics linking the experimental settings to the observed droplets. The strong mapping between physical composition and cluster assignment provides further evidence that the unsupervised clusters are not only mathematically separable but also physically interpretable.

VII. UNCERTAINTY ANALYSIS

The supervised-learning framework relies on experimentally measured fluid properties as ground truth, and the achievable prediction accuracy is therefore dependent on the uncertainty of these measurements. Nevertheless, the consistently high R^2 values suggest that the relationship between morphology and material properties remains robust, and the method provides a rapid characterization approach with an inherent trade-off between accuracy and simplicity. Experimental uncertainty in the training data introduces label noise, which can affect the learned mapping between morphology and fluid properties. Associating standard errors with machine-learning predictions is not a common practice (to the best of our knowledge, it is largely unexplored). To address this issue, in this section, we propose a simple, yet formal method to associate uncertainties to the predictions of the models. The idea is simple, and it follows the standard method of contrasting predicted and true values. Figures 7 and 8 show various predicted values plotted against known properties. As with any other method, we observe that the prediction is not perfect, with the data showing some scatter around the known values, i.e., the ground-truth values. When presented in this form, the predicted and true ground (experimentally measured) values can be linked by a linear fit, with a regression analysis able to provide us with average and error values for the slope and intercept. In brief, predicted and true values form the function

$$\zeta_{\text{predicted}} = m\zeta_{\text{true}} + b, \quad (1)$$

where $\zeta_{\text{predicted}}$ is the predicted property, which in our case can be either the surface tension or the viscosity; ζ_{true} is the true (measured, or ground-truth) value; m is the slope; and b is the value taken by the prediction at a condition where the true value is zero (or the point where the line crosses the y axis). In a perfect scenario, m would take a value of one

and b the value of zero; i.e., every prediction matches the true value. However, our predictions are not perfect; m is not one, and b is not zero. In fact, our true values are also not uncertainty-free, as they were obtained experimentally with error; this error is known as ground-truth uncertainty in machine learning. Propagation of uncertainty theory tells us that the error on the prediction [Eq. (1)] is given by

$$\delta\zeta_{\text{predicted}} = \sqrt{(\zeta_{\text{true}}\delta m)^2 + (m\delta\zeta_{\text{true}})^2 + \delta b^2}, \quad (2)$$

where $\delta\zeta_{\text{predicted}}$ is the error of the predicted value, $\delta\zeta_{\text{true}}$ is the experimental error of the true value (ground-truth uncertainty), δm is the error of the slope, and δb is the error of the y -axis crossing. Combining Eqs. (1) and (2), we obtain

$$\delta\zeta_{\text{predicted}} = \sqrt{\left(\frac{\zeta_{\text{predicted}} - b}{m}\right)^2 (\delta m)^2 + \delta b^2 + (m\delta\zeta_{\text{true}})^2}. \quad (3)$$

As seen, the first two terms of this equation depend on the fitting coefficients (m and b), and the third term depends on the ground-truth uncertainty. Applying this relationship to, for example, our viscosity predictions from the MLP model 1, Fig. 7(a)—i.e., $m = 0.91$, $\delta m = 0.02$, $b = 2$ mPa s, $\delta b = 4$ mPa s, and $\delta\mu_{\text{true}} = 0.1$ mPa s (ground-truth uncertainty)—we find that the error on the prediction is given by

$$\frac{\delta\mu_{\text{predicted}}^2}{(\text{mPa s})^2} = 4 \times 10^{-4} (\mu_{\text{predicted}} - 2)^2 + 16 + 8 \times 10^{-3}.$$

A simple analysis of this equation indicates that, for very low viscosities, the error on the prediction is largely dominated by δb , or the inaccuracy of the MLP method to predict very low viscosities. In fact, for viscosities near that of water, the error on the prediction is on the order of around 4 mPa s. However, for higher viscosities, the fractional error quickly decreases [see Appendix F, Fig. 15(a)], and at medium viscosities, i.e., $\mu_{\text{predicted}} > 100$ mPa s, the error associated with the MLP prediction only accounts for 5%. For even higher viscosities, i.e., greater than 500 mPa s, the total error only amounts to 2%. Consequently, the MLP model is well suited to predict viscosities above 100 mPa s. We also note that the third term of the equation, associated with the ground-truth uncertainty, is never relevant to the overall uncertainty.

In contrast, the error analyses of models 1 and 3 from XGBoost, e.g., Fig. 8(a) ($m = 0.977$, $\delta m = 0.004$, $b = 0.5$ mPa s, $\delta b = 0.8$ mPa s, and $\delta\mu_{\text{true}} = 0.1$ mPa s) and Fig. 8(d) ($m = 0.999$, $\delta m = 0.002$, $b = 0.05$ mN/m, $\delta b = 0.1$ mN/m s, and $\delta\sigma_{\text{true}} = 0.2$ mN/m), produce much more accurate results. The error on the predicted

viscosity is given by

$$\frac{\delta\mu_{\text{predicted}}^2}{(\text{mPa s})^2} = 1.7 \times 10^{-5} (\mu_{\text{predicted}} - 0.5)^2 + 0.64 + 0.01.$$

Here, the analyses [Fig. 15(b) in Appendix F] also shows that the error associated with the XGBoost model at low viscosities is also largely associated with δb . However, the error on the prediction only accounts for $\mu_{\text{predicted}} = 0.8$ mPa s at low viscosities, 5 times smaller than in the MLP. In fact, XGBoost predicts, within a single-digit percentile, viscosities in the range $\mu_{\text{true}} > 10$ mPa s, and the total fractional error is below 1% for viscosities $\mu_{\text{true}} > 100$ mPa s. Therefore, we conclude the MLP model is well suited to predict viscosities above 10 mPa s and excellent for viscosities beyond 100 mPa s.

Finally, the error associated with predictions of surface tension by XGBoost is given by

$$\frac{\delta\sigma_{\text{predicted}}^2}{(\text{mN/m})^2} = 4 \times 10^{-6} (\sigma_{\text{predicted}} - 0.05)^2 + 0.01 + 0.04.$$

Here, the dominating source of error is the ground-truth uncertainty; i.e., the third factor in the equation is always the largest. In other words, the error associated with the experimental measurements exceeds the uncertainty introduced by the XGBoost model. We note $\delta\sigma_{\text{predicted}}$ stays below 1% [Fig. 15(c) in Appendix F], making the XGBoost model excellent for predicting surface tension.

VIII. CONCLUSIONS

A central outcome of this work is that the interface geometry near pinch-off retains sufficient information to infer fluid parameters, such as surface tension and viscosity, from a single experimental snapshot. While asymptotic similarity predicts that, sufficiently close to singularity, the dynamics are governed by universal exponents depending only on ratios of material parameters, finite-time breakup dynamics do not collapse instantaneously onto this state. Instead, the interface retains information about fluid properties and flow conditions.

