

Observation of a Halo Trimer in an Ultracold Bose-Fermi Mixture

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The quantum mechanics of three interacting particles gives rise to fundamental and universal phenomena, such as the staircase of Efimov trimers predicted in the context of nuclear physics and observed in ultracold gases. Here, we observe a novel type of halo trimer using radio-frequency spectroscopy in an ultracold mixture of ^{23}Na and ^{40}K atoms. The trimers consist of two light bosons and one heavy fermion, and they have the structure of a Feshbach dimer weakly bound to one additional boson. We find that the trimer peak closely follows the dimer resonance over the entire range of explored interaction strengths and across an order-of-magnitude variation of the dimer energy, as reproduced by our theoretical analysis. The presence of this halo trimer is of direct relevance for many-body physics in ultracold mixtures and the association of ultracold molecules.

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I. INTRODUCTION

Three-body bound states in quantum mechanics have a rich history, including the molecular hydrogen ion [1,2], the nuclei of tritium and ^3He [3,4], and the helium trimer [5,6]. The three-body problem is of fundamental theoretical interest since it displays phenomena that are markedly different from the physics of just two particles. For pure contact interactions, Thomas predicted a collapse [7] since the effective potential—an inverse square law in the hyper-radius—supports infinitely deep bound states. Efimov studied particles in the regime of large two-body scattering lengths and found a universal “staircase” of trimer bound states, characterized by a constant ratio between successive energy levels [8].

Efimov’s scenario, originally proposed for nuclear physics, was studied in detail both experimentally and theoretically in the field of ultracold gases [9]. Efimov states were first observed in a gas of bosonic cesium [10], and

later, several successive states were demonstrated [11–14]. The majority of observations were via particle loss at particular values of the scattering length. Exceptions are studies of three distinguishable fermions of ^6Li [15,16] and three identical ^7Li bosons [17], where Efimov states were directly observed in radio-frequency spectra, allowing the determination of binding energies.

The properties of the Efimov trimers greatly depend on the relative masses of the involved particles, especially when not all mutual two-body interactions are large [9]. For one light atom interacting with two heavy ones, the Efimov effect becomes more prominent: Successive energy levels are more closely spaced and easier to observe [13,14]. For the opposite heavy-light-light combination that we study here, experimental observation of even the first Efimov resonance is nearly impossible since its scattering length a_- closely approaches unitarity. Excited Efimov states only appear exceedingly close to unitarity and at vanishing binding energies.

However, here we show that a “halo” trimer [18] can still exist over a wide and accessible range of positive scattering lengths. We directly observe such a halo trimer bound state using rf spectroscopy as illustrated in Fig. 1. The trimer is formed by two light ^{23}Na bosons surrounding a heavy “impurity” atom, fermionic ^{40}K . The fermion interacts resonantly with the bosons near a Feshbach resonance, with scattering length a , while the bosons weakly repel

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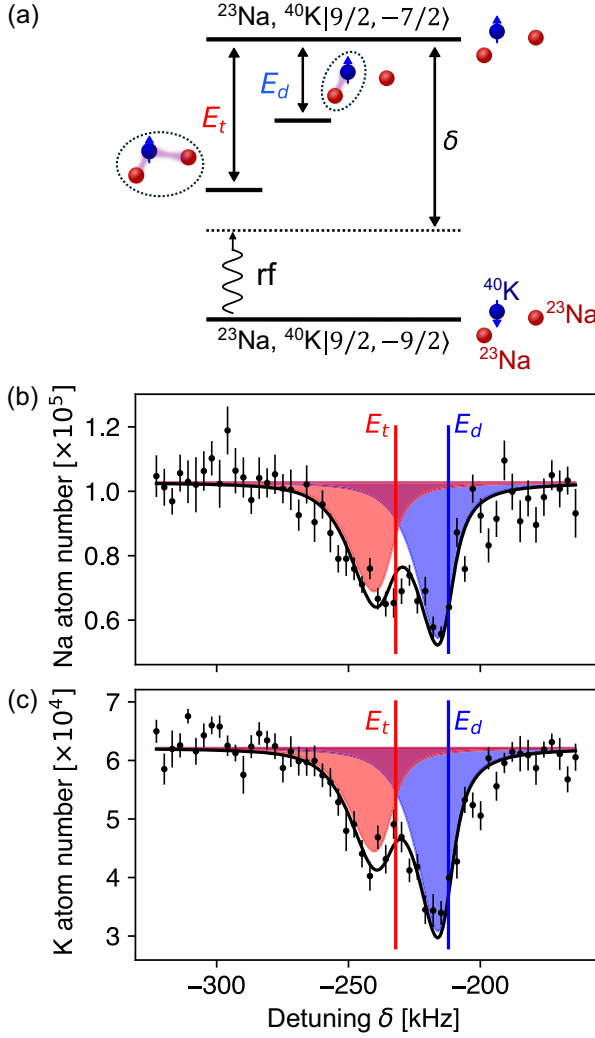


FIG. 1. Detection of halo trimers using radio-frequency spectroscopy. (a) Schematic representation of free-to-bound transitions into Na-K dimer and Na_2 -K trimer states with binding energies E_d and E_t , respectively, red-detuned from the bare atomic hyperfine transition. (b),(c) Depletion spectra taken at 101.8 G, where losses due to resonant coupling to the dimer state and trimer state are present in (b) Na and (c) K. The underlying dimer and trimer fitted line shapes are shown as blue and red shaded regions, respectively, with solid lines marking the binding energies and solid curves showing the summed line shape. Error bars show sample standard deviations obtained from typically 12 experimental repetitions per detuning.

each other (Bose-Bose scattering length $a_{\text{BB}} = 56.5a_0$, based on Ref. [19]).

We may understand the existence of this weakly bound trimer state by first considering the extreme limit of an infinitely heavy impurity. For positive scattering length, a dimer bound-state exists at energy E_d . In the absence of interactions between the light bosons, the impurity simply acts as an external potential in which both bosons can occupy the same energy level, giving this trimer a binding energy of $2E_d$. When bosons instead repel at short range,

they cannot simultaneously occupy the bound state, and the trimer energy is suppressed. Universal properties of trimers and larger clusters in this scenario have been studied in Refs. [20,21] for an effective three-body repulsion, finding that a universal trimer state always persists at least for near-unitary interactions.

In our case, the fermionic ^{40}K atom is only about twice as heavy as the boson, but in combination with the modest interboson repulsion, this mass imbalance is enough to yield a scenario qualitatively similar to the infinite mass case. The trimer bound state we observe has an energy close to E_d , implying that the second boson is, on average, found significantly further away from the heavy atom than the first. We can reproduce the measured binding energy curve of the trimer theoretically.

In Sec. II, we lay out the experimental methodology and the experimental evidence for the observation of the trimer state. In Sec. III, we discuss the properties of the trimer state—such as line shape, energy, and wave function—based on both experimental and theoretical analyses. After discussing the relevant loss processes and the relative dimer-trimer signal strengths (Sec. IV), we conclude our work in Sec. V.

II. EXPERIMENTAL OBSERVATIONS

We prepare a Bose-Fermi mixture of ^{23}Na and ^{40}K , trapped in an optical dipole trap with Na trapping frequencies $2\pi \times (18, 118, 125)$ Hz in their respective hyperfine ground states $|F=1, m_F=1\rangle_{\text{Na}}$ and $|F=9/2, m_F=-9/2\rangle_{\text{K}} \equiv |\downarrow\rangle_{\text{K}}$, with typical initial peak ^{23}Na densities of $0.4\text{--}0.88 \mu\text{m}^{-3}$ and ^{40}K densities of $1.1\text{--}2.4 \mu\text{m}^{-3}$. To probe the few-body bound states of interest, without complications due to many-body effects, we prepare the mixture in the nondegenerate regime at approximately 200 nK. Thus, for our densities, we have $T/T_C \sim 1.4\text{--}1.8$ and $T/T_F \sim 0.7\text{--}0.9$, where T_C is the critical temperature for Bose-Einstein condensation and T_F is the Fermi temperature. Although the peak density of ^{40}K is higher than that of ^{23}Na , the total atom number is 1.5–3 times larger for ^{23}Na because the ^{23}Na cloud is more spatially extended. Since the atom numbers are of the same order, we are able to observe the creation and subsequent loss of molecules in the population depletion of both species.

