

Quasicrystal Topological Hydrodynamics

Jun-liang Duan,^{1,2,*} Chuanjie Hu,^{2,3,*} Jiayun Ning,^{4,*} Xu-jie Chai,¹ Li-Wei Wang,^{5,6,7} Yang Dong,¹ Jianjun Liu,⁴ Shan Zhu,⁸
Huanyang Chen^{2,8,†}, Jian-Hua Jiang^{5,6,7,‡} and Jin-hui Chen^{1,2,8,§}

¹*Institute of Electromagnetics and Acoustics, Key Laboratory of Electromagnetic Wave Science and Detection Technology, Xiamen University, Xiamen 361005, China*

²*Shenzhen Research Institute of Xiamen University, Shenzhen 518000, China*

³*College of Physics, Nanjing University of Aeronautics and Astronautics, Key Laboratory of Aerospace Information Materials and Physics (NUAA), MIIT, Nanjing 211106, China*

⁴*Key Laboratory for Micro/Nano Optoelectronic Devices of Ministry of Education, School of Physics and Electronics, Hunan University, Changsha 410082, China*

⁵*State Key Laboratory of Bioinspired Interfacial Materials Science, Suzhou Institute for Advanced Research, University of Science and Technology of China, Suzhou 215123, China*

⁶*School of Physical Sciences, University of Science and Technology of China, Hefei 230026, China*

⁷*Institute for Quantum Science, School of Physical Science and Technology, Soochow University, Suzhou 215006, China*

⁸*Key Laboratory of Underwater Acoustic Communication and Marine Information Technology of the Ministry of Education, Department of Physics, Xiamen University, Xiamen 361005, China*



(Received 8 March 2026; accepted 29 July 2026; published 29 September 2026)

Quasicrystals extend topological concepts beyond periodic systems, but their exploration in mechanical wave systems remains elusive. Here, we experimentally realize two-dimensional quasicrystal water waves with fivefold rotational symmetry via the interference of multiple quasilattice waves. Although topologically indistinguishable in real space, these patterns are uniquely classified by higher-dimensional topological charge vectors that obey an angular-momentum-like conservation law. Using the water-wave amplitude and its in-plane gradient, we construct a synthetic three-dimensional vector field and demonstrate the emergence of skyrmions in water-wave quasicrystals. These skyrmions exhibit dynamically evolving boundaries and orientations, i.e., deformation and propagation due to phasonlike motion during the evolution of the quasicrystal water-wave field. Remarkably, the associated phase singularities can generate forces and torques that stably trap and drive the rotation of floating particles. We demonstrate in experiments the simultaneous manipulation of particles at multiple trapping sites in different ways, opening a path toward programmable control of particle dynamics. By bridging higher-dimensional topology with water-wave quasicrystals and skyrmions, our work not only advances the physics of topological hydrodynamics but also establishes a platform for on-demand particle control in aqueous environments, which is promising for applications in biomedicine, microfluidics, and lab-on-a-chip technologies.

DOI: [10.1103/rfgr-n77g](https://doi.org/10.1103/rfgr-n77g)

Subject Areas: Fluid Dynamics,
Interdisciplinary Physics

I. INTRODUCTION

The discovery of quasicrystals four decades ago modified the foundational paradigm of crystallography, revealing a

class of matter that exhibits long-range order and forbidden rotational symmetries such as five- and eightfold rotations, yet lacks translational periodicity [1–5]. This unique interplay of order and aperiodicity has since spawned a rich landscape of exotic phenomena, from wave localizations [6–14] and nontrivial topological states [15–18], to unconventional superconductivity [19–22]. A profound understanding of quasicrystals emerges from the higher-dimensional projection approach, in which they are viewed as projections of periodic lattices in higher-dimensional spaces [23–25]. This framework not only rationalizes their crystallographic properties but also establishes their inherent higher-dimensional geometry and topology [16,26]. Therefore, topological invariants defined in abstract

*These authors contributed equally to this work.

†Contact author: kenyon@xmu.edu.cn

‡Contact author: jhjiang3@ustc.edu.cn

§Contact author: jimchen@xmu.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

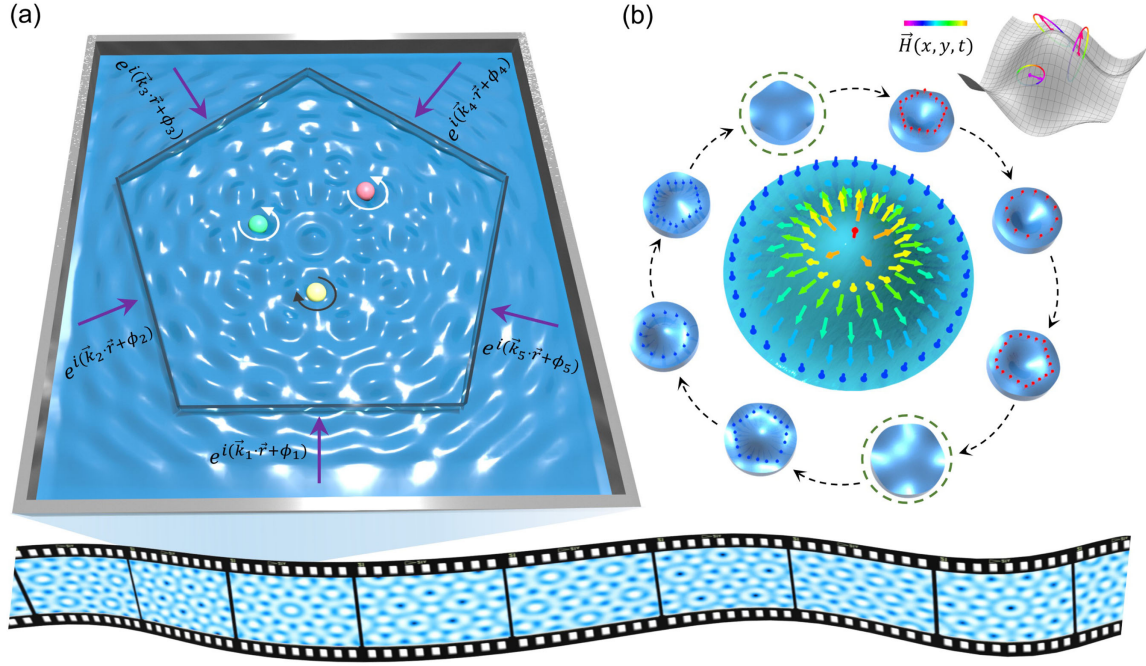


FIG. 1. Quasicrystal topological hydrodynamics based on multiple water-wave interference. (a) Schematic of quasicrystal water waves generated by five quasi-plane-wave interferences within a pentagonal structure. Multiple particles are trapped and rotated in the water-wave lattices. The rapid dynamics of quasicrystal lattices are revealed using the fast checkerboard demodulation method. (b) Theoretical results of the dynamic skyrmions embedded in quasicrystal water-wave lattices constructed by the 3D displacement field $[\vec{H}(x, y, t)]$ of water waves. The outer trajectory shows the time-dependent skyrmionic boundaries in a pentagonal quasicrystal over one evolution cycle. Two green-dashed circles mark the tracking-lost moment. The inset shows the local elliptical trajectories of the water-surface particles traced by their 3D displacement dynamics.

hyperspace can manifest themselves as physical phenomena in real-world quasicrystals—a principle recently confirmed through the manipulation of topological properties of bands [16,26] and charge vectors [27]. However, the exploration of such topological physics has remained largely confined to quantum and optical systems [16,26–28]. Moreover, the realization and stabilization of sophisticated three-dimensional topological defects such as skyrmions [29–37] in quasicrystals is also a desirable and nascent frontier [27].

Here, we report the emergence of unconventional topological structures in quasicrystal water waves engineered by sculpting surface water waves into programmable pentagonal quasiperiodic lattices [Fig. 1(a)]. Time-resolved snapshots of the complex amplitude of structured water waves are analyzed using a fast checkerboard demodulation (FCD) method. We demonstrate that the two-dimensional (2D) water-wave quasicrystals are isomorphic, as confirmed by their identical phase singularities (vortices) and spatial correlations. Nevertheless, these patterns are distinguishable by higher-order topological charge vectors interpreted as the Burgers vectors that obey an angular-momentum-related conservation law. Intriguingly, the underlying physics is found to be associated with topology in the emergent four-dimensional (4D) parameter space of water-wave quasicrystals. Benefiting from the slow

coherent dynamics of water waves, our experimental platform is able to track the evolution of quasicrystal water waves in real time using a conventional camera. In this way, we can observe persistent hydrodynamic skyrmions embedded in water-wave quasicrystals with orientation and boundary evolving periodically in time, continuously and globally (i.e., over all regions) over many wave cycles [Fig. 1(b)]. Moreover, the phase vortices associated with the quasicrystal lattices generate coherent wave-induced forces and torques that are strong enough to robustly trap and rotate particles floating on the water surface [38]. Remarkably, such torques vary with trapping locations and can even flip their directions from one location to another, enabling simultaneous yet distinct manipulation of multiple particles, as shown in Fig. 1(a). Our platform provides on-demand control of particle dynamics by quasicrystal water waves and opens a path toward precise single-particle manipulation and emergent collective dynamics of multiple particles.

II. EXPERIMENT RESULTS

A. Reconfigurable quasicrystal water waves

The interference of multiple directional plane waves provides a physical realization of the higher-dimensional projection method for the construction of quasicrystals [24,25,27,39]. Specifically, we employ a five-wave

interference scheme to create pentagonal quasicrystal water waves (Appendix A), as shown in Fig. 1(a). The out-of-plane displacement amplitude of the water surface $Z(\vec{r})$ due to such an interference is described by

$$Z(\vec{r}) = \text{Re} \sum_{n=1}^5 |A_n| e^{i\phi_n} e^{i\vec{k}_n \cdot \vec{r}}, \quad (1)$$

where $|A_n|$, ϕ_n , and $\vec{k}_n$ are the amplitude, phase, and wave vector of each incident water wave. Despite its simplicity, Eq. (1) allows extremely rich water-wave dynamics as unveiled in this work. First, the effective degree of freedom for the five-wave interference pattern is 4 since the amplitude and wave vectors are fixed in each experimental setup. Only the five phases are tunable. Taking into account that the overall phase does not affect the physical consequence of the water-wave interference, there are only 4 effective degrees of freedom spanning a 4D parameter space. One of the most intriguing findings is that multiple water-wave vortices can emerge by tuning the water-wave interference in the 4D parameter space. Remarkably, these water-wave vortices can have distinct properties: some of them are clockwise rotating, while others are counterclockwise rotating, as reflected directly by the rotation of floating particles on the water surface in our experiments, as discussed later.

Although water waves are mostly regarded as scalar waves characterized by the displacement profile in the z direction, these water-wave vortices clearly indicate the complexity of water waves. As shown in Ref. [37], the three-dimensional (3D) trajectory of displacement dynamics in water waves can even be elliptical, indicating local orbital angular momentum [see illustration in the inset of Fig. 1(b)]. To characterize this property, a 3D vector field was proposed for the water wave [37]: Starting with the out-of-plane displacement field $Z(x, y)$, the in-plane displacement vector fields can be created as $(X, Y) = \nabla_2 Z/k$, where $\nabla_2 = (\partial_x, \partial_y)$, and k is the absolute value of the wave vector [37,40]. With these components, a complex 3D vector field can be created as [37]

$$\vec{H} = X\hat{x} + Y\hat{y} + Z\hat{z}, \quad (2)$$

where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are the unit vectors in the x , y , and z directions. With this 3D vector field, water-wave vortices carrying orbital angular momentum can be characterized by skyrmion patterns [see Fig. 1(b)]. Intriguingly, in quasicrystal lattices, such skyrmion patterns are not static. They evolve cyclically due to the quasicrystal interference dynamics [Fig. 1(b)]. Notably, during such an evolution, the local skyrmion topological number is not conserved. However, there are conserved topological invariants in the 4D parameter space that govern the cyclical evolution of the skyrmion dynamics in water waves—another central

finding in this work that will be elaborated later. To study such a pattern dynamics, we record videos or take snapshots of the water waves. Because the water-wave dynamics are slow enough—significantly slower than photonic, acoustic, and other waves—using a high-speed camera, we can track the dynamics in quasicrystal lattices with high spatiotemporal resolution. From the recorded data, we analyze the rich water-wave dynamics and the underlying physics in quasicrystal lattices.

Experimentally, multiple quasi-plane-waves are generated within a 3D-printing pentagonal structure [Fig. 2(a)], where the five walls act as independent sources (Appendix B). These waves interfere coherently to form the quasicrystal lattices of water waves. We keep the excited waves of the same amplitude and frequency, but different propagation directions and phases (ϕ_n). Specifically, we used a water depth of 1.5 cm and set the excitation source frequency to 7.2 Hz (corresponding to a wavelength of approximately 3 cm; see Appendix C). The phase of each input plane wave is defined as $\phi_n = [(2\pi nQ)/5]$ and $Q \in \{0, \pm 1, \pm 2\}$. Consequently, the sum of these phases ($\Gamma = \sum_{n=1}^5 \phi_n$) is up to an integer of 2π [24,27]. The resultant out-of-plane displacement field of water wave $Z(x, y, t)$ is quantitatively measured using the FCD method (see Appendix B) [41]. The relatively low frequency and slow motion of water wave enable real-time probing and the reconstruction of complex water-wave dynamics by taking videos or snapshots of the water-wave displacement profile, as illustrated in the right panel of Fig. 2(a).

