

Entropy of Strongly Correlated Electrons in a Partially Filled Landau Level

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We use high-resolution chemical potential measurements to extract the entropy of monolayer and bilayer graphene in the quantum Hall regime via the Maxwell relation $[(d\mu)/(dT)]_N = -[(dS)/(dN)]_T$. Measuring the entropy from $T = 300$ K down to $T = 200$ mK, we identify the sequential emergence of quantum Hall ferromagnetism, fractional quantum Hall states, and various charge orders by comparing the measured entropy in different temperature regimes with theoretical models. At the lowest temperature of $T \approx 200$ mK we perform a detailed study of the entropy near even-denominator fractional quantum Hall states in bilayer graphene, and comment on the possible topological origin of the observed excess entropy.

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I. INTRODUCTION

In a correlated electron system, the temperature-dependent entropy gives direct experimental access to the degrees of freedom the system can explore at a given energy scale. At high temperatures, the entropy is typically dominated by the motional degrees of freedom of individual particles. However, at lower temperature, the entropy is determined by the long-wavelength collective excitations. Entropy thus provides a unique fingerprint of the energetic hierarchy of a many-body system. Entropy measurements are of particular interest in flat electronic bands where strong interactions can give rise to a rich variety of electronic orders hosting a correspondingly rich array of excitations. For example, in moiré- and rhombohedral graphene systems, entropy measurements have been used to probe the spin and valley excitations, revealing a regime of strongly fluctuating magnetic moments at finite temperature [1–4].

Landau levels offer an even more ideal flat band system in which entropy measurements can directly probe the nature of the emergent excitations. For example, entropy

measurements are predicted to provide a signature of the topological ground state degeneracy expected in the vicinity of non-Abelian fractional quantum Hall states, as are thought to occur in a half-filled first orbital Landau level. Specifically, it was proposed [5,6] that for electron density tuned near such fillings—where the ground state is expected to be a Wigner crystal of non-Abelian anyons with charge $e/4$ —the non-Abelian statistics lead to an additional entropy of $\ln(\sqrt{2})$ per fractionally charged quasiparticle. This entropy arises from the state space of the quantum bits delocalized between each pair of quasiparticles. When the motional entropy of the anyons is quenched, the entropy is expected to saturate at $S_{\text{topo}} \approx k_B N_q \ln(\sqrt{2})$, where N_q is the number of non-Abelian quasiparticles. Equating $dN_q/dN = 4$ (where N is the number of electrons), this should result in a low- T plateau of $dS/dN = 4k_B \ln \sqrt{2}$, revealing the topological degeneracy native to such states.

Previously, several experiments reported entropy measurements in partially filled Landau levels (LL), including in silicon [7] and GaAs quantum wells [8–10]. However, these experiments were typically restricted to a narrow range of filling factors and temperature. In the current work, we probe the entropy of quantum Hall states in mono- and bilayer graphene over 4 orders of magnitude of temperature, and across a range of density spanning several LLs. To determine the entropy, we first measure the temperature dependent chemical potential $\mu(N, T)$ using a

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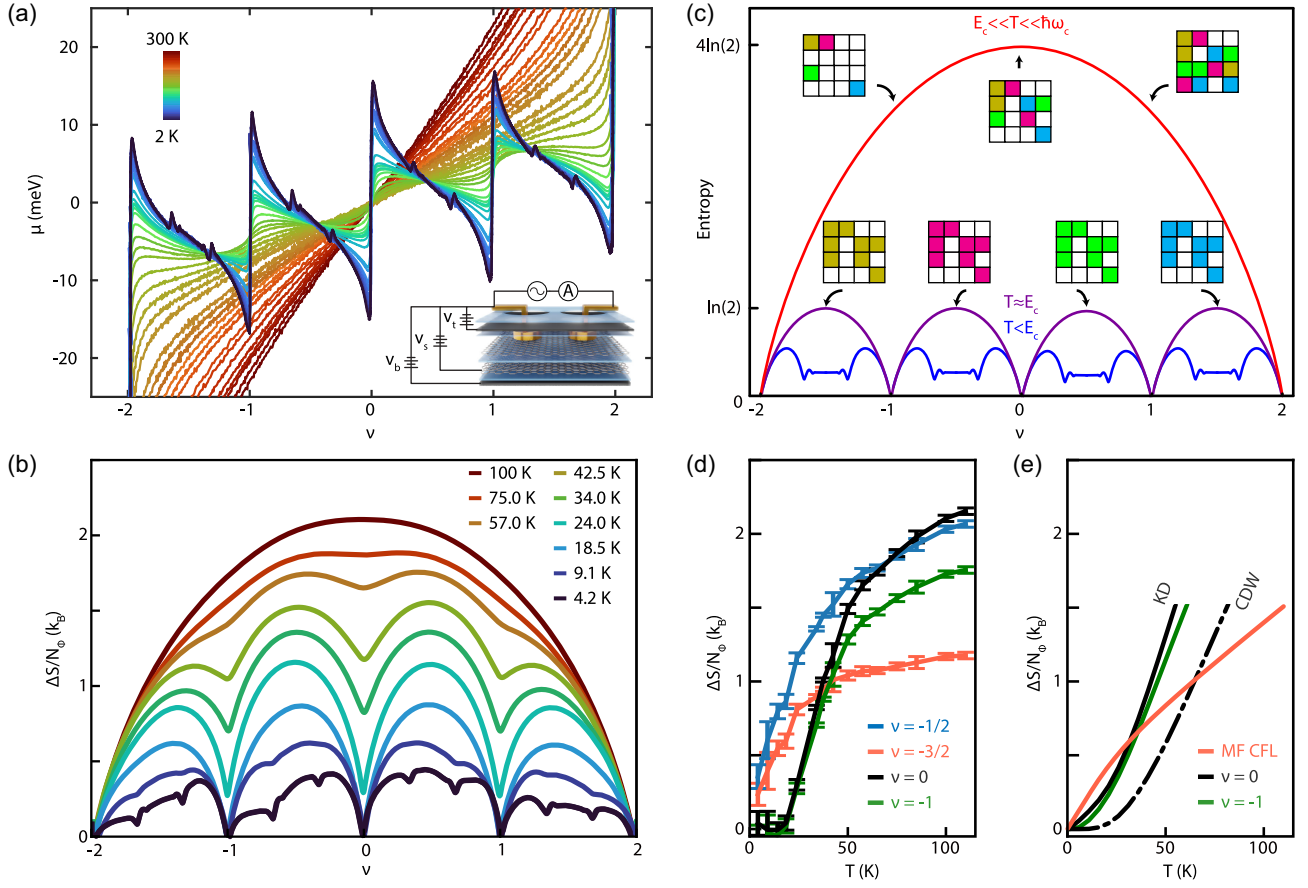


FIG. 1. Entropy in a partially filled monolayer graphene Landau level. (a) The chemical potential μ of monolayer graphene at $B = 11.5$ T at temperatures ranging from 2 to 300 K. (b) The entropy per flux determined from the temperature dependence of $\mu(\nu)$ in panel (a). (c) Schematic of the entropy per flux in the high (red), intermediate (purple) and low (blue) temperature regimes. The domes are associated with the occupational entropy of four isospin flavors ($T \gg E_C$) or a single isospin flavor ($T \sim E_C$). Available motional states are depicted on a grid, with the colors representing isospin flavors. (d) Measured T-dependent entropy at integer and half-integer fillings. (e) Temperature dependence of the entropy from the noninteracting magnon model at integer fillings, and from a mean-field composite Fermi liquid theory at half-integer filling. The dash-dotted line assumes SU(4) symmetric Coulomb interactions, while the solid line accounts for realistic isospin anisotropy [17].

proximal charge sensor [1,2,11–16]. As shown in the inset of Fig. 1(a), our experimental setup uses a multilayer heterostructure that includes a sample layer consisting of either mono- or bilayer graphene with applied voltage v_s , and a sensor layer consisting of a grounded graphene monolayer in which Corbino transport is measured. Adjusting the top and bottom gate voltages (v_t and v_b , respectively) to keep the density in the sensor layer fixed at a fractional quantum Hall conductance minimum allows the chemical potential μ of the sample layer to be measured as a function of Landau level filling ν and T , as described previously [13,16]. The entropy S then follows from the Maxwell relation,

$$\left. \frac{d\mu}{dT} \right|_N = - \left. \frac{dS}{dN} \right|_T, \quad (1)$$

where in the experiment is approximated by the finite difference between μ curves taken at different temperatures.