To measure the binding energies of the NaK dimer and Na_2K trimer states, we perform rf association spectroscopy by driving a free-to-bound hyperfine transition of K, from $|\downarrow\rangle_{\text{K}}$ to $|\uparrow\rangle_{\text{K}} \equiv |F=9/2, m_F=-7/2\rangle_{\text{K}}$, in the presence of Na, as shown schematically in Fig. 1(a) [22]. We work near a broad interspecies Feshbach resonance at about 110 G between $|F=1, m_F=1\rangle_{\text{Na}}$ and the final state $|\uparrow\rangle_{\text{K}}$ by applying magnetic fields from 100.5 G to 105 G (see Fig. 8). This approach changes the scattering length from approximately $600 a_0$ to $1600 a_0$, which allows us to measure the trimer's binding energy as it closely follows the dimer's over an order of magnitude. We then probe the

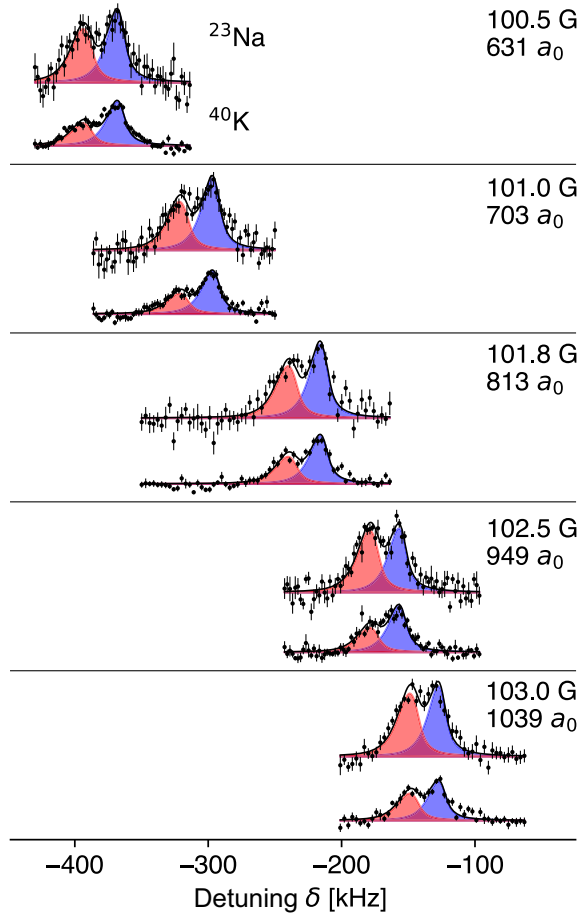


FIG. 2. Radio frequency association of dimers and trimers. Depletion spectra of Na (K), shown as the upper (lower) trace within each panel, taken at magnetic fields between 100.5 and 103 G, in proximity to the Feshbach resonance at 110 G. The trimer feature (red-shaded area) closely tracks the dimer (blue-shaded area) at each field. The initially prepared K:Na ratios prior to the rf pulse vary between data sets from 0.3 to 0.65. Error bars show sample standard deviations obtained from typically 12 experimental repetitions per detuning.

transitions via rf spectroscopy [23]. Subsequently, we detect the populations of Na and the two spin states of K using resonant absorption imaging. For the rf pulse, we choose a variable duration between 3 and 250 ms to keep loss fractions similar at the various fields. Strong losses in the initial-state populations indicate the presence of resonant rf coupling to a bound state.

The initial interspecies interaction between Na and K in the initial hyperfine state $|\downarrow\rangle$ is strongly attractive, ranging from $-1400 a_0$ to $-1100 a_0$. This attraction enhances two-body correlations and, therefore, wave-function overlap to the bound states of interest, facilitating their observation.

Radio-frequency loss spectra taken at 101.8 G are shown in Figs. 1(b) and 1(c). The presence of the trimer state is established in the observed double-peak structure, featuring an extra peak below the dimer association line. Figure 2 shows a compilation of rf depletion spectra at various

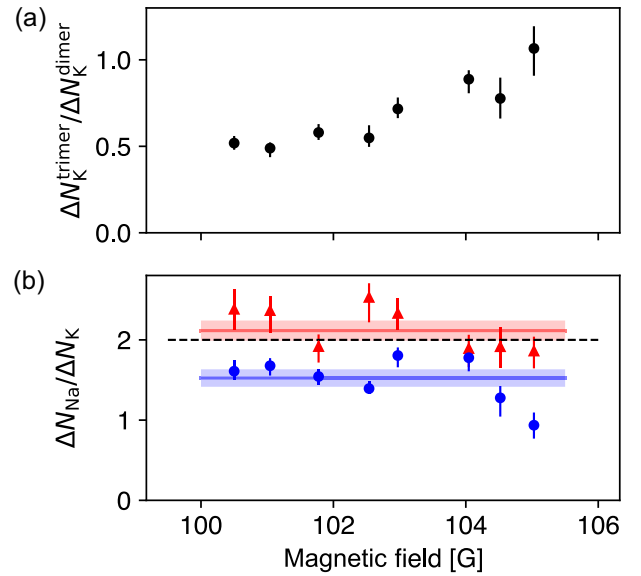


FIG. 3. (a) Ratio between the fitted trimer and dimer feature heights, in the K spectra. (b) Interspecies ratio of the fitted trimer (red triangles) and dimer (blue circles) peak depletions vs applied magnetic field. The trimer data cluster around a 2:1 Na-K loss ratio (dashed line), whereas the dimer data lie between 1:1 and 2:1. The solid lines (shaded regions) show the weighted mean and standard deviation of the ratios, across magnetic fields. Error bars indicate the 68% confidence intervals on fit parameters, as computed by bootstrapping.

magnetic fields, where, from top to bottom, the Feshbach resonance is approached. All the spectra clearly display the trimer peak below the dimer line. The trimer feature has comparable weight to the dimer peak for the whole range of magnetic fields. The relative peak weights as a function of magnetic field are shown in Fig. 3(a). We find that the trimer weight substantially increases as the resonance is approached, which can be understood from the decreasing relative binding energy leading to a larger wave-function extent and, therefore, Franck-Condon overlap (see Sec. III).

To fully understand our spectra, and to unambiguously establish the different nature of the dimer and trimer peaks, we need to consider not only the rf association processes but also possible loss processes. Indeed, without loss of the dimer and trimer populations, the signal detected in the Na imaging channel should remain unchanged. This is because our Na imaging probe is equally sensitive to the initial atom population and to the dimer and trimer populations since the binding energies of dimer and trimer states (about 100s of kHz) are small relative to the linewidth (about 10 MHz) of the Na optical transition used for imaging. This feature allows the near-resonant Na imaging light to scatter and dissociate the molecules.

