

Floquet-Based Ising Machines Escape Local Minima in QUBO Problems

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Solving large-scale quadratic unconstrained binary optimization (QUBO) problems is critical in various fields, including physics, finance, and engineering. However, these problems remain intractable on conventional computing architectures. Alternative solvers, such as Ising machines (IMs) based on networks of coupled electronic, mechanical, or photonic parametric oscillators (POs), have recently been developed. PO-based IMs aim to find the ground state of an Ising Hamiltonian, which encodes the solution to a QUBO problem. However, their analog nature and their energy-minimization process based on gradient descent make PO-based IMs inherently susceptible to identifying inaccurate solutions. In this work, we introduce and validate a QUBO solver—the analog Floquet solver (AFS)—which enhances the dynamics of PO-based IMs by leveraging Floquet states that emerge spontaneously in POs coupled to high-quality-factor resonances. These states enable the AFS to embed periodic time modulation into its energy-minimization process, allowing it to escape local minima during the search for QUBO problem solutions. As a result, the AFS significantly increases the likelihood of identifying accurate solutions compared to conventional PO-based IMs. More generally, this work defines a new paradigm in analog computing—spanning both classical and quantum realms—that can be physically realized with existing technologies across diverse physical domains.

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I. INTRODUCTION

In the past years, scientists have been looking for ways to solve large-scale quadratic unconstrained binary optimization (QUBO) problems with high accuracy and high speed [1]. Many such QUBO problems are nondeterministic-polynomial-time- (NP) hard combinatorial optimization problems (COPs) with binary variables, meaning they are at least as computationally difficult as the hardest problems in the NP complexity class. The resolution of these problems holds the potential to drive significant advancements across a wide range of critical disciplines [2]. For instance, the ability to solve large-scale QUBO problems may enable a more efficient allocation of available healthcare resources [3,4], and it may facilitate the identification of financial portfolios aligned with specific objectives, such as maximizing expected returns or

minimizing financial risks [5]. Being able to solve large-size QUBO problems can also streamline the sharing of the electromagnetic spectrum among growing numbers of users with diverse needs and mutable channel conditions [6] and can assist with drug and new material synthesis and discovery [7,8]. Unfortunately, the computational complexity of QUBO problems heavily increases with their size. As a result, traditional von Neumann computing architectures using brute-force approaches struggle to determine exact solutions to QUBO problems with more than a few dozen variables in a reasonable amount of time, owing to the exponential scaling of the solution space [9]. Consequently, there has been significant research effort around alternative algorithmic [10,11], heuristic [6,12], and computational methods capable of transcending the current limitations of von Neumann machines when tackling QUBO problems [13–15]. This has led to the creation of a variety of computing machines [13–15], including Ising machines (IMs) [16–19], which are hardware implementations of the Ising model. This model was initially conceived within the framework of condensed matter physics to study phase transitions in materials exhibiting ferromagnetism [20]. More recently, it was shown that all QUBO problems can be mapped onto networks of Ising spins [21], thus being

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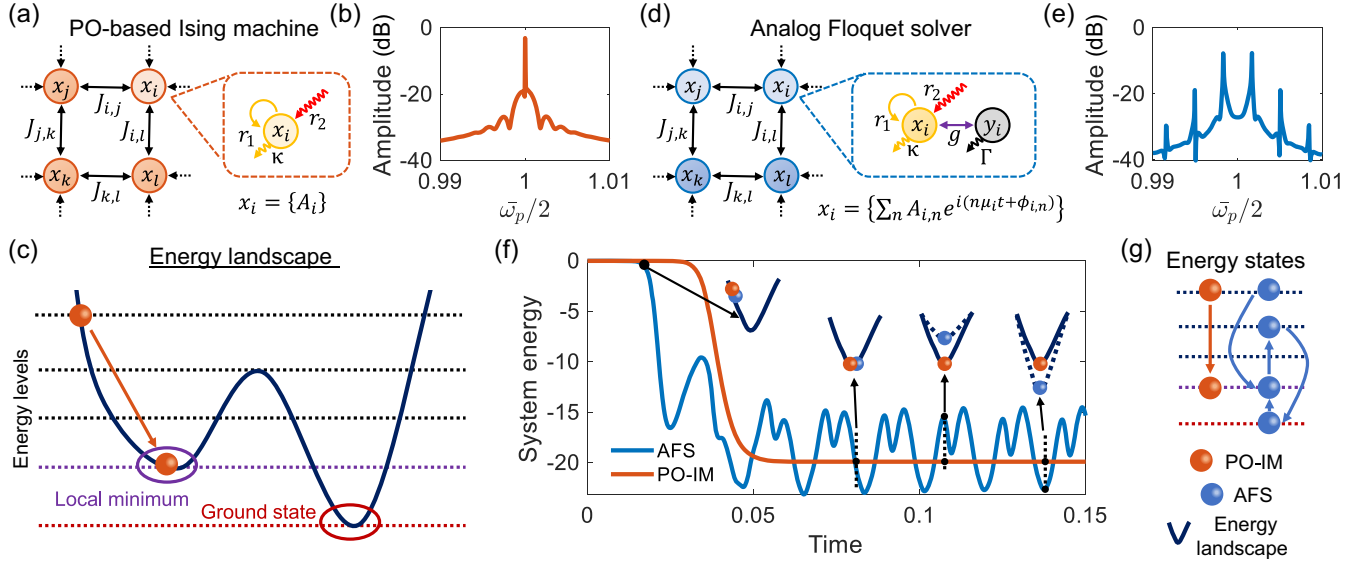


FIG. 1. (a) Schematic representation of a PO-based IM featuring a system of coupled POs. POs are represented with a single mode x . (b) Representation of the amplitude dynamics in the frequency domain of a single PO operating in the period-doubling regime under a rotating frame of $\omega_p/2$. (c) Symbolic representation of the energy minimization via gradient descent implemented by PO-based IMs. As can be seen, such an approach leaves the PO-based IM prone to convergence to suboptimal solutions. (d) Diagram representing a spin solver adopting Floquet nodes as computational elements. The x modes describing POs are coupled with interaction strength $J_{i,j}$ to map the targeted QUBO problem, while each x mode is coupled to its own unique y mode with strength g . (e) The simulated frequency spectrum normalized around $\omega_p/2$ of the mode x in the AFS operating with Floquet states. The periodic modulations of the system generate a frequency comb. (f) Symbolic comparison of the energy-minimization profiles of the PO-based IM and the AFS in normalized time. The periodicity of the AFS's energy-minimization profile illustrates the energetic oscillatory behavior endowed to the system. (g) Representation of the coordinated cycling of a subset of Ising energies undertaken by the AFS during the final stages of the computation compared with those explored by IMs.

addressable by IMs. This mapping allows an IM to read the correct solution of a QUBO problem from the collective spin configuration with the lowest energy (the “ground-state” energy).

