

Emulating 2D Materials with Magnons

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(Received 28 August 2025; revised 17 December 2025; accepted 30 January 2026; published 24 February 2026)

Spin waves (magnons) in two-dimensional (2D) materials have received increasing interest due to their unique states and potential for tunability. However, many interesting features of these systems, including Dirac points and topological states, occur at high frequencies, where experimental probes are limited. Here, we study a crystal formed by patterning a hexagonal array of holes in a perpendicularly magnetized thin film. Through simulation, we find that the magnonic band structure imitates that of graphene, but additionally has some kagomelike character and includes a few flat bands. Surprisingly, its nature can be understood using a nine-band tight-binding Hamiltonian. This clear analogy to 2D materials enables band-gap engineering in 2D, topological magnons along 1D phase boundaries, and spectrally isolated modes at 0D point defects. Interestingly, the 1D phase boundaries allow access to the valley degree of freedom through a magnonic analog of the quantum valley Hall insulator. These approaches can be extended to other magnonic systems, but are potentially more general due to the simplicity of the model, which resembles existing results from electron, phonon, photon, and cold-atom systems. This finding brings the physics of spin waves in 2D materials to more experimentally accessible scales, augments it, and outlines a few principles for controlling magnonic states.

DOI: [10.1103/PhysRevX.16.011034](https://doi.org/10.1103/PhysRevX.16.011034)

Subject Areas: Condensed Matter Physics, Magnetism, Metamaterials

I. INTRODUCTION

The isolation of single layers of carbon in 2004 essentially gave birth to the experimental field of two-dimensional (2D) materials [1] and immediately prompted further study of the quasirelativistic nature of charge carriers in graphene [2–4]. The catalog of 2D materials grew to include a zoo of electronic phases, including semiconductors [5] and insulators [6] at first, but later more exotic interacting phases [7–9]. Among the later species to join the zoo are 2D magnetic materials [10–12]. For example, ferromagnetism resembling both Ising-like and XY spins can be found within the family of chromium halides [13,14] and anti-ferromagnetism can be found in nickel halides [15,16]. However, much of this research goes beyond the equilibrium states of 2D magnets and focuses also on novel magnetization dynamics.

In magnetically ordered solids, including 2D materials, the low-energy spin excitations are usually wavelike collective excitations called spin waves (whose quanta are magnons—the terms will be used interchangeably). Magnons are promising as charge-free information carriers for low-power computing devices in a field of study referred to as magnonics [17–19]. Unique applications of magnons include signal guiding using broken time-reversal symmetry and inference tasks that exploit their inherent nonlinearities [20–23]. Magnons in 2D and layered magnets bear similarities to their electronic counterparts and can host, for example, Dirac points [24], topological gaps [25], and flat bands [26]. Although some properties of these states are attractive, their applications seem distant because of their high frequency (on the order of terahertz) and limited number of experimentally readily accessible probes.

One strategy of controlling magnons is given by magnonic crystals, analogous to the more well-known photonic crystals [27–29]. By patterning a magnetic material into an artificial lattice, one may modify dispersion relations and open band gaps at the Brillouin zone (BZ) edges [30–32]. This is a powerful control scheme for magnon transport that may find application in next-generation microwave and information technologies [33]. In this study, we outline a

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general strategy toward mapping between a few 2D materials and magnonic crystals supported by magnetization dynamics simulations and simple models. Understanding this mapping allows engineering of magnonic band structures using principles from 2D electronics. This brings some of the physics of 2D materials to more accessible frequency and length scales in a system that allows the addition of tailored defects and other spatial inhomogeneities. Therefore, we provide a simple experimentally feasible platform that allows one to probe directly the emerging physics of 2D materials, while also providing a design strategy for functional devices at microwave frequencies.

II. SPIN DYNAMICS SIMULATIONS

Phenomenologically, classical magnetization dynamics are described by the Landau-Lifshitz-Gilbert (LLG) equation [34,35]

$$\partial_t \mathbf{M} = -\gamma \mu_0 \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_S} (\mathbf{M} \times \partial_t \mathbf{M}), \quad (1)$$

where $\mathbf{M}$ is the magnetization with magnitude M_S , γ is the gyromagnetic ratio, and $\mathbf{H}_{\text{eff}}$ is an effective magnetic field having contributions from externally applied fields, the exchange interaction, crystalline anisotropies, the demagnetizing field, etc. This first term gives rise to the well-known precessional motion of the magnetization $\mathbf{M}$. α , the contribution of Gilbert, is a phenomenological damping parameter that weakens precession over time. A magnetic material may be thought of as being composed of many volume elements, known as “macrospins,” for which the internal magnetization profiles are roughly uniform and which each individually follow this equation of motion. It is possible to perform simulations based on this principle, in which $\mathbf{H}_{\text{eff}}$ is calculated for each macrospin and time is evolved in steps. This approach to numerical simulations, referred to as “micromagnetism,” [36] is rich in phenomena and is particularly helpful in studying magnetization switching, complex ground states, and nonlinear and soliton dynamics. Here, we use the GPU-accelerated micromagnetics program MUMAX3 [37] to study spin waves [Figs. 1(a) and 1(b)]. We choose realistic material parameters of the prototypical magnonics material yttrium iron garnet (YIG), which is widely used in magnonic devices due to its low dissipation. See Supplemental Material Sec. I for details [38].

A. Geometry

Any periodic magnetic structure can be considered a magnonic crystal. However, geometries like arrays of disks face a disadvantage in being coupled only by the relatively weak magnetic dipole-dipole interaction. One approach is to embed magnetic components in a different magnetic matrix so the components may be loosely thought of as

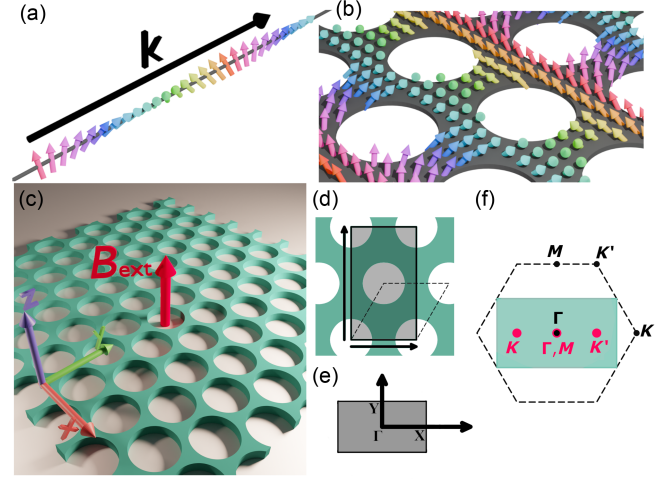


FIG. 1. Spin waves in (a) 1D and (b) 2D. Each arrow indicates a macrospin and its color indicates the direction of the in-plane magnetization and hence the phase of the precession. System geometry: (c) The hexagonal antidot lattice with a perpendicular field, leaving a film similar to a honeycomb lattice. (d) Our rectangular supercell (shaded), with orthogonal lattice vectors 333×589 nm, and (dashed) the typical unit cell of the hexagonal lattice. (e) Corresponding BZ for the rectangular unit cell, with a few special points $\mathbf{X}$ and $\mathbf{Y}$ marked. (f) Relationship between special points in the typical BZ (dashed, black letters) and their placement in the rectangular BZ (shaded, colored letters).

resonators coupled through spin waves in the matrix [39–42]. Another approach uses the so-called “antidot” lattice, in which a continuous thin film is patterned with an array of holes [43–47]. The antidot lattice is different because it cannot generally be thought of as a clear set of coupled resonators. For instance, a lattice with vanishingly small holes should approach the behavior of a pristine film. Note that similar geometries have been explored in the artificial spin ice community, mostly for their interesting ground states [48,49].