II. HIGH-TEMPERATURE POSITIONAL ENTROPY

To probe the formation of correlated phases in a partially filled Landau level, we focus first on monolayer graphene at high temperature. We choose monolayer because of its large cyclotron gap, which enables us to access a broad regime where temperature is large compared to the Coulomb energy but the active degrees of freedom are confined to the zero energy Landau level only. Figure 1(a) presents μ for monolayer graphene at $B = 11.5$ T across temperatures ranging from 2.7 to 300 K. At low temperatures, μ shows numerous well-understood signatures of electronic correlations including sudden jumps in μ at Landau level fillings factor $\nu = -2, -1, 0, 1, 2$ associated with quantum Hall ferromagnetic gaps and negative electronic compressibility ($\partial\mu/\partial\nu < 0$) [12]. To extract the entropy, we approximate the derivative in Eq. (1) by the difference between μ curves obtained at slightly different

temperatures, $dS/dN = -d\mu/dT \approx -\delta\mu/\delta T$, where $\delta\mu = \mu_{\text{hot}} - \mu_{\text{cold}}$ and $\delta T = T_{\text{hot}} - T_{\text{cold}}$.

We then obtain the entropy difference between two filling factors via integration, $\Delta S/N_\phi = \int_{\nu_1}^{\nu_2} (dS/dN) d\nu$. Here N_ϕ is the number of flux quanta piercing the sample. This procedure involves two arbitrary constants that must be constrained experimentally. First, a reference filling must be chosen such that ΔS provides a good representation of the total entropy S . Second, strictly speaking we measure the difference in chemical potential between the sample and the “sensor” layer, $\mu(\nu, T) - \mu_{\text{sens}}(T)$. Since the sensor layer is kept at fixed ν , $\mu(\nu, T)$ is known up to a density-independent but temperature-dependent constant, $\mu_{\text{sens}}(T)$. Upon integration, this results in a linear background added to the entropy, $S/N_\phi \rightarrow S/N_\phi - (\nu - \nu_1) \partial\mu_{\text{sens}}(T)/\partial T$. To remove this background, we require that the entropy is negligible for $T < 100$ K at $\nu = \pm 2$. This is justified by theoretical calculations using the MLG Landau level spectrum, which give $S(\nu = \pm 2, T = 100 \text{ K})/N_\phi = 0.02k_B$. As a check, we note that the S resulting from integration shown in Fig. 1(b) displays a high degree of particle-hole symmetry about charge neutrality; this expected symmetry would be spoiled if the offset $\mu_{\text{sens}}(T)$ were chosen incorrectly.

At 100 K, S exhibits a single dome with maximum at $\nu = 0$ as shown in Fig. 1(b). As T is reduced, the entropy is strongly suppressed at integer fillings, so that by $T \approx 20$ K the measured S shows four approximately identical domes centered at each half-integer ν . At $T = 4.2$ K, additional entropy minima develop at values of ν associated with the onset of fractional quantum Hall states. These findings are consistent with the energy hierarchy expected within an interacting partially filled Landau level [18], illustrated in Fig. 1(c). Our experiment operates at T well below the cyclotron gap $\Delta_c \sim 1500$ K, so that electronic states are restricted to the monolayer graphene zero energy Landau level (zLL). When T is comparable to the Coulomb interaction scale $E_C \sim 500$ K, electrons can freely occupy the nearly degenerate $4N_\phi$ orbitals of the zLL, where N_ϕ represents the number of magnetic flux quanta and the factor of four accounts for the combined spin and valley degeneracy of each orbital state. At these high temperatures, the $S(\nu)$ corresponds to the number of ways to distribute the $(\nu + 2)N_\phi$ electrons among the $4N_\phi$ available states:

$$S/N_\phi = k_B \ln \Omega/N_\phi = k_B \ln \left(\frac{4N_\phi}{(\nu + 2)N_\phi} \right) / N_\phi \quad (2)$$

$$= k_B [4 \ln 4 - (2 + \nu) \ln(2 + \nu) - (2 - \nu) \ln(2 - \nu)], \quad (3)$$

giving a single “dome” across the entire zLL. In this model, the entropy per flux quantum peaks at $S/N_\phi = 4 \ln(2)k_B \approx 2.8k_B$ at $\nu = 0$. At 100 K, the experimental data show a value somewhat below this, $S/N_\phi \approx 2.1k_B$. This reduction

is to be expected, as E_C is not entirely negligible in comparison to T .

As we lower the temperature, the entropy becomes sensitive to correlation effects which favor certain many-body states. Notably, even when isospin symmetry is preserved by the Hamiltonian—so that E_C is the only energy scale relevant to the partially filled LL energetics—a separation of energy scales occurs between ferromagnetic correlations at integer filling and the formation of more subtle fractional quantum Hall phases. In a simplified picture, for temperature somewhat below E_C , electrons near the Fermi level occupy only one isospin flavor, with other isospin flavors separated by an energy gap. This energy gap arises through a combination of exchange, Zeeman energies, and SU(4)-breaking anisotropies of the interaction. In this limit, the problem is reduced to filling $\tilde{\nu}N_\phi$ states out of N_ϕ within that flavor, where $\tilde{\nu} = \nu - \lfloor \nu \rfloor$, and the entropy is described by four distinct domes with peak entropy per flux quantum of $S/N_\phi = \ln(2)k_B$:

$$S/N_\phi = -k_B [\tilde{\nu} \ln \tilde{\nu} + (1 - \tilde{\nu}) \ln(1 - \tilde{\nu})]. \quad (4)$$

As the temperature is reduced by another order of magnitude, fractional quantum Hall insulators, Wigner crystals, or other correlated states begin to emerge at fractional fillings, further suppressing the entropy.

To allow a quantitative comparison to theoretical modeling, we focus the remainder of the paper on specific regimes of ν and T of progressively lower energy scale, beginning with integer filling within the ZLL.

III. INTEGER QUANTUM HALL FERROMAGNETISM

Figure 1(d) shows $\Delta S/N_\phi$ as a function of T at $\nu = -1$ and $\nu = 0$ as well as at $\nu = -1/2$ and $\nu = -3/2$. At the integer fillings, the entropy falls below our noise limit at low temperatures, becoming undetectable below $T \approx 10$ K. Notably, the entropy at $\nu = 0$ and $\nu = 1$ shows quantitatively similar behavior at low temperatures. In contrast, the half-integer states exhibit significantly higher entropy at low temperatures, consistent with gapless composite Fermi liquid (CFL) phases, which we will return to below.