Since their introduction, several types of IMs have been developed using digital [22–25] (discussed more thoroughly in Supplemental Material Sec. I [26]; see also references [27–67] therein) and analog [16–19] approaches. Analog IMs have received significant attention in the last decade due to their vast parallelization and energy-efficient physics-based computing approaches [68–71]. In this context, there have been significant efforts made toward developing quantum annealers as analog IMs, because they offer compelling scaling advantages. However, they suffer from energy overhead in maintaining near-cryogenic temperatures, and they require $1/f$ noise corrections [72]. Furthermore, they often struggle to implement all-to-all connectivity and even experience spin-bias leakage between adjacent nodes, often making them impractical for solving large-scale problems. As such, there have been considerable advancements in developing analog IMs in the form of networks of coupled oscillators, which explains why they are often dubbed “oscillator-based Ising machines” (OIMs, [18,19]). Each oscillator in OIMs can behave as an artificial spin under appropriate driving

conditions, which makes OIMs able to minimize the energy of the Ising model.

Most reported OIMs rely on free-running electronic oscillators driven by a strong injection signal to enter an operating regime in which their phases become bistable and they start functioning as artificial spins [18,19]. However, multiple scientific communities have recently developed a strong interest in a different type of OIM known as parametric-oscillator- (PO) based IMs [73–76]. PO-based IMs utilize POs as artificial spins. POs naturally exhibit the required bistability needed for computation when operating in their period-doubling regime [68,73,77,78], without requiring a high-power injection signal [see Fig. 1(a) for a depiction of a PO-based IM using the tight-binding model [79] and Fig. 1(b) for the frequency domain spectra of such POs operating in their period-doubling regime]. This makes PO-based IMs more energy efficient than their OIM counterparts that require a strong injection-locking signal for all their oscillators [18,19]. Furthermore, POs can be implemented by just using passive devices that exhibit a $\chi^{(2)}$ nonlinearity [77,80,81]. This has led to the development of PO-based IMs in several domains beyond electronics, including mechanical [82] and photonic [74]. Also, it has permitted the use of PO-based IMs even for quantum computing [16,71,83,84]—an application space wherein

amplifiers are typically avoided to prevent coherence losses due to their significant added noise. However, like any other OIM, the accuracy achievable by PO-based IMs is severely impacted by convergence to local minima of the energy landscape due to their gradient-descent-like energy-minimization process [85,86]. Figure 1(c) presents a symbolic illustration of the gradient-descent-based energy-minimization process employed by conventional PO-based IMs on a nonconvex energy landscape, starting from random initial conditions. Under these operational conditions, PO-based IMs can reach and become trapped in a local minimum.

The impact of local minima convergence on the accuracy of PO-based IMs becomes even more significant when PO-based IMs suffer from strong amplitude heterogeneity (AH) [73,87]. AH affects all OIMs, and its impact strongly depends on the problem being tackled [88–90]. In PO-based IMs, AH captures the discrepancies in the oscillation amplitude that the various POs typically exhibit when driven by the same pump signal. These discrepancies introduce an additional degree of freedom in the dynamics ruling the evolution of POs in PO-based IMs, causing an increase of the number of local minima in such IMs’ energy landscape compared to the ideal (and often unrealistic) case in which the amplitude dynamics are homogeneous or negligible [88,89]. This distortion of the energy landscape, known as “broken mapping” [17], heavily increases the likelihood of convergence to local minima [87,90,91] and degrades PO-based IMs’ accuracy in determining the ground-state Ising energy. It is also worth noting that the occurrence of a broken mapping also constitutes the primary limitation of PO-based IMs that operate in the quantum regime, typically referred to as “coherent Ising machines” (CIMs) [16]. In this regard, CIMs can benefit from much lower noise levels than PO-based IMs operating in the classical regime. Nonetheless, CIMs’ accuracy is ultimately limited by AH [16,84], which reduces the benefits of performing computation in the quantum regime rather than in the classical realm. This explains why developing approaches to enhance PO-based IMs has fundamental implications.

To this note, recent efforts have attempted to lower the impact of AH on the accuracy of PO-based IMs by recurring to higher-dimension (D) PO nodes, in combination with dimensional annealing [68,92,93]. This approach permits us to attack a combinatorial optimization problem from lower energy levels than the targeted ground state, thereby reducing the likelihood of convergence to local minima. Unfortunately, using higher-dimensional PO nodes introduces a host of additional challenges for the physical realization of these spin solvers compared to conventional IMs. For instance, a multidimensional PO-based spin solver with dimension D requires D times as many POs as a conventional (one-dimensional) PO-based IM, heavily increasing routing complexity, and

boosting the power consumption per computational node by a factor equal to D [68]. Other approaches to address the challenge of minimizing AH in OIMs involve feedback controls over the systems’ spin amplitudes [87,94]. These approaches require the implementation of Zeeman terms or modulations in the coupling weight, which introduce significant energy overhead [87,94] and heavily increase OIMs’ programming complexity when different problems must be tackled. Additionally, all the aforementioned approaches do not offer a computational mechanism to assist IMs in escaping local minima [68]. In summary, there is no approach that can improve the accuracy of PO-based IMs by mitigating the impact of AH and, as a result, reducing the likelihood of local minima convergence, without causing significant power, size, and implementation overheads.

In a different area of condensed matter physics, remarkably exotic material responses have been shown within the realm of Floquet matter [95–98]. Floquet states, associated with periodic temporal modulation of a material, unveil new forms of light-matter interactions, strategically controlled through Floquet engineering [95], with relevance in quantum materials [99–101], topological insulators [102], time crystals [103], and photonics [104–107]. Quite remarkably, as we show in the following, the unusual dynamics of Floquet states can be leveraged to enhance PO-based Ising machines and enable retrieval of more accurate solutions when tackling QUBO problems.

In this article, we introduce and numerically validate a new solver for QUBO problems, called the analog Floquet solver (AFS), wherein each computational node excites a Floquet state in the form of a frequency comb [95–97]. Each node of the AFS consists of a PO coupled to a passive resonant device with resonance frequency that closely matches the PO’s output frequency, effectively forming a “Floquet node” consisting of a PO-resonator pair [see Fig. 1(d) for a representation of a generic AFS and its computational nodes using the tight-binding model [77,81], and Fig. 1(e) for the typical spectrum of a generic AFS’s PO’s amplitude x_i]. The dynamics of a single Floquet node have been experimentally demonstrated in an electronic platform and used for wireless sensing in Ref. [81]. Other approaches to generate time-modulation dynamics in PO-based systems have also been reported in Refs. [78,108,109]. However, unlike AFS, these approaches rely on antisymmetric coupling in networks of POs to generate additional time modulation, which is an approach that adds significant implementation complexity [108] compared to Floquet nodes that require only one resonator symmetrically coupled to one PO. Also, prior work has neither studied the computational principle of dynamical systems relying exclusively on time-modulation dynamics nor evaluated the ability of such systems to identify the ground-state solution of large-scale benchmarking problem graphs. For the AFS to tackle a QUBO

problem, a coupling scheme between all Floquet nodes must be engineered to reflect the problem graph. In this regard, a coupling matrix $[J]$ can be assembled, defining the interactions between all computational nodes [73], as depicted in Figs. 1(a) and 1(d). Each coefficient $J_{i,j}$ captures the coupling strength between two specific nodes (with indexes i and j). As required for PO-based IMs, the AFS requires all its nodes to be continuously driven by a single-tone pump signal during the computation, which causes all the AFS's nodes to collectively pursue lower energy states as soon as the computation starts. Furthermore, the Floquet dynamics in the AFS do not result from the beating between a continuous wave signal and a lower-frequency modulation signal, as in conventional Floquet systems [105]. Instead, they emerge from the activation of frequency combs in all the AFS's nodes, caused by the same parametric nonlinearities that induce period doubling in PO-based IMs.