In general, a spin-wave dispersion relation depends upon the magnetic ground state. Even for pristine thin films, the dispersion is anisotropic for an in-plane magnetization due to the dipole-dipole interaction [50]. To emulate the isotropic dispersion that is natural for electrons in 2D, we choose an out-of-plane (OOP) ground state by applying an external field $B_{\text{ext}} \parallel \hat{z}$ sufficient to saturate the thin film [Fig. 1(c)]. Alternatively, one could also assume magnetic thin films that have perpendicular magnetic anisotropy. We choose antidot spacings of 100s of nanometers to balance experimental feasibility with large frequency bandwidths. Because of the rectangular shape of micromagnetic cells, it is tremendously convenient to work with a rectangular unit cell. We define a rectangular supercell containing two primitive cells [see Fig. 1(d)]. As a result, the BZ is halved and contains double the number of bands [Fig. 1(e)]. Note that special points in the hexagonal BZ ($\mathbf{K}$ and $\mathbf{K}'$, for instance) now fall inside the rectangular BZ, as shown in

Fig. 1(f). These are technically new points, but we will refer to them as $\mathbf{K}$ and $\mathbf{K}'$ for simplicity.

For numerical simulations, a rectangular unit cell tens of micromagnetic cells in width is tiled tens of times; these two sizes determine real- and inverse-space resolution, respectively. The simulation contains only one micromagnetic cell in the $\hat{z}$ direction—this is a good approximation for very thin films, in which the exchange interaction penalizes $\hat{z}$ direction texturing and standing waves (see Supplemental Material Sec. I [38]). A film thickness of 15 nm is chosen to ensure accuracy. After the structure is relaxed to its OOP equilibrium state with periodic boundary conditions in the x and y directions, excitation can be applied to probe the magnonic band structure.

B. Excitation and analysis

An excitation meant to probe a band structure should couple to many wave vectors and frequencies—in other words, it should resemble a $\delta(\mathbf{r}, t)$ function. Therefore, within a small region, a radio frequency (rf) pulse is applied of the form $B_x(t) = B_0(\sin[2\pi f(t - t_0)]/2\pi f(t - t_0))$, whose spectrum contains all frequencies between $-f$ and $+f$. B_0 is chosen to be small (approximately 0.1 mT or less) to make sure we stay in the linear regime. We time evolve the system under the LLG equation and record the magnetization state $\mathbf{m}(t, x, y, z)$ as a unit vector. Precession about $\hat{z}$ is assumed, so the complex field

$$\Psi \equiv m_x + im_y \quad (2)$$

is a convenient representation that will be used frequently. This field can be interpreted as a wave function in some sense. Assuming circular precession and neglecting damping, eigenmodes (by definition) should keep their shape $|\Psi(\mathbf{r}, t)|^2$ and time evolve only by a phase: $\partial_t \Psi(\mathbf{r}, t) = i\omega \Psi(\mathbf{r}, t)$ for a specific angular frequency $\omega = 2\pi f$. To give a relevant example, the linearized Landau-Lifshitz equation, including an exchange interaction of strength D_{ex} and an externally applied field $H_z(\mathbf{r})$ can be rewritten (see the Appendix) in a Schrödinger-like form assuming a $\hat{z}$ -polarized ground state [51–53],

$$\partial_t \Psi(\mathbf{r}, t) = i\gamma\mu_0(H(\mathbf{r}) - D_{\text{ex}}\nabla^2)\Psi(\mathbf{r}, t). \quad (3)$$

This equation is not used here except to provide an intuitive interpretation of mode profiles as wave functions. In the simulations of this study, the dipole-dipole interaction is included, which, of course, deviates from this form. The Ψ representation is not necessary, but is convenient and will be enlightening in drawing parallels to electrons in real 2D materials.

The time evolution of the magnetization after excitation is written in terms of $\Psi(\mathbf{r}, t)$. Because it is a result of the excitation, we will refer to it as a response. Bloch's theorem can be used to extract the magnonic band structure from the

response: The signal is folded into $\Psi(t, x, y, i, j)$, where (i, j) are unit cell indices and (x, y) label positions within the unit cell. Then, the signal is Fourier transformed along the t , i , and j axes, resulting in the complex amplitude $\Psi(f, x, y, k_x, k_y)$ associated with frequency f and crystal momentum (k_x, k_y) . This approach is similar to Ref. [54].

III. MAGNONIC BAND STRUCTURES

Though our analysis methods are general, we focus mainly on the YIG hexagonal antidot lattice. A few magnonic band structures are plotted in Fig. 2. After summing over the unit cell (x, y) , the magnitude of the response $|\Psi(f, k_x, k_y)|$ is a function of frequency and wave vector, so it can be displayed as a projection on the f, k_x plane or plotted in 3D as volumetric information. As expected, a continuous film [see Fig. 2(a)] has a magnonic band structure equivalent to the zone-folded representation of a free dispersion—the “nearly free magnon.” Small holes [see Fig. 2(b)] significantly change the dispersion, but do not open spectral gaps. However, large holes open spectral gaps and bring about some curious features [see Figs. 2(c) and 2(d)]: Near 1.5 GHz, the band structure resembles that of graphene. Near 2.5 GHz, the band structure seems to

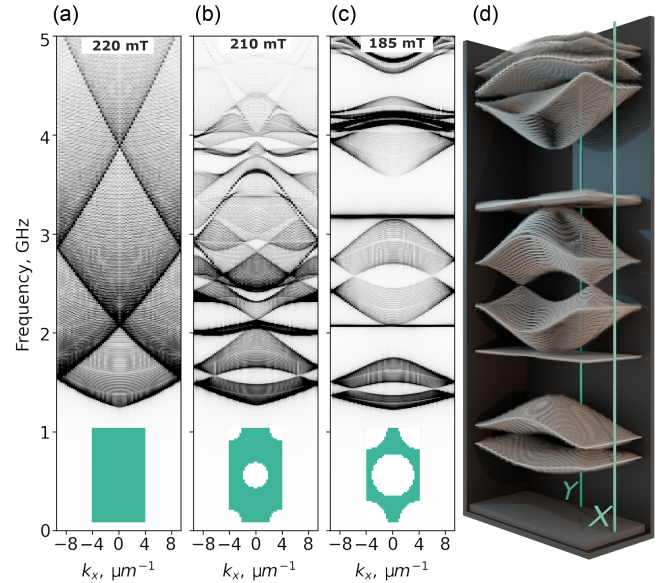


FIG. 2. The effect of hole diameter d on YIG films with lattice parameter $a = 333$ nm, thickness $t = 15$ nm. Band structures are plotted as projections in the (k_x, f) plane for: (a) $d/a = 0.0$ (no holes; equivalent to the free spin-wave spectrum), (b) $d/a = 0.4$, and (c) $d/a = 0.8$. Rectangular unit cells appear as insets. $B_{\text{ext}}||\hat{z}$ is varied to keep the lowest-frequency mode constant. (d) A 3D volumetric plot of the projection in (c), more clearly showing the isotropic character of flat bands. The $d/a = 0.8$ geometry is the main subject of this study. To be more precise, the plots are the magnitude of the response $|\Psi(f, x, y, k_x, k_y)|$ to a δ -like excitation, displayed as projections onto only a few axes: (k_x, f) for 2D or (k_x, k_y, f) for 3D.

have a second set of Dirac points and, additionally, has a few flat bands that meet dispersive bands at the Γ point. The nature of the transition between Figs. 2(b) and 2(c) is not obvious; see Supplemental Material Sec. II [38] for an expanded figure.