To understand the low- T IQH entropy, we appeal to the theory of quantum Hall ferromagnetism, which predicts that electrons spontaneously break the approximate SU(4) symmetry relating spins and valleys, resulting in low-energy magnon modes. When the SU(4) symmetry is exact, these modes are gapless, with a k^2 dispersion that results in linear- T entropy. However, in the experiment SU(4) is broken by spin and valley Zeeman fields, as well as small SU(4) anisotropies of the Coulomb interaction, all of which can gap out or modify the magnon dispersion. When considering *only* the Zeeman effects, the magnons [19–23] will be gapped by the spin Zeeman energy ($\Delta_Z = 15$ K at 11.5 T) and the $A - B$ sublattice splitting

induced by the hBN substrate, which acts as a valley Zeeman field in the zLL [24] ($\Delta_{AB} = 80$ K for this device, which was measured previously in [13]). At $\nu = -1$, the state would thus spin and valley polarize, with minimal magnon gap Δ_Z , while at $\nu = 0$, the state would be a valley-polarized, spin-singlet (referred to in the literature as the charge density wave state [25]) with a magnon gap equal to $\Delta_{AB} = 80$ K. To make this comparison quantitative, Fig. 1(e) shows the entropy calculated in a low-temperature expansion based on the thermal population of magnon modes. To do so we first use the single-mode approximation to obtain the magnon dispersion $\epsilon_n(k)$, and then in the spirit of the Debye model we compute the resulting free-boson entropy, keeping modes up to a momentum scale set by the inverse magnetic length. As shown in Fig. 1(e), when accounting only for Zeeman effects, the entropy of a valley-polarized $\nu = 0$ charge density wave state is suppressed relative to $\nu = -1$, in line with the above discussion and in contrast to experiment [26,27].

The comparatively large entropy at $\nu = 0$ may be accounted for by considering the anisotropy of the Coulomb interaction, which is known to favor a Kekulé or canted-antiferromagnetic order at charge neutrality, rather than the valley-polarized state, under similar experimental conditions [25,26,28–31]. These phases spontaneously break U(1) valley and/or spin rotation symmetry, and thus support gapless magnon (i.e., Goldstone) modes even in the presence of a sublattice splitting. As shown in Fig. 1(e), where we assume a spin-singlet Kekulé order, accounting for the contribution of this gapless mode to the entropy significantly increases the entropy at $\nu = 0$ to a similar level as at $\nu = -1$, consistent with the experiment [17]. Note that these calculations are valid only in the low temperature limit where magnon interactions can be neglected; however, in this regime, they agree with the $\Delta S/N_\phi \approx 0.25k_B$ measured at $T \approx 25$ K. For the lowest temperatures ($T \lesssim 20$ K), the measured $\nu = 0$ entropy falls below the theoretical expectation for a Goldstone mode (though in this regime the expected signal is also comparable or smaller than the measurement uncertainty). This may indicate that the Goldstone mode is weakly gapped by short-range disorder, which breaks the U(1) valley symmetry [32]. Indeed, low- T STM studies observe slow gradients in the phase of the Kekulé order that distort near disorder sites [31].

IV. COMPOSITE FERMION ENTROPY: COMPOSITE FERMION LIQUIDS AND DEGENERATE Λ LEVELS

To further investigate the effect of electronic correlations, we extend the thermodynamic measurements to the partially filled Landau levels. We begin by focusing on the half filled Landau level, where the system remains gapless at low temperature and the entropy remains comparatively large. Within a composite fermion picture, $\nu \rightarrow 1/2$

corresponds to zero magnetic field, with the ground state described by a composite Fermi liquid (CFL) [33]. As can be seen in Fig. 1(d), at low T the $1/2$ states exhibit larger entropy than the integer states, with no sign of a gap. Indeed, at the mean-field level (when ignoring gauge fluctuations) the CF Fermi surface has a density of states per flux quantum $\rho = m_* \ell_B^2 / \hbar^2 = 1/(xE_C)$, where x is a dimensionless constant that depends on the form of the interaction. The Fermi surface contributes a linear- T entropy, $S = (\pi^2/3)k_B T \rho = (\pi^2/3)k_B T / (xE_C)$. Figure 1(e) shows the entropy calculated for a single-component CFL treated at this mean-field level using the CF effective mass estimated in Ref. [33], $m_* \approx 3.3\hbar^2/(E_C \ell_B^2) \approx 0.1m_e$ where m_e is the electron mass. The magnitude of the evaluated entropy is qualitatively similar to experiment.

Going beyond free CF approximations, gauge fluctuations are expected to both renormalize the CF mass (and thereby the density of states) and contribute additional entropy. As first analyzed by Halperin, Lee, and Read (HLR) [33], these fluctuations result in sublinear scaling as $T \rightarrow 0$ (i.e., entropy greater than that of a Fermi liquid as $T \rightarrow 0$). For a long-range Coulomb interaction, the CF effective mass acquires a logarithmic renormalization—characteristic of a marginal Fermi liquid—resulting in the mean-field prediction $S \propto [(k_B T)/E_C] \ln(T_0/T)$. In contrast, for a short-range interaction the fluctuations are stronger, leading to non-Fermi liquid behavior with $S \propto [(k_B T)/E_C](T_0/T)^{1/3} \propto T^{2/3}$ at the mean-field level. In our system, screening from the graphite gates, described by $V(q) = [(2\pi)/q] \tanh(qd)$, introduces a new length scale d and prompts a crossover from long-range to short-range scaling as the temperature is lowered. Within the HLR framework, one may infer a crossover temperature of order $k_B T_* \sim E_C (\ell_B/d)^2 \sim 3$ K; since our lowest measured temperature is 4 K, we thus expect the entropy to scale between $T^{2/3}$ and T . While the calculations presented here are based on RPA, we note that calculations based on perturbative expansions also consistently indicate sublinear scaling of entropy with T [34–36].

In Fig. 2, we present a log-log plot of the measured entropy in monolayer graphene at $\nu = 1/2$ and $\nu = 3/2$, with T -linear and $T^{2/3}$ scalings shown for comparison. Although the experimental uncertainty is large and the accessible temperature range is limited, the data suggest that the low temperature entropy is sublinear in T , and more consistent with $T^{2/3}$ scaling. Future experiments at lower temperatures and with higher precision may enable a definitive quantitative comparison with the $T^{2/3}$ scaling and reveal additional corrections [35,36].