The breakup morphology can therefore be interpreted as encoding the route to singularity, rather than merely the final similarity exponent. Classical similarity analysis attempts to eliminate nonuniversality; the approach presented here exploits it. Machine-learning models can extract this high-dimensional morphological information without assuming universal prefactors. The morphology at breakup reflects the entire route to singularity not only the final asymptotic scaling law.

To test this approach, we have systematically analyzed the pinch-off dynamics of Newtonian liquids—including water, methanol, silicone oils, ethanol, and aqueous ethanol and glycerol-water mixtures—using high-speed imaging

across 840 experimental conditions spanning $0.001 < \text{Re} < 200$ and $0.01 < \text{Oh} < 20$. The frame immediately preceding droplet breakup was captured to record the pendant drop shape at detachment. The contour of each droplet was extracted and standardized for subsequent machine-learning analysis. Supervised regression models were used to predict viscosity and surface tension from these contours and known flow parameters. We further built an inverse model that predicts the shape of the drop near the pinch-off time from physical inputs and employed unsupervised clustering to reveal a morphology-physics structure in the data. Thus, we have created a bidirectional, image-driven pipeline linking droplet geometry and material properties. On supervised tests, single-task models achieve $R^2 = 0.9428$ (MLP) and 0.9978 (XGBoost) for viscosity, and $R^2 = 0.9843$ (MLP) and 0.9996 (XGBoost). In the multitask setting, we obtained $R^2 = 0.9361$ (MLP) and 0.9843 (XGBoost) for viscosity, and $R^2 = 0.9854$ (MLP) and 0.9995 (XGBoost) for surface tension. For the inverse mapping, the test-set coefficient of determination in latent space reaches $R^2 = 0.9843$ with XGBoost (MLP: 0.9145), which is significantly better than the MLP model, indicating that the tree-based model performs better than the simple MLP in these cases. In addition, our analyses demonstrate the XGBoost models perform very well in terms of errors, with surface tension predictions achieving a maximum percentage error, less than 2%. These results demonstrate that the instantaneous pinch-off profile encodes fluid properties with high fidelity and that a compact latent representation can be robustly regressed from physical parameters. Unsupervised analyses on latent shape features identify five physically interpretable clusters. The clusters separate cleanly across the Re-Oh and Bo-Oh planes, indicating that the latent representation captures genuine fluid-mechanical differences. This finding could provide new insights into the classification of droplet flow regimes. Our uncertainty analysis demonstrates that the models work particularly well for surface tension predictions and for high viscosities, having limited success for low viscosity liquids.

Previous studies have demonstrated that the simultaneous measurement of surface tension and viscosity is possible using specialized techniques [69], with most traditional characterization protocols relying on separate tensiometers and viscometers, which often demand substantial sample volumes [70] and/or are limited by their measurement range [16]. Our approach provides a single-step measurement of both surface tension and viscosity, alleviating the need for multiple instruments [71]. Our results demonstrate a generalizable route to rapid, multiparameter characterization of liquids from a single pinch-off snapshot, which is suited for integration into automated or in-line monitoring systems. The findings open up new possibilities in inkjet printing (e.g., real-time

characterization of ink viscosity and/or surface tension during jetting [46,72]), biomedical diagnostics with rapid small-volume property checks [73], or other industrial processes with on-the-fly materials characterization [74].

We note that our method is validated for gravity-driven droplet formation from circular nozzles into air and is restricted to the $0.001 < \text{Re} < 200$ and $0.01 < \text{Oh} < 20$ ranges. Within this parameter space, the cases considered span several orders of magnitude. We agree, however, that transferability beyond this geometry is not guaranteed. Likewise, the surrounding medium is air, such that the approach is most appropriate when density and viscosity contrasts remain large and the dynamical influence of the outer fluid is relatively weak.

By making our dataset (and code) openly available to the community, we expect that the database and models will be extended and potentially improved by including numerically obtained results and additional experiments under conditions not attainable in our facilities, thereby broadening validation and improving generalization.

ACKNOWLEDGMENTS

This work was supported by UCL's Engineering and Physical Sciences Research Council (EPSRC) Impact Acceleration Account 2022-25. A. A. C-P. acknowledges funding support from the UK Natural Environment Research Council (UKRI1271) and the UK Engineering and Physical Sciences Research Council (EP/W016036/1 and UKRI424). We also wish to acknowledge both reviewers and the editor, whose thoughtful comments and suggestions helped us to improve our manuscript.

DATA AVAILABILITY

The data and codes used in this study are openly available at [75].

APPENDIX A: CONVOLUTIONAL AUTOENCODER ARCHITECTURE

The droplet breakup silhouettes used in this study contain fine-scale interface curvatures that must be encoded in a compact numerical representation. Therefore, we trained a convolutional autoencoder that maps each padded grayscale droplet image of size 875×875 pixels into a 14-dimensional latent vector, which is later used as an input feature for the supervised and unsupervised-learning models.

Table II lists the encoder-decoder parameters. The encoder progressively downsamples the image through six stride-2 convolutional layers, increasing the channel depth while reducing the spatial resolution. After flattening, a fully connected layer outputs a latent representation $z \in \mathbb{R}^{14}$. The decoder mirrors this structure: A fully connected expansion restores the spatial dimensions, followed

TABLE II. Convolutional autoencoder architecture used for the droplet silhouette compression. The encoder reduces the input image to a 14-dimensional latent vector, and the decoder reconstructs the full-resolution image from the latent vector.