Because of its curious features, this third geometry appearing in Fig. 2(c) is the main subject of this study. Our primary goals are as follows:

- (1) Point out and understand the curious features that naturally occur in this antidot lattice.
- (2) Propose potential uses of its unique properties.
- (3) Use understanding from the field of 2D materials to engineer these excitations in ways not possible with natural van der Waals systems.

The isotropic flat bands of this system are particularly interesting. A truly flat band is one that has zero group velocity everywhere; its excitations are immobile. Figure 3 illustrates this immobility. A rectangular region is continuously excited at the first flat band frequency. In the nonflat cases [see Figs. 3(a) and 3(b)], the power radiates away as usual. However, the flat band state cannot propagate and is instead excited to approximately $1000\times$ the density of its counterparts [see Fig. 3(c)]. This is a rare feature, making this system potentially useful in nonlinear magnonics experiments (magnon Bose-Einstein condensation, for instance, in which large magnon densities must be achieved) [55,56]. Outside magnonics, interactions in flat

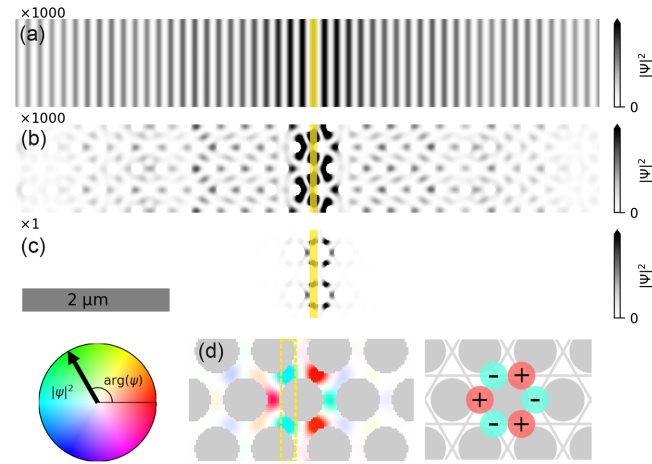


FIG. 3. Under periodic boundary conditions, a rectangular region (yellow) is continuously excited at the first flat band frequency of 2.08 GHz. (a)–(c) Scaled profiles of $|\Psi|^2$ are plotted after 50 ns for the geometries corresponding to (a)–(c) in Fig. 2. The strong localization in (c) demonstrates the exceptional flatness of the band. Note the scale differences: the flat band mode is excited to approximately $1000\times$ the density of its counterparts, making it potentially useful in easily reaching the nonlinear regime. (d) Enlarged plot of the mode in (c), with phase encoded in color. Key for color plots (left) and the localized wave function of the kagome lattice (right), which this mode closely resembles. This is an example of how Ψ can be useful to interpret as a wave function.

bands on the kagome lattice is a popular topic [57]—given the inherent nonlinearity of magnons, it is possible that this may provide a platform for an analog of interacting phases when this localized state is excited to large magnon numbers [58].

Interestingly, the reason behind localization here can be understood from existing principles in 2D materials. Figure 3(d) is an enlarged version of Fig. 3(c), plotted using a phase-sensitive scheme, which reveals that the excited mode closely resembles the famous localized electronic wave function of the kagome lattice, whose localization property can be understood as the result of destructive interference between neighboring sites [59–61]. This spin-wave mode is also similar to localized spin excitations observed in the flat bands of layered kagome magnets, which can be understood by the same means [26,62].

To understand the real nature of the excitations, $\Psi(x, y)$ can be examined at a few (f, k_x, k_y) points. This cell-periodic part of the response can be interpreted as a Bloch function. The magnon band structure is plotted along a few paths in Fig. 4(a), and some Bloch-like functions for

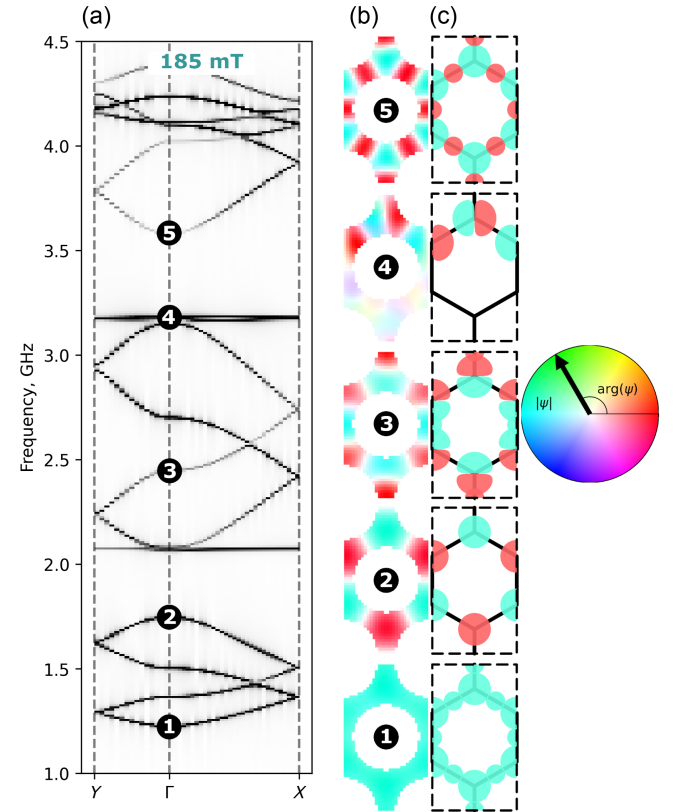


FIG. 4. (a) Magnonic band structure for $d/a = 0.8$ and $B_{\text{ext}} = 185$ mT along a few paths in the rectangular BZ. (b) A few examples of Bloch function responses at the Γ point, and (c) their reconstructions using a small number of basis “orbitals,” discussed in more detail in Sec. IV. Right: key for color plots of Ψ . The main finding is that the Bloch functions are simple, suggesting a tight-binding model.

Γ -point modes are plotted in Fig. 4(b). One finding is that the lowest set of bands is indeed graphenelike: This figure demonstrates the Dirac point more clearly, but perhaps more incriminating is the fact that Γ -point modes 1 and 2 in Fig. 4(b) are given by acoustic and optical modes of the honeycomb lattice. Actually, there exists a tiny gap at the Dirac point due to the dipole-dipole interaction, which is not easy to resolve (see Supplemental Material Sec. III [38]). This is interesting, but will be the primary subject of another study; here, we focus on a simple picture of the band structure.

