

Spin Squeezing with Itinerant Magnetic Dipoles

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Entanglement can improve the measurement precision of quantum sensors beyond the shot noise limit. Neutral atoms, the basis of some of the most precise and accurate optical clocks and interferometers, do not naturally exhibit the all-to-all interactions traditionally used to generate such entangled states. On the other hand, these systems exhibit exceedingly high degrees of experimental control over parameters such as temperature, spatial entropy, and itinerancy. In this work, we investigate spin squeezing in a highly coherent itinerant system of neutral atoms with magnetic dipole-dipole interactions. We achieve 7.1 dB of metrologically useful squeezing using finite-range spin-exchange interactions in an erbium quantum gas microscope, and we demonstrate that introducing atomic motion, realizing a dipolar t - J model, protects the spin sector coherence at low fillings, significantly improving the achievable spin squeezing in a 2D dipolar system. This work's protocol can be implemented with most neutral atoms, opening the door to quantum-enhanced metrology in other itinerant dipolar systems, such as molecules or optical lattice clocks, and serves as a novel method for studying itinerant quantum magnetism with long-range interactions.

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I. INTRODUCTION

Entangled states allow quantum sensors to overcome the standard quantum limit (SQL) in measurement precision and have been successfully applied to enhance the precision of optical atomic clocks [1,2], atom interferometers [3], and gravitational wave detectors [4]. One prominent method to generate large-scale entanglement is spin squeezing, where uncertainty is redistributed between quadratures of the collective spin of an atomic, solid-state, or molecular ensemble, improving metrological precision [5–8]. In cold atom systems, spin-squeezing generation has been extensively studied in the context of all-to-all interactions or measurements, such as in single-mode cavities [9–13] and single-mode Bose Einstein condensates [14–19]. Of particular interest to optical atomic clocks, there have also been some proposals for engineering all-to-all squeezing interactions in quantum gas microscopes with degenerate Fermi gases [20,21].

Recently, it was predicted that scalable spin squeezing can be achieved with finite-range interactions [22–24]. This

is remarkable from a fundamental perspective, as the nonequilibrium squeezing dynamics may be linked to equilibrium symmetry-broken phases with long-range order [25]. Additionally, squeezing from finite-range interactions is very attractive for metrological applications, as such interactions are inherently present in many cold atom, molecule, and solid-state systems [26–29]. Squeezing has already been realized through induced Rydberg interactions, of dipolar [30] or van der Waals [31,32] nature, and in trapped-ion platforms with tunable power-law interactions [33].

In this work, we demonstrate a new method for spin squeezing in a quantum gas microscope with a tunable dipolar t - J model. Itinerancy is enabled by a variable-depth optical lattice with nearest-neighbor hopping, and the spin-spin interaction arises from magnetic dipole-dipole interactions natively available in most atomic species. We carry out our experiment in an erbium quantum gas microscope, in which we prepare approximately 10^3 itinerant XY interacting fermions in one vertical layer of a 3D optical lattice. First, we describe novel methods for generating and imaging magnetically insensitive dipolar interacting spin states in fermionic ^{167}Er , inspired by Zou *et al.* [34]. While benchmarking the platform, we observe indications of the ferromagnetic ordering expected of the many-body ground state at high filling, predicted to lead to an extensive squeezed state [22,24,25,35]. Next, we generate and directly measure metrologically useful squeezing up to

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(7.1 ± 1.0) dB with no corrections for experimental imperfections. We then investigate the effects of itinerancy, which has recently been proposed to enhance squeezing in finite-range interacting systems [36,37]. Although finite lattice fillings impede the formation of an ordered ground state [38] and, thus, a squeezed state, tunneling was recently numerically shown to help overcome this effect [37,39]. However, it remains an open question whether this enhancement survives at finite temperature and finite tunneling, where the partial delocalization of defects may impede local ordering. Additionally, the study of the interplay between tunneling and power-law interactions has been of substantial recent interest due to advances in ultracold molecule experiments [40–42]. Remarkably, we observe that, in dilute systems where there is no metrological gain when the spins are static, the introduction of itinerancy protects the long-range order and leads to metrologically useful squeezing of (5.2 ± 0.6) dB.

II. EXPERIMENTAL PLATFORM AND SQUEEZING THEORY

We generate spin squeezing using a magnetic dipolar spin-exchange, or XY model, between two hyperfine states [Fig. 1(a)]. For sufficiently high fillings, and for sufficiently slowly decaying power-law interactions, the system chooses a global phase, creating a U(1) symmetry-broken ferromagnet [38,43]. This ferromagnet is suspected to protect the permutational symmetry of the ensemble, enabling spin squeezing [25]. For short times, the dipolar XY model has recently been shown numerically to produce

squeezing comparable to the all-to-all one-axis twisting (OAT) model [22,44].

We implement the dipolar XY model in an erbium quantum gas microscope (QGM) [45]. Although QGMs are typically designed as quantum simulators, they have many advantages when used for metrology. Most notably, QGMs can natively scale to large atom numbers, support site-resolved imaging, and can project arbitrary spatial potentials on atoms. We use tightly spaced 266-nm optical lattices in the x and y directions to maximize the dipolar interaction and trap our ^{167}Er atoms in one vertical plane of a 532-nm spacing z lattice. ^{167}Er has a rich hyperfine structure, with 104 total F, m_F levels. We perform all of our experiments in a single two-level qubit manifold: $|\downarrow\rangle = |F = 19/2, m_F = 1/2\rangle$, and $|\uparrow\rangle = |17/2, 1/2\rangle$. We choose hyperfine states to maximize the product of the qubit's coherence time with its interaction strength. Rather than choosing the fastest interacting states, we use a transition that is first-order magnetically insensitive at 10.6 G, which allows us to effectively eliminate magnetic field noise as a source of decoherence [see Appendix D and Fig. 1(c)]. Additionally, our chosen states have similar or lower interactions than less magnetic atoms, allowing for direct extension of our work to other neutral atoms (see Appendix A). Our quantum simulator realizes the effective 2D XY dipolar Hubbard model with itinerant fermions [Fig. 1(a)]. The on-site interaction between different spin species is much larger than all other scales (see Appendix J); therefore, we truncate the Hamiltonian to one particle per site. The effective Hamiltonian is then

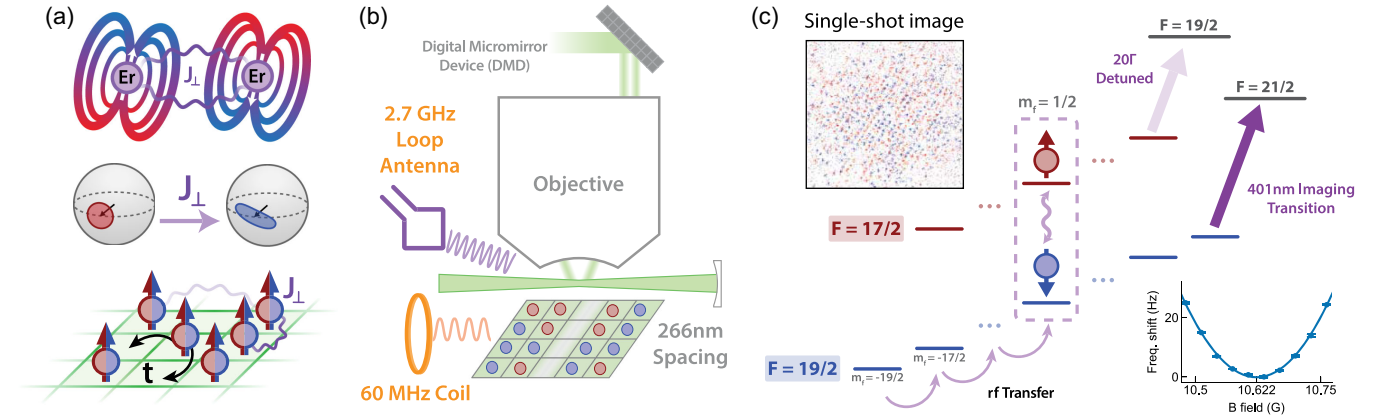


FIG. 1. Spin squeezing using magnetic dipole interactions between erbium atoms in a quantum gas microscope. (a) Our experiment utilizes the intrinsic magnetic dipole moment of ^{167}Er to generate entanglement. The interaction shears an initial coherent state, squeezing it along one axis. We load a 2D layer of atoms in our optical lattice, where the finite-range spin-exchange interaction $J_{\perp}$ is used to generate entanglement. Optionally, the atoms can tunnel to adjacent sites with tunable rate t . (b) Atoms in a 266-nm spacing lattice are loaded into tunable shapes by a digital micromirror device (DMD). The atom's spin states are manipulated with a 2.7-GHz loop antenna and an in-vacuum coil at 60.8 MHz. (c) The atoms are prepared in the $m_F = -19/2$ state and are transferred to $m_F = 1/2$ (the qubit manifold) through successive Landau-Zener sweeps. The magnetic field is set such that the qubit frequency is first-order insensitive to magnetic field fluctuations, as shown in the bottom right. We selectively image the two spin manifolds by a differential 20Γ detuning of the imaging transition. The two spin states are shown as red and blue dots in a single-shot image on the left. Error bars are SEM.

$$H = -t \sum_{\langle i,j \rangle, \sigma} (\hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + \text{H.c.}) + J_\perp \sum_{i \neq j} \frac{a^3}{r_{i,j}^3} \hat{S}_i^+ \hat{S}_j^- \quad (1)$$

Here, t describes the tunneling of hard-core fermions along nearest-neighbor bonds $\langle i, j \rangle$ and $J_\perp$ the characteristic dipolar exchange energy between adjacent sites, with a fixed value of 1.09 Hz throughout this work. $r_{i,j}$ is the distance between sites i and j , and $a = 266$ nm is the characteristic lattice spacing. Finally, $\hat{c}_{i,\sigma}^\dagger$ ($\hat{c}_{i,\sigma}$) are the fermionic creation (annihilation) operators with spin index σ , and $\hat{S}_i^+ = \hat{c}_{i,\uparrow}^\dagger \hat{c}_{i,\downarrow}$ ($\hat{S}_i^- = \hat{S}_i^{+\dagger}$) the spin-raising (-lowering) operators. The quantization axis is kept perpendicular to the plane, which ensures isotropic dipolar interactions. The states we choose have minimal differential Stark shifts due to lattice light at our chosen polarization, leading to millihertz-level disorder in the spin sector (see Appendix C). We also keep t either much larger than the site-to-site disorder of 3 Hz rms or set tunneling much smaller than any other energy scale (< 0.3 Hz), allowing us to ignore disorder in the kinetic sector.

Each experiment begins by adiabatically loading a spin-polarized band insulator from a degenerate Fermi gas of the stretched hyperfine state $|g\rangle = |19/2, -19/2\rangle$. The insulator is shaped into various spatial configurations by tuning the local chemical potential with a digital micromirror device (DMD) during the loading [Fig. 1(b)]. We then perform a series of Landau-Zener sweeps to transfer the spin state from $|g\rangle$ to $|\downarrow\rangle$, using an in-vacuum coil at 60.8 MHz [Fig. 1(c) and Appendixes B and I]. Next, we transfer the atoms from $|\downarrow\rangle$ to $|\uparrow\rangle$ using 2.7-GHz microwave pulses. We ensure spin purity by removing atoms in all states in the $F = 19/2$ manifold with a 583-nm resonant beam, retaining only the atoms transferred to $|\uparrow\rangle$. At the

end of this process, we are left with peak fillings of about 90% and atom numbers near 1000, dominated by initial loading [Fig. 1(c)].