Encoder	Operation
Input	$875 \times 875 \times 1$ grayscale image (padded)
Conv Layer 1	Conv2d(1, 32, kernel = 5, stride = 2, padding = 2), ReLU
Conv Layer 2	Conv2d(32, 64, kernel = 5, stride = 2, padding = 2), ReLU
Conv Layer 3	Conv2d(64, 128, kernel = 5, stride = 2, padding = 2), ReLU
Conv Layer 4	Conv2d(128, 256, kernel = 5, stride = 2, padding = 2), ReLU
Conv Layer 5	Conv2d(256, 256, kernel = 5, stride = 2, padding = 2), ReLU
Conv Layer 6	Conv2d(256, 256, kernel = 5, stride = 2, padding = 2), ReLU
Flatten	Converts $256 \times 14 \times 14$ to vector
FC Layer	Linear($256 \cdot 14 \cdot 14 \rightarrow 14$) $\Rightarrow$ latent vector z
Decoder	Operation
Input	Latent vector $z \in \mathbb{R}^{14}$
FC Layer	Linear($14 \rightarrow 256 \cdot 14 \cdot 14$), ReLU
Reshape	$256 \times 14 \times 14$
Transposed Conv 1	ConvTranspose2d(256, 256, kernel = 5, stride = 2, padding = 2, output_padding = 1), ReLU
Transposed Conv 2	ConvTranspose2d(256, 256, kernel = 5, stride = 2, padding = 2, output_padding = 1), ReLU
Transposed Conv 3	ConvTranspose2d(256, 128, kernel = 5, stride = 2, padding = 2, output_padding = 1), ReLU
Transposed Conv 4	ConvTranspose2d(128, 64, kernel = 5, stride = 2, padding = 2, output_padding = 1), ReLU
Transposed Conv 5	ConvTranspose2d(64, 32, kernel = 5, stride = 2, padding = 2, output_padding = 1), ReLU
Transposed Conv 6	ConvTranspose2d(32, 1, kernel = 6, stride = 2, padding = 2, output_padding = 1), Sigmoid
Output	Reconstructed image ($875 \times 875 \times 1$)

by six transposed convolution layers that reconstruct the original 875×875 image.

The training and validation loss curves seen in Fig. 11 indicate a stable convergence. Early stopping is triggered at epoch 140 to obtain an accurate reconstruction without overfitting.

APPENDIX B: SUPERVISED-LEARNING MODELS

Four supervised models were trained to map droplet images and physical parameters to quantitative fluid properties. Models 1–3 predict viscosity and/or surface tension from measured physical variables and the

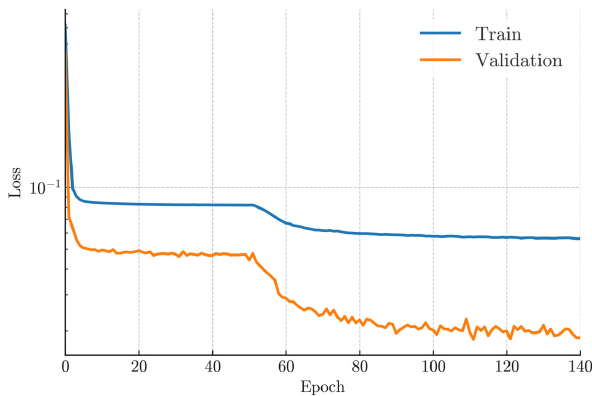


FIG. 11. Training and validation loss curves of the autoencoder. Early stopping is triggered at epoch 140.

14-dimensional latent vectors produced by the convolutional autoencoder. In contrast, model 4 performs the inverse task: It predicts the droplet's latent geometric descriptor directly from five physical variables, enabling comparisons between experimental outcomes and shape embeddings. Figure 12 shows a schematic MLP framework used for models 1–4. Each model uses a fully connected architecture with seven hidden layers and ReLU activation functions but differs in the dimensionality of its input and output spaces. Meanwhile, Fig. 13 shows a XGBoost framework used for models 1–4.

APPENDIX C: CLUSTERING ALGORITHMS

Here, we summarize the mathematical formulation of the clustering methods, i.e., the K -means and the Gaussian mixture (GMM) algorithms. Both methods operate on the latent representations $z_i \in \mathbb{R}^{14}$ that were extracted from the convolutional autoencoder.

1. K -means clustering

The K -means algorithm partitions latent vectors $\{z_i\}_{i=1}^N$ into K clusters by minimizing the within-cluster sum of squared distances:

$$J(\{\mu_k\}, \{c_i\}) = \sum_{i=1}^N \|z_i - \mu_{c_i}\|^2, \quad (\text{C1})$$

where $c_i \in \{1, \dots, K\}$ is the assignment of sample i and $\mu_k \in \mathbb{R}^d$ is the centroid of cluster k .

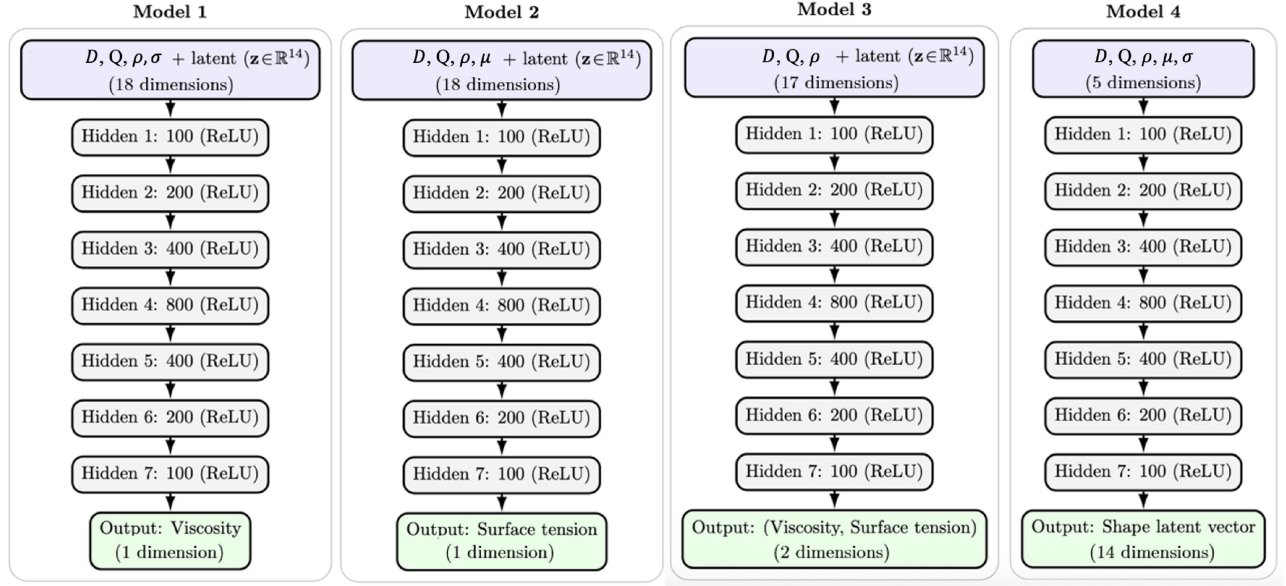


FIG. 12. Architecture summary of the supervised MLP models used in this work. Depending on the task, the inputs consist of physical parameters and, when required, the 14-dimensional latent droplet representation. The models predict viscosity (model 1), surface tension (model 2), viscosity and surface tension (model 3), or the 14-dimensional shape descriptor (model 4).

