

Teleportation and Entanglement Swapping of Continuous Quantum Variables of Microwave Radiation

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Quantum communication is needed to build powerful quantum computers and establish reliable quantum networks. At its basis lies the ability to generate and distribute entanglement to separate quantum systems, which can be used to run remote quantum operations on them or teleport quantum states from one system to another with the help of classical channels. To this end, it is useful to harness the resource of continuous-variable (CV) entanglement, since it can be efficiently and unconditionally produced by squeezing light in a nonlinear medium and can be easily manipulated, distributed, and measured using standard components. While various aspects of CV-based quantum communication have been successfully demonstrated in the optical domain, some key capabilities, such as entanglement swapping, have been lacking in the microwave domain. Here, we demonstrate three key elements of CV-based microwave quantum communication: (i) a Josephson mixer operating as a nondegenerate two-mode entangler with maximum measured logarithmic negativity $E_N = 1.5$, (ii) a quantum teleportation apparatus, capable of teleporting vacuum and coherent states with a maximum fidelity of 73%, which exceeds the 50% classical limit and is mainly limited by intermediate losses in the setup, and (iii) an entanglement-swapping system which generates entanglement between two remote noninteracting modes via entanglement-swapping operations applied to input vacuum and coherent states with maximum measured logarithmic negativity $E_N = 0.53$. Such hardware-efficient CV entanglement building blocks that are based on nondegenerate Josephson mixers could enable wide-ranging applications in modular quantum computation, quantum cryptography, and quantum communication.

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I. INTRODUCTION

Building superconducting quantum computers capable of running error correction codes [1] and solving applied computational problems that are intractable using classical computation [2,3] would require the formation of a hierarchy of quantum networks, which integrate thousands of quantum modules spread over multiple distances [4]—in particular, those that range between several millimeters to tens of meters. However, for such a large distributed architecture to work, it must have the capability of forming fast, reliable, high-fidelity remote entanglement between various modules of the system, where ideally the performance of the entangling operations is unaffected by the relative locations of the interacting modules.

Recently, there has been tremendous progress in demonstrating remote entanglement in the short range, i.e., millimeters to meters. Most of the high-fidelity methods deployed in this range use direct gates [5–7], parametric devices [8–10], or exchange discrete quantum variables (DV), such as single photons and qubit excitation, between quantum nodes connected via quantum links [11–14]. However, it remains an outstanding challenge to extend this level of success to the medium range [15,16], i.e., tens of meters, where, for example, the processors are in separate fridges interconnected by superconducting lines passing through cryogenic links [17,18], and the long range, i.e., tens of kilometers, where, for example, the modules are in distant fridges interconnected by optical fibers [19–22].

While the long-range connectivity challenge continues to be hindered by the lack of fast, high-efficiency, low-noise optical-to-microwave quantum converters to date [23], both medium and long ranges are expected to benefit from recent theoretical proposals that show potential upsides for using entanglement-based schemes especially those that rely on continuous quantum variables (CV) [24–27] or a hybrid of

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CV and DV [28], where CV refer to the continuous quadrature amplitudes of the quantized electromagnetic field.

Among the useful advantages of CV-based entanglement schemes [29,30] are (i) the existence of nonlinear devices, i.e., parametric amplifiers, which can reliably and efficiently produce CV entanglement by squeezing light [31–35], (ii) the entanglement generated by these sources is unconditional and gets produced every inverse bandwidth time [29], and (3) the entanglement distribution does not strongly depend on the length of the quantum link due to its reliance on propagating coherent states rather than on stationary or resonant modes [11–14]. On the other hand, two of the main vulnerabilities of such schemes are sensitivity to intermediate losses and imperfect entanglement due to the finite squeezing that can be achieved in practice [29].

Since the discovery of quantum teleportation by Bennett *et al.* [36], which remarkably enables the disembodiment of an unknown quantum state at Alice and its full reconstruction at Bob, using shared nonclassical Einstein-Podolsky-Rosen (EPR) correlations and transmission of purely classical information, teleportation has been extended to CVs [37] and successfully demonstrated in various DV and CV settings, including single-photon polarization states [38,39], photonic qubits [40], Schrödinger's cat [41], and squeezed [42] and coherent states [43–46] in the optical domain, as well as microwave coherent states [47,48]. Moreover, a key requirement of quantum repeaters, i.e., entanglement swapping, in which half of an entangled state is teleported [49], has been experimentally shown using single-photon polarization states [50] and demonstrated for optical coherent states [51,52].

In this work, we demonstrate three key capabilities of CV-based quantum communication in the microwave domain: (i) a fast, low-loss, two-mode squeezer based on nondegenerate Josephson mixers (JMs), which are typically used as quantum-limited amplifiers for high-fidelity qubit readout [53–57]—in particular, we show that such devices can serve as two-mode entangling sources, which are analogous to EPR-particle sources [58–61]—(ii) a high-fidelity quantum teleportation apparatus for coherent states that employs two JMs, one serving as an EPR source while the other as a which-path-information eraser, and (iii) a fast and unconditional entanglement-swapping scheme that utilizes three JMs, where the third serves as a second EPR source. In both latter setups, we demonstrate teleportation and entanglement swapping on vacuum and coherent states.

II. TWO-MODE SQUEEZING

To produce two-mode squeezed states, we pump a JM, i.e., nondegenerate parametric amplifier with $\chi^{(2)}$ nonlinearity. The input and output bosonic field operators of the amplifier are given by the scattering relations [58]

$$\begin{aligned} a_{\text{out}} &= S^\dagger a_{\text{in}} S = \cosh(r) a_{\text{in}} + e^{i\varphi_p} \sinh(r) b_{\text{in}}^\dagger, \\ b_{\text{out}}^\dagger &= S^\dagger b_{\text{in}}^\dagger S = \cosh(r) b_{\text{in}}^\dagger + e^{-i\varphi_p} \sinh(r) a_{\text{in}}, \end{aligned} \quad (1)$$

where $S = \exp(re^{i\varphi_p} a^\dagger b^\dagger - re^{-i\varphi_p} ab)$ is the unitary two-mode squeeze operator, a and b are the annihilation operators associated with modes a and b , respectively, of the amplifier, and $re^{i\varphi_p}$ is the complex squeezing parameter, where φ_p is the phase of the pump and $r \geq 0$ depends on the pump amplitude and sets the amplifier gain $G = \cosh^2(r)$. In particular, when acting on input vacuum states, the amplifier produces two-mode squeezed vacuum state $|\text{TMS}\rangle = S|0\rangle_a |0\rangle_b = \cosh^{-1}(r) \sum_n (\tanh(r))^n |n\rangle_a |n\rangle_b$, which represents a superposition of twin Fock states with distinct frequencies traveling on separate transmission lines.

The operation of the two-mode squeezing (TMSQ) device is illustrated in Fig. 1(a). Input vacuum states $|0\rangle_j$ that lie within the bandwidths of modes a ($j = 1$) and b ($j = 2$) get amplified and reflected toward the output. In the dimensionless quadrature representation of the fields expressed as conjugate position and momentum variables satisfying the commutation relations $[x_j, p_j] = i/2$, where $x_1 = \text{Re}(a)$, $p_1 = \text{Im}(a)$, and $x_2 = \text{Re}(b)$, $p_2 = \text{Im}(b)$, the output fields for each mode, when viewed separately, mimic amplified thermal states; however, non-local two-mode squeezing emerges when inspecting combinations of output field quadratures, i.e., $x_{\text{out},1} - x_{\text{out},2} = e^{-r}(x_{\text{in},1} - x_{\text{in},2})$ and $p_{\text{out},1} + p_{\text{out},2} = e^{-r}(p_{\text{in},1} + p_{\text{in},2})$, which are analogous to the relative position and total momentum, respectively, of a pair of EPR particles. Thus, while the noise variance of each output mode $\langle x_j^2 \rangle + \langle p_j^2 \rangle = \cosh(2r)/2$ increases with r , the output noise variance for certain orthogonal linear quadrature combinations undergoes either squeezing below the vacuum noise level, e.g., $\text{Var}[x_-] = \langle (x_1 - x_2)^2 \rangle = e^{-2r}/2$, or anti-squeezing, e.g., $\text{Var}[x_+] = \langle (x_1 + x_2)^2 \rangle = e^{2r}/2$ [29].

As illustrated in Fig. 1(a), losses in the TMSQ device or at its output are modeled as beam-splitter interactions with uncorrelated cold thermal baths. Thus, the output fields can be written as

$$\begin{aligned} a_{\text{out}} &= \sqrt{\bar{\alpha}} a_{\text{out}} + \sqrt{\alpha} a_{\text{th}}, \\ b_{\text{out}} &= \sqrt{\bar{\beta}} b_{\text{out}} + \sqrt{\beta} b_{\text{th}}, \end{aligned} \quad (2)$$

where a_{th} and b_{th} represent bosonic modes of the thermal baths at frequencies f_1 and f_2 and α and β are the effective power losses, while $\bar{\alpha} = 1 - \alpha$ and $\bar{\beta} = 1 - \beta$.

Moreover, since the two-mode squeezed state belongs to the class of Gaussian states, its Wigner function is a bipartite normalized Gaussian distribution given by [29]

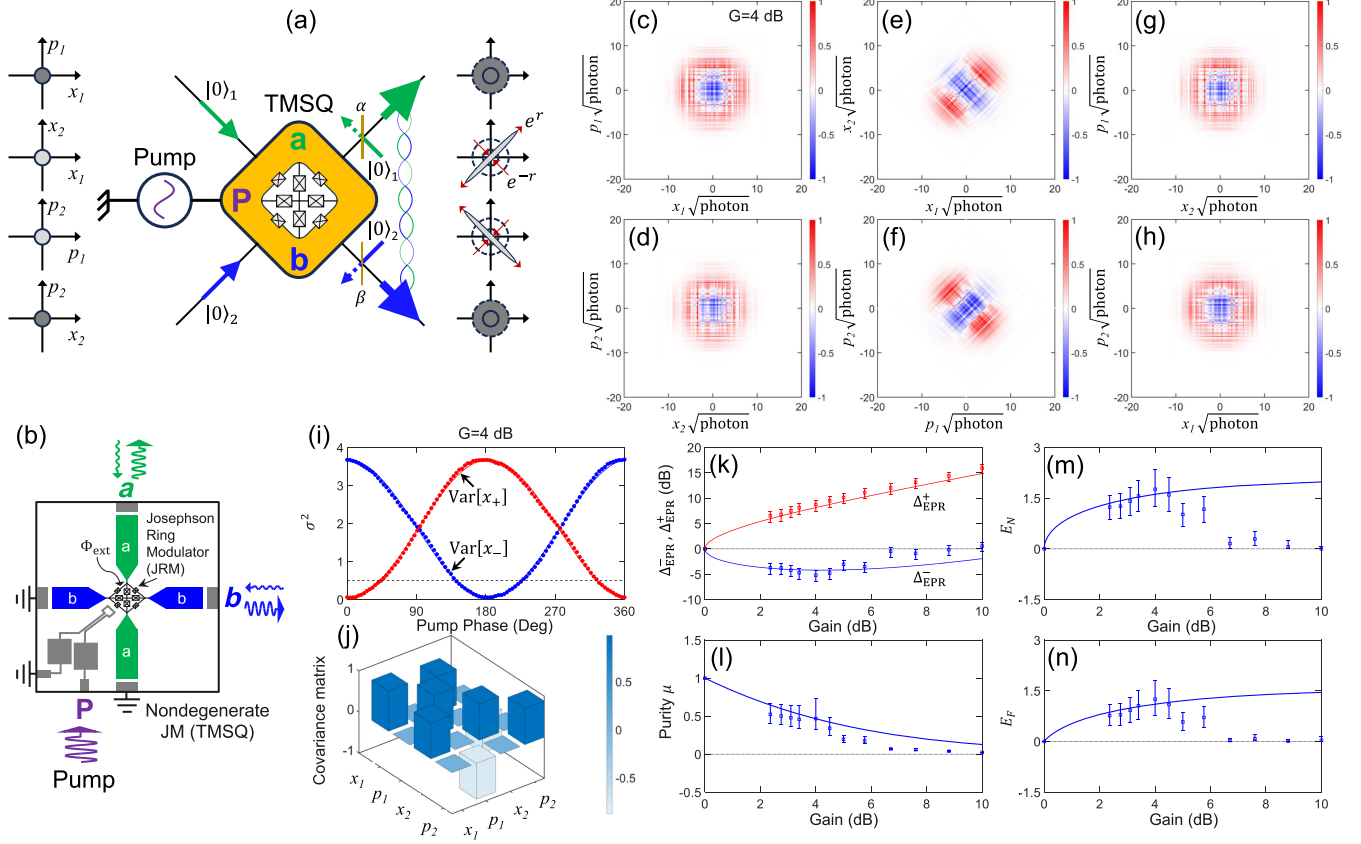


FIG. 1. Demonstration of two-mode squeezing and entanglement generated by a nondegenerate three-wave Josephson mixer. (a) Illustration of two-mode squeezing at the output of a three-wave Josephson mixer. When driving the Josephson mixer with a monochromatic pump tone at the frequency sum of modes a and b , input vacuum noise characterized by the orthogonal quadrature pairs $x_1 = (a + a^\dagger)/2$, $p_1 = (a - a^\dagger)/2i$ and $x_2 = (b + b^\dagger)/2$, $p_2 = (b - b^\dagger)/2i$ get reflected and amplified at the output, mimicking a thermal state as shown in the top and bottom cases on the right side. Two-mode squeezing and entanglement between the modes emerge, however, upon examining cross-correlations between the generalized position and momentum of the two modes as shown in the two middle cases on the right. Potential losses within or following the TMSQ device are represented by beam-splitter interactions with the thermal baths at the output of modes a and b , having power reflectivity (i.e., loss) of α and β , respectively. (b) Schematic representation of the nondegenerate Josephson mixer which serves as a two-mode squeezer. (c)–(h) Histogram difference between pump on and off plotted for the six pairs of dimensionless position and momentum coordinates of modes a and b measured at the output of the TMSQ device at $G = 4$ dB. Squeezing below the vacuum level is observed for the EPR-like coordinates of relative position $x_- = x_2 - x_1$ and total momentum $p_+ = p_2 + p_1$ as seen in (e) and (f). (i) The variance of EPR-like coordinates x_- (blue filled circles) and x_+ (red filled circles) as a function of the pump phase. The pump phase axis is shifted to yield a minimum of $\text{Var}[x_-]$ at 180° . Solid blue and red lines represent sinusoidal fits. The dashed black line represents the variance of vacuum noise. (j) Bar graph representing the covariance matrix elements of the TMSQ device for the minimum $\text{Var}[x_-]$ working point in (i). (k)–(n) exhibit various measured figures of merit for the two-mode entanglement as a function of the device gain. (k) shows Duan’s criterion for entanglement. (l)–(n) display the state purity μ , logarithmic negativity E_N , and entanglement of formation E_F , respectively. The solid curves in (k)–(n) are calculated fits based on the TMSQ model with asymmetric power transmission $\bar{\alpha} = 0.62$, $\bar{\beta} = 1$.

$$W(\xi) = \frac{1}{4\pi^2 \sqrt{\det V}} \exp \left\{ -\frac{1}{2} \xi V^{-1} \xi^T \right\}, \quad (3)$$

which is fully characterized by its covariance matrix V whose elements are given by $V_{ij} = \langle \xi_i \xi_j + \xi_j \xi_i \rangle / 2 - \langle \xi_i \rangle \langle \xi_j \rangle$, where ξ_i and ξ_j are components of the vector $\xi = (x_1, p_1, x_2, p_2)$.

The circuit diagram of the JM, which implements the TMSQ, is shown in Fig. 1(b). It consists of two orthogonal

half-wavelength microstrip resonators denoted a and b with different resonance modes, i.e., $f_b > f_a$. The two resonators are joined at the center by a Josephson ring modulator (JRM) that is flux tunable and serves as a dispersive three-wave mixing element [54,58]. Resonators a and b are capacitively coupled to external feedlines that carry the incoming and outgoing fields. A strong coherent microwave tone, i.e., the pump, is applied to the JM through a third filtered on-chip line that is capacitively coupled to the JRM nodes. When applying a pump at the

frequency sum $f_p = f_a + f_b$, the JM acts as a non-degenerate quantum-limited amplifier for quantum signals [55,56] and a two-mode squeezer [58].

In Figs. 1(c)–1(h), we exhibit the histogram difference between pump on ($G = 4$ dB) and off ($G = 0$ dB) measured for the six unique pairs of the position and momentum quadratures of the two modes at the output of the TMSQ. In this experiment, we flux tune the JM to have $f_a = 7.231$ GHz and $f_b = 9.695$ GHz and apply a pump tone at $f_p = 16.926$ GHz. As expected, when inspecting mixed quadrature pairs of position and momentum of the same mode [Figs. 1(c) and 1(d)] or belonging to different modes [Figs. 1(g) and 1(h)], we observe a uniform amplified vacuum noise in the former case and uncorrelated amplified noise in the latter, whereas, when considering cross-correlations between position [Fig. 1(e)] and momentum [Fig. 1(f)] quadratures of the two modes, we observe squeezing of the noise variance below the vacuum level for EPR-like coordinates x_- and p_+ and antisqueezing for the orthogonal joint coordinates x_+ and p_- . In Fig. 1(i), we show the periodic dependence of the variance of the EPR-like coordinates x_- and x_+ on the pump phase. As seen in the plot, two-mode squeezing occurs when $\text{Var}[x_-]$ (or $\text{Var}[x_+]$) goes below the vacuum noise variance of $1/2$, indicated by the dashed black line. Maximum squeezing of 9.2 dB is attained at the minima of $\text{Var}[x_-]$. Furthermore, using the quadrature histograms data measured for $G = 4$ dB, we reconstruct the covariance matrix V of the TMSQ device as outlined in Appendix A and display its elements in Fig. 1(j) using a bar graph format.

In the remaining panels [Figs. 1(k)–1(n)], we plot four key entanglement measures, derived from the reconstructed covariance matrix, which characterize the two entangled modes of the TMSQ device as a function of its gain. Namely, we plot the EPR measures $\Delta_{\text{EPR}}^{\pm} = 2(V_{11} + V_{33} \mp 2V_{13})$ [Eqs. (A17) and (A18)], the state purity $\mu = 1/[16\sqrt{\det(V)}]$ [Eq. (A27)] [62,63], the logarithmic negativity $E_N = \max[0, -\log_2(4\nu_-)]$ [58,63–65], and the entanglement of formation (EOF) $E_F = \max[0, h(4\nu_-)]$ [58,66,67] in Figs. 1(k)–1(n), respectively, where ν_- is the smallest symplectic eigenvalue of the partially transposed covariance matrix [59], given by Eq. (A22), and the function $h(x)$ is given by Eq. (A26). In addition, we depict in Figs. 1(k)–1(n) using solid curves the calculated entanglement measures of the TMSQ whose theoretical model incorporates the effect of the asymmetric losses experienced by modes a and b .

In the case of $\Delta_{\text{EPR}}^{\pm}$, which also characterize the amount of two-mode squeezing and antisqueezing, the violation of the inequality $\Delta_{\text{EPR}}^{\pm} \geq 1$ for a bipartite Gaussian system implies the inseparability of the two subsystems and, hence, their entanglement. This inequality can be derived from the general criterion for entanglement proposed by Duan *et al.* [29,68]. In the case of E_N , its positivity, i.e., $E_N > 0$, serves a dual purpose. It constitutes a necessary

and sufficient condition for entanglement and gives an upper bound on the number of equivalent entangled bits (ebits) that can be distilled from the entangled state. Similarly, E_F gives the minimum number of ebits required to create the quantum state as an ensemble of pure states [66].

In particular, in our case [as seen in Figs. 1(k)–1(n)], we find that the JM exhibits two-mode entanglement up to about $G = 8$ dB, with optimal entanglement values observed around 4 dB, i.e., $\Delta_{\text{EPR}}^- = -5$ dB, $E_N = 1.5$, and $E_F = 1.25$, followed by a rapid decline above 5 dB. While the calculated entanglement measures, e.g., E_N and E_F , yield a good agreement with the measured data in the low-gain range, they notably do not capture the decline observed above 5 dB. Although the origin of such decline in our system remains an open question [69], it underscores the increased fragility of entangled states as they grow larger in size (i.e., become more "nonclassical") in the presence of dissipation [70]. It also seems to follow the decline of two-mode squeezing, e.g., Δ_{EPR}^- , which is known to be sensitive to asymmetric losses [63].

Another useful measure for characterizing the generated entanglement is the entanglement bit rate [58], given by $R = B \times E_F$, where B is the dynamical bandwidth of the device. At 4 dB, we obtain an estimated maximum rate of 70 Mebit/sec, where E_F is maximal and $B = 56$ MHz.

Notably, the present JM exhibits slightly larger maximum squeezing and logarithmic negativity compared to the JM device characterized in Ref. [58], i.e., 5 versus 3.4 dB and 1.5 versus 1.15, respectively, but more importantly it yields a maximum ebit rate that is one order of magnitude higher, i.e., 70 versus 6 Mebit/sec, which stem from attaining a higher E_F and having a larger linear bandwidth in the case of the present JM.

III. TELEPORTATION OF COHERENT STATES

Equipped with this valuable quantum resource of two-mode entanglement which is produced by nondegenerate JMs allows us to readily demonstrate two important quantum communication capabilities in the microwave domain, i.e., quantum teleportation and entanglement swapping.

However, prior to reviewing our teleportation apparatus scheme shown in Fig. 2, it is instructive to highlight the four key elements that are typically used for the teleportation of coherent states in the optical domain [71,72], namely, (i) an EPR source which generates two-mode entanglement (for example, by interfering two orthogonal single-mode squeezed light on a 50:50 beam splitter), half of which is sent to Alice and the other to Bob, (ii) a 50:50 beam splitter combined with photon detectors at Alice, which allow her to perform a joint Bell-state measurement on her entangled mode and the input state she wishes to teleport, (iii) a classical communication channel between Alice and Bob through which Alice communicates the

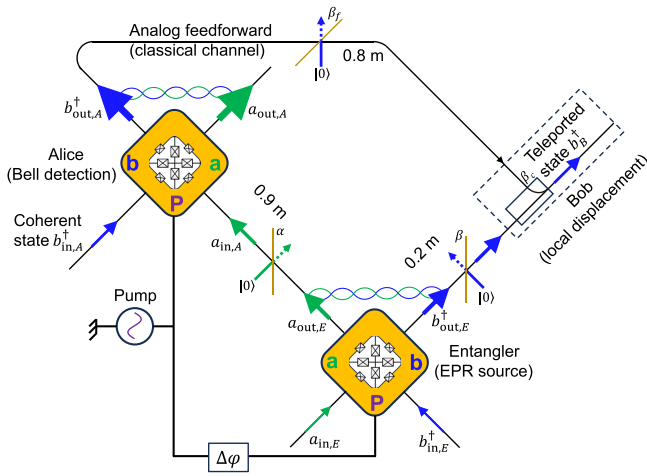


FIG. 2. Quantum teleportation scheme for microwave coherent states using two nondegenerate Josephson mixers. A nondegenerate JM, serving as an EPR source, sends entangled continuous-variable beams to Alice and Bob. Alice employs a second nondegenerate JM, acting as a which-path-information eraser, to perform a Bell-state-like measurement on the incoming coherent input state to be teleported to Bob, i.e., $b_{in,A}^\dagger$, and on half of the entangled state received from the EPR source, i.e., $a_{in,A}$. Following the “measurement,” Alice sends an analog feedforward signal to Bob containing the measurement result via a classical channel. Bob completes the teleportation process by locally displacing his half of the entangled state received from the EPR source using the feedforward signal sent by Alice. Approximate cable lengths between the parties are listed next to the connecting paths. Losses in the setup are modeled using beam-splitter interactions with thermal (i.e., vacuum noise) baths and given by $\bar{\alpha} = 0.62$, $\bar{\beta} = 0.93$, $\bar{\beta}_f = 0.4$, and $\beta_c = 0.1$, where β_c is the power coupling factor of the directional coupler. The gain on Alice is set to about 14 dB to match the total attenuation in the feedforward channel and directional coupler.

measurement results to Bob, and (iv) a local displacement capability at Bob which he applies to his entangled mode based on the results he receives from Alice, which, in turn, reconstructs the original state destroyed by Alice’s measurement.