At odd fractional fillings $\nu = p/(2p \pm 1)$, two dimensional electron systems forms incompressible fractional quantum Hall phases corresponding to integer quantum Hall effects of the composite fermions. To study these phases in detail, we move away from studying S/N_ϕ , which requires referencing our data to a state of fixed

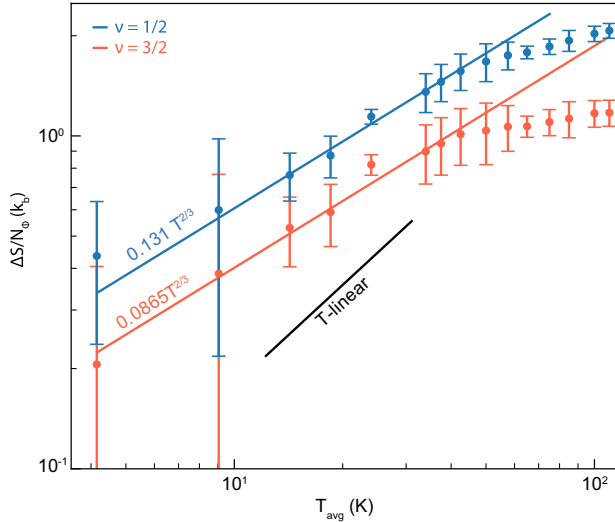


FIG. 2. Entropy per flux at $B = 11.5$ T of monolayer graphene half-filled $N_L = 0$ Landau level, which is expected to host a composite Fermi liquid (CFL). While a Fermi liquid features T -linear entropy, gauge fluctuations in the CFL further contribute, with $S \propto T^{2/3}$ expected for short-range interactions [33]. While the experimental uncertainty is too large to fit an exponent, the $S(T)$ data below the CF Fermi temperature (≈ 40 K), is consistent with T^α for $\alpha < 1$.

entropy. This fails at sufficiently low temperatures, where the integer quantum Hall gaps become so large that the sample does not charge on timescales of the experiment (practically, the RC time of the sample becomes larger than several minutes). Instead, we focus on measuring dS/dN , which can be determined without the need to integrate. We also transition to studying graphene bilayers, motivated by the existence of both odd- and even-denominator fractional quantum Hall states [37–39], the latter of which are predicted to host non-Abelian anyon excitations and will be the focus of the final part of this paper.

Figure 3(a) shows μ in a regime of ν and applied displacement field D where the valence electrons occupy an $N = 0$ orbital state [38,40] in a bilayer graphene device. Sharp jumps in μ correspond to the Jain sequence of incompressible states [41]. Analyzing the change in μ between $T_{\text{hot}} = 600$ and $T_{\text{cold}} = 100$ mK, we extract dS/dN , shown in Fig. 3(b). While this temperature difference is large even compared to the absolute averaged temperature (350 mK), we note that decreasing the temperature difference does not appreciably change the overall dS/dN , as shown in Supplemental Material [17] Fig. S5, suggesting S/T is temperature independent in this regime to within our experimental resolution.

dS/dN changes sign across the incompressible fractional quantum Hall (FQH) states, with enhanced entropy for small doping that mimics the integer case of Fig. 1. To understand the entropy in this regime, we consider the thermodynamics of composite fermions in a reduced, density-dependent effective magnetic field $B_* = B(1 - 2\nu)$. This problem

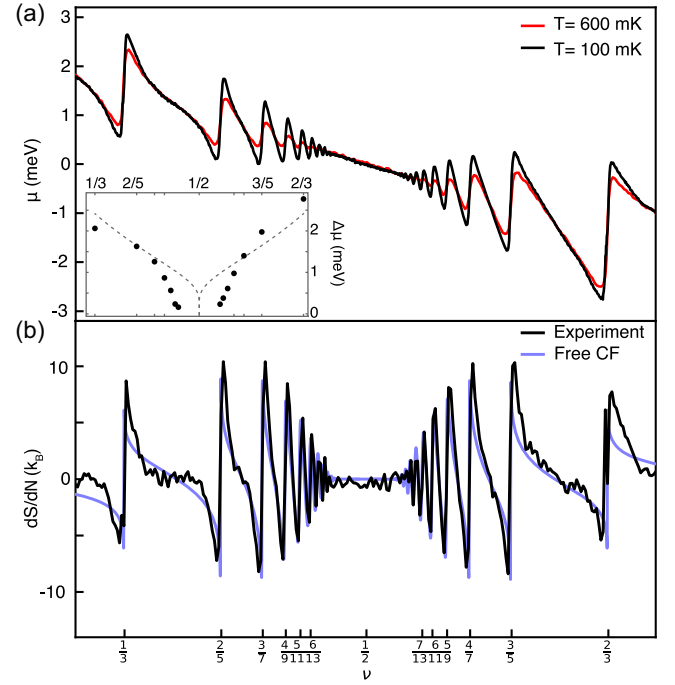


FIG. 3. Entropy in the fractional quantum Hall regime of bilayer graphene. (a) Chemical potential μ in the partially filled $N_L = 0$ Landau level at $B = 13.8$ T at $T = 600$ mK (red) and $T = 100$ mK (black). Inset: Thermodynamic energy gaps determined from the chemical potential jumps at 100 mK. Dashed line is a guide for the eye showing expected $1/\ln(2p+1)$ scaling. (b) Entropy per electron dS/dN obtained from the data in panel (a) (black curve). Also shown (in blue) is the calculated entropy from a noninteracting composite fermion model at $T = 350$ mK, assuming a Gaussian Landau level broadening $\Gamma = 150$ μ eV.

has previously been within the random-phase approximation [42], assuming an unscreened Coulomb interaction and in the absence of disorder. Within this approximation, the jump in the chemical potential at $T \rightarrow 0$ (typically referred to as the thermodynamic gap) is predicted to scale as $\Delta \propto \{1/[\ln(2p+1)]\}$ at $\nu = [p/(2p+1)]$, due to the renormalization of the CF effective mass. The experimentally observed jumps in μ fall off much faster than this at $T = 100$ mK, as shown in the inset of Fig. 3(a). Three effects likely contribute to this discrepancy, including finite temperature, screening of the Coulomb interactions by the gates, and disorder. It is an interesting challenge for theory to account for all three, but we note that the distance to the gates $d_g = 64$ nm is still sizable when compared with the effective CF magnetic length $\ell_B^{\text{eff}} = \sqrt{2p+1} \ell_B \approx 21$ nm at $2p+1 = 9$, $B = 13.8$ T. Consequently, we expect that the gates are not the dominant correction [17]. Similarly, $T = 100$ mK is 2 orders of magnitude smaller than the gap size, where the deviations from the clean theory occur, and little temperature dependence is seen below 100 mK.

We thus focus first on the effects of disorder. We consider noninteracting CF filling the Λ levels, with a

phenomenological Gaussian broadening to account for disorder. In this approach, the problem reduces to the thermodynamics of free fermions, and the CF entropy can be obtained via

$$S_{\text{CF}}(N_{\text{CF}} = N, B_* = B(1 - 2\nu)) = S_e(N, B). \quad (5)$$

Figure 3(b) shows the resulting entropy, assuming a CF effective mass of $m_* = 0.25m_e$ and a Gaussian broadening $\Gamma = 155 \mu\text{eV}$ [17]. In the absence of broadening, at $\nu = p/(2p + 1)$ the CFs occupy an integer number p of completely filled Λ levels and the entropy vanishes in the low-temperature limit. As ν is detuned from these fillings, the system becomes compressible, and the entropy arises from the motional degrees of freedom of the CFs in a partially occupied Λ level, leading to a pronounced sign change in dS/dN across the gap. The magnitude of this gap is nonmonotonic as a function of ν due to the competition between two effects. For $\nu = p/(2p + 1)$, the addition of one electron introduces $2p + 1$ quasiparticles, resulting in a larger entropy per electron as $\nu \rightarrow 1/2$. Conversely, the CF cyclotron gap scales as $B_* \propto (1 - 2\nu)B$, which eventually suppresses quantum oscillations when disorder becomes comparable to the gap.

Notably, within this simplified model, the entropy is completely dominated by a single Λ level for $p = 1, 2, 3$ [17]. In this regime, interparticle interactions within the nondispersing Λ level are expected to play a dominant role in determining the low-temperature entropy. Indeed, discrepancies between experiment and the free-CF model are apparent—for example, between $\nu = 1/3$ and $\nu = 2/5$ —where the experimentally measured entropy is detectably lower than that predicted by the noninteracting model. Because of the Coulomb interaction, the ground state in this regime is expected to form a Wigner crystal of either electrons or fractionally charged quasiparticles, with the residual entropy dominated by density fluctuations (magnetophonons) of the charge-ordered state. We analyze the entropy in this regime in detail in the next section.

