

Tomonaga-Luttinger Liquid Behavior in a Rydberg-Encoded Spin Chain

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Quantum fluctuations can disrupt long-range order in one-dimensional systems and replace it with the universal paradigm of the Tomonaga-Luttinger liquid (TLL), a critical phase of matter characterized by power-law decaying correlations and linearly dispersing excitations. Using a Rydberg quantum simulator, we study how TLL physics manifests in the low-energy properties of a spin chain, interacting under either the ferromagnetic or the antiferromagnetic dipolar XY Hamiltonian. Following quasiadiabatic preparation, we directly observe the power-law decay of spin-spin correlations in real space, allowing us to extract the Luttinger parameter. In the presence of an impurity, the chain exhibits tunable Friedel oscillations of the local magnetization. Moreover, by utilizing a quantum quench, we directly probe the propagation of correlations, which exhibit a light-cone structure related to the linear sound mode of the underlying TLL. Our measurements demonstrate the influence of the long-range dipolar interactions, renormalizing the parameters of TLL with respect to the case of nearest-neighbor interactions. Finally, comparison to numerical simulations exposes the high sensitivity of TLLs to doping and finite-size effects.

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I. INTRODUCTION

Quantum physics in one dimension (1D) can show radically new phenomena compared to higher dimensions. This comes from the enhanced role of quantum fluctuations, which generically suppress classical long-range order [1]. The low-energy properties of gapless 1D systems with short-range interactions are described by a universal harmonic quantum field theory of free massless bosons, the Tomonaga-Luttinger liquid (TLL) [1–5]. Remarkably, TLL theory predicts that the universal long-wavelength properties of the system can be related to the knowledge of only two numbers: the dimensionless

stiffness (Luttinger parameter) K and the sound velocity u [6]. This theory not only describes the low-energy physics of bosonic systems [7] but also of spin chains [1,2] and interacting fermions in 1D [3–5].

The universality associated with TLLs has rendered them crucial test beds for both theory and experiment. TLL physics has been widely explored in experiments in condensed matter [8–17] and in cold atoms [18–28]. In the context of quantum spin systems—the focus of this work—TLL physics has been extensively probed in spin-chain and spin-ladder materials [29]. In these systems, residual couplings between the chains or ladders inevitably lead to long-range order at low temperatures, preventing the observation of ground-state TLL physics. The physics of TLLs can still be probed at sufficiently high temperature, where the dynamical response functions at equilibrium can reveal the value of the u and K parameters [16,29]. However, some fundamental features of the TLL, such as the emergence of power-law correlations upon approaching its ground state, remain experimentally elusive. At the same time, theoretical

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investigations into the entanglement [30–34] and non-equilibrium dynamics [35–40] of the TLL continue to illuminate new facets of this prototypical example of a quantum critical phase of matter.

Over the past decades, experimental progress has made it possible to create and manipulate one-dimensional systems with single-particle control [25,41–45], enabling precise tests of our understanding of 1D physics. In this work, we explore the low-energy, TLL physics of a 1D ring of *dipolar-interacting* Rydberg atoms. The controllability and single-particle resolution of our experiments enable direct measurements of real-space correlations, local susceptibilities, and real-time dynamics. Our experiment realizes a power-law interacting spin-1/2 XY chain, with either ferromagnetic (FM) or antiferromagnetic (AFM) couplings. Much like the paradigmatic XY chain with nearest-neighbor (NN) interactions—which is dual to free fermions with $K = 1$ [46]—our system is in both cases predicted to realize a TLL at low energies [47–50]. However, the extended dipolar interactions either reinforce (FM) or frustrate (AFM) each other, modifying K and u from their NN values. Notably, we find that for the FM the stiffness is enhanced, $K_{\text{FM}} > 1$, similarly to the behavior of attractively interacting fermions, while for the AFM, $K_{\text{AFM}} < 1$, as with repulsive fermions.

We perform three different experiments, each one probing a different aspect of TLL behavior. First, we use local atomic control to adiabatically prepare low-energy states for both the FM and the AFM, and then measure the resulting spin-spin correlations in real space. In multiple measurement bases, the spatial profiles of these correlations feature a power-law decay, from which we directly estimate the Luttinger parameters K_{FM} and K_{AFM} . Second, we cut the AFM chain by removing a single atom, and observe Friedel-like oscillations [51] of the local magnetization after adiabatic preparation; moreover, we demonstrate the ability to linearly tune this oscillation wave vector by adjusting the total magnetization of the chain. Finally, for both FM and AFM chains, we probe the real-time dynamics upon quenching from a low-energy product state, and observe the ballistic propagation of correlations. For the AFM this provides a precise measurement of the sound velocity u . Interestingly, for the FM, proximity to a continuous symmetry-breaking phase transition [47,48] yields strong finite-size effects and we measure an effective velocity significantly smaller than theoretically predicted.

II. DIPOLAR XY SPIN CHAIN

Using optical tweezers generated by a spatial light modulator (SLM) [52,53], we trap $N = 24$ rubidium atoms in a circular geometry with a distance of $16.2 \mu\text{m}$ between nearest neighbors [Fig. 1(a)] [54]. We encode a pseudo-spin 1/2 using the two Rydberg states

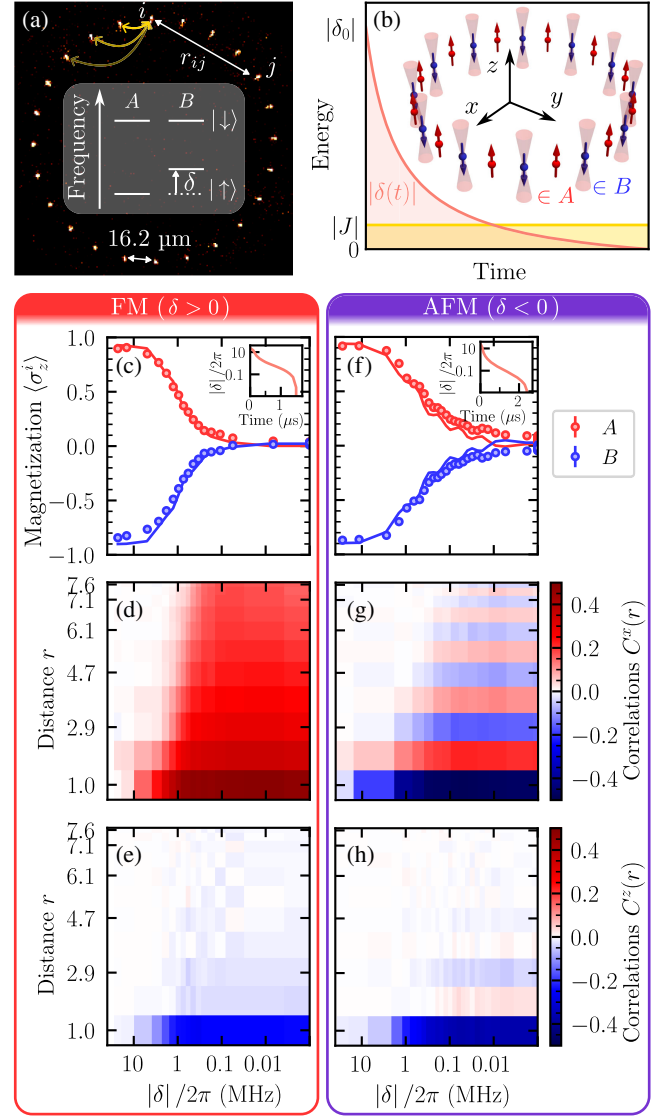


FIG. 1. Quasiadiabatic preparation of XY FM and AFM. (a) Fluorescence image of the $N = 24$ chain of Rydberg atoms. Yellow arrows between atoms indicate the dipolar XY interaction. Inset: definition of the light shift δ used to shift the energy of the state $|\uparrow\rangle$ on a given set of atoms, defining two sublattices A (non-addressed atoms) and B (addressed atoms). (b) Sketch of the experimental sequence for adiabatic preparation. The addressing light shifts $[\delta(t)]$, pink line, are ramped down in the presence of the dipolar XY interactions (J , gold line). Inset: representation of the initial state, along with the position of the addressed sites from sublattice B (pink laser spots). (c) Evolution of the z magnetization per sublattice with the light shift, showing the melting down of the staggered order along z . Solid lines are simulations of the dynamics that include experimental errors. Inset: light shift as a function of time. (d) Evolution of the x correlations $C^x(r)$ for all distances r , revealing the construction of the FM order. (e) Evolution of the z correlations $C^z(r)$. (f)–(h) Same as (c)–(e) for the AFM case.

$|\uparrow\rangle = |70S_{1/2}, m_J = 1/2\rangle$ and $|\downarrow\rangle = |70P_{1/2}, m_J = -1/2\rangle$. The resonant dipole-dipole interaction realizes a dipolar XY spin Hamiltonian:

$$H_{XY} = -\frac{\hbar J}{2} \sum_{i < j} \frac{1}{r_{ij}^3} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y). \quad (1)$$

Here, $J \approx 2\pi \times 0.55$ MHz is the nearest-neighbor interaction strength, r_{ij} is the distance between atoms i and j (in units of the nearest-neighbor distance), and σ_i^μ are the Pauli matrices acting on spin i in direction μ . This Hamiltonian exhibits a U(1) symmetry, and consequently the total z magnetization $M_z = \sum_i \sigma_i^z$ is conserved.

There are two noteworthy features of H_{XY} compared to the spin Hamiltonians of many solid-state systems. First, our interactions are highly spin anisotropic, at the easy-plane limit rather than near the SU(2)-isotropic point. Consequently, we avoid certain peculiarities associated with that non-Abelian symmetry [55], such as the long-range magnetic order that occurs in ferromagnetic Heisenberg chains, which are effectively the $K \rightarrow \infty$ limit of the TLL [56]. By contrast, U(1)-symmetric couplings, as we have here, can generically support a finite- K TLL for either FM or AFM sign. Second, the dipolar interactions extend beyond the finite range usually considered in the study of spin chains, and one might wonder whether they qualitatively alter the physics of the TLL. Indeed, in the presence of long-range XY FM couplings, asymptotically decaying as $1/r^\alpha$, renormalization group analyses indicate that the TLL is unstable to magnetic ordering when $\alpha < 3 - 1/(2K)$ [47,48]. Our dipolar ($\alpha = 3$) profile lies close to this ordering transition, but is definitively on the TLL side; numerical studies find that H_{XY} with FM coupling ($J > 0$) leads to $K \approx 1.7$ [49]. Power-law AFM interactions are even less destabilizing, due to frustration [57], and for H_{XY} with $J < 0$ one again expects a TLL [50], with a weakly renormalized $K \approx 0.86$ (Appendix I).

On top of the naturally occurring interactions, we create an effective magnetic field with controlled amplitude in space and time [58]. To this end, we use an addressing laser at 1014 nm with a detuning Δ from the atomic transition $70S_{1/2} - 6P_{3/2}$. This induces a light shift δ on the level $|\uparrow\rangle$ of the addressed atoms: $\delta \approx \Omega^2/(4\Delta)$ with Ω the Rabi frequency of the addressing light [inset of Fig. 1(a)]. A dedicated SLM generates addressing spots on the desired atoms, and an acousto-optic modulator (AOM) controls the overall amplitude in time. The addressing Hamiltonian reads $H_Z(t) = \sum_{i \in B} \hbar \delta(t) (1 + \sigma_i^z)/2$, where the sublattice B denotes the addressed atoms. The sign of δ , fixed during a sequence, is given by that of Δ , which we can choose arbitrarily. From the TLL perspective, the staggered field H_Z is a relevant perturbation when $K < 2$, so that for any nonzero δ the system becomes, in the thermodynamic limit, a gapped paramagnet [59]. Moving through this gapped phase provides a favorable route for preparing the gapless, critical TLL at $\delta = 0$ [60,61].

III. ADIABATIC PREPARATION OF A CLOSED RING

We first consider a closed ring geometry that realizes periodic boundary conditions (PBC). This geometry reduces finite-size effects compared to an open-boundary condition (OBC) chain, and enables improved statistics of observables by averaging over sites, which are all equivalent. We aim at preparing a low-energy state, close to the ground state of the Hamiltonians $\pm H_{XY}$. To do so, we use a quasiadiabatic scheme similar to Refs. [58,62]. First, we apply large light shifts $|\delta_0| \approx 2\pi \times 23$ MHz $\gg |J|$ on a staggered sublattice B [inset of Fig. 1(b)]; then, we prepare the ground state of the addressing Hamiltonian H_Z at half filling ($M_z = 0$), which is the product state $|\psi_0\rangle = |\uparrow\downarrow \cdots \uparrow\downarrow\rangle$ [inset of Fig. 1(b)]; finally, we ramp down the light shifts $\delta(t)$ using an adiabatic profile [44,63], in order to end up in a state close to the ground state of H_{XY} . The sequence is illustrated in Fig. 1(b), and is described in more details in Appendix C. The duration and shape of the ramp were experimentally optimized to limit the effects of decoherence, while preserving adiabaticity. The same protocol is used to prepare a state close to the AFM ground state of $-H_{XY}$, by changing the sign of $\delta(t)$: With $\delta < 0$, the initial state $|\psi_0\rangle$ is the highest-energy state of H_Z , and an adiabatic preparation connects $|\psi_0\rangle$ to the highest-energy state of H_{XY} , i.e., the ground state of $-H_{XY}$.