The atoms are imaged with single-site resolution by transferring them to an accordion lattice [46]. We image both spin states with sequential images, transferring states between images, as imaging light is mostly resonant with the $|\downarrow\rangle$ state [Fig. 1(c)]. The fidelity is further enhanced by shelving atoms in states that couple primarily to opposite polarizations of light. We measure a 99.5% fidelity to distinguish between 0 and 1 atom and a loss of the second state due to off-resonant scattering from the first image of less than 0.25% (see Appendix B).

III. SPIN CONTROL AND MEAN-FIELD INTERACTION

The loading procedure initializes a high-filling array of spin-polarized, static fermions in a short spacing lattice. We then use the magnetic dipolar interaction between states in the atomic hyperfine manifold to realize squeezing dynamics. Such interactions were previously observed as a mean-field shift in ^{87}Rb [34]. To study the interaction and show that it can generate coherent many-body dynamics, we prepare the atoms in an initial coherent superposition in the x, y plane with $\langle \hat{S}_z \rangle = \cos \theta$, $\langle \hat{S}_x \rangle = \sin \theta$, for $\theta \in [0, \pi]$. We observe the shearing effect of the effective OAT Hamiltonian by measuring $\langle \hat{S}_y \rangle$ to track the precession of the mean spin vector. This is done by exchanging $\hat{S}_y$ and $\hat{S}_z$ with a $\pi/2$ pulse before readout. Figure 2(a) shows the onset of the interaction at short time and at a time far beyond the optimal squeezing time to characterize the coherence of the state evolution. We compare our results with simulations implemented with a parameter-free

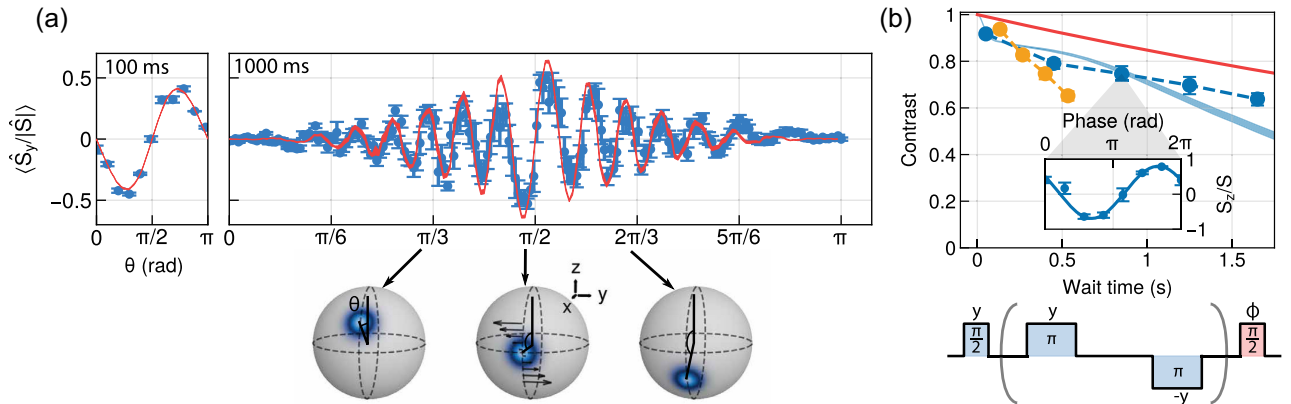


FIG. 2. Characterization of the spin-spin interaction. (a) We observe the XY interaction by the shearing it induces in the generalized Bloch sphere of 600 (1100) atoms at 100 ms (1 s). Each blue point corresponds to a different initial state, parametrized by θ , as shown in the Bloch spheres below the data. Error bars are SEM. (b) Ramsey contrast decay over time in a cloud of 1000 atoms with peak fillings of approximately 85% (blue), 85 atoms with center filling approximately 20% (orange), and the fitted interaction-free T_2 coherence (red). The orange and blue data, taken without decoupling the interaction, lie below the red curve, showing that our experimental coherence is interaction limited. Each point is the fitted amplitude of a Ramsey fringe (see the inset), and the shaded curve is DTWA. Error bars are 16th and 84th percentiles obtained from bootstrapping the data.

implementation of the DTWA. DTWA is a semiclassical method for capturing the many-body dynamics of ensembles of spins. It involves sampling spin states in a discrete phase space from a distribution that captures quantum-mechanical fluctuations, followed by evolution under a classical Hamiltonian and averaging over the resulting trajectories (see Appendix H). Using the theoretically predicted exchange rate of 1.09 Hz, we observe agreement between our experimental results and DTWA at all times sampled in this work.

A prerequisite for useful spin squeezing is a high degree of coherence or, equivalently, a ferromagnetic ordering of the constituent spins. In particular, with a dipolar XY interaction, squeezing is achievable with states prepared near the ferromagnetic ground state [25,30]. Thus, by probing the dynamics of its coherence, we validate that our system can spin squeeze via an XY model. For no interaction, noise in the classical control field will limit the lifetime of a spin-polarized state, representing the upper bound of our coherence. We implement an interaction decoupling Waugh-Huber-Haberlen (WAHUHA) + spin-echo pulse sequence [47] and report a T_2 of (6.0 ± 0.2) s [see Appendix F, Fig. 8(a)]. Next, we exclusively decouple frequency fluctuations using spin-echo pulses and compute the Ramsey contrast $C \equiv [|\langle \hat{S} \rangle|/(N/2)]$, which depends on both the presence of the interaction and density [Fig. 2(b)]. We expect that, for stochastic and dilute fillings of the optical lattice, the interaction inhomogeneity will cause the coherence to rapidly decay. At higher densities, the interaction is sufficiently homogeneous to force the atoms to all precess together, resulting in long-lived coherence [38]. For a dataset with a lower atom number ($N = 85$) and lower filling [approximately 20%, orange data points in Fig. 2(b)], we observe rapid decay out to 0.5 s, about one interaction time. For a cloud of $N = 1000$ atoms (blue data points), two regimes are present in the decay of C : First, for times less than 0.2 s, a rapid relaxation that we attribute to local equilibration from the interaction, dominated by finite filling and edge effects; and second, a plateau in the coherence, where the system maintains ferromagnetic order over timescales of $1/J_\perp$, with the contrast decaying principally due to the generation of antisqueezing as predicted with DTWA. The rapid relaxation comes from the proliferation of finite momentum spin waves during equilibration. However, recent work has shown that these decouple from the dynamics of the zero momentum spin waves that we expect to generate squeezing and preserve coherence [48].

IV. SPIN SQUEEZING

The observed dipolar shearing spin dynamics together with the preservation of coherence under the dipolar interaction create an excellent starting point for metrological spin squeezing. We now directly measure the squeezing in our system by sampling the distribution of the collective

spin of the atoms from QGM snapshots. A set of N uncorrelated spins $\hat{S} = \sum_{i=1}^N \hat{s}_i$ when sampled along an axis $\mathbf{n}_\perp$ perpendicular to the mean spin $\langle \hat{S} \rangle$ will have fluctuations due to quantum projection noise. The degree of squeezing is typically quantified with the squeezing parameter ξ^2 , which we refer to as noise squeezing:

$$\xi^2 \equiv \frac{4 \min_{\mathbf{n}_\perp} \text{Var}[\hat{S} \cdot \mathbf{n}_\perp]}{N}, \quad (2)$$

where the minimum is over all axes perpendicular to the mean spin direction [5]. A value of $\xi^2 < 1$ indicates that fluctuations are squeezed below the SQL, as shown in the inset in Fig. 3(a). While $\xi^2 < 1$ implies entanglement for spin-polarized states, the phase precision of an atomic sensor is improved by squeezing only if the reduction in the spin fluctuations is greater than the reduction in the contrast C . Thus, metrologically relevant squeezing is typically quantified through the Wineland squeezing parameter $\xi_R^2 \equiv \xi^2/C^2$ [7].

To generate and measure a squeezed state, we begin by preparing a coherent state on the equator of the generalized Bloch sphere with a $\pi/2$ pulse. After some evolution time τ , we apply a readout pulse of angular length θ and phase 90° offset from the initialization pulse. During evolution, spin-echo pulses are applied every 66 ms to decouple phase errors from our microwave drive and from lattice Stark shifts. At the end of the sequence, we measure in the z basis via population number measurements, which effectively reads out $\langle \cos(\theta) \hat{S}_z + \sin(\theta) \hat{S}_y \rangle$, the variance of which is used to estimate ξ^2 . This sequence is pictured in Fig. 3(a).

To account for drifts in the final pulse phase that lead to nonzero shot-dependent $\langle \hat{S}_z \rangle$ and other sources of classical noise, we measure a differential observable of two spatially separated clouds subjected to the same drive fields as depicted in Fig. 3(b). The clouds remain slightly coupled, leading to the optimal squeezing angle for the differential observable to decrease over time, even past zero (see Appendix G).

To extract the Wineland parameter, we measure the noise squeezing ξ^2 at four values of the evolution time τ and 21 values of the readout angle θ in two clouds with 260 atoms each and peak fillings of 80% in Fig. 3(c), and we measure the Ramsey contrast C at the same time points, in Fig. 3(d). Combining both measurements at $\tau = 0.27$ s, we plot ξ_R^2 as a function of θ and extract its minimum by fitting the data to a sin function with an offset in Fig. 3(e). In this manner, we measure a minimum Wineland squeezing of $\xi_R^2 = (-6.4 \pm 1.0)$ dB and fit a minimum of $\xi_R^2 = (-7.1 \pm 1.0)$ dB, in good agreement with our parameter-free DTWA theory, represented by the shaded curve on the same plot. In another experiment with two clouds of $N \approx 500$ each, we measure roughly $\xi_R^2 = -4$ dB. This decrease in squeezing is explained by microwave control errors