Analogously, the scheme of Fig. 2 includes microwave devices that fulfill similar roles. The JM, denoted as the entangler, plays the role of the EPR source which produces two-mode entangled vacuum states at modes a and b and sends them to Alice and Bob, respectively. The second JM, marked as Alice, effectively plays the role of the Bell-state detector, though with three key differences compared to the beam splitter and photon detectors used in the optical domain. First, it amplifies the combined signals. But, since the amplification is phase preserving, the JM acts in the high-gain limit as a which-path-information eraser [73] for the input state and the entangled mode. Second, the amplified output signal at port b of Alice forms an analog feedforward signal that can be directly sent to Bob without

any further processing. However, since the feedforward signal is amplified and the teleportation process requires a unity-gain classical signal, Bob uses a directional coupler to simultaneously attenuate the feedforward signal and displace his entangled mode with it. Third, since the scheme in Fig. 2 relies on wave interference, successful teleportation occurs only for relative pump phases between the entangler and Alice, denoted $\Delta\phi$, that minimize the noise at Bob’s output (see Appendixes B and C for further details). Note that Fedorov *et al.* [47] have successfully teleported coherent states using a similar microwave scheme based on single-mode squeezers, i.e., Josephson parametric amplifiers. However, the scheme of Fig. 2 which instead employs nondegenerate JMs has the advantage of cutting the cryogenic hardware overhead of the teleportation setup (both parametric and passive devices) by more than half, which, in turn, leads to additional benefits such as higher stability, simpler tuneup process, and fewer matching requirements between the various components.

Using the teleportation scheme of Fig. 2, we first demonstrate the quantum teleportation of vacuum states which represent Gaussian states with zero mean. In this experiment and what follows, we independently flux tune the different JMs to have the same resonance frequencies $f_a = 7.23$ GHz and $f_b = 9.707$ GHz and applied pump tone $f_p = 16.937$ GHz. In Figs. 3(a)–3(c), we plot for different entangler gains (G_E), the normalized noise power (solid dark blue), which we measure on resonance at Bob’s output as a function $\Delta\phi$. The solid black curves represent theory fits based on Eq. (C3). As seen in Figs. 3(a)–3(c), the measured noise is periodic in $\Delta\phi$ and its oscillation amplitude grows with G_E . Quantum teleportation occurs in these measurements at the minima points which asymptotically approach the average of the measured vacuum noise level at Bob without pumps (i.e., 0.51) represented by the dash-dotted dark red line. For reference, we show in gray the measured noise at Bob without pumps. We also display, for comparison, the average noise level (i.e., 1.56), obtained via classical teleportation (dashed magenta line) when no entanglement is shared between Alice and Bob, i.e., $G_E = 0$ dB, and the average amplified noise level at Bob when only the entangler is on (dashed green line), which increases with G_E as expected. At the bottom and top corners in Figs. 3(a)–3(c), we list the G_E employed in each measurement and the resultant quantum teleportation fidelity F_q , respectively, where the teleportation fidelity quantifies the amount of overlap between the input and teleported states [Eq. (J14)]. In Fig. 3(d), we plot the quantum teleportation fidelity (filled blue circles) measured for input vacuum noise versus G_E . The blue curve represents a theoretical model calculation that accounts for the losses in the setup. The black curve, serving as a reference, represents the ideal F_q achievable in a lossless setup [Eq. (K7)]. It is worth noting that the maximum

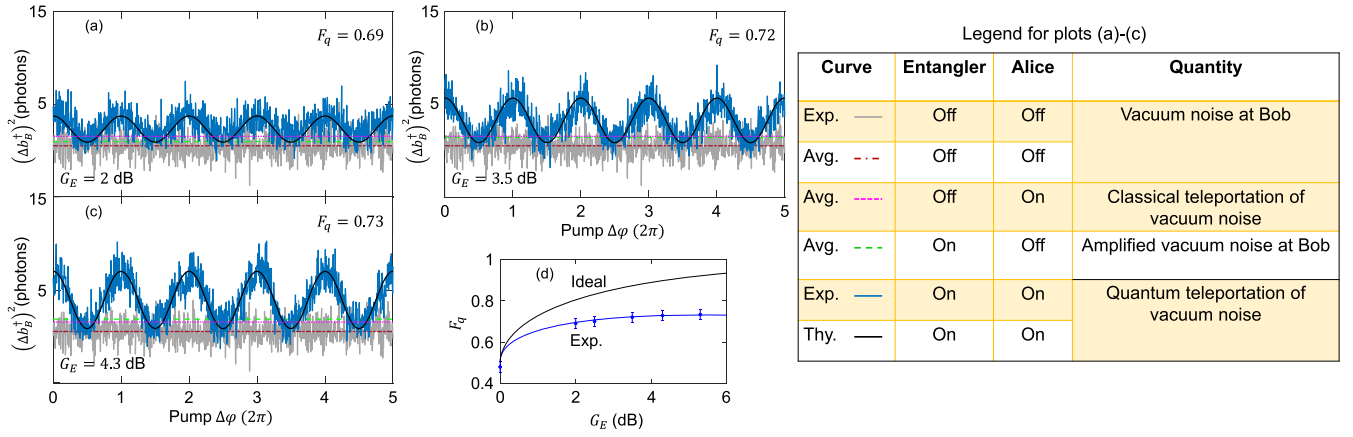


FIG. 3. Quantum and classical teleportation of vacuum noise. (a)–(c) exhibit quantum teleportation of vacuum noise at Bob (solid dark blue line) measured as a function of the pump phase difference between the pumps at the entangler and Alice $\Delta\phi$, plotted for ascending power gain on the entangler G_E . The solid black curves are theoretical fits to the data. The gray curves correspond to the measured vacuum noise at Bob without teleportation whose average is plotted as a dash-dotted dark red line. The dashed green line depicts the amplified vacuum noise at Bob received from the entangler, whereas the dashed magenta line shows the average level for the classically teleported vacuum noise (when the entangler is off). (d) depicts the measured quantum teleportation fidelity for vacuum noise as a function of the entangler gain. The solid blue curve represents a theoretical model calculation based on the estimated losses in the setup. In comparison, the solid black curve depicts the calculated quantum teleportation fidelity for a lossless setup.

measured teleportation fidelity $F_q = 0.73$ exceeds the classical limit $F_c = 1/2$.

In Fig. 4, we further use the scheme of Fig. 2 to perform quantum and classical teleportation of general coherent states (displaced vacuum). In this experiment, we apply a $1 \mu\text{s}$ pulse of a coherent tone on resonance to port b of Alice and measure the teleported result at Bob. The reconstructed input coherent signal is depicted in Fig. 4(a) using the standard x - p (I - Q) quadrature representation, where the distance between the origin and the 2D Gaussian histogram center corresponds to $\sqrt{n_{\text{in}}}$, where n_{in} is the average photon number of the coherent state and the angle with the x axis θ_{in} represents the coherent state phase. Using the same representation, we plot in Figs. 4(b) and 4(c) the 2D Gaussian histograms of the measured output field at Bob following quantum ($G_E = 2.1$ dB) and classical ($G_E = 0$ dB) teleportation which yield teleportation fidelities F_{tel} of 0.68 and 0.5, respectively, where only the former surpasses, beyond the margin of error ± 0.01 , the 50% barrier which bounds the fidelity of classical teleportation of coherent states [74]. We also list in the inset the measured average photon number n_{tel} and phase θ_{tel} of the teleported signal. In Fig. 4(d), we plot a histogram cut along the p quadrature of the input state (red curve), the quantum-mechanically teleported state (blue curve), and the classically teleported state (magenta curve), which clearly show a noise reduction in the quantum teleportation case versus the classical one. The solid curves in Fig. 4(d) are Gaussian fits whose variances are listed in the inset. In Fig. 4(e), we keep n_{in} constant [as in Fig. 4(a)] and measure θ_{tel} of the quantum-mechanically teleported coherent signal as we vary θ_{in} over one period. As indicated by the unity-slope

dashed line, θ_{tel} follows θ_{in} quite well. Similarly, in Fig. 4(f), we compare n_{tel} versus n_{in} for a fixed θ_{in} . The solid blue line represents a fit to the data, while the dashed black line is a guide to the eye with a unity slope. Based on the fit, we obtain a deviation of up to 13.6% between n_{tel} and n_{in} . In Fig. 4(g), we plot for different n_{in} the distribution of quantum teleportation fidelities measured for 20 input phases θ_{in} spanning one period. As expected, the measurement shows that F_q is largely independent of θ_{in} , and, with the exception of $n_{\text{in}} = 0.03$, F_q mostly lies in the range 0.6–0.7 for the various n_{in} .

One likely cause for the observed variation in the measured F_q versus n_{in} is deviation in the gain of the classical channel, which in the ideal case is set to unity; i.e., the gain at Alice balances out the subsequent attenuation in the path to Bob. However, in this experiment, setting this analogue condition *in situ* can be imprecise ± 0.9 dB due to uncertainty in the measured gain and extracted attenuation. Moreover, after manually setting the gain on Alice's JM at the beginning of the measurement, it can drift in the same range during the measurement or change as the input photon number is varied. In principle, both technical issues can be resolved by optimizing the teleportation fidelity as a function of the gain at Alice for each input state and applying a feedback loop to adjust the gain if necessary. In Appendix K 1, we theoretically address the impact of using a classical channel with a nonunity gain on the teleportation fidelity in the quantum and classical cases.

To further illustrate this point, we employ Eq. (J14) and the teleportation model exhibited in Fig. 13 (Appendix J) to calculate and plot using cyan and red lines the teleportation fidelity versus n_{in} in the quantum ($G_E = 2.1$ dB) and

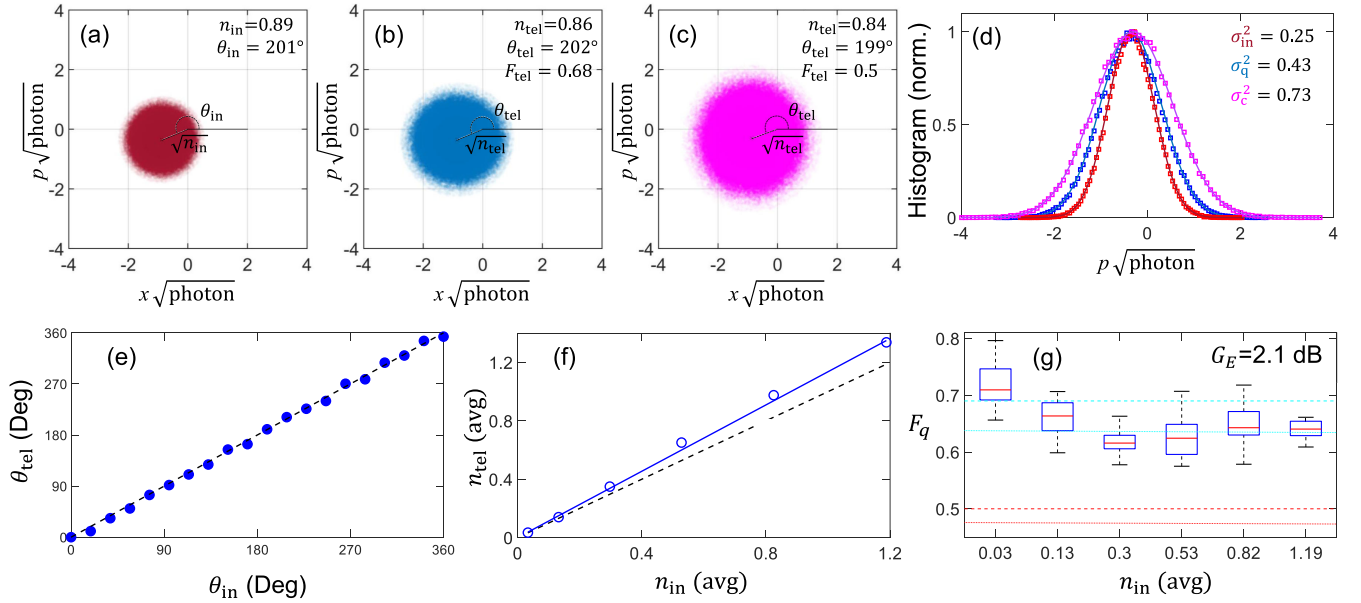


FIG. 4. Quantum and classical teleportation of coherent microwave states. (a) I-Q (x - p) quadrature representation of an applied coherent state at Alice's input port b (to be teleported to Bob) whose center is specified by the polar coordinates $(\sqrt{n_{\text{in}}}, \theta_{\text{in}})$. (b) and (c) depict the coherent state measured at Bob's output upon applying a quantum and classical teleportation, respectively. (d) Normalized histograms of the measured input coherent state (red curve) and quantum-mechanically teleported (blue curve) and classically teleported (magenta curve) coherent states along the p quadrature. The solid curves in the plot correspond to Gaussian fits. (e) The measured phase of the quantum-mechanically teleported coherent state θ_{tel} plotted versus θ_{in} (filled blue circles). The dashed black line corresponds to the ideal relation $y = \theta_{\text{in}}$. (f) The average photon number of the quantum-mechanically teleported coherent state n_{tel} versus n_{in} (open blue circles). The solid blue line represents a fit to the data, while the dashed black line is a guide to the eye for the ideal relation $y = n_{\text{in}}$. (g) Distribution of quantum teleportation fidelities measured over 20 input coherent state phases θ_{in} spanning one period taken for different average input photon number n_{in} . The inner red lines mark the mean fidelities, while the box edges represent the 25th and 75th percentiles, and the whiskers indicate the lowest and highest data points. The quantum teleportation in this experiment is performed using an entangler gain of 2.1 dB. Explanation of the cyan and red lines (dashed and dotted) is included in the main text.

classical ($G_E = 0$ dB) cases, respectively, given the setup parameters listed in the caption of Fig. 2. In each case, the dashed (top) and dotted (bottom) lines correspond to the calculated fidelity when the classical channel gain is 0 (optimal) and 0.55 dB (taken as an example), respectively.

Importantly, the results in Fig. 4(g) show that the teleportation setup shown in Fig. 2 is capable of teleporting random coherent states with high enough fidelities, which is required for quantum cryptography applications that employ CVs for quantum key distribution [47,75].

Finally, it is important to point out in this context that the well-known upper bound on the fidelity of classical teleportation of coherent states $F_{c,\text{max}} \leq 1/2$, which we use as a reference here and employed in many experimental demonstrations of CV teleportation [43–45,47], assumes that the set of input coherent states $|\alpha_s\rangle$ that we draw from for teleportation are flatly distributed in phase space. In the general case in which the available input coherent states are distributed in phase space according to a Gaussian distribution $p(\alpha_s) = (1/\pi N) \exp(-|\alpha_s|^2/N)$, the maximum fidelity of any possible classical strategy is given by $F_{c,\text{max}} \leq (1+N)/(2+1/N)$ [76,77]. Note that in this case flat distribution is attained in the limit $N \rightarrow \infty$.

As one example, for a probability distribution with width $N = 1$, which should be within reach based on the results in Fig. 4(g), we obtain $F_{c,\text{max}} = 2/3$. While some of the measured quantum teleportation fidelities shown in Fig. 4(g) lie above that limit and many could be improved by monitoring and maintaining a near-unity transmission in the feedforward channel, it is quite feasible to further lower this upper bound by, for example, replacing the JM used by Alice with an enhanced version capable of processing a higher number of incoming photons [78].

IV. ENTANGLEMENT SWAPPING

Here, we show that our quantum teleportation scheme is capable of teleporting not only coherent states, but also genuine quantum states such as the entanglement source itself, i.e., one-half of an entangled state, also known as entanglement swapping, which is a prerequisite for realizing quantum repeaters that are vital in long-range communications [79]. Using the extended teleportation setup shown in Fig. 5(a), we perform an entanglement-swapping operation which results in the entanglement of two remote coherent fields at Alice and Bob that have never

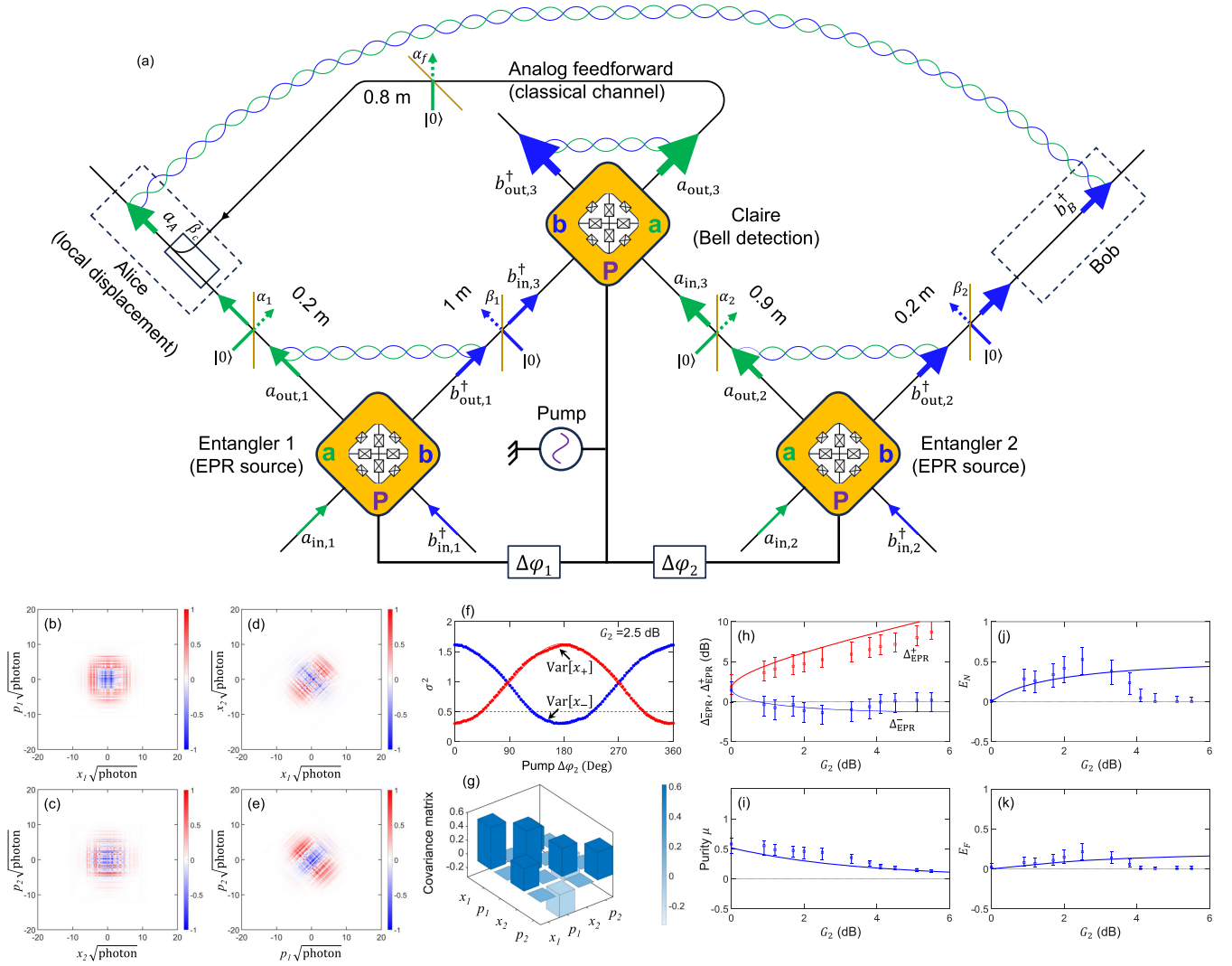


FIG. 5. Entanglement swapping of continuous quantum variables with nondegenerate Josephson mixers. (a) Entanglement-swapping scheme between two remote nodes, i.e., Alice and Bob, mediated by a “Bell-state” detector, i.e., Claire. Two EPR sources, entangler 1 and 2, generate two-mode CV entanglement. One-half of the entangled states generated by the entanglers are sent to Alice and Bob, while the other halves are sent to Claire, who erases their which-path information and forwards the mixing result to Alice through a classical channel. Alice then completes the entanglement-swapping process by locally displacing the one-half of the entangled state in her possession using the feedforward signal. Approximate cable lengths between the parties are listed next to the connecting paths. In (b)–(k), entangler 1 gain is fixed at $G_1 = 1.4$ dB. (b)–(e) Quadrature histogram difference between pump turned on ($G_2 = 2.5$ dB) and off plotted for the four main pairs of dimensionless position and momentum quadratures belonging to modes a (i.e., x_1 and p_1) and b (i.e., x_2 and p_2) measured at Alice and Bob. Squeezing below the vacuum level is observed for the EPR-like coordinates of relative position x_- and total momentum p_+ as seen in (d) and (e). (f) The variance of x_- (blue filled circles) and x_+ (red filled circles) versus the relative pump phase of entangler 2. The relative pump phase axis is shifted to yield a minimum of $\text{Var}[x_-]$ at 180° . The solid blue and red lines represent sinusoidal fits. The dashed black line represents the variance of vacuum noise. (g) Bar graph representing the covariance matrix elements of modes a and b measured at Alice and Bob for the minimum $\text{Var}[x_-]$ working point in (f). (h)–(k) exhibit various measured figures of merit of the remote entanglement between Alice and Bob versus G_2 . (h) Duan’s criterion for entanglement extended to the entanglement-swapping case. (i)–(k) display μ , E_N , and E_F , characterizing the two-mode entanglement between Alice and Bob generated via entanglement swapping. The solid curves in (h)–(k) are calculated fits based on an entanglement-swapping scheme model, which incorporates intermediate losses characterized by $\bar{\alpha}_1 = 0.9$, $\bar{\alpha}_2 = 0.72$, $\bar{\beta}_1 = 0.62$, $\bar{\beta}_2 = 0.97$, $\bar{\alpha}_f = 0.85$, and $\beta_c = 0.1$. In this setup, Claire’s gain is set to about 10.7 dB.

directly interacted. The setup consists of three nondegenerate Josephson mixers, two of which function as two-mode entanglement sources denoted as entangler 1 and 2, while the third, denoted as Claire, is located between Alice and Bob and mimics the role of a Bell-state detector [49,80]. In this scheme, entangler 1 (2) sends one-half of its entangled state, i.e., mode a (mode b), to Alice (Bob), while sending the other half, i.e., mode b (mode a), to Claire. By operating her nondegenerate Josephson mixer in the high-gain limit, Claire erases the which-path information of her inputs and forwards the amplified output mixed signal to Alice through a classical channel. Alice then completes the teleportation of one-half of the entangled state generated by entangler 2 (mode a) by attenuating the classical signal she receives from Claire down to unity transmission level and displacing with it her own half of the entangled state (received from entangler 1). Thus, although modes a and b received by Alice and Bob from entanglers 1 and 2 have not directly interacted, they become entangled following the entangling operation performed by Claire on the other entangled modes sent by the entanglers. Similar to the teleportation setup in Fig. 2, we match in this experiment the resonance frequencies of the three mixers (using magnetic flux), phase lock the pumps feeding the Josephson mixers, and fix the relative pump phase $\Delta\varphi_1$ such that the noise at Alice's output is minimized when both entangler 1 and Claire's JM are on and entangler 2 is off.

The results exhibited in Figs. 5(b)–5(k) are measured for vacuum-state input on entanglers 1 and 2. They reveal two-mode vacuum squeezing and entanglement generated at Alice and Bob due to the entanglement-swapping operation. The gain on entangler 1 is fixed at $G_1 = 1.4$ dB, while the gain on entangler 2 is set to $G_2 = 2.5$ dB in Figs. 5(b)–5(g) and varied in Figs. 5(h)–5(k). In Figs. 5(b)–5(e), we display the quadrature histogram difference with the pump applied to entangler 2 turned on and off, plotted for the four main pair combinations of x_1 , x_2 , p_1 , and p_2 , where x_1 (x_2) and p_1 (p_2) are the position and momentum quadratures, respectively, measured at Alice (Bob). As seen in Figs. 5(b) and 5(c), when inspecting the quadrature histograms of modes a and b separately, we observe amplified thermal states. However, when inspecting histograms of cross position and momentum quadratures as shown in Figs. 5(d) and 5(e), strong correlations and squeezing below the vacuum noise level are revealed along EPR-like coordinates, i.e., x_- and p_+ . In Fig. 5(f), we plot the variance of EPR-like coordinates x_- (blue filled circles) and x_+ (red filled circles) versus the pump phase difference between entangler 2 and Claire $\Delta\varphi_2$, which shows TMSQ around 180° as $\text{Var}[x_-]$ drops below the variance of vacuum noise indicated by the dashed horizontal line. Similar to the single JM case, maximum squeezing of 2.2 dB is obtained at the minima of $\text{Var}[x_-] = 0.3$. In Fig. 5(g), we plot a bar graph

representation of the covariance matrix elements of modes a (Alice) and b (Bob) taken at the minima working point in Fig. 5(f). Lastly, in Figs. 5(h)–5(k), we depict four key entanglement measures for Alice's and Bob's fields as a function of G_2 , i.e., $\Delta_{\text{EPR}}^\pm$, μ , E_N , and E_F . The solid curves represent theoretical calculations of the corresponding measures based on the covariance matrix of the entanglement-swapping scheme, which incorporates the effect of losses in the system (see Appendix D).