Figures 2(b)–2(d) present the time-averaged out-of-plane field amplitude distribution $|Z(x, y)|$ confirming the unique fivefold rotational symmetry of the quasicrystal lattice of water waves, which agrees well with the simulated results (see Appendix D). Intriguingly, these wave patterns are distinguishable by their higher-order topological charge vectors interpreted as the Burgers vectors that obey an angular-momentum-related conservation law (see Appendix F). The corresponding phase distributions (right panels) reveal that all possible phase singularities ($q = 0, \pm 1, \pm 2$) exist in each different type of lattice [27], although their spatial positions can be varied. While the central phase singularities can be used to identify the excitation Q values, the overall configuration of the phase singularities is identical across different types of quasicrystal lattices. This underscores that these 2D quasicrystals are globally isomorphic and cannot be totally characterized by conventional scalar phase topology alone, thus motivating the need for higher-dimensional topological charges for complete description, as will be discussed below.

To illustrate the similarity and difference of these 2D quasicrystal lattices of water waves, we measure and simulate the spatiotemporal correlations among topologically different quasicrystal water-wave lattices

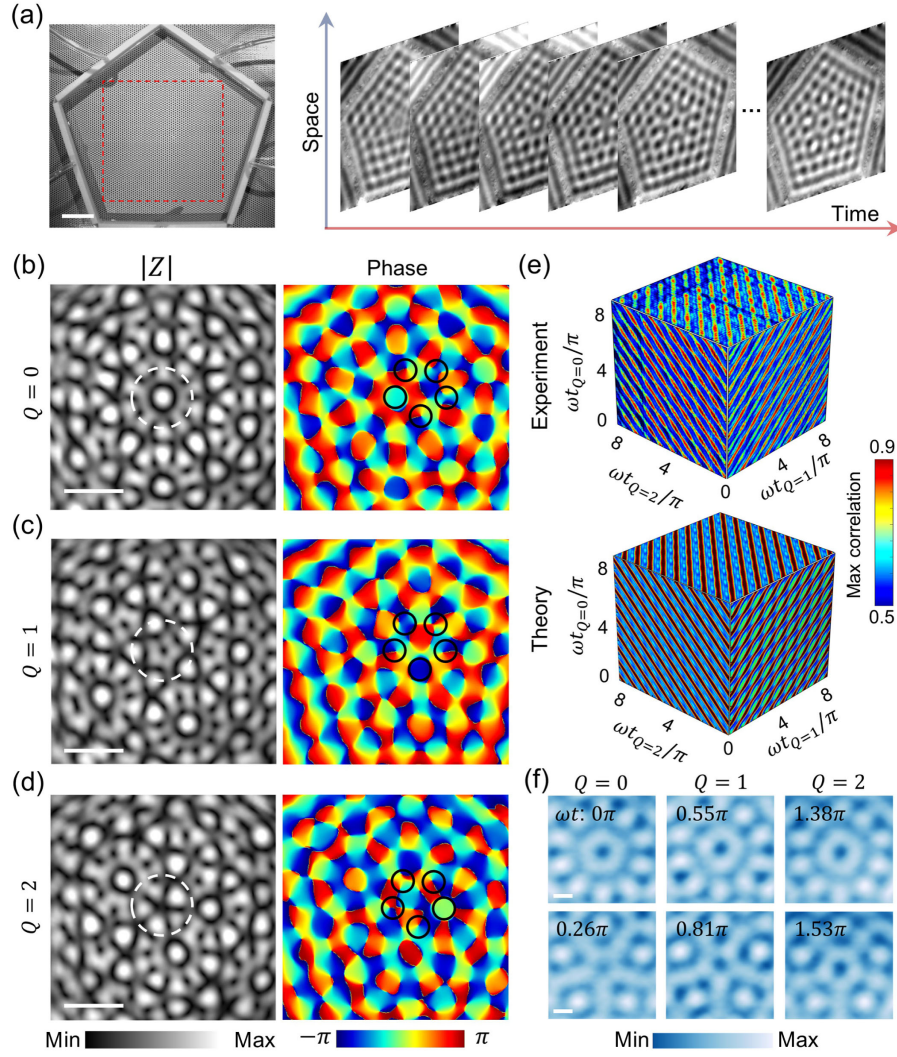


FIG. 2. Demonstration of reconfigurable quasicrystal water-wave lattices. (a) Experimental setup for generating five plane waves with the same amplitude and frequency but different propagation directions and phases (ϕ_n). Scale bar is 6 cm. The right panel shows the time evolution of five interacting water waves within a pentagonal structure. (b)–(d) The measured (absolute) amplitude and phase of out-of-plane displacement fields of water waves for different excitation Q values: (b) $Q = 0$, (c) $Q = 1$, (d) $Q = 2$. The white-dashed circles mark the central interference region, and the black circles mark all possible phase singularities ($q = 0, \pm 1, \pm 2$) in each of the quasicrystal lattices. The displayed field region is marked by the red-dashed box in (a). Scale bar is 6 cm. (e) The measured (upper panel) and simulated (lower panel) time-dependent correlation cube among three topologically distinct quasicrystal water-wave lattices with $Q = 0, 1, 2$. (f) Typical out-of-plane wave displacement fields with maximal correlation among the three quasicrystal lattices at different temporal phases. Scale bar is 2 cm.

corresponding to Q values 0, 1, 2, as shown in Fig. 2(e). The maximum correlations in the time domain reveal that quasicrystals with distinct topological landscapes can evolve into nearly identical patterns at certain times, with a constant recurrence period (see Appendix E). This feature is directly visualized by experiments in Fig. 2(f), which shows the temporal snapshots of the out-of-plane displacement field, confirming that different excitation Q values produce the same 2D quasicrystal lattices at different evolving times. This dynamical isomorphism provides clear evidence that the topology of a pentagonal quasicrystal remains hidden in conventional 2D spatial space.

B. Topological structures of the quasicrystal lattices

The form in Eq. (2) allows one to define two circular bases in the x - y plane: the left-circular field $H_+ = [(X + iY)/\sqrt{2}]$ and the right-circular field $H_- = [(X - iY)/\sqrt{2}]$. Projection theory can further assign a topological charge vector in hyperspace to each field component: $\vec{b}_Z$, $\vec{b}_+$, and $\vec{b}_-$ for Z , H_+ , and H_- , respectively (see Appendix F) [27,31]. For an out-of-plane topological charge $\vec{b}_Z = \vec{b}_Q$, the in-plane circular components carry distinct charge vectors $\vec{b}_\pm$: (i) when $\vec{b}_Z = \vec{b}_{Q=0}$, $\vec{b}_+ = \vec{b}_{Q=1}$, $\vec{b}_- = \vec{b}_{Q=-1}$,

(ii) when $\vec{b}_Z = \vec{b}_{Q=1}$, $\vec{b}_+ = \vec{b}_{Q=2}$, $\vec{b}_- = \vec{b}_{Q=0}$, and
 (iii) $\vec{b}_Z = \vec{b}_{Q=2}$, $\vec{b}_+ = \vec{b}_{Q=-2}$, $\vec{b}_- = \vec{b}_{Q=6} = \vec{b}_{Q=1}$, since Q is a modulo-5 integer. Intriguingly, these topological charge vectors conform to the angular-momentum-related conservation law such that $\vec{b}_+ + \vec{b}_- = 2\vec{b}_Z$ (see Appendix F). This is due to the different rotation directions of the two in-plane fields and the fact that the out-of-plane field does not rotate. This effect can be analogous to the spin-orbit interaction of light in near fields [27]. The distinct yet conserved relationships between $\vec{b}_Z, \vec{b}_+, \vec{b}_-$ thereby establish a comprehensive descriptor of the topology of water-wave quasicrystals.

In the emergent 4D parameter space of quasicrystals, the Burgers vector $\vec{b} = (b_1, b_2, b_3, b_4)$ describes a dislocation. Because only the b_2 and b_4 components in this 4D vector are nonzero (see Appendix A for details), all topological charge vectors can be conveniently represented in the effective 2D parameter space (b_2, b_4) . For each quasicrystal lattice, the triplet of topological charge vectors $(\vec{b}_Z, \vec{b}_+, \vec{b}_-)$ forms either a triangle or a degenerate line segment in vector space (see Appendix F). A line integral around the area yields exactly the Q value for the excitation, revealing that topological charge conservation manifests geometrically as area conservation in this vector representation.

Skyrmions are topologically stable quasiparticles that have been predicted and demonstrated across various platforms, such as solid-state matter, liquid crystals, and electromagnetic waves [30,31,33,36,42–44] but only recently observed in water waves in a limited number of studies [35,37,45]. In our platform, the skyrmion number N for the constructed water-wave vector field is defined as

$$N = \frac{1}{4\pi} \iint \vec{\bar{H}} \cdot (\partial_x \vec{\bar{H}} \times \partial_y \vec{\bar{H}}) dx dy, \quad (3)$$

where $\vec{\bar{H}}(\bar{X}, \bar{Y}, \bar{Z})$ is the normalized 3D water-wave vector field. Here, the skyrmion boundary is defined as the closed contour associated with the local extremum of the vertical component $\bar{Z}$ of the normalized three-dimensional displacement field $\vec{\bar{H}}$, where $\bar{Z}$ approaches $+1$ or -1 [36,46]. An active-contour algorithm can be used to extract this physically defined closed boundary [47]. Theoretical analysis shows that in quasicrystal lattices, each skyrmion can have three distinct boundary geometries: the pentagonlike, inverted pentagonlike, and circular shapes in a cycle of the harmonic-wave evolution [see Fig. 3(a) and Appendix G]. Interestingly, the pentagonlike and circular boundaries can be interconverted dynamically over time in a cycle. During a trackable period, the skyrmion boundary is initially pentagonlike, evolves gradually into a circular shape, then deforms into an inverted pentagonlike shape, and finally vanishes, reappearing with the skyrmion sign flipped, and repeating over a trackable period.

In experiments, without loss of generality, we measure and reconstruct the temporal 3D fields for $Q = 2$ quasicrystal to visualize the skyrmion distribution and track their boundary evolution, as shown in Figs. 3(bi)–3(di). It is observed that these skyrmions emerge only around a zero-phase-singularity ($q = 0$) region. The three typical skyrmionic boundaries—pentagonlike, circular, and inverted pentagonlike shapes—are measured and observed in accordance with the theoretical predictions. These unique phasonlike dynamics within the quasicrystal lattices are induced by the temporal phases of the interfering quasicrystal plane-waves [24,48]. Unexpectedly, although the local skyrmion texture in a given region can transiently become ill-defined or annihilate during the temporal evolution, well-defined skyrmion textures continuously exist in other regions of the quasicrystal lattice, as shown in Figs. 3(e) and 3(f). For time-periodic harmonic-wave vector fields, the instantaneous skyrmion number can reverse sign since the time-reversal symmetry is conserved. Additional calculations comparing a periodic hexagonal lattice and the pentagonal quasicrystal lattice show that, while the periodic lattice mainly exhibits sign reversal of the integrated skyrmion number over a large region, the quasicrystal lattice allows a more continuous temporal evolution because of its phasonlike dynamics and the absence of translational unit cells (see Appendix G). We further experimentally verify that a local skyrmion texture remains robust against particle-induced perturbation (see Appendix G). This sustained existence of skyrmions in water-wave quasicrystals, together with their dynamic evolution, is in stark contrast with the observed skyrmions in periodic systems in Ref. [37], highlighting an unprecedented topological phenomenon in water-wave quasicrystals. In addition to individual skyrmions, we discover skyrmion clusters emerging in water-wave quasicrystals with complex topological landscapes (see Appendix H). While this work focuses on the skyrmion dynamics of the 3D vector field $\vec{H}$, similar concepts can be naturally extended to the spin-angular-momentum field of water waves [37], which warrants future investigations.