The observed depletion of the Na atoms can be attributed to three-body loss processes: the relaxation of Feshbach dimers into a deeper rovibrational state upon collision or interaction with a third atom. The excess energy is released

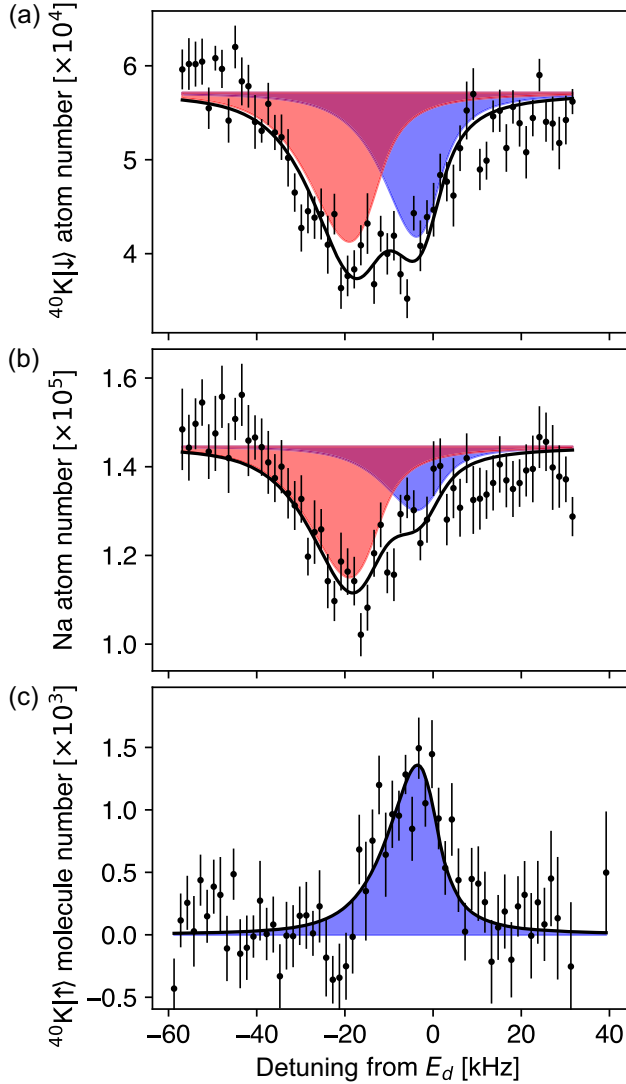


FIG. 4. (a),(b) Depletion spectra taken with a 30-ms Blackman-envelope rf pulse, at $B = 105$ G ($a \approx 1633 a_0$). The loss in Na population on the dimer feature is strongly suppressed here. (c) Arrival spectrum of K in the final hyperfine state, yielding a measure of the molecule number. Here, the rf pulse is a 3-ms square pulse, short enough to observe dimers within their lifetime. Only the dimer spectral feature is present, and no trimer arrivals are detected, indicating that the trimer lifetime is much shorter than the rf pulse duration. Error bars show sample standard deviations obtained from typically 12 experimental repetitions per detuning.

as kinetic energy, leading to the ejection of both the atom and the molecule from the trap [24]. For trimers, this process can happen internally, leading to relaxation into a deeply bound dimer and free atom ($\text{Na}_2\text{K} \rightarrow \text{NaK} + \text{Na}$). This result is consistent with our observation that the association and decay of trimers depletes the Na and K initial populations in a 2:1 ratio, as shown in Fig. 3(b). In contrast, the dimers can be lost through dimer-atom collisions with free atoms of either species (NaK-K and

NaK-Na collisions). The presence of these two loss mechanisms—with 2:1 and 1:2 ratios for Na to K depletion, respectively—is also consistent with the observed depletion ratio of less than 2:1 for the dimer feature.

The difference in the loss dynamics between dimers and trimers is most striking near the Feshbach resonance, as shown for $B = 105$ G in Fig. 4 (scattering length of approximately $1600 a_0$). Here, we see both the dimer and trimer features with roughly equal weights in the K depletion, whereas the Na depletion is strongly suppressed near the dimer resonance, indicating a lower rate of dimer-Na collisional loss.

A consequence of the longer dimer lifetime is that, near the Feshbach resonance, we can directly detect the surviving dimers, as shown in Fig. 4(c). For this purpose, the rf pulse duration is chosen comparable to the dimer decay time. Our measurement of the surviving dimer population after applying the rf pulse allows us to bound its $1/e$ decay time at greater than or approximately 1.4 ms at this magnetic field.

We cannot detect any arrival signal on the trimer feature within our accessible experimental parameter space and our signal-to-noise ratio (SNR), indicating a significantly shorter trimer lifetime. Given the length of the pulse and the SNR, we can estimate the trimer lifetime to be less than 1 ms. A lower bound of the lifetime is obtained from the width of the trimer spectral features, giving a lifetime of greater than or equal to $20 \mu\text{s}$. This finding is likely an underestimate of the lifetime, however, since there are known factors such as technical noise that also contribute to broadening.

In conclusion, the difference in lifetimes close to the resonance, the different binding energies, and the Na/K loss ratios are three physical features that allow us to clearly distinguish between the products, the dimer and trimer, created with the rf drive.

III. TRIMER PROPERTIES

A. Line shapes

Having established the existence of the halo trimer experimentally, we now study its properties with the help of a theoretical analysis. To obtain the dimer and trimer binding energies from the data, a good knowledge of the dimer and trimer line shapes is required. In previous works on the ^6Li trimer, the trimer line shapes were fitted using phenomenological fit functions such as Lorentzian or Gaussian line shapes [15,16]. Here, we attempt to determine the energy more accurately by employing line shapes based on theoretical modeling. The total line shape of the rf-association feature can be modeled as the weighted sum of the dimer and trimer lines. Both lines are intrinsically asymmetric due to the finite temperature and the free-to-bound nature of the rf transition.

In a thermal gas, the dimer line shape in the universal limit is well known [25], and it is given by (setting $\hbar = 1$)

$$F_d(\omega) \sim \frac{2}{\pi} \frac{k\kappa_d(1 - \kappa_d a_i)^2}{(k^2 + \kappa_d^2)^2 (a_i^2 k^2 + 1)} \exp\left(-\frac{E}{k_B T}\right), \quad (1)$$

where ω is the transition frequency and $\hbar\omega = E_d - E$. We further define the initial scattering length a_i , temperature T , the initial-state relative momentum $k = \sqrt{2\mu_{\text{NaK}}E}$ (with the Na-K reduced mass μ_{NaK}), and the dimer momentum scale $\kappa_d = \sqrt{2\mu_{\text{NaK}}|E_d|}$.

The trimer line shape cannot be captured in a simple analytical form, but the low-energy threshold behavior can be deduced straightforwardly. We first briefly revisit the dimer case. The two-particle s-wave scattering wave function $f_d(k, r)$ is given by

$$f_d(k, r) = \sqrt{\frac{2\mu_{\text{NaK}}}{\pi k}} \sin(kr + \delta_k), \quad (2)$$

where r is the interparticle distance and δ_k is the phase shift, which, for short-range interactions, is given by

$$\lim_{k \rightarrow 0} k \cot(\delta_k) = -\frac{1}{a} + \frac{r_{\text{eff}} k^2}{2} + \mathcal{O}(k^4) \quad (3)$$

for the effective range r_{eff} . Importantly, δ_k is linear in k for small k . The Franck-Condon factor determining the line shape is now given by the overlap of the scattering wave function f_d with the localized bound-state wave function ψ_d . This argument yields

$$\lim_{k \rightarrow 0} \left| \int_0^\infty dr \psi_d^*(r) f_d(k, r) \right|^2 \sim k \sim \sqrt{E}, \quad (4)$$

which we recognize from Eq. (1). In the experimentally relevant regime, the dimer is strongly bound compared to the initial thermal energy, so the line shape is well approximated by $F_d \sim \sqrt{E} \exp(-E/k_B T)$. The momentum dependence of the denominator in Eq. (1) plays a minor role at the experimental temperature, setting the prefactor of the expression to ensure the correct normalization of the wave function.