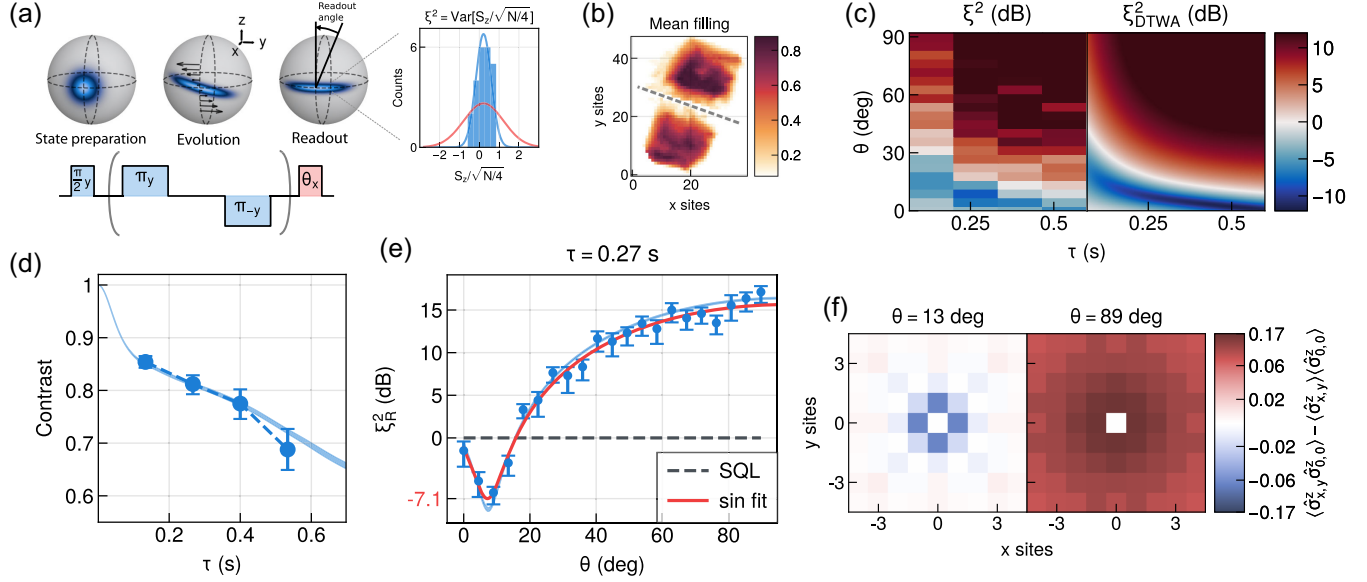


FIG. 3. Spin squeezing from finite-range magnetic XY interactions. (a) The interaction shears the collective spin along the black arrows on the Bloch sphere. We estimate the noise squeezing ξ^2 by sampling S_z . A histogram of $S_z/\sqrt{N/4}$ from the most squeezed point in (e), with a variance of $\xi^2 = -8.2 \pm 1.0$ dB below the SQL, is shown. A Gaussian with the measured (SQL) variance is shown in blue (red). (b) We perform differential measurements of ξ^2 with two spatially separated 260 atom clouds to reject common-mode noise. (c) The noise squeezing is sampled over evolution time and readout angle. The left square is experimental data; the right is a numerical simulation (DTWA) based on the filling in (b). (d) We see excellent agreement in the Ramsey contrast with DTWA (shaded curve) at all sampled times. Error bars are 16th and 84th percentiles obtained from bootstrapping the data. (e) ξ_R^2 after $\tau = 0.27$ s of evolution. The optimal squeezing measured was -6.4 ± 1.0 dB and fitted to be -7.1 ± 1.0 dB. The blue shaded curve is DTWA. Error bars are 16th and 84th percentiles obtained from bootstrapping the data. (f) We visualize the squeezing and antisqueezing by computing site-to-site two-particle spin correlations radially averaged over the clouds at a squeezed and an antisqueezed readout angle.

which introduce variance that scales linearly with atom number relative to the SQL. In addition, we notice that the measured optimal squeezing angle as a function of time in Fig. 3(e) agrees with DTWA, strongly implying that the system dynamics remain coherent and generate substantially more squeezing that could be measured with improved microwave control.

The site-to-site correlations of these highly squeezed states elucidate to what degree our system's magnetic dipole-dipole interactions mimic an effective OAT model. We exploit our QGM's single-site and spin-resolved imaging to directly measure two-particle spin correlations $g_{i,j}^2 \equiv \langle \hat{\sigma}_i^z \hat{\sigma}_j^z \rangle - \langle \hat{\sigma}_i^z \rangle \langle \hat{\sigma}_j^z \rangle$ at a squeezed and an antisqueezed point in Fig. 3(f). Averaging correlations over the atomic cloud, we observe correlations that decay over the region of interest, directly related to the limited propagation rate of the finite-range $1/r^3$ interaction. This precludes the effective model from being all to all for the early times sampled, despite the mean observables replicating OAT.

We reconcile these pictures by considering the local interpretation in which each spin's twisting rate depends on its neighboring spins, allowing local correlations to build up. At finite filling, this introduces a variance in the precession rate based on inevitable microscopic details that add to ξ^2 , limiting squeezing. We assume that there are

no spin-aligning terms at short times protecting global uniformity, unlike some recent theoretical work [22]. Despite this, we prepare a system [Fig. 3(c)] where the variance in precession rates is negligible, and twisting rates still mimic global OAT dynamics, due to an effective decoupling of finite momentum spin waves from the dynamics of global observables [48].

V. SPIN SQUEEZING WITH ITINERANCY

Having successfully generated spin squeezing at high fillings, we now investigate how itinerancy may enhance the viability of lower-density finite-range interacting platforms. Realizing high filling can be challenging for many up-and-coming molecular platforms with similar dipolar interactions, necessitating the exploration of low-filling regimes. Kinetic degrees of freedom can alleviate interaction inhomogeneity and local disorder in dilute systems. On the other hand, itinerancy may delocalize detrimental defects, introducing time-varying local noise that may disrupt squeezing, as has been observed in recent work with nearest-neighbor interactions [49]. However, provided the noise spectrum does not couple to the dynamics, tunneling promises to mitigate the limiting factors of finite-range interactions by spatially averaging out imperfections and improving coherence [36,37].

Here, we explore the interplay between motional degrees of freedom and the spin squeezing and contrast decay generated by dipolar XY interactions. By tuning the depth of our optical lattice, we probe three regimes of itinerancy where the nearest-neighbor tunneling parameter t is 0, 10, and 50 Hz, with peak fillings below 35%. We measure contrast and ξ^2 after four evolution times ranging from 133 to 533 ms and readout angle θ sampled from 0 to 30° . We observe a significant improvement in ξ^2 , most notably in the later time points, a result reinforced by our simulations [Fig. 4(b)]. The effects on the Ramsey contrast are even more prominent, as we see the contrast is improved already in the $t = 10$ Hz case but vastly so for $t = 50$ Hz [Fig. 4(c)], where it remains above 98% at all sampled evolution times.

For a fixed evolution time of 533 ms, we compute the Wineland parameter ξ_R^2 as a function of readout angle and fit a quadratic to extract its minimum [Fig. 4(d)]. At this low filling, without tunneling, the Ramsey contrast has sufficiently decayed such that ξ_R^2 is never below the SQL. Adding tunneling reduces ξ_R^2 by (3.8 ± 0.6) dB for $t = 10$ Hz and as much as (5.2 ± 0.6) dB for $t = 50$ Hz, shifting the curve to well within the regime of useful squeezing.

The spatial distribution of the entanglement across the cloud is measured by the two-particle spin correlations $g_{i,j}^2$ [Fig. 4(e)]. The spin correlations fall off rapidly for $t = 0$ Hz. At $t = 10$ Hz, we see significant delocalization, although the strongest correlations are still nearest neighbor. At $t = 50$ Hz, we observe substantially lower correlations in the window. However, the total squeezing and antisqueezing across the clouds are similar for each value of the tunneling. Therefore, the correlations extend much further for $t = 50$ Hz, and indeed we do not observe substantial decay as a function of distance across the averaging window; thus, tunneling redistributes the correlations from being appreciable only between neighboring spins to spanning the entire cloud, as would be expected for all-to-all OAT.

We attempt to capture the physics of these effects by modifying our DTWA simulations. For the intermediate tunneling regime ($t = 10$ Hz), we incorporate classical diffusive stochastic hopping of atoms localized to individual sites (see Appendix H). With just this modification, we qualitatively predict both the observed effects of tunneling on the Ramsey contrast and the effects on the noise squeezing ξ^2 . We understand the improvement in ξ^2 as the reduction of the effective variance in the local collective interaction strength through rapid averaging. For the high-tunneling regime ($t = 50$ Hz), we experimentally observe significantly higher contrast than is predicted by our stochastic tunneling-enabled DTWA, suggesting that the

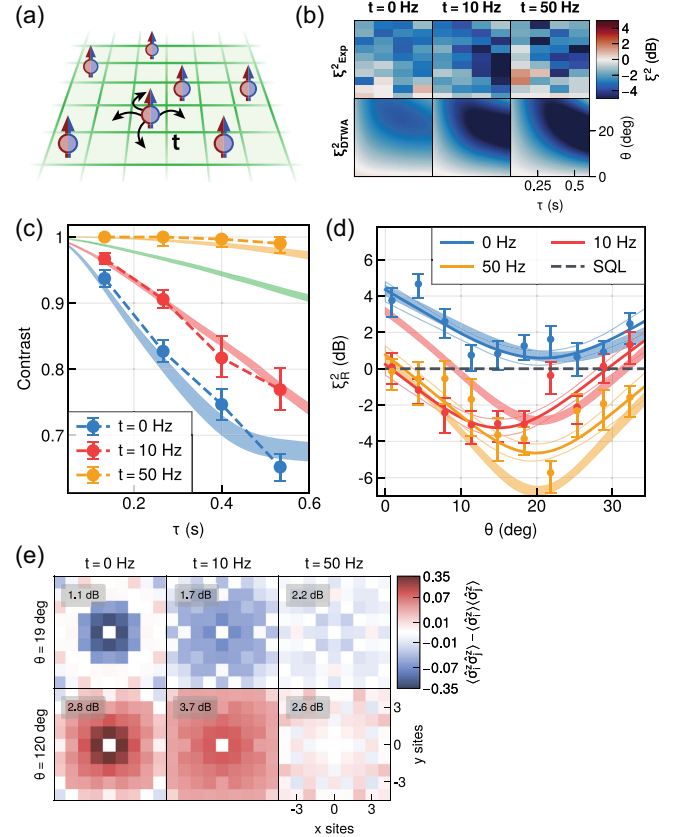


FIG. 4. Itinerancy at low lattice fillings. (a) Our system supports tunable nearest-neighbor tunneling with rate t . We focus on relatively low fillings (20%–30%) to ensure the hard-core fermions do not block each other's movement. (b) Noise squeezing as a function of evolution time τ , tunneling t , and readout angle θ . Tunneling improves noise squeezing in both theory and experiment. Each square shows the same 2D sweep of τ and θ . Experimental data are in the top row, compared to DTWA in the bottom row with the same color scale. The effects of tunneling are incorporated into DTWA with stochastic hopping for 10 Hz and OAT dynamics for 50 Hz. (c) Ramsey contrast decay as a function of τ and t . At high tunneling (yellow), the contrast remains above 98% as the system exhibits OAT-like dynamics. In this regime, stochastic tunneling (green curve) is not an accurate model. At intermediate tunneling (red), the contrast is still significantly improved from the no-tunneling case (blue) and is captured by stochastic hopping in DTWA (red curve). Error bars are 16th and 84th percentiles obtained from bootstrapping the data. (d) Wineland squeezing parameter, after an evolution time of 0.53 s, as a function of tunneling. The squeezing is significantly improved for both $t = 10$ and $t = 50$ Hz. The data are fitted to quadratic functions. Error bars are 16th and 84th percentiles obtained from bootstrapping the data. (e) Radially averaged correlations $g_{i,j}^2$ for $\tau = 200$ ms as a function of tunneling. While they fall off rapidly beyond nearest neighbor at $t = 0$, the correlations spread across the lattice for larger t . The noise squeezing (antisqueezing) for the whole system at each tunneling is inset in decibels for the top (bottom) row.

diffusive interpretation of the many-body dynamics breaks down. This may be caused by a lower effective motional temperature not represented in our simulation (see Appendix H). Instead, we compare our high-tunneling data to an all-to-all OAT model with shearing strength equal to the average interaction across all pairs of atoms in the experimentally observed cloud. We simulated this OAT model with DTWA and obtained good agreement on the contrast and squeezing.