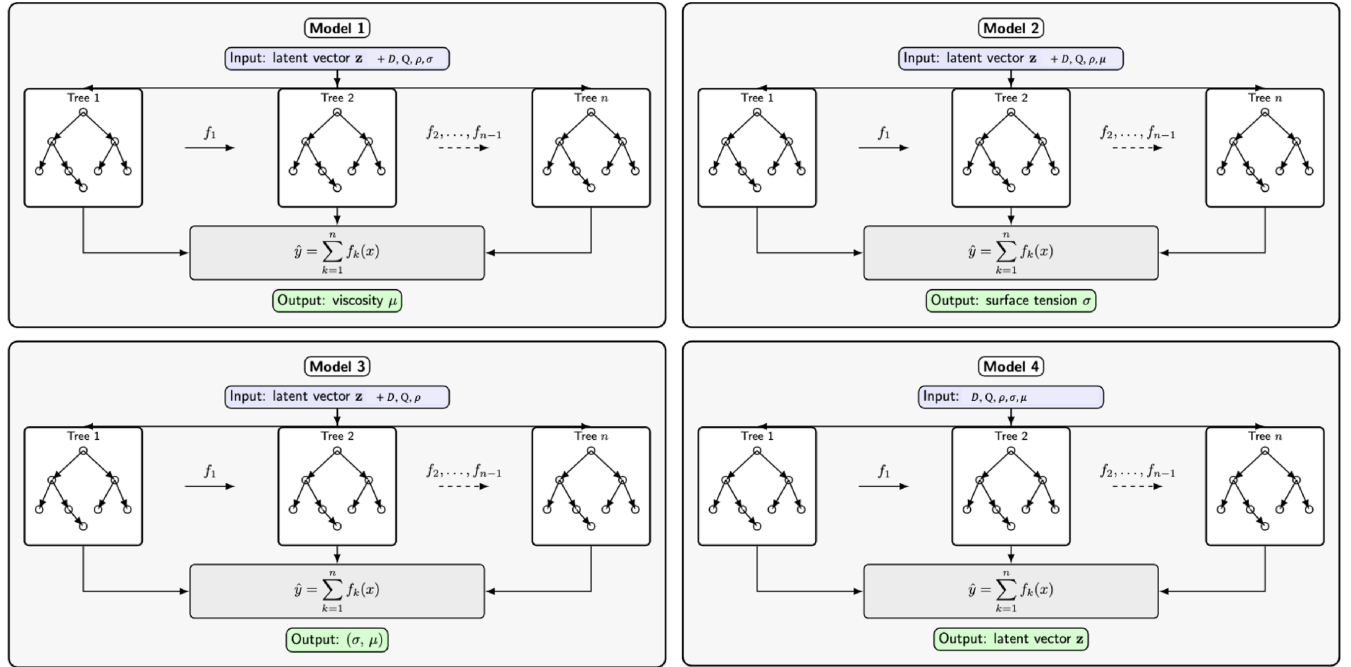


FIG. 13. Supervised-learning framework (XGBoost). Model inputs consist of physical parameters and, when required, the 14-dimensional latent droplet representation. The models predict viscosity (model 1), surface tension (model 2), viscosity and surface tension (model 3), or the 14-dimensional shape descriptor (model 4).

a. Assignment step

Each data point is assigned to the nearest centroid:

$$c_i^{(t+1)} = \arg \min_k \|z_i - \mu_k^{(t)}\|^2. \quad (C2)$$

b. Update step

Each centroid is updated as the mean of its assigned samples:

$$\mu_k^{(t+1)} = \frac{\sum_{i: c_i^{(t+1)}=k} z_i}{|\{i: c_i^{(t+1)}=k\}|}. \quad (C3)$$

Iterations terminate when neither assignments nor centroids change appreciably.

2. Gaussian mixture models and the EM algorithm

A Gaussian mixture model with K components represents the probability density of a latent vector $z \in \mathbb{R}^d$ as

$$p(z|\Theta) = \sum_{k=1}^K \pi_k \mathcal{N}(z|\mu_k, \Sigma_k), \quad (\text{C4})$$

where π_k are the mixture weights ($\pi_k \geq 0$, $\sum_k \pi_k = 1$), $\mu_k \in \mathbb{R}^d$ are the component means, and $\Sigma_k \in \mathbb{R}^{d \times d}$ are covariance matrices. The parameters $\Theta = \{\pi_k, \mu_k, \Sigma_k\}_{k=1}^K$ are estimated by maximizing the log-likelihood

$$\mathcal{L}(\Theta) = \sum_{i=1}^N \log p(z_i|\Theta). \quad (\text{C5})$$

Because the likelihood admits no closed-form maximizer, the EM algorithm is used to construct a sequence of estimates $\Theta^{(t)}$ that monotonically increases $\mathcal{L}$.

a. E step

The posterior responsibility assigned to each mixture component k for a data point z_i is given by

$$\gamma_{ik}^{(t)} = \frac{\pi_k^{(t)} \mathcal{N}(z_i|\mu_k^{(t)}, \Sigma_k^{(t)})}{\sum_{j=1}^K \pi_j^{(t)} \mathcal{N}(z_i|\mu_j^{(t)}, \Sigma_j^{(t)})}, \quad (\text{C6})$$

with these values satisfying $0 \leq \gamma_{ik} \leq 1$ and $\sum_{k=1}^K \gamma_{ik} = 1$.

b. M step

The effective membership of component k is

$$N_k^{(t+1)} = \sum_{i=1}^N \gamma_{ik}^{(t)}. \quad (\text{C7})$$

The updated mixture weights, means, and covariances are given by

$$\pi_k^{(t+1)} = \frac{N_k^{(t+1)}}{N}, \quad (\text{C8})$$

$$\mu_k^{(t+1)} = \frac{\sum_{i=1}^N \gamma_{ik}^{(t)} z_i}{N_k^{(t+1)}}, \quad (\text{C9})$$

$$\Sigma_k^{(t+1)} = \frac{\sum_{i=1}^N \gamma_{ik}^{(t)} (z_i - \mu_k^{(t+1)})(z_i - \mu_k^{(t+1)})^T}{N_k^{(t+1)}}. \quad (\text{C10})$$

Iterations continue until the increase in log-likelihood drops below a specified threshold.

APPENDIX D: CLUSTER EVALUATION RESULTS

We evaluated two standard metrics for both the K -means and the Gaussian mixture models. The first is the silhouette score [76],

$$s(z_i) = \frac{b(z_i) - a(z_i)}{\max\{a(z_i), b(z_i)\}},$$

where $a(z_i)$ denotes the mean distance from z_i to other points in the same cluster, and $b(z_i)$ is the mean distance to points in the nearest different cluster.