To consider the threshold scaling for the trimer wave function, we employ the adiabatic hyperspherical approach (see Appendix A 1). One can show that the three-body analog of the s-wave scattering wave function is asymptotically given by

$$f_t(K, R) \rightarrow \sqrt{\mu R} [\cos \Delta_K J_2(KR) - \sin \Delta_K Y_2(KR)], \quad (5)$$

where μ is the three-body reduced mass, R and K are the hyperradius and the hyperradial momentum ($K = \sqrt{2\mu E}$ for collision energy E), $J_2(KR)$ and $Y_2(KR)$ are regular and irregular Bessel functions of second order, and Δ_K is the three-body phase shift. The noninteracting solution

$J_2(KR)$ scales as K^2 for $KR \rightarrow 0$, but in the presence of interactions the analysis is complicated by the fact that the adiabatic potential energy curve asymptotically scales as R^{-3} [26]. This results in a threshold law $\tan(\Delta_K) \sim K$, so the second term in Eq. (5) scales differently. However, the leading-order phase shift occurs in the long range, and the wave function in the region overlapping with the bound state is suppressed by the centrifugal barrier in the hyper-radial potential [see Eq. (A6)]. As a result, the relevant short-range wave function still scales as K^2 [27], and the overlap between the scattering states and the localized trimer wave function ψ_t thus scales as

$$\lim_{K \rightarrow 0} \left| \int_0^\infty dr \psi_t^*(R) f_t(K, R) \right|^2 \sim K^4 \sim E^2. \quad (6)$$

The threshold behavior is therefore markedly different from the dimer case, leading to a larger temperature shift of the trimer peak than the dimer peak (in the low-temperature limit). The different energy dependence of the trimer line shape can be understood from a phase-space argument: For the low-energy contributions to the line shape, all the particles in the bound state need to be simultaneously at a small momentum, which is much less likely for the trimer than the dimer.

Whereas the dimer is strongly bound compared to the thermal energy scale, this is no longer true for the halo trimer. Here, the binding energy of the third atom to the dimer is comparable to the thermal energy. Hence, it is important to consider the leading-order corrections to the threshold scaling. Tscherbul and Rittenhouse [28] derive a trimer line shape for Efimov states close to the atom-dimer threshold, and for $Ka \ll 1$,

$$F_t(\omega) \sim \frac{(2\kappa_t^2 + K^2 - 2\kappa_t \sqrt{\kappa_t^2 + K^2})^2}{K^4} \exp\left(-\frac{E}{k_B T}\right), \quad (7)$$

where $\omega = E_t - E$ and $\kappa_t = \sqrt{2\mu(E_d - E_t)}$. Such Efimov states consist of a dimer weakly bound to an additional atom, and therefore, we expect a similar line shape in the present case of the Na₂K trimer. We indeed find good agreement of this line shape with the result from our hyperspherical calculations for the relevant low-energy regime. We do not account for the initial-state scattering length in the line shape because it is small compared to the thermal de Broglie wavelength. While the line shape itself is mostly unaffected by the attraction in the initial state, the strength of the signal is substantially enhanced by it.

For the final fit function, we convolve the above-mentioned line shape with a 8–10-kHz FWHM Lorentzian kernel and observe good agreement with the data, as can be viewed in Figs. 1, 2, and 4. The width of the experimental broadening is comparable to the intrinsic linewidth of the dimer and trimer features. The predominant source of broadening is given by magnetic field noise. A measure of

this noise is the width of the atomic transition, which is about 5 kHz. The dimer and trimer peaks are wider, pointing at additional broadening, which we expect to be predominantly caused by the time variation of the density profile and the temperature caused by the rf association and loss. The trimer and dimer lines further experience many-body shifts and broadening, which we expect to be of the order of a kHz.

B. Trimer binding energy

The fitted dimer and trimer binding energies are summarized in Fig. 5(a). Their spacing, shown in Fig. 5(b), is a small fraction (5%–20%) of the dimer binding energy across all our measurements. This finding strongly contrasts with the simplified fictitious case discussed in the introduction of a trimer state where an infinitely heavy impurity binds two noninteracting bosons, thus having a binding energy twice that of the dimer. The impurity having a finite mass only lowers the energy compared to the infinite-mass case due to the Efimov interaction mechanism [9,29], meaning that the suppression of the trimer energy is predominantly due to the interboson repulsion.

Interestingly, the interboson repulsion does not have to be resonant to suppress the trimer binding energy. In our case, the modest interboson repulsion of approximately $56.5 a_0$ already has this effect, even though the boson-impurity scattering length is more than an order of magnitude larger (ranging from 600–1600 a_0 for experimental parameters). As discussed in Refs. [20,21], multichannel effects can also give rise to an additional effective repulsion, but their influence is strongest for narrow resonances. In these works, the same qualitative picture of the trimer state lying slightly below the dimer was predicted.

We theoretically compute the dimer and trimer energies based on input from realistic Na-K and Na-Na interaction potentials [19,30] (see Appendixes A 1 and A 2 for more details). In short, we construct real-space model potentials using Gaussians, which match the scattering length and effective range of the realistic interaction potentials. All the parameters are chosen without fitting or matching to the experimental data. We find that, for the magnetic fields under consideration, the theoretical dimer energy is accurately represented by the universal expression

$$E_d = -\frac{1}{2\mu_{\text{NaK}} r_{\text{eff}}^2} \left(1 - \sqrt{1 - \frac{r_{\text{eff}}}{a}} \right)^2, \quad (8)$$

where a and r_{eff} both depend on B . The effective range varies approximately from 100 a_0 to 125 a_0 over the experimental magnetic field range, which gives corrections to the dimer energy up to 20% for $B = 100.5$ G compared to the leading-order term $(2\mu_{\text{NaK}} a^2)^{-1}$. The next order correction is about 1 kHz at this magnetic field and much smaller closer to the resonance.

The theoretical dimer and trimer energies are plotted along the experimental data in Figs. 5(a) and 5(b). From Fig. 5(a),

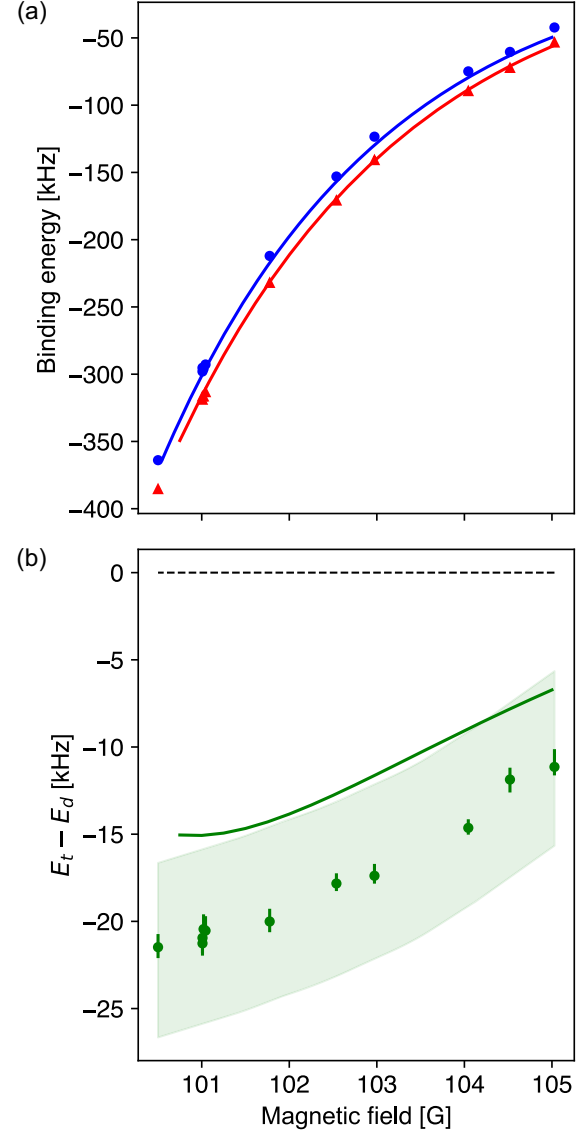


FIG. 5. (a) Measured binding energies of the dimer (blue) and trimer (red) vs applied magnetic field. Error bars showing 68% confidence intervals are found by bootstrapping, and they lie within the markers. (b) Difference in measured binding energies of the dimer and trimer (green circles) along with the theoretical prediction (solid line). The shaded region shows the FWHM of the Lorentzian convolution kernel used to fit the line shapes, setting a frequency scale for systematic error of the fit model. Error bars indicate the 68% confidence intervals on fit parameters, as computed by bootstrapping.

we see that the theoretical dimer and trimer energies agree with the experimental results up to 5–10 kHz. However, in the difference plot, we see that the theoretical dimer-trimer splitting is clearly smaller than the splitting obtained from the fits to the experimental data. Since the absolute difference is on the order of 5 kHz, which is smaller than the experimental linewidth, there are a variety of explanations for this difference.