C. Multiple particle manipulation in quasicrystal lattices

As surface matter waves, water waves exert radiation forces on floating particles [Fig. 4(a)], in close analogy to their optical and acoustic counterparts [49–52]. Based on the established dipole models [37,53–55], we describe the particle-field interaction via the wave momentum density $\vec{P} = (\rho\omega/2) \text{Im}[\vec{H}^* \cdot (\nabla_2) \vec{H}]$ and kinetic energy density $W = \rho\omega^2 |\vec{H}|^2/4$, where ρ is the mass density of water, and ω is the angular frequency. Assume a particle with single complex polarizability α , the wave-induced force is given by (see Appendix I) [37]

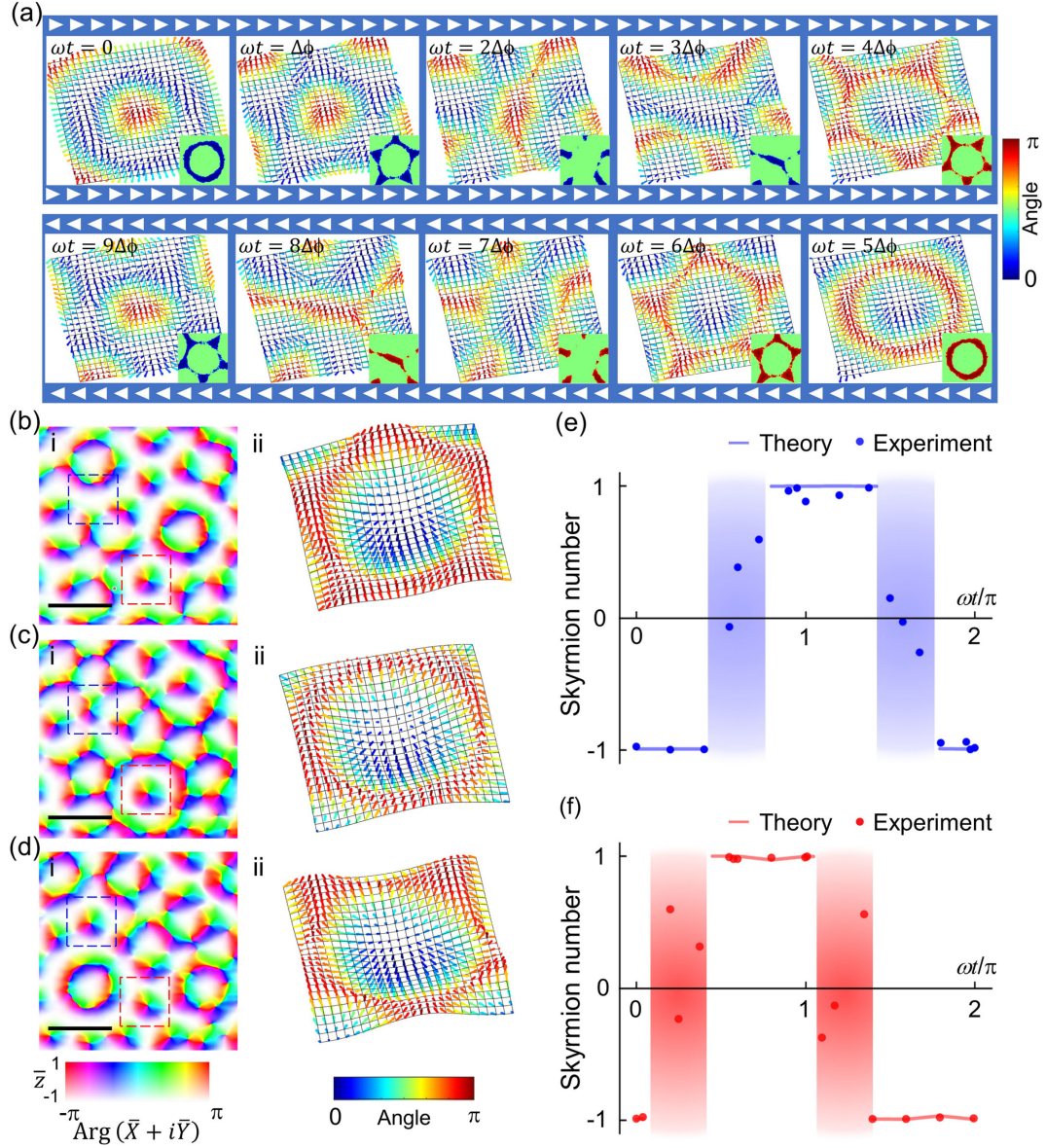


FIG. 3. Skyrmionic dynamics in water-wave quasicrystal lattices. (a) Simulated results of the phason-induced periodic skyrmion dynamics within a cycle (ten snapshots at equally separated times with $\Delta\phi = \pi/5$). Arrows represent the 3D normalized displacement field $\vec{H}(x, y, t)$ of the water wave. The color of the arrows represents the intersection angle between the $\vec{H}(x, y, t)$ and the z axis (color bar is on the right side of the figure). Insets in the lower-right corners of each snapshot illustrate the extracted skyrmion boundaries. (b)–(d) The measured $\vec{H}(x, y, t)$ for $Q = 2$ water-wave quasicrystal lattices at different times: (b) $\omega t = 1.6\pi$, (c) $\omega t = 1.8\pi$, (d) $\omega t = 2\pi$. The measured region is marked by a red-dashed box in the experimental setup illustrated in Fig. 2(a). Panels (i) are $\vec{H}(x, y, t)$ profiles encoded by transparency of the mask in white (for the out-of-plane component) and colors (for the azimuth angle of in-plane components). Panels (ii) are vectorial representations [the same way as in (a)] of the $\vec{H}(x, y, t)$ field enlarged for the subregions marked by the red-dashed boxes in panels (i) with the color of the arrows representing the intersection angle between the $\vec{H}(x, y, t)$ and the z axis (color bar is on the bottom of the figure). (e), (f) Theoretical and experimental results of time-dependent skyrmion number in different local regions: (e) for the blue-dashed boxes and (f) for the red-dashed boxes in (b)–(d). Here, the blue and red shadows represent durations without water-wave skyrmions, during which the water-wave skyrmions disappear. All scale bars (black) are 6 cm.

$$\vec{F} = \omega \text{Im}(\alpha) \vec{P} + \text{Re}(\alpha) \nabla_2 W \equiv \vec{F}_{\text{press}} + \vec{F}_{\text{grad}}. \quad (4)$$

$$T_{\text{spin}} = \omega \text{Im}(\alpha) S_z. \quad (5)$$

In turn, the out-of-plane torque on a small particle involves only the dipole moment and can be derived as

Here, $\vec{S} = (\rho\omega/2) \text{Im}(\vec{H}^* \times \vec{H})$ represents the spin density of the water-wave structure. We map the force field within

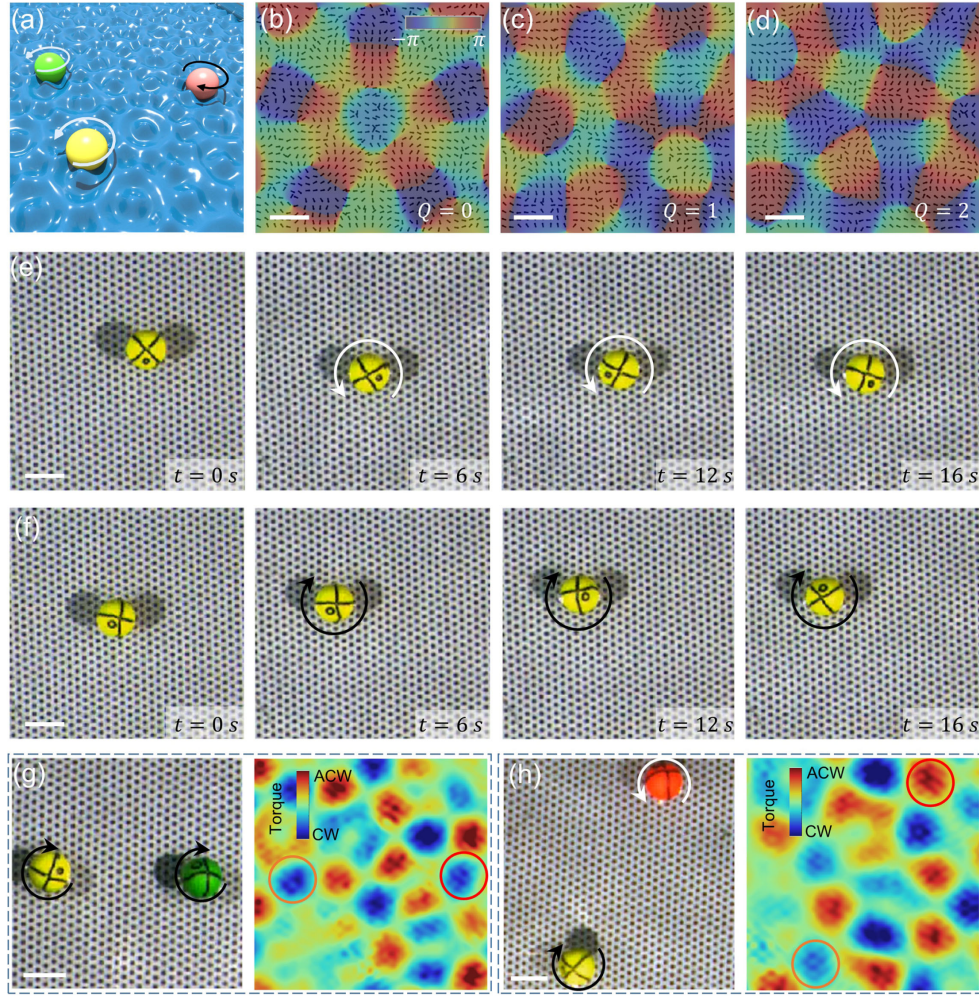


FIG. 4. Dynamics of floating particles in the quasicrystal water waves. (a) Schematic of rotating particles driven by the quasicrystal wave fields. The particles can be captured and rotated at specific locations within quasicrystal water-wave lattices. (b)–(d) Water-wave-induced force (black vectors) distributions in the central region of quasicrystal lattices of Q values: (b) $Q = 0$, (c) $Q = 1$, (d) $Q = 2$. The background-color plots are the phase distribution of the out-of-plane displacement fields showing the phase singularities. (e), (f) Snapshots of single particle (radius of 1 cm) being trapped and rotated by quasicrystal water waves: (e) anticlockwise rotation in $Q = 1$ lattices, (f) clockwise rotation in $Q = 2$ lattices. The particles are initially deployed around the central region of pentagonal lattices. (g), (h) Dual rotating particles trapped in different locations of quasicrystal lattices of $Q = 0$. The left panels in (g) [(h)] show clockwise-anticlockwise (clockwise-anticlockwise) rotating particles. The right panels in (g) and (h) show the torque distribution of water-wave quasicrystals in a selected region. The orange and red circles mark the positions of the corresponding particles. All scale bars (white) are 2 cm.

the central region of the pentagonal structures for the quasicrystal lattices of $Q = 0, 1, 2$ [Figs. 4(b)–4(d)]. For phase singularity of $q = 0$, the force field lacks a well-defined orientation, while $q = \pm 1$ generates robust, vortical force fields that enable both particle trapping and controlled driving. Higher phase singularities ($q = \pm 2$) are not topologically stable since their corresponding rotational fields are complicated, incompatible with global dynamics, and therefore unable to sustain stable collective motions. Consequently, robust and continuous rotation emerges exclusively around topological singularities with $q = \pm 1$. This property, in turn, indicates that the induced particle motion provides a

visually mechanical probe of the underlying topological structures in water waves.

In experiments, particle size needs to match the scale of the phase-singularity region, which mitigates parasitic forces and guarantees stable rotation. We used foam particles of a diameter of 2 cm to probe the mechanical responses of quasicrystal water waves at 6.1 Hz (corresponding to a wavelength of approximately 4 cm). Figure 4(e) shows the dynamic trajectory of a particle in the $Q = 1$ quasicrystal wave fields (see Supplemental Material Video 1 [56]). The particle is rapidly attracted to the center of the phase singularity with $q = 1$ and undergoes a stable anticlockwise self-rotation. The measured angular velocity is

approximately $[(2\pi)/3]$ rad/s. Figure 4(f) presents the corresponding dynamics in the $Q = 2$ lattice of water-wave quasicrystal (see Supplemental Material Video 2 [56]). Although the particle initially approaches the central singularity $q = 2$, it is quickly drawn to a neighboring singularity with $q = -1$, where it becomes stably trapped and self-rotates clockwise. These observed particle dynamics are in excellent agreement with the theoretical predictions for quasicrystal lattices (see more results in Appendix J and Supplemental Material Video 3 [56]).

As manipulation capabilities extend to multiple particles, this demonstrates our platform's versatile ability to trap and control particles floating on water surfaces. We observe stable corotation [Fig. 4(g) and Supplemental Material Video 4 [56]] and counterrotation [Fig. 4(h) and Supplemental Material Video 5 [56]] of particle pairs, with dynamics dictated by the theoretical torque distributions [right panels of Figs. 4(g) and 4(h), Appendix I]. The spinning dynamics of the particles are intrinsically determined by the local topological structures of the water-wave quasicrystal lattices. Most notably, we successfully achieved the simultaneous rotation of up to five particles introduced into a quasicrystal water-wave lattice of $Q = 0$, highlighting its potential for versatile hydrodynamic particle control (Appendix K and Supplemental Material Videos 6–8 [56]).

III. SUMMARY AND DISCUSSION

Our work demonstrates that classical water-wave systems provide a fertile, experimentally accessible platform for exploring higher-dimensional topological physics, a domain previously dominated by quantum and optical settings. By engineering pentagonal quasicrystal lattices through multiwave interference, we have realized a hydrodynamic analog of topological quasicrystals characterized not by conventional scalar phase singularities, but by a triad of higher-dimensional topological charge vectors—Burgers vectors in an emergent 4D hyperspace. This extends the framework of higher-dimensional topology to classical hydrodynamics, establishing a new paradigm for studying topological structures in water waves and other fluidic systems.

Compared with previously reported plasmonic quasicrystal platforms [27], the present water-wave system provides two distinctive capabilities enabled by its low-frequency, macroscopic, and mechanical nature. First, the slow hydrodynamic evolution allows the quasicrystal vector field and the associated skyrmion textures to be tracked directly in real time and real space. Second, because the water-wave field is a real mechanical displacement field carrying momentum and angular momentum, its local topological structures can generate forces and torques that are strong enough to drive observable motion of floating particles—a phenomenon potentially useful in biophysics and other related fields. Moreover, in contrast to periodic-

lattice wave interference systems [37], the quasicrystal interference fields studied here provide more complex non-periodic rotational vector-field textures for particle manipulation. The five-wave quasicrystal interference patterns provide more localized rotational confinement that gives rise to suppressed orbital motion and significant spinning motion. Our platform establishes a direct, experimentally accessible topology-dynamics coupling with high tunability through the strong wave-induced mechanical forces.