During the adiabatic ramp, we monitor the dynamics of the spins. Figures 1(c) and 1(f) show the z magnetization of each sublattice, respectively, in the FM and AFM cases. At the beginning of the preparation, the z magnetizations of sublattices A and B are opposite, reflecting the staggered spin pattern of the initial state $|\psi_0\rangle$. As we ramp down the light shifts, they merge to zero, signaling the meltdown of the initial pattern into a translation-invariant state in the xy plane. To further characterize the FM and AFM states prepared during the ramp, we measure the connected correlations between the σ^μ spin components ($\mu \in \{x, z\}$) for all spin pairs (i, j) , defined as $C_{i,j}^\mu = \langle \sigma_i^\mu \sigma_j^\mu \rangle - \langle \sigma_i^\mu \rangle \langle \sigma_j^\mu \rangle$. Figures 1(d), 1(e), 1(g), and 1(h) present the correlations $C^\mu(r)$ averaged over pairs separated by the same chord distance r [64]. As expected, we observe the progressive buildup of FM or AFM correlations along x as $\delta(t)$ is ramped down to 0 [Figs. 1(d) and 1(g)]. Along the z axis, we observe weak negative correlations in the FM case and weak staggered magnetization in the AFM case [Figs. 1(e) and 1(h)].

IV. CRITICAL CORRELATIONS

We now examine in detail the correlations of the final state, shown in Fig. 2. TLL theory predicts that these correlations are scale invariant, indicative of a quantum critical state. More specifically, C^x and C^z should decay with distance as sums of power laws, whose exponents are universally determined by the single Luttinger parameter K . We first focus on the FM case in the x basis, for which the

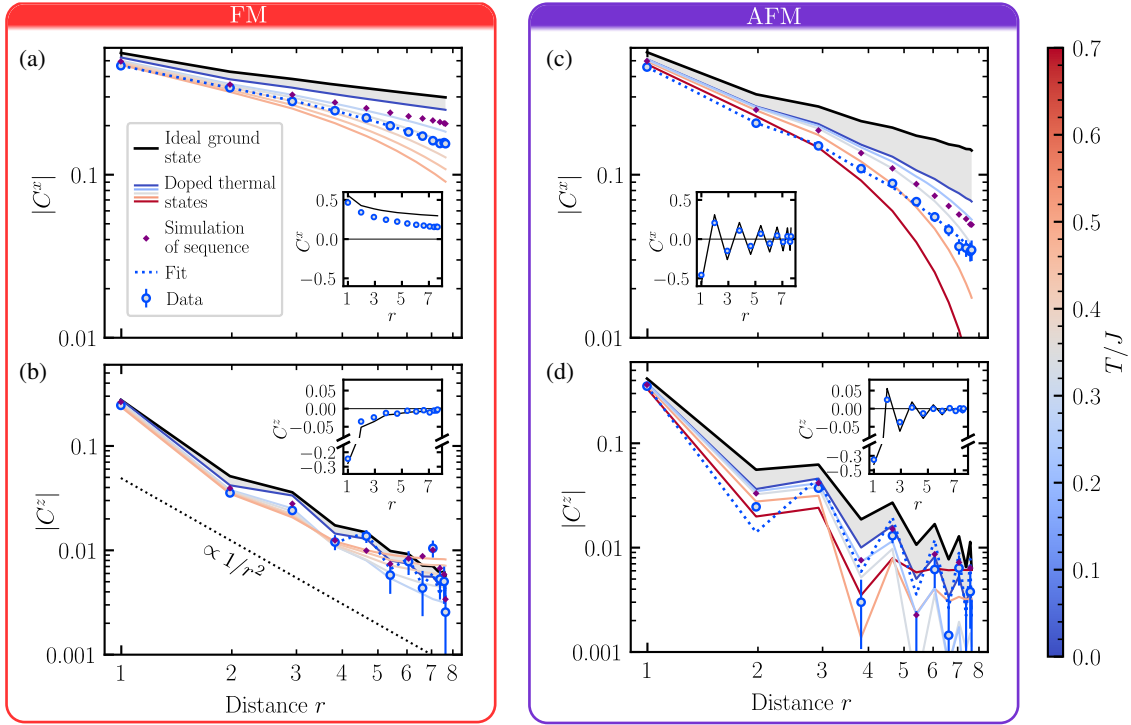


FIG. 2. Spatial profiles of the correlations in the ferromagnetic and antiferromagnetic ground states. The left-hand panels show the FM correlations at $t = 1600$ ns along x (a) and along z (b). The right-hand panels display the AFM correlations along x at $t = 3000$ ns (c) and along z at $t = 3200$ ns (d). The blue points represent experimental data, with statistical error bars estimated through bootstrapping (when not visible, error bars are smaller than the symbol size). The purple points correspond to simulated sequences with imperfections. The dotted lines indicate fits by the theoretical profiles (see text). The solid lines depict simulations of the ideal ground state (in black) and thermal states in the presence of randomly placed holes, with the color of the lines coding for the temperature. For a fair comparison with the data, all simulated correlations are multiplied by a factor of 0.89, corresponding to experimentally measured detection errors (see Appendix E). The doping with holes is 4% in the FM case and 6% in the AFM case, and its effect is highlighted by the gray region. Insets show the signed correlations in linear scale.

dominant power laws (i.e., at asymptotic distances $r \gg 1$) are expected to be [56]

$$C_{\text{FM}}^x(r) \approx A \left(\frac{1}{r} \right)^{1/2K} + B(-1)^{d(r)} \left(\frac{1}{r} \right)^{2K+1/2K}, \quad (2)$$

where A and B are nonuniversal amplitudes, and $d(r) \equiv (N/\pi) \arcsin[(\pi r)/N]$ is the perimeter distance in units of the lattice spacing (contrary to r which is a real number, d is an integer). Numerical calculations (see Appendix H 3) predict $K_{\text{FM}} = 1.85(1)$ for our $N = 24$ atom ring.

The FM experimental data feature a power-law decay of the x correlations up to ~ 5 sites [blue points in Fig. 2(a)]. At larger distances, the correlations decay faster than a power law; we attribute this deviation to preparation errors (see below, and Appendix E). To account for this deviation, we follow the approach of Refs. [44,65] and use a modified fit function for the data: $\tilde{C}_{\text{FM}}^x(r) = C_{\text{FM}}^x(r)e^{-r/\xi}$, where ξ is an empirical correlation length that accounts for finite temperature and other imperfections. We obtain $K_{\text{FM}} = 1.6(4)$, already close to the theoretical value, and a correlation length $\xi = 15(4)$ sites.

Next, we examine the FM z correlations, whose theoretical behavior at long distance takes the form [56,66]

$$C^z(r) \approx -\frac{2K}{\pi^2} \left(\frac{1}{r} \right)^2 + D(-1)^{d(r)} \left(\frac{1}{r} \right)^{2K}. \quad (3)$$

The structure of the first term, with an integer exponent and K appearing as a simple prefactor, encodes the special role of σ^z as the local density of a conserved quantity M_z . By contrast, the second term reflects the presence of emergent gapless fluctuations at wave vector $k = \pi$, which, similar to the x correlations, has a nonuniversal amplitude D and a K -dependent exponent. In the measured correlations, the $1/r^2$ term dominates, as highlighted by the black dotted line in Fig. 2(b). We find that Eq. (3) fits the experimental data well, without the need for an exponential correction, but we face two challenges in using it to determine K . First, for the FM, the staggered part has a small amplitude and a rapid decay ($2K \approx 3.6$), which makes fits of that exponent unstable even in ideal numerical data. Second, the uniform prefactor gives a reliable estimate of K in theoretical calculations, but experimental imperfections and readout

errors reduce the overall magnitude of measured correlations in a nonuniversal way. Accounting for readout errors (see Appendix G), our experimental fit to Eq. (3) yields $K_{\text{FM}} \approx 1.4(1)$.

Proceeding now to the AFM, we expect the same critical correlation structure, except with a global staggered sign $(-1)^{d(r)}$ multiplying the x correlations, and a different Luttinger parameter, which we estimate as $K_{\text{AFM}} = 0.85$ from our simulation of the ground state of a $N = 24$ chain. The measured z correlations are fitted reasonably well by Eq. (3), and we obtain $K_{\text{AFM}} \approx 0.90(1)$ [Fig. 2(d)]. However, the x correlations again require the introduction of an exponential decay, with a short correlation length of $\xi = 5(1)$ sites [Fig. 2(c)]; this leads to larger uncertainty in our estimate of the Luttinger parameter, $K_{\text{AFM}} \approx 1.0(3)$. To check the robustness of the analysis, we applied the same preparation protocol for different system sizes (ranging from $N = 16$ to $N = 28$) and obtained similar values of K and ξ . The small value of ξ indicates that the AFM is particularly sensitive to experimental imperfections.

Motivated by this observation, we carried out numerical simulations of the state-preparation procedure via matrix-product-state (MPS) approaches, including most known experimental imperfections (see Appendixes H and I). The results of these calculations (Fig. 2, purple diamonds) quantitatively reproduce much of the deviation between the measurements and the ideal ground state. Among the included errors, we find that a finite density p of holes is especially important, i.e., atoms lying outside the Rydberg manifold due to either a failed excitation or spontaneous decay. We estimate $p = 0.04$ for the FM at the end of the ramp, while $p = 0.06$ for the AFM due to a longer adiabatic preparation. Owing to the dipolar interactions, sites separated by a hole remain coupled with a reduced strength $J/8$. However, each hole leads to a slip of the sublattice structure, which causes snapshot-averaged measurements of staggered part of the correlations to decay over a perimeter distance $\xi_p = 1/|\ln(1 - 2p)|$ [67]. This disordered-readout effect is most significant in the AFM x correlations, where the staggered part dominates; the preceding formula for ξ_p corresponds to a chord distance of 6.5, which quantitatively accounts for the observed exponential decay. Additionally, when fitting the simulated FM correlations with holes by Eqs. (2) and (3), we obtain $K_{\text{FM}} = 1.55(1)$ from the C^x correlations and $K_{\text{FM}} = 1.44(1)$ from the C^z correlations, in good agreement with the experimental values. For the AFM, we expect that this hole density ultimately destabilizes the TLL phase in the thermodynamic limit (see Appendix I 2), although TLL-like behavior may still emerge at short distance. Finally, the other imperfections, such as nonadiabaticity, effectively raise the energy of the final prepared state: comparing to equilibrium calculations that include holes (based on both quantum Monte Carlo as well as MPS

techniques, see Appendix H 2), we find that both the FM and AFM experimental correlations are compatible with thermal correlations at a temperature of $T/J \approx 0.35$.

V. FRIEDEL OSCILLATIONS ON AN OPEN RING

The single-atom control of our experiment allows us to test another prediction of TLL theory, namely the existence of Friedel oscillations at the edges of an open chain [1]. To do so, we modify the geometry of the chain by removing one atom, which amounts to considering a chain of $N = 23$ spins with OBCs—modulo the weak dipolar coupling across the hole. The AFM ground state is then expected to exhibit spatial oscillations of the local z magnetization around the hole, akin to Friedel oscillations in fermionic systems [68,69]. Examples of such behaviors have been observed with scanning tunneling microscopy at the edges of carbon nanotubes [14] and near defects in WS_2 heterostructures [17]. For an open chain with an odd number of atoms N and a total magnetization M_z , the z magnetization at a site j away from the impurity takes the form (see Appendix J 1)

$$\langle \sigma_z^j \rangle \approx A \cos(2k_F j) \left[\frac{N}{\pi} \cos\left(\frac{\pi j}{N}\right) \right]^{-K(M_z)}. \quad (4)$$

Here A is an overall amplitude, $K(M_z)$ is the magnetization dependent AFM Luttinger parameter, and $2k_F$ is the Friedel wave vector. It obeys the kinematic relation

$$k_F = \frac{\pi}{2} \left(1 - \frac{M_z}{N} \right), \quad (5)$$

which in the dual fermionic picture [1] is simply the distance between Fermi points as a function of filling. Equivalently, it encodes a consistency condition between the microscopic symmetries (e.g., lattice translation) of H_{XY} and the long-wavelength symmetries of the TLL [70,71].