V. INTEGER AND FRACTIONAL WIGNER CRYSTALS

In the vicinity of both integer and fractional quantum Hall states, quasiparticles emerge with density proportional to the deviation of ν from the nearest quantized value, i.e., $n_{\text{qp}} \propto \nu - \nu_0$, where ν_0 corresponds to an incompressible integer or fractional Hall state. We first examine the behavior of dS/dN near $\nu_0 = \pm 2$ (see Fig. 4) in monolayer graphene. At high temperatures, the motional entropy of these quasiparticles in the lowest Landau predominates. As shown in Figs. 4(a) and 4(b), the dashed curves represent the high- T theoretical result computed via Eq. (4) from the motional entropy of a single Landau level, and the experimental data closely follow this prediction.

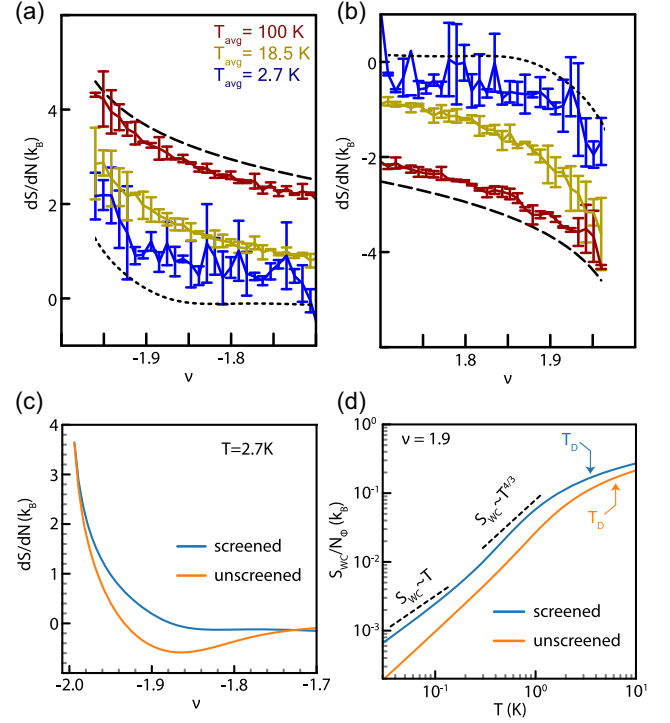


FIG. 4. Entropy per electron near integer fillings in monolayer graphene. (a) ν dependence of the entropy per added particle, dS/dN , near $\nu = -2$ and (b) $\nu = 2$ at various temperatures. The dashed curves indicate the high- T motional entropy of a four-flavor degenerate Landau level, while the dotted curve shows the entropy predicted by a classical Wigner crystal (WC) model at $T = 2.7$ K. The three temperature ranges are: $T_{\text{hot}} = 120$ to $T_{\text{cold}} = 80$ K; $T_{\text{hot}} = 20.9$ to $T_{\text{cold}} = 16.1$ K; and $T_{\text{hot}} = 3.4$ to $T_{\text{cold}} = 2$ K. (c) Theoretical predictions for dS/dN at $T = 2.7$ K are shown for an unscreened Coulomb interaction and for one screened by metallic gates and Landau-level mixing. (d) Calculated entropy as a function of temperature for an electron Wigner crystal at $\nu = -1.9$, computed with and without screening. For $T \lesssim T_D$, $dS/dN < 0$ due to WC stiffening with increasing density. This regime is more challenging to access when screening is present.

At high T , the reduction in dS/dN with increasing n_{qp} arises from the indistinguishability of electrons and holes, since the statistical factor $1/N!$ reduces the effective number of accessible states.

As the temperature is lowered, dS/dN decreases as interactions and disorder progressively freeze quasiparticle motion. The shape of dS/dN versus ν also changes, indicating a distinct state at low T . In the absence of disorder and at low densities and temperatures, we expect the quasiparticles to form a triangular-lattice Wigner crystal (WC) [43–47], as has been recently visualized via scanning probe microscopy measurements in bilayer graphene [48]. Within the WC regime, the decrease in dS/dN with increasing n_{qp} is attributed to the behavior of the magneto-phonon modes. At low n_{qp} , weak interactions yield soft magnetophonon modes and a high entropy per particle

S/N . As n_{qp} increases, strengthening interactions stiffen the WC, shifting the magnetophonon modes to higher energies and reducing S/N . The sign and magnitude of dS/dN are determined by the competition between the increased density of modes and the reduced entropy per mode; when the latter dominates (as at higher n_{qp}), dS/dN decreases and eventually reverses sign. This behavior is captured by our theoretical model, as indicated by the dotted curves in Figs. 4(a) and 4(b), which depict the predicted entropy of an electron WC at $T = 2.7$ K.

We now briefly describe the theoretical model for the entropy of the WC. Building on previous studies of magneto-phonon modes [43,49], we compute the entropy S from the phonon dispersion $\epsilon(\mathbf{k})$ by treating the phonon excitations as free bosons, akin to the Debye model. Although we expect the WC to melt in the dilute limit—particularly in the fractional regime where energy scales are low—Ref. [5] shows that magnetophonons remain the dominant source of entropy even above the melting temperature. Moreover, the same framework applies to a WC of fractional quasiparticles by replacing the elementary charge e with the fractional charge e^* and the magnetic length ℓ_B with $\ell_B^* = \sqrt{\hbar/(e^*B)}$, assuming exchange statistics are negligible in a localized crystal [5,6].

In the case of an unscreened Coulomb interaction, the high-field WC entropy is predicted to scale as [6]

$$\frac{S_{\text{WC}}}{N_\phi} \propto \left(\frac{T}{T_D(\nu)} \right)^{4/3}, \quad (6)$$

where the Debye temperature $T_D(\nu) \sim E_C \sqrt{e/e^*} |\nu - \nu_0|^{3/2}$. We find that this formula underestimates T_D by an order of magnitude compared to the more quantitative model introduced below. To calculate the WC entropy more quantitatively, we adopt a WC model that incorporates three screening effects; notably, our model allows us to quantitatively account for the behavior of μ at low temperatures [16]. First, screening from the dual graphite gates at a distance d_g renders the $q = 0$ portion of the Coulomb interaction finite. Consequently, the magnetophonon dispersion crosses over from $\epsilon(k) \propto k^{3/2}$ [43,49] for $kd_g \gtrsim 1$ to $\epsilon(k) \propto k^2$ for $kd_g \lesssim 1$. This crossover causes the magneto-phonon entropy $S_{\text{WC}}(T, \nu)$ to change from a $T^{4/3}$ to a linear- T dependence at the lowest temperatures, as shown in Fig. 4(d). Second, strong Landau-level (LL) mixing in monolayer and bilayer graphene, attributable to the filled Dirac sea, softens the interactions and lowers the Debye temperature T_D . We include LL mixing at the level of the static random-phase approximation [38,50–53]. Third, when applied to fractional quantum Hall liquids with small gaps, we model the static dielectric response of the FQH liquid as $\Pi_{\text{FQH}}(q) = -bq^2 + \dots$, where b is a fitting parameter determined by comparing to experimental

data [see Fig. 6(a)] [16], though its precise value plays a minor role in the resulting entropy.