By comparing the entanglement measures obtained using the entanglement-swapping system shown in Figs. 5(h)–5(k) to those measured using the single-stage nondegenerate Josephson mixer shown in Figs. 1(k)–1(n), we find that (i) like the single-stage JM, the entanglement-swapping system exhibits two-mode entanglement when the entanglers are turned on, i.e., $G_{1,2} > 0$, as evidenced by E_N , E_F , and Simon's criterion for entanglement shown in Fig. 7. This means that, in principle, the entanglement-swapping system can be deployed to perform coherent state teleportation using the scheme of Fig. 2 with the single-stage entangler replaced by the entanglement-swapping system. (ii) The entanglement-swapping system exhibits an unavoidable degradation in the quality of the entanglement as indicated by the lower values achieved for E_N , E_F , and μ compared to the single-stage JM. (iii) Similar to the JM case, the measured two-mode entanglement for the entanglement-swapping case deteriorates and vanishes above certain entangler gains, e.g., $G_2 > 4$ dB, which yields a deviation from the theoretical model prediction beyond the margin of error. (iv) Unlike the case of JM, in the entanglement-swapping system, due to the finite gain on entangler 1, i.e., $G_1 = 1.4$ dB, Δ_{EPR}^- and Δ_{EPR}^+ lie above 0 dB for $G_2 = 0$ dB as seen in Fig. 5(h), and, in turn, Δ_{EPR}^- drops below 0 dB above a certain threshold $G_2 = 0.7$ dB [see Eq. (D28)] and reaches a minimum of -1.4 dB at $G_2 = 2.5$ dB.

Moreover, similar to the single JM case, we evaluate the effective maximum entanglement bit rate for the entanglement-swapping system. To get a conservative estimate, we consider the smallest dynamical bandwidth of the three JMs (belonging to Claire) of about $B = 32$ MHz at the maximal $E_F = 0.21$ working point, which together yield $R = 6.7$ Mbit/sec.

As yet another demonstration of the entanglement capability of the entanglement-swapping system, we set $G_1 = 1.2$ dB and $G_2 = 3.8$ dB and apply a coherent state with $\bar{n}_{\text{in}} = 0.92$ to port b of entangler 2, plotted in Fig. 6(a) in the $x-p$ quadrature plane, and measure the resultant amplified and entangled output fields at Bob and Alice shown in Figs. 6(b) and 6(c), respectively. Note that the field measured at Bob is largely a phase-preserving amplified version of the input signal reflecting off port b of entangler 2, while the one measured at Alice represents a teleported state of the entangled amplified signal at the output of port a of entangler 2. Also, similar to the

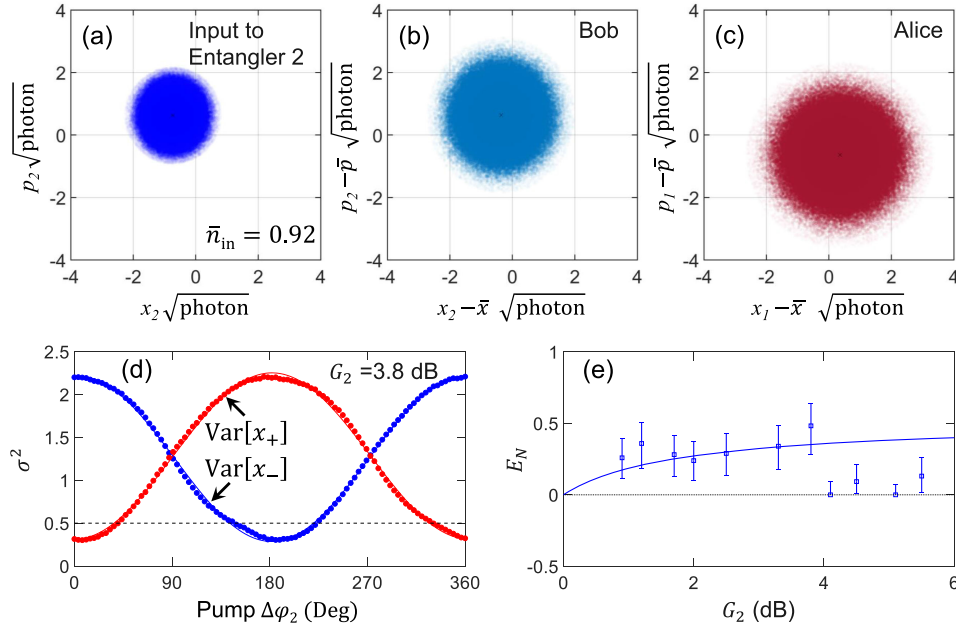


FIG. 6. Generating remote entangled coherent states at Alice and Bob using the entanglement-swapping setup. (a) I-Q (x - p) quadrature representation of a coherent state applied to the input b of entangler 2. (b),(c) The resulting entangled coherent states measured at Bob (mode b) and Alice (mode a), obtained for 1.2 and 3.8 dB of gain on entangler 1 and 2, respectively. The measured coherent states at Bob and Alice are plotted relative to their collective averages of the position $\bar{x} = (\bar{x}_1 + \bar{x}_2)/2$ and momentum $\bar{p} = (\bar{p}_1 + \bar{p}_2)/2$ quadratures. (d) The variance of EPR-like coordinates x_- (blue filled circles) and x_+ (red filled circles) plotted as a function of the relative pump phase of entangler 2. The relative phase between Claire and entangler 1 ($\Delta\phi_1$) is set to maximize the teleportation fidelity of Claire's input at port a . The relative pump phase axis is shifted to yield a minimum of $\text{Var}[x_-]$ at 180° . Solid blue and red lines represent sinusoidal fits. The dashed black line represents the variance of vacuum noise. (e) The logarithmic negativity of the coherent states at Alice and Bob measured while fixing the gain on entangler 1 at 1.2 dB and varying the gain on entangler 2.

vacuum-state input case [i.e., Figs. 5(f) and 5(j)], we show that TMSQ [Fig. 6(d)] and two-mode entanglement [Fig. 6(e)] can also be attained for applied coherent states in the entanglement-swapping setup.

V. DISCUSSION

Following the above demonstrations, we highlight three key aspects that can be improved in order to enhance the overall performance of our CV entanglement setups. First is reducing or eliminating the intermediate losses between the JMs and between the JMs and the directional couplers. Improving this aspect would be impactful, since our theoretical modeling and fits to the data show that the various measured entanglement metrics and teleportation fidelities are primarily limited by such losses. One concrete example which shows the detrimental effect of loss on CV entanglement can be seen in Fig. 3(d). While the calculated and measured teleportation fidelities are limited to about 0.73 due to intermediate losses in the system, the predicted fidelity in the lossless case exceeds 0.95 for $G_E = 6$ dB. However, eliminating these losses can be challenging. This is because considerable portions of the intermediate losses in the teleportation and entanglement-swapping setups originate from commercial ferrite-based circulators which are needed to separate incoming and outgoing

signals propagating on the same transmission lines and provide enough isolation between the different JMs (see Appendix E). Thus, eliminating these losses would require the development and deployment of lossless on-chip circulators [81,82] and nondegenerate through Josephson amplifiers that are capable of generating TMSQ in transmission rather than reflection [83] (a diagram of the proposed device is shown in Fig. 17). A similar functionality to the latter device could possibly be achieved using traveling-wave parametric amplifiers [64,84,85] and lossless diplexers designed to filter out the pump tones while allowing entangled modes at different frequencies, generated by the amplifiers, to travel on separate transmission lines. Second is enhancing the saturation power of JMs, especially those that fulfill the role of which-path-information erasers such as Alice and Claire in the teleportation and entanglement-swapping setups, respectively. Having JMs in these roles with high saturation powers enables them to process larger coherent states and tolerate higher power gains on the entanglers, which ideally correspond to larger squeezing and entanglement. While the present saturation power of JM ranges between -140 and -130 dBm at 20 dB (which amounts to a few photons) has not considerably limited the teleportation fidelity and entanglement-swapping metrics achieved in this work,

it limited the gains on the erasers to less than 15 dB, which, in turn, limited the maximum allowable attenuation of the directional couplers to 10 dB. It also limited the maximum input power of coherent states that can be teleported or subjected to entanglement-swapping operations. One promising venue for enhancing the saturation power of JMs is to optimize certain inductance ratios that characterize their JRMs and resonators [78]. Third is minimizing the arrival time difference of the various analog signals in the setup by optimizing the distances or cable lengths between the different parties, namely, Alice, Bob, Claire, and the entanglers. In our experiments, these distances were predominantly set by the relative locations of the various components inside the fridge and result in a few nanosecond difference in the propagation time along the different paths. Another likely contributor to arrival time differences is the signal processing time at the which-path-information erasers, i.e., Alice and Claire in the teleportation and entanglement-swapping setups, respectively, which is inversely proportional to their bandwidths and, therefore, falls in the range of a few tens of nanoseconds. To minimize the effect of such timing differences in our experiment, we measure the output signals in steady state averaged over 1–2 microseconds.

Furthermore, it is worth noting that using CV-based entanglement methods which employ broadband two-mode squeezers, such as traveling wave parametric amplifiers (TWPAs) or large-bandwidth JMs realized via impedance matching techniques [86–88], could offer a key scalability advantage over DV-based methods, in particular, if they enable frequency-multiplexed entanglement operations between arrays of qubits, which result in fewer links between modules than DV-based methods which typically require one quantum link for each pair of qubits. Such reduction in the number of links could help to alleviate possible connectivity bottlenecks in large systems.

Finally, it is important to emphasize that to generate high-fidelity remote qubit entanglement using CV entanglers similar to the ones demonstrated in this work requires the realization of efficient, high success rate CV to DV conversion and distillation schemes. To this end, several promising theoretical protocols have been proposed recently [26,28,89]. Further innovations will likely be necessary, though, to realize high-performance CV to DV distillation processes that enable large distributed quantum computers as well as experimental realizations of such schemes to uncover the practical limitations on the rate and fidelity of the distilled DV pairs.

VI. SUMMARY

In this work, we show that JMs can generate fast, high-quality CV entanglement between two different microwave modes traveling on separate transmission lines. At the optimal working point, we measure for the entangled

modes logarithmic negativity of 1.5 and generation rate of 70 Mebit/sec. By taking advantage of this built-in nondegenerate property, we utilize JMs in their roles as two-mode entanglers and which-path-information erasers to form hardware-efficient apparatus capable of performing quantum teleportation and entanglement swapping of input vacuum and coherent states. With the teleportation scheme, we achieve average quantum teleportation fidelities in the range 0.62–0.73 for coherent states having up to 1.2 input photons and a maximum fidelity of 0.73 for vacuum states, which in both cases surpass the classical teleportation limit of $1/2$ and are primarily limited by intermediate losses in the setup. Likewise, we demonstrate with the entanglement-swapping apparatus entanglement generation between two remote modes that lack any prior interaction with one another. At the optimal working point, we measure for the entangled modes produced by the entanglement-swapping setup logarithmic negativity of 0.53 and effective generation rate of 6.7 Mebit/sec.

In addition to potentially enabling quantum communication in the microwave domain and medium-range remote entanglement between quantum modules, the CV entangling building blocks presented here could serve as a verification and optimization test bed for a variety of theoretical protocols [26,28] designed for establishing long-range links via optical and microwave mode squeezers, especially since such squeezers are still in their early stage of development [31]. Furthermore, such CV entanglement schemes could find numerous applications in the areas of quantum cryptography [90–92] and quantum illumination [93,94].

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B. A. conceived and performed the experiments, developed the theory, and analyzed the data, W. S. developed the data-taking software, O. J. and J. R. R. made significant contributions to the experimental setup, J. O. supervised the project, and B. A. wrote the paper with input from the other authors.

The authors declare no competing interests.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available because they contain commercially sensitive information. The data are available from the authors upon reasonable request.

APPENDIX A: NONDEGENERATE TWO-MODE SQUEEZER

To obtain the output fields of the JM, we apply the unitary two-mode squeeze operator $S = \exp(re^{i\varphi_p} a_{\text{in}}^\dagger b_{\text{in}}^\dagger - re^{-i\varphi_p} a_{\text{in}} b_{\text{in}})$ to the input fields a_{in} and $b_{\text{in}}^\dagger$ [29,58]:

$$\begin{aligned} a_{\text{out}} &= S^\dagger a_{\text{in}} S = \cosh(r) a_{\text{in}} + e^{i\varphi_p} \sinh(r) b_{\text{in}}^\dagger, \\ b_{\text{out}}^\dagger &= S^\dagger b_{\text{in}}^\dagger S = \cosh(r) b_{\text{in}}^\dagger + e^{-i\varphi_p} \sinh(r) a_{\text{in}}, \end{aligned} \quad (\text{A1})$$

where φ_p is the phase of the pump and the squeezing parameter $r \geq 0$ relates to the device power gain by $G_J = \cosh^2(r)$.

In particular, when acting on the vacuum state at the input, the two-mode squeeze operator turns it into a two-mode squeezed vacuum state at the output $|\text{TMS}\rangle = S|0\rangle_a|0\rangle_b = \cosh^{-1}(r) \sum_n (\tanh(r))^n |n\rangle_a |n\rangle_b$, which represents a superposition of twin Fock states at the frequencies of modes a and b propagating on different physical channels.

Writing the field quadratures of the bosonic modes a and b in terms of the annihilation and creation operators, we have

$$\begin{aligned} I_a &= \text{Re}(a) = \frac{a + a^\dagger}{2}, & Q_a &= \text{Im}(a) = \frac{a - a^\dagger}{2i}, \\ I_b &= \text{Re}(b) = \frac{b + b^\dagger}{2}, & Q_b &= \text{Im}(b) = \frac{b - b^\dagger}{2i}, \end{aligned} \quad (\text{A2})$$

which are also equivalent to the dimensionless position and momentum coordinates of the harmonic oscillators associated with modes a and b or analogous to the same conjugate coordinates characterizing the motion of a pair of EPR particles 1 and 2:

$$\begin{aligned} x_1 &= \text{Re}(a) = \frac{a + a^\dagger}{2}, & p_1 &= \text{Im}(a) = \frac{a - a^\dagger}{2i}, \\ x_2 &= \text{Re}(b) = \frac{b + b^\dagger}{2}, & p_2 &= \text{Im}(b) = \frac{b - b^\dagger}{2i}, \end{aligned} \quad (\text{A3})$$

whose commutation relations for the input fields satisfy $[x_j, p_j] = i/2$, where $j = 1, 2$. It follows that $\langle(\Delta x_j)^2\rangle = \langle(\Delta p_j)^2\rangle = 1/4$ for input coherent states including vacuum and satisfy the Heisenberg limit $\langle(\Delta x_j)^2\rangle\langle(\Delta p_j)^2\rangle = 1/16$, where $\langle(\Delta x_j)^2\rangle \equiv \langle x_j^2 \rangle - \langle x_j \rangle^2$. In the expressions above, $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ refer to the real and imaginary parts, respectively.

While considering each output field separately, we obtain an amplified thermal state with variance $\langle(\Delta a_{\text{out}})^2\rangle = \langle(\Delta x_{\text{out},1})^2\rangle + \langle(\Delta p_{\text{out},1})^2\rangle = \cosh(2r)/2$. We uncover, however, nonlocal two-mode squeezing effects when evaluating the combinations of the output fields given by

$$a_{\text{out}} \pm e^{i\varphi_p} b_{\text{out}}^\dagger = e^{\pm r} (a_{\text{in}} \pm e^{i\varphi_p} b_{\text{in}}^\dagger). \quad (\text{A4})$$

In particular, for $\varphi_p = 0$, the relations given by Eq. (A4) exhibit cross-correlations between $x_{\text{out},1}$ and $x_{\text{out},2}$ and $p_{\text{out},1}$ and $p_{\text{out},2}$:

$$\begin{aligned} x_{\text{out},1} + x_{\text{out},2} &= e^r (x_{\text{in},1} + x_{\text{in},2}), \\ x_{\text{out},1} - x_{\text{out},2} &= e^{-r} (x_{\text{in},1} - x_{\text{in},2}), \\ p_{\text{out},1} + p_{\text{out},2} &= e^{-r} (p_{\text{in},1} + p_{\text{in},2}), \\ p_{\text{out},1} - p_{\text{out},2} &= e^r (p_{\text{in},1} - p_{\text{in},2}), \end{aligned} \quad (\text{A5})$$

which demonstrate squeezing in the amplitude of the relative position $x_{\text{out},1} - x_{\text{out},2}$ and total momentum $p_{\text{out},1} + p_{\text{out},2}$ of the EPR pair and antisqueezing along the orthogonal combinations. Furthermore, if we calculate the variance for the relative position and total momentum $\langle(x_1 - x_2)^2\rangle = \langle(p_1 + p_2)^2\rangle = e^{-2r}/2$, we find that, for any $r > 0$, the noise variance is squeezed below the standard vacuum limit of $1/2$, while for $\langle(x_1 + x_2)^2\rangle = \langle(p_1 - p_2)^2\rangle = e^{2r}/2$ the noise variance is amplified above that limit.

In the phase space spanned by the vector $\xi = (x_1, p_1, x_2, p_2)$, the squeezing operation can be described by

$$S(r, \varphi_p) = \begin{pmatrix} \cosh(r)I_2 & \sinh(r)ZR(\varphi_p) \\ \sinh(r)ZR(\varphi_p) & \cosh(r)I_2 \end{pmatrix}, \quad (\text{A6})$$

where

$$\begin{aligned} I_2 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, & Z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, & \text{and} \\ R(\varphi_p) &= \begin{pmatrix} \cos \varphi_p & \sin \varphi_p \\ -\sin \varphi_p & \cos \varphi_p \end{pmatrix}. \end{aligned} \quad (\text{A7})$$

The resulting two-mode squeezed state which belongs to the class of Gaussian states can be expressed by the Wigner function, which takes the form of normalized Gaussian distributions [29]:

$$W(\xi) = \frac{1}{4\pi^2 \sqrt{\det V}} \exp \left\{ -\frac{1}{2} \xi V^{-1} \xi^T \right\}, \quad (\text{A8})$$

which is fully characterized by its covariance matrix for the bipartite system denoted V , whose elements are given by

$$V_{ij} = \frac{1}{2} \langle \xi_i \xi_j + \xi_j \xi_i \rangle - \langle \xi_i \rangle \langle \xi_j \rangle, \quad (\text{A9})$$

where ξ_i and ξ_j are components of ξ .

More explicitly, we can cast the two-mode squeezing covariance matrix V in the form [31]

$$V = \begin{pmatrix} V_{aa} & V_{ab} \\ V_{ab}^T & V_{bb} \end{pmatrix} = \begin{pmatrix} V_{11} & 0 & \tilde{V}_{13} & \tilde{V}_{14} \\ 0 & V_{11} & \tilde{V}_{14} & -\tilde{V}_{13} \\ \tilde{V}_{13} & \tilde{V}_{14} & V_{33} & 0 \\ \tilde{V}_{14} & -\tilde{V}_{13} & 0 & V_{33} \end{pmatrix}, \quad (\text{A10})$$

where V_{aa} , V_{bb} , and V_{ab} are real 2×2 matrices. The diagonal parts of V , i.e., V_{aa} and V_{bb} , represent the variances of the quadratures of modes a and b (such as $x_{\text{out},1}$ and $p_{\text{out},1}$ for V_{aa}), while the off-diagonal part, i.e., V_{ab} , represents the covariance of the quadratures from both modes (such as $x_{\text{out},1}$ and $x_{\text{out},2}$). Since the nondegenerate two-mode squeezer is a phase-preserving amplifier which equally amplifies both quadratures of the mode, we have $V_{11} = V_{22}$ and $V_{33} = V_{44}$ (in the ideal case, $V_{11} = V_{22} = V_{33} = V_{44}$). Furthermore, due to the orthogonality of the x and p quadratures of the same mode, we have $V_{12} = V_{21} = V_{34} = V_{43} = 0$. The off-diagonal part which embodies the two-mode squeezing can be expressed as

$$V_{ab} = V_{13}[\cos(\varphi_p)Z + \sin(\varphi_p)X], \quad (\text{A11})$$

where $V_{13} \equiv \sqrt{\tilde{V}_{13}^2 + \tilde{V}_{14}^2}$, $\tan(\varphi_p) \equiv \tilde{V}_{14}/\tilde{V}_{13}$ and

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (\text{A12})$$

To further bring the covariance matrix into one of the standard forms and demonstrate the inseparability of the two subsystems, we apply a unitary local operation $U_{\text{loc}} = U_a \otimes U_b$, where U_a and U_b are local operations on modes a and b , respectively. For our purpose, we choose U_{loc} that rotates only the b mode with phase φ without altering mode a . This operation translates to a symplectic transformation $S(\varphi) = I_2 \oplus R(\varphi)$, which acts on the off-diagonal part of the covariance matrix $V(\varphi) = S(\varphi)VS^T(\varphi)$, yielding

$$V_{ab}(\varphi) = V_{13}[\cos(\varphi_p - \varphi)Z + \sin(\varphi_p - \varphi)X]. \quad (\text{A13})$$

By choosing $\varphi_p - \varphi = m\pi$, where m is an integer, the covariance matrix V takes the standard form

$$V = \begin{pmatrix} V_{11} & 0 & V_{13} & 0 \\ 0 & V_{11} & 0 & -V_{13} \\ V_{13} & 0 & V_{33} & 0 \\ 0 & -V_{13} & 0 & V_{33} \end{pmatrix}. \quad (\text{A14})$$

1. Duan's criterion for entanglement

To demonstrate entanglement of modes a and b , we apply the sufficient condition presented in Refs. [29,68], which states that, for any separable system, there exists a lower bound for the total variance of a pair of EPR-type coordinates, such as $x_- = x_1 - x_2$ and $p_+ = p_1 + p_2$. Thus, if for some local operation U_{loc} applied to the two subsystems, the inequality for the total variance given by

$$\langle (\Delta x_-)^2 \rangle + \langle (\Delta p_+)^2 \rangle \geq 1 \quad (\text{A15})$$

is violated, then the two subsystems are inseparable and, therefore, entangled.

Using the same U_{loc} introduced above, which rotates only mode b by φ while leaving mode a unchanged, we get $x_-(\varphi) = (x_1 - x_2 \cos \varphi - p_2 \sin \varphi)$ and $p_+(\varphi) \equiv (p_1 - x_2 \sin \varphi + p_2 \cos \varphi)$, whose total variance can be written as

$$\begin{aligned} \Delta_{\text{EPR}} &= \langle (\Delta x_-(\varphi))^2 \rangle + \langle (\Delta p_+(\varphi))^2 \rangle \\ &= 2(V_{11} + V_{33} - 2V_{13} \cos(\varphi_p - \varphi)), \end{aligned} \quad (\text{A16})$$

where $\langle (\Delta x_-(\varphi))^2 \rangle = \langle (\Delta p_+(\varphi))^2 \rangle$.

Note that Δ_{EPR} is minimized for $\varphi_- = \varphi_p$:

$$\Delta_{\text{EPR}}^- = 2(V_{11} + V_{33} - 2V_{13}) \quad (\text{A17})$$

and corresponds to two-mode squeezing of modes a and b , whereas, for $\varphi_+ = \varphi - \pi$, Δ_{EPR} is maximized:

$$\Delta_{\text{EPR}}^+ = 2(V_{11} + V_{33} + 2V_{13}) \quad (\text{A18})$$

and corresponds to the case of antisqueezing.

2. Simon criterion for entanglement

Another useful criterion for verifying the entanglement of two-mode Gaussian states is Simon's continuous-variable version of the Peres-Horodecki partial-transpose criterion [95], which for the standard form of the covariance matrix presented in Eq. (A14) reads [29]

$$16(V_{11}V_{33} - V_{13}^2)^2 \geq (V_{11} + V_{33}) + 2V_{13}^2 - \frac{1}{16}. \quad (\text{A19})$$

Satisfying this inequality is a necessary and sufficient condition for the separability of any bipartite Gaussian states. Conversely, the violation of the inequality is a necessary and sufficient condition for inseparability and, therefore, entanglement of modes a and b .