The second is the Davies-Bouldin index [77],

$$\text{DBI} = \frac{1}{K} \sum_{i=1}^K \max_{j \neq i} \left(\frac{s_i + s_j}{d_{ij}} \right),$$

where s_i is the average distance between points in cluster i and its centroid, and d_{ij} is the distance between the centroids of clusters i and j . These metrics provide complementary assessments of how well the data partition into distinct clusters for a given choice of K .

A higher silhouette score, approaching 1, indicates that the clusters are well separated and compact. Lower DBI

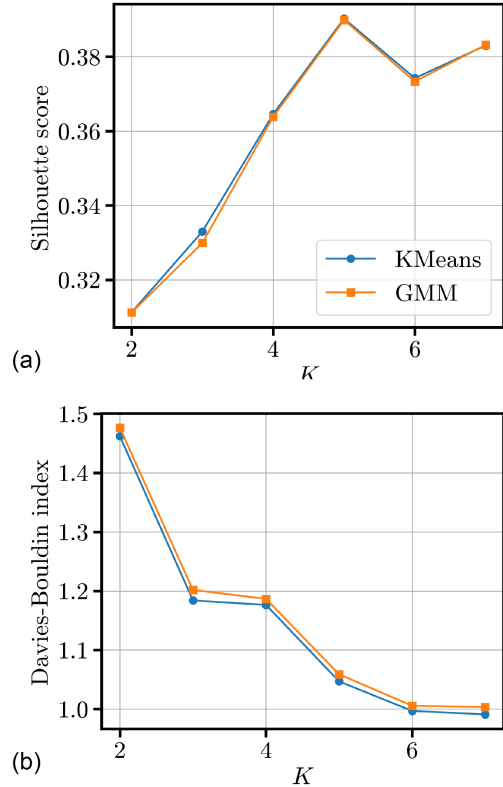


FIG. 14. Cluster evaluation results. (a) Silhouette scores for K -means and GMM across cluster sizes K . (b) DBI for K -means and GMM across K .

values indicate better clustering performance, favoring compact and well-separated clusters.

Figure 14(a) shows the silhouette score as a function of the cluster number K . The score measures the relative separation between clusters: Values closer to 1 indicate samples that are well matched to their own cluster and poorly matched to neighboring clusters. Both K -means and GMM exhibit nearly identical trends, with the score increasing from $K = 2$ to 5 and reaching a broad maximum in this range. This behavior suggests that the learned latent manifold naturally organizes the droplet silhouettes into a moderate number of distinct morphological families. Increasing K beyond this range yields only marginal improvement, indicating that additional splitting does not reveal further meaningful structure.

Figure 14(b) presents the DBI, which quantifies the average similarity between each cluster and its most similar neighbor. Lower DBI values correspond to better-defined, more-separated clusters. Consistent with the silhouette analysis, the DBI decreases sharply from $K = 2$ to $K = 5$ –6 for both algorithms, after which improvements saturate. The close agreement between K -means and GMM across both metrics reinforces the stability of the cluster structure and indicates that the latent space

provides a robust embedding in which fluid composition and interfacial behavior are reflected as geometric regularities.

These two metrics show that $K \approx 5$ provides an optimal balance between cluster compactness and separation, matching the visual grouping observed in the latent-space projections and supporting the use of five-cluster partitioning in the analysis of droplet breakup regimes.

APPENDIX E: K -MEANS AND GMM CLUSTER ASSIGNMENTS

In this appendix, we report the full cluster assignment counts for the liquids tested in the dataset. Tables III and IV provide a fine-grained view of liquid formulation distributions across the five clusters obtained from the K -means and Gaussian mixture models. As observed, highly viscous liquids—including pure glycerin and silicone oils of 350 cSt and 1000 cSt—largely occupy cluster 4, reflecting their well-separated breakup behavior in the latent and physical parameter spaces. Conversely, low-viscosity mixtures largely populate cluster 0, consistent with their overlapping regimes in the Bo-Oh and Re-Oh diagrams.

TABLE III. K -means cluster assignment counts for all liquid types.

Liquid type	Cluster				
	0	1	2	3	4
10% water + 90% ethanol	10	5	5	10	0
10% water + 90% glycerin	1	5	0	5	14
20% water + 80% glycerin	18	8	18	10	0
30% water + 70% ethanol	10	5	5	10	0
30% water + 70% glycerin	5	5	10	10	0
40% water + 60% glycerin	8	9	18	19	0
5% water + 95% ethanol	10	5	5	10	0
5% water + 95% glycerin	4	5	0	1	15
50% water + 50% ethanol	10	5	5	10	0
50% water + 50% glycerin	0	10	13	18	0
70% water + 30% ethanol	10	5	9	6	0
70% water + 30% glycerin	0	10	12	10	0
80% water + 20% ethanol	14	9	18	18	0
90% water + 10% ethanol	4	5	10	11	0
90% water + 10% glycerin	0	10	11	10	0
Pure glycerin	0	0	0	2	28
Pure methanol	19	9	9	17	0
Pure ethanol	20	9	9	16	0
Pure water	0	16	10	10	0
Silicone oil 1000 cSt	0	0	0	0	8
Silicone oil 2 cSt	15	5	5	5	0
Silicone oil 350 cSt	0	0	0	0	18
Silicone oil 50 cSt	0	9	9	0	36
Silicone oil 5 cSt	17	5	5	3	0

TABLE IV. GMM cluster assignment counts for all liquid types.

Liquid type	Cluster				
	0	1	2	3	4
10% water + 90% ethanol	10	5	5	10	0
10% water + 90% glycerin	1	5	0	5	14
20% water + 80% glycerin	18	9	18	9	0
30% water + 70% ethanol	10	5	5	10	0
30% water + 70% glycerin	5	5	10	10	0
40% water + 60% glycerin	8	9	18	19	0
5% water + 95% ethanol	10	5	5	10	0
5% water + 95% glycerin	4	5	0	1	15
50% water + 50% ethanol	10	5	5	10	0
50% water + 50% glycerin	0	10	13	18	0
70% water + 30% ethanol	10	5	9	6	0
70% water + 30% glycerin	0	10	12	10	0
80% water + 20% ethanol	14	9	18	18	0
90% water + 10% ethanol	4	5	10	11	0
90% water + 10% glycerin	0	10	11	10	0
Pure glycerin	0	0	0	2	28
Pure methanol	18	9	9	18	0
Pure ethanol	18	9	9	18	0
Pure water	0	16	10	10	0
Silicone oil 1000 cSt	0	0	0	0	8
Silicone oil 2 cSt	15	5	5	5	0
Silicone oil 350 cSt	0	0	0	0	18
Silicone oil 50 cSt	0	9	9	0	36
Silicone oil 5 cSt	17	5	5	3	0

APPENDIX F: ERROR ANALYSIS

In this appendix, we report the absolute and fractional prediction errors for viscosity and surface tension (see Fig. 15). These errors are calculated from the uncertainty propagation analysis presented in Sec. VII and are shown as a function of the predicted liquid property. The results provide a quantitative assessment of the expected prediction accuracy across the full range of viscosities and surface tensions considered in this work. As discussed in the main text, the prediction uncertainty decreases markedly for increasing viscosity, whereas the surface-tension predictions obtained with XGBoost remain below 1% over the entire range investigated. These results further illustrate the

robustness of the proposed ML framework and clarify the practical operating range over which the models provide the highest accuracy.