The most important finding demonstrated by Fig. 4 is that not just the graphenelike modes, but *all* the Γ -point modes may be understood with a simple set of basis functions (or orbitals). Figure 4(c) is an illustration of reconstructions using this set of functions, which consists of (1) *s*-like spin-wave modes on the honeycomb lattice, (2) *p*-like modes on the honeycomb lattice, and (3) *s*-like modes on the kagome lattice, for a total of 9 degrees of freedom per primitive unit cell or 18 per rectangular cell. Bands of 1–2 GHz appear to be composed of *s*-like modes and 2–3.5-GHz bands of *p*-like modes—we refer to these as *s*-bands and *p*-bands, respectively. A natural question is: If the Bloch functions can be reconstructed using these basis orbitals, can the band structure also be reproduced? The latter part of this study is dedicated to this type of understanding and the engineering it enables.

IV. TIGHT-BINDING-LIKE ANALYSIS

Taking a step back, a simple Heisenberg spin chain under an externally applied magnetic field has the Hamiltonian [63]

$$\mathcal{H} = -2\frac{\mathcal{J}}{\hbar^2} \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - \frac{g\mu_B B_0}{\hbar} \sum_i S_i^z \quad (4)$$

for an externally applied field $B_0 || \hat{z}$ and ferromagnetic exchange parameter $\mathcal{J}$, which couples neighboring spins $\langle ij \rangle$. This can be rewritten using spin raising and lowering operators $S^\pm \equiv S_x \pm iS_y$. The Holstein-Primakoff (HP) transformation [64] can then be used to rewrite this problem with magnon operators, assuming total spin *s* parallel to $\hat{z}$ and small magnon number,

$$\begin{aligned} S_i^+ &\approx \hbar\sqrt{2s}a_i, \\ S_i^- &\approx \hbar\sqrt{2s}a_i^\dagger, \end{aligned} \quad (5)$$

$$\begin{aligned} \mathcal{H} = & -2\mathcal{J}s \sum_{\langle ij \rangle} (a_i^\dagger a_j + a_i a_j^\dagger - a_i^\dagger a_i - a_j^\dagger a_j + s) \\ & + g\mu_B B_0 \sum_i (s - a_i^\dagger a_i). \end{aligned} \quad (6)$$

Defining constants $t = -2\mathcal{J}s$, $\varepsilon = g\mu_B B_0 + 4\mathcal{J}s$, and keeping only terms with magnon operators, this can be rewritten as

$$\mathcal{H} = t \sum_{\langle ij \rangle} (a_i^\dagger a_j + a_i a_j^\dagger) + \varepsilon \sum_i a_i^\dagger a_i. \quad (7)$$

This is the generic form of a tight-binding (TB) model for a 1D atomic chain, except that operators refer to magnons in the HP formalism, which suggests that TB models with appropriate parameters can generically describe exchange-only systems with ferromagnetic ground states. The interpretation is the same when these operators act on macrospins in nanostructures [65]. With this TB picture in mind, it is not a large stretch to expect that nonuniform spin-wave modes may also be represented by operators in this model, as long as the hopping parameters t_{ij} and energies ε_i are properly adjusted. For example, an array of exchange-coupled elements may be described by an effective TB model in which $a_i^\dagger$ creates a magnon in a nonuniform mode *i*, mainly confined to a single magnetic element [66]. To be more precise, the nonuniform mode operators can be thought of as superpositions of magnon operators on many macrospins, with new hopping parameters that are simple to calculate from old parameters. In this picture, the Hamiltonian becomes

$$\mathcal{H} = \sum_{ij} t_{ij} (a_i^\dagger a_j + a_i a_j^\dagger) + \sum_i \varepsilon_i a_i^\dagger a_i, \quad (8)$$

in which *i*, *j* may now refer to different types of nonuniform modes.

Motivated by the observation that the Bloch functions [see Fig. 4(b)] are made of *s* and *p* orbitals, we choose this basis for a TB model. Using only nearest-neighbor hoppings, we implement this model using the PYHTB package [67] and adjust parameters by hand. Fit parameters are listed in Table I in units of frequency. Figure 5(a) is the same band structure appearing earlier, and Fig. 5(b) is its corresponding model, displayed in a manner similar to the simulations, using basis orbitals shown in Figs. 5(c) and 5(d). More precisely, we plot the spectrum of a supercell

TABLE I. Tight-binding parameters and their values as used in Fig. 5. ε_i are “on-site” frequencies for orbital *i* and t_{ij} are hopping parameters between orbitals *i* and *j*. Subscripts *s* and *p* denote *s*-like and *p*_{*x,y*}-like modes on the honeycomb lattice. Subscripts *k* denote *s*-like modes on the kagome lattice.

ε_s	1.69	GHz
ε_p	3.15	GHz
ε_k	3.17	GHz
t_{sk}	−0.38	GHz
t_{pk}	−0.63	GHz

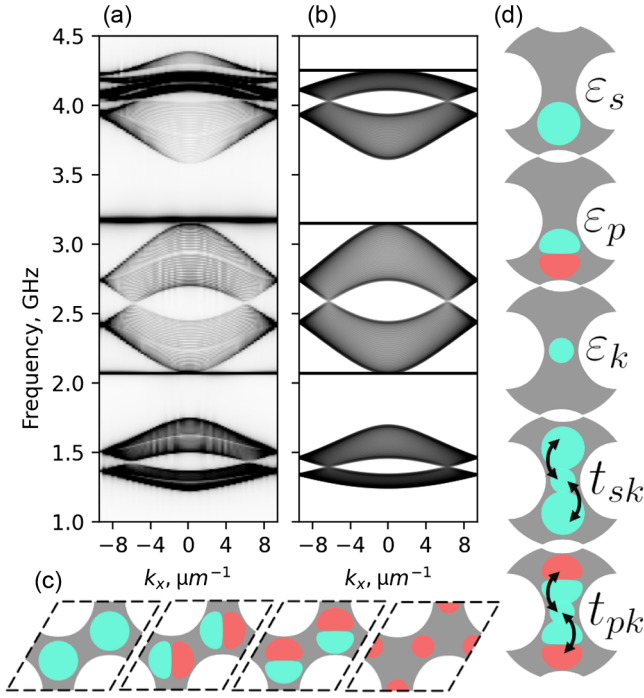


FIG. 5. (a) Simulated k_x -projected band structure as plotted in Fig. 2(c), and (b) its nine-band tight-binding model using fitting parameters listed in Table I. (c) Basis orbitals for the TB model (left to right): s , p_x , p_y orbitals on honeycomb sites and s orbitals on kagome sites. (d) Illustration of fitting parameters.

extended in the $\hat{y}$ direction with semitransparent coloring and plot only the k_x extent of the rectangular BZ.

It is remarkable that a TB model works well in an antidot lattice. The orbitals emerge naturally from the structure without any careful engineering. The fact that a simple model works suggests that this is a general feature of the geometry that has nothing to do with spin waves. Indeed, simply solving the Schrödinger equation for a massive particle in a potential resembling this antidot lattice can yield a similar band structure (see Supplemental Material Sec. IV [38]). This phenomenon can be understood from Eq. (3). In fact, as a representation of the simplicity of excitations in this system, similar band structures have been discussed and observed in electronic, photon-polariton, acoustic, and cold-atom systems [68–73].

Although the model appears to work, there are some inconsistencies. At the Γ point of the lowest band, the dispersion in the simulation is linear due to the dipole-dipole interaction [50], which is explicitly ignored by the TB model. Small midband gaps are also observable in the simulations, but this is only an artifact of the imperfect reconstruction of the hexagonal lattice using pixels, which breaks its C_6 symmetry down to C_2 . Such gaps are not expected for real lattices. In addition, the model is less accurate at high frequencies because the real wavelength of spin waves becomes small compared to the lattice features. For larger hole sizes (smaller lattice features), these high-frequency bands approach a more TB-like form

(see Supplemental Material Sec. II [38]). Other subtle inconsistencies exist, in part due to the TB model's exclusion of the dipole-dipole interaction.