To check this prediction, we use the quasiadiabatic preparation described above in order to drive the system close to the AFM ground state of the XY Hamiltonian on the open ring at fixed total magnetization M_z . To control the magnetization, we initialize $(N + M_z)/2$ spins in $|\uparrow\rangle$ (nonaddressed atoms) and $(N - M_z)/2$ spins in $|\downarrow\rangle$ (addressed atoms), in a way that distributes as uniformly as possible the net magnetization across the circle (see addressing patterns in Appendix J). Then, we ramp down the light shifts δ acting on sublattice B and measure the final state in the z basis.

The resulting z magnetization $\langle \sigma_z^i \rangle$ for each spin i is plotted in Fig. 3(a), after subtracting a PBC background (see Appendix J 2). We observe symmetric oscillations around the hole, consistent with the picture of Friedel oscillations. To probe their spatial frequency, we Fourier transform the z -magnetization profile: $\langle \sigma_z^q \rangle = \sum_j e^{iqj} \langle \sigma_z^j \rangle$

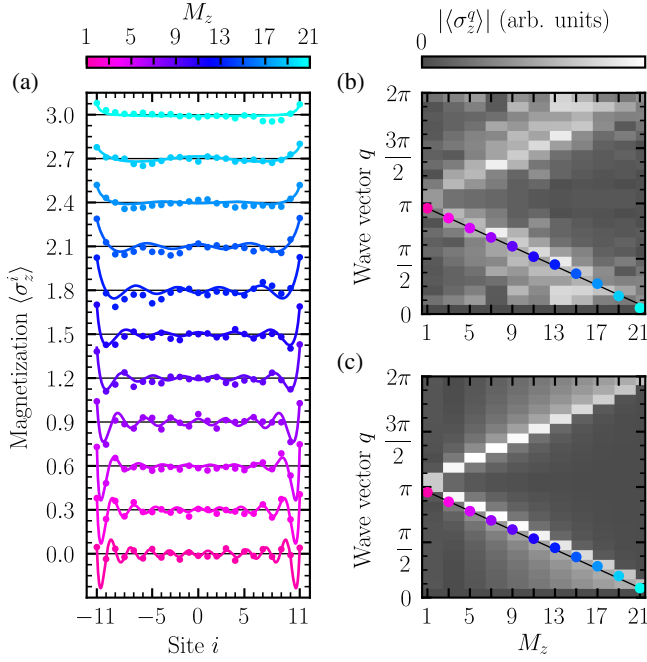


FIG. 3. Friedel oscillations at the edges of a spin chain. (a) z magnetization per site at the end of the adiabatic state preparation of the AFM ground state, for various M_z sectors. The y axis corresponds to the magnetization of the open chain, minus a background that was measured on the closed chain (see Appendix J). The curves are offset for clarity; the horizontal gray line displays the zero for each curve. Solid lines are fits to the data points using Eq. (4). (b) Background: Fourier transform of the z magnetization for each M_z sector, showing a linear shift of the spatial frequency. Colored circles: fitted values of the oscillation frequency $2k_F$ using Eq. (4). Solid black line: theory prediction given by Eq. (5), with no free parameter. (c) Same as (b) based on a density matrix renormalization group (DMRG) simulation of the open chain. The amplitude of the color map in (c) is larger by a factor 2 compared with the one in (b).

for $q \in \{2\pi n/N\}_{0 \leq n < N}$. In Fig. 3(b), we plot $|\langle \sigma_z^q \rangle|$ as a function of the wave vector q and the magnetization sector M_z . For each value of M_z , $|\langle \sigma_z^q \rangle|$ peaks at a given value of q ; this value is consistent with the wave vector of the Friedel oscillation $2k_F$, linearly shifting from π at small M_z to 0 at large M_z [Eq. (5)]. Extracting $K(M_z)$ from the data turned out to be unreliable: While the dominant Friedel frequency is solely controlled by the magnetization per spin, the amplitude decay is additionally sensitive to, e.g., residual finite energy due to lack of adiabaticity, and other experimental imperfections (see Appendix J 2).

VI. VELOCITY OF LOW-ENERGY EXCITATIONS

We conclude our study by probing the low-energy excitations of $\pm H_{XY}$, in order to measure the second parameter of the TLL Hamiltonian, namely the sound velocity u . Following a method we have used in 2D [72], we initialize the system in a low-energy product state

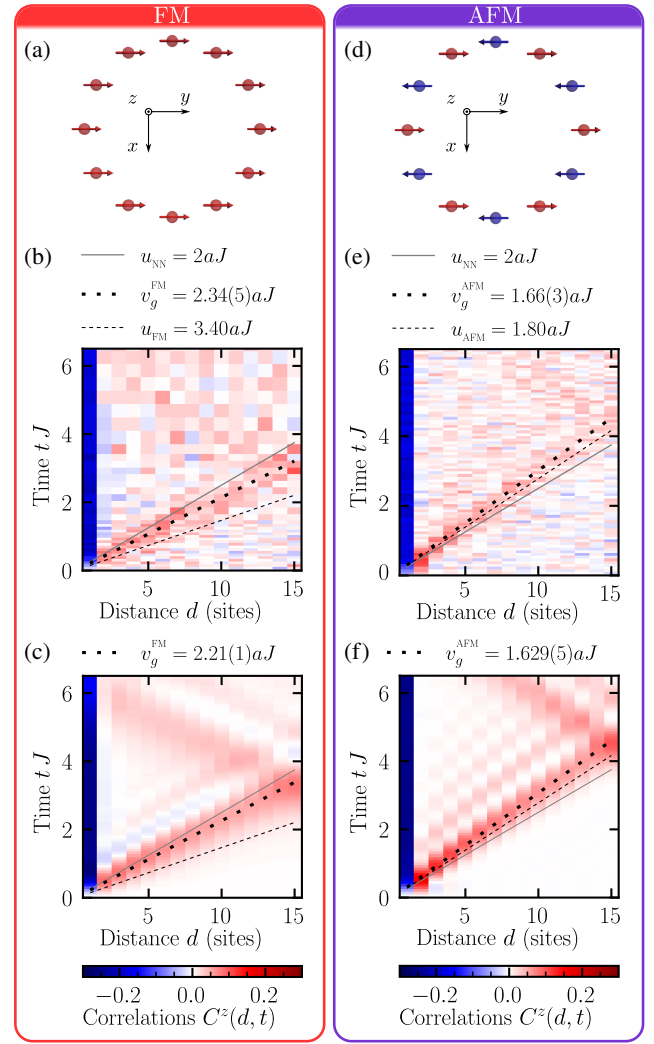


FIG. 4. Measurement of the sound velocity u , by monitoring the propagation of correlations in quench experiments. Data were taken with a chain of $N = 30$ atoms using a different mapping for the spins (see Appendix A). (a) Sketch of the initial state used to measure the FM dispersion relation: The spins are initialized in a coherent spin state along y and evolve freely under XY interactions. (b) Evolution in space and time of the measured z correlations $C^z(d, t)$. The black dotted line shows $d = 2v_g^{\text{FM}}t$ where v_g^{FM} is extracted from a fit of the positive linear wave front for distances $d > 1$ site. For comparison, the gray solid line shows $d = 2u_{\text{NN}}t$ with $u_{\text{NN}} = 2aJ$ the expected sound velocity assuming only nearest-neighbor interactions. The dashed line shows the predicted sound velocity u_{FM} of the TLL theory. (c) Evolution in space and time of the simulated z correlations, with no free parameter (see Appendix H). (d)–(f) Same as (a)–(c) in the AFM case.

which is not an eigenstate of H_{XY} , and monitor its free evolution under H_{XY} . This is similar to what was done in Ref. [22] for a 1D Bose gas, and alternative to, e.g., Bragg spectroscopy of 1D gases [23]. For the initial state we choose the coherent spin state $|\text{CSS}\rangle = |\rightarrow_y \rightarrow_y \cdots \rightarrow_y\rangle$ where all spins are pointing along y [Fig. 4(a)]. This state is a mean-field approximation of the FM ground state of H_{XY} .

We then let the system evolve for a time t , and measure the z correlations. The resulting evolution of $C^z(d, t)$ is shown in Fig. 4(b) where d is the perimeter distance along the ring. We observe two distinct behaviors depending on the distance. First, nearest-neighbor correlations ($d = 1$) build up in less than $1/J$ and reach a quasistationary negative value. Second, for $d > 1$ a positive correlation wave front spreads ballistically from $d = 2$ up to the largest distances.

The ballistic propagation occurs at a velocity $2v_g$, revealing the characteristic group velocity v_g of the excitations of the system [73]. Indeed, two entangled quasiparticles, emitted together with opposite velocities, will spread correlations through the system at their relative velocity $2v_g$ [74]. A two-dimensional fit of the data gives $v_g = 2.34(5)aJ$, which is larger than the sound velocity of a system with NN interactions $u_{\text{NN}} = 2aJ$. This reveals that dipolar FM interactions accelerate the dynamics compared to NN interactions. Also, contrary to the 2D case [72] or to 1D systems with longer-range interactions [41,42], they maintain a linear light-cone dynamics, as expected for a TLL. Yet the sound velocity expected theoretically for the dipolar FM chain $u_{\text{FM}} \approx 3.7aJ$ is significantly larger than the measured one. This discrepancy can be explained by inspecting the whole spectrum of excitations, which reveals the linear sound mode involves only very few wave vectors in our small ring (see Appendix I). As our quench protocol does not populate selectively those few wave vectors, the correlation dynamics is instead dominated by modes at intermediate wave vectors, which possess a smaller effective group velocity.

To study the correlation dynamics in the AFM chain, we perform the same protocol up to a sublattice rotation [72]: As shown in Fig. 4(e), the initial state is now a staggered coherent spin state $|\text{CSS}\rangle_{\text{stag}} = |\rightarrow_y \leftarrow_y \cdots \rightarrow_y \leftarrow_y\rangle$, which is the mean-field ground state of the AFM Hamiltonian $-H_{\text{FM}}$. The qualitative behavior of $C^z(d, t)$ is the same, but the propagation of correlations is now slower than the one with NN interactions: $v_g = 1.66(3)aJ$, a signature of frustration of dipolar AFM interactions. This time the observed light-cone velocity is much closer to the sound velocity predicted theoretically, $u_{\text{AFM}} \approx 1.8aJ$, reflecting the fact that in the AFM case the linear sound mode dominates the spectrum at small wave vectors (see Appendix I).

VII. CONCLUSION AND OUTLOOK

In this work, we have shown that a 1D chain of Rydberg-encoded spins interacting under the dipolar XY Hamiltonian realizes the physics of one-dimensional gapless spin liquids, belonging to the family of Tomonaga-Luttinger liquids. The measured FM and AFM correlation profiles reveal the expected power-law decays, and the discrepancies with the theoretical ground-state correlations can be explained by experimental imperfections. We have

observed Friedel oscillations around an impurity and have verified the expected dependence of the oscillation wave vector with the magnetization. Finally, the quench dynamics at low energy shows a linear light-cone propagation of correlations, which allows us to extract the sound velocity of a Tomonaga-Luttinger liquid when the sound mode dominates the low-energy spectrum. Our realization of this state in a quantum simulator opens a variety of directions for future exploration.

First, our work presents a new example of simulated quantum criticality. An interesting point of comparison is the $(1+1)\text{D}$ Ising critical point (associated with $\mathbb{Z}_2$ symmetry breaking), which also features power-law correlations as recently observed in analog and digital quantum simulation [44,75,76]. As in our system, such critical correlations are a consequence of an emergent conformal field theory (CFT) description of the low-energy physics; however, the TLL CFT is distinct from the Ising CFT, and is in some ways much richer. In particular, the microscopic $U(1)$ spin symmetry present in our model is further enhanced in the continuum description [77–79], leading to an infinite family of fundamental excitations (the so-called primary field operators)—by contrast, at Ising criticality there are just three such modes [80]. This structure is responsible for the appearance of four distinct critical exponents in the spin-spin correlation functions [Eqs. (2) and (3)], reflecting the contribution of four different primary excitations. A key aim for future experiments will be to reach enhanced precision (i.e., by increasing system size and reducing imperfections), so that each of these power laws is well resolved, as well as those present in more complex observables such as multi-spin correlation functions and entanglement measures.

Second, our system differs from many conventional spin models by way of its dominant power-law interactions, and its particular imperfections, e.g., dynamically formed holes due to spontaneous Rydberg decay. Our measurements, coupled with our multimethod numerical simulations, have begun to probe the TLL's robustness to such effects, although several fundamental questions remain open. For example, how does the strong velocity renormalization observed in the FM quench experiment converge to the thermodynamic limit as one varies the system size, or moves closer to the symmetry-breaking transition with longer-range tails? Does the slow formation of holes lead to interesting transient dynamics or nontrivial steady states in the TLL? How do the long-range FM and AFM interactions modify the stability of the TLL to weak disorder, the nature of the strong-disorder limit, and any phase transition between the two regimes? Systematically investigating the impact of each of these effects on the TLL, and especially their interplay, is likely to be insightful [81–84].