By defining a new variable

$$\Delta_S \equiv 16(V_{11}V_{33} - V_{13}^2)^2 - (V_{11} + V_{33}) - 2V_{13}^2 + 1/16, \quad (\text{A20})$$

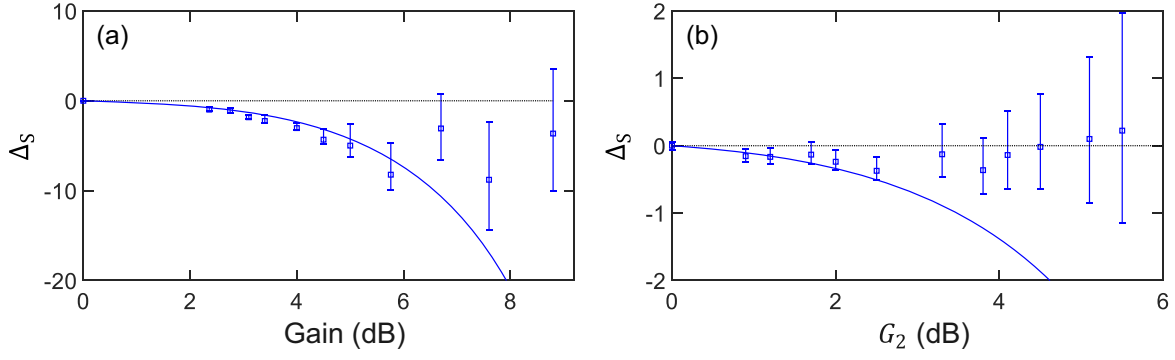


FIG. 7. Simon's criterion for entanglement. (a) Violation of Simon's criterion [Eq. (A21)] for the TMSQ device presented in Fig. 1, plotted as a function of the device gain. The solid blue curve represents a theoretical calculation of Δ_S which uses the entries of the covariance matrix in the presence of asymmetric loss [Eq. (A30)]. (b) The same as (a) but applies to the entanglement swapping experiment presented in Fig. 5, plotted as a function of gain on entangler 2 (while entangler 1 gain is fixed at $G_1 = 1.4$ dB). Similarly, the solid blue curve represents a theoretical calculation of Δ_S but uses the entries of the covariance matrix derived for the entanglement swapping case in the presence of loss [see Eqs. (D20)–(D23)].

we can express the inequality (A19) in a compact form:

$$\Delta_S \geq 0. \quad (\text{A21})$$

In Fig. 7, we exhibit two examples in which the above inequality is violated, thus indicating entanglement of the bipartite states. The examples presented in Figs. 7(a) and 7(b) belong to the TMSQ device and entanglement-swapping experiment whose two-mode squeezing and entanglement results are shown in Figs. 1 and 5, respectively. In Fig. 7(a), the parameter Δ_S is plotted as a function of the JM gain, whereas in Fig. 7(b) it is plotted versus the gain of entangler 2 while the gain on entangler 1 is fixed at 1.4 dB. The solid blue curves represent theoretical calculations of Δ_S using the entries of the covariance matrix given by Eq. (A30) for Fig. 7(a) and the entanglement-swapping covariance matrix given by Eqs. (D20)–(D23) for Fig. 7(b). Note that in both cases Simon's criterion for entanglement is well aligned with the other entanglement measures applied in the respective cases, in particular, the logarithmic negativity and EOF.

3. Entanglement measures

a. Logarithmic negativity

One of the important measures for quantifying entanglement of a bipartite state is the logarithmic negativity, which is directly related to the smallest symplectic eigenvalue of the partially transposed covariance matrix given by [59,64]

$$\nu_- = \sqrt{\frac{\Delta(V) - \sqrt{(\Delta(V))^2 - 4 \det(V)}}{2}}, \quad (\text{A22})$$

where

$$\Delta(V) = \det(V_{aa}) + \det(V_{bb}) - 2 \det(V_{ab}). \quad (\text{A23})$$

Using ν_- , the logarithmic negativity for Gaussian states can be evaluated by

$$E_N = \max[0, -\log_2(4\nu_-)]. \quad (\text{A24})$$

The two modes are, thus, entangled when $E_N > 0$, which is satisfied when $4\nu_- < 1$. The positivity of E_N serves not only as a necessary and sufficient condition for entanglement, but also as an upper bound on the number of equivalent entangled bits (i.e., Bell pairs) that can be extracted from the entangled quantum state using distillation procedures.

Note that in the symmetric case where $V_{11} = V_{33}$ the symplectic eigenvalue of the covariance matrix of Eq. (A14) simplifies to $\nu_- = V_{11} - V_{13}$.

b. Entanglement of formation

Another important measure for characterizing entanglement is the EOF [58,66,67], which quantifies the minimum amount of quantum bits (i.e., Bell pairs) required to create the quantum state as an ensemble of pure states. For a pure state, EOF reduces to the entanglement entropy which has an analytical solution for symmetric two-mode Gaussian states with a covariance matrix in the standard form of Eq. (A14) and $V_{11} = V_{33}$. In this case, the EOF reads

$$E_F = \max[0, h(\tilde{\nu}_-)], \quad (\text{A25})$$

where $h(\tilde{\nu}_-)$ is defined as

$$h(\tilde{\nu}_-) \equiv \frac{(1 + \tilde{\nu}_-)^2}{4\tilde{\nu}_-} \log_2 \left(\frac{(1 + \tilde{\nu}_-)^2}{4\tilde{\nu}_-} \right) - \frac{(1 - \tilde{\nu}_-)^2}{4\tilde{\nu}_-} \log_2 \left(\frac{(1 - \tilde{\nu}_-)^2}{4\tilde{\nu}_-} \right) \quad (\text{A26})$$

and $\tilde{\nu}_- = 4\nu_- = 4(V_{11} - V_{13})$.

c. Purity of the entangled state

The purity of two-mode Gaussian state with covariance matrix V reads [63]

$$\mu = \frac{1}{16\sqrt{\det(V)}}, \quad (\text{A27})$$

which suggests that the purity condition for the Gaussian state ($\mu = 1$) is met when $\det(V) = 1/16^2$.

It is worth noting that, unlike E_N and E_F which depend on both $\det(V)$ and $\Delta(V)$, which are invariant under the action of symplectic transformations, μ depends only on $\det(V)$. This implies that, in general, E_N and E_F provide a better characterization of the information content in the quantum state than μ .

4. Loss modeling

In the lossless case, the covariance matrix of the non-degenerate Josephson mixer reads [58]

$$V = S(r, 0)V_{\text{in}}S^T(r, 0) = \frac{1}{4} \begin{pmatrix} \cosh(2r) & 0 & \sinh(2r) & 0 \\ 0 & \cosh(2r) & 0 & -\sinh(2r) \\ \sinh(2r) & 0 & \cosh(2r) & 0 \\ 0 & -\sinh(2r) & 0 & \cosh(2r) \end{pmatrix}, \quad (\text{A28})$$

where $V_{\text{in}} = I_4/4$ is the covariance matrix of the input vacuum noise for modes a and b .

To quantify the effect of losses which the squeezed fields of modes a and b experience after leaving the device, we model the losses as beam splitters which couple uncorrelated cold thermal noise to each mode, such that

$$\begin{aligned} a_{\text{out}'} &= \sqrt{\bar{\alpha}}a_{\text{out}} + \sqrt{\alpha}a_{\text{th}}, \\ b_{\text{out}'} &= \sqrt{\bar{\beta}}b_{\text{out}} + \sqrt{\beta}b_{\text{th}}, \end{aligned} \quad (\text{A29})$$

where α and β are the effective lumped-element power losses in the path of modes a and b which satisfy the standard beam-splitter conditions $\bar{\alpha} = 1 - \alpha$ and $\bar{\beta} = 1 - \beta$, respectively, and a_{th} and b_{th} are the bosonic modes of the thermal baths at frequencies f_a and f_b , respectively.

Thus, the covariance matrix in the presence of asymmetric loss becomes [64]

$$V' = \frac{1}{4} \begin{pmatrix} \bar{\alpha} \cosh(2r) + \alpha & 0 & \sqrt{\bar{\alpha}\bar{\beta}} \sinh(2r) & 0 \\ 0 & \bar{\alpha} \cosh(2r) + \alpha & 0 & -\sqrt{\bar{\alpha}\bar{\beta}} \sinh(2r) \\ \sqrt{\bar{\alpha}\bar{\beta}} \sinh(2r) & 0 & \bar{\beta} \cosh(2r) + \beta & 0 \\ 0 & -\sqrt{\bar{\alpha}\bar{\beta}} \sinh(2r) & 0 & \bar{\beta} \cosh(2r) + \beta \end{pmatrix}. \quad (\text{A30})$$

In the symmetric loss case (i.e., $\alpha = \beta$), Eq. (A30) reads

$$V' = \frac{1-\alpha}{4} \begin{pmatrix} \cosh(2r) + \frac{\alpha}{1-\alpha} & 0 & \sinh(2r) & 0 \\ 0 & \cosh(2r) + \frac{\alpha}{1-\alpha} & 0 & -\sinh(2r) \\ \sinh(2r) & 0 & \cosh(2r) + \frac{\alpha}{1-\alpha} & 0 \\ 0 & -\sinh(2r) & 0 & \cosh(2r) + \frac{\alpha}{1-\alpha} \end{pmatrix}, \quad (\text{A31})$$

which verifies entanglement, since $4\nu_- = (1-\alpha)e^{-2r} + \alpha < 1$ for any gain $r > 0$ and limited loss $\alpha < 1$.

a. The effect of loss on the entanglement measures

In the presence of losses in modes a and b , the variance $\langle(\Delta x_-)^2\rangle$ of the joint quadrature $x_- = x_1 - x_2$ at its minimum value (i.e., squeezing) can be derived from Eq. (A17) by replacing the terms V_{11} , V_{33} , and V_{13} with the corresponding matrix elements of V' given by Eq. (A30) and using the relation $\langle(\Delta x_-)^2\rangle = \Delta_{\text{EPR}}^-/2$:

$$\langle(\Delta x_-)^2\rangle = \frac{1}{2} \left[\left(1 - \frac{\bar{\alpha} + \bar{\beta}}{2}\right) + \frac{e^{-2r}}{4} \left(\sqrt{\bar{\alpha}} + \sqrt{\bar{\beta}}\right)^2 + \frac{e^{2r}}{4} \left(\sqrt{\bar{\alpha}} - \sqrt{\bar{\beta}}\right)^2 \right], \quad (\text{A32}) \quad \text{and}$$

which is equivalent to the expression derived in Ref. [63].

Similarly, we can obtain the variance $\langle(\Delta x_+)^2\rangle$ of the orthogonal joint quadrature $x_+ = x_1 + x_2$ at its maximum value (i.e., antisqueezing) by substituting the corresponding matrix elements of V' in Eq. (A18) and using the relation $\langle(\Delta x_+)^2\rangle = \Delta_{\text{EPR}}^+/2$:

$$\langle(\Delta x_+)^2\rangle = \frac{1}{2} \left[\left(1 - \frac{\bar{\alpha} + \bar{\beta}}{2}\right) + \frac{e^{2r}}{4} \left(\sqrt{\bar{\alpha}} + \sqrt{\bar{\beta}}\right)^2 + \frac{e^{-2r}}{4} \left(\sqrt{\bar{\alpha}} - \sqrt{\bar{\beta}}\right)^2 \right]. \quad (\text{A33})$$

Likewise, to calculate E_N and E_F in the presence of asymmetric losses in modes a and b , we evaluate Eqs. (A24) and (A25) while using the covariance matrix V' [i.e., Eq. (A30)] to evaluate the symplectic eigenvalue ν_- of Eq. (A22). We also use V' in Eq. (A27) to evaluate the purity of the two-mode Gaussian state.

In the case of asymmetric losses and in the limit of large r , the logarithmic negativity approaches [63]

$$E_N \approx -\log_2 [e^{-2r} + \epsilon(1 - e^{-2r}) + \tanh(r)\epsilon^2\delta^2], \quad (\text{A34})$$

and the purity takes the approximate analytical form [63]

$$\mu \approx \frac{1}{1 + 2(1 - \epsilon)\epsilon[\cosh(2r) - 1]} - \left[\frac{\epsilon\delta}{2(1 - \epsilon)\epsilon} \right]^2 e^{-2r}, \quad (\text{A35})$$

where $\epsilon = (\alpha + \beta)/2$ represents the average loss while $\delta = (\alpha - \beta)/(\alpha + \beta)$ corresponds to the relative asymmetry in the loss.

In the symmetric loss case (i.e., $\alpha = \beta$), Eqs. (A32) and (A33) reduce to

$$\langle(\Delta x_-)^2\rangle = \frac{1}{2} [1 - \bar{\alpha} + \bar{\alpha}e^{-2r}] \quad (\text{A36})$$

$$\langle(\Delta x_+)^2\rangle = \frac{1}{2} [1 - \bar{\alpha} + \bar{\alpha}e^{2r}]. \quad (\text{A37})$$

Furthermore, the logarithmic negativity and state purity simplify to

$$E_N = -\log_2 [e^{-2r} + \alpha(1 - e^{-2r})] \quad (\text{A38})$$

and

$$\mu = \frac{1}{1 + 2\bar{\alpha}\alpha[\cosh(2r) - 1]}. \quad (\text{A39})$$

5. Reconstruction of the covariance matrix

Since the measured histograms of the output fields contain thermal noise added by the output chain, we can reconstruct the two-mode squeezing covariance matrix at the output of the device using the relation [59,64]

$$V = V^{\text{on}} - V^{\text{off}} + V_0, \quad (\text{A40})$$

where $V_0 = I_4/4$ corresponds to the covariance matrix for the input vacuum noise and V^{on} and V^{off} correspond to the covariance matrices referenced back to the output of the two-mode squeezer, measured when the pump is on and off, respectively:

$$\begin{aligned} V^{\text{on}} &= V + V^{\text{th}}, \\ V^{\text{off}} &= V_0 + V^{\text{th}}, \end{aligned} \quad (\text{A41})$$

where V^{th} represents the diagonal covariance matrix of the uncorrelated thermal noise added by the output chain:

$$V^{\text{th}} = \frac{1}{4} \begin{pmatrix} (1 + 2n_a^{\text{th,sys}}) & 0 & 0 & 0 \\ 0 & (1 + 2n_a^{\text{th,sys}}) & 0 & 0 \\ 0 & 0 & (1 + 2n_b^{\text{th,sys}}) & 0 \\ 0 & 0 & 0 & (1 + 2n_b^{\text{th,sys}}) \end{pmatrix} \quad (\text{A42})$$

and $n_a^{\text{th,sys}} = \langle h_a^\dagger h_a \rangle$, $n_b^{\text{th,sys}} = \langle h_b^\dagger h_b \rangle$.

6. Which-path-information eraser

The entanglement between the output fields of the non-degenerate two-mode squeezer originates from its action as a which-path-information eraser for the input fields impinging on its ports which are spectrally and physically distinct. To demonstrate this fact, we start with the input and output scattering relations of the JM, which read [96]

$$\begin{aligned} a_{\text{out}} &= \sqrt{G_J} a_{\text{in}} + \sqrt{G_J - 1} b_{\text{in}}^\dagger, \\ b_{\text{out}}^\dagger &= \sqrt{G_J} b_{\text{in}}^\dagger + \sqrt{G_J - 1} a_{\text{in}}, \end{aligned} \quad (\text{A43})$$

where we assume, without loss of generality, that $\varphi_p = 0$. Using Eqs. (A43) and (A2), the output I and Q quadratures of mode a when referred back to the input read

$$\begin{aligned} \tilde{I}_{\text{out},a} &= I_{\text{out},a} / \sqrt{G_J} = I_{\text{in},a} + (\sqrt{G_J - 1} / \sqrt{G_J}) I_{\text{in},b}, \\ \tilde{Q}_{\text{out},a} &= Q_{\text{out},a} / \sqrt{G_J} = Q_{\text{in},a} - (\sqrt{G_J - 1} / \sqrt{G_J}) Q_{\text{in},b}, \end{aligned} \quad (\text{A44})$$

which in the limit of large gain $G_J \gg 1$ reduces to

$$\begin{aligned} \tilde{I}_{\text{out},a} &= I_{\text{in},a} + I_{\text{in},b}, \\ \tilde{Q}_{\text{out},a} &= Q_{\text{in},a} - Q_{\text{in},b}. \end{aligned} \quad (\text{A45})$$

Equation (A45) shows that, in the high-gain limit, the quadratures of the output field, which constitute the measured physical observables of the field, contain symmetric combinations of the quadratures of its input fields. Hereby, measurement of the output field at either port does not convey information about the origin of the input signals. Also, note that $\tilde{I}_{\text{out},a}$ and $\tilde{Q}_{\text{out},a}$ commute, i.e., $[\tilde{I}_{\text{out},a}, \tilde{Q}_{\text{out},a}] = 0$. This means they can be measured simultaneously with arbitrary precision. A similar relation can be derived for the output quadratures I and Q of mode b .

APPENDIX B: QUANTUM TELEPORTATION OF COHERENT STATES WITH NONDEGENERATE TWO-MODE SQUEEZERS

The calculations carried out in this section and the next show how the scheme of Fig. 2 performs a quantum teleportation of an unknown coherent state from Alice to Bob.

We start with the entangled fields at the output of the entangler given by

$$\begin{aligned} a_{\text{out},E} &= \cosh(r_E) a_{\text{in},E} + e^{i\varphi_E} \sinh(r_E) b_{\text{in},E}^\dagger, \\ b_{\text{out},E}^\dagger &= \cosh(r_E) b_{\text{in},E}^\dagger + e^{-i\varphi_E} \sinh(r_E) a_{\text{in},E}. \end{aligned} \quad (\text{B1})$$

The entangler field that reaches Alice reads

$$a_{\text{in},A} = \sqrt{\alpha} a_{\text{out},E} + \sqrt{\alpha} a_{\text{th},E}, \quad (\text{B2})$$

where we use beam-splitter interactions with thermal baths to model the effect of losses experienced by modes a and b in their paths shown in Fig. 2. Namely, we define $\bar{\beta} \equiv 1 - \beta$, $\bar{\beta}_c \equiv 1 - \beta_c$, $\bar{\alpha} \equiv 1 - \alpha$, and $\bar{\beta}_f \equiv 1 - \beta_f$ and use a_{th} and b_{th} to represent the bosonic modes of the thermal baths introduced by the loss in the paths of modes a and b , respectively.

Let $b_{\text{in},A}^\dagger$ be the unknown input quantum state which Alice wants to teleport to Bob. The output of the effective projective joint measurement that Alice performs on the unknown input state and the entangler field (sent to her by the EPR source) reads

$$\begin{aligned} a_{\text{out},A} &= \cosh(r_A) a_{\text{in},A} + e^{i\varphi_A} \sinh(r_A) b_{\text{in},A}^\dagger, \\ b_{\text{out},A}^\dagger &= \cosh(r_A) b_{\text{in},A}^\dagger + e^{-i\varphi_A} \sinh(r_A) a_{\text{in},A}. \end{aligned} \quad (\text{B3})$$

Substituting Eq. (B2) in Eqs. (B3) while using Eqs. (B1) yields

$$\begin{aligned} b_{\text{out},A}^\dagger &= \cosh(r_A) b_{\text{in},A}^\dagger + e^{-i\varphi_A} \sqrt{\alpha} \sinh(r_A) \cosh(r_E) a_{\text{in},E} \\ &\quad + e^{i\varphi_{EA}} \sqrt{\alpha} \sinh(r_A) \sinh(r_E) b_{\text{in},E}^\dagger \\ &\quad + e^{-i\varphi_A} \sqrt{\alpha} \sinh(r_A) a_{\text{th},E} \end{aligned} \quad (\text{B4})$$

and

$$\begin{aligned} a_{\text{out},A} &= \sqrt{\alpha} \cosh(r_A) \cosh(r_E) a_{\text{in},E} \\ &\quad + e^{i\varphi_E} \sqrt{\alpha} \cosh(r_A) \sinh(r_E) b_{\text{in},E}^\dagger \\ &\quad + e^{i\varphi_A} \sinh(r_A) b_{\text{in},A}^\dagger + \sqrt{\alpha} \cosh(r_A) a_{\text{th},E}, \end{aligned} \quad (\text{B5})$$

where $\varphi_{EA} \equiv \varphi_E - \varphi_A$.

The field at Bob's output, which represents the teleported state, is given by

$$\begin{aligned} b_B^\dagger &= \sqrt{\beta_c} \left[\sqrt{\bar{\beta}_f} b_{\text{out},A}^\dagger + \sqrt{\beta_f} b_{\text{th},A}^\dagger \right] \\ &\quad + \sqrt{\bar{\beta}_c} \left[\sqrt{\bar{\beta}} b_{\text{out},E}^\dagger + \sqrt{\beta} b_{\text{th},E}^\dagger \right]. \end{aligned} \quad (\text{B6})$$

The first term in the above expression represents the projective measurement result, which Alice feedforwards to Bob, while the second represents the field sent to Bob by the entangler (i.e., the EPR source).

Substituting Eqs. (B1) and (B4) in Eq. (B6) gives

$$\begin{aligned} b_B^\dagger = & \sqrt{\beta_c \bar{\beta}_f} \cosh(r_A) b_{\text{in},A}^\dagger + e^{-i\varphi_E} \left[\sqrt{\beta_c \bar{\beta}_f} \bar{\alpha} e^{i\varphi_{EA}} \sinh(r_A) \cosh(r_E) + \sqrt{\bar{\beta} \bar{\beta}_c} \sinh(r_E) \right] a_{\text{in},E} \\ & + \left[\sqrt{\beta_c \bar{\beta}_f} \bar{\alpha} e^{i\varphi_{EA}} \sinh(r_A) \sinh(r_E) + \sqrt{\bar{\beta} \bar{\beta}_c} \cosh(r_E) \right] b_{\text{in},E}^\dagger \\ & + \sqrt{\beta_c \bar{\beta}_f} \alpha e^{-i\varphi_A} \sinh(r_A) a_{\text{th},E} + \sqrt{\beta_c \bar{\beta}_f} b_{\text{th},A}^\dagger + \sqrt{\bar{\beta} \bar{\beta}_c} b_{\text{th},E}^\dagger. \end{aligned} \quad (\text{B7})$$

Under the condition of unity transmission in the feedforward channel $\sqrt{\beta_c \bar{\beta}_f} \cosh(r_A) = 1$, large gain at Alice $\cosh(r_A) \gg 1$ (mimicking a projective measurement), and small coupling at Bob $\beta_c \ll 1$, we obtain

$$\begin{aligned} b_B^\dagger = & b_{\text{in},A}^\dagger + e^{-i\varphi_E} \left[\sqrt{\bar{\alpha}} e^{i\varphi_{EA}} \cosh(r_E) + \sqrt{\bar{\beta}} \sinh(r_E) \right] a_{\text{in},E} \\ & + \left[\sqrt{\bar{\alpha}} e^{i\varphi_{EA}} \sinh(r_E) + \sqrt{\bar{\beta}} \cosh(r_E) \right] b_{\text{in},E}^\dagger \\ & + \sqrt{\bar{\alpha}} e^{-i\varphi_A} a_{\text{th},E} + \sqrt{\bar{\beta}} b_{\text{th},E}^\dagger, \end{aligned} \quad (\text{B8})$$

where the approximations $\sqrt{\beta_c \bar{\beta}_f} \sinh(r_A) \cong 1$ and $\bar{\beta}_c \simeq 1$ have been used.