In the line-shape model, we have not accounted for many-body energy shifts of the lines. Because of the shallow trimer state, the atom-dimer scattering length is large and positive, which leads to a shift of the dimer line to smaller binding energies. The trimer-atom scattering length is not known. However, if it is negative, it will downshift the trimer line, and if it is positive, there exist larger bound states below the trimer that we do not resolve. In both cases, the spectral weight would shift to larger binding energies. Consequently, we expect these many-body effects to be responsible for an increase in the dimer-trimer splitting fitted from the experimental data on the kHz level as compared to the theoretically computed (few-body) energy splitting.

Other effects, such as saturation, also have the tendency to increase the observed dimer-trimer splitting. Taking this into consideration, the theoretical results are consistent with the experiment.

C. Trimer structure

Since the theoretical trimer energy is consistent with the experiment, we can use the theoretical model to further analyze the internal structure of the trimer, as shown in Fig. 6(a). This figure shows the numerical trimer wave-function amplitude for the magnetic field of 103 G as a function of the two Na-K distances r_1 and r_2 . Only the s-wave component (independent of the angle between r_1 and r_2 , see Appendix A 3) is shown, which constitutes more than 99% of the wave-function norm. From the plot, it is clear that the trimer can be viewed as consisting of a tightly bound dimer state, with a weakly bound additional third particle.

Inspired by this result, we construct a model wave function of the form

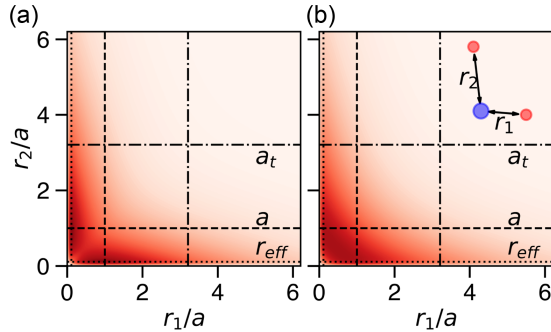


FIG. 6. Trimer wave-function amplitude $\psi_t(r_1, r_2)$ as a function of the two Na-K distances r_1/a and r_2/a at $B = 103$ G in units of the scattering length. (a) Full numerical wave function. (b) Result of the model from Eq. (9). The horizontal and vertical lines show the relevant length scales in the model: the Na-K scattering length a (dashed), the trimer size $a_t = 1/\kappa_t$ (dash-dotted), and the Na-K effective range r_{eff} (dotted).

$$\psi_t(r_1, r_2) = \frac{1}{\sqrt{2N}} [\psi_d(r_1)\psi_{ad}(r_2) + \psi_{ad}(r_1)\psi_d(r_2)]. \quad (9)$$

Here, N is a normalization factor, ψ_d is the dimer wave function, and ψ_{ad} is the atom-dimer wave function specifying the behavior of the third particle. Both ψ_d and ψ_{ad} can be modeled using a similar functional form [31]

$$\psi_{d/ad} \sim \left[\exp(-\kappa_{d/t} r) - \exp\left(-\frac{r}{\rho_{d/t}}\right) \right], \quad (10)$$

where

$$\rho_d = \frac{3a}{8} - \frac{a}{8} \sqrt{9 - 16 \frac{r_{\text{eff}}}{a}}. \quad (11)$$

The equation for ρ_t is the same upon replacement of the Na-K scattering length and effective range by their atom-dimer equivalents. The first term in Eq. (10) is the asymptotic solution outside the range of the potential, whereas the second term captures a suppression of the wave function at short range. The precise form of this term is less important, as another term with a similar magnitude and extent would capture the physics equally well. Note that the κ_t in this equation is defined using a different reduced mass than in Eq. (7)—namely, μ_{NaK} instead of the three-body reduced mass μ —because, here, the spatial coordinate r we are using corresponds to the boson-impurity distance instead of the hyperradius. The parameter $\rho_{d/t}$ characterizes the range at which the short-range wave function is suppressed. In particular, $\rho_d \approx r_{\text{eff}}/3$ for $r_{\text{eff}} \ll a$.

We find that a wave function of the form of Eq. (9) has an overlap of around 98% with the full numerical trimer wave function for the whole range of magnetic fields probed in the experiment. The atom-dimer scattering length and effective range entering in ρ_t are computed using the adiabatic hyperspherical approach. The result from this construction for $B = 103$ G is shown in Fig. 6(b). For large values of either r_1 or r_2 , the results in Figs. 6(a) and 6(b) match well. Since this is where most of the weight of the wave function is located, it leads to the large overlap that we find. However, as may be expected, the simplified wave function does not fully capture the short-range details of the wave function for $r_1, r_2 < a$. In particular, the trimer wave function in panel (a) shows a stronger suppression along the diagonal due to the interboson repulsion, which is not directly captured in panel (b).

The simplified wave function makes the interpretation of the trimer structure and its relevant length scales easier since they can directly be read off from Eq. (10). These relevant length scales are indicated by the horizontal and vertical lines in Fig. 6. The extent of ψ_d is set by the dimer binding energy, where for $B = 103$ G, $1/\kappa_d \approx 1.06a$. Similarly, the size of the trimer is set by $1/\kappa_t \equiv a_t$ (note that a_t is not a scattering length). The peak of the dimer

wave function is approximately set by the effective range r_{eff} . From Fig. 6, we see that the peak of the trimer wave function lies around a , showing that the atom-dimer effective range is approximated well by the Na-K scattering length. We find this to be generally true over the magnetic field range we study.

D. Universality of the halo trimer

An important open question is to which degree the properties of the observed halo trimers are universal. The trimer we observe exists outside the regime where the universal Efimov scaling relations apply. However, since the trimer is much more extended than the range of the potential, it should still exhibit short-range universality: The trimer properties should be insensitive to the microscopic details of the potentials and only depend on a few emergent parameters. For example, in the context of Efimov physics, a single emergent property called the three-body parameter fixes the entire Efimov spectrum [9]. Even though the typically chosen three-body parameter a_- almost diverges in our case, we hypothesize that an alternative three-body parameter fixing the trimer properties as a function of the Na-K scattering length still exists.

The three-body parameter is determined by the mechanism preventing the three particles from reaching the short range, stabilizing against the Thomas collapse [7]. Homonuclear Efimov trimers are stabilized by an effective three-body repulsion originating from the strong van der Waals attraction and the presence of many bound states in the potential [32]. Hence, barring some exceptions [33], the three-body parameter is found to be proportional to the van der Waals length [9,34]. In heteronuclear systems, the nonresonant interaction potential is also known to be important [35], but the parameter regime we study here has not been explored yet.

For the Na₂K halo trimer, the interboson repulsion is the predominant factor preventing stronger binding, and it is therefore expected to be the dominant scale setting the three-body parameter. The positive Na-Na scattering length originates from the quantum reflection off the van der Waals potential, which we mimic in our model potential with a strong repulsive core (see Appendix A 2). Thus, we can approximately capture the experimental trimer energies. The sensitivity of the three-body parameter to the shape of the potentials and multichannel effects, possibly explaining the difference between theory and experiment, is subject to further study [36].

IV. DISCUSSION: LOSS PROCESSES AND RELATIVE SIGNAL STRENGTHS

Since the formation processes of dimers and trimers require differing numbers of their constituent Na atoms, their association rates should depend differently on the

density of Na atoms. We have measured this dependence and shown the results in the spectra in Figs. 7(a) and 7(b).

To model the trimer formation, we use a simple rate equation with trimer formation coefficient Γ_t ,

$$\frac{dn_K}{dt} = \frac{1}{2} \frac{dn_{\text{Na}}}{dt} = -\Gamma_t n_K n_{\text{Na}}^2. \quad (12)$$

Here, the densities of K and Na are denoted by n_K and n_{Na} , respectively. We use the fact that the trimer decay rate is much faster than the association rate in our experiment.