From the introduction of itinerancy, the emergence of signatures consistent with all-to-all interactions is quite surprising. Contact interactions in Bose-Einstein condensates realize an infinite-range model in a single motional mode. However, for itinerant hard-core particles, as occurs for fermions, or bosons in the Mott limit, many motional modes are occupied and, therefore, can introduce time-varying disorder, disrupting squeezing. The magnetic dipole-dipole interaction, being sufficiently long range, is seemingly robust to these defects, leading to signatures of all-to-all couplings, while contact interactions are not [49]. Realizing and further exploring a system with apparent all-to-all interaction through itinerancy has several advantages. Since permutational symmetry is preserved, the sample remains spin polarized, preventing collisional decay. In addition, the complete averaging means that the itinerant squeezing is reversible by changing the sign of the interaction, allowing for Heisenberg scaling protocols, despite the randomization of the atomic positions [50].

VI. OUTLOOK

In this work, we have demonstrated that the native magnetic dipole-dipole interactions of neutral atoms, despite their finite-range nature and susceptibility to microscopic details, are excellent for generating strong spin squeezing with applications to quantum metrology. Our mean-field observations indicate that the observed squeezing should continue to scale to thousands of atoms given realistic improved phase and amplitude control of the microwave field. We have also investigated the effects of itinerancy, an open question in the context of quantum-enhanced metrology, and found that it can be harnessed to improve both spin coherence and squeezing, even reaching the all-to-all OAT limit for dipolar systems. Furthermore, our work requires only itinerant dipoles, suggesting a path to quantum-enhanced metrology in ultracold molecular platforms.

The realized interaction and subsequent squeezing are generalizable to most neutral atoms and scale with system size. A spin-exchange interaction on a magnetically insensitive transition is present in any atomic state with nonzero electronic and nuclear spins (see Appendix A). Such an interaction has already been observed through its effect on the microwave clock transition frequency of ^{87}Rb [34]. Importantly, this is also applicable to optical clocks, as the $^3P_2$ metastable states of ^{87}Sr and ^{173}Yb host magnetically

insensitive qubits with twice the interaction strength used in this work (see Appendix A) [51]. Furthermore, squeezing in a 2D system with $1/r^3$ interactions (such as in this work) is believed to scale with atom number; for as few as 4000 atoms, well beyond -20 dB of squeezing is predicted [22], presenting a substantial potential improvement in clock precision.

The platform presented here has broad utility in quantum simulation and metrology. With tunable itinerancy, site-resolved imaging, and high-filling fractions, we can explore the dynamics and phases of itinerant XXZ interacting fermions such as the t - J model. Further comprehensive investigation regarding the effect of kinetic temperature on squeezing and ordering may inform strategies for adding itinerancy to systems for metrological improvement. For example, there are untapped regimes where tunneling energies are comparable to the single-particle dipolar interaction, achievable by choosing states in ^{167}Er with up to 4 Hz (9 Hz) interaction strength with (without) a magic B field, that should generate entanglement between the kinetic and spin sectors.

Finally, we show that, for dipolar interactions, the addition of large tunnelings generates longer-range correlations than purely $1/r^3$, replicating, at least to second order, spin dynamics of interactions with different functional forms. This work's entanglement generation scheme can be combined with erbium's optical clock transitions that possess attractive properties or extended to higher spin models [52]. Given the presence of transitions at telecom frequencies with a blackbody radiation shift 10^3 times lower than current standards, erbium lattice clocks emerge as exciting candidates for modern metrology experiments [53,54].

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A. D., V. K., L. S., M. S., and O. M. ran experiments and analyzed experimental data. A. D., V. K., L. S., M. S., V. S., and O. M. contributed to the experimental apparatus. A. D. and V. K. conceived the experiment. V. K. performed numerical simulations. M. G. supervised all works. All authors discussed the results and contributed to the manuscript.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [55].

APPENDIX A: ORIGIN OF THE SPIN-EXCHANGE INTERACTION

The interaction Hamiltonian between two magnetic dipoles μ_1 and μ_2 separated by distance r and oriented along the unit vector $\hat{r}$ is given by

$$H_{dd} = -\frac{\mu_0}{4\pi r^3} [3(\mu_1 \cdot \hat{r})(\mu_2 \cdot \hat{r}) - \mu_1 \cdot \mu_2], \quad (\text{A1})$$

where μ_0 is the vacuum permeability.

For magnetic atoms, the dipole moment operator is

$$\mu = -\frac{\mu_B}{\hbar} (g_S S + g_L L) - \frac{\mu_N}{\hbar} g_I I, \quad (\text{A2})$$

where μ_B and μ_N are the Bohr and nuclear magnetons, respectively, and S , L , and I are the spin, orbital, and nuclear angular momentum operators, respectively.

At the magnetic field strengths used in our experiment, the spin-orbit coupling ($L \cdot S$) in erbium dominates, allowing the electron spin and orbital contributions to be combined into a single term:

$$\mu \approx -\frac{\mu_B g_J}{\hbar} \mathbf{J}, \quad (\text{A3})$$

where $g_J \approx 1.16$ in the ground state of erbium [56]. The nuclear contribution is suppressed by the ratio $\mu_N/\mu_B \sim 1/2000$ and is, thus, neglected. Substituting into Eq. (A1), the dipole-dipole interaction simplifies to

$$H_{dd} \approx -\frac{\mu_0 g_J^2 \mu_B^2}{4\pi r^3 \hbar^2} [3(\mathbf{J}_1 \cdot \hat{r})(\mathbf{J}_2 \cdot \hat{r}) - \mathbf{J}_1 \cdot \mathbf{J}_2]. \quad (\text{A4})$$

In our experiment, ^{167}Er atoms are loaded into a 2D square optical lattice in the xy plane with spacing $a = 266$ nm, and a quantization field $\mathbf{B} = B\hat{z}$ is applied. For low magnetic fields ($B < 100$ G), the hyperfine coupling $\mathbf{I} \cdot \mathbf{J}$ dominates over the Zeeman term, such that the eigenstates are well approximated by total angular momentum states $|F, m_F\rangle$.

We define a qubit using two states with $\Delta F = 1$ and $\Delta m_F = 0$, enabling a “magic” magnetic field where the qubit becomes first-order magnetically insensitive. In this manifold, since $\Delta m_F = 0$, the matrix elements $\langle F+1, m_F | J_{\pm} | F, m_F \rangle = 0$, and, thus, transverse components J_x and J_y do not contribute to coupling. Only the longitudinal term $J_{1z} J_{2z}$ contributes in Eq. (A4).

The nearest-neighbor spin-exchange interaction is then given by

$$C \equiv \langle F+1 | {}_1 \langle F | {}_2 \frac{\mu_0 g_J^2 \mu_B^2}{4\pi a^3 \hbar^2} J_{1z} J_{2z} | F \rangle {}_1 | F+1 \rangle {}_2, \quad (\text{A5})$$

which simplifies to

$$C = \frac{\mu_0 g_J^2 \mu_B^2}{4\pi a^3} \left| \langle F+1, m_F | \frac{J_z}{\hbar} | F, m_F \rangle \right|^2. \quad (\text{A6})$$

Evaluating the prefactor numerically for our lattice spacing, we find

$$\frac{1}{\hbar} \frac{\mu_0 g_J^2 \mu_B^2}{4\pi a^3} \approx 2\pi \times 0.934 \text{ Hz}. \quad (\text{A7})$$

The matrix element

$$C_G \equiv \left| \langle F+1, m_F | \frac{J_z}{\hbar} | F, m_F \rangle \right|^2 \quad (\text{A8})$$

is evaluated with Clebsch-Gordan coefficients:

$$C_G = \left(\sum_{m_J=-J}^J m_J \mathcal{C}_{m_J, (m_F-m_J), m_F}^{J, I, F} \mathcal{C}_{m_J, (m_F-m_J), m_F}^{J, I, F+1} \right)^2. \quad (\text{A9})$$

In our experiment, we use $F = 17/2$, $F+1 = 19/2$, and $m_F = 1/2$. For these states, the sum evaluates to $C_G = \frac{420}{361}$, yielding a nearest-neighbor spin-exchange interaction of approximately 1.087 Hz.

At the magic magnetic field, the expectation value of any single-particle $\hat{Z}$ operator vanishes, so no ZZ-type Ising interaction appears. Note that the above derivation assumes zero magnetic field, so the actual interaction strength at the magic field may differ slightly due to state mixing.

We plot the interaction strengths for this class of magnetically insensitive qubits in ^{167}Er and compare them

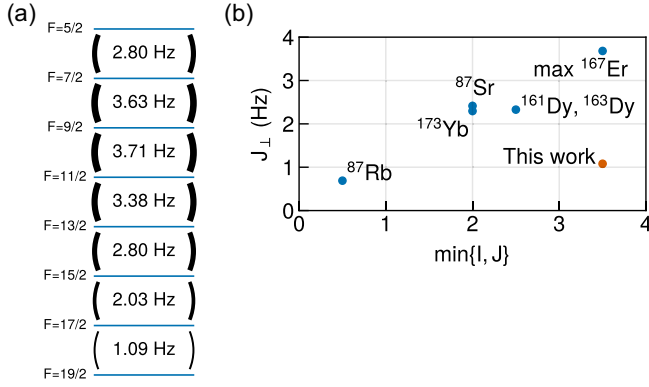


FIG. 5. Magnetically insensitive dipolar interaction strengths across atomic species. (a) Maximum interaction strength between adjacent hyperfine manifolds F and $F + 1$ in ^{167}Er , consistently achieved in the $m_F = \pm 1/2$ sublevels. (b) Maximum nearest-neighbor transverse interaction strength $J_{\perp}$, of the form used in this work, for various fermionic isotopes (and ^{87}Rb) relevant to cold atom experiments. All values assume a lattice spacing of 266 nm. The interaction magnitude scales with $\min\{I, J\}$, where I and J are the nuclear and electronic spin quantum numbers, respectively.

with other atomic species relevant to cold atom experiments in Fig. 5.

APPENDIX B: EXPERIMENTAL PROCEDURE

1. Band insulator preparation

We begin by preparing a degenerate Fermi gas of ^{167}Er , following the same two-stage magneto-optical trap and forced evaporation sequence described in our previous work [57]. The combination of a narrow-line magneto-optical trap and dipolar collision-dominated evaporation naturally produces a spin-polarized three-dimensional (3D) bulk gas in the hyperfine spin state $|g\rangle = |F = 19/2, m_F = -19/2\rangle$.

This gas is then loaded into a 532-nm wavelength accordion lattice and compressed, similar to our previous procedure with bosonic ^{168}Er [45]. Reducing the accordion power after compression induces vertical evaporation, selectively removing atoms occupying higher bands of the vertical lattice. Unlike the bosonic species, transferring from the accordion to the vertical lattice with 532-nm spacing proves significantly more challenging for fermionic ^{167}Er . This transfer demands improved alignment and approximately double the trap frequency. We attribute this increased difficulty to the incompressibility of the degenerate Fermi gas.