A striking manifestation of this topology is the persistent yet dynamically evolving skyrmionic textures embedded within the water-wave quasicrystal lattice. These hydrodynamic skyrmions exhibit real-time periodic-boundary-transformation dynamics between pentagonal and circular shapes, while remaining continuously observable at the global-field level throughout the temporal evolution. This spatiotemporally persistent behavior enriches the broader understanding of skyrmion dynamics and wave physics in aperiodic systems.

Beyond fundamental insights, this platform enables precise, programmable manipulation of floating particles via wave-induced forces and torques arising from phase singularities. We have demonstrated stable trapping, controlled rotation, and orchestrated multiparticle dynamics—including corotation and counterrotation pairs—directly governed by the underlying topological structures of the quasicrystal. Furthermore, such capabilities enable analogs of optical and acoustic tweezers in water waves, opening a new avenue for topological particle manipulation. The applications of this approach are particularly promising in biomedical and lab-on-chip environments, as water waves offer a noninvasive, scalable, and energy-efficient means to handle cells, particles, and biological aggregates in aqueous media.

ACKNOWLEDGMENTS

The authors thank Professor Yi Xu for helpful discussions. This work is supported by National Key Research and Development Program of China (Grants No. 2023YFA1407104 and No. 2022YFA1404400), National Natural Science Foundation of China (Grants No. 12274357, No. 12125504, No. 62475223, and No. 12374410), the “Hundred Talents Program” of the Chinese Academy of Sciences, Shenzhen Science and Technology Program (Grants No. JCYJ20240813145614019 and No. JCYJ20230807091300001), Fujian Provincial Natural Science Foundation of China (Grant No. 2023J06011), and the Priority Academic Program Development of Jiangsu Higher Education Institutions.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not

technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: THEORY OF QUASICRYSTAL LATTICES AND TOPOLOGICAL CHARGE VECTORS

The charge-density-wave model is developed in Refs. [23–25,27,39] to describe quasicrystals through interference of scalar waves. Based on Landau-Lifshitz theory of condensed matter, the order parameter generating the phase of a material must transform as an irreducible representation of the symmetry group. Such a representation can come in the form of a Fourier series of the charge density [24,39], whose spatial frequencies are wave vectors in the reciprocal lattice of the material, with N_R elementary lattice vectors. In regular crystals, $N_R = d$, where d is the spatial dimension of the crystal, whereas quasicrystals have n_i incommensurate length scales, resulting in $N_R = n_i d$. Thus, a 2D pentagonal quasicrystal can be described as

$$\rho(\vec{r}) = \text{Re} \sum_{n=0}^{N_R} |\rho_n| e^{i\phi_n} e^{i\vec{k}_n \cdot \vec{r}}, \quad (\text{A1})$$

where $|\rho_n|$, ϕ_n , and $\vec{k}_n$ are the amplitude, phase, and wave vector of each plane wave in the interference pattern, respectively; $\vec{r}$ is the spatial position vector. This equation is exactly the same as Eq. (1) in the main text. The material is stationary and results in the condition of $\sum_{n=0}^{N_R} \vec{k}_n = 0$. The phase of the crystal and any transitions to and from it can be described by the Landau free energy, which can be expanded as a power series of the density ρ_r [23–25,27,39]. Close to the ground state, we can approximate that only the leading term in the lattice symmetry contributes to the free energy (E) [23–25,27,39]:

$$E \approx \prod_{n=0}^{N_R} |\rho_n| e^{i\phi_n} + \text{c.c.} \propto \cos(\Gamma), \quad (\text{A2})$$

where $\Gamma = \sum_{n=0}^{N_R} \phi_n$ is the sum of phases. Therefore, the sum of the phases determines the free energy of the quasicrystal, up to a constant multiplying factor. If Γ is constrained to be a constant, i.e., the same free energy for all solutions, we reach that only the phases $\phi_n, n = 1, \dots, N_R$ are necessary to describe the system. For a 2D pentagonal quasicrystal, this constraint leaves 4 independent-phase degrees of freedom, consistent with the two incommensurate length scales ($n_i = 2$) in two spatial dimensions ($d = 2$).

The point-group symmetry of a 2D pentagonal quasicrystal embodied by the condition $\sum_{n=0}^4 \vec{k}_n = 0$ fixes the orientations of the five reciprocal wave vectors as

$$\vec{G}_n = G[\cos(2\pi n/5), \sin(2\pi n/5)], \quad n = 0, \dots, 4, \quad (\text{A3})$$

where $G = |\vec{k}|$ sets the characteristic length scale of the interference pattern. For a pentagonal quasicrystal, the reciprocal lattice has four basis vectors, implying that a higher-dimensional representation is required to parametrize the independent phase degrees of freedom. Accordingly, in addition to the two-dimensional reciprocal vectors $\vec{G}_n$, we introduce another 2D vector set $\vec{G}_{\langle 3n \rangle_5}$ and combine them to form a four-dimensional vector:

$$\begin{aligned} \tilde{\vec{G}}_n &= \vec{G}_n \oplus \alpha \vec{G}_{\langle 3n \rangle_5} \\ &= G[\cos(2\pi n/5), \sin(2\pi n/5), \alpha \cos(2\pi \langle 3n \rangle_5/5), \\ &\quad \alpha \sin(2\pi \langle 3n \rangle_5/5)], \end{aligned} \quad (\text{A4})$$

where α is a scale factor controlling the relative weight of the two-phase subspaces, and $\langle a \rangle_b$ stands for $\text{mod}(a, b)$.

To construct a convenient real-space basis associated with reciprocal vectors, we introduce a set of elementary real-space vectors $\vec{R}_n = R[-\sin(2\pi n/5), \cos(2\pi n/5)]$, each of which is chosen to be perpendicular to the corresponding reciprocal vector $\vec{G}_n$. Their magnitudes are fixed by the characteristic length scale of the pattern. The four-dimensional real-space basis is then constructed as $\tilde{\vec{R}}_n = \vec{R}_n \oplus \beta \vec{R}_{\langle 3n \rangle_5}$, where β is a scale factor controlling the relative weight of the two subspaces. A Burgers vector $\vec{b}$ describing a dislocation can then be expanded in this real-space basis as $\vec{b} = [[b_1, b_2][b_3, b_4]] = r_1 \tilde{\vec{R}}_1 + r_2 \tilde{\vec{R}}_2 + r_3 \tilde{\vec{R}}_3 + r_4 \tilde{\vec{R}}_4 = (\vec{b}_{\text{phonon}}, \vec{b}_{\text{phason}})$, where $\vec{b}_{\text{phonon}}$ represents displacements in physical space, while $\vec{b}_{\text{phason}}$ corresponds to displacements in the additional internal degrees of freedom. Then we have

$$\begin{aligned} \phi_{j,\mu} &= \tilde{\vec{G}}_j \cdot \tilde{\vec{R}}_\mu \\ &= GR \left\{ \sin \left[\frac{2\pi(j-\mu)}{5} \right] + \alpha \beta \sin \left[\frac{6\pi(j-\mu)}{5} \right] \right\}. \end{aligned} \quad (\text{A5})$$

Along with $\phi_{j,j} = 0$, these parameters R , α , and β are determined by requiring $\phi_{j,j+2} = -\phi_{j,j+3} = 2\pi$ and $\phi_{j,j+1} = -\phi_{j,j+4} = 0$. This yields $GR = 8\pi \sin[(4\pi)/5]/5$ and $\alpha\beta = \sin[(2\pi)/5]/\sin[(4\pi)/5]$. In order to make the Burgers lattice as simple as possible, we set $\beta = 1$, then $\alpha = \sin[(2\pi)/5]/\sin[(4\pi)/5]$.

We can parametrize the four independent components of ϕ_n with two two-component fields:

$$\phi_n = \vec{G}_n \cdot \vec{b}_{\text{phonon}} + \alpha \vec{G}_{(3n)5} \cdot \vec{b}_{\text{phason}} + \frac{\Gamma}{5} = \vec{G}_n \cdot \vec{b} + \frac{\Gamma}{5}, \quad (\text{A6})$$

where $\Gamma = \sum_{n=0}^{N_R} \phi_n$ is the sum of all phases. In this work, the phase of each input port is $\phi_n = [(2\pi nQ)/5]$, thus $\Gamma = 0$ or integers of 2π . The phase shifts induced by a Burgers vector are defined modulo 2π . Accordingly, we impose the constraint

$$\phi_n \equiv \left\langle \frac{2\pi nQ}{5} \right\rangle_{2\pi}, \quad (\text{A7})$$

which specifies a representative phase configuration corresponding to the topological charge Q . Returning to Eq. (A6), we can obtain $[b_1, b_2, b_3, b_4]$ by solving Eqs. (A6) and (A7). Finally, by solving equation $\phi_n = \vec{G}_n \cdot (r_1 \tilde{R}_1 + r_2 \tilde{R}_2 + r_3 \tilde{R}_3 + r_4 \tilde{R}_4) + \Gamma/5 = \langle (2\pi nQ)/5 \rangle_{2\pi}$, we can obtain the coefficients of each dislocation basis vector $\{r_1, r_2, r_3, r_4\}$. Therefore, the Burgers vectors can be obtained:

$$\begin{aligned} \vec{b}_{Q=0} &= (0, 0, 0, 0), \\ \vec{b}_{Q=\pm 1} &= \pm \frac{R}{\sqrt{5}}(0, \varphi, 0, \varphi - 1) = \mp \frac{2}{5} \tilde{R}_1 \pm \frac{1}{5}(\tilde{R}_3 + \tilde{R}_4), \\ \vec{b}_{Q=\pm 2} &= \pm \frac{R}{\sqrt{5}}(0, 1, 0, -1) = \pm \frac{1}{5} \tilde{R}_1 \pm \frac{2}{5}(\tilde{R}_3 + \tilde{R}_4), \end{aligned} \quad (\text{A8})$$

where $\varphi = [(1 + \sqrt{5})/2]$. The above constraint admits infinitely many equivalent Burgers vectors differing by elements of the Burgers-vector lattice. Among these, we select minimal-norm representatives, which correspond to elementary Burgers vectors and yield the simplest phase-winding configurations.

According to Eq. (A6), the configuration of the phases ϕ_n of the five waves determines the dislocation vector in four-dimensional space and influences the two-dimensional projection of the quasicrystal. We utilize Q to adjust the total phase sum of the five waves, that is, $\phi_n = [(2\pi nQ)/5]$, $Q \in \{0, \pm 1, \pm 2\}$. Then, $\vec{b}$ represents the four-dimensional topological charge or topological charge vector and obeys the conservation law of the topological charge vector [27]

$$\vec{b}_{Q_1} - \vec{b}_{Q_2} = \vec{b}_{Q_1 - Q_2}, \quad (\text{A9})$$

where $\vec{b}_{Q_1}$, $\vec{b}_{Q_2}$, $\vec{b}_{Q_1 - Q_2}$ are different topological charge vectors. Moreover, since there are two incommensurate length scales in the quasicrystalline system [23,25], the four-dimensional topological charge vector can be chosen such that it has only two nonzero components: one with the topology of the $\vec{b}_{\text{phonon}}$ and the actual two-dimensional space, and the other with the topology of the two additional dimensions $\vec{b}_{\text{phason}}$ of the quasicrystal. Below is a list of all possible permutations of Eq. (A9):

$$\begin{aligned} \vec{b}_{Q=0} - \vec{b}_Q &= \vec{b}_{-Q}, \\ \vec{b}_{Q=\pm 1} - \vec{b}_{Q=\mp 1} &= 2\vec{b}_{Q=\pm 1} = \mp \frac{4}{5} \tilde{R}_1 \pm \frac{2}{5}(\tilde{R}_3 + \tilde{R}_4) = \pm \frac{\langle 1 \rangle_5}{5} \tilde{R}_1 \pm \frac{2}{5}(\tilde{R}_3 + \tilde{R}_4) = \pm \frac{1}{5} \tilde{R}_1 \pm \frac{2}{5}(\tilde{R}_3 + \tilde{R}_4) = \vec{b}_{Q=\pm 2}, \\ \vec{b}_{Q=\pm 2} - \vec{b}_{Q=\mp 2} &= 2\vec{b}_{Q=\pm 2} = \pm \frac{2}{5} \tilde{R}_1 \pm \frac{4}{5}(\tilde{R}_3 + \tilde{R}_4) = \pm \frac{2}{5} \tilde{R}_1 \mp \frac{\langle 1 \rangle_5}{5}(\tilde{R}_3 + \tilde{R}_4) = \pm \frac{2}{5} \tilde{R}_1 \mp \frac{1}{5}(\tilde{R}_3 + \tilde{R}_4) = \vec{b}_{Q=\mp 1}, \\ \vec{b}_{Q=1} - \vec{b}_{Q=2} &= -\frac{3}{5} \tilde{R}_1 - \frac{1}{5}(\tilde{R}_3 + \tilde{R}_4) = \frac{\langle 2 \rangle_5}{5} \tilde{R}_1 - \frac{1}{5}(\tilde{R}_3 + \tilde{R}_4) = \frac{2}{5} \tilde{R}_1 - \frac{1}{5}(\tilde{R}_3 + \tilde{R}_4) = \vec{b}_{Q=-1} = -\vec{b}_{Q=1} = -(\vec{b}_{Q=2} - \vec{b}_{Q=1}) \\ \vec{b}_{Q=-1} - \vec{b}_{Q=-2} &= \frac{3}{5} \tilde{R}_1 + \frac{1}{5}(\tilde{R}_3 + \tilde{R}_4) = -\frac{\langle 2 \rangle_5}{5} \tilde{R}_1 + \frac{1}{5}(\tilde{R}_3 + \tilde{R}_4) \\ &= -\frac{2}{5} \tilde{R}_1 + \frac{1}{5}(\tilde{R}_3 + \tilde{R}_4) = \vec{b}_{Q=1} = -\vec{b}_{Q=-1} = -(\vec{b}_{Q=-2} - \vec{b}_{Q=-1}) \\ \vec{b}_{Q=1} - \vec{b}_{Q=-2} &= -\frac{1}{5} \tilde{R}_1 + \frac{3}{5}(\tilde{R}_3 + \tilde{R}_4) = -\frac{1}{5} \tilde{R}_1 + \frac{\langle -2 \rangle_5}{5}(\tilde{R}_3 + \tilde{R}_4) \\ &= -\frac{1}{5} \tilde{R}_1 - \frac{2}{5}(\tilde{R}_3 + \tilde{R}_4) = \vec{b}_{Q=-2} = -\vec{b}_{Q=2} = -(\vec{b}_{Q=-2} - \vec{b}_{Q=1}), \\ \vec{b}_{Q=-1} - \vec{b}_{Q=2} &= \frac{1}{5} \tilde{R}_1 - \frac{3}{5}(\tilde{R}_3 + \tilde{R}_4) = \frac{1}{5} \tilde{R}_1 + \frac{\langle 2 \rangle_5}{5}(\tilde{R}_3 + \tilde{R}_4) = \frac{1}{5} \tilde{R}_1 + \frac{2}{5}(\tilde{R}_3 + \tilde{R}_4) = \vec{b}_{Q=2} = -\vec{b}_{Q=-2} = -(\vec{b}_{Q=2} - \vec{b}_{Q=-1}). \end{aligned} \quad (\text{A10})$$