Finally, how might the TLL realized here be modified and augmented using other capabilities of the dipolar Rydberg apparatus? Two potentially useful tuning knobs

are our fine control of the atom geometry, and the ability to incorporate additional interactions [85,86]. Indeed, it was recently predicted that one can use such features to reach a deconfined critical point at the edge of the TLL phase [50]. An alternative direction is to study multiple interacting TLL chains, as in the theoretical coupled-wire constructions of two-dimensional topological phases [87]. We note that, for van der Waals (vdW) Rydberg chains at Ising criticality, this has been argued to be a potential route to a gapless $\mathbb{Z}_2$ spin liquid [88]. We suspect that there are similarly complex phases to be built out of the dipolar TLL [89].

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: MAPPING OF ATOMIC STATES ONTO A SPIN MODEL

All experiments were performed with arrays of ^{87}Rb atoms trapped in optical tweezers, using the setup described in previous works [52,58,90]. The mapping from two Rydberg states onto a spin $1/2$ is $|\uparrow\rangle = |nS_{1/2}, m_J = 1/2\rangle$ and $|\downarrow\rangle = |nP_{1/2}, m_J = -1/2\rangle$ with n the principal quantum number. A 45 G magnetic field perpendicular to the array ensures isotropic interactions. Spin rotations are performed using microwave pulses with Gaussian temporal envelope and the magnetic field isolates the effective spin states from other Zeeman levels.

The atoms in those states interact under the dipole-dipole Hamiltonian, which in spin language translates into the following Hamiltonian [91]:

$$H_{\text{tot}} = H_{\text{XY}} + H_{\text{vdW}}. \quad (\text{A1})$$

Here, H_{XY} is the first-order contribution of effective Hamiltonian theory and is given by Eq. (1) of the main text; the second-order van der Waals contribution H_{vdW} reads

$$H_{\text{vdW}} = \sum_{i<j} \frac{1}{r_{ij}^6} \sum_{(st) \in \{\uparrow, \downarrow\}^2} U_{s,t} n_i^s n_j^t, \quad (\text{A2})$$

with $n_i^\uparrow = (1 + \sigma_i^z)/2$ and $n_i^\downarrow = (1 - \sigma_i^z)/2$. The vdW Hamiltonian leads to small corrections compared to the pure XY Hamiltonian. In particular, the TLL is expected to remain stable under this translation-invariant and M^z -conserving interaction, with only a small renormalization of K and u [48]. It is taken into account in all our numerical simulations. The values of the interaction energies are estimated from Ref. [92] and are summarized in Table I. The vdW term can be written in terms of spin operators as $H_{\text{vdW}} = -J_{\text{vdW}}/2 \sum_{i<j} (1/r_{ij}^6) \sigma_i^z \sigma_j^z$, up to uniform field terms which are irrelevant when the magnetization is conserved.

The choice of n results from a trade-off between having the longest possible Rydberg lifetimes (scaling as n^3) and a not-too-large XY interaction energy J (scaling as n^4) to make initial-state preparation and detection easier (see

TABLE I. Values of the interaction energies for the various experimental sequences, for a 45 G magnetic field perpendicular to the array to guarantee isotropic interactions.

Sequence	Adiabatic preparation	Quench experiment
Principal quantum number	$n = 70$	$n = 60$
Nearest-neighbor distance	$16.2 \mu\text{m}$	$13 \mu\text{m}$
XY interaction energy [Eq. (1)]	$J = 2\pi \times 0.55 \text{ MHz}$	$J = 2\pi \times 0.62 \text{ MHz}$
van der Waals interaction energies [Eq. (A2)]	$U_{\uparrow,\uparrow} = 2\pi \times 0.051 \text{ MHz}$ $U_{\downarrow,\downarrow} = -2\pi \times 0.007 \text{ MHz}$ $U_{\uparrow,\downarrow} = U_{\downarrow,\uparrow} = 2\pi \times 0.058 \text{ MHz}$	$U_{\uparrow,\uparrow} = 2\pi \times 0.030 \text{ MHz}$ $U_{\downarrow,\downarrow} = -2\pi \times 0.006 \text{ MHz}$ $U_{\uparrow,\downarrow} = U_{\downarrow,\uparrow} = 2\pi \times 0.009 \text{ MHz}$

details in Sec. E). For the adiabatic preparation schemes, we choose $n = 70$ in a $N = 24$ atoms chain with nearest-neighbor distance $16.2 \mu\text{m}$, whereas for the quench experiments we choose $n = 60$ to have more atoms ($N = 30$) in the same area, with a similar interaction energy (NN distance $13 \mu\text{m}$).

APPENDIX B: EXPERIMENTAL SEQUENCE

The beginning of the experimental sequence, as well as the readout, are common to all experiments. Starting from a cloud of ^{87}Rb atoms in a magneto-optical trap, single atoms are loaded into optical tweezers and rearranged into a defect-free array with the desired geometry [53]. Atoms are Raman sideband cooled down to $\sim 10 \mu\text{K}$, and optically pumped into the state $|5S_{1/2}, F = 2, m_F = 2\rangle$. After that, we adiabatically ramp down the tweezers' power to reduce the velocity dispersion of the atoms; next, we switch off the tweezers, and excite all atoms to the Rydberg state $|\uparrow\rangle$ using a stimulated Raman adiabatic passage (STIRAP) via the intermediate state $|6P_{3/2}, F = 3, m_F = 3\rangle$. The following of the sequence depends on the type of experiment that

we perform: either an adiabatic preparation of XY ground states (Secs. III and V of the main text) or a quench experiment from the mean-field ground state (Sec. VI).

At the end of the sequence, we perform a projective measurement of each atom's state, and repeat it more than 1000 times with the same sequence to acquire statistics. Each projective measurement consists of four steps.

- (1) A global microwave pulse defines the measurement basis. In the absence of this pulse, the measurement basis is z . To measure spins in the xy plane, we apply a global $\pi/2$ pulse, whose phase determines the basis (x or y or any combination of those bases).
- (2) To prevent the spins from evolving during the following of the readout process, the spin dynamics is stopped by a “freezing” pulse which removes the atoms in $|\downarrow\rangle$ faster than the typical evolution time $2\pi/J \sim 1 \mu\text{s}$. Depending on the atomic states used in the mapping, we send the atoms either to the state $|69D_{5/2}, m_J = -1/2\rangle$ with a single-photon Gaussian pulse [Figs. 5(a) and 5(b)] or to the hydrogenic manifold $58G$ via a three-photon square pulse [Figs. 5(c) and 5(d)].

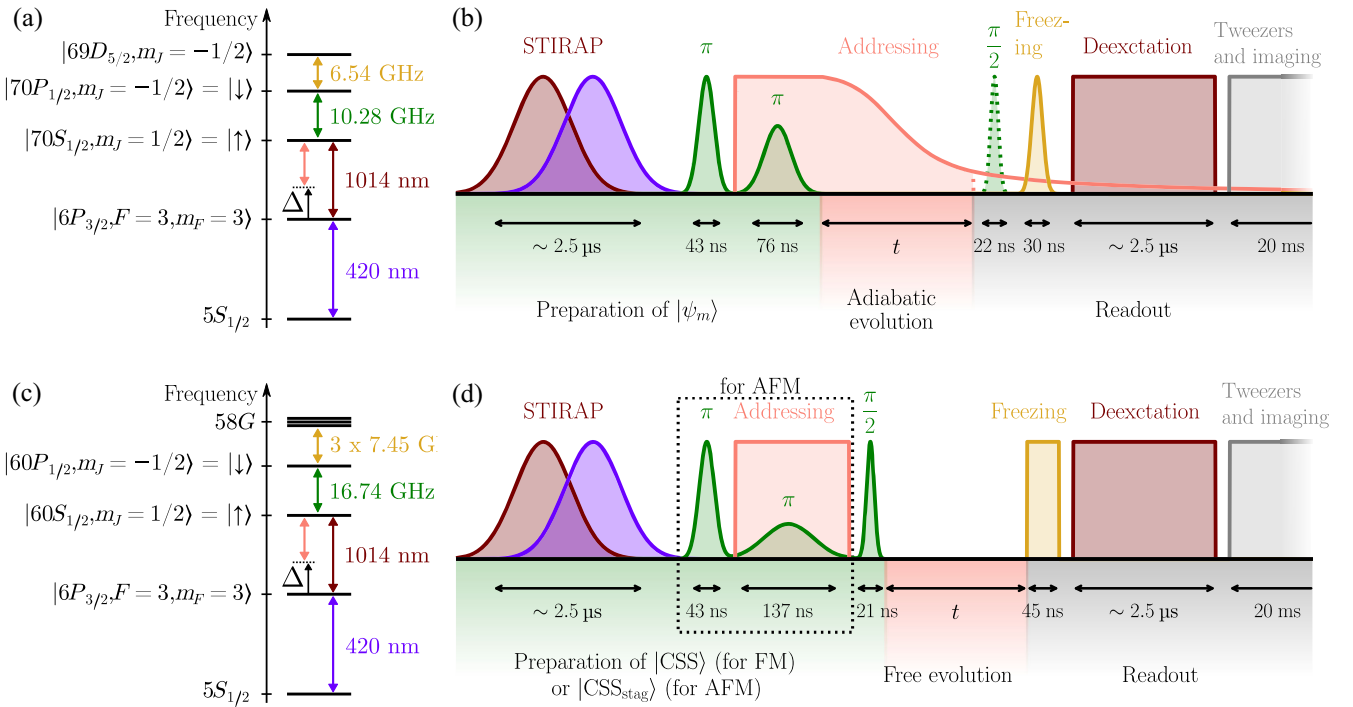


FIG. 5. Experimental sequence for adiabatic preparation of XY ground states (Secs. III and V of the main text) and for the quench experiment (Sec. VI). (a) Atomic energy levels involved in the adiabatic sequence and associated transitions: 420 and 1014 nm are the wavelengths used for Rydberg excitation, and 10.28 and 6.54 GHz are microwave fields for driving Rydberg-Rydberg transitions. (b) Sketch of the experimental sequence for adiabatic preparation of the XY FM and AFM ground state of $\pm H_{XY}$. For clarity the timings of the pulses are not to scale, and their colors refer to the transition energies in (a). The readout depends on the basis of measurement. To measure spins in the z basis, we leave the addressing light shifts on (pink solid line), freeze the interaction dynamics (yellow pulse), and read the atomic state of each atom (red and gray pulses). To measure the spins in the xy plane, we switch off the addressing light shifts (pink dotted line) and perform a global microwave $\pi/2$ pulse (green dotted line), before freezing and reading of the atomic state. (c) Atomic energy levels involved in the quench sequence. (d) Sketch of the experimental sequence for the quench experiment from the XY mean-field ground state of $\pm H_{XY}$. The dotted black rectangle shows the part of the sequence which is specific to the AFM case.

- (3) Atoms in $|\uparrow\rangle$ are deexcited to the ground-state manifold $5S_{1/2}$ by shining a pulse of 1014-nm light on resonance with the short-lived state $|6P_{3/2}, F=3, m_F=3\rangle$.
- (4) Tweezers are switched back on to recapture the atoms in $5S_{1/2}$ and eject those remaining in the Rydberg states by the ponderomotive force. Finally, we perform a global fluorescence imaging of atoms in $5S_{1/2}$, and map the imaged atoms on $|\uparrow\rangle$, whereas the lost atoms are considered as $|\downarrow\rangle$.

APPENDIX C: EXPERIMENTAL METHODS FOR ADIABATIC PREPARATIONS

To prepare the ground state of $\pm H_{XY}$, we use the same adiabatic protocol as in Ref. [58]. The sequence is shown in Fig. 5(b). A first global microwave π pulse transfers all atoms from $|\uparrow\rangle$ to $|\downarrow\rangle$. Then, we switch on the addressing light shifts on sublattice B , and perform another π pulse on resonance with the atoms of sublattice A only, thus preparing the state $|\psi_m\rangle$ with $(N+m)/2$ spins in $|\uparrow\rangle$ (sublattice A) and $(N-m)/2$ spins in $|\downarrow\rangle$ (sublattice B). Next, we ramp down the addressing light shifts $\delta(t)$ by controlling the addressing optical power with an AOM. In the particular case where the energy gap between the ground state and the first excited state depends linearly on δ , one can derive the following analytical expression for the adiabatic ramp (up to the response time of the AOM) [44,63]:

$$\delta(t) = \frac{E_0\delta_c t + E_c\delta_0(T-t)}{E_0 t + E_c(T-t)}, \quad (\text{C1})$$

with T the duration of the ramp, δ_0 the initial light shift, δ_c the light shift at the critical point, E_0 the energy gap for $\delta = \delta_0$, and E_c the gap for $\delta = \delta_c$. For the 1D XY model, the critical point occurs at $\delta_c = 0$, so that Eq. (C1) can be simplified into

$$\delta(t) = \delta_0 \frac{T-t}{T-(1-\alpha)t}, \quad (\text{C2})$$

where $\alpha = E_0/E_c$ (E_c is a finite-size energy gap, expected to vanish in the thermodynamic limit). The values of T and α were optimized empirically to maximize the experimentally measured x correlations, and the data from the main text were taken with $(T, \alpha) = (1.5 \mu\text{s}, 20)$ in the FM case and $(T, \alpha) = (2.5 \mu\text{s}, 100)$ in the AFM case. The value of α is larger in the AFM case, in agreement with the fact that the critical gap E_c is smaller in the AFM case, due to the weak frustration induced by next-nearest neighbor couplings.