Unlike PO-based IMs, the energy-minimization process of the AFS does not follow a gradient descent. Instead, with all the AFS's nodes supporting a Floquet state, the process is governed by a combination of gradient descent and periodic oscillations of the overall energy landscape. In this regard, the AFS's energy landscape periodically oscillates following the periodic drive of the Floquet states [see the inset of Fig. 1(f)]. Like PO-based IMs, the AFS encodes the solution of the associated QUBO problem into the ground state of an Ising Hamiltonian, which represents the energy landscape $[H_{\text{Ising}}(t) = \frac{1}{2} \sum_{i,j}^N \sigma_i(t) J_{i,j} \sigma_j(t)]$, where σ_i and σ_j are the digitized spin values representing the amplitude and phase solutions for the AFS nodes of indices i and j respectively, and N is the total number of nodes in the problem graph]. The periodic variations of the AFS's energy landscape induced by the Floquet drive results in synchronized flips of all σ values (from a “spin-up” or “+1” configuration to a “spin-down” or “−1” configuration and vice versa). These flips result in deterministic and periodic changes in the discretized Ising energy level reached during the computation. This allows us to reach more accurate solutions than those identified by PO-based IMs when tackling large-size QUBO problems. In turn, the AFS never settles to a final steady-state collective spin configuration, as seen in Fig. 1(g). This marks an important distinction between the AFS and conventional PO-based IMs [see Fig. 1(f)].

In the following, we discuss the computational principles of the AFS, we show how the AFS's Floquet dynamics allow us to solve QUBO problems with varying densities, i.e., we use the BIQ MAC and GSET libraries in our analysis [69,110,111], and we show that the AFS can solve them more accurately than conventional PO-based IMs. We also demonstrate how the ability to escape local minima originates from a reduction of the impact of AH on the solution accuracy and produces substantially improved capabilities to ascertain optimal or near-optimal solutions

to QUBO problems than what is achievable when using PO-based IMs. Such improved computational performance is attained while requiring only marginal amounts of space and power overhead to generate the Floquet states (e.g., only one extra passive resonator per PO is needed compared to traditional PO-based IMs).

II. METHODS

A. Nonlinear dynamics of Floquet nodes

To begin, we apply temporal coupled-mode theory [112–115] to analyze the dynamics of a PO-resonator pair forming the AFS's computational node. Considering the analytical model depicted in Fig. 1(d), a Floquet node can be treated as a triply resonant mode, containing a pump mode x_p and a signal mode x , which are parametrically coupled through a second-order $\chi^{(2)}$ nonlinearity, and an external resonant mode y with a loss rate Γ . Like any resonant mode studied using temporal coupled-mode theory, the evolution of y can be simply modeled by using a first-order differential equation: $\dot{y}(t) = -(\Gamma/2)y(t)$. When neglecting y , the behavior of the system in Fig. 1(d) coincides with that of a PO and can be modeled, after applying adiabatic elimination, using a single differential equation: $\dot{x}(t) = -[(\kappa/2) + r_1|x(t)|^2]x(t) + r_2x^*(t)$ [77,81]. More information about the dynamics of POs using temporal coupled-mode theory can be found in Supplemental Material Sec. II [26].

On the other hand, in Floquet nodes, x is strongly coupled to y via a linear coupling term g , but the resonant mode y is uncoupled to the pump mode x_p , as seen in Fig. 1(d). Additionally, to synthesize the strong coupling between x and y modes that is required to generate the Floquet state, g must exceed $\Gamma/2$ [81]. This permits us to once again use adiabatic elimination to express the dynamics of a single Floquet node by using a system of two equations that describe the temporal evolution of the x and y modes [Eqs. (1a) and (1b)] when relying on rotational frames [e.g., $x(t) \rightarrow x(t)e^{-i(\pi/4)}$, $x(t) \rightarrow x(t)e^{-i(\omega_p/2)t}$, and $y \rightarrow y(t)e^{-i(\omega_p/2)t}$],

$$\dot{x}(t) = -\left(\frac{\kappa}{2} + r_1|x(t)|^2\right)x(t) + r_2x^*(t) - igy(t), \quad (1a)$$

$$\dot{y}(t) = -\frac{\Gamma}{2}y(t) - igx(t). \quad (1b)$$

In this system, r_1 is the gain saturation coefficient, r_2 is the small-signal gain parameter, and κ is the loss rate of the x mode. Similar to a conventional PO, a Floquet node can also access a period-doubling regime for sufficiently large values of r_2 ; however, it enters an additional nonlinear regime for lower r_2 values in which x and y manifest modulations of their amplitudes. Because of these modulations, x and y have a frequency spectrum featuring a

frequency comb centered at $\omega_p/2$, as seen in Fig. 1(e) [81]. In this regime, the x and y slowly-varying complex amplitudes can be represented using Floquet theory as $x(t) = \sum_{n=-\infty}^{\infty} A_n(t) e^{i(n\mu t + \phi_n(t))}$ and $y(t) = \sum_{n=-\infty}^{\infty} B_n(t) e^{i(n\mu t + \theta_n(t))}$, where n represents the index of the Floquet multipliers $e^{i[n\mu t + \phi_n(t)]}$ and $e^{i[n\mu t + \theta_n(t)]}$ that elegantly capture the Floquet node's comb-like behavior. $A_n(t)$, $B_n(t)$, $\phi_n(t)$, and $\theta_n(t)$ describe the amplitudes and phases of the n th Floquet harmonic of the x and y modes, respectively, while μ , which varies with respect to g and r_2 , is the Floquet exponent. The periodicity of the x and y modes indicates that the Floquet node operating in this frequency comb regime features a classical Floquet regime [96]. Considering x (through which we extract σ) to be purely real due to the rotational frame and lack of detuning [81], we can express this mode as $x = \sum_{n=1}^{\infty} A_n \cos(n\mu t + \phi_n)$ [Fig. 1(e)]. For the sake of brevity, we will omit the time dependence of the mode amplitudes and phases in the following analysis. It is important to note that μ varies inversely with respect to r_2 , eventually converging to 0 for a sufficiently large r_2 [81] values. More information about the dynamics of Floquet nodes and how to convert a PO into a Floquet node in the electronic domain, can be found in Supplemental Material Sec. II [26].