In the limit of vanishing insertion losses $\bar{\alpha}, \bar{\beta} \rightarrow 1$ ($\alpha, \beta \rightarrow 0$), Eq. (B8) simplifies to

$$\begin{aligned} b_B^\dagger = & b_{\text{in},A}^\dagger + e^{-i\varphi_E} [e^{i\varphi_{EA}} \cosh(r_E) + \sinh(r_E)] a_{\text{in},E} \\ & + [e^{i\varphi_{EA}} \sinh(r_E) + \cosh(r_E)] b_{\text{in},E}^\dagger. \end{aligned} \quad (\text{B9})$$

In the special case when the relative pump phase between the entangler and Alice's amplifier is set to $\varphi_{EA} = 0$ (or integer multiples of 2π), we get the original input state $b_{\text{in},A}^\dagger$ superimposed with an amplified input vacuum noise of the entangler that is enhanced by an amplitude gain factor of e^{r_E} :

$$b_B^\dagger = b_{\text{in},A}^\dagger + e^{r_E} [e^{-i\varphi_E} a_{\text{in},E} + b_{\text{in},E}^\dagger]. \quad (\text{B10})$$

On the other hand, when the relative pump phase between the entangler and Alice's amplifier is set to $\varphi_{EA} = \pi$ (or odd integer multiples of π), we get the original input state $b_{\text{in},A}^\dagger$ superimposed with a squeezed input vacuum noise of the entangler that is diminished by the same amplitude gain factor e^{r_E} :

$$b_B^\dagger = b_{\text{in},A}^\dagger + e^{-r_E} [e^{-i\varphi_E} a_{\text{in},E} + b_{\text{in},E}^\dagger]. \quad (\text{B11})$$

In turn, in the limit of large two-mode squeezing of the entangler $r_E \gg 0$, we obtain at Bob's output a complete and faithful teleportation of the unknown coherent state input at Alice, that is,

$$b_B^\dagger = b_{\text{in},A}^\dagger. \quad (\text{B12})$$

In comparison, when classical teleportation is conducted in which no entanglement is shared between Alice and Bob ($r_E = 0$), Eq. (B9) becomes

$$b_B^\dagger = b_{\text{in},A}^\dagger + e^{-i\varphi_E} a_{\text{in},E} + b_{\text{in},E}^\dagger. \quad (\text{B13})$$

From these results, we see that while in quantum teleportation it is possible to fully teleport the unknown coherent quantum state from Alice to Bob [Eq. (B12)], in the classical case, even in the absence of losses, the input state is teleported with unavoidable excess noise corresponding to vacuum noise added by Alice's amplifier (measurement device) and vacuum noise added at Bob's location [which are, respectively, represented by the second and third terms in Eq. (B13)].

APPENDIX C: NOISE POWER OF THE TELEPORTED COHERENT STATE

The noise power measured with a spectrum analyzer is proportional to the symmetrized variance of the field operator

$$(\Delta b_B^\dagger)^2 = \frac{\langle \{b_B, b_B^\dagger\} \rangle}{2} - |\langle b_B^\dagger \rangle|^2, \quad (\text{C1})$$

where $\{.,.\}$ denotes the anticommutator.

Under the conditions of unity transmission in the feedforward classical channel, i.e., $\sqrt{\beta_c \bar{\beta}_f} \cosh(r_A) = 1$, large gain approximation at Alice $\sqrt{\beta_c \bar{\beta}_f} \sinh(r_A) \cong 1$, and small coupling at Bob, i.e., $\beta_c \ll 1$, substituting Eq. (B8) in Eq. (C1) yields

$$\begin{aligned} (\Delta b_B^\dagger)^2 = & (\Delta b_{\text{in},A}^\dagger)^2 \\ & + \left| \sqrt{\bar{\alpha}} e^{i\varphi_{EA}} \cosh(r_E) + \sqrt{\bar{\beta}} \sinh(r_E) \right|^2 (\Delta a_{\text{in},E})^2 \\ & + \left| \sqrt{\bar{\alpha}} e^{i\varphi_{EA}} \sinh(r_E) + \sqrt{\bar{\beta}} \cosh(r_E) \right|^2 (\Delta b_{\text{in},E}^\dagger)^2 \\ & + \alpha (\Delta a_{\text{th},E})^2 + \beta (\Delta b_{\text{th},E}^\dagger)^2, \end{aligned} \quad (\text{C2})$$

which expands into

$$\begin{aligned}
 (\Delta b_B^\dagger)^2 &= (\Delta b_{\text{in},A}^\dagger)^2 + \left[\bar{\alpha} \cosh^2(r_E) + \bar{\beta} \sinh^2(r_E) + \sqrt{\bar{\alpha}\bar{\beta}} \sinh(2r_E) \cos(\varphi_{EA}) \right] (\Delta a_{\text{in},E})^2 \\
 &\quad + \left[\bar{\alpha} \sinh^2(r_E) + \bar{\beta} \cosh^2(r_E) + \sqrt{\bar{\alpha}\bar{\beta}} \sinh(2r_E) \cos(\varphi_{EA}) \right] (\Delta b_{\text{in},E}^\dagger)^2 \\
 &\quad + \alpha (\Delta a_{\text{th},E})^2 + \beta (\Delta b_{\text{th},E}^\dagger)^2.
 \end{aligned} \tag{C3}$$

Assuming balanced losses in the path of the entangled fields, i.e., $\bar{\alpha} = \bar{\beta}$, we get

$$(\Delta b_B)^2 = (\Delta b_{\text{in},A}^\dagger)^2 + \bar{\alpha} [\cosh(2r_E) + \sinh(2r_E) \cos(\varphi_{EA})] [(\Delta a_{\text{in},E})^2 + (\Delta b_{\text{in},E}^\dagger)^2] + \alpha [(\Delta a_{\text{th},E})^2 + (\Delta b_{\text{th},E}^\dagger)^2]. \tag{C4}$$

When the relative phase between the pumps is set to $\varphi_{EA} = 0$ (or integer multiples of 2π), we get

$$\begin{aligned}
 (\Delta b_B^\dagger)^2 &= (\Delta b_{\text{in},A}^\dagger)^2 + \bar{\alpha} e^{2r_E} [(\Delta a_{\text{in},E})^2 + (\Delta b_{\text{in},E}^\dagger)^2] \\
 &\quad + \alpha [(\Delta a_{\text{th},E})^2 + (\Delta b_{\text{th},E}^\dagger)^2].
 \end{aligned} \tag{C5}$$

In this case, the noise power of the teleported state $(\Delta b_B^\dagger)^2$ exceeds that of the original input state $(\Delta b_{\text{in},A}^\dagger)^2$, and the added noise by Alice and Bob is amplified by the factor e^{2r_E} .

On the other hand, when the relative phase between the pumps is set to $\varphi_{EA} = \pi$ (or odd integer multiples of π), we obtain

$$\begin{aligned}
 (\Delta b_B^\dagger)^2 &= (\Delta b_{\text{in},A}^\dagger)^2 + \bar{\alpha} e^{-2r_E} [(\Delta a_{\text{in},E})^2 + (\Delta b_{\text{in},E}^\dagger)^2] \\
 &\quad + \alpha [(\Delta a_{\text{th},E})^2 + (\Delta b_{\text{th},E}^\dagger)^2],
 \end{aligned} \tag{C6}$$

which in this case exhibits suppression of the added noise by Alice and Bob by the same factor e^{2r_E} .

If we further assume lack of losses in the paths of the entangled fields $\bar{\alpha} = 1$ ($\alpha = 0$), then, in the limit of large squeezing by the entangler $r_E \gg 0$, the relation (C6) reduces to

$$(\Delta b_B^\dagger)^2 = (\Delta b_{\text{in},A}^\dagger)^2. \tag{C7}$$

This result demonstrates that in the quantum domain it is possible to teleport the coherent state faithfully from one place to another without adding any noise throughout the process.

In contrast, in classical teleportation, when Alice and Bob do not share any entangled fields from an EPR source (such as the entangler), corresponding to the condition $r_E = 0$, we obtain in the ideal lossless case

$$(\Delta b_B^\dagger)^2 = (\Delta b_{\text{in},A}^\dagger)^2 + (\Delta a_{\text{in},E})^2 + (\Delta b_{\text{in},E}^\dagger)^2. \tag{C8}$$

Equation (C8) shows that classical teleportation inevitably adds excess noise to the teleported state manifested by the second and third terms of the expression, which can be attributed to vacuum noise added by Alice during measurement and Bob during construction of the state, respectively.

To illustrate the general behavior of the noise variance at Bob resulting from the teleportation setup (shown in Fig. 2), we plot in Fig. 8 the variance $(\Delta b_B^\dagger)^2$ given by Eq. (C4) as a function of the relative pump phase between the entangler and Alice. The different curves depicted in Fig. 8(a) correspond to different power transmission parameters, i.e., $\bar{\alpha}$, while the entangler gain is fixed $G_E = 3$ dB, whereas in Fig. 8(b) the curves correspond to different G_E , while $\bar{\alpha} = 0.9$ is kept constant. The dashed black lines in both panels correspond to the vacuum variance equal to $1/2$. The horizontal blue line in Fig. 8(b) corresponds to the classical teleportation scenario in which no entanglement is shared, i.e., $G_E = 0$ dB, which yields $3/2$ in agreement with the result of Eq. (C8). As seen in Fig. 8, the noise variance is periodic with the pump phase. It is maximized (minimized) at even (odd) number multiples of π , where teleportation with maximum fidelity occurs at the minima points. Also, as expected, the achievable minimum of the teleported noise decrease as the loss in the setup is decreased [Fig. 8(a)] or as the entangler gain is increased [Fig. 8(b)].

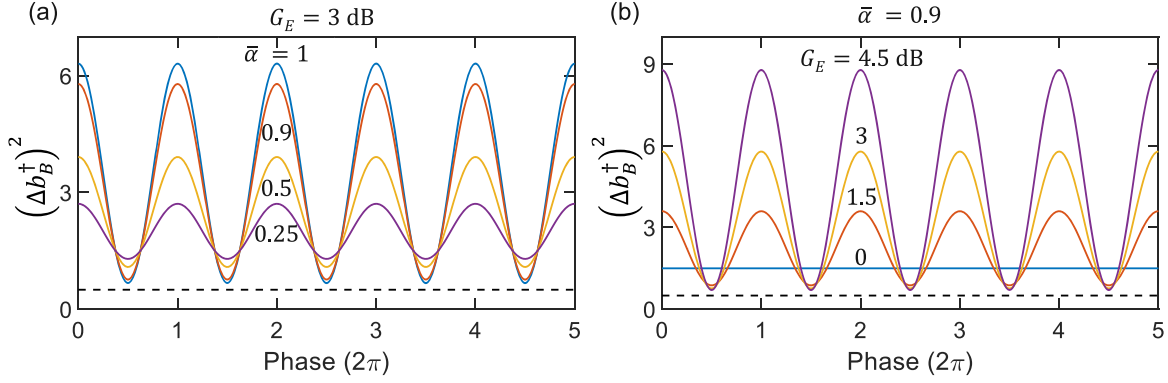


FIG. 8. Calculated noise at Bob (in photons) as a function of the relative pump phase between the entangler and Alice. The curves in (a) and (b) are calculated in response to a vacuum noise at Alice's input in the quantum teleportation setup (depicted in Fig. 2). Quantum teleportation occurs at the minima points of the curves corresponding to odd multiples of π , where the teleported state approaches the vacuum state (i.e., $1/2$) indicated by the dashed black line. In both (a) and (b), the transmission loss is assumed to be symmetric, i.e., $\bar{\alpha} = \bar{\beta}$. While the curves in (a) are calculated for a constant $G_E = 3$ dB with different transmission losses $\bar{\alpha}$, the curves in (b) are calculated for a constant $\bar{\alpha} = 0.9$ with different entangler gains G_E . The blue constant line in (b) corresponds to the classical teleportation case for which $G_E = 0$ dB and equals $3/2$ as expected.

APPENDIX D: ENTANGLEMENT SWAPPING WITH NONDEGENERATE TWO-MODE SQUEEZERS

The calculation carried out in this section demonstrates how the entanglement-swapping scheme of Fig. 5(a) performs its operation.

We start with the entangled fields at the output of entangler 1 given by

$$\begin{aligned} a_{\text{out},1} &= \cosh(r_1)a_{\text{in},1} + e^{i\varphi_1} \sinh(r_1)b_{\text{in},1}^\dagger, \\ b_{\text{out},1}^\dagger &= \cosh(r_1)b_{\text{in},1}^\dagger + e^{-i\varphi_1} \sinh(r_1)a_{\text{in},1}. \end{aligned} \quad (\text{D1})$$

Likewise, the entangled fields at the output of entangler 2 are given by

$$\begin{aligned} a_{\text{out},2} &= \cosh(r_2)a_{\text{in},2} + e^{i\varphi_2} \sinh(r_2)b_{\text{in},2}^\dagger, \\ b_{\text{out},2}^\dagger &= \cosh(r_2)b_{\text{in},2}^\dagger + e^{-i\varphi_2} \sinh(r_2)a_{\text{in},2}. \end{aligned} \quad (\text{D2})$$

The input fields at Claire's amplifier can be written as

$$\begin{aligned} a_{\text{in},3} &= \sqrt{\bar{\alpha}_2}a_{\text{out},2} + \sqrt{\bar{\alpha}_2}a_{\text{th},2}, \\ b_{\text{in},3}^\dagger &= \sqrt{\bar{\beta}_1}b_{\text{out},1}^\dagger + \sqrt{\bar{\beta}_1}b_{\text{th},1}^\dagger, \end{aligned} \quad (\text{D3})$$

which, in turn, yield the following entangled fields at the output:

$$\begin{aligned} a_{\text{out},3} &= \cosh(r_3)a_{\text{in},3} + e^{i\varphi_3} \sinh(r_3)b_{\text{in},3}^\dagger, \\ b_{\text{out},3}^\dagger &= \cosh(r_3)b_{\text{in},3}^\dagger + e^{-i\varphi_3} \sinh(r_3)a_{\text{in},3}. \end{aligned} \quad (\text{D4})$$

Substituting Eqs. (D1)–(D3) in Eq. (D4) gives

$$\begin{aligned} a_{\text{out},3} &= \sqrt{\bar{\alpha}_2} \cosh(r_2) \cosh(r_3) a_{\text{in},2} + \sqrt{\bar{\alpha}_2} e^{i\varphi_2} \sinh(r_2) \cosh(r_3) b_{\text{in},2}^\dagger + \sqrt{\bar{\beta}_1} e^{i\varphi_3} \cosh(r_1) \sinh(r_3) b_{\text{in},1}^\dagger \\ &\quad + \sqrt{\bar{\beta}_1} e^{i\varphi_3} \sinh(r_1) \sinh(r_3) a_{\text{in},1} + \sqrt{\bar{\alpha}_2} \cosh(r_3) a_{\text{th},2} + \sqrt{\bar{\beta}_1} e^{i\varphi_3} \sinh(r_3) b_{\text{th},1}^\dagger. \end{aligned} \quad (\text{D5})$$

The output field at Alice is given by

$$a_A = \sqrt{\bar{\beta}_c} [\sqrt{\bar{\alpha}_f} a_{\text{out},3} + \sqrt{\bar{\alpha}_f} a_{\text{th},3}] + \sqrt{\bar{\beta}_c} [\sqrt{\bar{\alpha}_1} a_{\text{out},1} + \sqrt{\bar{\alpha}_1} a_{\text{th},1}], \quad (\text{D6})$$

while the output field at Bob reads

$$b_B^\dagger = \sqrt{\bar{\beta}_2} b_{\text{out},2}^\dagger + \sqrt{\bar{\beta}_2} b_{\text{th},2}^\dagger. \quad (\text{D7})$$

Substituting Eqs. (D1) and (D5) in Eq. (D6) gives

$$\begin{aligned}
 a_A = & \sqrt{\beta_c \bar{\alpha}_f \bar{\alpha}_2} \cosh(r_2) \cosh(r_3) a_{\text{in},2} + \sqrt{\beta_c \bar{\alpha}_f \bar{\alpha}_2} e^{i\varphi_2} \sinh(r_2) \cosh(r_3) b_{\text{in},2}^\dagger \\
 & + e^{i\varphi_1} \left[\sqrt{\beta_c \bar{\alpha}_f \bar{\beta}_1} e^{i\varphi_{31}} \cosh(r_1) \sinh(r_3) + \sqrt{\bar{\beta}_c \bar{\alpha}_1} \sinh(r_1) \right] b_{\text{in},1}^\dagger \\
 & + \left[\sqrt{\beta_c \bar{\alpha}_f \bar{\beta}_1} e^{i\varphi_{31}} \sinh(r_1) \sinh(r_3) + \sqrt{\bar{\beta}_c \bar{\alpha}_1} \cosh(r_1) \right] a_{\text{in},1} \\
 & + \sqrt{\beta_c \bar{\alpha}_f \alpha_2} \cosh(r_3) a_{\text{th},2} + \sqrt{\beta_c \bar{\alpha}_f \beta_1} e^{i\varphi_3} \sinh(r_3) b_{\text{th},1}^\dagger + \sqrt{\beta_c \alpha_f} a_{\text{th},3} + \sqrt{\bar{\beta}_c \alpha_1} a_{\text{th},1}. \quad (\text{D8})
 \end{aligned}$$

Requiring unity transmission through the feedforward classical channel, given by $\sqrt{\beta_c \bar{\alpha}_f} \cosh(r_3) = 1$ and large gain at Claire given by $\sqrt{\beta_c \bar{\alpha}_f} \sinh(r_3) \cong 1$, yields

$$\begin{aligned}
 a_A = & \sqrt{\bar{\alpha}_2} \cosh(r_2) a_{\text{in},2} + \sqrt{\bar{\alpha}_2} e^{i\varphi_2} \sinh(r_2) b_{\text{in},2}^\dagger + e^{i\varphi_1} \left[\sqrt{\bar{\beta}_1} e^{i\varphi_{31}} \cosh(r_1) + \sqrt{\bar{\beta}_c \bar{\alpha}_1} \sinh(r_1) \right] b_{\text{in},1}^\dagger \\
 & + \left[\sqrt{\bar{\beta}_1} e^{i\varphi_{31}} \sinh(r_1) + \sqrt{\bar{\beta}_c \bar{\alpha}_1} \cosh(r_1) \right] a_{\text{in},1} + \sqrt{\bar{\alpha}_2} a_{\text{th},2} + \sqrt{\bar{\beta}_1} e^{i\varphi_3} b_{\text{th},1}^\dagger + \sqrt{\beta_c \alpha_f} a_{\text{th},3} + \sqrt{\bar{\beta}_c \alpha_1} a_{\text{th},1}. \quad (\text{D9})
 \end{aligned}$$

In the absence of loss in the path between entanglers 1 and 2 and Claire, i.e., $\bar{\alpha}_{1,2} = 1$, $\bar{\beta}_{1,2} = 1$ ($\alpha_{1,2} = 0$, $\beta_{1,2} = 0$), and small coupling at Alice $\beta_c \simeq 0$ ($\bar{\beta}_c \simeq 1$), Eq. (D9) reduces to

$$a_A = a_{\text{out},2} + \left[a_{\text{out},1} + e^{i\varphi_3} b_{\text{out},1}^\dagger \right], \quad (\text{D10})$$

which can also be cast in the form

$$\begin{aligned}
 a_A = & a_{\text{out},2} + [\cosh(r_1) + e^{i\varphi_{31}} \sinh(r_1)] a_{\text{in},1} \\
 & + [e^{i\varphi_{31}} \cosh(r_1) + \sinh(r_1)] e^{i\varphi_1} b_{\text{in},1}^\dagger. \quad (\text{D11})
 \end{aligned}$$

Setting φ_{31} , the relative phase of the pumps between Claire and entangler 1, to 0 (or integer multiples of 2π) gives

$$a_A = a_{\text{out},2} + e^{r_1} \left[a_{\text{in},1} + e^{i\varphi_1} b_{\text{in},1}^\dagger \right], \quad (\text{D12})$$

whereas setting φ_{31} to π (or odd integer multiples of π) yields

$$a_A = a_{\text{out},2} + e^{-r_1} \left[a_{\text{in},1} - e^{i\varphi_1} b_{\text{in},1}^\dagger \right]. \quad (\text{D13})$$

In the limit of large gain on entangler 1, corresponding to $r_1 \gg 0$, Eq. (D13) becomes

$$a_A = a_{\text{out},2}. \quad (\text{D14})$$

This means that the output field of mode a of entangler 2 is fully teleported to Alice. Moreover, since the output field at Bob in the lossless case is equal to $b_{\text{out},2}^\dagger$ [see Eq. (D7)],

the output fields of Alice and Bob become entangled. Thus, even though the fields of entanglers 1 ($a_{\text{out},1}$) and 2 ($b_{\text{out},2}^\dagger$) never interact directly, they become entangled through the joint measurement of $b_{\text{out},1}^\dagger$ and $a_{\text{out},2}$ performed by Claire and the classical signal she sends to Alice containing the measurement outcome. Therefore, this scheme effectively passes the entanglement generated at Claire between the received fields on to their distant noninteracting counterparts.

1. Covariance matrix of entanglement swapping

Here, we derive the covariance matrix that characterizes the correlations between the output fields at Alice and Bob upon applying the entanglement swapping operation depicted in Fig. 5(a).

To calculate the covariance matrix elements given by Eq. (A9), we express the generalized position x_A and x_B and momentum p_A and p_B coordinates associated with the output fields at Alice and Bob, respectively, as

$$\begin{aligned}
 x_A &= \frac{a_A + a_A^\dagger}{2}, & p_A &= \frac{a_A - a_A^\dagger}{2i}, \\
 x_B &= \frac{b_B + b_B^\dagger}{2}, & p_B &= \frac{b_B - b_B^\dagger}{2i}, \quad (\text{D15})
 \end{aligned}$$

where a_A is given by Eq. (D9) (assuming a large gain is applied at Claire and unity transmission in the classical channel) and $b_B^\dagger$ by Eq. (D7).

Further assuming for simplicity that the phases of the pumps feeding the two entanglers and Claire satisfy $\varphi_1 = 0$, $\varphi_2 = 0$, and $\varphi_3 = \pi$ and that the noise entering the two entanglers is uncorrelated and has zero mean value,

we get

$$\begin{aligned}
 V_{11} &= \langle x_A^2 \rangle \\
 &= \bar{\alpha}_2 \cosh^2(r_2) \langle x_{\text{in},a2}^2 \rangle + \bar{\alpha}_2 \sinh^2(r_2) \langle x_{\text{in},b2}^2 \rangle + \left[\sqrt{\bar{\beta}_c \bar{\alpha}_1} \sinh(r_1) - \sqrt{\bar{\beta}_1} \cosh(r_1) \right]^2 \langle x_{\text{in},b1}^2 \rangle \\
 &\quad + \left[\sqrt{\bar{\beta}_c \bar{\alpha}_1} \cosh(r_1) - \sqrt{\bar{\beta}_1} \sinh(r_1) \right]^2 \langle x_{\text{in},a1}^2 \rangle + \alpha_2 \langle x_{\text{th},a2}^2 \rangle + \beta_1 \langle x_{\text{th},b1}^2 \rangle + \beta_c \alpha_f \langle x_{\text{th},a3}^2 \rangle + \bar{\beta}_c \alpha_1 \langle x_{\text{th},a1}^2 \rangle, \quad (\text{D16})
 \end{aligned}$$

$$\begin{aligned}
 V_{33} &= \langle x_B^2 \rangle \\
 &= \bar{\beta}_2 \cosh^2(r_2) \langle x_{\text{in},b2}^2 \rangle + \bar{\beta}_2 \sinh^2(r_2) \langle x_{\text{in},a2}^2 \rangle + \beta_2 \langle x_{\text{th},b2}^2 \rangle, \quad (\text{D17})
 \end{aligned}$$

$$\begin{aligned}
 V_{13} &= \langle x_A x_B \rangle \\
 &= \sqrt{\bar{\beta}_2 \bar{\alpha}_2} \cosh(r_2) \sinh(r_2) \langle x_{\text{in},b2}^2 \rangle + \sqrt{\bar{\beta}_2 \bar{\alpha}_2} \cosh(r_2) \sinh(r_2) \langle x_{\text{in},a2}^2 \rangle, \quad (\text{D18})
 \end{aligned}$$

$$\begin{aligned}
 V_{24} &= \langle p_A p_B \rangle \\
 &= -\sqrt{\bar{\beta}_2 \bar{\alpha}_2} \cosh(r_2) \sinh(r_2) \langle p_{\text{in},b2}^2 \rangle - \sqrt{\bar{\beta}_2 \bar{\alpha}_2} \cosh(r_2) \sinh(r_2) \langle p_{\text{in},a2}^2 \rangle, \quad (\text{D19})
 \end{aligned}$$

where the subscripts of the noise variances, e.g., $\langle x_{\text{in},b2}^2 \rangle$, indicate the origin of the noise, i.e., input (in), thermal bath (th), its mode frequency a or b , and whether it traces back to fields from entangler 1, 2, or Claire 3. Similarly, it is straightforward to show that $V_{22} = \langle p_A^2 \rangle = V_{11}$, $V_{44} = \langle p_B^2 \rangle = V_{33}$, $V_{31} = \langle x_B x_A \rangle = V_{13}$, and $V_{42} = \langle p_B p_A \rangle = V_{24}$ and that $V_{12} = V_{21} = V_{23} = V_{32} = V_{14} = V_{41} = V_{34} = V_{43} = 0$.