The fact that TB works well is curious but does not teach us anything fundamentally new. However, understanding this band structure in terms of graphenelike s and $p_{x,y}$ excitations enables engineering using existing principles from the field of 2D materials.

V. SPIN-WAVE ENGINEERING

A. Inversion-broken crystals

In graphene, the masslessness of Dirac electrons is protected in part by inversion symmetry. Hexagonal boron nitride (h -BN), another 2D compound, is isostructural but with different atoms on each honeycomb sublattice. The presence of two different atoms in the hexagonal unit cell breaks the inversion symmetry of the unit cell and the Dirac points gap (for this reason, these symmetry-breaking terms are sometimes called “mass terms”). As a result, h -BN is an insulator, which is shown schematically in Fig. 6(b). Even stacking graphene on h -BN has this result because of the small amount of symmetry breaking [74,75]. This principle is powerful because, unless the system is fine-tuned, any method of breaking inversion symmetry should result in a gap.

To mimic the relationship between graphene and h -BN, we create new unit cells that break inversion symmetry and perform simulations again. Figure 6(a) displays the resulting band structures in the same manner as before.

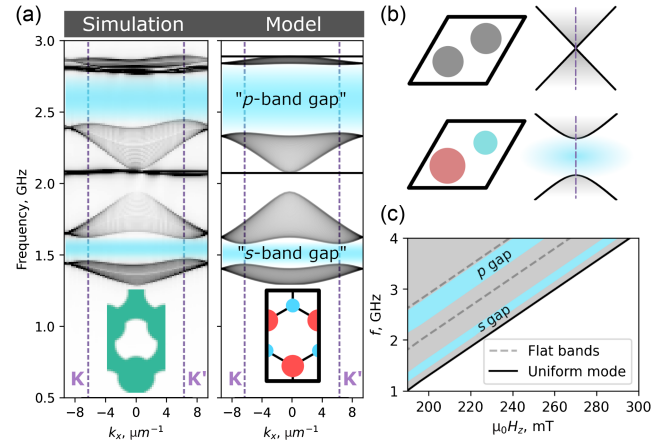


FIG. 6. The effect of inversion symmetry breaking on the s and p bands. (a) Band structure for a unit cell with broken symmetry, plotted in the same manner as before. The spectrum can still be modeled with TB. Symmetry breaking enters the model as terms that discriminate between the honeycomb sublattices $\Delta\epsilon_s$ and $\Delta\epsilon_p$, resulting in band gaps for both the s and p bands. (b) Analogy to graphene and its symmetry-broken sister h -BN: a gap (highlighted in blue) opens at $\mathbf{K}$ and $\mathbf{K}'$ points. Fit parameters are listed in Table II. (c) The dependence of induced gaps and flat band frequencies on externally applied field, extracted from simulations. This tunability is one of the unique benefits of magnonic systems.

TABLE II. Tight-binding parameters and their values as used in Fig. 6, following the same labeling as in Table I.

ϵ_s	1.93	GHz
ϵ_p	2.83	GHz
ϵ_k	3.28	GHz
$\Delta\epsilon_s$	0.12	GHz
$\Delta\epsilon_p$	0.35	GHz
t_{sk}	-0.45	GHz
t_{pk}	-0.49	GHz

As predicted, a gap is present in the inversion-broken geometries. In analogy to *h*-BN, the symmetry breaking can be described in TB by a difference $\Delta\epsilon \equiv \frac{1}{2}(\epsilon_B - \epsilon_A)$ in the energies of the two sublattices. For simplicity, this is the only addition made to the model. Controllable spin-wave band gaps are desirable for rf applications; it is interesting to note that this is an example of gaps whose width is tunable using guidance from the analogy to 2D materials. Furthermore, the frequency of these gaps (along with all other features) can be tuned by an externally applied magnetic field as shown in Fig. 6(c). This tunability is natural to magnonic crystals and, from a functionality perspective, is a benefit compared to photonic and phononic crystals.

In graphenelike systems, these symmetry-breaking mass terms do more than simply open a gap. For small-mass terms, opening such a gap generates sharply peaked Berry curvature near the valleys $\mathbf{K}$ and $\mathbf{K}'$. Integrating the Berry curvature over the vicinity of a single valley yields an approximately quantized value, often referred to as a valley Chern number (VCN) [76]. The two valleys carry opposite VCNs, so the total Chern number over the full Brillouin zone is zero. Nevertheless, a change in VCN across a domain wall implies valley-polarized boundary modes [77,78], which remain robust as long as intervalley scattering remains weak [76]. TB calculations in this *s, p_{x,y}* system show that symmetry breaking has this effect for *both* the *s*- and the *p*-band gaps. At a single valley, the sign of the Berry curvature is opposite above and below the gap for both *s* and *p* bands, but notably has a different sign in each (*s, p*) case. Upon changing the sign of the symmetry-breaking terms $\Delta\epsilon_s$ and $\Delta\epsilon_p$, the sign of curvature flips (see Supplemental Material Sec. V [38]), schematically illustrated in Fig. 7(a).

This magnetic thin film geometry, unlike van der Waals systems, is continuously tunable between distinct gapped phases. Partially inspired by the Jackiw-Rebbi model [79], we investigate a boundary between these gapped phases. Figure 7(b) shows a gradual phase boundary in which the unit cell changes shape over a few lattice spacings, making a smooth transition between the two phases. In the corresponding TB model, we create a long supercell with varying mass terms [Fig. 7(c)]. The band structure of the

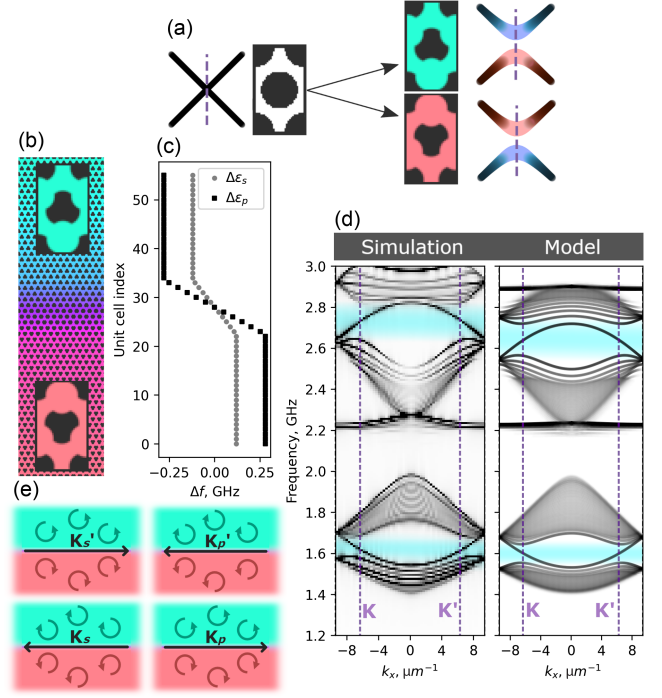


FIG. 7. A boundary between different gapped phases. (a) Two distinct gapped phases with different Berry curvature at a given valley. (b) The simulated geometry, including a boundary between the phases. (c) The corresponding TB setup, in which the symmetry-breaking terms change sign along the $\hat{y}$ direction. (d) Simulated and modeled band structures. In the simulation, the excitation is at the boundary. To mimic this, the TB spectrum is plotted with opacity given by amplitude at the boundary. Magnon states bridging the gaps (highlighted in blue) indicate two instances of quantum valley Hall insulator behavior. However, the states bridging the two gaps have opposite propagation: (e) *s* modes (in the lower gap) have $\mathbf{K}'$ -polarized waves moving to the right, while *p* modes (in the upper gap) have $\mathbf{K}$ -polarized waves moving to the left.