Following this, we further evaporate the gas in two dimensions by inducing “spillover” in the x, y plane using a magnetic field gradient. The final atom number is tuned by adjusting the size and power of the x, y confining optical dipole trap dimple during this spillover stage. We observe that a spillover duration of 2.5—approximately 10 times

longer than for the bosonic isotope ^{168}Er —yields the lowest temperatures even though the elastic cross section is only about 10% smaller [58,59]. This discrepancy is likely due to Pauli blocking, though it may also reflect a higher initial temperature following 2D loading.

During spillover, the atoms are highly sensitive to the magnetic field due to the dense spectrum of Feshbach resonances [60]. We choose to operate at a magnetic field of 0.55 G, which offers a resonance-free range of ± 0.05 G.

To impose arbitrary geometries for the atoms, as shown in Fig. 3, we ramp up a DMD while simultaneously ramping down the dimple, performing this while the system remains a bulk gas after the secondary spillover. This minimizes heating from transport. We then prepare a band insulator by adiabatically ramping up the x, y lattices while ramping down the dimple over 400 ms. Each stage in this sequence is optimized using M-Loop [61].

The resulting filling of approximately 95% corresponds to an entropy per particle of around $0.2k_B$ at the center of the cloud.

2. Spin state preparation

a. Experimental sequence for spin preparation and squeezing

The experimental control sequence is shown in Fig. 6. We begin by transferring atoms from the initial state $|g\rangle = |F = 19/2, m_F = -19/2\rangle$ to $|\downarrow\rangle = |F = 19/2, m_F = 1/2\rangle$ via a single Landau-Zener sweep through ten consecutive $\Delta F = 0, \Delta m_F = +1$ transitions (indicated by the red box in Fig. 6), following the protocol described in Ref. [62]. These transitions are driven by an in-vacuum rf coil with a natural resonance at 60.8 MHz. The rf tone is held at a fixed frequency, while the magnetic field is swept upward by 2 G over a 100 ms interval to successively bring each transition into resonance.

A Rabi frequency of approximately 2 kHz yields an overall state transfer fidelity exceeding 97%. The large quadratic Zeeman shift at 60 G, which introduces an approximately 90-kHz detuning between neighboring $m_F \rightarrow m_F + 1$ and $m_F + 1 \rightarrow m_F + 2$ transitions, is crucial for isolating individual steps in the sweep and achieving high fidelity. However, this field range is dense with Feshbach resonances, approximately 26 per Gauss, posing a risk of inelastic loss. To mitigate this, we first load the atoms into a band insulator and then ramp the magnetic field rapidly and diabatically with respect to tunneling, thereby suppressing two-body collision processes [60].

After completing state preparation, the magnetic field is set to the magnetically insensitive value of 10.622 G. We then perform a Landau-Zener sweep between $|\downarrow\rangle$ and $|\uparrow\rangle = |F = 17/2, m_F = 1/2\rangle$ using a 2.689 GHz rf tone (dark green box in Fig. 6). This sweep is executed with a Rabi frequency of 870 Hz over 20 ms and a total frequency sweep of 30 kHz. Residual atoms in the $F = 19/2$ manifold (primarily those not transferred to $m_F = 1/2$)

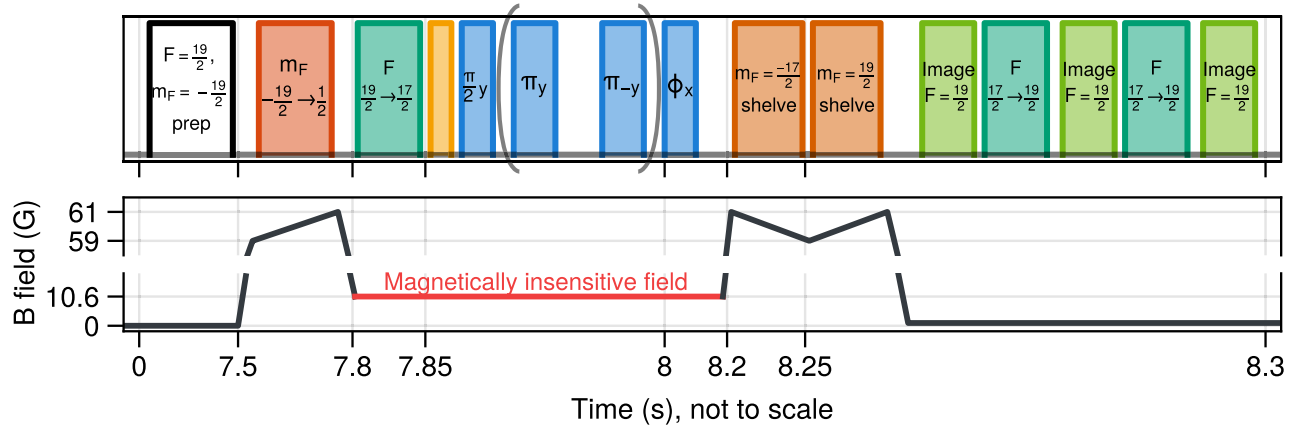


FIG. 6. Experimental control sequence. Experimental spin manipulation sequence for ^{167}Er . Atoms are initially cooled and spin polarized into the hyperfine state $|F = 19/2, m_F = -19/2\rangle$. The top shows the control pulse sequence: microwave pulses (blue), microwave Landau-Zener sweeps (dark green), rf Landau-Zener sweeps (red and orange), imaging optical pulses (lime), and a blowout pulse (yellow) used to remove atoms in undesired hyperfine states. The sequence in the gray parentheses is repeated every 133 ms over the duration of the squeezing evolution. The bottom shows the magnetic field versus time. The red-highlighted region marks the magnetically insensitive point, where the main squeezing evolution occurs. The magnetic field is ramped to 60 G to enable high-fidelity $m_F \rightarrow m_F + 1$ Landau-Zener sweeps and subsequently lowered below 1 G for spin-resolved imaging within the $F = 19/2$ manifold.

are removed using a resonant 583-nm optical “blowout” pulse scanned across all m_F states (yellow box in Fig. 6).

With the atoms fully polarized in the $|\uparrow\rangle$ state, we proceed to apply the squeezing pulse sequence (blue boxes in Fig. 6). The sequence begins with a $\pi/2$ pulse to place each atom in an equal superposition state, $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$, followed by spin-echo π pulses of alternating sign applied every 66 ms. The sequence concludes with a final readout pulse, applied with a 90° phase shift relative to the initial $\pi/2$ pulse.

3. Spin-selective imaging

a. Spin-resolved imaging procedure

Following the squeezing dynamics, we quench any residual tunneling by ramping the lattice depth to suppress the tunneling rate to 0.3 Hz within 100 μs . The atoms are then loaded into a phase-matched 488-nm accordion lattice. This lattice is expanded from the science lattice spacing of 266 nm to an imaging spacing of 2.5 μm over a duration of 60 ms, resulting in atom loss below 1%. Importantly, potential depth disorder, interactions, and tunneling during this expansion process conserve σ_z , ensuring that subsequent spin measurements remain unaffected.

To enhance spin-resolved detection, we ramp the magnetic field to 60 G and perform Landau-Zener sweeps to map the $|\uparrow\rangle$ atoms into the stretched state $|F = 17/2, m_F = -17/2\rangle$ and the $|\downarrow\rangle$ atoms into the opposite stretched state $|F = 19/2, m_F = 19/2\rangle$ [Fig. 7(a)]. Next, we ramp down the magnetic field and apply a weak optical pulse (intensity approximately $0.3I_{\text{sat}}$) resonant with the 401-nm $F = 19/2 \rightarrow F = 21/2$ cycling

transition (linewidth 30 MHz), while remaining detuned by approximately 20Γ from the $F = 17/2 \rightarrow F = 19/2$ transition. This selectively images the $|\downarrow\rangle$ atoms [Fig. 7(b)]. The use of stretched states, due to Clebsch-Gordan suppression, reduces off-resonant scattering from $F = 17/2$ by an additional factor of approximately 50.

To image the other spin state, we invert the in-plane magnetic field—effectively switching the handedness of the circularly polarized imaging light—so that the negative m_F stretched state gains more favorable Clebsch-Gordan coefficients. A subsequent Landau-Zener sweep transfers atoms in the $F = 17/2$ manifold into the $F = 19/2$ manifold [Fig. 7(c)], after which we apply another resonant optical pulse to image these atoms [Fig. 7(d)]. To verify completeness of atom detection, we reapply the Landau-Zener sweep and use a power-broadened resonant pulse (approximately $20I_{\text{sat}}$) to capture any remaining atoms in either hyperfine manifold. Fewer than 0.2% of atoms typically remain in this final step.

On average, we collect just 13 photons per atom to suppress the likelihood of off-resonant excitations. To minimize dark counts and imaging delay, we operate our EMCCD camera (Andor iXon Ultra 897 EXF) at a fast readout rate of 17 MHz, yielding a background noise equivalent to 2.3 photons per site. Multiplication noise is reduced by setting the gain to maximum and binarizing each of the 144 pixels per atom, a method effective in the limit of $1 \gg$ photon per pixel [46]. Under these imaging conditions, we achieve a 99.5% fidelity for distinguishing between 0 and 1 atom per site. The total atom number imaging difference between the $|\uparrow\rangle$ and $|\downarrow\rangle$ populations, as extracted from fits to the Rabi oscillation in Fig. 10, is less than 0.25%.

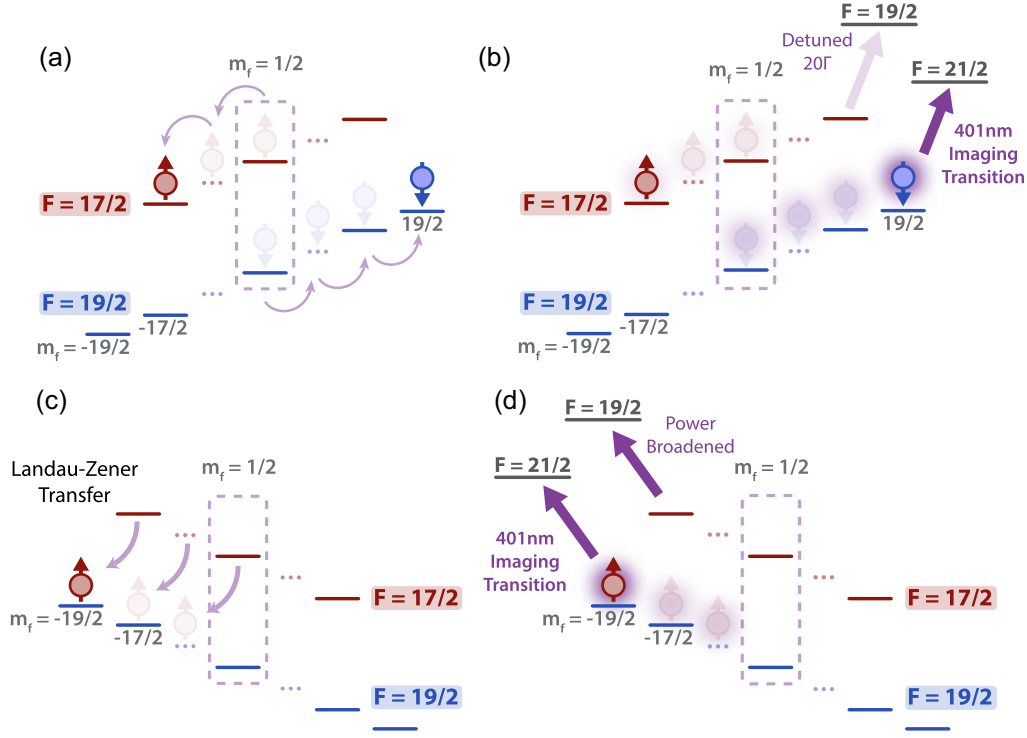


FIG. 7. Imaging sequence. (a) To prepare for imaging, the atoms in $F = 17/2$ and $F = 19/2$ are shelved in opposite stretched m_F states. (b) Atoms in $F = 19/2$ are imaged first with a resonant 401-nm optical pulse to $F = 21/2$. This pulse is 20Γ detuned from the corresponding transition for the atoms in $F = 17/2$, and crosstalk is a further 50 times suppressed by the shelving due to Clebsch-Gordan coefficients. (c) Atoms in each sublevel of $F = 17/2$ are transferred to $F = 19/2$. (d) The atoms formerly in $F = 17/2$ are imaged similarly to (b), with higher power and opposite quantization axis direction.