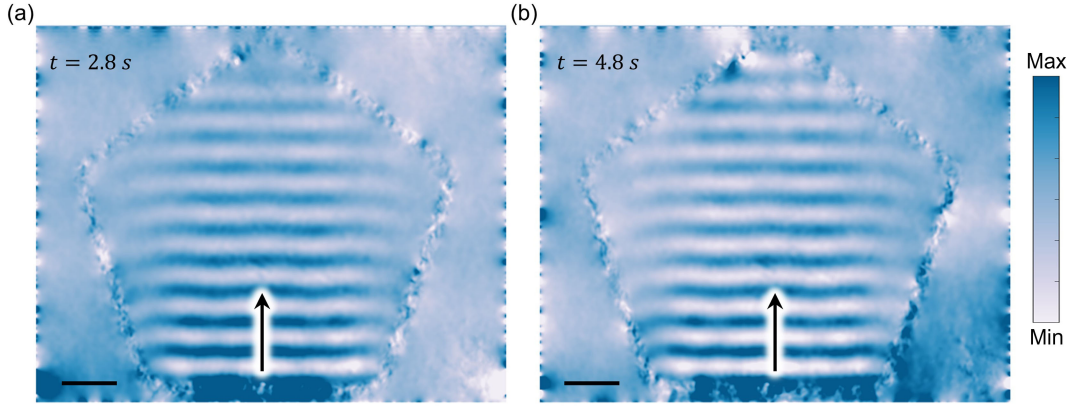


FIG. 5. Single-edge excitation experiment in the pentagonal tank. (a) The plane wave generated from the bottom edge of the pentagonal structure propagates upward and reaches the upper edges at $t = 2.8$ s. (b) The stable water-wave fields in the pentagonal tank region at $t = 4.8$ s with no significant reflected-wave interference observed. All scale bars (black) are 6 cm.

APPENDIX B: EXPERIMENTAL SETUP

The interference field was generated inside a 3D-printed pentagonal structure with a side length of 24 cm. The source sides are connected with tubing to speakers controlled by signal generators (RIGOL DG4062) and an amplifier (FPA2000-30W). We used a sinusoidal signal at 7.2 Hz (6.1 Hz) for characterizing the topological properties (particle manipulations) of quasicrystal water waves. The out-of-plane displacement field $Z(x, y, t)$ of the water wave is reconstructed by the FCD method. Specifically, a hexagonal lattice pattern composed of dots (dot diameter 3 mm, dot spacing 3 mm) is placed at the bottom of the transparent tank. A camera (MER2-041-608U3C-HS, Daheng or AIC-133GC, JCOPTIX) is deployed above the tank to record distortions of the lattice caused by surface waves. By comparing the distorted frames with the undistorted reference pattern, a demodulation algorithm was applied to reconstruct the out-of-plane displacement of water waves $Z(x, y, t)$. All FCD reconstruction and experimental data processing are performed with MATLAB 2024b. The simulated out-of-plane-displacement fields in Fig. 2(e) are obtained from COMSOL Multiphysics.

In the experiments, the five source edges in our device do not behave as rigidly fixed reflecting walls. Instead, they touch the water surface only slightly. Such a configuration provides a more compliant boundary condition than an ideal hard wall and is expected to reduce the abrupt impedance mismatch that would otherwise occur at a rigid reflecting boundary. Consequently, when an incident wave reaches the opposite source edges, part of the wave energy can be coupled into the source boundary and dissipated through mechanical losses, rather than being fully reflected back into the pentagonal region. Figures 5(a) and 5(b) show the single-edge excitation experiment used to examine the boundary reflections in the pentagonal structure. Plane waves were generated from the bottom edge of the pentagon and propagated toward the two upper edges. During propagation, the wave amplitude was

substantially attenuated because of viscous damping and wave dissipation in the shallow-water system [57,58]. As a result, when the incident wave reached the two upper edges at $t = 2.8$ s, the reflected waves from these boundaries were extremely weak and could hardly be directly observed. Therefore, although weak residual boundary reflections may still introduce small perturbations near the peripheral region of the tank, the quasicrystal patterns analyzed in this work are mainly governed by the designed multiwave interference rather than by cavity-mode resonances.

APPENDIX C: DISPERSIONS OF THE SURFACE WATER WAVES

The dispersion relation of the water waves is $\omega^2 = [gk + (\sigma/\rho)k^3] \tanh(kh)$, where k , σ , ρ , h , and g are the wave number of the water wave, surface-tension coefficient, density of water, undisturbed water depth, and gravitational acceleration, respectively. Then, we can obtain the relationship between the frequency and

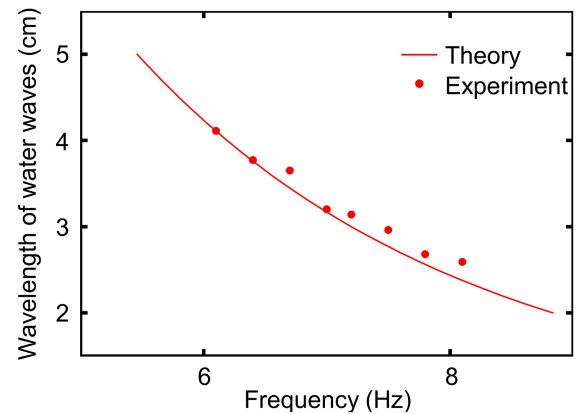


FIG. 6. Theoretical and experimental results of water-wave dispersion between the wavelength and frequency. The water depth is set as 1.5 cm.

wavelength λ , as shown in Fig. 6. In the experiment, we chose 7.2 Hz for the quasicrystal lattice and 6.1 Hz for the floating particle trapping.

APPENDIX D: ANALYTICAL RESULTS OF QUASICRYSTAL WATER-WAVE LATTICES

Figures 7(a) and 7(b) present the analytical results of the time-averaged out-of-plane displacement and phase distributions of quasicrystals under different Q values. Compared with Fig. 2 in the main text, the analytical results match the experimental data well in the central interference region. Note that due to the limited size of the practical device, the boundary effect induces some perturbations to the quasicrystal lattice beyond the central region.

APPENDIX E: SIMILARITIES AMONG QUASICRYSTAL WATER-WAVE LATTICES

In each topological quasicrystal pattern, we select a square region of 230×230 pixels (corresponding to an actual area of $6 \times 6 \text{ cm}^2$) centered on the phase singularity of $q = 0$, and performed their temporal correlations. We used a high-speed camera with a frame rate of 500 frames/s to capture the water-wave fields. The similarity among different quasicrystal lattices is characterized based on the maximal value of correlation (MVC):

$$\text{MVC} = \max_{k,l} \left| \frac{\sum_{m=1}^M \sum_{s=1}^S Z_{Q=a}[m,s] Z_{Q=b}[m-k,s-l]}{\|Z_{Q=a}\| \|Z_{Q=b}\|} \right|, \quad (\text{E1})$$

where M and S are all 230, which represents the maximum pixel values in the x and y directions, respectively. (m,s) and (k,l) are used to describe a specific pixel. The symbols $|\cdot|$ and $\|\cdot\|$ are the absolute value and norm value, respectively. The results of the MVC mapping are shown in Fig. 2(e) of the main text. Figure 8 shows more results of the quasicrystal patterns at more moments; the difference between adjacent images is $\frac{1}{8}$ of a cycle. The evolutions of the quasicrystals' out-of-plane displacement field for different topological charge are identical.

APPENDIX F: TOPOLOGICAL CHARGE VECTORS CONFORMING TO THE ANGULAR-MOMENTUM-RELATED CONSERVATION LAW

The in-plane fields of constructed water waves can be derived from the gradient of their out-of-plane fields [40]:

$$[X, Y] = \nabla_2 Z / k, \quad (\text{F1})$$

where $\nabla_2 = (\partial_x, \partial_y)$, and k is the wave number of the water wave. The amplitude and phase distributions of the in-plane quasicrystal lattices are shown in Figs. 9(a)–9(c). Depending on the rotation of the in-plane field, they

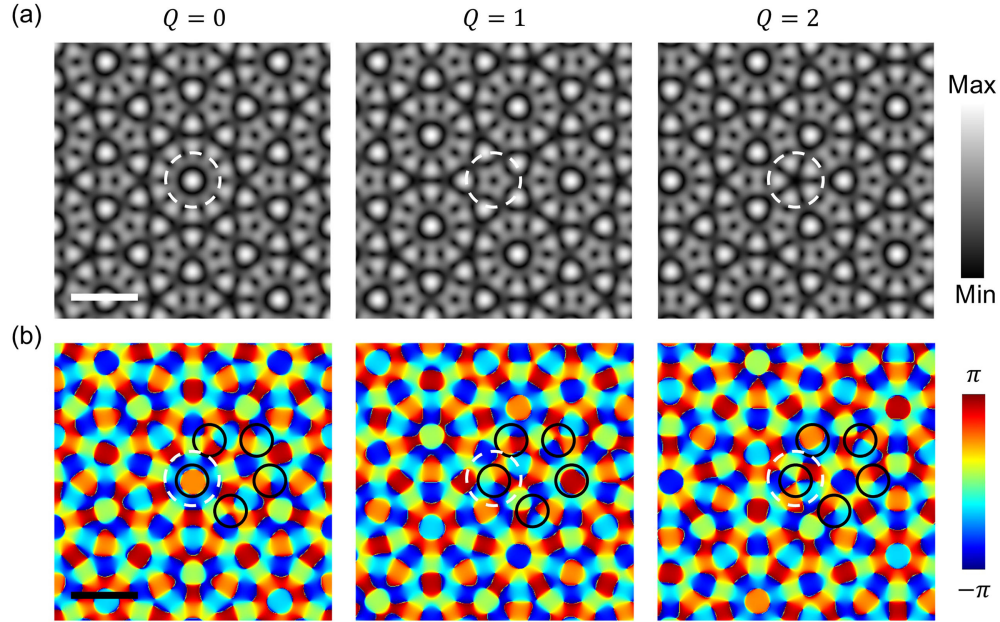


FIG. 7. Analytical results of the time-averaged out-of-plane displacement and phase distributions of pentagonal quasicrystal lattices. (a) Time-averaged out-of-plane displacement ($|Z|$) of quasicrystal lattices for different Q values. (b) Phase distributions of quasicrystal lattices for different Q values. The white-dashed circles mark the central interference region. The black circles mark all possible phase singularities ($q = 0, \pm 1, \pm 2$) in each of the quasicrystal lattices. Scale bar is 6 cm.