The dimer formation and decay processes are modeled with the relevant two-body processes,

$$\frac{dn_K}{dt} = -\Gamma_d n_K n_{\text{Na}} - \Gamma_{d,K} n_d n_K, \quad (13)$$

$$\frac{dn_{\text{Na}}}{dt} = -\Gamma_d n_K n_{\text{Na}} - \Gamma_{d,\text{Na}} n_d n_{\text{Na}}, \quad (14)$$

$$\frac{dn_d}{dt} = \Gamma_d n_K n_{\text{Na}} - \Gamma_{d,\text{Na}} n_d n_{\text{Na}} - \Gamma_{d,K} n_d n_K, \quad (15)$$

where n_d denotes the density of dimers, Γ_d is the density-normalized rf association rate, and $\Gamma_{d,\{\text{species}\}}$ is the normalized loss rate due to atom-dimer collisions. Because $dn_d/dt \approx 0$ for most data sets in our experiment, we have a single timescale for population decay, set by the rate $\Gamma_d n_K n_{\text{Na}} \approx \Gamma_{d,\text{Na}} n_d n_{\text{Na}} + \Gamma_{d,K} n_d n_K$. We thus find that when fitting the rate coefficients, we are primarily sensitive to variations in Γ_d , which sets the timescale, and the ratio (but not the absolute values) of $\Gamma_{d,\text{Na}}/\Gamma_{d,K}$, which determines the ratio of population loss between species, as shown in Fig. 3(a).

Notice that for short rf pulse durations and small depletions $\Delta n/n \ll 1$ these two models predict the intuitive linear and quadratic Na density dependence we expect for the instantaneous formation rate of Na-K dimers and Na₂-K trimers, respectively. We find that each of these models captures the qualitative trends of the peak heights observed in experiment, with fitted coefficients $\Gamma_t/\Omega_{\text{rf}}^2 = 9 \times 10^{-9} \text{ s } \mu\text{m}^6$, $\Gamma_d/\Omega_{\text{rf}}^2 = 1 \times 10^{-8} \text{ s } \mu\text{m}^3$, and $\Gamma_{d,\text{Na}}/\Gamma_{d,K} = 2.5$, where Ω_{rf} is the bare atomic Rabi frequency. This is shown in the fit lines of Figs. 7(c) and 7(d), as well as the time dependence of population depletion with the rf pulse tuned to the dimer and trimer resonant features in Figs. 7(e) and 7(f).

However, not all observed effects can be explained by this simple rate equation model. First, we observe in Fig. 7(c) and 7(d) that the results with a different pulse length deviate from the expected curve. Second, the relative magnitude of the trimer peak compared to the dimer peak as shown in Fig. 3(a) is larger than the theoretical predictions. Experimentally the dimer-to-trimer peak ratio varies from one to three. However, in a homogeneous system, the trimer signal is theoretically expected to be 5–10 times lower than the dimer signal.

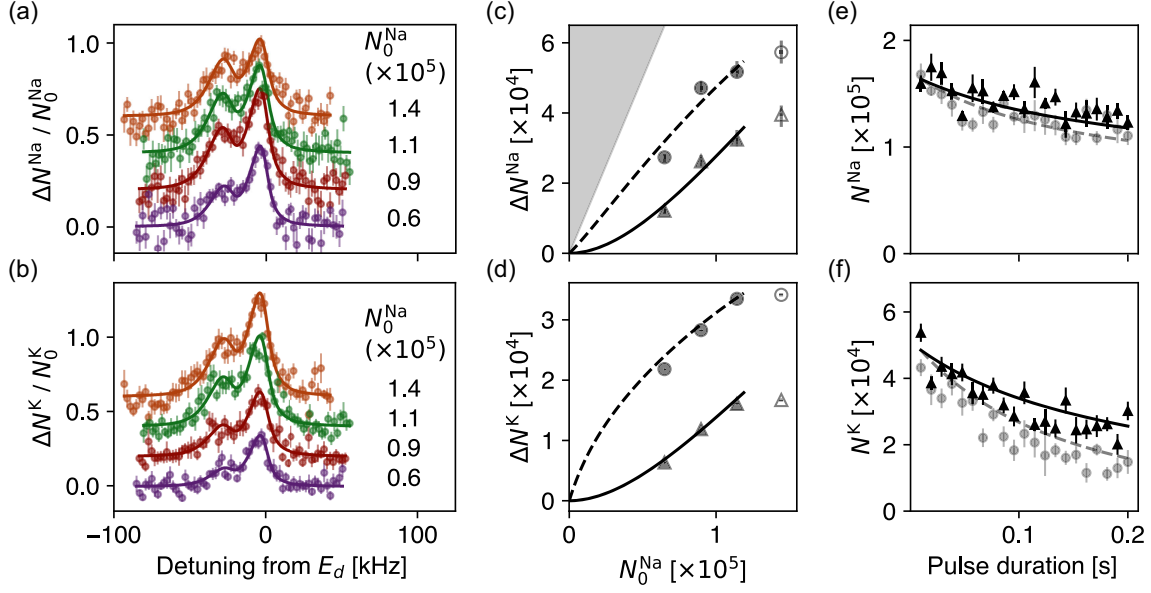


FIG. 7. Dimer and trimer spectra as a function of the Na density. (a),(b) Depletion spectra taken at 101 G, with initial corresponding peak Na density ranging from 0.4 to 0.88 μm^{-3} , shown as fractional loss of the initial population. Offsets have been added between spectra for visual clarity. (c),(d) Peak heights of the fitted underlying dimer (circles) and trimer (triangles) features vs initial Na density. Solid (dashed) lines show fits to the two- and three-body collision models described in Eqs. (12)–(14). The open-face markers represent the spectrum taken at the highest initial Na density, with a shorter rf pulse duration (100 ms, as opposed to 200 ms for the other spectra) to avoid unity depletion of the K population, and therefore deviate from the curve fits computed for 200-ms pulses. The gray shaded area in the upper plot represents an upper bound corresponding to maximum loss of Na. (e),(f) Initial-state population depletion vs duration of rf pulse, with rf frequency chosen on the peak of the dimer (trimer) feature, corresponding to the circular (triangular) markers. Solid (dashed) lines show the prediction using the models described in panel (b). Error bars in panels (a), (b), (e), and (f) show sample standard deviations obtained from experimental repetitions. Error bars in panels (c) and (d) indicate the 68% confidence intervals on fit parameters, as computed by bootstrapping.

Both of these effects can qualitatively be explained by saturation effects, heating, and trap dynamics. Especially since the peak K density is about 3 times higher than the peak Na density, the Na can be depleted in the center of the trap. Since the interspecies scattering length is large, the trap dynamics is expected to be diffusive. As a result, the repopulation of areas where the Na is depleted is slow, forming a limiting factor for the dimer and trimer association rates. The dimer creation rate is the most affected since (theoretically) the dimer association rate is much higher than the trimer association rate. In addition, heating will have a strong effect on the density profiles of both species. We explore the effects of the trap dynamics and heating on rf association experiments in future work.

Further from the resonance than the experimentally studied regime, the trimer is expected to disappear into the atom-dimer continuum [37], leading to an atom-dimer scattering resonance [38]. In another experiment using the same Na-K mixture, but a different Feshbach resonance, indications of such a feature were observed [39].

V. CONCLUSION AND OUTLOOK

We have reported the first direct production of heteronuclear halo trimers, consisting of two light Na bosons

bound to one heavy K atom. From our rf-spectroscopy measurements, we extracted the trimer binding energy, lifetime, and the density dependence of the trimer formation rate. These features allowed us to clearly distinguish the trimer peak in the rf spectrum from the familiar Feshbach dimer signal. Furthermore, we presented a theoretical analysis of the trimer energy and structure.