Altogether, the screened interaction is given by

$$V_{\text{scr}}(q) = \frac{V(q)}{1 - \Pi(q)V(q)}, \quad (7)$$

where $V(q) = E_C \ell_B [(2\pi)/q] \tanh(qd_g)$, and $\Pi(q)$ is either the noninteracting polarization $\Pi_{\text{LL}}(q)$ due to inter-LL transitions for graphene [38,50–53] or $\Pi_{\text{FQH}}(q)$ for FQH Wigner crystals [see Supplemental Material [17] for the precise form of $\Pi_{\text{LL}}(q)$ and $\Pi_{\text{FQH}}(q)$]. In Figs. 4(c) and 4(d), we compare the calculated dS/dN near $\nu = -2$ with and without accounting for these screening effects. As shown in Fig. 4(c), incorporating screening is essential to capture the experimentally observed trends. In particular, screening reduces T_D and therefore raises the entropy.

Finally, we comment on the physical meaning of dS/dN and how it can be used to infer T/T_D . At or above the Debye temperature, $dS/dN > 0$ because the system follows equipartition and so the entropy follows the density of magneto-phonon modes, which increases with N . At $T \ll T_D$, however, these modes partially freeze out. In this case, $dS/dN < 0$ for large n_{qp} because the Wigner crystal becomes more rigid with increasing N , suppressing the entropy. Including screening brings $T = 2.7$ K close to T_D , making both theory and experiment yield $dS/dN \approx 0$. As illustrated in Fig. 4(d), screening also makes it more challenging to reach the $T < T_D$ regime. Currently, the limited conductivity of the WC phase restricts the achievable charging rates at the lowest temperatures. Future IQHE experiments that use alternative device geometries or charging methods may reach the $T < 1$ K regime. If observed, $dS/dN < 0$ would provide further confirmation for the Wigner crystal interpretation.

VI. ANYONIC WIGNER CRYSTALS AND THE EVEN-DENOMINATOR STATE

Obtaining a quantitative understanding of the entropy of dilute fractional quantum Hall (FQH) quasiparticles is especially important for experimentally identifying non-Abelian anyons via thermodynamic measurements. As discussed in the introduction, a collection of N_a well-separated non-Abelian anyons is predicted to contribute a topological entropy

$$S_{\text{topo}} = N_a k_B \ln(\mathcal{D}), \quad (8)$$

where $\mathcal{D} = \sqrt{2}$ for the Ising anyons anticipated near filling $\nu_0 = -1 + 1/2$ in bilayer graphene [37–39,53]. Taking $dN_a/dN = \pm 4$ for quasiparticles of charge $\pm e/4$ then implies

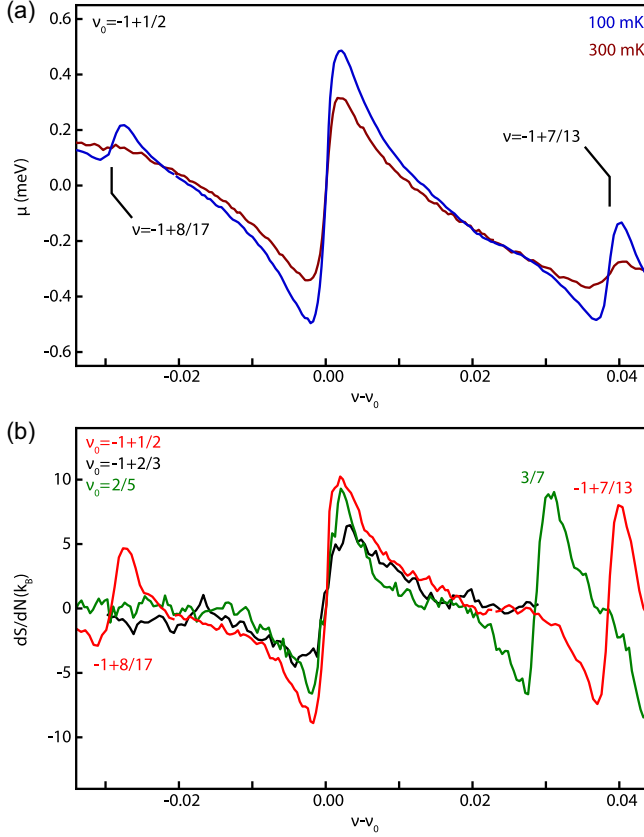


FIG. 5. Entropy of the bilayer graphene $\nu = -1 + 1/2, -1 + 2/3$, and $2/5$ states at $B = 13.8$ T. (a) Measured μ at $T = 300$ and $T = 100$ mK near $\nu_0 = -1/2$ at $B = 13.8$ T. (b) Measured dS/dN near $\nu_0 = -1/2, -1/3$, and $2/5$ extracted from the measured μ using $T_{\text{hot}} = 300$ to $T_{\text{cold}} = 100$ mK.

$$\frac{dS_{\text{topo}}}{dN} = 4k_B \ln \sqrt{2} \text{sgn}(\nu - \nu_0), \quad (9)$$

which is independent of temperature [6].

This topological contribution arises *in addition* to the motional entropy of the anyons. Ideally, the motional entropy freezes out at low temperatures, leaving only S_{topo} . In practice, however, whether there exists an accessible regime where S_{topo} dominates the measured entropy is subtle. For example, at sufficiently low quasiparticle density N_a , the motional entropy may remain significant at experimental temperatures, concealing the topological contribution. Conversely, at higher quasiparticle densities, interanyon interactions can lift the topological degeneracy and entirely suppress the topological entropy. In the extreme limit, these interactions can drive the formation of new incompressible fractional quantum Hall “daughter” states at sufficiently high N_a [54].

Figure 5(a) shows the chemical potential μ measured near $\nu_0 = -1 + 1/2$ in bilayer graphene at $T = 100$ and 300 mK, at half filling of the $N = 1$ orbital Landau level. As previously reported [16], this energy gap is considerably

larger in bilayer graphene than reported in other material platforms [16], further motivating our shift of focus from monolayer to bilayer. Figure 5(b) displays the corresponding dS/dN extracted from these measurements, alongside data for $\nu_0 = -1 + 2/3$ and $\nu_0 = 2/5$ in the same temperature range. These two additional states, with anticipated quasiparticle charges $\pm e/3$ and $\pm e/5$, respectively, offer useful comparisons to the $\pm e/4$ excitations of the $\nu = -1 + 1/2$ state. The measured thermodynamic gaps are also comparable: $\Delta\mu_{-1+2/3} = 1.3$ meV, $\Delta\mu_{-1+1/2} = 1.0$ meV, and $\Delta\mu_{2/5} = 1.5$ meV.

As with conventional FQH phases in Fig. 3, a sizable dS/dN appears near ν_0 , reflecting residual motional entropy of the low-density quasiparticle gas. Close to the $\nu = -1 + 1/2$ state, large dS/dN also appears in the vicinity of $\nu = -1 + 8/17$ and $\nu = -1 + 7/13$, consistent with the formation of Levin-Halperin fractional quantum Hall states [54]. These states emerge as interacting liquids of the $e/4$ quasiparticles from the $\nu = -1 + 1/2$ state and thus mark an upper limit on the anyon density at which the topological entropy can be detected. Similarly, the presence of the $3/7$ state sets a limit on how many charge- $e/5$ anyons can be added to the $2/5$ state before their interactions drive the formation of a new incompressible phase with entirely different elementary excitations. Consequently, if a density range exists where the topological entropy S_{topo} dominates, it would likely occur in the window $0.005 < |\nu - \nu_0| < 0.02$, where Coulomb repulsion can localize anyons into a Wigner crystal without fully lifting the topological degeneracy.