To demonstrate the computational capabilities of the AFS, we analyze the dynamics of large-scale, dissipatively coupled networks of identical (i.e., $\kappa_i = \kappa$, $\Gamma_i = \Gamma$, and $g_i = g$) Floquet nodes. In addition, we introduce stochastic Brownian noise in our system of equations, analogous to the Johnson noise introduced in the electrical PO-based IM studied in Ref. [73]. Such a process is represented with a scalar N_b , modeling the magnitude of the noise, while $\zeta_{i,x}$ and $\zeta_{i,y}$ represent Gaussian white noise with zero mean for the i th x and y modes, respectively [116]. Overall, this modeling approach permits us to describe the dynamics of N Floquet nodes with $2N$ equations.

$$\dot{x}_i = -\left(\frac{\kappa}{2} + r_1 |x_i|^2\right) x_i + r_2 x_i^* - i g y_i - \sum_{j \neq i}^N J_{i,j} x_j + N_b \zeta_{i,x}, \quad (2a)$$

$$\dot{y}_i = -\frac{\Gamma}{2} y_i - i g x_i + N_b \zeta_{i,y}. \quad (2b)$$

By excluding Eq. (2b) and setting $y_i = 0$, a similar system of equations can be devised to model the dynamics of N -coupled POs. More information about these coupled systems can be found in Supplemental Material Sec. II [26].

B. Principle of computation

To verify that the AFS can efficiently tackle NP-hard QUBO problems, we must first determine the cost function

that is minimized by the system's collective dynamics. Then, we must demonstrate that the cost function's interaction term is structurally compatible with the Ising Hamiltonian [68,85]. In this regard, the interaction term of a cost function describes the system's collective behavior. Hence, an interaction term closely aligned with the Ising Hamiltonian indicates that the minimization of the system's energy corresponds to the minimization of the Ising Hamiltonian, thereby yielding the potential for the system to solve QUBO problems. To this end, we report a cost function $F_C(x_1, x_1^*, y_1, y_1^*, \dots, y_N, y_N^*)$ [117,118] derived from Eqs. (2a) and (2b), which is effectively a $4N$ -dimensional space comprised of each mode and its complex conjugate. When excluding the noise terms $N_b \zeta_{i,x}$ and $N_b \zeta_{i,y}$, this function takes the form

$$F_C = \sum_i^N \left[\left(\frac{\kappa}{2}\right) |x_i|^2 + \left(\frac{\Gamma}{2}\right) |y_i|^2 + \frac{r_1}{2} (|x_i|^4) - \frac{r_2}{2} (x_i^2 + x_i^{*2}) + i g (x_i^* y_i + x_i y_i^*) \right] + \frac{1}{2} \sum_{i \neq j}^N x_i^* J_{i,j} x_j. \quad (3)$$

F_C also obeys $-(\partial F_C)/(\partial x_i^*) = \dot{x}_i$, $-(\partial F_C)/(\partial x_i) = \dot{x}_i^*$, $-(\partial F_C)/(\partial y_i^*) = \dot{y}_i$, and $-(\partial F_C)/(\partial y_i) = \dot{y}_i^*$ [117,118]. This property ensures that $\partial F_C/\partial t$ is negative semidefinite ($[(\partial F_C)/(\partial t)] = -2 \sum_i^N [|\dot{x}_i|^2 + |\dot{y}_i|^2] \leq 0$) throughout the trajectories of x_i and y_i [117,118], which is important because this property affirms indeed that F_C (and therefore, the Ising Hamiltonian) is minimized by the AFS throughout its evolution. In Supplemental Material Sec. II [26], we provide more rigorous analysis relating to the structure of F_C and the role of the Floquet states in the minimization of F_C ; for comparison, F_{PO} , the cost function of a typical PO-based IM, is also discussed in Supplemental Material Sec. II [26].

The minimization of F_C occurs through two processes: through the self-sustaining oscillations of each individual Floquet node and through the coupled interactions between nodes [117]. The interaction term, which we will refer to as $\eta = \frac{1}{2} \sum_{i \neq j}^N x_i^* J_{i,j} x_j$, encodes the AFS's energy landscape [21] (or energy potential) and is intended to map the QUBO problem being solved via the coupling scheme configured by the matrix $[J]$ [1,21]. When substituting the periodic solutions of x_i and x_j into η , we can express it in Eq. (4) as

$$\eta = \frac{1}{2} \sum_{i,j} \left[J_{i,j} \sum_{n=1}^{\infty} A_{i,n} \cos(n\mu_i t + \phi_{i,n}) \times \sum_{m=1}^{\infty} A_{j,m} \cos(m\mu_j t + \phi_{j,m}) \right]. \quad (4)$$

This expression can be further simplified by assuming $\mu_i = \mu_j = \mu$ (all Floquet nodes have the same characteristic Floquet exponent for simplicity), and $n = m = 1$ (we consider only the first Floquet harmonic since its amplitude

is much larger than the amplitude of the other harmonics). In this scenario, η can be decomposed into two terms representing parallel interaction dynamics expressed separately as η_0 and η_1 in Eqs. (5a) and (5b) such that $\eta = \eta_0 + \eta_1$,

$$\eta_0 = \frac{1}{4} \sum_{i \neq j}^N [J_{i,j} A_{i,1} A_{j,1} \cos(\phi_{i,1} - \phi_{j,1})], \quad (5a)$$

$$\eta_1 = \frac{1}{4} \sum_{i \neq j}^N \{J_{i,j} A_{i,1} A_{j,1} \cos[2\mu t + (\phi_{i,1} + \phi_{j,1})]\}. \quad (5b)$$

The term η_0 retains a structure similar to a typical PO-based IM's interaction term ($\eta_{\text{PO}} = \frac{1}{2} \sum_{i \neq j}^N J_{i,j} x_i x_j$ [68,87]), imposing a gradient-descent-like evolution akin to what is leveraged by PO-based IMs [85]. On the other hand, η_1 introduces a slow periodic oscillation in η due to the modulation of the nodes' amplitudes [see Fig. 1(f)]. Furthermore, the instantaneous contribution of η_1 to η may be either positive or negative, leading to a Floquet modulation of the AFS's energy over time. It is η_1 that endows the AFS with the ability to escape local minima in H_{Ising} as it evolves to minimize η .