TB model contains the essential features of the simulation, most notably the presence of states that bridge the gap at each valley, highlighted in Fig. 7(d). These will be referred to as boundary modes.

In our TB model, as we increase mass terms, Berry curvature is no longer localized to the valleys and the VCN loses its quantization (Supplemental Material Sec. V [38]). This behavior is common in realistic systems [80–82], where the absolute value of the VCN is smaller than its quantized value in the small-gap limit and varies smoothly with system parameters. However, since the bulk gap does not close as we vary mass parameters, the boundary modes we observe are adiabatically connected to those in the small-gap limit. They remain (for example, in our simulations) visible well beyond the regime where the VCN is sharply quantized. This phenomenon of boundary-localized states has been seen before in many other graphenelike systems [83–89], but occurs here for both the *s* and *p* bands. Because the $\Delta\epsilon$ terms affect the *s* and *p* gaps in

different ways, the corresponding magnon boundary modes bridge the gaps with opposite chiralities: in the lower gap, $\mathbf{K}(\mathbf{K}')$ -valley magnons propagate to the left (right), while in the upper gap, $\mathbf{K}(\mathbf{K}')$ -valley magnons propagate to the left (right). The direction of this valley-polarized magnon current can be reversed by changing the sign of the symmetry-breaking terms (see Supplemental Material Sec. VI [38]). By calculating the VCNs of the bulk bands in the small-gap limit, we thus identify these boundary states as the magnonic analog of quantum valley Hall (QVH) edge states [76,90], as illustrated in Fig. 7(e). This is a more accessible realization of the type of state proposed for 2D honeycomb magnets with controllable staggered anisotropies [91]. For a more complete discussion including Berry curvature maps, see Supplemental Material Sec. V [38].

B. Topological magnon waveguiding

The QVH-like states that are localized to the boundary may be used as a frequency-selective waveguide for valley-polarized magnons. In the same geometry as before, shown again in Fig. 8(a), an excitation is applied inside a localized region. At the in-gap frequency of 2.7 GHz, spin waves are launched along *only* the phase boundary, as shown in Fig. 8(b). Because of the valley-specific topological origin of these modes, left- and right-moving modes are oppositely valley polarized—an interesting feature because valley polarization is occasionally thought

of as an information-carrying degree of freedom in discussions of next-generation information technologies [92]. The transition between the insulating phases can be reduced in width, and the states persist, even following sharp turns made by the phase boundary. This phenomenon occurs for both s and p boundary modes [see Figs. 8(c) and 8(d), respectively]. In both cases, magnon transport remains completely localized to the boundary after 500 ns of continuous excitation. It is worth noting a few limitations here. Geometry does not affect only the shape of the band structure; there can be a constant frequency offset as a result of the demagnetizing field, which is the same reason B_{ext} is varied for the different geometries in Fig. 2. When geometry is varied over space, this varying frequency offset can result in a spectral overlap of the localized mode with other bulk modes. In this case, a large-area excitation like that in Fig. 8(a) would excite both boundary and bulk modes. In other words, the appealing boundary-specific transport demonstrated by Fig. 8 may not exist in all geometries with phase boundaries. Also note that these modes are not protected against backscattering, making them fundamentally different from more famous examples of topological edge states in metamaterials, in Refs. [93,94] for example. Scattering between valley modes caused by a point defect is demonstrated in Figs. 8(e)–8(g) for both s - and p -gap modes, returning differing reflection amplitudes for each case. This is discussed in more detail in Supplemental Material Sec. VII [38].

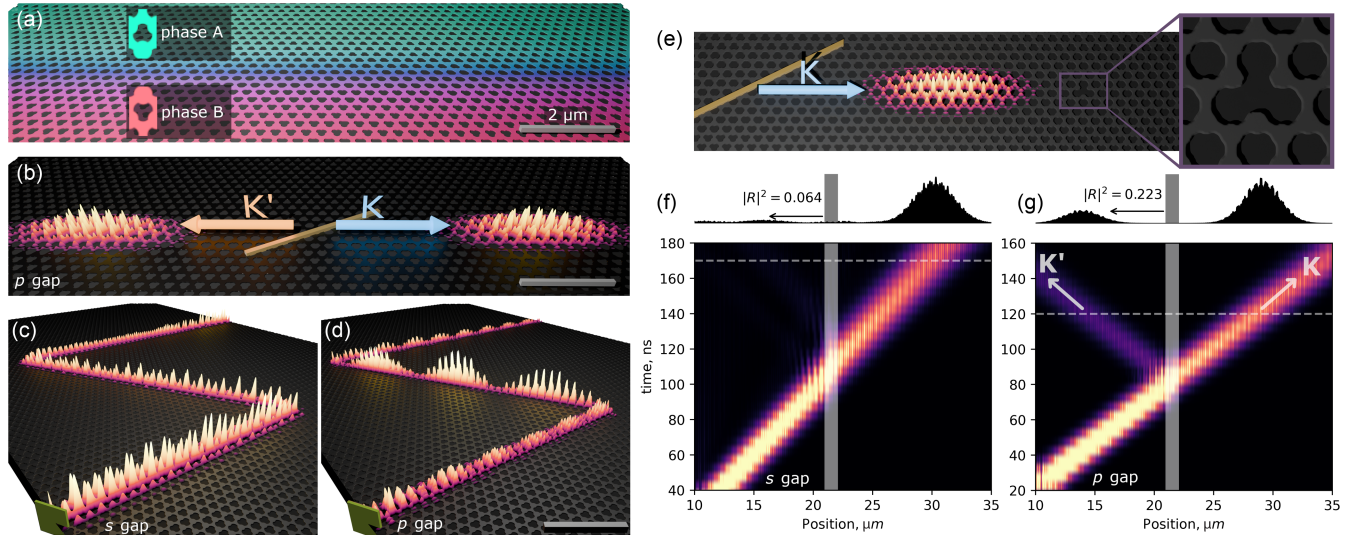


FIG. 8. Demonstration of spin-wave transport along phase boundaries. (a) Simulation geometry, the same as that in Fig. 7. (b) Spin-wave profile 30 ns after the peak of a 2.7-GHz pulse (inside the p -band gap). Left- and right-moving modes are valley polarized and move only along the boundary. The excitation (yellow) is tilted to match the phase profile of the spin waves and allow efficient coupling. The boundary can be reduced in width and still allow directed transport, even along sharp turns. Profiles are plotted after 500 ns of continuous excitation at (c) 1.55 GHz in the s -band gap and (d) 2.55 GHz in the p -band gap. Unlike the edge states of Chern insulators, because there exist colocalized left- and right-moving edge states, intervalley scattering is allowed. To demonstrate this, we show (e) Gaussian wave packets scattering from a point defect formed by removing a honeycomb site. Space-time plots of the spin-wave profiles are shown for (f) the s -gap mode and (g) the p -gap mode with the defect position marked in gray. Profiles are examined after the collision and give calculated reflections of approximately 6% and 22%, respectively.