Over the temperature range $T = 100$ – 300 mK, the $\nu = -1/2$ data do not show a clear plateau at S_{topo} . Nonetheless, dS/dN near $\nu = -1/2$ exceeds that of both $\nu = -1 + 2/3$ and $\nu = 2/5$, remaining above S_{topo} for all $|\nu - \nu_0| < 0.016$. These observations leave open the possibility of detecting non-Abelian entropy at lower temperatures, while indicating that motional entropy still plays a significant role under current experimental conditions. In line with the integer WC results, we observe $dS/dN \gtrsim 0$ in the Abelian states and $dS/dN > dS_{\text{topo}}/dN$ near $\nu = -1/2$, suggesting that $T \gtrsim T_D$. Reaching $T < T_D$ may thus be necessary to fully isolate S_{topo} , which would provide an unambiguous thermodynamic signature of non-Abelian anyons.

In order to assess whether future experiments at lower temperatures and higher sensitivity could isolate S_{topo} , we compare our results with theoretical modeling of $S(\nu, T)$. In the vicinity of the $\nu_0 = -1 + 1/2, -1 + 2/3$, and $2/5$ states, we assume fractional quasiparticles crystallize into a triangular lattice WC, enabling us to compute the entropy via the framework outlined in the previous section. For $\nu - \nu_0 = 0.01$ and $B = 14$ T, the lattice spacing a reaches $\sim 13\ell_B \approx 90$ nm, where exchange effects are negligible [5,6]. At higher densities $\nu - \nu_0 \gtrsim 0.02$, exchange effects become non-negligible, leading to the formation of new FQH liquids; the WC calculation is thus not expected to

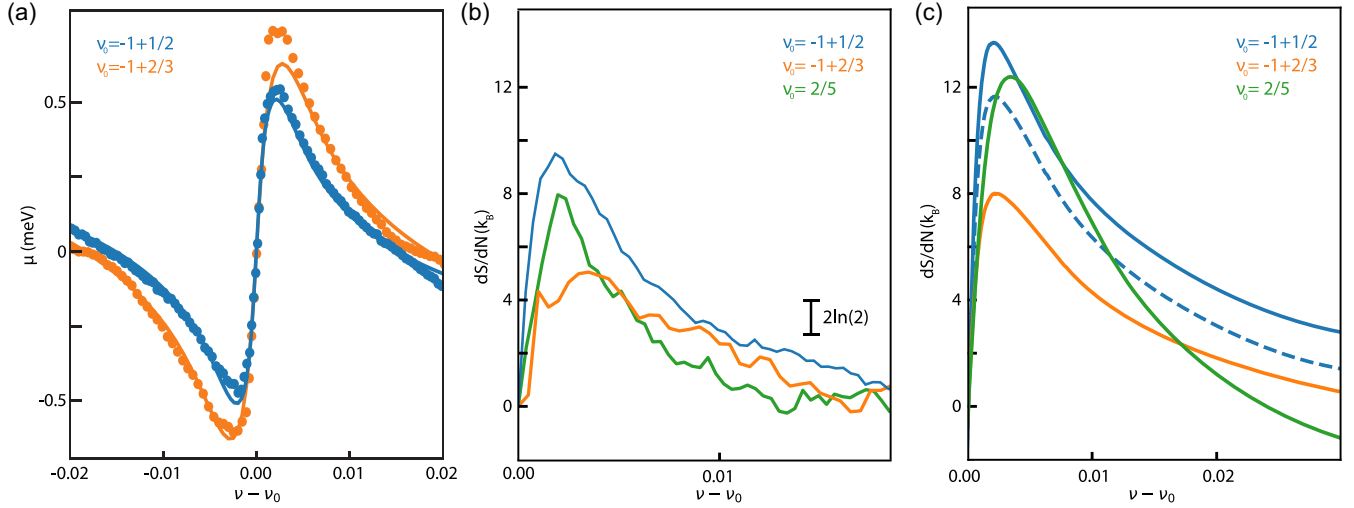


FIG. 6. Comparison of theoretical bilayer graphene FQH Wigner model with experiment. (a) Wigner crystal model fit to the $T = 100$ mK chemical potential data, from which model parameters—including the gap size Δ , screening parameter b , and disorder broadening Γ —were extracted. Dots indicate the experimental data, and solid lines represent the model fit. (b) Experimental dS/dN data from Fig. 5, antisymmetrized about ν_0 . (c) Theoretically predicted Wigner crystal entropy in the vicinity of different fractional fillings. For the $\nu_0 = -1 + 1/2$ curve, the non-Abelian contribution to the entropy [6] is either accounted for (solid), or neglected (dashed).

hold in this regime. At lower densities, disorder must be incorporated into the model to quantitatively capture the behavior. We identify three distinct ways in which disorder influences $\mu(\nu)$ and, consequently, dS/dN . First, the long-wavelength component of the disorder potential causes inhomogeneous broadening: even within the gap, $\pm e^*$ excitations exist in the system, thus rounding the jump in μ . We account for this broadening in the same manner as Ref. [16], treating its width Γ as a phenomenological parameter. By fitting $\mu(T = 100 \text{ mK}, \nu)$ near $\nu_0 = -1 + 1/2, -1 + 2/3$ to the fractional Wigner crystal model, we fix Δ , b , and Γ as shown in Fig. 6(a). The fitting procedure and resulting parameters are discussed in Supplemental Material [17]. Since the same bilayer graphene device was used in Ref. [16], we adopt identical parameters here. Next, the inhomogeneous broadening also complicates the extraction of S_{topo} and $\mathcal{D}$. The total density change dN reflects the difference between the number of $+e^*$ and $-e^*$ anyons, whereas S_{topo} depends on their sum. Consequently, if disorder induces quasiparticles in some regions and quasiholes in others, ΔN may not track the true change in the total number of excitations N_a . This mismatch remains negligible for $|\nu - \nu_0| \gtrsim 0.005$ but can matter closer to ν_0 [17]. Finally, short-range disorder can pin the Wigner crystal, generating a “pinning gap” T_{pin} [49]. In principle, for $T < T_{\text{pin}}$, the WC would be immobilized and its entropy strongly reduced. However, lacking a microscopic understanding of the disorder distribution and any direct experimental evidence for such a gap, we adopt the minimal assumption $T_{\text{pin}} = 0$.

With the theoretical parameters thus fixed by experiment, we may compare the predicted dS/dN , shown in

Fig. 6(c), with the experimental data in Fig. 6(b). To facilitate this comparison, we antisymmetrize the experimental dS/dN with respect to a particle-hole transformation about each ν_0 . We find overall qualitative agreement between theory and experiment: the relative magnitudes of dS/dN near the $\nu_0 = -1/3, 2/5$, and $-1/2$ states match well, the predicted crossing of the $-1/3$ and $2/5$ curves is observed, and dS/dN remains positive throughout the range examined. Figure 6(c) also shows theoretical results for $\nu_0 = -1/2$ both with and without the topological contribution dS_{topo}/dN . As the experimental data lack the requisite precision to distinguish these two cases, we cannot definitively conclude whether S_{topo} has been observed.