Note that in our discussion so far, η is analog in the sense that it considers the amplitude and phase components of the node solutions as its inputs, whereas QUBO problems are digital, with binarized values as inputs. To properly understand the relationship between minimizing η and solving QUBO problems, we must digitize the information encoded in each Floquet node as a value σ_i in $\{+1, -1\}$. Such a digitization permits us to convert the continuous solutions obtained for η into discrete solutions for H_{Ising} . The digitization process entails replacing the analog amplitudes x_i in η with the corresponding spin values computed as $\sigma_i = \text{sign}(\sum_n A_{i,n}(t) \cos(n\mu t + \phi_{i,n}))$. This permits us to transform $\eta = \frac{1}{2} \sum_{i \neq j}^N x_i^* J_{i,j} x_j$ into $H_{\text{Ising}} = \sum_{i \neq j}^N \sigma_i J_{i,j} \sigma_j$. The digitized σ values exhibit the same time periodicity of the continuous x . As a result, while the AFS minimizes η_0 in η to optimize H_{Ising} , all σ values keep changing from $+1$ and -1 (and vice versa) at a rate equal to μ . Such periodic changes of σ can help the AFS to escape local minima in H_{Ising} by correspondingly cycling through different sets of spin configurations, including degenerate ground states of H_{Ising} . Please refer to Supplemental Material Sec. II [26] for more information about the relationship between the modulations of the AFS's nodes and the AFS's energy modulation. Additionally, Supplemental Material Sec. III [26] includes a more detailed discussion about the respective interaction terms of the AFS and the PO-based IMs.

III. RESULTS

A. Energy-minimization dynamics with randomized initial conditions

In this section, we compare the AFS to a PO-based IM when solving a randomly generated 20-node 3-regular MAX-CUT problem [73,75,119] (with a precomputed ground-state Ising energy of -22). We begin with a small-scale problem because it allows for a clearer investigation of the computational dynamics compared to large-scale problems. Section II and Supplemental Material Sec. II [26] contain more information about the system of equations that must be assembled to analyze the behavior of the AFS and PO-based IMs when tackling this problem. After configuring the matrix $[J]$ in accordance with the MAX-CUT problem to be solved, we activated both the AFS and the PO-based IM by applying $r_2 = 0.7\kappa$. Then, we tracked all nodes' digitized spin states during the computation and used them to extract H_{Ising} at each time step, as shown in Fig. 2(a). During this computation, we see that the PO-based IM (represented by the orange line) converges to a local minimum (-20) as it reaches its steady state. Differently, the AFS undergoes its own gradient descent augmented by Floquet modulations in its energy. This allows the AFS to access a diverse range of energy states and, most importantly, to periodically reach the ground state. To visualize how η_1 influences the Ising energy level reached during the computation and helps avoid getting trapped in a local minimum of H_{Ising} , we must examine the temporal evolution of the identified spin configuration and the corresponding H_{Ising} value for both the AFS and the PO-based IM. To this end, we plotted in Figs. 2(b) and 2(c) the Ising energy landscape associated with the problem being solved. We did this by determining the value of the Ising energy achieved when considering different configurations of $N = 20$ spin values ranging from $\{-1-1-1, \dots, -1-1-1\}$ to $\{+1+1+1, \dots, +1+1+1\}$. Overlaid on the landscape, we chronologically marked the specific spin configurations identified by the PO-based IM and the AFS throughout their computation.

As already observed in Fig. 2(a), the value of H_{Ising} identified by the PO-based IM follows a gradient-descent profile, with some noise-induced fluctuations in energy during the early stages of its computation. On the other hand, the energy evolution profile of the AFS demonstrates some bearing to a classical gradient-descent routine during its early computational stages. However, once the identified H_{Ising} has reached the low-energy portion of the landscape (thus, the region where the PO-based IM is liable to get stuck into local minima), the AFS's Floquet dynamics becomes more evident and, as a result, the AFS begins cycling through a recurring set of low-energy spin configurations. Evidently, during this phase, the AFS identifies a broad range of spin configurations corresponding to energy levels that are lower than the lowest energy obtained

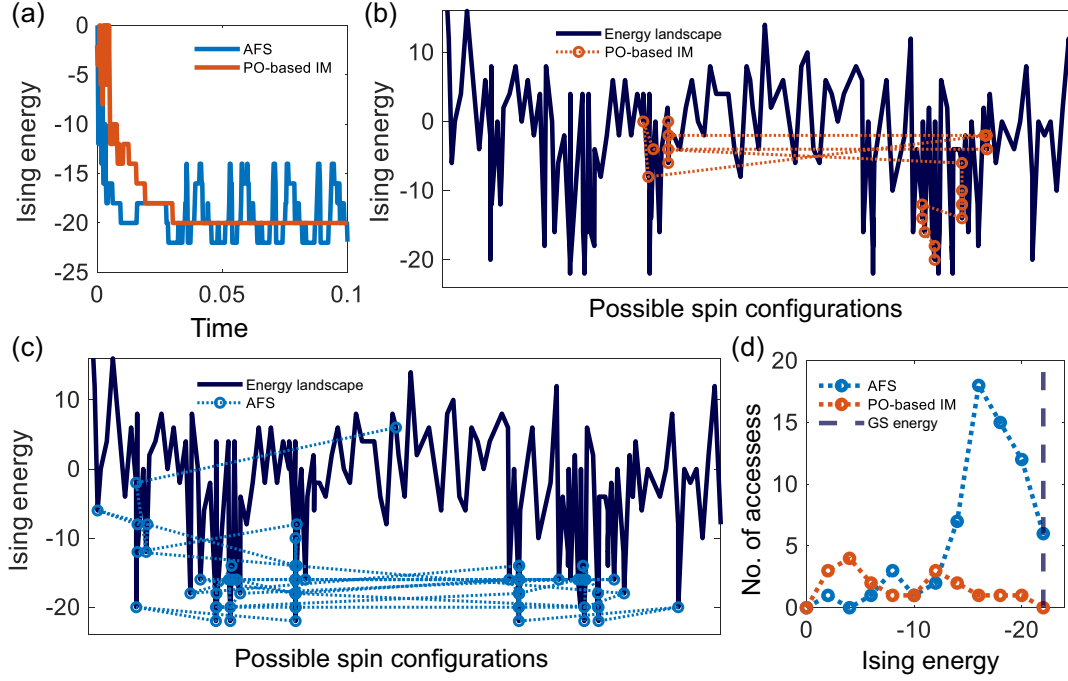


FIG. 2. (a) A plot showing the Ising energy over time (H_{Ising}) of the PO-based IM and the AFS in normalized time. (b) A plot of the spin states accessed by the PO-based IM throughout the entirety of its computation. In this instance, the spin solver fails to reach the ground-state energy value. (c) A plot of the states accessed by the AFS throughout the entirety of its computation. In this case, the spin solver identifies all degenerate ground states. (d) A plot showing the number of times a certain energy level was accessed by the PO-based IM and the AFS. The PO-based IM had only a limited exposure to lower energy levels, while the AFS operated mostly in the lower energy levels.

by the PO-based IM, including all six degenerate ground-state solutions [16] of the targeted problem. An additional quantitative comparison between the analyzed PO-based IM and the AFS can be performed by evaluating the quality of the solutions they identify over the course of the entire computation—defined, for the AFS, as the interval from the start of the computation to the end of the second period of the temporal modulation observed in H_{Ising} .