APPENDIX C: LATTICE INHOMOGENEITY IN THE SPIN SECTOR

For the data presented in Fig. 2, we employed approximately circular polarization for the science lattice to suppress backreflections using a “poor man’s isolator.” The in-plane lattices (x and y) typically had depths of approximately $35E_R$, where $E_R = 4.2$ kHz, while the vertical lattice had a depth of approximately $31E_R$, with $E_R = 1.05$ kHz. The differential polarizability between the $|\downarrow\rangle$ and $|\uparrow\rangle$ states was measured to be $1.71 \text{ Hz}/E_R$ and $2.36 \text{ Hz}/E_R$ for the x and y lattices, respectively, and $3 \text{ Hz}/E_R$ for the vertical lattice, in the polarization configuration used for Fig. 2. For the squeezing experiments shown in Figs. 3 and 4, we optimized the polarization to minimize the differential polarizability. This was achieved by rotating the x and y lattice polarizations to adjust the tensor and vector light shifts, reducing the differential polarizability to below 0.1 and $0.2 \text{ Hz}/E_R$, respectively. It is not possible to create a magic polarization condition for the vertical lattice in our dipole configuration due to its retroreflected geometry. However, the resulting spin-dependent potential disorder is negligible: The peak-to-peak disorder in the total potential is approximately 30 ppm, corresponding to about 9 mHz disorder in the spin sector—dominated by the vertical lattice. We also estimate a global

harmonic confinement in the spin sector of approximately $1.7 \text{ mHz}/r^2$ for Fig. 2 and $1.1 \text{ mHz}/r^2$ for the polarization-tuned configuration used in the squeezing experiments, where r is the distance from the trap center in lattice site units. Our spin-echo sequences decouple the mean-field shifts from this harmonic confinement. The more relevant figure of merit is the maximum site-to-site potential gradient, which is 43 mHz for a system of 1600 atoms—sufficiently small to have negligible impact on dynamics, according to our simulations. Nevertheless, we included the spin-dependent potential in the DTWA simulations for Fig. 3.

APPENDIX D: QUBIT CHOICE AND MAGNETIC INSENSITIVITY

The chosen qubit consists of the states $|F = 19/2, m_F = 1/2\rangle$ and $|F = 17/2, m_F = 1/2\rangle$, which form a magnetically insensitive transition at first order. At an out-of-plane quantization field of 10.622 G [see Fig. 1(c)], the second-order magnetic sensitivity is $1.2 \text{ mHz}/\text{mG}^2$. While this magnetic insensitivity is essential for achieving long coherence times, it also prohibits the presence of effective Ising interactions. This is because atoms in magnetically insensitive states do not experience a shift from the mean local magnetic field produced by other

atoms. Nevertheless, due to the nonzero nuclear spin of ^{167}Er , the native dipolar Ising interaction between distinct m_J projections is transformed into a spin-exchange interaction in the hyperfine manifold [34] (see Appendix A).

This constraint substantially reduces the interaction strength. For instance, the maximum dipolar spin-exchange interaction for ^{167}Er on our lattice is 9.3 Hz, which occurs between the states $|F = 19/2, m_F = \pm 1/2\rangle$. In contrast, magnetically insensitive transitions exhibit interaction strengths ranging from 1.09 to 3.7 Hz (see Fig. 5). Our measured root-mean-square (rms) magnetic field noise is on the order of 1 mG, which would limit the coherence time of the $|F = 19/2, m_F = \pm 1/2\rangle$ qubit to approximately 1 ms. This would result in an interaction-coherence time product that is 700 times smaller than what we achieve using the magnetically insensitive qubit.

APPENDIX E: TUNNELING METHODS

We probe the effects of tunneling on spin-squeezing dynamics in three regimes: (i) static, with $t \approx 0$ Hz; (ii) intermediate, with $t = 10$ Hz; and (iii) large, with $t = 50$ Hz. In the intermediate regime ($t = 10$ Hz), the tunneling rate into a single neighboring site exceeds the collective dipolar interaction energy experienced by an individual atom by approximately a factor of 4. Moreover, the tunneling significantly exceeds the site-to-site disorder, which has an rms value of 3 Hz.

To eliminate additional disorder contributions, we turn off the DMD. However, this also prevents the use of two spatially separated atomic clouds for technical noise subtraction. To ensure the integrity of the squeezing measurements under these conditions, we restrict our analysis to clouds containing fewer than 100 atoms.

To enable direct comparison across different tunneling regimes, we maintain a constant atomic density. This is accomplished by adiabatically ramping down the lattice depth from a band insulator, thereby preparing a low-entropy initial state at $t = 50$ Hz, and then quenching the lattice to the desired tunneling strength for the squeezing sequence. This procedure preserves constant site filling across all measurements. However, in the $t = 50$ Hz case, the absence of a quench results in a kinetic sector that is substantially colder than in the quenched cases.

Despite the expectation that increased tunneling would lead to more double occupancies at finite polarization—potentially resulting in inelastic collisions or dipolar relaxation—we observe no tunneling-dependent atom loss. This is confirmed by summing both spin populations shown in Fig. 4(c). Even if collisional loss terms are present in the effective Hamiltonian, the high contrast observed—particularly for $t = 50$ Hz—suggests that the cloud remains highly spin polarized throughout the evolution, thereby suppressing collision-induced loss.

APPENDIX F: T_2 AND LATTICE LOSS MEASUREMENTS

In Fig. 8(a), we present a measurement of the transverse coherence time T_2 without spin-resolved imaging. To account for atom loss from the lattice, we compute a lower bound on the state polarization by normalizing the observed spin contrast with respect to the total atom number.

The measurement proceeds in two steps. First, we independently characterize atom loss by fitting the decay of total atom number to an exponential function [Fig. 8(b)]. Next, we perform the spin-echo + WAHUA pulse sequence on the same cloud to dynamically decouple dipolar interactions and measure the resulting spin contrast $S_z/|S|$. We normalize this measured contrast by dividing it by the atom-loss fit curve, yielding the normalized polarization. An exponential fit to this normalized signal allows us to extract the T_2 time.

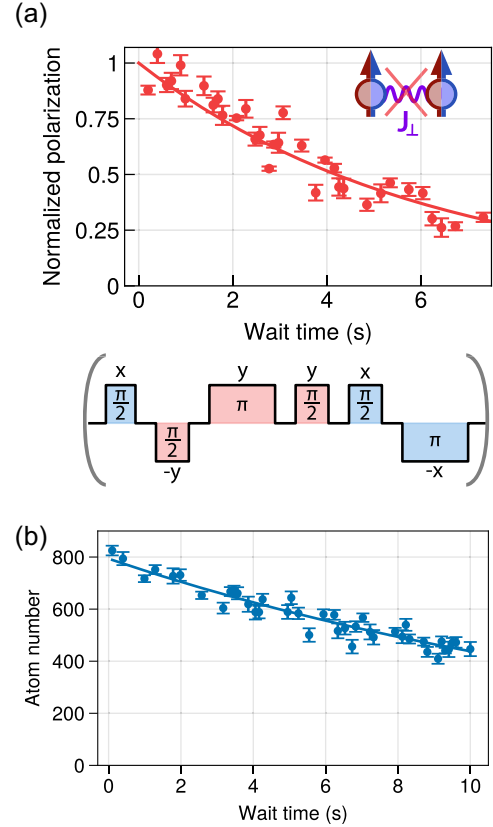


FIG. 8. Lattice lifetime and T_2 coherence (a) T_2 coherence of a cloud of 900 atoms with disorder and interaction decoupled by a spin-echo + WAHUA pulse sequence. We measure a T_2 of (6.0 ± 0.2) s. The pulse sequence is shown below the data; the sequence within the gray parentheses is repeated every 66 ms. Error bars are SEM. (b) We measure the single-particle lattice loss and fit to an exponential to obtain a lattice lifetime of (16.8 ± 0.5) s. Error bars are SEM.

We observe that atom loss from the lattice is independent of the initial spin state. Furthermore, the loss rate remains unchanged when we deliberately dephase the atoms by applying a magnetic field gradient or a large optical potential gradient.

APPENDIX G: DATA PROCESSING

From each experimental shot, we extract spin-dependent lattice occupation matrices using the site-resolved imaging protocol described in Appendix B 3. These matrices yield the total spin-resolved occupation numbers $N_\uparrow$ and $N_\downarrow$ for each individual shot.

For each scan group (i.e., sets of shots analyzed together under consistent conditions), we apply atom number filtering by discarding all shots whose total atom numbers fall outside the 12.5th to 87.5th percentiles within that group. Additionally, across all included shots, we ignore lattice sites whose average occupation is less than 10% of the maximum mean site occupation observed within the scan group. This ensures robust statistics across the cloud while avoiding low-signal regions.

Shot-to-shot atom number fluctuations greater than one standard deviation are primarily attributed to slow long-term drifts in loading efficiency or lattice alignment. Each observable is computed using the filtered data, as described in the analysis sections below.

1. Contrast

The Ramsey contrast C is measured by preparing the spin state on the equator of the generalized Bloch sphere and applying a $\pi/2$ readout pulse with variable phase ϕ . We then record the normalized magnetization $S_z/S = (N_\uparrow - N_\downarrow)/N$ as a function of ϕ , where $N = N_\uparrow + N_\downarrow$, and fit the resulting data to a sinusoidal function. The standard error of the mean (SEM) is used to estimate the uncertainty of each data point.

The amplitude of the fitted sinusoid yields the Ramsey contrast C . For datasets where C is close to 1, the distribution of S_z/S values deviates significantly from normality, making the fit sensitive to small perturbations. To account for this, we apply a bootstrap procedure: 500 resamples of the dataset are generated, and a sinusoidal fit is performed for each. From this, we obtain a distribution of contrast values, which we clip to the interval $[0, 1]$ to reflect physical constraints.

We report the median of the bootstrap distribution as our estimate of C , rather than using the amplitude from the fit to the original dataset, to reduce sensitivity to outliers or skew in the data. Error bars are defined by the 16th and 84th percentiles of the bootstrap distribution. In cases where $S_z/S = 1$ for all shots at a particular value of ϕ , we manually assign an SEM of 0.01 to that point to ensure fit convergence.