Reasonable technical improvements will allow the association of trimers within their decay time and thus direct measurements of their intrinsic lifetime, an active area of both experimental [40–42] and theoretical [43] research. In addition, enhanced signal-to-noise ratios will allow us to more accurately measure the trimer energy, thus stringently testing theoretical models. This will help to advance our understanding of the universality of these trimers and the nature of the three-body parameter. It may even be possible to detect bound states with more particles, such as halo tetramers, whose existence we cannot confirm or rule out with the present data.

Our study is of direct relevance for several other research avenues in the domain of ultracold gases. The appearance of halo trimers should not be restricted to Na-K mixtures because the conditions of having heavy fermions (or bosons) immersed in a Bose gas with modest positive scattering lengths are met in other mixtures as well.

Particularly interesting is what happens when such mixtures are cooled into the degenerate regime.

The phase diagram of strongly interacting Bose-Fermi mixtures is rich and both experimentally and theoretically challenging to explore [44–50]. However, it is clear from studies of Fermi-Fermi mixtures and unitary Bose gases that bound-state physics should play a crucial role. For Fermi-Fermi mixtures, the appearance of a bosonic dimer state drives the BCS-to-BEC crossover [51]. For strongly interacting Bose gases, higher-order correlations originating from the Efimov effect take center stage [52–55], which is, for example, predicted to lead to an exotic Efimov liquid [56]. Our detection of the halo trimer state indicates that higher-order correlations are also expected to be crucial in the phase diagram of Bose-Fermi mixtures, which may contain a phase of trimers. At the same time, the trimer binding energy is smaller than twice the dimer energy. Hence, at equal densities, the system would most likely prefer to form a gas of dimers [50], rather than clusters of multiple bosons around some of the fermions. A scenario of a Fermi liquid of pairs is quite distinct from the case of a noninteracting Bose gas interacting with fermions, as then the formation of large clusters is favorable [29,57]. This finding, once again, stresses the importance that even a modest interboson repulsion can have.

In the limit of a single fermionic impurity in a BEC, the problem reduces to the Bose polaron problem [58–61]. Here, the role of three-body correlations has been investigated in some detail [37,62–65], giving rise to rich physics, strongly dependent on the mass ratio and the interboson interactions. By performing our rf-spectroscopy experiments at lower temperatures, in a BEC of Na, the direct impact of the halo trimer state on the Bose polaron properties can be studied.

The presence of the halo trimer state will also affect the association procedure of bi-alkali molecules. The commonly used magneto-association protocol for creating dimers by ramping across a Feshbach resonance [50] may partially lead to the formation of short-lived trimers instead, thus reducing the efficiency. In particular, in the case of fermionic molecules, this leads to hole heating of the resulting molecular Fermi gas [66]. Knowledge of the presence of the trimer state can therefore help us to design better protocols to reach colder molecular samples.

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APPENDIX: THEORY

1. Adiabatic hyperspherical approach

To model the trimer properties, we employ the adiabatic hyperspherical approach. For an elaborate discussion, view, for example, Ref. [67]. Here, we give an overview of the main points.

The first step in the construction is to define the mass-weighted Jacobi coordinates,

$$\rho_1 = \sqrt{\frac{\mu_{12}}{\mu}}(\mathbf{r}_2 - \mathbf{r}_1), \quad (\text{A1})$$

$$\rho_2 = \sqrt{\frac{\mu_{12,3}}{\mu}}\left(\mathbf{r}_3 - \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}\right), \quad (\text{A2})$$

where $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3$ are the positions of particles 1, 2, 3 with masses m_1, m_2, m_3 . We have further defined

$$\mu = \sqrt{\frac{m_1 m_2 m_3}{m_1 + m_2 + m_3}}, \quad (\text{A3})$$

$$\mu_{12} = \frac{m_1 m_2}{m_1 + m_2}, \quad (\text{A4})$$

$$\mu_{12,3} = \frac{(m_1 + m_2)m_3}{m_1 + m_2 + m_3}. \quad (\text{A5})$$

We can now define the hyperradius $R = \sqrt{\rho_1^2 + \rho_2^2}$, and its value is independent of the choice of the set of Jacobi coordinates. Together with two “hyperangles” describing the internal configuration of the three-body system, and the three Euler angles describing its overall orientation, the three-body coordinate system is fully specified.

The Schrödinger equation can now be brought into the following hyperradial form,

$$\left[-\frac{1}{2\mu} \frac{\partial^2}{\partial R^2} + \frac{15}{8\mu R^2} + \frac{\hat{\Lambda}^2}{2\mu R^2} + V(R, \theta, \phi) \right] \psi = E\psi. \quad (\text{A6})$$

Here, $\hat{\Lambda}$ is the grand angular momentum operator, which describes the kinetic energy corresponding to the hyperangular motion and overall rotation for a given hyperradius. In the *adiabatic* hyperspherical approach, one solves this Schrödinger equation by first solving the hyperangular problem for every R , by diagonalizing $[\hat{\Lambda}^2/(2\mu R^2)] + V(R, \theta, \phi)$. In practice, we solve this problem, by expanding the hyperangular wave function using an adaptive basis of B-splines.

The scattering wave function is now expanded in terms of the hyperangular channels at every hyperradius R . This results in the following Schrödinger equation in terms of the adiabatic channels [68]

$$\left[-\frac{1}{2\mu} \frac{\partial^2}{\partial R^2} + U_\nu(R) - E \right] F_\nu(R) - \left[\frac{1}{2\mu} \sum_{\nu'} 2P_{\nu,\nu'} \frac{d}{dR} + Q_{\nu,\nu'} \right] F_{\nu'}(R) = 0. \quad (\text{A7})$$

Here, $F_\nu(R)$ represents the expansion coefficient of the wave function corresponding to the hyperangular channel ν at hyperradius R , and $U_\nu(R)$ are the hyperradial adiabatic potential energy curves. The matrices P and Q contain the nonadiabatic couplings, and their definitions can be found in Ref. [68].

Directly from Eq. (A6), one can extract the form of the asymptotic solutions. Indeed, for the lowest hyperangular momentum state, and outside the range of the potential, only the first two terms of the Schrödinger equation remain. The energy-normalized scattering solutions hence take the form of Bessel functions of the second order, as in Eq. (5). However, the analysis of the phase shifts is complicated by the fact that the lowest asymptotic hyperradial curve U_0 asymptotically scales as R^{-3} [26], meaning that the usual Wigner threshold laws are modified [27]. However, the part of the wave function reaching the short range is damped by the centrifugal barrier. The overlap of the threshold scattering wave functions and the localized bound-state wave function [see Eq. (6)] scales the same as the three-body recombination rate, which is known to scale as K^4 .

2. Interaction potentials

Accurate interaction potentials for Na-K and Na-Na are available in the literature [19,30]. Directly employing these potentials, including the full spin structure of the particles in our three-body calculations, would be challenging [33]. However, since the trimers found experimentally are weakly bound, we expect that model potentials having the same low-energy scattering properties as the realistic potentials should give similar trimer energies and wave functions. Since the Feshbach resonance under consideration is broad, we further assume that multichannel effects are not essential for the description of the trimer properties if the magnetic field dependence of the scattering length and effective range is used as input for the calculations. A more thorough investigation of the multichannel effects and the shape of the potential on the trimer wave function is left for future work [36].