APPENDIX D: EXPERIMENTAL SEQUENCE FOR THE QUENCH EXPERIMENT

For the quench experiment discussed in Sec. VI, we use the same protocol as in Ref. [72]. The sequence is summarized in Fig. 5(d), and it differs for FM and AFM. In the FM

case, a global microwave $\pi/2$ pulse after the STIRAP directly prepares the targeted state $|\text{CSS}\rangle = |\rightarrow_y \rightarrow_y \cdots \rightarrow_y\rangle$. In the AFM case, we add a set of microwave and addressing pulses [shown inside the black dotted frame of Fig. 5(d)] to first prepare the state $|\psi_0\rangle = |\uparrow\downarrow \cdots \uparrow\downarrow\rangle$, then we rotate it with a global $\pi/2$ pulse, thus preparing the state $|\text{CSS}\rangle_{\text{stag}} = |\rightarrow_y \leftarrow_y \cdots \rightarrow_y \leftarrow_y\rangle$. The initial-state preparation is followed by a free evolution under H_{tot} .

APPENDIX E: EXPERIMENTAL IMPERFECTIONS

Several imperfections limit the fidelities of the previously described adiabatic protocol.

1. Finite fidelity of the initial-state preparation

The initial-state preparation suffers from two limitations. First, the Rydberg excitation has a finite efficiency $1 - \eta_{\text{STIRAP}}$, leading to a small portion of atoms $\eta_{\text{STIRAP}} \approx 2\%$ which are left in the ground state and do not interact with the other atoms. Second, the spin rotations are performed in the presence of interactions and with finite light shifts, which reduce their efficiency by $\sim 1\%$ – 2% and thus increase the energy of the initial state compared with the ideal ground state of H_Z .

2. Nonadiabaticity and decoherence

Several decoherence phenomena affect the time evolution of the system, especially the adiabaticity.

A major limitation is the finite Rydberg lifetimes, which induce leakages from the ideal isolated two-level system ($|\uparrow\rangle, |\downarrow\rangle$) to other atomic states. There are two contributions to the Rydberg lifetimes [93]: spontaneous emission to the low-lying energy states, at a rate $1/\tau_0$, and depopulation by absorption and stimulated emission of blackbody radiation (BRR) in our room-temperature setup, at a rate $1/\tau_{\text{BRR}}$. Those contributions add up to an effective decay rate $1/\tau_{\text{eff}} = 1/\tau_0 + 1/\tau_{\text{BRR}}$ that depends on the considered Rydberg level. For $n = 70$, we estimate [94]: $\tau_{\text{eff}}^{|\uparrow\rangle} \approx 144 \mu\text{s}$ and $\tau_{\text{eff}}^{|\downarrow\rangle} \approx 172 \mu\text{s}$.

The lifetime of the state $|\uparrow\rangle$ can also be reduced in the presence of the addressing, which can induce transitions to the short-lived state $6P_{3/2}$ that then decays to the ground-state manifold $5S_{1/2}$. We limited this depumping effect by initializing the addressed atoms in the state $|\downarrow\rangle$, such that the depumping can happen only during the adiabatic ramp, when the addressing light shifts are still on and the addressed atoms are partially in $|\uparrow\rangle$. We checked experimentally that the percentage of depumped atoms due to the addressing is smaller than 0.5% at the end of the adiabatic ramp.

Another decoherence effect comes from the fluctuations of the atomic positions. During the Rydberg sequence, atoms are in free flight: The standard deviation of their

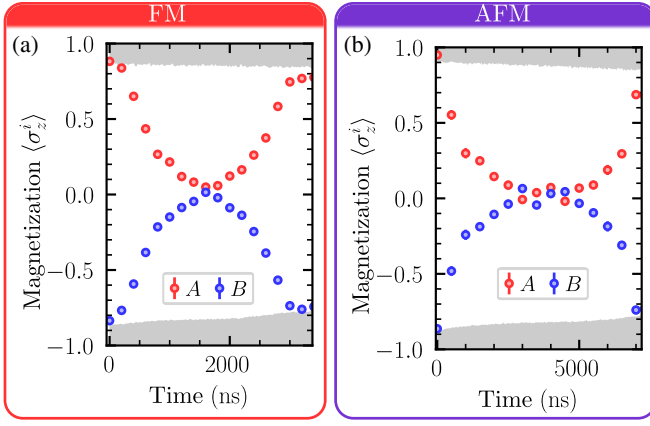


FIG. 6. Adiabatic preparation of XY ground states and back. Time evolution of the magnetization per sublattice in the FM case (a) and in the AFM case (b), in an experiment where light shifts are ramped down and then ramped up in a symmetric way. The ramp-up starts at $t = 1.5 \mu\text{s}$ in the FM case and at $t = 2.5 \mu\text{s}$ in the AFM case. The gray regions indicate the expected loss of contrast from single-atom imperfections, due to finite Rydberg lifetimes and depumping of addressed atoms (see text), starting from the experimentally measured contrast.

position along μ increases as $\sigma_\mu^{\text{tot}}(t) = \sqrt{\sigma_\mu^2 + (\sigma_{v_\mu} t)^2}$, with σ_μ the position uncertainty of the atoms in their the tweezer, and σ_{v_μ} their velocity uncertainty. We estimate that in the radial directions, $\sigma_{x,y} \sim 100 \text{ nm}$ and $\sigma_{v_{x,y}} \sim 25 \text{ nm}/\mu\text{s}$, whereas in the axial direction $\sigma_z \sim 800 \text{ nm}$ and $\sigma_{v_z} \sim 40 \text{ nm}/\mu\text{s}$. This leads to fluctuations of the couplings between spins, which are averaged over many slightly disordered geometries.

The level of adiabaticity can be quantified by performing a back-and-forth adiabatic preparation: Starting from the state $|\psi_0\rangle$, we first ramp down δ to 0, and then ramp it up in a symmetric way to get back to the initial state $|\psi_0\rangle$. Figure 6 shows the resulting z magnetization in both FM and AFM cases. Whereas we would ideally expect the z magnetization to come back to its initial value, we observe a reduction of the contrast of about 12% in the FM case (from 86% at $t = 0$ to 76% at $t = 3.4 \mu\text{s}$), and about 21% in the AFM case (from 90% at $t = 0$ to 71% at $t = 7.3 \mu\text{s}$).

The observed loss of contrast may come from adiabatic imperfections, but also from single-particle errors: finite Rydberg lifetimes and depumping of addressed atoms. To estimate those contributions, we perform Monte Carlo simulations for the recapture probabilities of addressed and nonaddressed atoms: First, the atomic state is randomly sampled according to the calibrated preparation errors, then its time evolution is computed accounting only for *single-particle* errors. The resulting magnetization for addressed and nonaddressed atoms is shown as the gray regions in Fig. 6. At the end of the ramp, the largest accessible contrast due to single-particle errors is 80% in the FM case and 82% in the AFM case, partially explaining the contrast

reduction of the experimental data. The remaining contribution is attributed to adiabatic imperfections.

3. Readout errors

Spin states can be misread during the measurement phase. A spin $|\uparrow\rangle$ has a probability $\varepsilon_\uparrow = 2.5 \pm 1\%$ to be measured as $|\downarrow\rangle$, owing to mechanical losses and finite deexcitation efficiency. Conversely, a spin $|\downarrow\rangle$ has a probability $\varepsilon_\downarrow = 3 \pm 1\%$ to be measured as $|\uparrow\rangle$ due to spontaneous emission to $5S_{1/2}$ before the atom is kicked out from the tweezers. The values of $|\downarrow\rangle$ and $|\uparrow\rangle$ are experimentally measured using a set of independent measurements. The effect of detection errors on the measured observables can be computed. The average magnetization $\langle \sigma_\mu^i \rangle$ and correlations $C_{i,j}^\mu$ are related to the same quantities $\langle \tilde{\sigma}_\mu^i \rangle$ and $\tilde{C}_{i,j}^\mu$ without detection errors by the relations (valid to first order in $\varepsilon_{\uparrow,\downarrow}$):

$$\langle \sigma_\mu^i \rangle = (1 - \varepsilon_\downarrow - \varepsilon_\uparrow) \langle \tilde{\sigma}_\mu^i \rangle + \varepsilon_\downarrow - \varepsilon_\uparrow, \quad (\text{E1})$$

$$C_{i,j}^\mu = (1 - 2\varepsilon_\downarrow - 2\varepsilon_\uparrow) \tilde{C}_{i,j}^\mu. \quad (\text{E2})$$

APPENDIX F: U(1) SYMMETRY OF THE PREPARED STATES

In this appendix, we check if the states prepared using the adiabatic protocol of Sec. III satisfy the expected U(1)

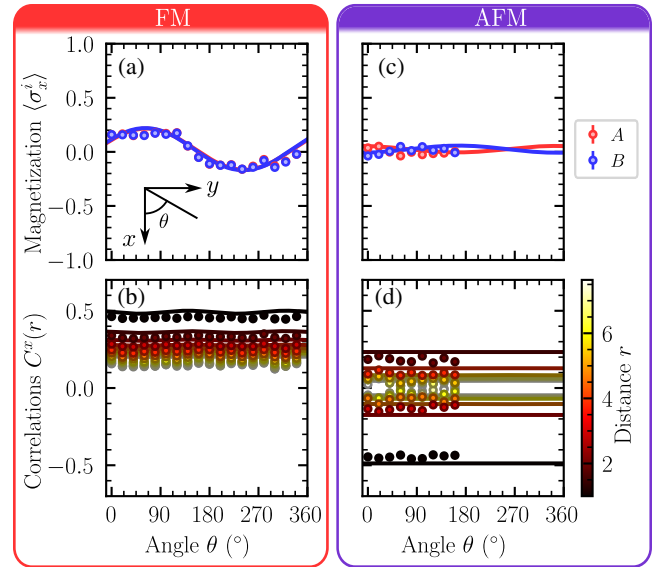


FIG. 7. Angular dependence of the magnetization and correlations in the xy plane. (a) Magnetization per sublattice, as a function of the angle θ in the xy plane (inset), in the FM case, measured at $t = 1.6 \mu\text{s}$. The variation of the angle is obtained by scanning the phase of the measurement pulse. (b) Correlations as a function of θ , in the FM case, at $t = 1.6 \mu\text{s}$. (c, d) Same as (a, b) in the AFM case, at $t = 3.5 \mu\text{s}$.

symmetry of H_{XY} . For this purpose, we perform a measurement at the end of the ramp along a given direction $\cos(\theta)x + \sin(\theta)y$, and we scan the angle θ . The resulting magnetization $\langle \sigma_\theta^i \rangle$ shows a residual oscillation [Figs. 7(a) and A3(e)], which we attribute to the repercussion of a small U(1) symmetry breaking in the initial state. The amplitude of the oscillation is larger in the FM case, in agreement with the numerical simulations. However, the connected correlations $C^\theta(r)$ are isotropic up to the statistical noise, and simulations confirm the weak dependence of $C^\theta(r)$ with θ [Figs. 7(b) and A3(f)]: The connected character of the correlations tends to compensate the symmetry breaking observed in the magnetization.

APPENDIX G: FITTING PROCEDURE FOR THE CORRELATION PROFILES

Here, we explain our protocol to extract the power-law exponent of the correlations from their spatial profile shown in Fig. 2 of the main text. This procedure may suffer from three biases.

- (1) At large distances, any finite energy density turns the power-law decay into an exponential decay.
- (2) At short distances, the theoretical expression from the TLL theory [Eqs. (2) and (3) of the main text] may not describe accurately the system, since this field theory is valid only at asymptotic distances $r \gg 1$.
- (3) Independent detection errors rescale the correlation profile by a global multiplicative factor $1 - 2\varepsilon_\downarrow - 2\varepsilon_\uparrow = 0.89$, whatever the distance between the spins [Eq. (E2)]. This does not modify the power-law exponent of the correlations, but it affects their amplitude which depend on K in the case of the z correlations [Eq. (3)].

To limit the large-distance bias, we choose to incorporate a finite correlation length ξ into our fitting functional form, as already explained in the main text: $\tilde{C}^x(r) = C^x(r)e^{-r/\xi}$, in both FM and AFM cases. This is done only along x , since the measured z correlations appear less sensitive to finite-temperature effects.