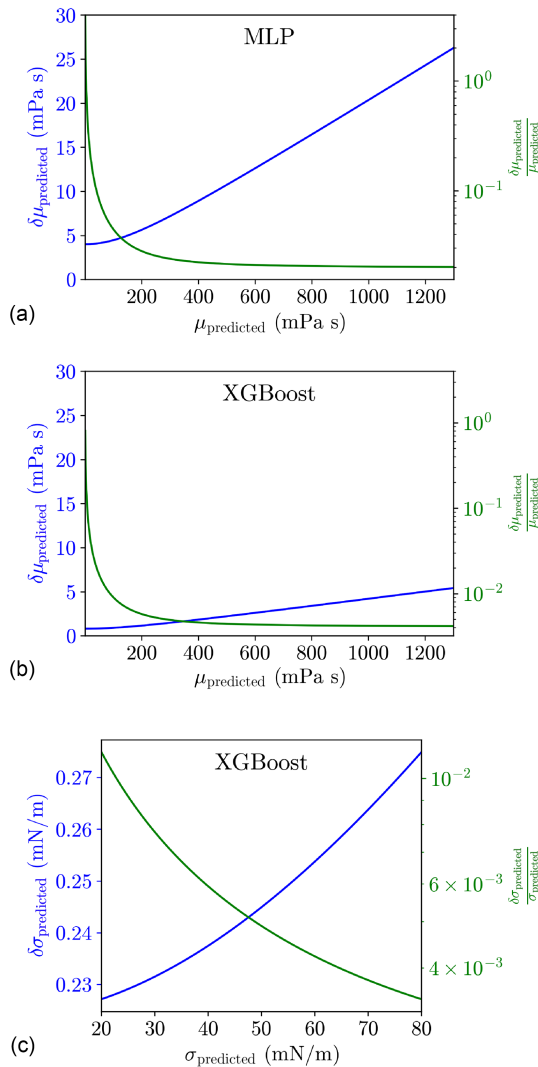


FIG. 15. Overall error and fractional error in terms of the predicted value. (a) Prediction error in terms of the viscosity value; MLP method. (b) Prediction error in terms of the viscosity value; XGBoost method. (c) Prediction error in terms of the surface tension value; XGBoost method.

- [1] A. L. Dearden, P. J. Smith, D.-Y. Shin, N. Reis, B. Derby, and P. O'Brien, *A low curing temperature silver ink for use in ink-jet printing and subsequent production of conductive tracks*, *Macromol. Rapid Commun.* **26**, 315 (2005).
- [2] J. Stringer and B. Derby, *Limits to feature size and resolution in ink jet printing*, *J. Eur. Ceram. Soc.* **29**, 913 (2009).
- [3] S. Pan, R. Guo, M. Björnmalm, J. J. Richardson, L. Li, C. Peng, N. Bertleff-Zieschang, W. Xu, J. Jiang, and F. Caruso, *Coatings super-repellent to ultralow surface tension liquids*, *Nat. Mater.* **17**, 1040 (2018).
- [4] T. Carey, C. Jones, F. Le Moal, D. Deganello, and F. Torrisi, *Spray-coating thin films on three-dimensional surfaces for a semitransparent capacitive-touch device*, *ACS Appl. Mater. Interfaces* **10**, 19948 (2018).
- [5] C. E. Stauffer, *The measurement of surface tension by the pendant drop technique*, *J. Phys. Chem.* **69**, 1933 (1965).
- [6] S. M. Saad, Z. Policova, and A. W. Neumann, *Design and accuracy of pendant drop methods for surface tension measurement*, *Colloids Surf. A* **384**, 442 (2011).
- [7] F. Bashforth and J. C. Adams, *An Attempt to Test the Theories of Capillary Action: By Comparing the Theoretical and Measured Forms of Drops of Fluid* (Cambridge University Press, Cambridge, United Kingdom, 1883).
- [8] F. Hansen and G. Rødsrud, *Surface tension by pendant drop: I. A fast standard instrument using computer image analysis*, *J. Colloid Interface Sci.* **141**, 1 (1991).
- [9] J. D. Berry, M. J. Neeson, R. R. Dagastine, D. Y. Chan, and R. F. Tabor, *Measurement of surface and interfacial tension using pendant drop tensiometry*, *J. Colloid Interface Sci.* **454**, 226 (2015).
- [10] S.-Y. Lin, L.-J. Chen, J.-W. Xyu, and W.-J. Wang, *An examination on the accuracy of interfacial tension measurement from pendant drop profiles*, *Langmuir* **11**, 4159 (1995).
- [11] S.-Y. Lin, W.-J. Wang, L.-W. Lin, and L.-J. Chen, *Systematic effects of bubble volume on the surface tension measured by pendant bubble profiles*, *Colloids Surf. A* **114**, 31 (1996).
- [12] A. Morita, D. Carastan, and N. Demarquette, *Influence of drop volume on surface tension evaluated using the pendant drop method*, *Colloid Polym. Sci.* **280**, 857 (2002).
- [13] W. D. Harkins and H. F. Jordan, *A method for the determination of surface and interfacial tension from the maximum pull on a ring*, *J. Am. Chem. Soc.* **52**, 1751 (1930).
- [14] M. Simon, *Recherches sur la capillarité*, in *Annales de Chimie et de Physique*, 3rd Ser. (Victor Masson, Paris, 1851), Vol. 32, pp. 5–87.
- [15] N. Dixit, D. L. Zeng, and D. S. Kalonia, *Application of maximum bubble pressure surface tensiometer to study protein-surfactant interactions*, *Int. J. Pharm.* **439**, 317 (2012).