Considering vacuum noise at the input of entanglers 1 and 2 and the same added by the beam splitters representing the losses between the stages, we obtain

$$\begin{aligned}
 V_{11} &= \frac{1}{4} \bar{\alpha}_2 \cosh(2r_2) + \frac{1}{4} [\bar{\beta}_1 + \bar{\beta}_c \bar{\alpha}_1] \cosh(2r_1) \\
 &\quad - \frac{1}{2} \sqrt{\bar{\beta}_c \bar{\alpha}_1 \bar{\beta}_1} \sinh(2r_1) \\
 &\quad + \frac{1}{4} [\alpha_2 + \beta_1 + \beta_c \alpha_f + \bar{\beta}_c \alpha_1], \quad (\text{D20})
 \end{aligned}$$

$$V_{33} = \frac{1}{4} \bar{\beta}_2 \cosh(2r_2) + \frac{1}{4} \beta_2, \quad (\text{D21})$$

$$V_{13} = \frac{1}{4} \sqrt{\bar{\beta}_2 \bar{\alpha}_2} \sinh(2r_2), \quad (\text{D22})$$

$$V_{24} = -\frac{1}{4} \sqrt{\bar{\beta}_2 \bar{\alpha}_2} \sinh(2r_2). \quad (\text{D23})$$

In the absence of loss, i.e., $\beta_{1,2} = 0, \alpha_{1,2,f} = 0$, and having a small coupling $\beta_c \rightarrow 0$, Eqs. (D20)–(D23) become

$$V_{11} = \frac{1}{4} \cosh(2r_2) + \frac{e^{-2r_1}}{2}, \quad (\text{D24})$$

$$V_{33} = \frac{1}{4} \cosh(2r_2), \quad (\text{D25})$$

$$V_{13} = \frac{1}{4} \sinh(2r_2), \quad (\text{D26})$$

$$V_{24} = -\frac{1}{4} \sinh(2r_2). \quad (\text{D27})$$

Equations (D24)–(D27) show that (i) in the limit of infinite gain on entangler 1, i.e., $r_1 \rightarrow \infty$, the covariance matrix associated with the entanglement swapping becomes indistinguishable from the covariance matrix of an ideal two-mode squeezer [see Eq. (A28)] with a squeezing parameter r_2 of entangler 2; (ii) when entangler 1 is idle, i.e., $r_1 = 0$, corresponding to classical teleportation of mode a of entangler 2 to Alice, both quadratures x_A and p_A acquire excess noise of half a photon as seen in Eq. (D24), representing V_{11} and V_{22} , which is due to both Claire and Alice adding half a photon of noise to the processed signal as required by classical teleportation; (iii) for finite two-mode squeezing on entangler 1, such as small r_1 , the entanglement-swapping scheme exhibits certain degradation in performance compared to an ideal two-mode squeezer even in the absence of losses as implied by the presence of the second term in Eq. (D24).

To characterize the resulting entanglement between the output fields in the entanglement-swapping scheme, we apply the same measures introduced earlier for two-mode squeezers. Equipped with Eqs. (D20)–(D23),

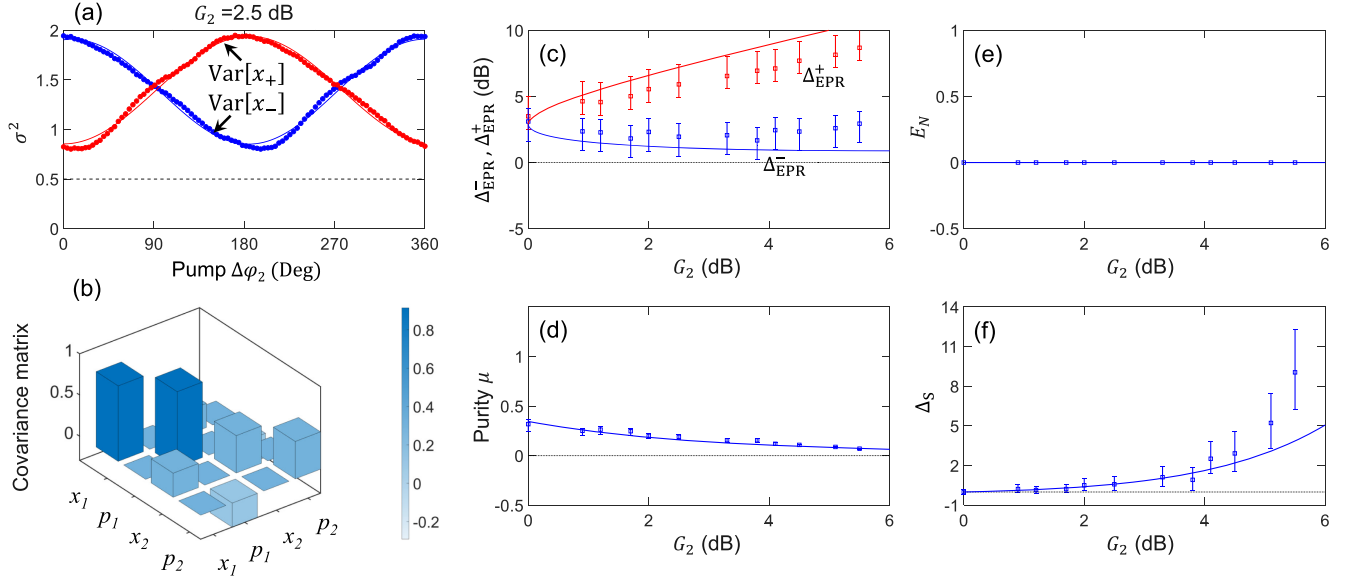


FIG. 9. Entanglement-swapping measurement with entangler 1 off. (a)–(f) show the lack entanglement and squeezing when only one entangler in the entanglement-swapping scheme is on ($G_2 > 0$ dB) while the other is off ($G_1 = 0$ dB). (a) The variance of EPR-like coordinates x_- (blue filled circles) and x_+ (red filled circles) are plotted as a function of the pump phase of entangler 2 relative to Claire's. The relative pump phase axis is shifted to yield a minimum of $\text{Var}[x_-]$ at 180° . Solid blue and red lines represent sinusoidal fits. The dashed black line represents the variance of vacuum noise. Since the minimum of $\text{Var}[x_-]$ is above the vacuum noise, there is no two-mode squeezing between Alice and Bob. (b) Bar graph representing the covariance matrix elements of the modes a and b measured at Alice and Bob for the minimum $\text{Var}[x_-]$ working point in (a). (c)–(f) exhibit various measured figures of merit for characterizing the remote entanglement (or lack of in this case) between Alice and Bob as a function of entangler 2 gain (G_2). (c) Duan's criterion for entanglement. (d)–(f) display the state purity μ , logarithmic negativity E_N , and Simon's criterion for entanglement. The solid curves in (c)–(f) are calculated fits based on an entanglement-swapping scheme model which incorporates intermediate losses. (c), (e), and (f) show the absence of any two-mode entanglement between Alice and Bob when there is no shared entanglement between Alice and Claire.

we can evaluate Simon's criterion, logarithmic negativity, entanglement of formation, and state purity using Eqs. (A19), (A24), (A25), and (A27), respectively.

Revisiting Duan's criterion given by Eq. (A15), we recalculate here Δ_{EPR}^- and Δ_{EPR}^+ given by Eqs. (A17) and (A18), which yield for the case of lossless entanglement swapping the simple result

$$\Delta_{\text{EPR}}^- = e^{-2r_2} + e^{-2r_1}, \quad (\text{D28})$$

$$\Delta_{\text{EPR}}^+ = e^{2r_2} + e^{-2r_1}. \quad (\text{D29})$$

What is notable about Eq. (D28) is that in the case of classical teleportation, i.e., $r_1 = 0$, we get $\Delta_{\text{EPR}}^- > 1$ regardless of r_2 . Also, for any positive r_1 , there exists a threshold value for r_2 above which $\Delta_{\text{EPR}}^- < 1$.

In Fig. 9, we show another crucial test of the entanglement-swapping setup shown in Fig. 5(a). Specifically, we show that two-mode squeezing and entanglement are extinguished when one of the entanglers in the setup is turned off. In this measurement, we turn off entangler 1, whose role is to enable the quantum teleportation of one-half of the entangled state produced by entangler 2. Thus,

when it is turned off, only classical teleportation is allowed to take place. Like Fig. 5(f), we show in Fig. 9(a) the dependence of $\text{Var}[x_-]$ and $\text{Var}[x_+]$ on the relative pump phase $\Delta\phi_2$ for $G_2 = 2.5$ dB. But, in contrast to Fig. 5(f), the result here does not exhibit two-mode squeezing, since both variances do not drop below the vacuum state noise variance indicated by the dashed black line. The solid curves in the plot represent sinusoidal fits. Similarly, in reference to Fig. 5(g), we show in Fig. 9(b) a bar graph representation of the covariance matrix elements of the modes at Alice and Bob, obtained for the same gain as Fig. 9(a) and relative pump phase that minimizes $\text{Var}[x_-]$. In Figs. 9(c)–9(f), we plot various measures of two-mode entanglement as a function of G_2 , i.e., Duan's criterion of entanglement [Fig. 9(c)], μ [Fig. 9(d)], E_N [Fig. 9(e)], and Simon's criterion of entanglement [Fig. 9(f)]. The lack of generated two-mode entanglement is particularly obvious in this case, since both Duan's and Simon's inequalities are not violated $\Delta_{\text{EPR}}^- > 0$ dB [Fig. 9(c)] and $\Delta_S \geq 0$ [Fig. 9(f)] and E_N is zero [Fig. 9(e)]. The solid curves in Figs. 9(c)–9(f) represent theoretical calculations of the various measures based on the covariance matrix derived for the entanglement-swapping case.

APPENDIX E: EXPERIMENTAL SETUP

The datasets presented in this work which correspond to two-mode squeezing, teleportation of coherent states, and entanglement swapping were taken separately in the course of three cooldowns. Certain modifications were made to the setup between cooldowns.

A schematic diagram of the experimental setup used in the teleportation and entanglement-swapping experiments is shown in Fig. 10. The experimental setup above the mixing chamber was largely unchanged between the three cooldowns, which is mostly shown in Fig. 10(a). The main changes were introduced in the components and connections below the mixing chamber, which are, respectively, depicted in Figs. 10(b) and 10(c) for the teleportation and entanglement-swapping experiments.

The setup includes four input and output lines denoted I_1 , I_2 , I_3 , and I_4 and O_1 , O_2 , O_3 , and O_4 , respectively. It also includes three pump lines denoted P_1 , P_2 , and P_3 , which are used for driving the three JMs. Both input and output lines incorporate standard commercial microwave components that are commonly used in superconducting qubit experiments such as cryogenic attenuators and low-noise HEMT amplifiers mounted inside the fridge and I - Q mixers and fast switches located outside the fridge.

The pump lines include wideband 20 dB directional couplers at the base stage, which attenuate the pump tones that drive the JMs. The attenuation is realized by directing a large portion of the pump power toward the 4 K stage, where it gets dissipated in a 50 Ohm termination.

The blue dots on the input, output, and pump lines shown at the room-temperature stage mark the locations at which the various lines get connected or disconnected to microwave generators and measuring devices, such as a vector network analyzer, spectrum analyzer, and heterodyne setup that is commonly used in dispersive readout of superconducting qubits. All microwave sources and measurement devices are phase locked to a 10 MHz reference oscillator of a rubidium atomic clock.

With the exception of the measurement results shown in Fig. 3 and some in Fig. 11, which are taken with the spectrum analyzer, all other results are taken using the heterodyne setup, in which output noise or coherent states exiting the fridge are down-converted, sampled, and digitized using a dual-channel Alazar card.

The black lines connecting the various components inside and outside the fridge correspond to normal-metal coax cables, whereas the cyan lines inside the fridge correspond to NbTi superconducting coax cables. The latter lines are primarily employed to minimize insertion losses between JMs and at sections of the output lines that precede the HEMTs which are known to degrade CV entanglements and the SNR of output lines.

The three JMs used in this work are mounted on the same oxygen-free high-conductivity (OFHC) copper bracket at the bottom of the dilution fridge and reside deep inside

a 4-inch-diameter cryoperm magnetic shield can (not shown). The commercial cryogenic circulators, which connect between the JMs or between the JMs, and the input and output lines are mounted on a different bracket located outside the cryoperm magnetic shield to minimize magnetic interference between the circulators and the JMs.

Likewise, to save space and decrease the footprint of the setup, we mount the directional couplers inside the magnetic shield can near the top and mount the 180° hybrids on the same bracket as the circulators. Thus, it is worth emphasizing that the approximate cable lengths between the various components or parties in our experiments [indicated in Figs. 2 and 5(a)] are largely set by the relative locations of these components rather than by performance considerations.

Small superconducting magnetic coils (not shown) are attached to the OFHC copper packages of the JMs, which enable us to match the resonance frequencies of modes a and b of the JMs by flux tuning each JM individually.

The red 50 Ohm terminations shown in the figure at the mixing chamber in the teleportation and entanglement-swapping setups correspond to cryogenic 50 Ohm terminations that are thermalized with tight OFHC copper clamps that are attached to the base plate of the mixing chamber via thin narrow copper braids fixed by small washers and screws at both ends. On the surface of the clamp holding the termination, we mount with screws a heater in the form of a 500 Ohm resistor with voltage and current leads soldered across it and a ruthenium oxide thermometer, which we thermally calibrated in a separate cooldown.

Using this configuration, we are able to vary *in situ* the steady-state temperature of the 50 Ohm terminations by applying a dc current through the heaters and record the resultant temperature using the thermal sensors. We connect these variable-temperature 50 Ohm terminations to the third port of auxiliary circulators that we mount following the directional couplers at the output of port a of entangler 1 and port b of entangler 2. The 50 Ohm terminations are connected to the circulators via a 4-inch coax cables. The length of the braid connecting the clamps to the base plate is set experimentally to be long enough such that it provides some thermal isolation so that the termination can be heated locally without considerably raising the mixing chamber plate temperature but also short enough so that the termination can be cooled back down relatively quickly once we turn off the dc current flowing through the heater.

We use these variable-temperature 50 Ohm terminations to vary the thermal noise at the bottom of the fridge and measure the resultant noise power emitted from the output lines O_1 and O_4 . The JMs in these measurement are turned off, thus acting as perfect mirrors, which reflect the generated thermal noise into the output chains.

Such noise measurements enable us to extract the total gain and effective added noise associated with the output chains and verify, after accounting for the intermediate

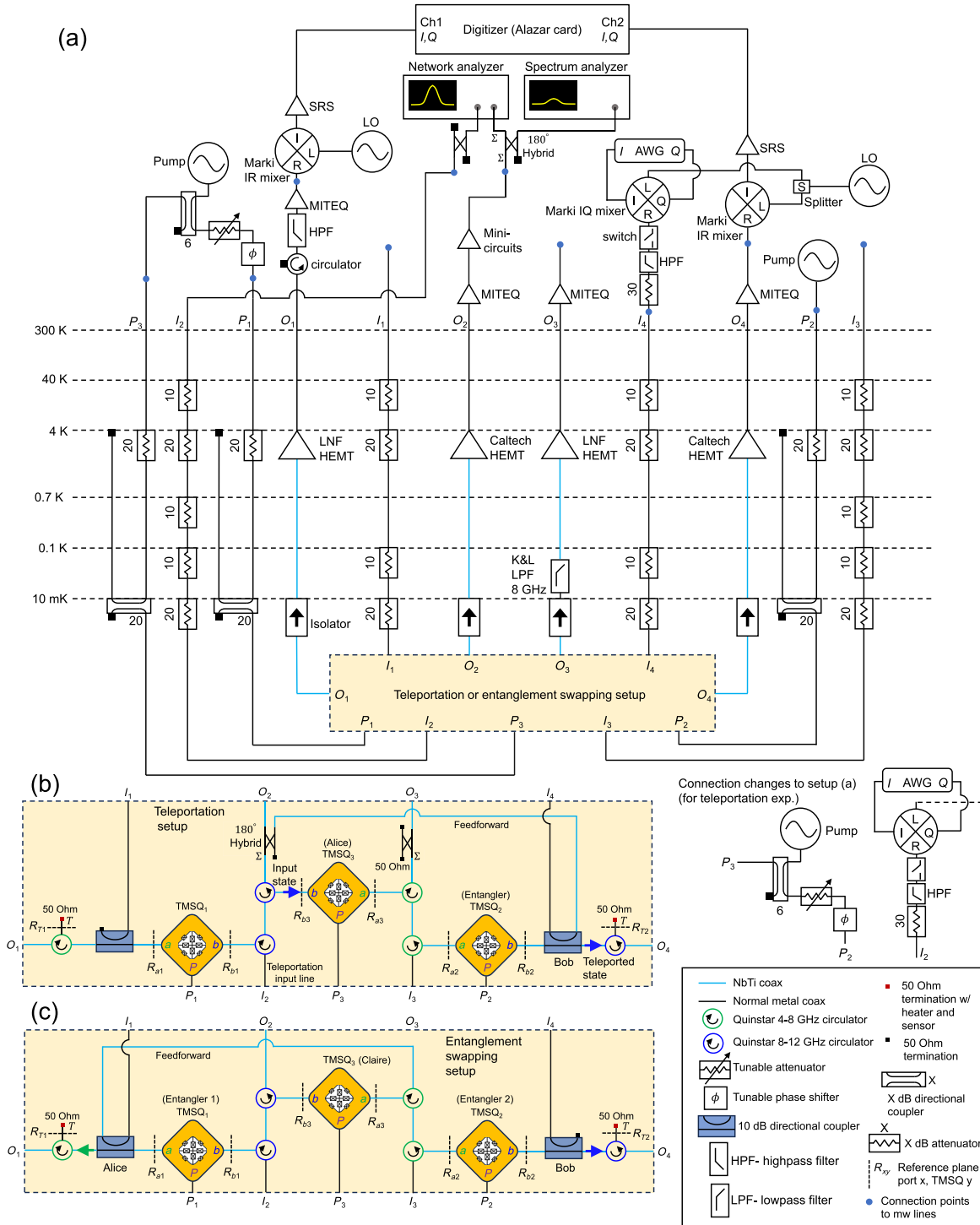


FIG. 10. Teleportation and entanglement-swapping setup. (a) The fridge and room-temperature measurement setups used in the teleportation and entanglement-swapping experiments. The bottom right side in (a) shows two changes to the room-temperature setup relevant to the teleportation experiments. (b) Circuit components and configuration at base used in the teleportation experiments. (c) Circuit components and configuration at base used in the entanglement-swapping experiments.

losses between the circulators and the JM planes, that the extracted parameters are in agreement with the values obtained by varying the gain of the JMs without heating the terminations.

Finally, the setup used for taking the two-mode squeezing data is very similar to the teleportation setup, with four main differences: (i) the cables connecting the different components are normal-metal coaxes, (ii) JM1 measured in

the experiment is mounted where JM2 is shown in the schematic, (iii) the heterodyne setup is connected to the output lines O_3 and O_4 , and (iv) an additional isolator is incorporated in each output line.

APPENDIX F: TMSQ PARAMETERS

The three JMs employed in this work, which we denote as JM1, JM2, and JM3, are nominally identical. They share the same design and fabrication process. In Table I, we list the maximum resonance frequencies for resonators a and b , i.e., $f_{a,\max}$ and $f_{b,\max}$, measured for the three JMs as well as their linear bandwidths, i.e., $\gamma_{a,m}/2\pi$ and $\gamma_{b,m}/2\pi$, measured at the maximum resonance frequencies and the bandwidths at the working points employed in the main text, i.e., $\gamma_a/2\pi$ and $\gamma_b/2\pi$, which correspond to the resonance frequencies $f_a = 7.23$ GHz and $f_b = 9.707$ GHz. Furthermore, for documentation purposes, we specify in Table II the role played by each JM in the different experimental setups reported in the main text.

The dynamical bandwidth of the JM at power gain G , which we denote as $B = \gamma/2\pi$, is given by [53]

$$\gamma = \frac{\gamma_0}{\sqrt{G}}, \quad (\text{F1})$$

where $\gamma_0 = 2\gamma_a\gamma_b/(\gamma_a + \gamma_b)$. Also, as mentioned in the main text, the JMs are flux tunable. Thus, by applying external flux to the JRM loop, the resonance frequencies of modes a and b can be tuned over about 500 MHz.

APPENDIX G: CALIBRATION OF THE GAIN AND NOISE OF THE OUTPUT CHAIN

The thermal photon population corresponding to angular frequency ω and temperature T at the bottom of the output line is given by the Bose-Einstein statistics given by $1/2 \coth(\hbar\omega/2k_B T)$. The measured noise power of the output chain at room temperature is given by $P_{N,k} = (\sigma_{I,k}^2 + \sigma_{Q,k}^2)/R$ [64,84], where the index $k = a, b$ corresponds to modes a and b of the JM, $\sigma_{I,k}^2$ and $\sigma_{Q,k}^2$ are the voltage noise variance of the I and Q quadratures of mode k , respectively, and $R = 50$ Ohm is the input resistance of the measurement device.

When the JM is off and the temperature T on the cryogenic termination is varied, we get

$$\begin{aligned} P_{N,k}(T, G_J = 1) &= P_{0,k} \times N_{\text{ph},k}(T) \\ &= P_{0,k} \times \left(\frac{1}{2} \coth\left(\frac{\hbar\omega_k}{2k_B T}\right) + N_{\text{sys},k} \right), \end{aligned} \quad (\text{G1})$$

where $P_{0,k} = G_{\text{sys},k} \times \text{BW} \times \hbar\omega_k$, $G_{\text{sys},k}$ is the total gain or loss of the output chain for mode k , BW is the measurement bandwidth, ω_k is the angular frequency of mode k , T is the temperature of the variable-temperature 50 Ohm termination at the input of the output chain, $N_{\text{sys},k}$ is the total added noise photons by the output chain for mode k , and $N_{\text{ph},k}(T)$

TABLE I. Bandwidths and maximum resonance frequencies of the JMs. $f_{a,\max}$ and $f_{b,\max}$ are the measured maximum resonance frequencies of modes a and b of the JM, respectively. $\gamma_{a,m}/2\pi$ and $\gamma_{b,m}/2\pi$ are the linear bandwidths of modes a and b of the JM, respectively, measured at their maximum frequencies, while $\gamma_a/2\pi$ and $\gamma_b/2\pi$ are the bandwidths measured at the teleportation and entanglement-swapping working points in the main text.

Qubit	$f_{a,\max}$ (GHz)	$f_{b,\max}$ (GHz)	$\gamma_{a,m}/2\pi$ (MHz)	$\gamma_{b,m}/2\pi$ (MHz)	$\gamma_a/2\pi$ (MHz)	$\gamma_b/2\pi$ (MHz)
JM1	7.363	9.968	103	80	103	78
JM2	7.364	9.967	107	79	97	90
JM3	7.36	9.964	103	85	78	84

TABLE II. The roles played by the three JMs in the experiments in the main text.

JM	TMSQ experiment	Teleportation experiment	Entanglement-swapping experiment
JM1	TMSQ (Fig. 1)	...	Entangler 1 (Fig. 5)
JM2	...	Entangler (Fig. 2)	Entangler 2 (Fig. 5)
JM3	...	Alice (Fig. 2)	Claire (Fig. 5)

is the total output noise equivalent photon number for mode k as a function of T .

In the limit $\hbar\omega \gg K_B T$, we obtain $1/2 \coth(\hbar\omega/2k_B T) \cong 1/2$, which is the default case when the termination is not heated, since the temperatures of the base stage and termination are 10 and 52 mK, respectively, and the mode frequencies are $\omega_a/2\pi = 7.23$ GHz and $\omega_b/2\pi = 9.707$ GHz.

Similarly, if the termination is kept cold while the gain on the JM is varied, we can write the measured noise power of the output chain at room temperature as

$$\begin{aligned} P_{N,k}(T \cong 0, G_J) &= P_{0,k} \times N_{\text{ph},k}(G_J) \\ &= P_{0,k} \times \left(G_J \frac{1}{2} + (G_J - 1) \frac{1}{2} + N_{\text{sys},k} \right), \end{aligned} \quad (\text{G2})$$

where G_J is the power gain of the JM and $N_{\text{ph},k}(G_J)$ is the total output noise equivalent photon number for mode k as a function of G_J .