In this comparison, a solution is considered to be of higher quality if its energy value is lower and closer to the ground state. Likewise, a solver is deemed more accurate if it consistently identifies spin configurations with H_{Ising} values nearer to the ground state. To quantify the superiority of the AFS to the PO-based IM architecture, we recorded every unique instance when either solver accessed an Ising energy level between 0 and -22 , as shown in Fig. 2(d). We found that the AFS predominantly operates in low-energy regions of the landscape, permitting the AFS to identify many spin configurations with Ising energy close to the ground state. On the other hand, the PO-based IM progressively accesses fewer spin configurations (especially those with low energy) as the spin solver approaches the ground state. This bias in the AFS toward identifying lower energy values occurs due to the modulation process, instantiated by η_1 , generally occurring after the initial energy minimization mediated by η_0 . In summary, the

AFS identifies a significantly larger quantity of optimal and near-optimal solutions than the PO-based IM during the same computation time. This capability can be especially valuable when solving QUBO problems in time- or power-sensitive applications where optimal or near-optimal solutions are sufficient [17].

B. Floquet amplitude modulations to escape local minima

As another validation of the AFS's capabilities to escape local minima in H_{Ising} and achieve lower-energy solutions than PO-based IMs, we leverage the AFS ability to operate as a PO-based IM for significantly larger pump-power levels (i.e., for higher r_2 values) than those needed to trigger the Floquet states in its nodes. As before, we tackled a 20-node 3-regular MAX-CUT problem [the same analyzed in Fig. 2(d)]. Different from before, we initially configured r_2 to a sufficiently large value to activate the period-doubling regime in all the AFS's nodes. After enough time had elapsed for the system to reach its steady state [at a normalized time (t) equal to 0.5; see Figs. 3(a) and 3(b)], we abruptly reduced r_2 and activated Floquet states in all the AFS's nodes. We monitored the spin states and noted that the activation of Floquet states corresponded with a synchronized change of all the AFS's nodes' spin values, as

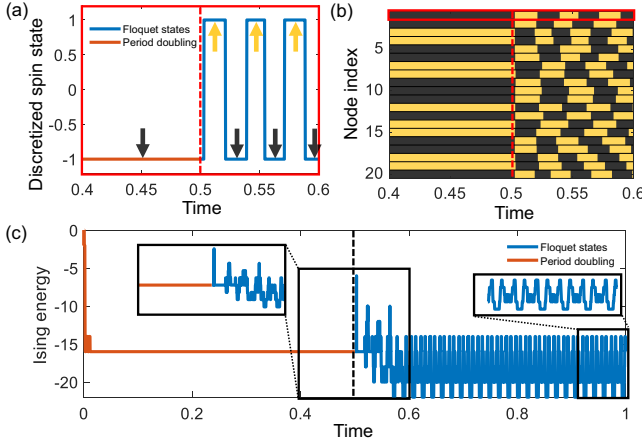


FIG. 3. (a) The extracted spin state of the x_1 node σ_1 shown throughout a small range of the simulation time. As can be seen, once the AFS switches from operating in the period-doubling regime and begins to exploit its nodes' Floquet states, σ_1 regularly flips its spin state. (b) The extracted spin states σ_i of all the nodes showing how the system's spins collectively undergo periodic changes of state. (c) The Ising energy of the system throughout the entire simulation. While the system initially converges to a local minimum while operating as a PO-based IM, the activation of Floquet states enables the system to eventually identify the problem's ground-state spin configuration by escaping the previously identified local minimum. Time, in these plots, is normalized over the entire simulation window.

seen in Fig. 3(b). Then, we computed H_{Ising} and determined that the AFS, while operating in the period-doubling regime, reached a local minimum Ising energy of -16 , as seen in Fig. 3(c).

Once the Floquet states are activated through the abrupt reduction of r_2 , H_{Ising} starts varying periodically, which, notably, significantly enhances the AFS's ability to identify the ground-state Ising energy (-22) and helps the system escape the previously attained energy local minimum [see Fig. 3(c)]. This demonstrates that the energetic oscillations in η induced by the activation of the Floquet states provides a path for the AFS to identify the ground-state spin configuration, allowing us to overcome the computational limits of PO-based IMs. As expected, during the period-doubling regime, the system energy undergoes a very sharp minimization before converging to a local minimum, indicative of the behavior of a gradient-descent search. The subsequent emergence of the Floquet states and synchronized collective spin flips leads the AFS to undergo substantial movements throughout the energy landscape, mostly localized proximate to optimal or near-optimal solutions.

C. Benchmarking the AFS against PO-based IMs in mid- and large-scale problems

The analysis discussed in Figs. 2 and 3 allowed us to study and visualize the main differences in the computation

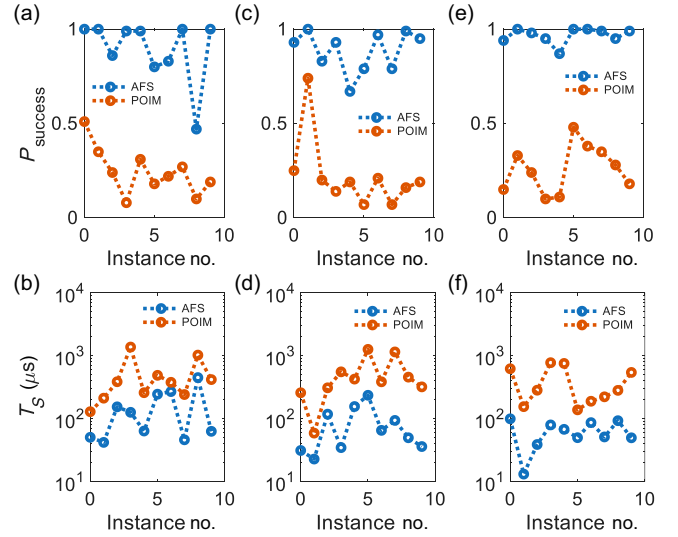


FIG. 4. Numerically computed trends for the PO-based IM and the AFS. These trends show the obtained P_{success} and T_S values over 100 iterations for (a),(b) the set of 60-node BIQ MAC MAX-CUT problems, (c),(d) the set of 80-node BIQ MAC MAX-CUT problems, and (e),(f) the set of 100-node BIQ MAC MAX-CUT problems. Agnostic to the problem instance, the AFS achieves higher P_{success} with lower T_S than the PO-based IM counterpart.

between the AFS and the PO-based IM. Nonetheless, a more meaningful quantitative comparison of these two QUBO solvers requires testing their performance when attempting to solve larger-scale and more densely connected QUBO problems. To do so, we compared the performance of the AFS with those of a PO-based IM when tackling midscale MAX-CUT problems from the BIQ MAC [111,120] library and large-scale MAX-CUT problems from the GSET library. These problems are commonly used for benchmarking the performance of IMs. In particular, we solved the BIQ MAC “g” series problems, with graph sizes of 60 nodes, 80 nodes, and 100 nodes. We numerically solved each of these problems 100 times using both the AFS and the PO-based IM when considering the nodes' initial amplitudes to be randomly distributed between -10^{-5} and 10^{-5} . For both machines, we applied a fixed pump signal amplitude of 0.7κ without running any annealing, and we computed the instantaneous cut size identified by both machines during their computation.