To avoid the second bias, we define a cutoff distance r_c and fit correlations $C(r)$ on distances $r \geq r_c$. We set the value of r_c by repeating the fits for different values of r_c on the ideal ground state (simulated by DMRG), and choose the smallest cutoff that gives a satisfying convergence. This analysis is shown in Fig. 8. The only case which is sensitive to the cutoff is the FM correlations along z , for which we set $r_c = 3$ sites; in the three other cases we set $r_c = 0$.

Finally, to take into account the effect of independent detection errors, we included the rescaling factor due to the detection errors in the fitting functional form for the z correlations: $\tilde{C}^z(r) = 0.89C^z(r)$. Since the rescaling factor is fixed by independently calibrated detection errors, it preserves the number of free parameters used in the fit.

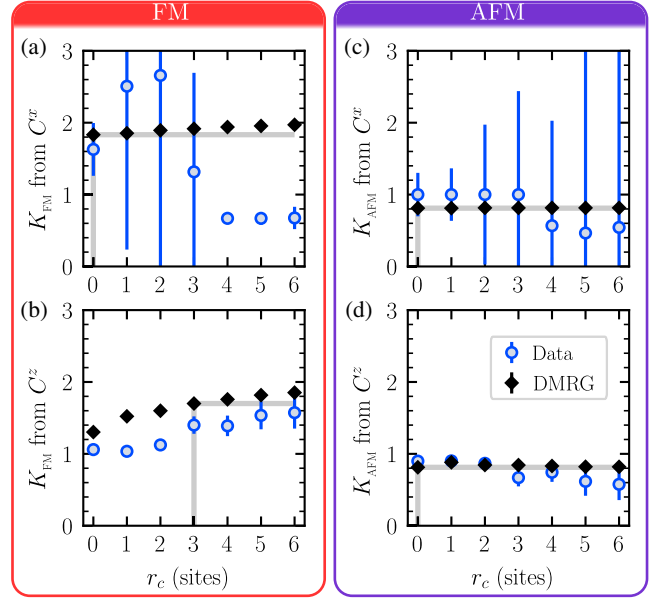


FIG. 8. Effect of a lower cutoff on the fitted Luttinger parameter K . Each panel corresponds to one of the four spatial correlation profiles shown in Fig. 2 of the main text: $C^x(r)$ in the FM case (a) and in the AFM case (c); $C^z(r)$ in the FM case (b) and in the AFM case (d). For each correlation profile, we fit the correlations for distances $r \geq r_c$, for both the ideal ground state (simulated with DMRG, black diamonds) and the data (blue points), and we repeat the fit for various cutoff distances r_c . The chosen value of r_c is indicated by the gray vertical line.

APPENDIX H: NUMERICAL METHODS

1. Simulation of the adiabatic ramp with imperfections

The dynamical simulations of the ramp were performed using the time-dependent variational principle (TDVP) with MPSs, implemented via the ITENSOR package [95] with truncation error $< 10^{-10}$ and bond dimension up to 200. The simulations contain no free parameter.

The state following the laser Rydberg excitation consists of all spins pointing up, where each spin is replaced by a hole with a probability of 0.02% to describe the STIRAP errors. A hole represents a spin that does not interact with other spins and thus does not contribute to the observables. Moreover, during the time evolution, new holes appear due to the finite lifetime of the Rydberg states. They are incorporated into the simulation using the quantum trajectory method [96] with four distinct decay channels: transitions from the $|\uparrow\rangle$ or $|\downarrow\rangle$ states to either the atomic ground state $|g\rangle$ or another Rydberg state $|r'\rangle$. Consequently, the local Hilbert space is expanded to $\dim(\mathcal{H}_i) = 4$. We introduce $4N$ jump operators $c_{i,m}$, where $m = 1, \dots, 4$ defined as

$$\begin{aligned} c_{i,1} &= \sqrt{\gamma_1}|g\rangle_i\langle\uparrow|, & c_{i,2} &= \sqrt{\gamma_2}|r'\rangle_i\langle\uparrow|, \\ c_{i,3} &= \sqrt{\gamma_3}|g\rangle_i\langle\downarrow|, & c_{i,4} &= \sqrt{\gamma_4}|r'\rangle_i\langle\downarrow|, \end{aligned} \quad (\text{H1})$$

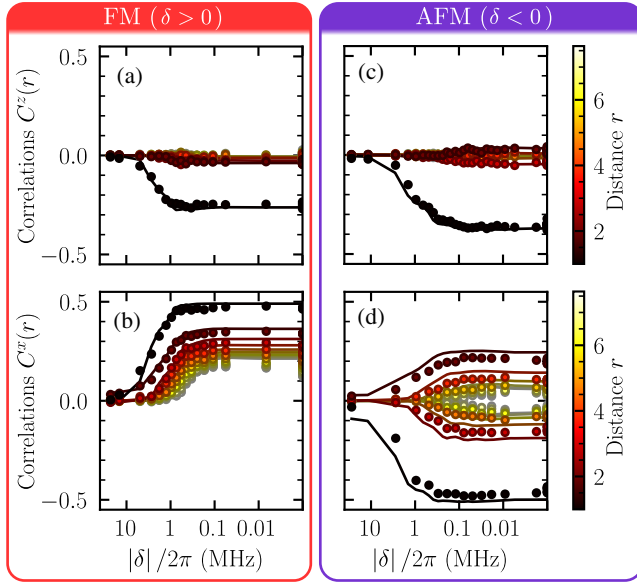


FIG. 9. Comparison of the measured correlations (data points) with numerical simulations of the dynamics (solid lines). The first row (a), (c) is the correlations along z for all distances, and the second row (b), (d) is the correlations along x .

where γ_m represents the decay rate associated with the decay channel m and $i = 1, \dots, N$ is the atomic index. We evaluate at each step of the evolution the probability associated to every decay channel $\delta p_{i,m} = \delta t \langle \Psi(t) | c_{i,m}^\dagger c_{i,m} | \Psi(t) \rangle$, where $|\Psi(t)\rangle$ denotes the state at time t . Calling $\delta p = \sum_{i,m} \delta p_{i,m}$ the probability that a quantum jump occurs, the state is stochastically evolved with probability δp to $|\Psi(t + \delta t)\rangle = \sqrt{\delta t / \delta p_{i,m} c_{i,m}} |\Psi(t)\rangle$ where one particular channel is chosen with probability $\delta p_{i,m} / \delta p$, otherwise to $|\Psi(t + \delta t)\rangle = \exp[-iH_{\text{eff}}\delta t] |\Psi(t)\rangle / \sqrt{1 - \delta p}$, with the effective Hamiltonian given by $H_{\text{eff}} = H_{\text{tot}} - (i/2) \sum_{i,m} c_{i,m}^\dagger c_{i,m}$. We assumed state- and site-independent decay rates $\gamma_m = 0.0037$. This significantly simplifies the picture, as the non-Hermitian Hamiltonian is then proportional to the identity.

We also simulate the microwave pulses to prepare the Néel state and to measure observables in the x, y plane. During those pulses, the spins still interact via H_{tot} . For a meaningful comparison of observables in the xy plane between experiment and theory, we need to align the x and y axis of the theoretical calculations with those of the experiment. This is done by aligning the direction of maximum magnetization in the xy plane between simulation and experiment.

The results of the simulations of the dynamics are shown in Fig. 1(c) for the magnetization and Fig. 9 for the correlations. They are in very good agreement with the data, except for the AFM x correlations, which grow slightly faster than observed.

2. Ground-state and thermal state calculations

We simulate ground states and thermal states of the Hamiltonian H_{tot} using the quantum Monte Carlo (QMC) method based on the stochastic series expansion approach [97] for the FM. For the AFM, we use the DMRG provided by the ITENSOR package [95] with truncation error $< 10^{-10}$ and bond dimension up to 200 (QMC method has a sign problem in that case). For both the FM and AFM, we average over a few hundred realizations of holes, with a concentration p which accounts for both the holes in the initial state due to the imperfect excitation of atoms to Rydberg states and the holes appearing during the ramp.

In the FM case, both the experimental and numerical data for the ramp dynamics show a significant residual magnetization in the xy plane $m_{xy} \approx 0.18$, resulting from an imperfect preparation of the initial state of the sequence. We include this element in the finite-temperature calculations by adding a small transverse field $h/J \approx 10^{-2}$, adjusted so as to stabilize the experimentally observed magnetization. For the AFM chains, we use the purification method applied to MPS [98]. The final residual magnetization in the xy plane is very small in the AFM case, and there we do not include any transverse field.

In an ideal state-preparation sequence, the system would be prepared in a zero-magnetization state at the beginning of the ramp, and it would remain in that sector throughout the evolution, since the Hamiltonian conserves magnetization. As a consequence, the variance of the magnetization $\text{Var}(M_z) = \sum_{ij} C^z(i, j)$ would be strictly zero in the final state—at variance with a thermal state with a fluctuating magnetization. This constraint is global and is important for observables sensitive to scales comparable to the system size, such as the long-distance $C^z(i, j)$ correlations. In practice, however, the initial state in the experiment has a finite uncertainty on M^z due to the finite fidelity of the initial-state preparation. This variance is then propagated throughout the ramp sequence, although it remains significantly smaller than that of thermal states at the temperatures relevant for the experiment. In principle one would need to constrain the simulated thermal states to reproduce the experimentally observed $\text{Var}(M_z)_{\text{exp}}$, adding therefore an additional Lagrange multiplier to the Gibbs ensemble beside temperature (and transverse field for the FM). We opt for a simpler approach, by leaving the simulations unconstrained, and correcting the simulated correlations $C^z(i, j)$ by an offset C_{off} , $\tilde{C}^z(i, j) = C^z(i, j) + C_{\text{off}}$, such that $\sum_{ij} \tilde{C}^z(i, j) = \text{Var}(M_z)_{\text{exp}}$. This correction affects primarily the long-distance correlations. Even though empirical, we verified its quantitative validity using exact diagonalization in small systems.

As for the temperature, it could be determined in principle by matching the experimental energy to that of the numerical simulations. Yet such an approach is highly sensitive to uncertainties in the experimental reconstruction

of correlations at short range, which dominate the energy. We use a more global approach, searching for the temperature which gives the best match between experiments and simulations for the whole spatial structure of the C^x and C^z correlations. This leads to the temperature cited in the main text.

3. Simulation of the quench experiment

The simulations were done using the software ITENSOR [95]. We used the TDVP algorithm with time step 0.02 μ s (i.e., $0.0051/J$), truncation error $< 10^{-5}$, and bond dimension 1600. They do not include experimental imperfections.

APPENDIX I: LUTTINGER-LIQUID PROPERTIES VS LOCALIZATION OF DIPOLAR CHAINS

We analyze here the ground-state correlations of dipolar chains to compare them to the expectations from Luttinger-liquid theory, in the ideal case and in the presence of holes.

1. Dipolar chains without holes

Correlation functions and compressibility. Figure 10 shows QMC results for the ground state of the dipolar FM chain. We compare two system sizes, $N = 24$ (the experimental one) and a larger one ($N = 100$) to capture the asymptotic properties of the correlations. The calculations were performed at temperatures $T \lesssim 10^{-2}J$ where thermal effects are negligible. We observe that already for $N = 24$ the correlations decay as $1/r^{1/(2K)}$ —and the value of K does not change significantly when fitting by the more

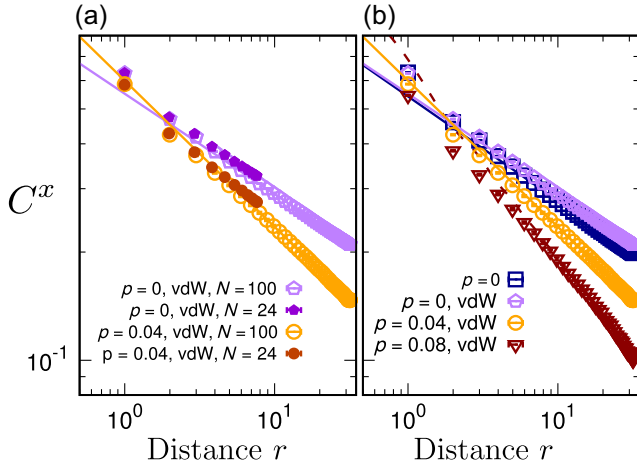


FIG. 10. Ground-state correlations of the dipolar FM chain. (a) Correlations C^x for the ground state of the dipolar FM Hamiltonian (including vdW interactions) for $p = 0$ and $p = 0.04$ hole doping. We compare the experimental system size $N = 24$ to a larger system size $N = 100$. (b) Same as in (a) for $N = 100$, including the case of a chain without holes and vdW interactions, and a chain with doping $p = 0.08$. In both (a) and (b) r is the chord distance. The solid and dashed lines are power-law fits to $A/r^{1/(2K)}$.

involved Eq. (2) of the main text. The fit leads to a value of $K_{\text{FM}} = 1.85(1)$, which is larger than the better estimates for the longer chain. Indeed, for $N = 100$, we obtain $K_{\text{FM}} \approx 1.72$ for the chain with purely dipolar interactions, in agreement with the estimate of Ref. [49], and $K_{\text{FM,vdW}} \approx 1.79$ when including the vdW interactions. The increase of K in the presence of ferromagnetic vdW interactions is consistent with what is observed in the XXZ model, in which the K parameter is a monotonically increasing function of the (ferromagnetic) interactions for the z spin components [1].