Concretely, we use coupled-channels calculations to compute the Na-K scattering length and effective range as a function of magnetic field (see Fig. 8). We find that the dimer energy in this magnetic field regime is described up to high accuracy using the universal formula (8). For every

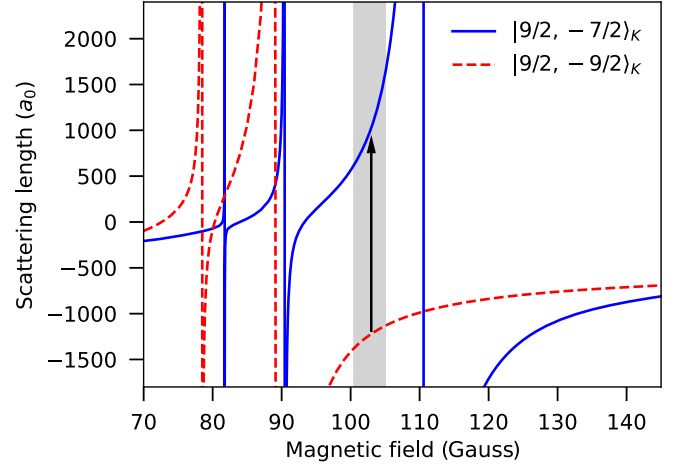


FIG. 8. Scattering length (in a_0) of scattering between ^{23}Na and ^{40}K as a function of magnetic field. For ^{23}Na , the internal state is $|F=1, m_F=1\rangle$, and for ^{40}K , the internal state is $|F=9/2, m_F=-9/2\rangle$ (red dashed) and $|F=9/2, m_F=-7/2\rangle$ (blue solid). The gray shaded area indicates the magnetic field range probed in the experiment. The curves are based on coupled-channels calculations with the Na-K interaction potentials from Ref. [30].

magnetic field, we now construct a Gaussian model potential with the same scattering length and effective range:

$$V_{\text{NaK}}(r) = \frac{g}{2L_g^2} \exp\left(-\frac{r^2}{L_g^2}\right). \quad (\text{A8})$$

For simplicity, we use a model potential without any deeper bound states in the potential aside from the dimer state under consideration. With this procedure, we find that, over the experimentally probed magnetic field range, L_g varies between 70 and 95 a_0 and the dimensionless value of $m_{\text{NaK}}g$ varies from -4.9 to -4.5 .

The interboson scattering properties are also extracted using coupled-channels calculations, but they have negligible variation over the experimentally probed magnetic field range. Since the interboson scattering length is modest and positive, we model it with a repulsive potential without bound states. There does not exist a single repulsive Gaussian potential with the same scattering length and effective range as the realistic interboson potential. This can simply be remedied by choosing a sum of two Gaussian potentials,

$$V_{\text{NaNa}}(r) = \frac{U_1}{2L_{U1}^2} \exp\left(-\frac{r^2}{L_{U1}^2}\right) + \frac{U_2}{2L_{U2}^2} \exp\left(-\frac{r^2}{L_{U2}^2}\right). \quad (\text{A9})$$

Now, the scattering length and effective range are not enough to uniquely specify the parameters. Therefore, we choose the potential by considering the phase shift over a

collision energy range from 0 to 50 MHz. We construct several different potentials with a similar energy-dependent phase shift over this range of collision energies, and we find the difference of the trimer energy between these potentials to be below 0.1 kHz (see Ref. [69] for more details). For the results presented in this paper, we used $L_{U1} = 54.45 a_0$, $L_{U2} = 157.35 a_0$, $m_{\text{Na}}U_1 = 30.51$, and $m_{\text{Na}}U_2 = -0.458$. Even though the interboson scattering length is much smaller than the Na-K scattering length, the peak value of the interboson potential is substantially larger than the well depth of the Na-K potential. The strong repulsive core

of the potential mimics the quantum reflection of the wave function around the van der Waals radius.

3. Wave-function computations

To compute the trimer wave function shown in Fig. 6, it is more convenient to solve the three-body problem in the reference frame of the K atom (see Refs. [65,69]) because this gives us direct access to the wave function in terms of the coordinates r_1 and r_2 such as in Eq. (9). The Hamiltonian in this frame can be written as

$$\hat{\mathcal{H}} = \frac{1}{(2\pi)^3} \int d^3k \frac{k^2}{2\mu_{\text{NaK}}} \hat{b}_k^\dagger \hat{b}_k + \frac{1}{2m_K} \frac{1}{(2\pi)^6} \int d^3k \int d^3k' \mathbf{k}' \cdot \mathbf{k} \hat{b}_k^\dagger \hat{b}_k^\dagger \hat{b}_{k'} \hat{b}_k + \int d^3r V_{\text{NaK}}(\mathbf{r}) \hat{b}_r^\dagger \hat{b}_r + \frac{1}{2} \int d^3r \int d^3r' V_{\text{NaNa}}(\mathbf{r}' - \mathbf{r}) \hat{b}_r^\dagger \hat{b}_r^\dagger \hat{b}_{r'} \hat{b}_{r'}. \quad (\text{A10})$$

We have used second quantization, with the bosonic creation and annihilation operators $\hat{b}^\dagger, \hat{b}$. The wave function can be written in the form

$$|\psi\rangle = \frac{1}{\sqrt{2}} \int d^3r_1 d^3r_2 \frac{\psi(\mathbf{r}_1, \mathbf{r}_2)}{r_1 r_2} \hat{b}_{r_1}^\dagger \hat{b}_{r_2}^\dagger, \quad (\text{A11})$$

which we parametrize numerically as

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = r_1 r_2 \sum_{lij} \sum_{m=-l}^l (-1)^m \psi_{lij} \chi_{lm}(\sigma_i, \mathbf{r}_1) \chi_{l-m}(\sigma_j, \mathbf{r}_2), \quad (\text{A12})$$

$$\chi_{lm}(\sigma, \mathbf{r}) = Y_{lm}(\theta_r, \phi_r) i^{-l} r^l \exp(-\sigma r^2). \quad (\text{A13})$$

The matrix elements of the spherical Gaussian basis functions χ_{lm} with respect to the Gaussian potentials and kinetic energy terms can be computed analytically. A set of exponents σ_i is chosen to efficiently represent the wave function, as is conventional in quantum chemistry. The problem of finding the variational parameters ψ_{lij} minimizing the energy can now be cast into a generalized eigenvalue problem, which can efficiently be solved numerically. This wave-function representation centered on the impurity accurately captures the wave function close to the impurity, but describing the interboson scattering far from the impurity requires fine angular and radial grids. Therefore, the relatively small energy contribution of the interboson scattering far from the impurity converges slowly with the number of angular and radial modes. To remedy this issue, we replace the full interboson potential with an effective three-body potential

$$V'_{\text{NaNa}}(\mathbf{r}_1, \mathbf{r}_2) = V_{\text{NaNa}}(\mathbf{r}_1 - \mathbf{r}_2) [1 - c_{\text{Born}}] \times \exp\left(-\frac{r_1^2}{L_W^2}\right) \exp\left(-\frac{r_2^2}{L_W^2}\right) + c_{\text{Born}} V_{\text{NaNa}}(\mathbf{r}_1 - \mathbf{r}_2), \quad (\text{A14})$$

where r_1 and r_2 are the boson-impurity distances because the impurity is at the origin. The coefficient $c_{\text{Born}} \approx 0.167$ depends on the potential and is calculated so that the rescaled potential $c_{\text{Born}} V_{\text{NaNa}}(\mathbf{r}_1 - \mathbf{r}_2)$ gives the correct interboson scattering length on the level of the Born approximation. This approximation is chosen so that the interboson scattering gives the right energy contribution when the variational wave function does not resolve the Na-Na scattering wave function. The coefficient L_W should be calibrated for a fixed basis set size with more accurate calculations. We find that, for our chosen basis set (including 15 Gaussian basis functions per angular momentum mode, and $l_{\text{max}} = 9$), the optimal value of $L_W = 11.4 L_g$ gives a trimer energy agreeing within 0.03 kHz with the hyperspherical calculations over the entire experimental magnetic field range. The fact that a single value of L_W can correctly fix the trimer energy over the whole magnetic field range indicates the validity of the approximation. For reference, if we take $L_W \rightarrow \infty$, the energy difference is of the order 1–3 kHz.

The wave function in Fig. 6(a) shows the $l = 0$ contribution of the wave function (A12), which constitutes 99.4% of the wave-function norm for this magnetic field. The wave function defined in Eq. (9), by definition, only has an $l = 0$ component because it is independent of the orientations of r_1 and r_2 . The wave function in Eq. (9), with the atom-dimer scattering length and effective range

extracted from hyperspherical calculations, has more than 98% wave-function overlap with the full numerical wave function (including the higher angular momentum modes) for all the considered magnetic fields.

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