2. Spin squeezing

For direct squeezing measurements (Fig. 4), we estimate the spin projection noise ξ^2 by sampling m individual experimental shots. From each shot, we extract the spin populations $N_\uparrow$ and $N_\downarrow$ and compute the squeezing parameter as

$$\xi^2 = \frac{\text{Var}[N_\uparrow - N_\downarrow]}{\hat{\sigma}_{\text{SQL}}^2}, \quad (\text{G1})$$

where the estimated SQL is defined as

$$\begin{aligned} \hat{\sigma}_{\text{SQL}}^2 &\equiv 4\langle p \rangle(1 - \langle p \rangle)\langle N \rangle, \\ \langle p \rangle &\equiv \frac{1}{2} \left(1 + \left\langle \frac{N_\uparrow - N_\downarrow}{N} \right\rangle \right). \end{aligned} \quad (\text{G2})$$

Here, $\langle \cdot \rangle$ denotes an average over the ensemble of m shots. This form of $\hat{\sigma}_{\text{SQL}}^2$ accounts for possible systematic pulse errors that may cause a nonzero mean spin projection at readout.

The variance is computed as the standard sample variance. To estimate uncertainty in ξ^2 , we employ a nonparametric bootstrap with 1000 resamples and report the 16th and 84th percentiles of the resulting distribution.

For differential squeezing measurements (Fig. 3), we prepare two spatially separated atomic ensembles, A and B , and extract spin populations for each by manually defining a separation boundary in the lattice image. The differential squeezing parameter is then computed as

$$\xi^2 = \frac{\text{Var}[(N_\uparrow^{(A)} - N_\downarrow^{(A)}) - (N_\uparrow^{(B)} - N_\downarrow^{(B)})]}{\hat{\sigma}_{\text{SQL}}^2{}^{(A)} + \hat{\sigma}_{\text{SQL}}^2{}^{(B)}}, \quad (\text{G3})$$

where $\hat{\sigma}_{\text{SQL}}^2{}^{(A,B)}$ are computed independently for each ensemble using the same method as above.

To compute error bars for the interferometric squeezing parameter ξ_R^2 , we combine bootstrapped distributions of both ξ^2 and the contrast C to produce a distribution of ξ_R^2 via 500 bootstrap resamples. The 16th and 84th percentiles of this distribution are used as confidence intervals.

To determine the minimum squeezing from data taken as a function of readout angle θ , we fit the measured ξ^2 or ξ_R^2 to an appropriate model: either a sinusoidal function (Fig. 3) or a quadratic function (e.g., Fig. 4) when the sampled angle range is narrow and does not capture the full sinusoidal behavior. In general, any quantum state rotated about the Bloch sphere exhibits elliptic spin noise contours, leading to a $\sin^2$ dependence of the measured variance. Antisqueezed points help fix the phase of the fitted $\sin^2$ function, and interpolated squeezed points are used to extract the offset corresponding to the minimum variance.

The dominant source of deviation from the sinusoidal fit would be systematics in the final rotation pulse. However,

Rabi oscillations and π -pulse fidelity measurements confirm that such effects are negligible in our experiment.

The uncertainty of the points for the fits are set to half the difference between the 84th and 16th percentiles of the bootstrapped distributions of ξ^2 or ξ_R^2 . To obtain the upper and lower fit error lines, we sample 1000 curves from the covariance matrix of the fitted parameters and plot the 84th and 16th percentiles for every readout angle. The final reported squeezing is extracted from the fit parameters, with error bars from the fit covariance matrix. This error bar is then propagated in the standard manner into decibels.

In differential measurements, the optimal squeezing readout angle θ_{opt} approaches zero as the evolution time τ increases. However, for an isolated cloud with XY-symmetric interactions, the variance along S_z is preserved, and, thus, θ_{opt} should not reach zero. Both theoretical modeling and experimental observations indicate that, for $\tau > 500$ ms, θ_{opt} shifts anomalously toward zero. We attribute this behavior to finite dipolar interactions between the two clouds. This hypothesis is supported by numerical simulations in which the effect disappears upon removing one of the clouds.

3. Correlations

We compute the two-particle correlations using the definition

$$g_{i,j}^{(2)} \equiv \langle \hat{\sigma}_i^z \hat{\sigma}_j^z \rangle - \langle \hat{\sigma}_i^z \rangle \langle \hat{\sigma}_j^z \rangle, \quad (\text{G4})$$

where $\langle \hat{\sigma}_i^z \rangle$ denotes the mean spin projection along z at site i , averaged over all experimental shots in which site i is occupied. Similarly, $\langle \hat{\sigma}_i^z \hat{\sigma}_j^z \rangle$ is computed by averaging over all shots in which both sites i and j are occupied; all other shots are excluded from the calculation. For each site i , we select all sites j within a 9×9 square region centered on i and compute $g_{i,j}^{(2)}$ for each such pair. In such a 9×9 square region, we refer to the i th site by the x, y coordinates $(0,0)$ and to the j th site by its x, y coordinates relative to i . These local correlation maps are then averaged over all possible such 9×9 subsections in the cloud to generate a full two-dimensional map. To suppress noise, we radially symmetrize the resulting correlation map by averaging over all four 90° rotations of the 9×9 array and then further average it with its transpose to enforce mirror symmetry across the diagonal. Finally, we display the correlation matrix using a stitched linear-logarithmic color scale. This approach preserves the visual structure of the correlation features even in cases where the absolute magnitude of the correlations is small.

APPENDIX H: COMPARISONS TO THEORY (DTWA)

Throughout this paper, we compare our results to numerics implemented with the DTWA [63]. This semiclassical

method captures the buildup of quantum correlations through the evolution of an entangling Hamiltonian from an initially uncorrelated, classical state. In our system, we represent each atom as a single spin-1/2 degree of freedom. At the beginning of every experimental sequence, all atoms are spin polarized, and squeezing is produced through evolution under the spin-spin interaction Hamiltonian. We describe the procedure to capture this evolution with DTWA below.

Given the initial spin state described by a unit vector $\mathbf{s}_0 = (\langle \hat{\sigma}_x \rangle, \langle \hat{\sigma}_y \rangle, \langle \hat{\sigma}_z \rangle)$ in the Bloch sphere (where $\hat{\sigma}_i$ are the Pauli spin operators), choose unit vectors $\mathbf{s}_1$ and $\mathbf{s}_2$ such that $\mathbf{s}_0, \mathbf{s}_1$, and $\mathbf{s}_2$ form an orthonormal basis. Then, sample $\hat{\mathbf{s}}$ from one of the four vectors $\{\mathbf{s}_0 \pm \mathbf{s}_1, \mathbf{s}_0 \pm \mathbf{s}_2\}$ with equal probability. These four vectors represent the four points of a discrete phase space that, with the inclusion of $\mathbf{s}_1$ and $\mathbf{s}_2$, capture to first order the quantum fluctuations in the initial spin states around $\mathbf{s}_0$. For every spin-1/2 particle in the system, labeled with the index i , perform this sampling to obtain a set of initial conditions $\hat{\mathbf{s}}_i(t=0)$. Finally, evolve the $\hat{\mathbf{s}}_i$ as classical trajectories with the classical Hamiltonian of a coupled spin system:

$$\frac{d}{dt} \hat{\mathbf{s}}_i = \mathbf{B}_{\text{eff},i} \times \hat{\mathbf{s}}_i, \quad \mathbf{B}_{\text{eff},ik} \equiv R_{ik} + J_{ijkl} \hat{\mathbf{s}}^j. \quad (\text{H1})$$

$\mathbf{B}_{\text{eff},i}$ is the effective magnetic field felt by particle i due to on-site terms and interactions with neighboring spins. The component R_{ik} gives the on-site magnetic field along the k axis felt for the i th particle; for example, a local detuning in the qubit frequency at site i would be captured by $R_{iz} \neq 0$. The component J_{ijkl} gives the coupling along axes k and l for particles i and j ; the dipolar XY interaction in our simulation is captured with

$$J_{ijxx} = J_{ijyy} = \frac{1}{2} \frac{(2\pi)1.087 \text{ Hz}}{|\mathbf{r}_i - \mathbf{r}_j|^3/a^3}, \quad (\text{H2})$$

where $\mathbf{r}_i$ is the position of particle i and $a = 266$ nm is the lattice spacing.

We perform this sampling procedure many times, obtaining a set of independent classical trajectories for each atom i . The fluctuations in $\hat{\mathbf{s}}_i$ across these trajectories capture the quantum-mechanical fluctuations of the many-body state. We then extract an estimate for a single or multiparticle observable by averaging the observable's value across all sampled classical trajectories. For example, we extract the variance of the i th particle's spin vector along $\mathbf{s}_1$ as $\langle (\hat{\mathbf{s}}_i \cdot \mathbf{s}_1)^2 \rangle - \langle \hat{\mathbf{s}}_i \cdot \mathbf{s}_1 \rangle^2$, where $\langle \cdot \rangle$ denotes an average over the classical trajectories. For each experimental dataset that we wish to reproduce with DTWA, we randomly sample ten measured shots. For each of these ten shots, we sample 100 classical trajectories. From these trajectories, we extract the relevant spin observables shown in this work, including squeezing and contrast. Error bars, shown via the

thickness of shaded curves in the figures, are calculated as standard errors of the mean over the ten sampled shots. When comparing metrological squeezing ξ_R^2 results to DTWA, we compute ξ_R^2 from the DTWA simulations of the squeezing dataset (rather than the contrast dataset).

We capture experimental imperfections, such as nonuniform filling, by sampling the initial classical state for our simulations from a set of experimentally observed fillings. The finite beam width of our vertical lattice also introduces a light shift in the qubit frequency that is harmonic (quadratic) along both axes of the atomic cloud's plane. We compute this light shift to be 1 MHz/site² and include it in our simulations as a position-dependent Pauli Z term. We replicate our dynamical decoupling sequence of one spin-echo π pulse every 67 ms by flipping all spins around the y axis every 67 ms in the simulation. Furthermore, we find our simulation results to be unchanged with the addition of a 1-Hz standard deviation Gaussian site-to-site Pauli Z disorder term. Lastly, due to the finite width of the atomic Wannier functions in the z axis compared to the x , y lattice spacing, the atoms do not experience the full dipolar interaction at short distances.

We account for the reduction in the mean squeezing field by rescaling the total interaction to the predicted 86% [90.6% in Fig. 2(a)] but do not incorporate the change in spatial dependence of the potential.