From the magnetization curve of the $N = 100$ system at small fields (not shown), we calculate a spin susceptibility along the z axis of $\kappa \approx 0.33$, from which we estimate the sound velocity using $u_{\text{FM}}/(2Ja) = \pi\kappa K$ [1]. We obtain $u/(2Ja) \approx 1.85$ for the system with vdW interactions. This represents a large increase with respect to the case of nearest-neighbor interactions ($K = 1$). Thus, compared to the case of NN interactions the ferromagnetic dipolar interactions lead to an enhancement of ground-state correlations, as well as to an acceleration of the propagation of modes at low wave vector.

Figure 11 shows instead the DMRG results for the correlations C^x and $|C^z|$ of the AFM chain, including antiferromagnetic vdW interactions. A fit using Eqs. (2) and (3) of the main text gives a consistent picture of a Tomonaga-Luttinger liquid with $K_{\text{AFM}} \approx 0.865$, reduced with respect to nearest-neighbor interactions because of frustration. The analysis of the magnetization curve gives $\kappa \approx 0.33$, leading to a sound velocity $u_{\text{AFM}}/(2Ja) \approx 0.9$, reduced as well with respect to the nearest-neighbor XY chain.

It is interesting to compare these values to a perturbative estimate one can make for a simpler short-range model, with just nearest-neighbor (J_1) and next-nearest-neighbor (J_2) couplings, when $J_2 \ll J_1$ [99]. In that limit, u and K

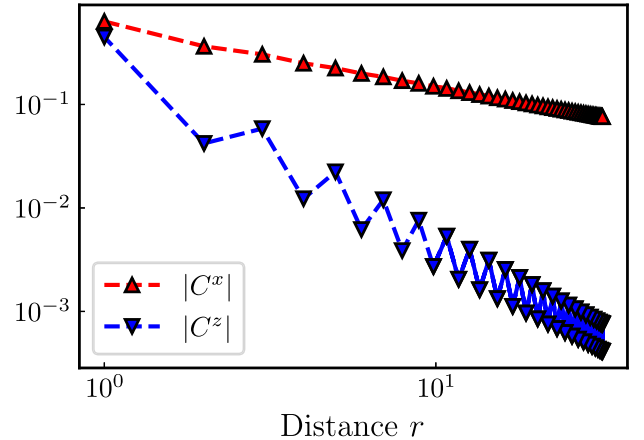


FIG. 11. Ground-state correlations of the dipolar AFM chain. The dashed lines connect the points, while the solid lines are fits of the long-distance behavior to the expected power-law decay in an LL.

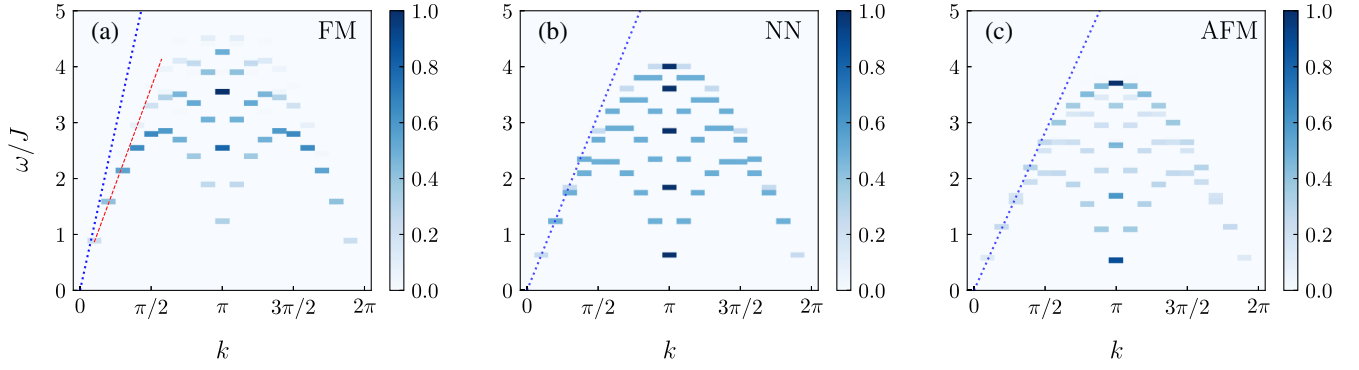


FIG. 12. Dynamical structure factors of dipolar and NN chains. (a) Dipolar FM chain, (b) NN XY chain, (c) dipolar AFM chain. All results have been obtained for a $N = 20$ chain via exact diagonalization. The color bar is normalized to the peak value. Blue dashed lines: dispersion of the LL sound mode $\omega = uk$, with u estimated from the ground-state calculations. In panel (a), the dashed red line indicates the effective sound mode $\omega = v_g k$ observed in the experiment, with $v_g = 2.2Ja$.

are both renormalized by the same factor, $\sqrt{1 - (8/\pi)(J_2/J_1)}$, which for us estimates $K_{\text{AFM}} \approx 0.83$ and $K_{\text{FM}} \approx 1.15$. Compared to our numerical values for the full dipolar model, we see that this short-range contribution already accounts for the AFM shift in K (in fact overshooting it slightly), whereas reproducing the shift in K_{FM} requires additional contributions from the longer-range tail.

Dynamical structure factor and sound mode. The excitation spectrum of a Tomonaga-Luttinger liquid is composed of a sound mode with dispersion relation $\omega = uk$ [1]. In the case of a lattice spin system, the sound mode captures only the long-wavelength excitations, while at shorter wavelengths the spectrum exhibits a continuum of excitations for each wave vector, as revealed, e.g., by neutron scattering experiments on quantum magnets [15]. We test the importance of the sound mode with respect to the other modes of the spectrum of the lattice model by calculating the dynamical structure factor in the ground state:

$$S(q, \omega) \sim \sum_n |\langle \psi_n | S_q^z | 0 \rangle|^2 \delta(\omega - \omega_{n0}), \quad (11)$$

where $|\psi_0\rangle, |\psi_n\rangle$ are the ground state and a generic excited state of the Hamiltonian, respectively, with corresponding energies E_0, E_n ; $S_q^z = (1/\sqrt{N}) \sum_j e^{iqr_j} \sigma_j^z$ and $\omega_n = (E_n - E_0)/\hbar$. Figure 12 shows the dynamical structure factor from exact diagonalization (using the QUSPIN package [100]) for the dipolar FM and AFM chains with $N = 20$ sites, as well as for NN interactions. For the FM [Fig. 12(a)] we observe that the sound mode contains essentially one wave vector, due to the significant curvature of the dispersion relation. We understand this curvature from the fact that, compared to the system with NN interactions, the sound velocity is renormalized by the dipolar interactions much more strongly than the bandwidth of the excitations. Compared to the NN XY chain

[Fig. 12(b)], for which $u/(2Ja) = 1$ and the maximum energy $\omega_{\text{max}} = 4J$, the dipolar FM chain has $u/(2Ja) \approx 1.85$ while $\omega_{\text{max}} \lesssim 5J$, i.e., an 85% increase in the sound velocity for a $\approx 20\%$ increase in the bandwidth. The latter can be understood simply as an effect of the integral of the dipolar interactions in 1D, $\sum_{r>0} 1/r^3 \approx 1.2$. The sound velocity is much more renormalized, since the model is close to a quantum phase transition to long-range order, occurring for interactions $1/r^\alpha$ with $\alpha \lesssim 2.8$, at which both u and K diverge [47].

In the case of the dipolar AFM chain [Fig. 12(c)], both the bandwidth and the sound velocity are reduced by $\approx 10\%$, compatible with the fact that $\sum_{r>0} (-1)^{r+1}/r^3 \approx 0.9$. As a consequence, the sound mode is as pronounced in the spectrum as in the case of the NN interactions [Fig. 12(b)]. The Luttinger-liquid sound mode is thus visible in the quench dynamics of the correlations in the AFM case, as shown in the main text. For the dipolar FM, the sound mode with the highest group velocity is not clearly visible on small systems, as it is masked by modes at intermediate k with a weaker group velocity. Figure 12(a) shows that the observed light-cone velocity in the FM experiment, $u/(2Ja) \approx 1.2$ —corresponding to the slope of the red dashed line—is compatible with the group velocity of the dispersion relation at wave vectors just above the one(s) associated with the actual sound mode.

2. Dipolar chains with holes

As discussed in the main text, the imperfect preparation of the initial state and the decay of atoms during dynamics leads to holes appearing in the chain. They break the chain into disconnected segments for NN interactions, destroying the power-law correlations of the undoped system. Power-law interactions allow instead for a finite concentration of holes, while retaining a full connectivity, and possibly preserving the Luttinger-liquid nature of the ground state at long distances.

Hole doping in the FM chain. We probe the effect of a finite hole concentration for the FM via QMC calculations,

averaged over > 100 hole realizations, and at temperatures $T \lesssim 10^{-2}J$ to eliminate thermal effects. Figure 10(b) shows the effect of two hole concentrations, $p = 4\%$ (the one at the end of the ramp for the FM) and $p = 8\%$. The C^x correlations with 4% of holes are consistent with those of a Tomonaga-Luttinger liquid with exponent $K_{\text{FM,vdW}}(p = 0.04) \approx 1.22$, while the analysis of the C^z correlations (not shown) gives $K_{\text{FM,vdW}}(p = 0.04) \approx 1.28(3)$. This suggests that 4% of hole doping preserves the LL physics in the FM chain, albeit renormalizing significantly the Luttinger parameters. To compare to the experiment, we calculate K on an $N = 24$ ring, thus providing an upper bound to the actual K value [see Fig. 10(a)]. We find that for $p = 4\%$, $K_{\text{FM}}(N = 24) = 1.55(1)$ from the C^x correlations and $K_{\text{FM}}(N = 24) = 1.44(1)$ from the C^z correlations.

The simulation for $p = 8\%$ leads instead to $K_{\text{FM,vdW}}(p = 0.08) \approx 0.93$ from the C^x correlations and $K_{\text{FM,vdW}}(p = 0.08) \approx 1.20(4)$ from C^z correlations. The fact that one value lies below 1 and that the C^x correlations deviate from a strict power-law behavior suggests that the chain with doping sits in the vicinity of a quantum phase transition from Luttinger-liquid physics to a different, disorder-induced phase. This new phase could be a localized one such as a Bose glass [81] or a Mott glass [83]. The latter may be favored by the fact that hole doping leads to bond disorder for the remaining spins, which is accompanied by a net zero magnetization, i.e., a commensurate (one-half) filling in the bosonic language. A third possibility is that the bond-disordered chain realizes a random-singlet phase, which retains power-law correlations [82].

In fact, we cannot exclude that even the smaller dilution we considered, $p = 4\%$, leads to a stabilization of a random-singlet phase in very large systems. The fact that we observe an effective LL behavior on the system sizes we simulated may be due to a crossover from a LL-like behavior at short distance to a random-singlet behavior at long distance, for which all two-point correlations are expected to decay as $1/r^2$ [82]. This crossover, observed in microscopic calculations on bond-disordered chains [101,102], is expected to occur over length scales which diverge with the inverse strength of disorder, controlled in our case by the density of holes.

Hole doping in the AFM chain. Contrary to the FM chain, hole doping in the AFM is expected to affect strongly LL physics. First, hole doping leads to a disruption of the sublattice structure in a 1D AFM with power-law interactions: every hole leads to a one-site slip of the sublattice structure, so that correlations between two sites at the same distance can be of opposite signs if a hole appears or not in between the two sites [see Fig. 13(a)]. This effect is the one mainly responsible for the observation, shown in Fig. 2(f) of the main text, that even in the ground states the C^x correlations of the hole-doped AFM exhibit a faster-than-algebraic decay. Yet this sublattice slip does not affect the correlations at long distance, provided that one knows

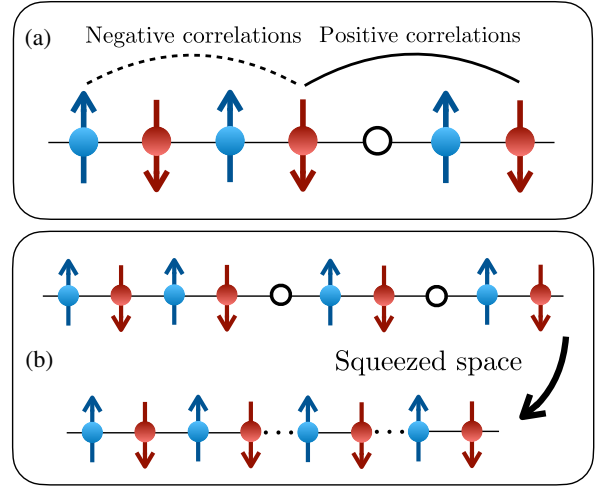


FIG. 13. Hole doping in the dipolar AFM. (a) Hole doping leads to a slip of the sublattice structure, such that spins at the same distance can be correlated or anticorrelated, depending on the even or odd number of holes between them. (b) The sublattice slip can be removed by going to “squeezed space,” i.e., eliminating the holes (provided that one knows their positions). Nonetheless, this leaves behind some randomness in the couplings: For example, the NN couplings—indicated in the lower part of the figure—are bimodally distributed (if one neglects two or more adjacent holes), with values J (solid lines) and $J/8$ (dotted lines).

the positions of the holes: We could define correlations in “squeezed space” [Fig. 13(b)] [25], eliminating the holes from the picture. These correlations may retain a power-law behavior—this would be true if the spin couplings in squeezed space were the same as the couplings in a clean system. In the case of dipolar coupling, the interactions are reduced by a factor of $1/8$ across a hole with respect to those between nearest neighbors. As a consequence, the spin model that emerges in squeezed space is an XY model with bond disorder.