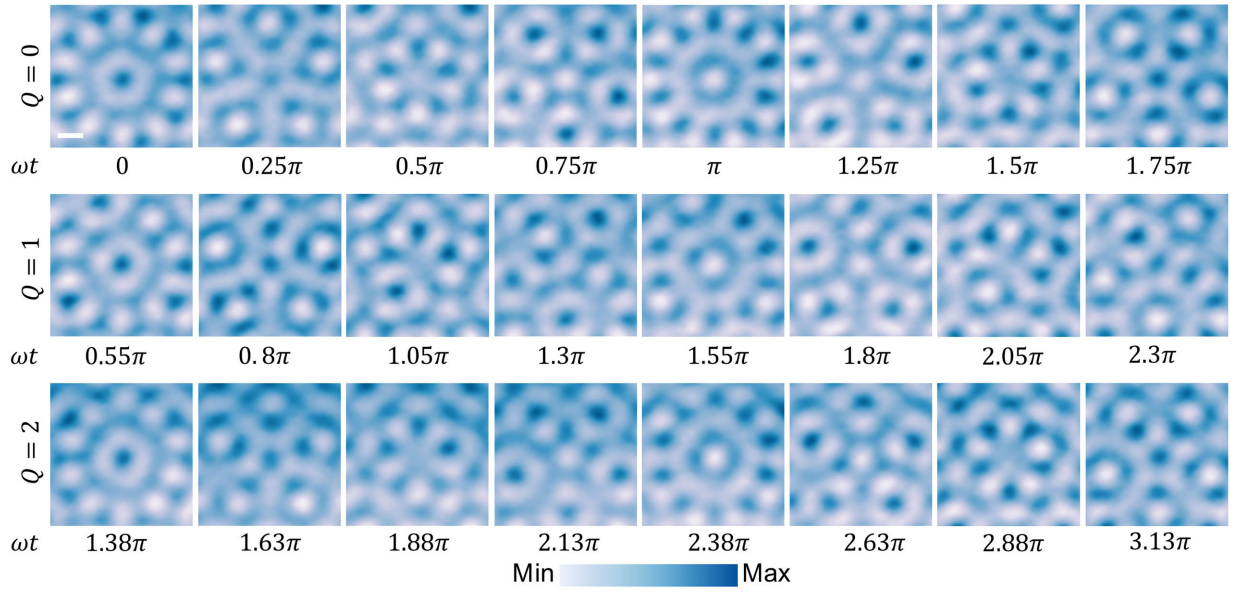


FIG. 8. The typical out-of-plane water-wave displacement fields with maximal correlation among three quasicrystal lattices ($Q = 0, 1, 2$) at different temporal phases. Scale bar is 2 cm.

contain a quasilattice with a topological charge vector that is different from that of the out-of-plane field, conforming to the relation $\vec{b}_+ + \vec{b}_- = 2\vec{b}_Z$. This relation is, again, a manifestation of angular momentum conservation—a direct consequence of the opposite in-plane chirality, the nonrotating out-of-plane field, and their coupling into a unified vector mode.

The topologies of $\vec{b}_+$ and $\vec{b}_-$ related to different topological $\vec{b}_Z$ are summarized as follows ($2\vec{b}_Z = \vec{b}_+ + \vec{b}_-$):

$$\begin{aligned}
 2\vec{b}_{Q=0} &= \vec{b}_{Q=1} + \vec{b}_{Q=-1}, \\
 2\vec{b}_{Q=1} &= \vec{b}_{Q=2} + \vec{b}_{Q=0}, \\
 2\vec{b}_{Q=2} &= \vec{b}_{Q=-2} + \vec{b}_{Q=6} = \vec{b}_{Q=-2} + \vec{b}_{Q=1}, \\
 2\vec{b}_{Q=-1} &= \vec{b}_{Q=-2} + \vec{b}_{Q=0}, \\
 2\vec{b}_{Q=-2} &= \vec{b}_{Q=2} + \vec{b}_{Q=-6} = \vec{b}_{Q=2} + \vec{b}_{Q=-1}. \quad (\text{F2})
 \end{aligned}$$

Since all three topological charge vectors describe the full information about the topological properties of the quasicrystal water-wave lattice, it is instructive to place them on a plane and view the shape that they create. Interestingly, all possible combinations of shapes created by the topological charge vectors lie on a single plane, with one axis corresponding to a “phononic” (physical plane) degree of freedom, and the other corresponding to a “phasonic” (higher-dimensional plane) one. All the possible shapes created by quasicrystal lattices are plotted in Fig. 9(d). Moreover, their area can be calculated via a simple line integral. Though the direction of integration is by and large arbitrary, keeping it constant will preserve any sign

differences between the resulting values for different quasicrystal lattices (in accordance with Stokes’s law). Thus, this procedure may allow us to learn something new about the higher-dimensional topology. By normalizing the area of the generated shapes (triangles) with the area of the projected 4D unit cell onto the plane on which the topological charge vectors reside, the line integration result is exactly the value Q (including the sign for the proper choice of integration direction) [27].

APPENDIX G: EVOLUTION AND ROBUSTNESS OF SKYRMION TEXTURES

The boundaries of skyrmions can be obtained analytically: $\sum_{n=1}^5 e^{i[k_n \cos(\theta_n)x + k_n \sin(\theta_n)y] + i\varphi_n - i\omega t} = \text{Lm}_{[x,y]}$, where $\text{Lm}_{[x,y]}$ is a local maximum or minimum in the selected region $[x, y]$, and θ_n is the intersection angle of each input wave with x axis. The analytical results are shown in Figs. 10(a)–10(c). It is clear that the boundaries of skyrmions evolve from a regular pentagonlike shape to a circle, and then to an inverted pentagonlike shape within trackable time intervals, which agree with the simulated results.

To further clarify the dynamical nature of the skyrmion textures discussed in the main text, we calculate the time-dependent skyrmion number over large regions for both a periodic hexagonal lattice and the pentagonal quasicrystal lattice, as shown in Figs. 10(d) and 10(e). In the periodic lattice, the integrated skyrmion number mainly flips its sign during the harmonic-wave cycle, reflecting the synchronized temporal evolution of the periodic skyrmion texture. In contrast, in the pentagonal quasicrystal lattice, the integrated skyrmion number evolves more continuously and smoothly in time. This continuous evolution originates

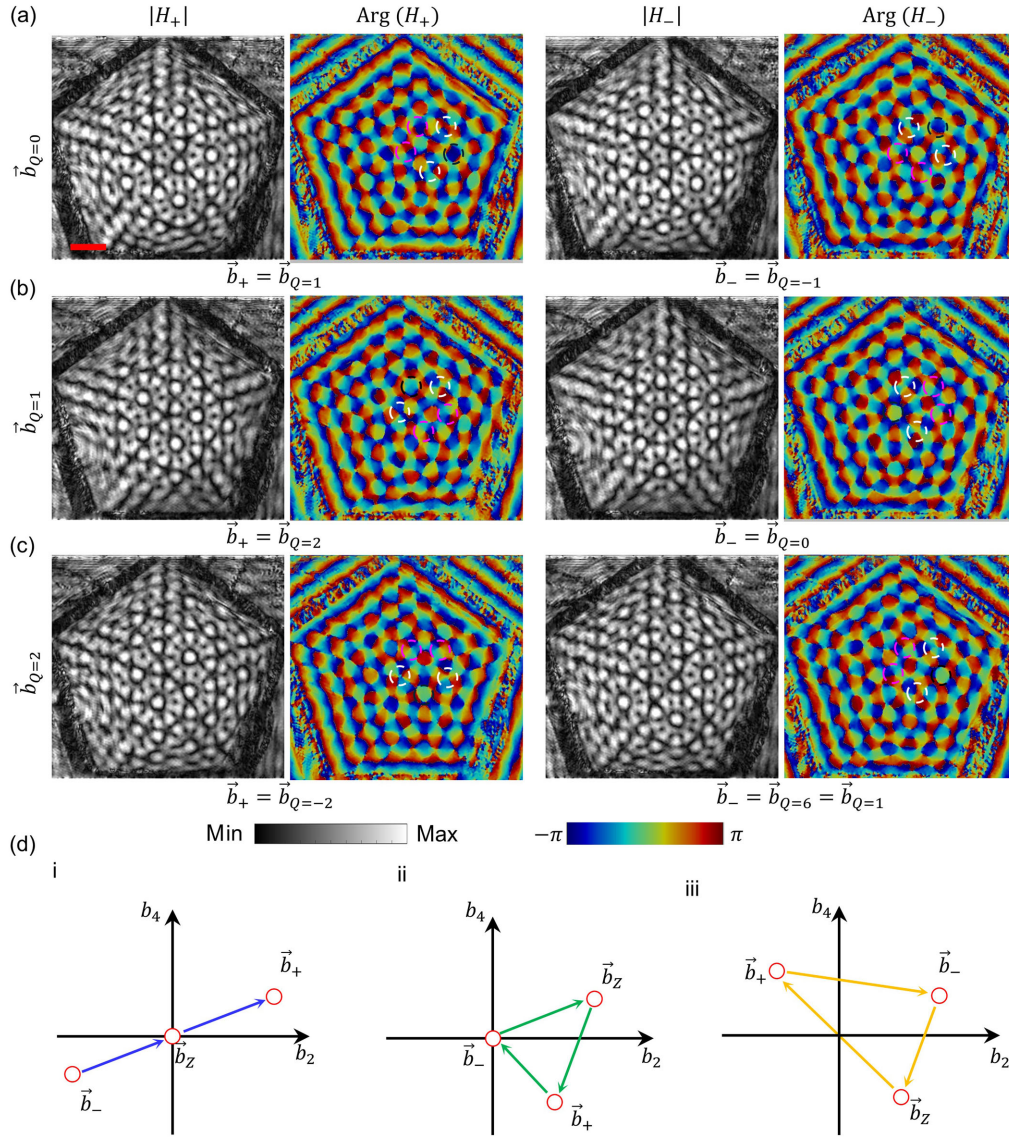


FIG. 9. The measured time-averaged amplitude and phase distribution of the in-plane displacement field (H_+ and H_-) of water waves under different excitation Q values: (a) $Q = 0$, (b) $Q = 1$, (c) $Q = 2$. The dashed circles mark all possible phase singularities. The scale bar is 6 cm. (d) Conservation of topological charge vectors in quasicrystal lattices. The three topological charge vectors ($\vec{b}_z$, $\vec{b}_+$, $\vec{b}_-$) of panels (i–iii) extracted from (a–c) form triangles or line segments in parameter space (b_2 , b_4) with $b_1 = b_3 = 0$.

from the phasonlike dynamics and the absence of translational unit cells in the quasicrystal lattice. During this process, local skyrmion textures can be created or annihilated in pairs with opposite charges or can cross the selected integration boundary.

We also examine the robustness of a local skyrmion texture against particle-induced perturbation. The floating particle acts as a localized scatterer and therefore provides a physical perturbation to the water-wave field. We calculate the skyrmion number in the same spatial region adjacent to the selected skyrmion with and without the particle, as marked by the black circles in Figs. 10(f) and 10(g). The skyrmion number changes only from $N = 0.960$ without the particle to $N = 0.932$ with the particle,

remaining close to unity in both cases. This confirms that the local skyrmion texture is robust against the particle-induced perturbation.

APPENDIX H: SKYRMION CLUSTERS IN QUASICRYSTAL WATER-WAVE LATTICES

We also discovered skyrmion clusters in the quasicrystals, which are composed of multiple small skyrmions surrounding a larger skyrmion. In Fig. 11, a slightly larger region is selected with the phase singularity at $q = 0$ as the center, with the very center being the single skyrmion described in Figs. 3(b)–3(d) in the main text. It is evident that several small skyrmions surround this central one, and

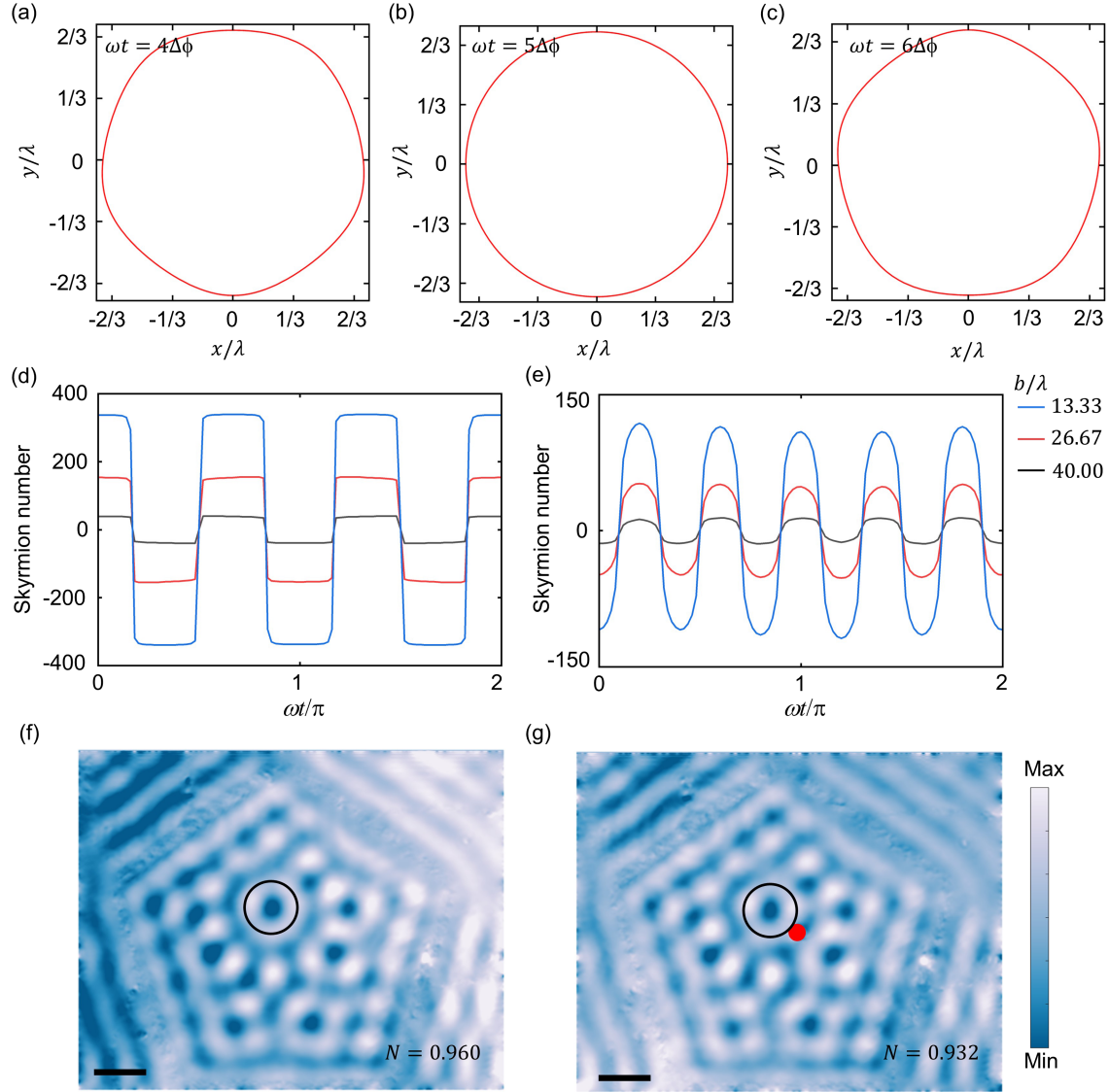


FIG. 10. Evolution of skyrmionic boundaries of quasicrystal water waves. (a)–(c) Analytical results of skyrmionic boundaries within trackable time intervals. The temporal phase ($\Delta\phi$) is set as $\pi/5$. (d) The time-dependent skyrmion number over large regions for a periodic hexagonal lattice and (e) the present pentagonal quasicrystal lattice. The line color represents areas of different sizes, and b is the side length of the integral area. (f), (g) Local skyrmion texture adjacent to the selected region without and with a floating particle, respectively. The calculated skyrmion numbers are $N = 0.960$ and $N = 0.932$, showing that the local skyrmion texture remains robust against the particle-induced perturbation. Scale bars are 6 cm.

their number changes over time. As shown in Figs. 11(a) and 11(b), ten skyrmions surround the central skyrmion, while in Figs. 11(c) and 11(d), there are a few skyrmions. It is noted that in the experiment, some skyrmion boundaries are not entirely closed [37].