For the intermediate tunneling regime ($t = 10$ Hz), we incorporate diffusive tunneling in DTWA using stochastic hopping based on the Metropolis-Hastings algorithm. While we expect coherent tunneling in the experiment, we assume that we start from a high-temperature occupation of momentum modes because of the lattice depth quench and, therefore, experience effectively classical motion. We consider only nearest-neighbor tunneling at a rate t (in hertz), in a DTWA simulation with step size dt (in seconds). Each time step, we propose for an atom in lattice site i to hop to a random empty adjacent site along the x axis with probability $4mt dt$, where m is the number of empty nearest-neighbor sites along the x axis. This procedure is later repeated for the y axis within the same time step. The factor of 4 comes from the rate of ballistic expansion of a free particle in a lattice, where $1/(2t)$ is the time to do a full oscillation for a two-site system [64]. We then accept this proposal with probability

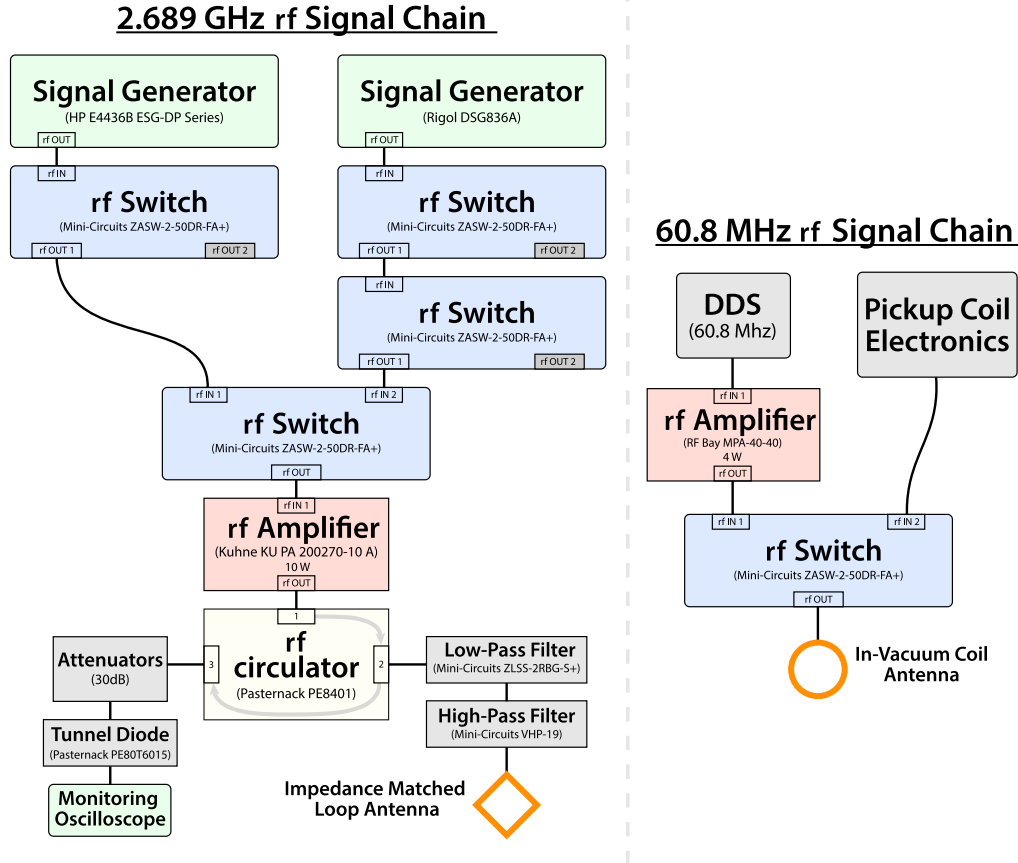


FIG. 9. rf and microwave delivery system used in the experiment. Microwave tones at 2.689 GHz are delivered via a loop antenna (gold square) positioned outside the science chamber. Radio-frequency (rf) tones at 60.8 MHz are transmitted using a pair of large in-vacuum coils (gold circles). These coils also passively receive and monitor the 2.689- GHz microwave signals emitted by the loop antenna, allowing for indirect diagnostics of the microwave delivery to the atoms.

$\min\{1, P(j)/P(i)\}$, where P is the experimentally observed averaged lattice filling of the dataset being simulated to account for harmonic confinement and disorder. The acceptance factor ensures the lattice filling of the simulation approximates the filling observed experimentally even while the atoms undergo hopping. To smooth discontinuities in the acceptance function, we apply a one-site standard deviation Gaussian filter over the experimental filling distribution to obtain P . We benchmark our algorithm by ensuring that the final simulated lattice filling matches that observed experimentally at the end of the simulation.

For the high tunneling regime ($t = 50$ Hz), we simulate an all-to-all OAT model with DTWA by replacing the strength of the $XX + YY$ interaction between every pair of atoms with the average value of this strength over the experimentally observed cloud. That is, for a set of N atoms at locations $\mathbf{r}_i$, $i \in [1, N]$, we simulate the Hamiltonian

$$H = -\chi S_z^2, \quad \chi = \frac{4}{N(N-1)} \sum_{i \neq j} \frac{(2\pi)1.087 \text{ Hz}}{|\mathbf{r}_i - \mathbf{r}_j|^3/a^3}. \quad (\text{H3})$$

Further theoretical investigations can be modified to include quantum fluctuations and effective temperature in the motion using the fermionic truncated Wigner approximation [65].

APPENDIX I: MICROWAVE AND RF SETUP

To precisely manipulate the hyperfine states of our atoms, we generate a range of microwave pulses and rf frequency sweeps. The full configuration of the microwave and rf delivery system, including part numbers and signal pathways, is shown in Fig. 9.

For transitions between m_F levels within a given hyperfine manifold, we generate a 60.8-MHz rf signal that drives an in-vacuum magnetic field coil acting as an antenna. This rf tone is synthesized using an in-house built direct digital synthesizer, gated with a fast rf switch, and subsequently amplified with a 4-W amplifier driven to supply a 1-W signal.

To manipulate atoms within the spin-squeezing qubit manifold, we generate a 2.689-GHz microwave signal corresponding to the energy splitting between the states of our hyperfine qubit. This signal directly couples to the magnetic dipole moment of the qubit with a Rabi frequency of 870 Hz, driving a 10-W amplifier so as to output a 3-W signal [see Fig. 10(a)], yielding a π -pulse fidelity lower bound of $(0.999\,981 \pm 0.000\,003)$. This fidelity is measured through 2000 repeated π pulses, all applied with the same phase, separated by 2-ms waits over a total duration of 5 s. The extracted fidelity is determined from the amplitude of a Gaussian profile centered at the optimal π -pulse duration, with a FWHM of $(6.90 \pm 0.06) \mu\text{s}$, consistent with the measurement shown in Fig. 10(b). This effective Gaussian shape arises due to phase noise

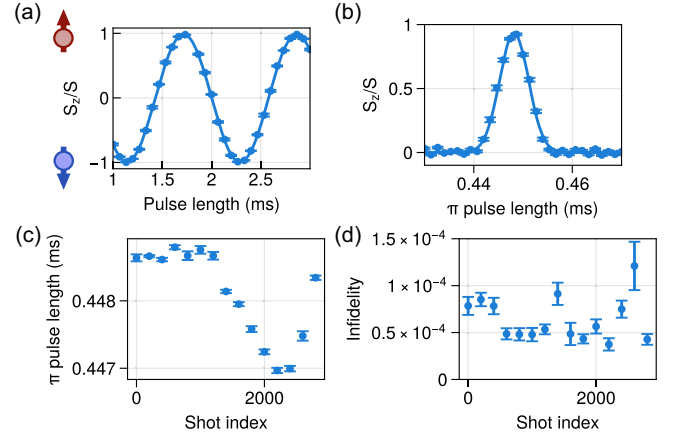


FIG. 10. Coherent microwave spin control. (a) Rabi oscillations at $\Omega = 870$ Hz of the hyperfine qubit, fitted to a sin function with amplitude (0.9882 ± 0.0007) and offset (0.0051 ± 0.0006) . Error bars are SEM. (b) Spin polarization after 400 consecutive π pulses as a function of pulse duration, fitted to a Gaussian with a FWHM of $(6.90 \pm 0.06) \mu\text{s}$. Error bars are SEM. (c) Drift in the π pulse length (inversely proportional to the Rabi frequency) measured over a 12-hour period. Error bars are the standard deviation of the estimated fit parameter (pulse length). (d) Drift in the π -pulse infidelity measured over a 12-hour period. Error bars are computed from the standard deviation of the estimated fit parameter (maximum spin polarization).

and dephasing from the XY interaction, which primarily affects atoms whose qubit states are not confined near the poles of the Bloch sphere.

To monitor long-term stability, we repeatedly perform 400 π pulses (with 4-ms intervals), tracking the peak of the Gaussian over 3000 shots grouped into batches of 200. This analysis reveals a 12-hour Rabi frequency drift of approximately 0.4% [Fig. 10(c)], which corresponds to a long-term average π -pulse fidelity of $(0.999\,84 \pm 0.000\,01)$ without active feedback [Fig. 10(d)]. We note that the instantaneous infidelity is lower than the approximately 10^{-4} quoted above due to our 200-shot batching procedure.

Both discrete microwave rotations and Landau-Zener transfers are employed for state preparation, squeezing, and readout between the two qubit states. We utilize two dedicated signal sources for these operations. For discrete microwave rotations, we derive the signal from a stable frequency reference. For Landau-Zener transitions, we employ a separate arbitrary waveform generator to produce frequency sweeps. These two signals are routed through an rf switch, amplified with a low-noise amplifier, and delivered to a loop antenna mounted externally to the vacuum chamber. This antenna is impedance matched via a shunt network, achieving over 99% transmission at the target frequency.

To avoid unintended qubit rotations or precession, a high extinction ratio is required between the signal generators and the antennae during idle periods. This is achieved by cascading multiple rf switches, providing sharp and reliable

signal cutoff. Furthermore, the 2.689-GHz qubit signal must meet stringent phase noise requirements: A phase jitter of just 1 degree would limit the maximum achievable squeezing to approximately -10 dB for 500 atoms in the absence of differential detection. To suppress such noise, both microwave sources are phase locked to a low-phase-noise 10-MHz reference oscillator (Wenzel 501-31686).

For monitoring the out-of-vacuum loop antenna, we use two diagnostics. First, a circulator and tunnel diode are used to monitor reflected power. Second, when not in use for rf frequency sweeps, the in-vacuum magnetic field coil is repurposed to detect the transmitted 2.689-GHz signal, verifying in-vacuum delivery.

APPENDIX J: INTERSPIN ON-SITE INTERACTION

1. Attempted measurement of on-site interspin interactions

We attempted to measure the on-site interspin interaction energy by preparing a coherent superposition of $|\uparrow\rangle$ and $|\downarrow\rangle$ at a bias magnetic field of approximately 1 G, far from the magnetically insensitive operating point. To dephase the spins—preventing two identical spin-polarized fermions from occupying the same site—we applied a magnetic field gradient for 600 ms. After dephasing, we modulated the amplitude of one of the science lattice axes for 2 s, aiming to resonantly drive a transition from two atoms on two separate sites to two atoms occupying a single site, which would manifest as heating.

However, no resonance features were observed in the amplitude modulation frequency range from 0 to 32 kHz that persisted in spin-polarized samples. The absence of such features suggests either that the interaction energy lies outside the explored frequency range or that the transition rate is suppressed. Although it seems unlikely that the on-site interaction energy is far beyond this range, we defer a detailed investigation to future work. Based on this outcome, we conclude that on-site interspin interactions are unlikely to play a significant role in the dynamics presented in this paper.

We estimate the on-site interaction energy to be repulsive and in the range of 900–3600 Hz. For atoms confined in our tight optical lattice, we estimate a contact interaction strength of approximately $30 \text{ Hz}/a_0$. Assuming the relevant background scattering length is comparable to that measured between $|F = 19/2, m_F = -19/2\rangle$ and $|F = 19/2, m_F = -17/2\rangle$, which was found to be $90a_0$ [66], the interaction energy would fall within our estimated range.

However, since magnetic insensitivity constrains us to operate at fixed bias fields, we cannot systematically tune the interaction via Feshbach resonances. Given the large number of such resonances in the lanthanides, the actual scattering length may differ considerably from the background value, further complicating interpretation.

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