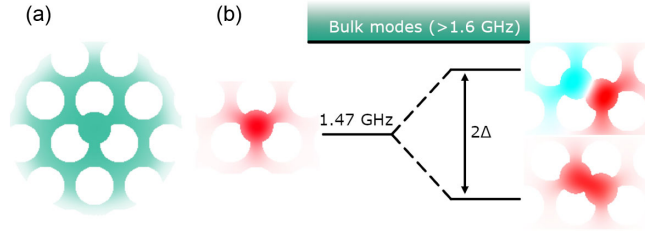


FIG. 9. Level structure of point-defect modes under $B_{\text{ext}} = 200 \text{ mT}\hat{z}$. (a) Simulation geometry: A single defect resembling that used in the inversion-broken structures of Sec. V A. (b) The localized modes of a single defect versus two adjacent defects. When coupled, they split by $2\Delta \approx 0.18 \text{ GHz}$, producing a two-level magnon spectrum that appears entirely below the continuum. These modes are plotted in the same fashion as the Bloch-like functions of Fig. 4.

C. Isolated defect modes

The unique power of patterned crystals lies in the ability to control its properties in a spatially varying way. In the previous section, this ability was used to exploit valley band topology and waveguide magnons along a one-dimensional boundary. It is also worth discussing magnons that are confined to zero-dimensional point defects. Here we demonstrate a simple case of this: Only one honeycomb site is replaced with a disk similar to that in the inversion-broken unit cells [see Fig. 9(a)], analogous to a substitutional defect in graphene (nitrogen or boron, for example). In a phenomenon similar to spin-wave “edge modes,” [95,96], a single defect has a localized mode that lies *below* the bulk uniform precession mode. Two such defect modes couple when placed in adjacent sites. Using the TB interpretation, the level should split into symmetric and antisymmetric superpositions of the original modes, separated in frequency by twice the coupling parameter Δ . This is apparently a good interpretation, as evidenced by the mode profiles in Fig. 9(b). A single-defect mode at 1.47 GHz splits into a symmetric mode at 1.38 GHz and an antisymmetric mode at 1.56 GHz (for details, see Supplemental Material Sec. VIII [38]). It happens that this whole level structure appears below the continuum, which implies that these defect-localized magnons may be interacted with separately from propagating waves, suggesting further application in magnonics. For the chosen Gilbert damping parameter of $\alpha = 10^{-4}$, these modes have quality factors of approximately 4100.

VI. OTHER TB-LIKE SYSTEMS

In the previous sections, it was demonstrated that a TB-like understanding can be used to engineer magnonic band structures in analogy to a few popular 2D materials. It is worthwhile to show that this TB approach is applicable in at least a few other cases. Figure 10 shows two such examples: a square antidot lattice and an artificial kagome lattice. The magnonic structure of the perpendicularly

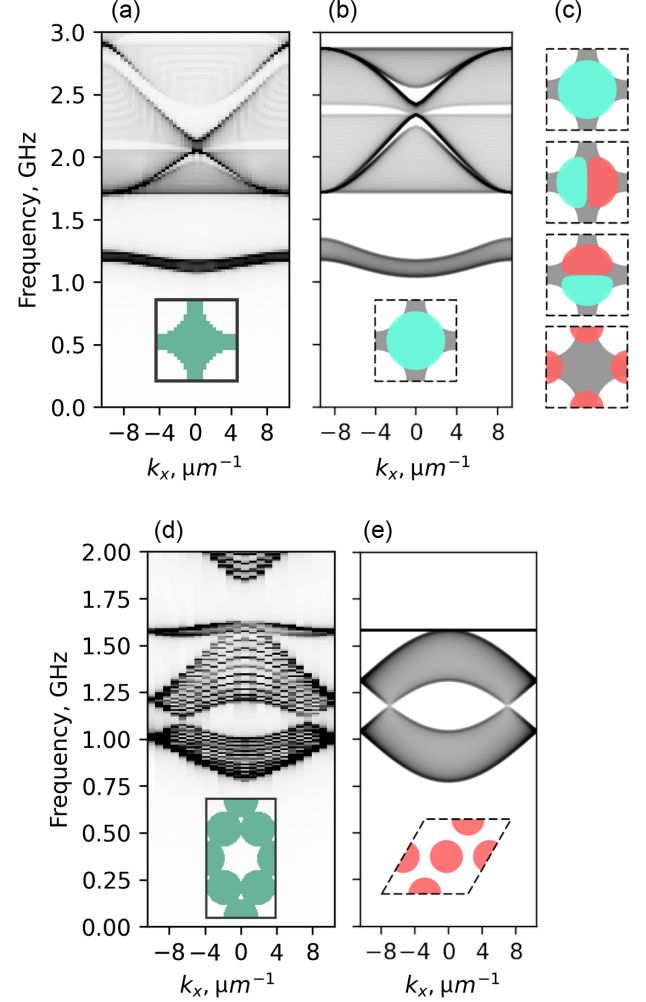


FIG. 10. Other geometries that can be thought of as TB-like, using 15-nm YIG films with a lattice spacing $a = 300 \text{ nm}$ and $B_{\text{ext}}||\hat{z} = 185 \text{ mT}$. (a) The square antidot lattice and (b) its TB model, using (c) basis orbitals similar to those used in the hexagonal antidot model. Some inconsistencies are due to hybridization with higher-order modes like d orbitals (see Supplemental Material Sec. IX [38]), which are not included for simplicity. (d) The artificial kagome lattice and (e) its TB model using only s orbitals on kagome sites.

magnetized square antidot lattice [see Fig. 10(a)] happens to be analyzable using the same approach. A TB model with s and p orbitals in the middle of the unit cell and s orbitals between these sites can approximate the band structure with the right fitting parameters. Its spectrum is plotted in Fig. 10(b), using the basis orbitals in Fig. 10(c). This square lattice is apparently another case of emergent “atomic orbitals” (see Supplemental Material Sec. I [38] for details). There is a simpler (albeit less experimentally feasible) approach to engineer magnonic band structures. Figure 10 also shows an artificial kagome lattice, where it is assumed that disks will host s -like modes, so the disks are arranged to mimic the kagome lattice [see Fig. 10(d)]. In this case, a simple model of s orbitals on the kagome lattice

can approximate the band structure [see Fig. 10(e)]. This is similar to results previously reported for magnetic rods embedded in a magnetic matrix [42,97]. In this case, the relative decrease in momentum-space resolution is due to the fact that the unit cell must be simulated with greater resolution, so the whole simulation field has fewer unit cells for computationally practical reasons.

VII. EXPERIMENTAL FEASIBILITY

The YIG antidot lattices studied in this work may be fabricated by standard electron beam lithography procedures including a hard mask and ion etch step or by focused ion beam (FIB) milling for the fabrication of single devices. There is already a significant amount of work on these structures [43–47], albeit with somewhat different dimensions. Preliminary work indicates that FIB can yield devices with sufficient scale and tolerances. In Supplemental Material Sec. X [38], we verify that lattices with a thin magnetic dead layer caused by ion damage are expected to yield similar physics. We also verify that, in the presence of edge roughness, flat modes, band gaps, defect-localized modes, and QVH-like modes persist with only moderate modification (Sec. XI in Ref. [38]). All simulations are conducted for a Gilbert damping parameter $\alpha = 10^{-4}$ and $M_S = 140$ kA/m, which is realistic for YIG [98,99]. Overall, the antidot lattice is a more realistic experimental route to these types of magnonic flat bands compared to existing proposals, which mainly focus on ferromagnetic rods embedded in a matrix [41,42,97].