To find $G_{\text{sys},k}$ and $N_{\text{sys},k}$ for each output chain, we measure the output noise power for varying T or G_J and extract their values by fitting the data using Eq. (G1) or (G2). While varying T of a variable-temperature termination is one of the widely used and trusted methods for extracting $G_{\text{sys},k}$ and $N_{\text{sys},k}$ of output chains, it has several drawbacks. (i) It requires adding auxiliary components that do not play an active role in the experiment, such as cold terminations, dedicated heaters, thermal sensors, extra dc wiring for applying heat and reading out the temperature, and circulators to direct the generated thermal noise. In experiments that employ multiple output lines, applying such overhead to all output lines could be challenging. (ii) It introduces some undesired loss in the signal path due to the need to couple the variable-temperature termination to the output chain, for example, via a circulator. And most important, (iii) it adds uncertainties to the extracted values of $G_{\text{sys},k}$ and $N_{\text{sys},k}$ due to the unavoidable separation and added intermediate loss between the variable-temperature termination and the reference plane of the device under test. For these reasons, we demonstrate in Fig. 11 that the two methods, i.e., varying T or G_J , can be used to extract $G_{\text{sys},k}$ and $N_{\text{sys},k}$ for two output lines, i.e., $O1$ and $O4$ (shown in Fig. 10). Furthermore, we show below that the extracted values obtained using the two methods are in agreement after accounting for the intermediate losses between the reference planes of the variable-temperature terminations and the JMs.

In the experiment, we measure the noise power using two measuring devices: an analog-to-digital converter digitizer card and a spectrum analyzer. Despite having certain differences in their amplification and mixing components at room temperature, both measuring devices yield similar results for $N_{\text{sys},k}$ of the output chains. It is worth noting that

when measuring the output noise power using a spectrum analyzer the limiting bandwidth is set by the resolution bandwidth, whereas when measuring using the digitizer card it corresponds to the inverse of the integration time $1/T_{\text{int}}$.

Another useful measure that we consider is the improvement in the signal-to-noise ratio of the output chain due to the JM given by $G_J/G_{N,k}$, where $G_{N,k}$ is the output noise ratio (or noise rise) corresponding to the JM being on versus off:

$$\begin{aligned} G_{N,k} &= \frac{P_{N,k}(G_J)}{P_{N,k}(G_J = 1)} \\ &= \frac{T_{\text{sys},k} + G_J T_{Q,k} + (G_J - 1) T_{Q,k}}{T_{\text{sys},k} + T_{Q,k}}, \end{aligned} \quad (\text{G3})$$

where $T_{Q,k} = \hbar\omega_k/2k_B$ is the equivalent temperature of the zero-point fluctuations at mode k and $T_{\text{sys},k} = N_{\text{sys},k} \hbar\omega_k/k_B$ is the effective noise temperature of the output chain at mode k in the absence of the JM. Note that the figure $G_{N,k}$, unlike $P_{N,k}$, is independent of the measurement bandwidth and system gain. It mainly depends on the total noise added by the system.

Using Eq. (G3), the signal-to-noise ratio improvement due to the JM reads

$$\frac{G_J}{G_{N,k}} = \frac{T_{\text{sys},k} + T_{Q,k}}{T_{\text{sys},k}/G_J + T_{Q,k}[1 + (G_J - 1)/G_J]}. \quad (\text{G4})$$

While the relations (G2) and (G4) are not independent, we, nevertheless, apply the extracted system noise from the fit of Eq. (G2) into Eq. (G4) and confirm that the resulting curve fits the signal-to-noise ratio improvement data as well.

In Fig. 11, we exhibit calibration measurements of output lines $O1$ and $O4$ used in the entanglement-swapping experiment. Figures 11(a)–11(d) exhibit measurement results for mode a taken for $O1$. The filled blue circles represent measured data, while the solid black curves are theory fits. Figures 11(a)–11(c) show noise power measurements taken using the digitizer setup. Figures 11(a) and 11(b) display $N_{\text{ph},a}$ as a function of G_J and T , respectively. Figure 11(c) plots the SNR improvement versus G_J . Figure 11(d) exhibits $N_{\text{ph},a}$ measured as a function of G_J using a spectrum analyzer. Figures 11(e)–11(h) are the same as Figs. 11(a)–11(d), measured instead for $O4$ and mode b . The extracted parameters from the fits, i.e., G_{sys} and N_{sys} , are listed in Table III. By calculating the total gain ratio for mode a (b) between G_{sys} extracted using the variable-temperature and the variable-gain methods, we obtain an estimate for the insertion loss between the reference planes R_{T1} and R_{a1} (R_{T2} and R_{b2}), outlined in Fig. 10, of about -1.1 dB (-1.4 dB), which corresponds to a gain ratio of 0.77 (0.72). Such estimates for the insertion losses between the variable-temperature terminations and

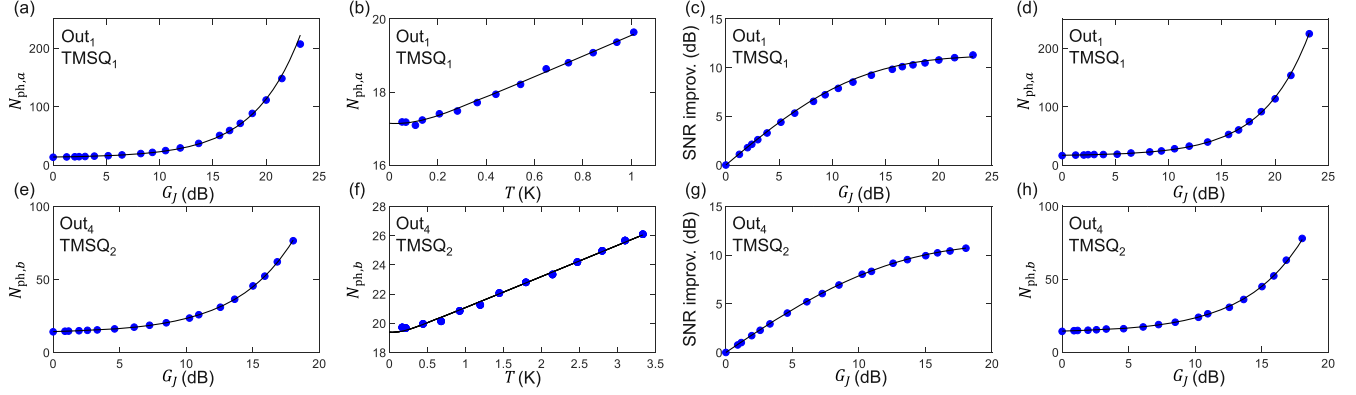


FIG. 11. Noise power measurements for extracting the total gain and added noise of output lines 1 and 4. The blue filled circles in (a) and (b) represent the noise equivalent photon number N_{ph} of mode a measured using the digitizer card setup as a function of TMSQ₁ gain (G_J) and the local temperature of the termination (T), respectively. The solid black curves are fits to the data which employ Eqs. (G2) and (G1), respectively. (c) exhibits SNR improvement data for mode a of TMSQ₁ versus G_J . The fitting curve employs Eq. (G4) and uses the same N_{sys} parameter extracted in (a). (d) N_{ph} of mode a measured at the output of line 1 versus G_J taken using a spectrum analyzer. (e)–(h) The same as (a)–(d), taken for output 4 and mode b of TMSQ₂. The extracted parameters from the fits, presented in Table III, are employed in the calibration of the entanglement-swapping experiment.

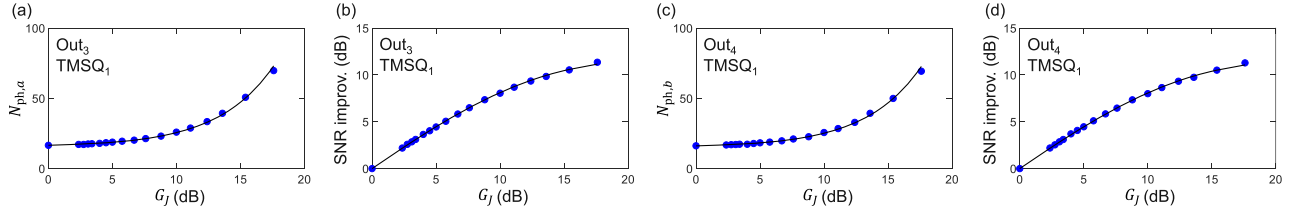


FIG. 12. Noise power measurements for calibrating the output lines in the two-mode squeezing experiment. (a) [(c)] exhibits the noise equivalent photon number N_{ph} of mode a (b) measured using the digitizer card setup as a function of TMSQ₁ gain G_J . The solid black curves are fits to the data which employ Eq. (G2). (b) [(d)] plots SNR improvement data for mode a (b) of TMSQ₁ versus G_J . The fitting curve employs Eq. (G4) and uses the same N_{sys} parameter extracted in (a) [(c)]. The extracted parameters from the fits, presented in Table IV, are employed in the calibration of the two-mode squeezing experiment.

JMs match the expected total loss of the intermediate microwave components, which include a short normal-metal cable, circulator, directional coupler, and a few normal-metal connectors.

Likewise, Fig. 12 depicts the calibration data and fits for the output lines used in the two-mode squeezing experiment. The extracted parameters from the fits, i.e., G_{sys} and N_{sys} , are listed in Table IV.

It is worth noting that similar calibration procedures and measurements are carried out for the teleportation experiment as well (data not shown).

1. Estimation of intermediate losses in the setup

To get an estimate for the intermediate losses between JM stages connected in series and attached to the same output line, we use an *in situ* calibration method in which we measure the noise power of the output chain as a function of the gain on each JM separately. Using these two measurements, we extract two different $G_{\text{sys},k}$ parameters for the two JMs whose ratio gives an estimate for the amount of loss present between their respective output reference planes denoted as $R_{x,y}$ in Fig. 10, where index x refers to port a or b and y identifies the JM.

TABLE III. Estimated total gain and added noise for output lines 1 and 4. G_{sys} and N_{sys} are extracted from fits to the data shown in Fig. 11. T_{sys} is calculated using $T_{\text{sys},k} = N_{\text{sys},k} \hbar \omega_k / k_B$. The extracted parameters are employed in the calibration of the entanglement-swapping experiment. The uncertainty in evaluating G_{sys} and N_{sys} is less than $\pm 10\%$.

Output, mode	Fig.	G_{sys}	N_{sys}	T_{sys} (K)	Fig.	G_{sys}	N_{sys}	T_{sys} (K)	Fig.	G_{sys}	N_{sys}	T_{sys} (K)
O1, a	11(a)	3.5×10^6	13.2	4.6	11(b)	2.7×10^6	16.6	5.8	11(d)	4.4×10^5	16.4	5.7
O4, b	11(e)	1.8×10^7	13.8	6.4	11(f)	1.3×10^7	18.9	8.8	11(h)	2.3×10^6	13.9	6.5

TABLE IV. Estimated total gain and added noise for output lines used in the two-mode squeezing experiment. G_{sys} and N_{sys} are extracted from fits to the data shown in Fig. 12. The uncertainty in evaluating G_{sys} and N_{sys} is less than $\pm 10\%$.

Output, mode	Fig.	G_{sys}	N_{sys}	T_{sys} (K)
$O3, a$	12(a)	6.8×10^6	16.1	5.6
$O4, b$	12(c)	1.3×10^7	15.7	7.3

To better see this relation, we rewrite Eq. (G2) for two JMs connected in series with an intermediate loss characterized by a power transmission amplitude $\bar{\alpha}$ between their output reference planes. The output noise power measured for the top JM with subscript p (closer to the output line) is given by

$$P_{N,k,p}(G_{J,p}) = G_{\text{sys},k,p} \times \text{BW} \times \hbar\omega_k \times \left(G_{J,p} \frac{1}{2} + (G_{J,p} - 1) \frac{1}{2} + N_{\text{sys},k,p} \right), \quad (\text{G5})$$

whereas the output noise power measured for the bottom JM with subscript m (farther away from the output line) is given by

$$P_{N,k,m}(G_{J,m}) = G_{\text{sys},k,m} \times \text{BW} \times \hbar\omega_k \times \left(G_{J,m} \frac{1}{2} + (G_{J,m} - 1) \frac{1}{2} + N_{\text{sys},k,m} \right), \quad (\text{G6})$$

where $G_{\text{sys},k,m} = \bar{\alpha} G_{\text{sys},k,p}$ and $N_{\text{sys},k,m} = N_{\text{sys},k,p} / \bar{\alpha} + (1 - \bar{\alpha}) / 2\bar{\alpha}$. The second term in $N_{\text{sys},k,m}$ assumes vacuum noise is introduced by the cold dissipation.

To demonstrate how we apply this calibration method to get estimates for intermediate losses, we give here two examples related to the teleportation experiment whose base-temperature stage configuration is shown in Fig. 10(b). In the first example, we estimate the insertion loss between the reference planes R_{a2} and R_{a3} denoted $\bar{\alpha}$ by measuring the noise power emitted from output line 3 twice—the first versus the gain on TMSQ₃ while TMSQ₂ is off and the second versus the gain on TMSQ₂ while TMSQ₃ is off. By fitting the data of the two measurements, we extract $G_{\text{sys},a,p} = 7.3 \times 10^6$ and $G_{\text{sys},a,m} = 4.5 \times 10^6$ obtained using TMSQ₃ and TMSQ₂, respectively. Calculating their ratio yields the estimate $\bar{\alpha} = G_{\text{sys},a,m} / G_{\text{sys},a,p} = 0.61$ (corresponding to -2.1 dB). In the second example, we use the same procedure to estimate the insertion loss of the feedforward channel in the teleportation experiment, denoted as $\bar{\beta}_f$. This time, however, we measure the noise power for mode b emitted from output line 4 as a function of the gain on TMSQ₂ while TMSQ₃ is off and as a

function of the gain on TMSQ₃ while TMSQ₂ is off. After extracting the corresponding parameters $G_{\text{sys},b,p} = 2.1 \times 10^7$ and $G_{\text{sys},b,m} = 8.2 \times 10^5$, we get $G_{\text{sys},b,m} / G_{\text{sys},b,p} = 0.04$, which corresponds to 14 dB of loss. After subtracting 10 dB of coupling loss introduced by the directional coupler, we obtain an estimate of $\bar{\beta}_f = 0.4$ (corresponding to -4 dB).

2. Calibration of the measured quadratures

As explained in Appendix A 5, to reconstruct the two-mode squeezing covariance matrix and calculate the various entanglement measures, we must refer the output field quadratures measured using the room-temperature electronics back to the output of the device. To do so, we employ the total gain and noise of the output chain measured for the different modes and lines (detailed in the previous subsections).

The experimentally measured four dimensionless quadratures of the output fields corresponding to modes a and b at room temperature X_a , P_a , X_b , and P_b can be written as [64]

$$X_k = \frac{A_k + A_k^\dagger}{2}, \quad P_k = \frac{A_k - A_k^\dagger}{2i}, \quad (\text{G7})$$

which are obtained from the measured raw quadratures in volts X_a^{raw} , P_a^{raw} , X_b^{raw} , and P_b^{raw} using the conversion relations

$$X_k = \sqrt{\gamma_k} X_k^{\text{raw}}, \quad P_k = \sqrt{\gamma_k} P_k^{\text{raw}} \quad (\text{G8})$$

with the conversion factor γ_k given by

$$\gamma_k = \frac{T_{\text{int}}}{R E_k^{\text{ph}}}, \quad (\text{G9})$$

where $R = 50$ Ohm, T_{int} is the integration time of the measurement, and $E_k^{\text{ph}} = \hbar\omega_k$ is the photon energy of mode k .

Finally, we refer the measured quadratures X_a , P_a , X_b , and P_b back to the output of the device using the amplitude scaling:

$$x_k = X_k / \sqrt{G_{\text{sys},k}}, \quad p_k = P_k / \sqrt{G_{\text{sys},k}}. \quad (\text{G10})$$

APPENDIX H: MEASUREMENT PARAMETERS

In all 2D histogram data shown in the paper, we measure $N = 10^5$ data points. In the digitizer setup, we apply 1 μ s measurement pulses and integration times T_{int} of the same duration. We also use a 10 MHz frequency offset in the up-conversion and down-conversion process.

For the teleportation measurement in Fig. 3, which we measure using a spectrum analyzer, we measure the output

at 9.707 GHz with 1001 data points, a sweep time of 0.1 s, resolution bandwidth and video bandwidth of 470 kHz, and 300 averages. We vary the pump phase by offsetting the frequency of the pump on one of the JMs by 50 Hz, while triggering the spectrum analyzer with 10 Hz, 50% duty cycle square pulses originating from an arbitrary wave generator.

APPENDIX I: ANALYSIS OF MEASUREMENT ERRORS

Errors in the measured entanglement and squeezing figures, namely, $\Delta_{\text{EPR}}^{\pm}$, μ , E_N , E_F , and Δ_S , stem from statistical and systematic errors associated with the measurement and reconstruction of the covariance matrix of the TMSQ device and the entanglement-swapping apparatus.

The statistical errors, which are dominant in our experiment, originate from uncertainties in the calculation of variances and covariance of a bivariate variable, which arise from having a finite number of observations of a random sample.

Thus, for a general bivariate covariance matrix of the form [31]

$$\Sigma = \begin{pmatrix} \sigma_{11}^2 & \rho\sigma_{11}\sigma_{22} \\ \rho\sigma_{11}\sigma_{22} & \sigma_{22}^2 \end{pmatrix}, \quad (\text{II})$$

the variance of the covariance matrix reads

$$\text{Var}(\Sigma) = \frac{1}{N-1} \begin{pmatrix} 2\sigma_{11}^4 & (1+\rho^2)\sigma_{11}^2\sigma_{22}^2 \\ (1+\rho^2)\sigma_{11}^2\sigma_{22}^2 & 2\sigma_{22}^4 \end{pmatrix}, \quad (\text{I2})$$

where σ_{ii}^2 is the variance of sample distribution sampled from a Gaussian variable and N is the number of samples.

When compared to the covariance matrices of the TMSQ device and the entanglement-swapping apparatus presented

in Appendixes A and D, we have $V_{11} = \sigma_{11}^2$, $V_{33} = \sigma_{22}^2$, and $\rho = V_{13}/\sqrt{V_{11}V_{33}}$.

The systematic errors, on the other hand, which play a lesser role in our system, originate from calibration errors of the vacuum noise levels or, more specifically, from estimation errors of the total gains and added noise of the output chains.

However, since the effect of systematic errors on the calculation of the covariance matrices is nontrivial, we employed a worst-case scenario approach for calculating the total errors, i.e., systematic and statistical, which is similar to the one employed in Ref. [31]. In this approach, we repeat the full calculation of the covariance matrix for the upper and lower bounds of the vacuum noise levels while accounting for the various statistical errors (± 1 standard deviation) in each run. We then calculate the various two-mode squeezing and entanglement measures using the different covariance matrices obtained in the previous step and set the total error bars of each quantity based on the maximum achievable range.

APPENDIX J: TELEPORTATION FIDELITY

The teleportation operation presented in Fig. 2 can be broken down into a sequence of quantum operators acting on propagating signals along three paths as shown in Fig. 13. In this representation, (i) the input and output fields are expressed as column vectors with three pairs of rows (six in total), where the first, second, and third pairs correspond to the I and Q quadratures of the microwave signals in paths 1, 2, and 3, respectively, and (ii) the quantum operators acting on the I and Q quadratures of these signals, indicated by the dashed vertical lines, are expressed as matrices of size 6×6 . The main three operators playing a role in this scheme are the two-mode squeeze (S), insertion loss (L) represented by beam-splitter interaction, and coupling (C). Using this representation, adapted from Ref. [47], allows us to calculate the output fields and added noise through multiplication of vectors and matrices and, in turn, to calculate the teleportation

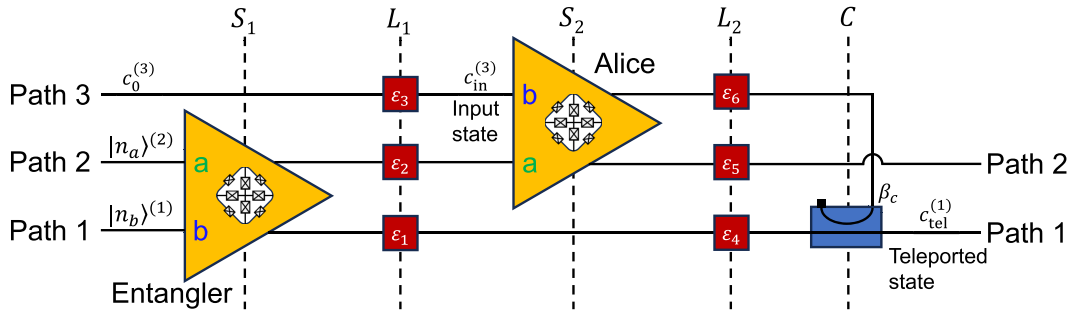


FIG. 13. Schematic diagram for calculating the teleported state and the teleportation fidelity of coherent states. The teleportation process is modeled as a sequence of operations, i.e., two-mode squeezing S , loss L , and coupling C , which act on signals propagating along paths 1, 2, and 3. The inputs to the entangler are typically vacuum states. At the end of the sequence, coherent states input on path 3 within the bandwidth of mode b of the entangler and Alice get teleported to the output of path 1.

fidelity by evaluating the overlap between the input and output states.

In Fig. 13, $c_0^{(3)}$ represents a coherent state propagating along path 3, while the input states $|n_b\rangle^{(1)}$ and $|n_a\rangle^{(2)}$ entering the entangler correspond to the thermal noise in modes b and a that are present in paths 1 and 2, respectively, with average noise photon numbers given by $n_{b,a} = 1/2 \coth(\hbar\omega_{b,a}/2k_B T)$. Note that in the quantum limit applicable here, i.e., $\hbar\omega_i/2k_B T \gg 1$, these input states correspond to the vacuum. Thus, the general covariance matrix for the input thermal noise of the system can be cast as

$$V_0 = \frac{1}{4} \begin{pmatrix} (1 + 2n_b^{\text{th}})I_2 & 0_2 & 0_2 \\ 0_2 & (1 + 2n_a^{\text{th}})I_2 & 0_2 \\ 0_2 & 0_2 & (1 + 2n_b^{\text{th}})I_2 \end{pmatrix}, \quad (\text{J1})$$

where 0_2 is a zero matrix of dimension 2×2 and $n_{a,b}^{\text{th}} = 1/(e^{\hbar\omega_{a,b}/2k_B T} - 1)$.

Using Eq. (A6), the two-mode squeezing operations at the entangler and Alice can be, respectively, written as

$$A_{1,2} = \frac{1}{4} \begin{pmatrix} (1 + 2n_b^{\text{th}})\varepsilon_{1,4}I_2 & 0_2 & 0_2 \\ 0_2 & (1 + 2n_a^{\text{th}})\varepsilon_{2,5}I_2 & 0_2 \\ 0_2 & 0_2 & (1 + 2n_b^{\text{th}})\varepsilon_{3,6}I_2 \end{pmatrix}. \quad (\text{J5})$$

Note that some of the losses shown in the sequence in Fig. 13 are redundant, but they are, nevertheless, included to express the loss matrix in a standard generic form that applies to the different paths and loss stages. In particular, the losses ε_3 , ε_4 , and ε_5 can be zero without loss of generality. We also note that, using the notations in Fig. 2, we have $\varepsilon_1 = \beta$, $\varepsilon_2 = \alpha$, and $\varepsilon_6 = \beta_f$.

Lastly, the directional coupler operation can be expressed as

$$C = \begin{pmatrix} \sqrt{\beta_c}I_2 & 0_2 & \sqrt{\beta_c}I_2 \\ 0_2 & I_2 & 0_2 \\ -\sqrt{\beta_c}I_2 & 0_2 & \sqrt{\beta_c}I_2 \end{pmatrix}. \quad (\text{J6})$$

Using these matrices, we can calculate the teleported state and teleportation fidelity of a coherent state propagating along path 3.