- [16] V. Fainerman and R. Miller, *Maximum bubble pressure tensiometry—an analysis of experimental constraints*, *Adv. Colloid Interface Sci.* **108**, 287 (2004).
- [17] C. Weber, *Zum zerfall eines flüssigkeitsstrahles*, *Z. Angew. Math. Mech.* **11**, 136 (1931).
- [18] A. Gaillard, R. Sijs, and D. Bonn, *What determines the drop size in sprays of polymer solutions?*, *J. Non-Newtonian Fluid Mech.* **305**, 104813 (2022).
- [19] J. R. Castrejón-Pita, A. A. Castrejón-Pita, S. S. Thete, K. Sambath, I. M. Hutchings, J. Hinch, J. R. Lister, and O. A. Basaran, *Plethora of transitions during breakup of liquid filaments*, *Proc. Natl. Acad. Sci. U.S.A.* **112**, 4582 (2015).
- [20] G. Schramm *et al.*, *A practical approach to rheology and rheometry* (1994), <https://library.wur.nl/WebQuery/titel/2030370>.
- [21] E. P. Koehler, D. W. Fowler, C. F. Ferraris, and S. Amziane, *A new, portable rheometer for fresh self-consolidating concrete*, *ACI Spec. Publ.* **233**, 97 (2005), https://www.researchgate.net/profile/Sofiane-Amziane/publication/265040129_A_NEW_PORTABLE_RHEOMETER_FOR_FRESH_SELF-CONSOLIDATING_CONCRETE_A_NEW_PORTABLE_RHEOMETER_FOR_FRESH_SELF-CONSOLIDATING_CONCRETE/links/555a264c08ae6fd2d8281e79/A-NEW-PORTABLE-RHEOMETER-FOR-FRESH-SELF-CONSOLIDATING-CONCRETE-A-NEW-PORTABLE-RHEOMETER-FOR-FRESH-SELF-CONSOLIDATING-CONCRETE.pdf.
- [22] A. Castrejón-Pita, E. Betton, N. Campbell, N. Jackson, J. Morgan, T. Tuladhar, D. Vadiello, and J. R. Castrejón-Pita, *Formulation, quality, cleaning, and other advances in inkjet printing*, *Atomization Sprays* **31**, 57 (2021).
- [23] A. A. Castrejón-Pita, J. R. Castrejón-Pita, and I. M. Hutchings, *Breakup of liquid filaments*, *Phys. Rev. Lett.* **108**, 074506 (2012).
- [24] O. A. Basaran, H. Gao, and P. P. Bhat, *Nonstandard inkjets*, *Annu. Rev. Fluid Mech.* **45**, 85 (2013).
- [25] O. A. Basaran, *Small-scale free surface flows with breakup: Drop formation and emerging applications*, *AIChE J.* **48**, 1842 (2002).
- [26] R. F. Day, E. J. Hinch, and J. R. Lister, *Self-similar capillary pinch-off of an inviscid fluid*, *Phys. Rev. Lett.* **80**, 704 (1998).
- [27] J. R. Castrejón-Pita, A. A. Castrejón-Pita, E. J. Hinch, J. R. Lister, and I. M. Hutchings, *Self-similar breakup of near-inviscid liquids*, *Phys. Rev. E* **86**, 015301 (2012).
- [28] J. B. Keller and M. J. Miksis, *Surface tension driven flows*, *SIAM J. Appl. Math.* **43**, 268 (1983).
- [29] A. U. Chen, P. K. Notz, and O. A. Basaran, *Computational and experimental analysis of pinch-off and scaling*, *Phys. Rev. Lett.* **88**, 174501 (2002).
- [30] I. M. Hauner, A. Deblais, J. K. Beattie, H. Kellay, and D. Bonn, *The dynamic surface tension of water*, *J. Phys. Chem. Lett.* **8**, 1599 (2017).
- [31] A. Deblais, M. A. Herrada, I. Hauner, K. P. Velikov, T. Van Roon, H. Kellay, J. Eggers, and D. Bonn, *Viscous effects on inertial drop formation*, *Phys. Rev. Lett.* **121**, 254501 (2018).
- [32] M. Corpart, M. Herrada, A. Deblais, and D. Bonn, *Approach to similarity in the pinch-off of a viscous liquid thread*, *J. Fluid Mech.* **1019**, A15 (2025).
- [33] J. D. Paulsen, J. C. Burton, S. R. Nagel, S. Appathurai, M. T. Harris, and O. A. Basaran, *The inexorable resistance of inertia determines the initial regime of drop coalescence*, *Proc. Natl. Acad. Sci. U.S.A.* **109**, 6857 (2012).
- [34] D. T. Papageorgiou, *On the breakup of viscous liquid threads*, *Phys. Fluids* **7**, 1529 (1995).
- [35] J. Eggers, *Universal pinching of 3d axisymmetric free-surface flow*, *Phys. Rev. Lett.* **71**, 3458 (1993).
- [36] Y. Li and J. E. Sprittles, *Capillary breakup of a liquid bridge: Identifying regimes and transitions*, *J. Fluid Mech.* **797**, 29 (2016).
- [37] B. Ambravaneswaran, H. J. Subramani, S. D. Phillips, and O. A. Basaran, *Dripping-jetting transitions in a dripping faucet*, *Phys. Rev. Lett.* **93**, 034501 (2004).
- [38] P. K. Notz and O. A. Basaran, *Dynamics and breakup of a contracting liquid filament*, *J. Fluid Mech.* **512**, 223 (2004).
- [39] D. Alveringh, D. Le, J. Groenesteijn, J. Schmitz, and J. Lötters, *Fluid classification with integrated flow and pressure sensors using machine learning*, *Sens. Actuators A* **363**, 114762 (2023).
- [40] Q. Chen and C. R. Constante-Amores, *Low-dimensional multiscale dynamics of intermittent reversals in turbulent rayleigh-benard convection*, *Chaos* **36**, 063131 (2026).
- [41] C. R. Constante-Amores, A. J. Linot, and M. D. Graham, *Dynamics of a data-driven low-dimensional model of turbulent minimal pipe flow*, *J. Fluid Mech.* **1020**, A58 (2025).
- [42] A. Gaikwad, T. Chang, B. Giera, N. Watkins, S. Mukherjee, A. Pascall, D. Stobbe, and P. Rao, *In-process monitoring and prediction of droplet quality in droplet-on-demand liquid metal jetting additive manufacturing using machine learning*, *Journal of Intelligent Manufacturing* **33**, 2093 (2022).
- [43] H. Chen, Y. Liu, S. Balabani, R. Hirayama, and J. Huang, *Machine learning in predicting printable biomaterial formulations for direct ink writing*, *Research* **6**, 0197 (2023).
- [44] F. S. Kratz and J. Kierfeld, *Pendant drop tensiometry: A machine learning approach*, *J. Chem. Phys.* **153**, 094701 (2020).
- [45] T. Li, L. Lu, Q. Zeng, and K. Liao, *Machine learning based multi-parameter droplet optimisation model study*, *Sci. Rep.* **15**, 25966 (2025).
- [46] S. Kim, M. Cho, and S. Jung, *The design of an inkjet drive waveform using machine learning*, *Sci. Rep.* **12**, 4841 (2022).
- [47] P. S. Clegg, *Characterising soft matter using machine learning*, *Soft Matter* **17**, 3991 (2021).
- [48] T. Orlova, A. Piven, D. Darmoroz, T. Aliev, T. M. T. A. Razik, A. Boitsev, N. Grafeeva, and E. Skorb, *Machine learning for soft and liquid molecular materials*, *Digital Discovery* **2**, 298 (2023).
- [49] M. H. Mobarak, M. A. Mimona, M. A. Islam, N. Hossain, F. T. Zohura, I. Imtiaz, and M. I. H. Rimón, *Scope of machine learning in materials research—a review*, *Appl. Surface Sci. Adv.* **18**, 100523 (2023).
- [50] J. Masci, U. Meier, D. Cireşan, and J. Schmidhuber, *Stacked convolutional auto-encoders for hierarchical feature extraction*, in *International Conference on Artificial Neural Networks* (Springer, New York, 2011), pp. 52–59.
- [51] J. W. Khor, N. Jean, E. S. Luxenberg, S. Ermon, and S. K. Tang, *Using machine learning to discover shape descriptors*