VIII. DISCUSSION AND CONCLUSION

Because the magnonic crystals discussed here demonstrate unique properties, it is worth discussing their potential applications.

First, the existence of isotropic flat bands of Fig. 3 is notable. In the TB picture, it is trivially easy to achieve flat bands in one sense: An array of decoupled resonators has no dispersion. However, here, the modes are strongly coupled and flatness emerges from the symmetry of the basis orbitals and the geometry of the lattice. Bloch functions in this band contain singularities at band touching points, a property that sometimes awards it the title of “topological flat band” in electronics [59,100]. Furthermore, these states may be of interest for exploring magnon-magnon interactions, particularly as analogs of interacting electronic phases on the kagome lattice. The localization property of these magnons indicates that they could easily be excited to large magnon numbers, potentially resulting in emergent modes that already have some theoretical description in the hot research area of interacting kagome flat bands. The excitation enhancement due to flat band magnons has been studied before, for example, in Ref. [101].

Second, the topological magnons of Sec. VB are, of course, attractive for precise control of magnon propagation. However, they are additionally interesting because we can exercise control over the valley degree of freedom. For example, in the setup of Fig. 8(a), any spin-wave device to the right of the antenna should only ever receive **K**-polarized spin waves at the frequency of 2.7 GHz. This work may prove an important step in realistic magnon valleytronics.

Third, additional control over the valley degree of freedom could reasonably be exercised here through an inhomogeneous pattern that mimics a strain field. Inhomogeneous strain fields can result in an effective gauge field for Dirac point excitations, commonly called a pseudomagnetic field [102,103]. The freedom associated with patterning in this system might be used to realize magnon valley filters, lenses, collimators, and (pseudo-) Landau levels for magnons [104–106].

Fourth, point defects such as those in Fig. 9 may be useful in magnonics because of their spatial localization and spectral isolation. This implies that a carefully placed waveguide could selectively excite these quasiuniform localized modes—making them potentially useful as a magnonic memory because spin waves will not radiate away from the defect. Additionally, the fact that a coupled two-mode structure of such defects still lies below the continuum is interesting. We envision classical spin-wave analogs of effects like induced transparency and Rabi oscillation. If sufficiently long magnon lifetimes can be achieved, possibly aided by the localization of the modes [107], then this may also be an interesting perspective for designing quantum magnonic systems.

Fifth, these findings are generalizable in a few ways. Section VI demonstrates that tight binding is somewhat accurate in a few other cases of perpendicularly magnetized YIG films, implying that the 2D-analog engineering done in the previous sections can be extended to other magnon systems. However, in Sec. IV, we mention that both tight binding and the Schrödinger equation can reproduce band structures similar to the ones observed here—implying that some of these engineering strategies are applicable not only to magnonic systems, but to any system that can reasonably be described by a simple Schrödinger equation.

Finally, because there exists similar physics in other analog systems [68–73], it is worth noting a few novelties and benefits associated with this magnonic system. The literature tends to focus on *s*-like modes, but we show that it is also possible to accomplish valley-specific physics with *p*-like modes, and we show that it can be understood through the same TB lens. This amplifies the potential for valleytronics because there are two channels through which valley information may propagate. There are also benefits that are unique to magnonic systems. Most importantly, the whole band structure is tunable not just by geometry, but also by externally applied field, as was demonstrated in

Fig. 6(c). This suggests that a constant microwave signal may be tuned on the fly to be blocked by band gaps, waveguided, valley or orbitally polarized, strongly localized in flat bands, interacting with defect modes, etc. simply by varying the external field. Spin waves also are associated with a unique combination of small wavelengths and low frequencies; it is a significant result that a microwave signal can be localized to these phase boundaries by some band topology manipulation.

In conclusion, patterning a hexagonal array of holes into perpendicularly magnetized YIG thin films is found to lead to some unexpected features. Namely, the band structure mimics a few unique properties of 2D systems and can be understood using a tight-binding model. This new understanding allows the engineering of magnonic states using existing principles from 2D materials to produce controllable band gaps, topological magnons along 1D channels, and spectrally isolated magnons at 0D point defects. These design principles seem to be generalizable to other geometries and even other excitations, but here represent an important step in achieving and manipulating physical states that are usually reserved for van der Waals systems.

ACKNOWLEDGMENTS

We thank Taylor Hughes for stimulating and fruitful discussions. All aspects of this work were mainly supported by the National Science Foundation through the Illinois Materials Research Science and Engineering Center (MRSEC) under Grant No. DMR-2309037. Additional contributions to data analysis and manuscript preparation were supported by the U.S. DOE, Office of Science, Basic Energy Sciences, Materials Science and Engineering Division under Contract No. DE-SC0022060. Other contributions to analysis were supported by the U.S. Office of Naval Research (ONR) Multidisciplinary University Research Initiative (MURI) Grant No. N00014-20-1-2325 on Robust Photonic Materials with High-Order Topological Protection.

DATA AVAILABILITY

The data that support the findings of this article are openly available [108].

APPENDIX: SCHRÖDINGER-LIKE FORM OF THE LLG EQUATION

Beginning with the Landau-Lifshitz-Gilbert equation,

$$\partial_t \mathbf{M} = -\gamma \mu_0 \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_S} (\mathbf{M} \times \partial_t \mathbf{M})$$

for magnetization $\mathbf{M}$ of magnitude M_S , gyromagnetic ratio γ , and permeability of free space μ_0 . Neglecting damping and rewriting in terms of the reduced magnetization vector $\mathbf{m} \equiv \mathbf{M}/M_S$,

$$\partial_t \mathbf{m} = -\gamma \mu_0 \mathbf{m} \times \mathbf{H}_{\text{eff}}.$$

Assuming $\mathbf{H}_{\text{eff}} \parallel \hat{z} \equiv H$, $\mathbf{m} \approx \hat{z}$, the linear limit,

$$\partial_t m_x = -\gamma \mu_0 H m_y,$$

$$\partial_t m_y = \gamma \mu_0 H m_x,$$

$$\partial_t (m_x + i m_y) = \gamma \mu_0 H (i m_x - m_y),$$

$$\text{defining: } \Psi \equiv m_x + i m_y,$$

$$\partial_t \Psi = i \gamma \mu_0 H \Psi.$$

Because we have introduced no $\mathbf{H}_{\text{eff}}$ term that couples spins over space, this should also be true for a set of macrospins organized in space,

$$\partial_t \Psi(\mathbf{r}) = i \gamma \mu_0 H(\mathbf{r}) \Psi(\mathbf{r}).$$

Including the exchange interaction of strength D_{ex} in the effective field (and assuming its direction is $+\hat{z}$), this can be written in a form [51–53]

$$\partial_t \Psi(\mathbf{r}) = i \gamma \mu_0 (H(\mathbf{r}) - D_{\text{ex}} \nabla^2) \Psi(\mathbf{r}),$$

which resembles a Schrödinger equation for a massive particle in a potential $\gamma \mu_0 H(\mathbf{r})$. See Ref. [53] for an application of this idea to quantized spin wave modes in finite-size systems.

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