We expect this bond disorder to strongly affect LL physics in the AFM chain for the hole concentrations p realized in the experiment. We motivate this conclusion by considering that, in the case of NN interactions, the model can be mapped onto free fermions [46], which in 1D are strongly susceptible to disorder. A disordered potential would lead to localization of the entire spectrum. Disordered hoppings with a broad distribution leads instead to a random-singlet phase [82], which, unlike localized phases, retains power-law correlations. Hole doping leads to bond disorder with a discrete (nearly bimodal) distribution; hence the relevance of the random-singlet phase is not immediately obvious. We hence probe numerically the correlations of the NN XY chain with bimodal bond disorder, with $p = 6\%$ of bonds with couplings $J/8$ randomly doped in the system, the other bonds having couplings J . Figure 14 shows the C^x correlations for this system, obtained by a mapping onto free fermions. We

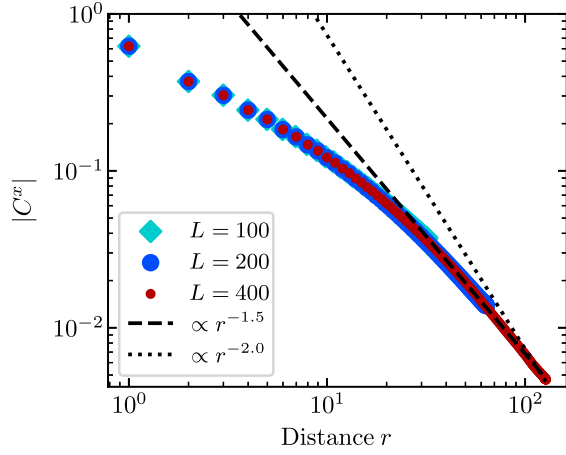


FIG. 14. Correlations in a bond-disordered NN XY chain. The calculation was done on chains with $N = 100, 200$, and 400 spins. The solid lines correspond to a power-law fit of the correlation tail, as well as the expected r^{-2} decay in the random-singlet phase.

observe correlations which are incompatible with a power-law decay, although an emergent power-law decay seems to appear at very large distances. This behavior is suggestive of a random-singlet phase, although further studies would be needed to precisely pinpoint the nature of the ground state.

In the case of the dipolar AFM chain, the undoped ideal system has $K < 1$, leading to weaker C^x correlations than the NN XY chain. Hence we expect that a fraction $p = 6\%$ of holes, as in the experiment, will lead *a fortiori* to a disorder-induced phase. We therefore conclude that the ground state of the hole-doped dipolar AFM is not a Tomonaga-Luttinger liquid for the hole concentrations relevant to the experiment, but rather a disorder-dominated phase.

APPENDIX J: FRIEDEL OSCILLATIONS

1. Analytic origin of the Friedel oscillation

The open-chain ground states, at each filling fraction, can be understood as those of a conformal field theory with open-boundary conditions—a so-called boundary CFT [103,104]. In this general setting, primary field excitations $\mathcal{O}$ of the CFT will develop an expectation value $\langle \mathcal{O} \rangle$ that decays away from the boundary as $r^{-\Delta_{\mathcal{O}}}$, where $\Delta_{\mathcal{O}}$ is that field's scaling dimension. (Under periodic boundary conditions, $\langle \mathcal{O} \rangle = 0$.) For the (1+1)D case at hand, the explicit functional form can be calculated via a conformal mapping to be $\langle \mathcal{O} \rangle = A_{\mathcal{O}} \{ (L/\pi) \cos[(\pi j)/L] \}^{-\Delta_{\mathcal{O}}}$ [69]. Here, $j \in \mathbb{Z} - [(N-1)/2, (N-1)/2]$ is the position of the spins (in units of sites), with $j = 0$ being the center of the chain. This equation holds when the long-wavelength scaling limit is meaningful, i.e., at some sufficiently large distance from the boundary. The coefficient $A_{\mathcal{O}}$ is

generically nonzero for all operators not forbidden by any remaining microscopic symmetry. We note that in some cases the critical exponents in the boundary CFT can differ from those with PBC, but in our instance they are the same.

The experimental measurements of $\langle \sigma_j^z \rangle$ probe two primary operators of the LL CFT: the conserved current $J = \partial_x \phi$ and the vertex operator $\mathcal{V}_{1,0} = e^{i\phi}$ [80]. The first corresponds to the zero-momentum component of σ^z , i.e., the average magnetization m_0 ; it has dimension $\Delta_J = 1$, and is responsible for the r^{-2} part of the C^z decay in the periodic system. The second has scaling dimension $\Delta_{1,0} = K$, and corresponds to oscillations of σ^z at a particular momentum which depends on m_0 . As noted in the main text, consistency requirements between the actions of microscopic and continuum-limit symmetries set this wave vector to be $\pi(1/2 - m_0)$ [70,71]. We therefore expect that

$$\begin{aligned} \langle \sigma_j^z \rangle - m_0 &= \langle \mathcal{V}_{1,0} \rangle + \dots \\ &= A \cos(2k_F j + \delta) \left[\frac{L}{\pi} \cos\left(\frac{\pi j}{L}\right) \right]^{-K} + \dots, \end{aligned} \quad (\text{J1})$$

where the ellipses denote more rapidly decaying terms, A is a nonuniversal prefactor, and δ is the overall phase for the oscillations. For our odd- N chain, reflection symmetry across the center site ($j = 0$) sets $\delta = 0$, leading to Eq. (4) in the main text.

2. Background subtraction for Friedel oscillations

The ideal ground state of a closed ring (PBC) does not break translational invariance, and in particular we expect no Friedel oscillations. To check this experimentally we use the same adiabatic sequence as in the open ring system. The resulting magnetization $\langle \sigma_z^i \rangle_{\text{PBC}}$ is shown in Fig. 15 (yellow points), for different addressing pattern corresponding to all target values of the total magnetization M_z . In contrast with the expectation for the ideal ground state, we observe a spatially inhomogeneous magnetization which depends on the addressing pattern. We also distinguish a small gradient of magnetization (which in a circular geometry translates into an oscillation with wavelength N). The amplitude of those inhomogeneities are on the same order as the expected Friedel signal in a chain with OBC ($\langle \sigma_z^i \rangle \sim 0.1$). Several effects could explain the inhomogeneities: nonadiabaticity of the ramp leading to a reminiscence of the initial addressing pattern, positional disorder on the average atomic position or inhomogeneous spin frequencies due to gradients of electric or magnetic field.

When we perform the same experiment on the open ring, the magnetization $\langle \sigma_z^i \rangle_{\text{OBC}}$ shows very similar inhomogeneities, on top of well-defined oscillations close to the edges [orange points of Fig. 15]. To mitigate the measured inhomogeneities and conserve only the contribution of the edges, we subtract the PBC background to the Friedel OBC

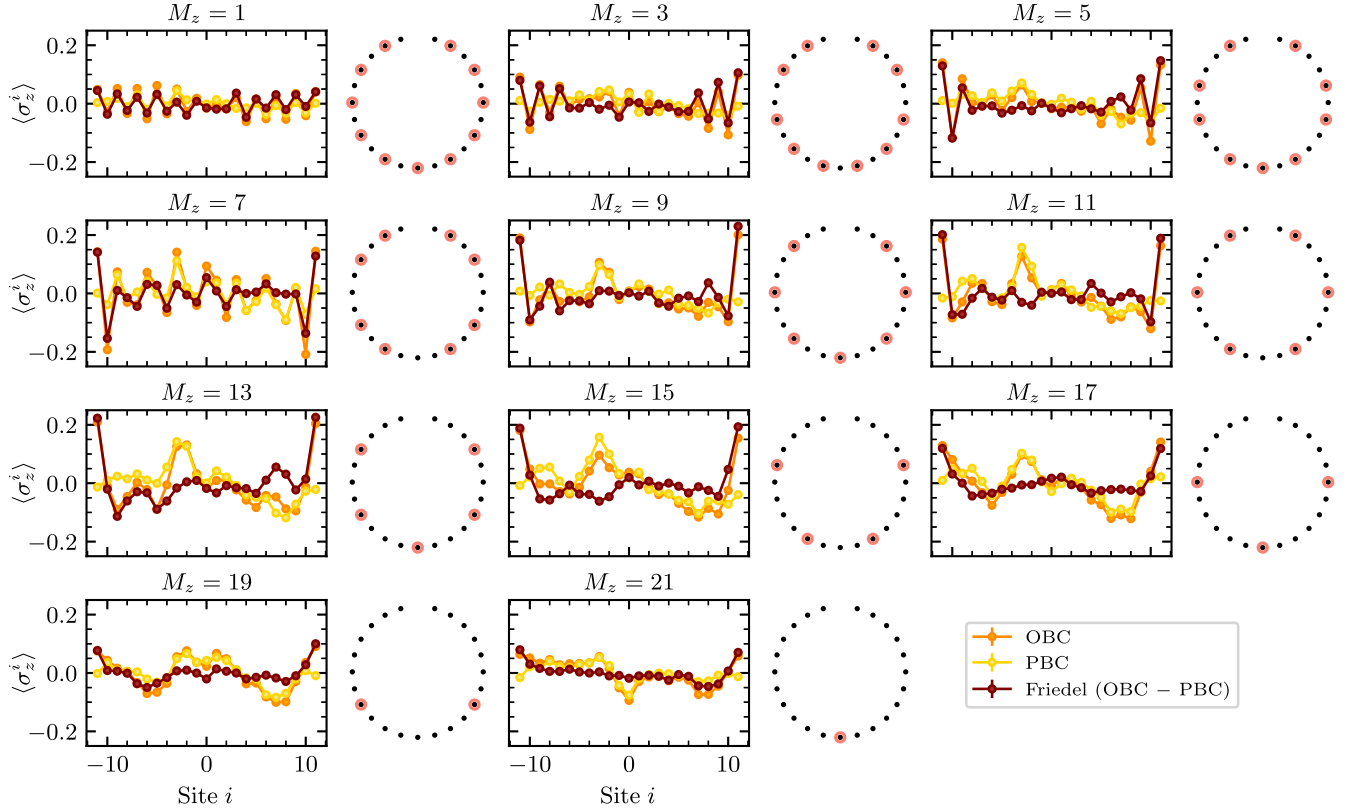


FIG. 15. Extraction of the magnetization for Friedel oscillations. Spatial dependence of the z magnetization $\langle \sigma_z^i \rangle$ with the site positions i (the average magnetization $\sum_{j=1}^N \langle \sigma_z^j \rangle / N$ is removed for clarity), for different M_z sectors. For each value m of the total magnetization, a different initial state $|\psi_m\rangle$ is used for the adiabatic preparation, with $(N + m)/2$ spins in $|\uparrow\rangle$ (nonaddressed atoms) and $(N - m)/2$ spins in $|\downarrow\rangle$ (addressed atoms); the position of the addressed atoms is indicated by pink circles. The Friedel signal (OBC-PBC) at site i is the difference of the OBC magnetization and the PBC magnetization measured with the same sequence and the same addressing pattern. The ramp times differ for the different M_z sectors ($T = 2500$ ns for $M_z = 1$; $T = 5000$ ns for $M_z \in [3, 5, 7, 9, 11, 13, 15]$; $T = 8000$ ns for $M_z \in [17, 19, 21]$), and the measurements were always taken 500 ns after the end of the ramp.

signal: $\langle \sigma_z^i \rangle_{\text{Friedel}} = \langle \sigma_z^i \rangle_{\text{OBC}} - \langle \sigma_z^i \rangle_{\text{PBC}}$ (red points of Fig. 15). The data shown in Fig. 3(c) of the main text correspond to the corrected data $\langle \sigma_z^i \rangle_{\text{Friedel}}$.

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