- for predicting emulsion stability in a microfluidic channel, *Soft Matter* **15**, 1361 (2019).
- [52] H. Zhang and S. K. Moon, *Reviews on machine learning approaches for process optimization in noncontact direct ink writing*, *ACS Appl. Mater. Interfaces* **13**, 53323 (2021).
- [53] L. Au-Yeung and P. A. Tsai, *Predicting impact outcomes and maximum spreading of drop impact on heated nanostructures using machine learning*, *Langmuir* **39**, 18327 (2023).
- [54] M. Tembely, D. C. Vaddillo, A. Dolatabadi, and A. Soucemarianadin, *A machine learning approach for predicting the maximum spreading factor of droplets upon impact on surfaces with various wettabilities*, *Processes* **10**, 1141 (2022).
- [55] M. Pierzyna, D. A. Burzynski, S. E. Bansmer, and R. Semaan, *Data-driven splashing threshold model for drop impact on dry smooth surfaces*, *Phys. Fluids* **33**, 123317 (2021).
- [56] S. Kim, R. Wenger, O. B rky, G. Balestra, U. Jeong, and S. Jung, *Predicting inkjet jetting behavior for viscoelastic inks using machine learning*, *Flexible Printed Electron.* **8**, 035007 (2023).
- [57] S. Kim, M. Cho, and S. Jung, *Reinforcement learning-based dynamic optimization of driving waveforms for inkjet printing of viscoelastic fluids*, *Langmuir* **41**, 10831 (2025).
- [58] K. J. Mysels, *Some limitations in the interpretation of the time dependence of surface tension measured by the maximum bubble pressure method*, *Langmuir* **5**, 442 (1989).
- [59] N. Otsu *et al.*, *A threshold selection method from gray-level histograms*, *Automatica* **11**, 23 (1975), https://engineering.purdue.edu/kak/computervision/ECE661.08/OTSU_paper.pdf.
- [60] D. P. Kingma, *Adam: A method for stochastic optimization*, [arXiv:1412.6980](https://arxiv.org/abs/1412.6980).
- [61] A. Creswell, K. Arulkumaran, and A. A. Bharath, *On denoising autoencoders trained to minimise binary cross-entropy*, [arXiv:1708.08487](https://arxiv.org/abs/1708.08487).
- [62] T. Chen and C. Guestrin, *Xgboost: A scalable tree boosting system*, in *Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, KDD '16* (ACM, 2016), pp. 785–794.
- [63] R. F. Day, E. J. Hinch, and J. R. Lister, *Self-similar capillary pinch-off of an inviscid fluid*, *Phys. Rev. Lett.* **80**, 704 (1998).
- [64] J. Eggers, *Universal pinching of 3d axisymmetric free-surface flow*, *Phys. Rev. Lett.* **71**, 3458 (1993).
- [65] D. T. Papageorgiou, *On the breakup of viscous liquid threads*, *Phys. Fluids* **7**, 1529 (1995).
- [66] J. R. Castrej n-Pita *et al.*, *Plethora of transitions during breakup of liquid filaments*, *Proc. Natl. Acad. Sci. U.S.A.* **111**, 1418541112 (2014).
- [67] M. C. Dallaston *et al.*, *Stability of similarity solutions of viscous thread pinch-off*, *Phys. Rev. Fluids* **6**, 104004 (2021).
- [68] I. Hauner, A. Deblais, J. K. Beattie, H. Kellay, and D. Bonn, *Dynamic surface tension from drop pinch-off*, *J. Phys. Chem. Lett.* **8**, 1599 (2017).
- [69] D. Lohse, *Fundamental fluid dynamics challenges in inkjet printing*, *Annu. Rev. Fluid Mech.* **54**, 349 (2022).
- [70] L. Bautista, D. C. Vargas, X. Luo *et al.*, *A simple capillary viscometer based on the ideal gas law*, *RSC Adv.* **8**, 30441 (2018).
- [71] N. Seimiya and K. Takei, *Simultaneous measurement of surface tension and viscosity using a liquid dynamics sensor*, *Small Methods* **9**, 2401983 (2025).
- [72] T. H. Phung, S. H. Park, I. Kim, T.-M. Lee, and K.-S. Kwon, *Machine learning approach to monitor inkjet jetting status based on the piezo self-sensing*, *Sci. Rep.* **13**, 18089 (2023).
- [73] P. S. Lenzen, F. Dingfelder, M. Muller, and P. Arosio, *Portable microfluidic viscometer for formulation development and in situ quality control of protein and antibody solutions*, *Anal. Chem.* **96**, 13185 (2024).
- [74] S. Tronci, P. Van Neer, E. Giling, U. Stelwagen, D. Piras, R. Mei, F. Corominas, and M. Grosso, *In-line monitoring and control of rheological properties through data-driven ultrasound soft-sensors*, *Sensors* **19**, 5009 (2019).
- [75] <https://github.com/CFTL-Illinois/Droplet-Properties-Prediction>.
- [76] P. J. Rousseeuw, *Silhouettes: A graphical aid to the interpretation and validation of cluster analysis*, *J. Comput. Appl. Math.* **20**, 53 (1987).
- [77] D. L. Davies and D. W. Bouldin, *A cluster separation measure*, *IEEE Trans. Pattern Anal. Mach. Intell.* **PAMI-1**, 224 (1979).