Our benchmarked theoretical model can be used to evaluate how future experiments might unambiguously detect S_{topo} . Figures 7(a) and 7(b) show dS/dN and S/N computed for $\nu - \nu_0 = 0.02$ for $\nu_0 = -1/2$ with and without screening effects and in the absence of disorder. Similar calculations for $\nu_0 = 1/3$ are also shown for comparison. Screening reduces T_D by roughly a factor of two. As a result, although T_D provides a useful baseline to estimate the temperatures required to suppress WC excitations, more refined criteria can be developed.

A useful temperature scale is defined by the crossover temperature T^* , where the entropy due to magneto-phonon modes equals the topological entropy $S_{\text{WC}}(T^*) = S_{\text{topo}}$. Below this temperature, the topological entropy comprises the majority of the total entropy. Measurements in the $T < T^*$ regime might hope to infer S_{topo} by extrapolation. As shown in Fig. 7(c), empirically $T^* \approx 0.1T_D$, well below 100 mK for the experimentally relevant range.

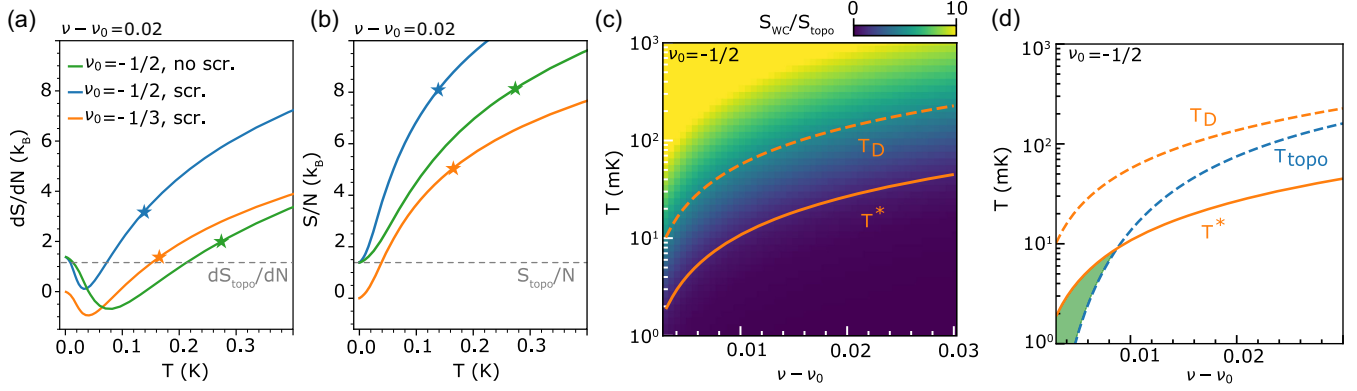


FIG. 7. Theoretically determined Wigner crystal entropy for bilayer graphene at $B = 13.8$ T. (a) Temperature dependent dS/dN and (b) S/N for $\nu - \nu_0 = 0.02$ near $\nu_0 = -1/2$ and $-1/3$ in different theoretical models described in the main text. The $\star$ symbols indicate the Debye temperature T_D for each curve, while the horizontal dashed line represents the expected topological contribution near $\nu_0 = -1/2$. (c) Ratio of Wigner crystal entropy S_{WC} and S_{topo} , plotted over the experimentally relevant range of $\nu - \nu_0$ and T . T^* , defined as the temperature where $S_{\text{WC}} = S_{\text{topo}}$, as well as T_D , are shown in overlay. (d) Comparison of T_D , T^* , and T_{topo} , the scale characterizing splitting of the topological degeneracy by anyon interactions. The desired $T_{\text{topo}} < T < T^*$ regime is indicated by the green shading.

Even if temperatures below T^* become experimentally accessible at moderate $\nu - \nu_0$, one encounters a second constraint: at larger $\nu - \nu_0$, the quasiparticle interaction energy may split the topological degeneracy, introducing an additional scale T_{topo} . To observe S_{topo} , the experiment must then satisfy $T_{\text{topo}} < T < T^*$. Although T_{topo} has not been directly measured, theoretical estimates based on model wave functions and unscreened Coulomb interactions suggest $k_B T_{\text{topo}} \sim 0.01 E_C e^{-a/\xi}$, where $E_C = e^2/(4\pi\epsilon\ell_B)$ is the Coulomb scale, a is the interanyon spacing, and $\xi \approx 2.3\ell_B$ is the neutral-fermion correlation length [55]. As shown in Fig. 7(d), this estimate indicates $T_{\text{topo}} < T^*$ only for $|\nu - \nu_0| < 0.01$, where T^* can fall below 10 mK. While these numbers appear challenging for near-term experiments, the same screening effects that reduces the WC energy scale very likely lower T_{topo} as well. However, the quantitative impact of screening on T_{topo} is not well established, and systematic numerical studies of realistic interaction potentials are needed to clarify the likely size of the temperature window.

Looking ahead, the primary obstacle to accessing the low- T regime in the current experiment is the resolution with which μ can be determined. Since entropy is obtained by differentiating the chemical potential with respect to temperature, the temperature difference δT must be large enough to produce a detectable change $\delta\mu$ that exceeds both systematic and random noise. This requirement sets a minimum δT of about 200 mK in our measurements. Overcoming this limitation will likely demand a more sensitive transducer than the graphene sensor layer; for instance, single-electron transistors, with input voltage noise in the $nV/\sqrt{\text{Hz}}$ range [56], may enable sufficiently precise detection of μ . One can also consider modifications to the sample which might help quench the motional

entropy. Disorder might accomplish this by generating a pinning gap T_{pin} , which our theoretical modeling has neglected. Alternatively, fabricating a small quantum dot could allow a single non-Abelian anyon to be localized [57]. Our present modeling indicates that, at least in an idealized (disorder-free) system, the energy scales needed to suppress Wigner-crystal entropy remain experimentally challenging, thereby motivating further studies of deliberate confinement.

VII. CONCLUSION

We have explored the entropy of monolayer and bilayer graphene in the quantum Hall regime across four orders of magnitude in temperature, ranging from 200 mK to 300 K. The entropy exhibits a clear energy hierarchy, with different physical phenomena dominating at distinct temperature scales. At 100 K, the occupational entropy of four-flavor electrons is dominant, forming a characteristic dome across the zero energy Landau level. As the temperature decreases, various correlated states emerge in sequence. At $T \approx 50$ K, quantum Hall ferromagnetism becomes dominant, splitting the zero energy Landau level into four distinct domes arising from the motional entropy of single-flavor electrons. At integer fillings in this temperature regime, thermally excited magnons become the primary contributors to the entropy, whose detailed temperature dependence provides entropic evidence for Kekulé or antiferromagnetic order at $\nu = 0$. As the temperature is further reduced to $T \approx 1$ K, charge order begins to manifest at small doping levels relative to integer fillings, indicated by deviations in the filling dependence from that of the full configurational entropy. The fractional quantum Hall states are revealed as dips in the entropy at fractional fillings. Using a free composite fermion model, we qualitatively reproduce the

observed entropy patterns. To probe the entropy associated with the quantum degeneracy of non-Abelian anyons, we focus on the entropy per electron at finite doping near the Pfaffian state at $\nu = -1/2$. At low doping, the entropy is dominated by density modes, while at higher doping, we observe excess entropy near $\nu = -1/2$ compared to the Jain fractions. We interpret this excess as a potential signal of the entropy associated with non-Abelian anyons. However, more precise measurements at lower temperatures and with higher sensitivity are needed to confirm this interpretation.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [58], embargo periods may apply.

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