Because the BIQ MAC library problems have known exact solutions expressed in terms of number of cuts, we were able to calculate the probability of success P_{success} , which we define as the number of runs that identify a cut solution within 0.5% of the analytical maximum cut divided by the total number of runs for each problem. We also calculated the time to solution T_S , which is the expected time required to find a solution within 0.5% of the optimal one. The computation for T_S required us to track τ_r , which is the run-time of the AFS or the PO-based IM. Both P_{success} and T_S are plotted in Fig. 4; particularly, the results of the 60-node

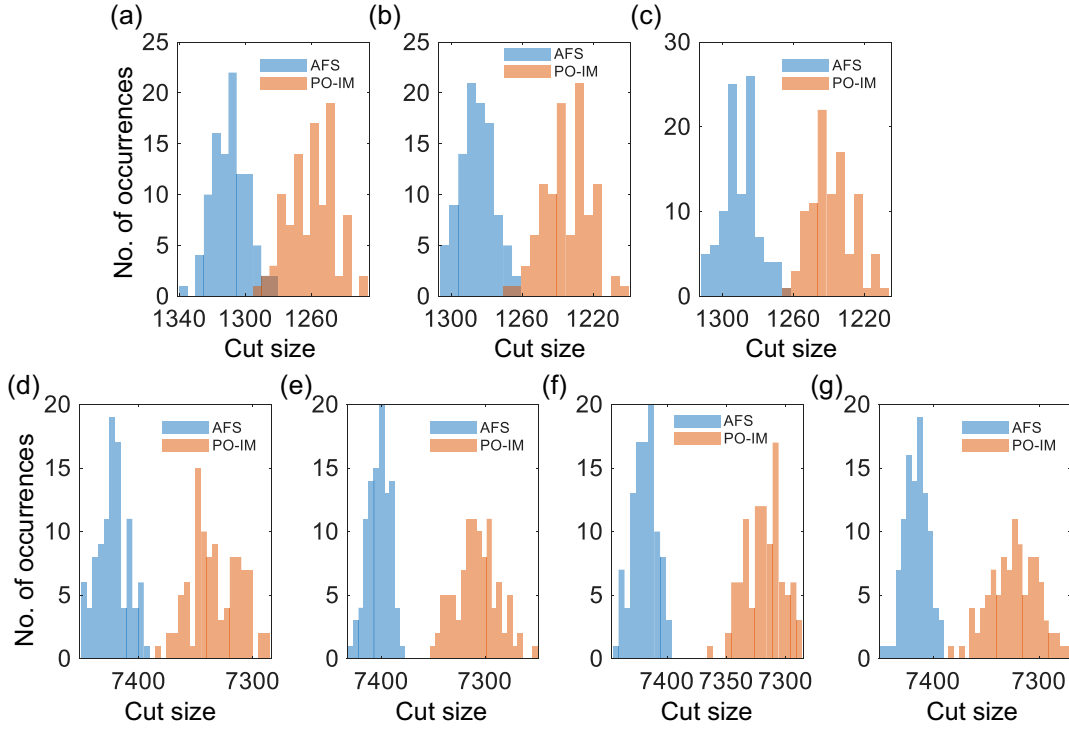


FIG. 5. Numerically computed cut distributions for the PO-based IM and the AFS when solving 2000-node GSET problems (a) *G32*, (b) *G33*, (c) *G34*, (d) *G35*, (e) *G36*, (f) *G37*, and (g) *G38*. As can be clearly seen, the AFS computes a dramatically higher cut size than the PO-based IM. In many of the observed instances, the distributions of cuts do not overlap at all, meaning that the worst solution obtained by the AFS is still better than the best solution obtained by the PO-based IM.

problems are shown in Figs. 4(a) and 4(b), the results of the 80-node problems are shown in Figs. 4(c) and 4(d), and the results of the 100-node problems are shown in Figs. 4(e) and 4(f). As can be seen, the AFS offers a significant improvement both in the form of an increased P_{success} and a reduced T_S compared to the PO-based IM across all problem instances and problem sizes.

In Supplemental Material Sec. IV [26], we plot the distribution of the obtained maximum cut values for both the AFS and the PO-based IM across all 30 BIQ MAC problem instances, showing that the AFS yields a much tighter spread of cuts with a higher mean cut size than PO-based IMs.

Next, the AFS and the PO-based IM were benchmarked against each other for various unweighted and binary-weighted problem graphs of different sizes and connectivity from the GSET library. In particular, all problems from *G01* to *G64*, as well as *G67*, are solved using the AFS and PO-based IM simulations 100 times. In Fig. 5, the histograms for the cut distributions for seven large-scale 2000-node problems are shown, which clearly demonstrates the improvement in solution quality when using AFS over PO-based IM. In fact, for some problems, the worst solution identified by the AFS is better than the best solution identified by the PO-based IM. Since the ground-state Ising energy or maximum cut value has not been analytically confirmed for many of these problems (with new

“best solutions” being discovered even to this day [121]), the cut values identified by the AFS and PO-based IM have been normalized to the best known solutions (to the authors’ knowledge) as N_S , which effectively gives the accuracy of the AFS and the PO-based IM compared to the best-known cuts for each problem. N_S , along with the range of solutions (also normalized to the best-known cut) is plotted in Fig. 6 for each of the investigated problems solved by the AFS and PO-based IM. As can be seen, the AFS exhibits a higher accuracy and a tighter spread of solutions than PO-based IMs for all of the considered GSET problems. To further quantify the performance of AFS

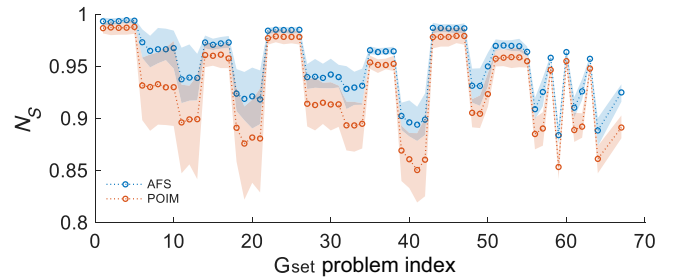


FIG. 6. Numerically extracted trends of N_S (normalized to the best-known solution) averaged over 100 runs for GSET problems *G01* – *G64* and *G67*. The shaded region shows the range of the minimum and maximum runs for each problem for both solvers.

against the state of the art with regard to projected energy consumption, an additional benchmarking study comparing the AFS and PO-based IM with GPU-based and p-bit-based simulated annealing algorithms is presented in Supplemental Material Sec. I [26], and the cut-size results of PO-based IM and AFS are shown in a table in Supplemental Material Sec. IV [26]. Additionally, running these large-scale problems also allowed us to confirm that AH is the limiting factor for the performance achieved by the PO-based IM (see Supplemental Material Sec. IV [26]).