APPENDIX I: WATER-WAVE-INDUCED FORCES AND TORQUES ON FLOATING PARTICLES

Here, we present a theoretical framework built on the analogy among the key dynamical properties of monochromatic (single-frequency) optical, acoustic, and water-wave fields [40,59]. These systems share similar

expressions for local energy, canonical momentum, and spin density. In addition, the dominant forces and torques acting on small isotropic particles take comparable forms in both optical and acoustic settings [55]. It is therefore reasonable to anticipate that small floating particles in a single-frequency water-wave field will experience analogous forces and torques.

This approach relies on the minimal coupling between the lowest-order multipole moments induced in a particle by an incident wave field. In acoustics, these take the form of monopole and dipole moments corresponding, respectively, to uniform compression or expansion, and to linear displacement of the particle. These monopole and dipole

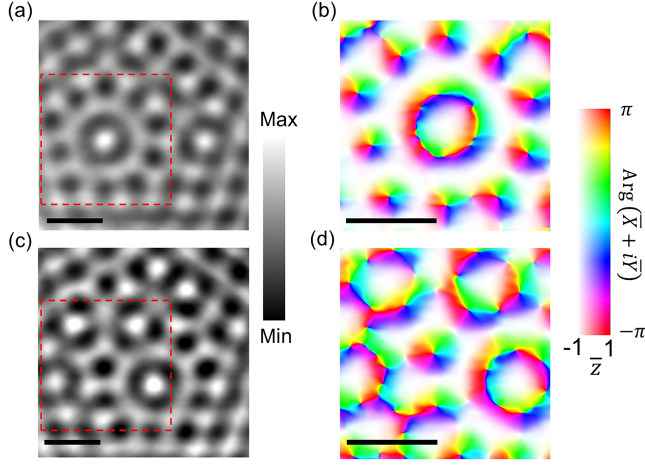


FIG. 11. Skymion clusters in quasicrystal water waves. (a),(c) Out-of-plane displacement fields of quasicrystal water waves in different time moments. (b),(d) Distribution of the reconstructed normalized three-dimensional displacement field (encoded by the transparency of the mask in white and colors) of the quasicrystal waves showing the skymion distribution. The spatial area is selected from the red-dashed box in (a) and (c). All scale bars are 6 cm.

responses naturally couple to the scalar pressure field and the vector velocity (or displacement) field in acoustics. For water waves, assuming both the fluid and the particle are incompressible, one might expect that only the dipole moment coupled to the vector displacement field would be relevant. However, unlike bulk acoustic waves, surface water waves are strongly anisotropic, with their out-of-plane (z) and in-plane (x, y) behaviors differing considerably. Indeed, the governing equations of water waves resemble two-dimensional acoustic equations when the out-of-plane displacement Z is treated as a scalar field (analogous to acoustic pressure) and the in-plane displacement $\vec{H}_2 = (X, Y)$ as a two-dimensional vector field (analogous to acoustic velocity) [40,60].

Thus, we can characterize a floating particle by the monopole and dipole moments induced by the out-of-plane and in-plane displacement wave fields:

$$M = \alpha_M Z, \quad \vec{D}_2 = \alpha_D \vec{H}_2. \quad (I1)$$

Here, α_M and α_D are the complex monopole and dipole polarizability of the particle. To find these parameters, one has to solve an exact water-wave scattering problem, like the Mie problem in optics and acoustics. This is beyond the scope of this work, and we will use the polarizability as unknown proportionality parameters with dimensions of volume. Assuming the minimal-coupling energy of the wave-particle interaction in the form

$$W_{\text{int}} = -\frac{\rho\omega^2}{2} \text{Re}(M^* Z + \vec{D}_2^* \cdot \vec{H}_2), \quad (I2)$$

we obtain the wave-induced force on the particle [54,55,61]:

$$\vec{F} = \text{Re}(\alpha_M) \nabla_2 W^{(z)} + \text{Re}(\alpha_D) \nabla_2 W^{(x,y)} + \omega \text{Im}(\alpha_M) \vec{P}^{(z)} + \omega \text{Im}(\alpha_D) \vec{P}^{(x,y)}. \quad (I3)$$

Here we introduce the parts of the kinetic energy density and of the wave momentum density related to the out-of-plane and in-plane displacement fields: $W^{(z)} = (\rho\omega^2/4)|Z|^2$, $W^{(x,y)} = (\rho\omega^2/4)|\vec{H}_2|^2$, $\vec{P}^{(z)} = (\rho\omega/2) \text{Im}[Z^* \nabla_2 Z]$, and $\vec{P}^{(x,y)} = (\rho\omega/2) \text{Im}[\vec{H}_2^* \cdot (\nabla_2) \vec{H}_2]$. In this work, we need only the simplest toy model that can explain the main features of the particle behavior in structured water-wave fields. Therefore, we set $\alpha_M = \alpha_D \equiv \alpha$, and in this case, Eq. (I3) is reduced to the force in Eq. (4) in the main text. This explains the particle dynamics observed in our experiments. In turn, the out-of-plane torque on a small particle involves only the dipole moment and can be derived as

$$T_{\text{spin}} = -\frac{\rho\omega^2}{2} \text{Re}(\vec{D}_2^* \times \vec{H}_2). \quad (I4)$$

For $\alpha_D = \alpha$, this yields

$$T_{\text{spin}} = \omega \text{Im}(\alpha) S_z. \quad (I5)$$

Here, $\vec{S} = (\rho\omega/2) \text{Im}(\vec{H}^* \times \vec{H})$ represents the spin density of the water-wave structure.

APPENDIX J: TRAJECTORY OF FLOATING PARTICLES IN QUASICRYSTAL WATER-WAVE LATTICES

Figure 12 shows the dynamic trajectory of a particle in the $Q = 0$ quasicrystal wave field (see Supplemental Material Video 3 [56]). The particle is rapidly trapped around the central phase singularity with $q = 1$ and undergoes a stable clockwise rotation about its axis. It should be noted first that while water waves drive the movement of floating particles, the floating particles also scatter the water waves. Figure 13(a) shows the time-averaged lattice of a water-wave quasicrystal, and Fig. 13(b) shows the time-averaged lattice of a water-wave quasicrystal within a floating particle. The out-of-plane displacement of water waves close to the floating particle is slightly affected; thus, the wave-induced force is also slightly affected. Therefore, the wave-induced force distribution with a floating particle differs slightly from that without floating particles.

To further examine the validity of the dipole approximation for the finite-size floating particles used in our experiments, we performed an additional scattering measurement under plane-wave excitation. As shown in Fig. 13(c), an incident plane wave propagates past a floating particle. Since the particle is gradually pushed by the incident wave during the measurement, a

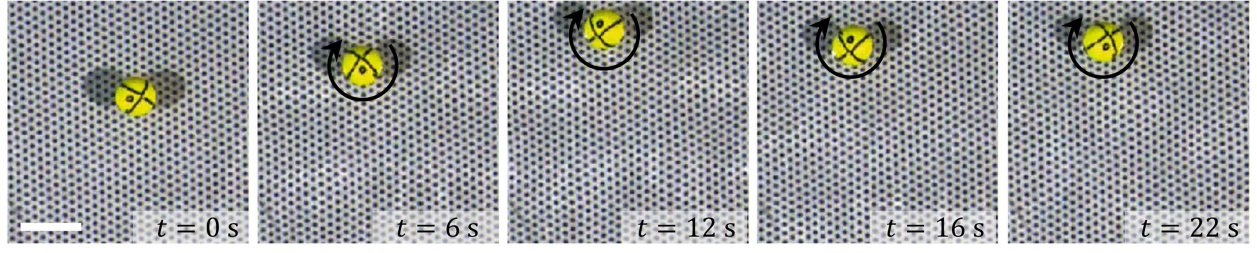


FIG. 12. Single particle (radius of 10 mm) is trapped and rotated by quasicrystal water waves when $Q = 0$. Scale bar is 3 cm.

representative single frame was selected for the scattering analysis. Figure 13(d) shows the scattered field obtained from the boxed region in Fig. 13(c) after subtracting the corresponding background field measured without the particle. The angular radiation pattern extracted along the black circle in Fig. 13(d) is shown in Fig. 13(e). We further performed a Fourier decomposition of the scattered field along the same circle and obtained the normalized multipole expansion intensity $|\bar{c}_n|^2$, as shown in Fig. 13(f).

The finite size of the particle indeed excites higher-order multipole components. For example, a finite monopole component is observed, with a normalized intensity of approximately 0.04 for $n = 0$, and weak quadrupole components also appear, with normalized intensity of approximately 0.004 and 0.002 for $n = \pm 2$. Nevertheless, the overall scattering response is dominated

by the dipole components $c_{\pm 1}$, whose normalized intensity is approximately 0.5. Therefore, the qualitative topological dynamics, including the robust trapping and steady rotation of the particle at the phase singularities, remain fundamentally governed by the dominant dipole-field coupling mechanism described by our model. This multipole analysis clarifies why the dipole-based model remains useful for interpreting the main experimental observations, while also explicitly identifying its limitation for predicting exact force and torque magnitudes in the finite-size particle regime.

APPENDIX K: MULTIPLE PARTICLE MANIPULATION

Here, we explore the dynamic motions of floating particles in quasicrystal water waves. In the case of the

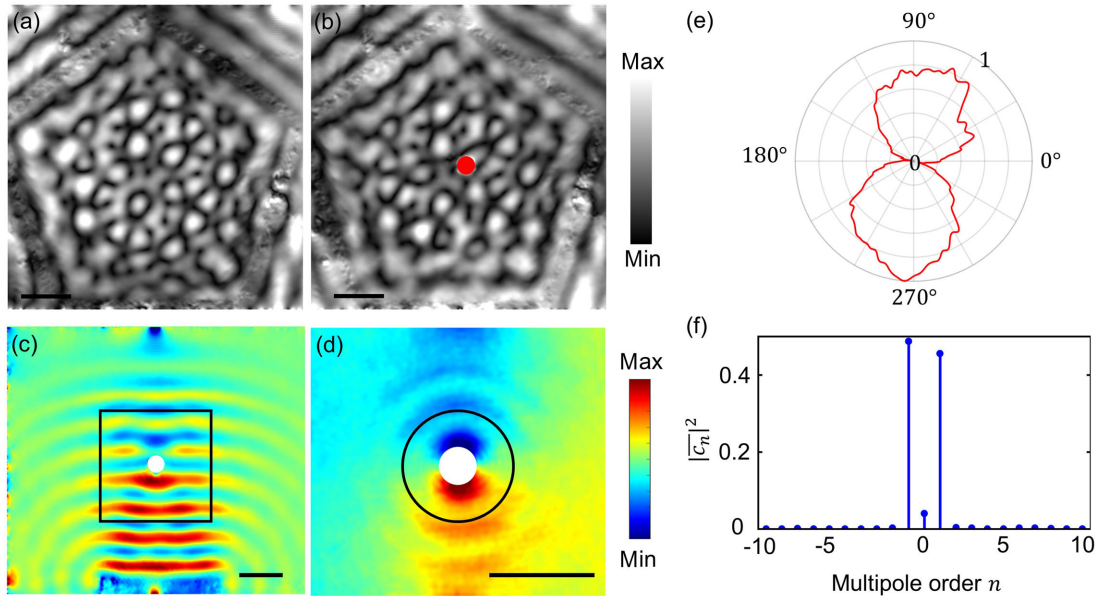


FIG. 13. Time-averaged lattice of water-wave quasicrystal and experimental characterization of the multipole scattering response of a floating particle. (a) Time-averaged quasicrystal lattice without a floating particle. (b) Time-averaged quasicrystal lattice with a floating particle (red circle). (c) An incident plane wave propagating past a floating particle. (d) Scattered field obtained from the boxed region in (c) after subtracting the corresponding background field measured without the particle. (e) Angular radiation pattern extracted along the black circle marked in (d). (f) Normalized multipole expansion intensity spectrum obtained by Fourier decomposition of the scattered field along the same circle. All scale bars (black) are 6 cm.