On the other hand, the AFS's performance is much less impacted by AH. More information about our other analysis of MAX-CUT problems up to 2000 nodes, as well as a detailed discourse comparing the impact of AH in PO-based IMs and the AFS, can be found in Supplemental Material Secs. IV and V [26]. It is worth emphasizing that the AFS's improved accuracy over the PO-based IM when tackling BIQ MAC MAX-CUT problems is achieved with identical pump-power levels. This suggests that any future hardware implementations of the AFS will present minimal power overhead compared to the corresponding PO-based IM attained by excluding the resonant cavity needed to activate the Floquet dynamics. To this note, the mechanism by which the Floquet states are activated in a PO-based system are implementable in a diverse range of domains, including photonic, acoustic, and mechanical domains. Furthermore, the AFS's increased resilience to AH also provides a pathway for improving the performance of CIMs [16,84], creating means for such quantum solvers to achieve solutions with much higher accuracy limited only by quantum noise in their POs.

IV. CONCLUSIONS

In this article, we introduce and analyze a new analog solver platform for QUBO problems—an AFS—which leverages Floquet phenomena to enhance conventional PO-based IMs. The AFS consists of a network of dissipatively coupled periodically modulated nodes featuring comblike dynamics, mapping the specific problem to solve. Our investigation of the reported AFS demonstrates that its ability to activate Floquet states enables a superior ability to tackle NP-hard QUBO problems compared to conventional PO-based IMs. This advantage originating from periodically varying minimization dynamics allows the AFS to escape local minima of the Ising Hamiltonian. This can be simply achieved by including a set of resonators (one per PO) in the existing PO-based IMs. The AFS paves the way toward higher accuracy than PO-based IMs when solving QUBO problems with minimal power overheads, especially compared to PO-based IMs that rely on higher-dimensional approaches to improve computational performance. More generally, this work shows how the exotic dynamics of Floquet states can be leveraged for improving the performance of analog computing engines when addressing combinatorial optimization problems.

In the future, we envision the AFS to be realizable with different types of hardware components exploiting various physical domains, including photonic, electromagnetic, or mechanical devices. However, for each of these physical domains, there remain numerous implementation challenges related to the deployment of large-scale analog Floquet solvers. In this regard, it would be useful to study the ability of a chain of Floquet nodes to realize a single “artificial” Floquet node, like what is done in Ising machines implemented with sparse connectivity networks. Additionally, it would be insightful to characterize the computational impact of fabrication variations across parametric oscillators and their respective resonant modes, as well as unwanted variations in the coupling weights between Floquet nodes, when considering both sparse connectivity networks and all-to-all connectivity networks, to determine the suitability of specific physical domains for large-scale AFS implementations. Finally, further studies characterizing the sensitivity of the achieved Floquet modulations in each physical domain to analog precision errors in the coupling between Floquet nodes, especially for all-to-all connectivity networks, for increasing N would be useful.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: NUMERICAL ANALYSIS AND SIMULATION PARAMETERS

During our numerical analysis, we often compare the performance of a PO-based IM operating in the period-doubling regime with the AFS operating with activated Floquet states. In this regard, we numerically solve the system of $2N$ equations described in Eqs. (2a) and (2b) to characterize the performance of the AFS. For the PO-based IM, we use a similar equation, Eq. (A1), derived in the same way as Eqs. (2a) and (2b) using temporal coupled-mode theory and the tight-binding model, to describe the collective dynamics of coupled POs. As can be seen, Eq. (A1) is very similar to Eq. (2a), but excluding the mode y from its system,

$$\dot{x}_i = -\left(\frac{\kappa}{2} + r_1|x_i|^2\right)x_i + r_2x_i^* - \sum_{i \neq j}^N J_{i,j}x_j + N_b\zeta_{i,x}. \quad (\text{A1})$$

Almost all identical parameters are used while solving Eqs. (2a), (2b), and (A1) to produce the data shown in Figs. 2–5. In this regard, $\Gamma = 500000$ rad/s, $\kappa = 5\Gamma$, $g = \Gamma$, $J_{i,j} = 0.5\Gamma$, and $\omega_p/2 = 100\kappa$, and $N_b = 0.1$. Thus, the PO has a quality factor of 100 while the resonator's quality factor is 500, which matches values achievable in the electronic and mechanical domains [73,122], and a value of $r_1 = 0.75\kappa$ was considered for the AFS. In Figs. 2 and 3, both the AFS and the PO-based IM used $r_2 = 0.7\kappa$, while an $r_2 = 1.2\kappa$ was used in Fig. 4 to drive the AFS into the period-doubling regime before being reduced to $r_2 = 0.7\kappa$ to activate the Floquet states. A study of the impact of r_2 on the computational performance of the AFS is included in Supplemental Material Sec. IV [26], and a study of the impact of noise and coupling weight errors is included in Supplemental Material Sec. VI [26].

APPENDIX B: DETAILS ON SPIN-STATE EXTRACTION

As in other PO-based IMs, the system's spin states are encoded in the phase values $[\Phi(t)]$ of the POs' complex amplitudes. To calculate $\Phi_i(t)$ of the i th PO, we would generally compute $\arctan(\text{Im}\{x_i(t)\}/\text{Re}\{x_i(t)\})$ and then translate the acquired phase to +1 (spin-up) or −1 (spin-down). Because of the previously implemented rotational frame, the expression used for extracting spin values from analog amplitudes becomes further simplified to $\text{sign}(\text{Re}\{x_i\})$. Naturally, spins with a phase of 0 (or having a positive amplitude) are considered as spin-up and assigned a value of +1, while those with a phase of π (or having a negative amplitude) are considered as spin-down and assigned a spin value of −1. The spin states in the AFS can be extracted in the same way despite the fact that the systems' spins exhibit periodic modulations of their amplitudes. Thus, the expression for the spin extraction in the AFS takes the following form: $\text{sign}[\sum_n A_{i,n}(t) \cos(n\mu_i t + \phi_{i,n})]$, where each spin deterministically flips with frequency μ . We wish to reiterate that it is through these regular and collectively coordinated spin flips that the system is endowed with capabilities to escape the local minima of the Ising energy landscape.

Like many PO-based IMs, the AFS require the regular extraction of their spin states and computation of H_{Ising} . However, a key difference between the computational engines is that PO-based IMs reach a steady-state spin configuration that indicates the completion of the computation and the final identification of the solution to the QUBO problem being mapped. On the other hand, the AFS can indefinitely cycle through sets of values of H_{Ising} ; as such, we have included a discussion about sampling and

solution extraction for the AFS and PO-based IMs in Supplemental Material Sec. III [26].

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