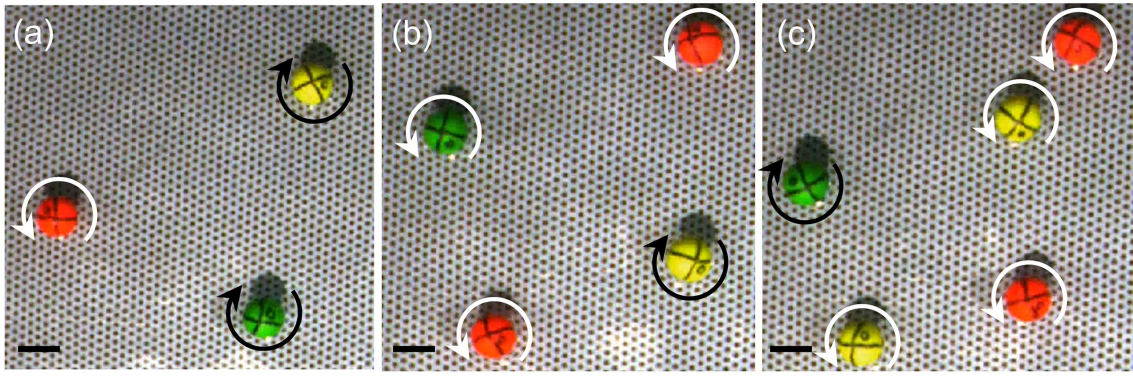


FIG. 14. Multiple floating particles trapping and rotations by quasicrystal water waves: (a) three particles (b) four particles, (c) five particles. The scale bar is 2 cm.

triple spheres illustrated in Fig. 14(a), the yellow and green spheres rotate clockwise, while the red sphere rotates counterclockwise. In Fig. 14(b), only the yellow sphere rotates clockwise among the four spheres, while the others rotate counterclockwise. In Fig. 14(c), only the green sphere rotates clockwise among the five spheres, while the others rotate counterclockwise. It is worth noting that in the upper right corner of Fig. 14(c), though two spheres (red and yellow) are very close to each other, approximately 2 cm apart, they can still rotate stably without affecting each other.

- [1] D. Shechtman, I. Blech, D. Gratias, and J. W. Cahn, *Metallic phase with long-range orientational order and no translational symmetry*, *Phys. Rev. Lett.* **53**, 1951 (1984).
- [2] W. Man, M. Megens, P. J. Steinhardt, and P. M. Chaikin, *Experimental measurement of the photonic properties of icosahedral quasicrystals*, *Nature (London)* **436**, 993 (2005).
- [3] A. Ledermann *et al.*, *Three-dimensional silicon inverse photonic quasicrystals for infrared wavelengths*, *Nat. Mater.* **5**, 942 (2006).
- [4] D. V. Talapin *et al.*, *Quasicrystalline order in self-assembled binary nanoparticle superlattices*, *Nature (London)* **461**, 964 (2009).
- [5] X. Zeng *et al.*, *A columnar liquid quasicrystal with a honeycomb structure that consists of triangular, square and trapezoidal cells*, *Nat. Chem.* **15**, 625 (2023).
- [6] W. Gellermann, M. Kohmoto, B. Sutherland, and P. C. Taylor, *Localization of light waves in Fibonacci dielectric multilayers*, *Phys. Rev. Lett.* **72**, 633 (1994).
- [7] Y. Chan, C. T. Chan, and Z. Y. Liu, *Photonic band gaps in two dimensional photonic quasicrystals*, *Phys. Rev. Lett.* **80**, 956 (1998).
- [8] M. Zoorob, M. Charlton, G. Parker, J. Baumberg, and M. Netti, *Complete photonic bandgaps in 12-fold symmetric quasicrystals*, *Nature (London)* **404**, 740 (2000).
- [9] T. Matsui, A. Agrawal, A. Nahata, and Z. V. Vardeny, *Transmission resonances through aperiodic arrays of sub-wavelength apertures*, *Nature (London)* **446**, 517 (2007).
- [10] S.-Y. Jeon, H. Kwon, and K. Hur, *Intrinsic photonic wave localization in a three-dimensional icosahedral quasicrystal*, *Nat. Phys.* **13**, 363 (2017).
- [11] P. Wang *et al.*, *Localization and delocalization of light in photonic moiré lattices*, *Nature (London)* **577**, 42 (2020).
- [12] S. Vaidya, C. Jörg, K. Linn, M. Goh, and M. C. Rechtsman, *Reentrant delocalization transition in one-dimensional photonic quasicrystals*, *Phys. Rev. Res.* **5**, 033170 (2023).
- [13] C. Han *et al.*, *Observation of dispersive acoustic quasicrystals*, *Nat. Commun.* **16**, 1988 (2025).
- [14] G. Xu *et al.*, *Hydrodynamic moiré superlattice*, *Science* **386**, 1377 (2024).
- [15] H. Huang and F. Liu, *Quantum spin Hall effect and spin Bott index in a quasicrystal lattice*, *Phys. Rev. Lett.* **121**, 126401 (2018).
- [16] Y. E. Kraus, Y. Lahini, Z. Ringel, M. Verbin, and O. Zilberberg, *Topological states and adiabatic pumping in quasicrystals*, *Phys. Rev. Lett.* **109**, 106402 (2012).
- [17] Z. Liu *et al.*, *Field-tunable valley coupling in a dodecagonal semiconductor quasicrystal*, *Nat. Phys.* **22**, 33 (2026).
- [18] A. Shi *et al.*, *Observation of topological corner state arrays in photonic quasicrystals*, *Laser Photonics Rev.* **18**, 2300956 (2024).
- [19] Y. Nagai, N. Nakai, and M. Machida, *Direct numerical demonstration of sign-preserving quasiparticle interference via an impurity inside a vortex core in an unconventional superconductor*, *Phys. Rev. B* **85**, 092505 (2012).
- [20] K. Kamiya *et al.*, *Discovery of superconductivity in quasicrystal*, *Nat. Commun.* **9**, 154 (2018).
- [21] Y. Tokumoto *et al.*, *Superconductivity in a van der Waals layered quasicrystal*, *Nat. Commun.* **15**, 1529 (2024).
- [22] A. Uri *et al.*, *Superconductivity and strong interactions in a tunable moiré quasicrystal*, *Nature (London)* **620**, 762 (2023).
- [23] P. Bak, *Symmetry, stability, and elastic properties of icosahedral incommensurate crystals*, *Phys. Rev. B* **32**, 5764 (1985).
- [24] D. Levine, T. C. Lubensky, S. Ostlund, S. Ramaswamy, P. J. Steinhardt, and J. Toner, *Elasticity and dislocations in pentagonal and icosahedral quasicrystals*, *Phys. Rev. Lett.* **54**, 1520 (1985).

- [25] J. E. Socolar, T. C. Lubensky, and P. J. Steinhardt, *Phonons, phasons, and dislocations in quasicrystals*, *Phys. Rev. B* **34**, 3345 (1986).
- [26] Y. E. Kraus, Z. Ringel, and O. Zilberberg, *Four-dimensional quantum Hall effect in a two-dimensional quasicrystal*, *Phys. Rev. Lett.* **111**, 226401 (2013).
- [27] S. Tsesses *et al.*, *Four-dimensional conserved topological charge vectors in plasmonic quasicrystals*, *Science* **387**, 644 (2025).
- [28] M. Lohse, C. Schweizer, H. M. Price, O. Zilberberg, and I. Bloch, *Exploring 4D quantum Hall physics with a 2D topological charge pump*, *Nature (London)* **553**, 55 (2018).
- [29] T. H. R. Skyrme, *A unified field theory of mesons and baryons*, *Nucl. Phys.* **31**, 556 (1962).
- [30] J. Fukuda and S. Žumer, *Quasi-two-dimensional skyrmion lattices in a chiral nematic liquid crystal*, *Nat. Commun.* **2**, 246 (2011).
- [31] S. Tsesses *et al.*, *Optical skyrmion lattice in evanescent electromagnetic fields*, *Science* **361**, 993 (2018).
- [32] T. J. Davis *et al.*, *Ultrafast vector imaging of plasmonic skyrmion dynamics with deep subwavelength resolution*, *Science* **368**, eaba6415 (2020).
- [33] L. Du, A. Yang, A. V. Zayats, and X. Yuan, *Deep-sub-wavelength features of photonic skyrmions in a confined electromagnetic field with orbital angular momentum*, *Nat. Phys.* **15**, 650 (2019).
- [34] H. Ge *et al.*, *Observation of acoustic skyrmions*, *Phys. Rev. Lett.* **127**, 144502 (2021).
- [35] D. A. Smirnova, F. Nori, and K. Y. Bliokh, *Water-wave vortices and skyrmions*, *Phys. Rev. Lett.* **132**, 054003 (2024).
- [36] Y. Shen *et al.*, *Optical skyrmions and other topological quasiparticles of light*, *Nat. Photonics* **18**, 15 (2024).
- [37] B. Wang *et al.*, *Topological water-wave structures manipulating particles*, *Nature (London)* **638**, 394 (2025).
- [38] J. Wang *et al.*, *Dynamic water-wave tweezers*, *Phys. Rev. Res.* **8**, L032033 (2026).
- [39] P. Bak, *Phenomenological theory of icosahedral incommensurate ('quasiperiodic') order in Mn-Al alloys*, *Phys. Rev. Lett.* **54**, 1517 (1985).
- [40] K. Y. Bliokh, H. Punzmann, H. Xia, F. Nori, and M. Shats, *Field theory spin and momentum in water waves*, *Sci. Adv.* **8**, eabm1295 (2022).
- [41] S. Wildeman, *Real-time quantitative schlieren imaging by fast Fourier demodulation of a checkered backdrop*, *Exp. Fluids* **59**, 97 (2018).
- [42] U. Al Khawaja and H. Stoof, *Skyrmions in a ferromagnetic Bose–Einstein condensate*, *Nature (London)* **411**, 918 (2001).
- [43] N. Nagaosa and Y. Tokura, *Topological properties and dynamics of magnetic skyrmions*, *Nat. Nanotechnol.* **8**, 899 (2013).
- [44] A. Fert, N. Reyren, and V. Cros, *Magnetic skyrmions: Advances in physics and potential applications*, *Nat. Rev. Mater.* **2**, 17031 (2017).
- [45] K. Y. Bliokh *et al.*, *Polarization singularities and Möbius strips in sound and water-surface waves*, *Phys. Fluids* **33**, 077122 (2021).
- [46] Y. Shen, E. C. Martínez, and C. Rosales-Guzmán, *Generation of optical skyrmions with tunable topological textures*, *ACS Photonics* **9**, 296 (2022).
- [47] C. Peters, N. Mata-Cervera, P. Ornelas *et al.*, *Extracting topological features of skyrmionic light: A tutorial*, *Adv. Photon.* **8**, 023001 (2026).
- [48] M. Lin *et al.*, *Photonic quasicrystal of spin angular momentum*, *Sci. Adv.* **11**, eadv3938 (2025).
- [49] D. G. Grier, *A revolution in optical manipulation*, *Nature (London)* **424**, 810 (2003).
- [50] D. Gao *et al.*, *Optical manipulation from the microscale to the nanoscale: Fundamentals, advances and prospects*, *Light Sci. Appl.* **6**, e17039 (2017).
- [51] A. Özcelik *et al.*, *Acoustic tweezers for the life sciences*, *Nat. Methods* **15**, 1021 (2018).
- [52] K. Dholakia, B. W. Drinkwater, and M. Ritsch-Marte, *Comparing acoustic and optical forces for biomedical research*, *Nat. Rev. Phys.* **2**, 480 (2020).
- [53] K. Y. Bliokh and F. Nori, *Transverse and longitudinal angular momenta of light*, *Phys. Rep.* **592**, 1 (2015).
- [54] K. Y. Bliokh, A. Y. Bekshaev, and F. Nori, *Extraordinary momentum and spin in evanescent waves*, *Nat. Commun.* **5**, 3300 (2014).
- [55] I. D. Toftul, K. Y. Bliokh, M. I. Petrov, and F. Nori, *Acoustic radiation force and torque on small particles as measures of the canonical momentum and spin densities*, *Phys. Rev. Lett.* **123**, 183901 (2019).
- [56] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/rfgr-n77g> for videos showing single-particle trapping and rotation, corotation and counter-rotation of two particles, and simultaneous manipulation of three to five particles in quasicrystal water-wave lattices.
- [57] Chiang C. Mei and L. F. Liu, *The damping of surface gravity waves in a bounded liquid*, *J. Fluid Mech.* **59**, 239 (1973).
- [58] M. Antuono and A. Colagrossi, *The damping of viscous gravity waves*, *Wave Motion* **50**, 197 (2013).
- [59] L. Burns, K. Y. Bliokh, F. Nori, and J. Dressel, *Acoustic versus electromagnetic field theory: Scalar, vector, spinor representations and the emergence of acoustic spin*, *New J. Phys.* **22**, 053050 (2020).
- [60] Y. Meng, Y. Hao, S. Guenneau, S. Wang, and J. Li, *Willis coupling in water waves*, *New J. Phys.* **23**, 073004 (2021).
- [61] I. Toftul, S. Golat, F. J. Rodriguez-Fortuno, F. Nori, Y. Kivshar, and K. Y. Bliokh, *Radiation forces and torques in optics and acoustics*, *Rev. Mod. Phys.* **98**, 025002 (2026).