Starting with the following column vector representing an input coherent state with an average photon number n_s and phase θ_s :

$$c_0 = (0, 0, 0, 0, \sqrt{n_s}\cos(\theta_s), \sqrt{n_s}\sin(\theta_s))^T, \quad (\text{J7})$$

$$S_1 = \begin{pmatrix} \cosh(r_E)I_2 & \sinh(r_E)ZR(\varphi_E) & 0_2 \\ \sinh(r_E)ZR(\varphi_E) & \cosh(r_E)I_2 & 0_2 \\ 0_2 & 0_2 & I_2 \end{pmatrix} \quad (\text{J2})$$

and

$$S_2 = \begin{pmatrix} I_2 & 0_2 & 0_2 \\ 0_2 & \cosh(r_A)I_2 & \sinh(r_A)ZR(\varphi_A) \\ 0_2 & \sinh(r_A)ZR(\varphi_A) & \cosh(r_A)I_2 \end{pmatrix}. \quad (\text{J3})$$

As to the loss operation, we can express the diagonal matrix representing the transmission amplitudes in the signal path as

$$L_{1,2} = \begin{pmatrix} \sqrt{\bar{\varepsilon}_{1,4}}I_2 & 0_2 & 0_2 \\ 0_2 & \sqrt{\bar{\varepsilon}_{2,5}}I_2 & 0_2 \\ 0_2 & 0_2 & \sqrt{\bar{\varepsilon}_{3,6}}I_2 \end{pmatrix}, \quad (\text{J4})$$

where $\bar{\varepsilon}_i = 1 - \varepsilon_i$ and ε_i corresponds to the power loss in element i of the circuit. From this result, it follows that the added noise due to losses in the signal paths (modeled as finite couplings to thermal baths) can be expressed as

we can express the input coherent state at Alice as

$$c_{\text{in}} = L_1 S_1 c_0 \quad (\text{J8})$$

and the corresponding covariance matrix of the input state as

$$V_{\text{in}} = L_1 S_1 V_0 S_1^\dagger L_1^\dagger. \quad (\text{J9})$$

Similarly, to get the output vector, denoted c_{tel} (which contains the teleported state), and the covariance matrix of the output, denoted V_{tel} , we calculate

$$c_{\text{tel}} = T c_0, \quad (\text{J10})$$

where T is the teleportation matrix given by the product

$$T = C L_2 S_2 L_1 S_1 \quad (\text{J11})$$

and

$$V_{\text{tel}} = T V_0 T^\dagger + A, \quad (\text{J12})$$

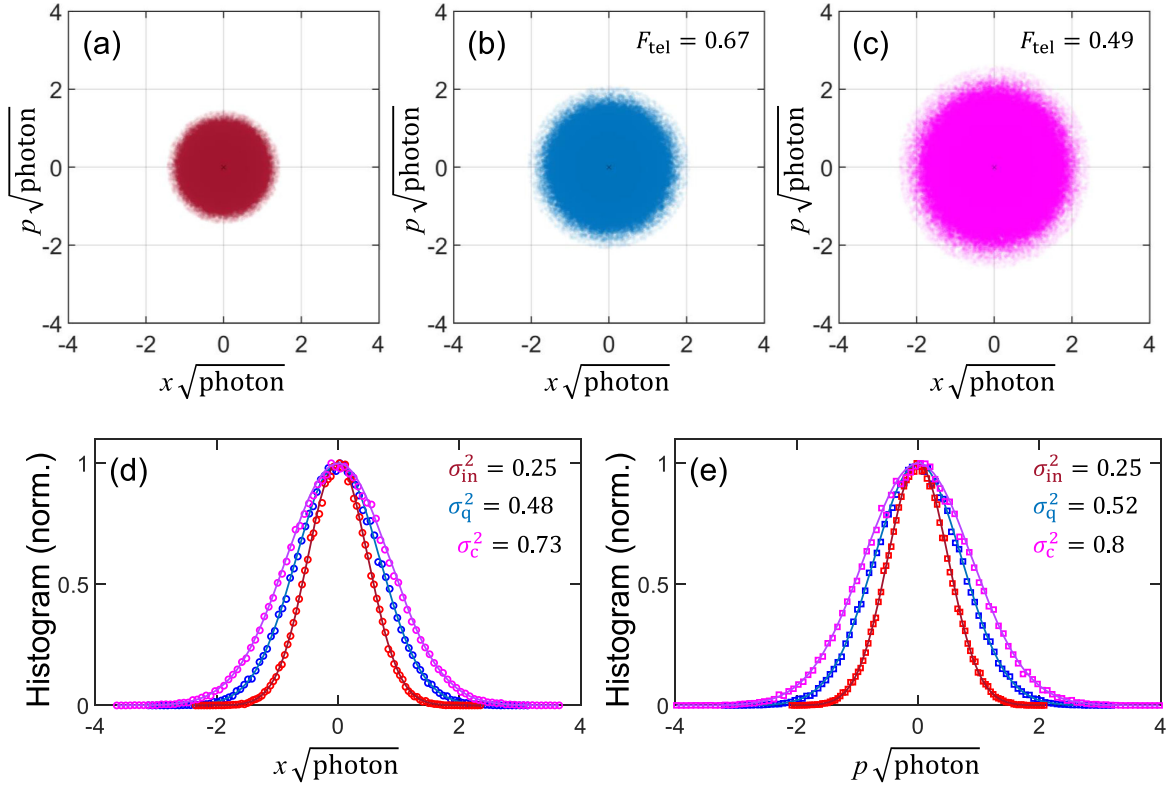


FIG. 14. Quantum and classical teleportation of vacuum state. (a) I-Q (x - p) quadrature representation of the vacuum state at the input of port b of Alice to be teleported to Bob via the teleportation scheme depicted in Fig. 2. (b), (c) I-Q (x - p) quadrature representation of the quantum-mechanically ($G_E = 2.1$ dB) and classically ($G_E = 0$ dB) teleported vacuum states at Bob, respectively. (d), (e) Normalized histograms of the scatter data in (a) (red), (b) (blue), and (c) (magenta) taken along the position and momentum axis, respectively. The solid curves in (d) and (e) are Gaussian fits.

where A represents the added noise of the whole sequence, which reads

$$A = CL_2S_2A_1S_2^\dagger L_2^\dagger C^\dagger + CA_2C^\dagger. \quad (\text{J13})$$

Focusing on the displacement coordinates of the input and teleported coherent states denoted $c_{\text{in}}^{(3)}$ and $c_{\text{tel}}^{(1)}$ in paths 3 and 1, respectively, and on the bottom-right 2×2 block of V_{in} , which we denote $V_{\text{in}}^{(3)}$ and the top-left 2×2 block of V_{tel} , which we denote $V_{\text{tel}}^{(1)}$, the teleportation fidelity between the input and teleported Gaussian states can be written as

$$F_q(c_{\text{in}}^{(3)}, V_{\text{in}}^{(3)}, c_{\text{tel}}^{(1)}, V_{\text{tel}}^{(1)}) = \frac{1}{2} \frac{\exp[-\beta_{\text{tel}}^T (V_{\text{in}}^{(3)} + V_{\text{tel}}^{(1)})^{-1} \beta_{\text{tel}}]}{\sqrt{\Lambda + \Delta} - \sqrt{\Delta}}, \quad (\text{J14})$$

where $\beta_{\text{tel}} = c_{\text{in}}^{(3)} - c_{\text{tel}}^{(1)}$, $\Lambda = \det(V_{\text{in}}^{(3)} + V_{\text{tel}}^{(1)})$, and $\Delta = 16(\det V_{\text{in}}^{(3)} - 1/16)(\det V_{\text{tel}}^{(1)} - 1/16)$.

In the special case of vacuum-state teleportation, where $\beta_{\text{tel}} = 0$ and $\Delta = 0$, Eq. (J14) reduces to

$$F = \frac{2}{\sqrt{(1 + 4\sigma_x^2)(1 + 4\sigma_p^2)}}, \quad (\text{J15})$$

where $\sigma_x^2 = \langle(\Delta x)^2\rangle$ and $\sigma_p^2 = \langle(\Delta p)^2\rangle$ are the variances of the I and Q quadratures of the teleported field.

Furthermore, in the common case of unsqueezed vacuum, where $\sigma_x^2 = \sigma_p^2 = \sigma^2$, we obtain

$$F = \frac{2}{(1 + 4\sigma^2)}. \quad (\text{J16})$$

In Fig. 14, we exhibit the results of quantum and classical teleportation of vacuum states obtained for $G_E = 2.1$ dB, which are quantitatively similar to the results in Fig. 4 obtained for coherent states with nonzero mean. In Fig. 14(a), we plot the x - p quadrature representation of the vacuum field at port b of Alice that is teleported to Bob via the teleportation setup shown in Fig. 2. Likewise, in Figs. 14(b) and 14(c), we plot the teleported fields measured at Bob, which correspond to quantum ($G_E = 2.1$ dB) and classical ($G_E = 0$ dB) teleportation operations, respectively. While the teleportation fidelity achieved by the latter is 0.49, a much higher fidelity 0.67 is

achieved using quantum teleportation, which notably beats the classical bound of 0.5 for the teleportation of coherent states. Furthermore, in Figs. 14(d) and 14(e), we plot normalized histograms of the scatter data shown in Figs. 14(a)–14(c) plotted using red, blue, and magenta markers taken along the position x and momentum p quadratures, respectively. We also list on the sides in Figs. 14(d) and 14(e) the histogram variances extracted from the corresponding Gaussian fits (solid curves).

APPENDIX K: UPPER BOUND ON TELEPORTATION FIDELITY

Here, we derive analytical expressions for the teleportation fidelity in the ideal lossless case.

Without loss of generality, we set $\varphi_E = 0$ for the entangler and $\varphi_A = \pi$ for Alice's amplifier. For these phases, the rotation matrices for the entangler and Alice become $R(\varphi_E) = I_2$ and $R(\varphi_A) = -I_2$. Thus, the two-mode squeezing matrices for the entangler (S_1) and Alice (S_2) become

$$S_1 = \begin{pmatrix} \cosh(r_E)I_2 & \sinh(r_E)ZI_2 & 0_2 \\ \sinh(r_E)ZI_2 & \cosh(r_E)I_2 & 0_2 \\ 0_2 & 0_2 & I_2 \end{pmatrix} \quad (\text{K1})$$

and

$$S_2 = \begin{pmatrix} I_2 & 0_2 & 0_2 \\ 0_2 & \cosh(r_A)I_2 & -\sinh(r_A)ZI_2 \\ 0_2 & -\sinh(r_A)ZI_2 & \cosh(r_A)I_2 \end{pmatrix}. \quad (\text{K2})$$

Using Eqs. (K1), (K2), and (J6), we calculate the teleportation matrix $T = CS_2S_1$, whose block elements read

$$\begin{aligned} T_{11} &= \sqrt{\bar{\beta}_c} \cosh(r_E)I_2 - \sqrt{\beta_c} \sinh(r_A) \sinh(r_E)I_2, \\ T_{12} &= \sqrt{\bar{\beta}_c} \sinh(r_E)Z - \sqrt{\beta_c} \sinh(r_A) \cosh(r_E)Z, \\ T_{13} &= \sqrt{\bar{\beta}_c} \cosh(r_A)I_2, \\ T_{21} &= \cosh(r_A) \sinh(r_E)Z, \\ T_{22} &= \cosh(r_A) \cosh(r_E)I_2, \\ T_{23} &= -\sinh(r_A)Z, \\ T_{31} &= -\sqrt{\bar{\beta}_c} \cosh(r_E)I_2 - \sqrt{\beta_c} \sinh(r_A) \sinh(r_E)I_2, \\ T_{32} &= -\sqrt{\bar{\beta}_c} \sinh(r_E)Z - \sqrt{\beta_c} \sinh(r_A) \cosh(r_E)Z, \\ T_{33} &= \sqrt{\bar{\beta}_c} \cosh(r_A)I_2. \end{aligned} \quad (\text{K3})$$

Using the relation Eq. (J10) to calculate the teleported state vector, we get for the input coherent state c_0 defined in Eq. (J7)

$$c_{\text{tel}} = \left([\sqrt{\bar{\beta}_c} \cosh(r_A)]\sqrt{n_s} \cos(\theta_s), [\sqrt{\bar{\beta}_c} \cosh(r_A)]\sqrt{n_s} \sin(\theta_s), *, *, *, * \right)^T, \quad (\text{K4})$$

where we listed only the two elements associated with the teleported state in path 1.

The covariance matrix for the input coherent state is $V_0 = I_6/4$, where I_6 is the unity matrix of size 6×6 . Hence, the covariance matrix of the teleported state is given by $V_{\text{tel}} = TV_0T^\dagger = TT^\dagger/4$.

The covariance elements associated with the teleported state in path 1 correspond to the top-left 2×2 block of V_{tel} , which read

$$\begin{aligned} V_{\text{tel}}^{(1)} &= \frac{1}{4} \left[\bar{\beta}_c \cosh(2r_E) + \beta_c \sinh^2(r_A) \cosh(2r_E) \right. \\ &\quad \left. - 2\sqrt{\bar{\beta}_c\beta_c} \sinh(r_A) \sinh(2r_E) + \beta_c \cosh^2(r_A) \right] I_2. \end{aligned} \quad (\text{K5})$$

In the limit of unity gain for the classical feedforward signal, i.e., $\beta_c \cosh^2(r_A) = 1$, large gain in Alice's which-path-information eraser, i.e., $G_A = \cosh^2(r_A) \gg 1$, small coupling in the directional coupler, i.e., $\beta_c \approx 0$, and the approximation $\beta_c \sinh^2(r_A) \cong 1$, we obtain for the top-left

2×2 block of the teleported covariance matrix representing path 1

$$V_{\text{tel}}^{(1)} = \frac{1}{4} (2e^{-2r_E} + 1)I_2. \quad (\text{K6})$$

We also get $\beta_{\text{tel}} = 0$, since $c_0^{(3)} = c_{\text{tel}}^{(1)}$, and $\Delta = 0$ because $\det V_0^{(3)} = 1/16$ for the input coherent state. Substituting these parameters and $\Lambda = \det(V_{\text{in}}^{(3)} + V_{\text{tel}}^{(1)}) = (e^{-2r_E} + 1)^2/4$ in Eq. (J14) reduces the quantum teleportation fidelity into

$$F_q = \frac{1}{e^{-2r_E} + 1}. \quad (\text{K7})$$

This simple result demonstrates that the quantum teleportation fidelity in the lossless case (i) depends only on the two-mode squeezing parameter r_E , (ii) F_q can reach unity in the limit of large squeezing $r_E \gg 0$, and, more importantly, (iii) F_q exceeds 1/2 for any nonzero two-mode squeezing $r_E > 0$, thus beating the upper bound on the

achievable fidelity in the classical case, i.e., $F_c = 1/2$, attained when $r_E = 0$.

1. Teleportation fidelity with nonunity-gain classical channel

In this subsection, we calculate the teleportation fidelity for the lossless case when the unity-gain requirement for the classical channel is not met, i.e., $\beta_c \cosh^2(r_A) \neq 1$.

Start with the classical teleportation scenario for which no entanglement is shared between Alice and Bob. Substituting $r_E = 0$ in Eq. (K5) gives

$$V_{\text{tel}}^{(1)} = \frac{1}{4} [1 + 2\beta_c (G_A - 1)] I_2, \quad (\text{K8})$$

where $G_A \equiv \cosh^2(r_A)$. Since $\beta_{\text{tel}} = c_0^{(3)} - c_{\text{tel}}^{(1)} = (1 - \sqrt{\beta_c} \cosh(r_A), 1 - \sqrt{\beta_c} \cosh(r_A))^T$, $\Delta = 0$, and $(V_{\text{in}}^{(3)} + V_{\text{tel}}^{(1)})^{-1} = I_2 / \sqrt{\Lambda}$, where $\Lambda = [1 + \beta_c (G_A - 1)]^2 / 4$, the classical teleportation fidelity can be written as

$$F_c = \frac{\exp \left[-|\alpha_s|^2 \frac{B(\beta_c, G_A)}{A(\beta_c, G_A)} \right]}{2A(\beta_c, G_A)}, \quad (\text{K9})$$

where $|\alpha_s|^2 = n_s$, $A(\beta_c, G_A) = [1 + \beta_c (G_A - 1)]/2$, and $B(\beta_c, G_A) = \beta_c G_A - 2\sqrt{\beta_c} G_A + 1$.

Note that in the unity-gain limit $\beta_c G_A = 1$ and low coupling $\beta_c \cong 0$ we recover the expected upper bound on classical teleportation fidelity for coherent states $F_c = 1/2$. Also, for vacuum noise without displacement $|\alpha_s|^2 = 0$ and Alice's amplifier off ($G_A = 1$), i.e., not performing any measurement, we get $F_c = 1$ as expected. Lastly, in the high-amplification limit of the classical channel $\beta_c G_A \gg 1$, the teleportation fidelity diminishes $F_c \rightarrow 0$ due to increase in the added noise originating from the amplification process.

Turning now to the case of quantum teleportation, we rewrite Eq. (K5) as

$$V_{\text{tel}}^{(1)} = \frac{1}{4} \left[(1 - 2\beta_c + k) \cosh(2r_E) - 2\sqrt{k - k\beta_c - \beta_c + \beta_c^2 \sinh(2r_E) + k} \right] I_2 \quad (\text{K10})$$

and $\beta_{\text{tel}} = c_0^{(3)} - c_{\text{tel}}^{(1)} = (1 - \sqrt{k}, 1 - \sqrt{k})^T$, where $k \equiv \beta_c G_A$. Using these relations in combination with $(V_{\text{in}}^{(3)} + V_{\text{tel}}^{(1)})^{-1} = I_2 / \sqrt{\Lambda}$ and $\Delta = 0$, we arrive at the following quantum teleportation fidelity:

$$F_q = \frac{\exp \left[-|\alpha_s|^2 \frac{B(k)}{C(r_E, k)} \right]}{2C(r_E, k)}, \quad (\text{K11})$$

where $B(k) = (\sqrt{k} - 1)^2$ and $C(r_E, k) \equiv \sqrt{\Lambda}$ given by

$$C(r_E, k) = \frac{1}{4} \left[1 + k + (1 - 2\beta_c + k) \cosh(2r_E) - 2\sqrt{k - k\beta_c - \beta_c + \beta_c^2 \sinh(2r_E) + k} \right]. \quad (\text{K12})$$

In the limit of very small coupling $\beta_c \rightarrow 0$ and large gain at Alice $G_A \gg 1$, Eq. (K12) simplifies to

$$C(r_E, k) = \frac{1}{2} [(1 + k) \cosh^2(r_E) - \sqrt{k} \sinh(2r_E)]. \quad (\text{K13})$$

Substituting $r_E = 0$ in Eq. (K11) and using Eq. (K13) allows us to rewrite the classical teleportation fidelity in the form

$$F_c = \frac{\exp \left[-|\alpha_s|^2 \frac{2(\sqrt{k}-1)^2}{(1+k)} \right]}{(1+k)}. \quad (\text{K14})$$

Note that for $k = 1$, i.e., corresponding to unity gain in the classical channel, Eq. (K14) yields the expected bound on classical teleportation, i.e., $F_c = 1/2$.

In Fig. 15, we plot the fidelity of quantum teleportation of vacuum states as a function of G_E , calculated using Eq. (J14) for multiple cases of interest. The solid curves correspond to cases where the classical channel has unity transmission $k = 1$. The solid black curve corresponds to the lossless case, where the fidelity simplifies to Eq. (K7). The solid cyan and blue curves correspond to cases of

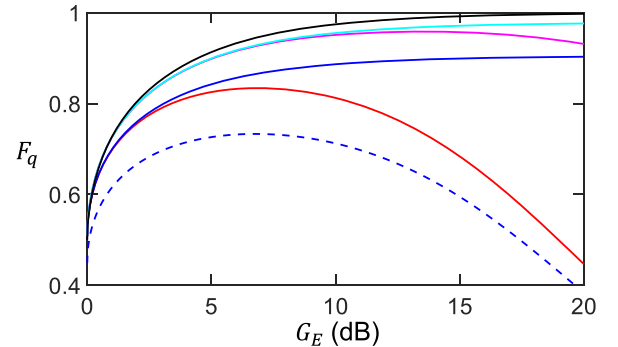


FIG. 15. Calculated fidelity of quantum teleportation of vacuum states versus entangler gain. The solid curves represent cases where $k = 1$ (i.e., unity-transmission classical channel, in particular, $G_A = 15$ dB and $\beta_c = -15$ dB). The black solid curve corresponds to the lossless case. The solid cyan and blue curves are calculated for the cases of symmetric loss of 0.1 and 0.5 dB, respectively, where the loss is between the entangler and Alice and the entangler and Bob. The solid magenta and red curves are calculated for the cases of asymmetric loss of 0.2 and 1 dB, respectively, where the loss is only between the entangler and Bob. The dashed blue curve represents an amplification case where $k > 1$ (i.e., $G_A = 16$ dB and $\beta_c = -15$ dB) and has a symmetric loss of 0.5 dB between the entangler and Alice and the entangler and Bob.

symmetric loss of 0.1 and 0.5 dB, respectively, present between the entangler and Alice and the entangler and Bob. The solid magenta and red curves correspond to cases of asymmetric loss of 0.2 and 1 dB, respectively, where the loss is present only between the entangler and Bob. Finally, the dashed blue curve corresponds to the case in which the classical channel amplifies $k > 1$ (i.e., $G_A = 16$ dB and $\beta_c = -15$ dB) and the two paths from the entangler to Alice and Bob have equal losses of 0.5 dB.

By inspecting these test cases, we observe that (i) F_q is monotonic and increases with G_E for unity-transmission classical channels and symmetric losses, (ii) F_q peaks for finite G_E in the cases of asymmetric losses combined with unity-transmission classical channels or symmetric losses combined with amplifying classical channels, and (iii) the maximum achievable fidelities decrease with increased losses.

Finally, in Fig. 16, we illustrate the effect of the transmission magnitude of the classical channel, i.e., k , on the quantum and classical teleportation fidelity of coherent states as a function of their average photon number $|\alpha_s|^2$. In particular, we consider the lossless case which we have analytical expressions for. The dashed blue and red lines represent the calculated quantum and classical teleportation fidelities, corresponding to $G_E = 5$ dB and $G_E = 0$ dB, respectively, for the case of unity-transmission classical channel, i.e., $k = 1$. As expected from Eqs. (K11) and (K14), the fidelities are constant and independent of $|\alpha_s|^2$, whereas, in the amplifying case in which $k > 1$, where we assume in this example $G_A = 10$ and $\beta_c = 1/8$, the

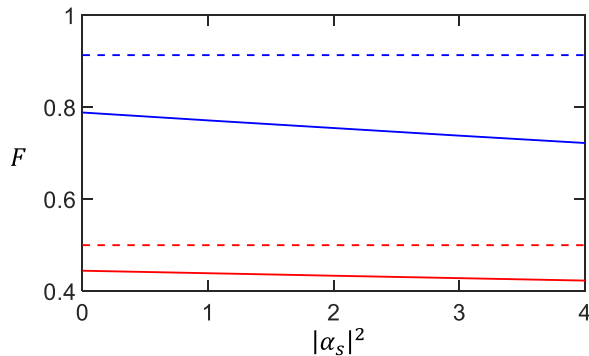


FIG. 16. Calculated teleportation fidelity of coherent states versus their average photon number in the absence of loss. The dashed blue and red lines represent the quantum and classical teleportation fidelity, respectively, in the case of unity-transmission classical channel, i.e., $k = 1$. The solid blue and red lines represent the quantum and classical teleportation fidelity in the case of nonunity-transmission classical channel, i.e., $k > 1$, where in this numerical example $G_A = 10$ and $\beta_c = 1/8$. In the quantum teleportation scenarios (blue lines) the fidelity is calculated for $G_E = 5$ dB. Unlike the $k > 1$ cases, the fidelities for $k = 1$ are independent of the average photon number of the input coherent state.

fidelities decrease with $|\alpha_s|^2$ and their maximum values are much lower than in the $k = 1$ case.

APPENDIX L: COMPARISON BETWEEN CV ENTANGLEMENT AND TELEPORTATION IN THE OPTICAL AND MICROWAVE DOMAIN

As highlighted in the paper, CV entanglement schemes can be advantageous in quantum communication owing to their unconditional and deterministic nature as well as their ability to operate at faster rates than their DV counterparts [29,97,98].

In the optical domain, CV entanglement serves as an important quantum resource in numerous quantum network applications including quantum teleportation of coherent states [43–46], entanglement swapping between CVs [51,52], entanglement swapping between DVs and CVs [99], and quantum dense coding [100].

Combining such capabilities with recent demonstrations of CV entanglement distribution at a telecommunication wavelength over 20 km of fiber [101] and teleportation of optical coherent modes through fiber channels that extend over 6 [46] and 10 km [102] underscore the feasibility of establishing metropolitan quantum networks that employ deterministic quantum teleportation.

Notably, such key results in the optical domain are largely enabled by the availability of high-performance squeezers, low-loss beam splitters, low attenuation channels, and efficient homodyne detectors.

While demonstrations of CV-based quantum teleportation in the microwave domain are limited so far to our present work and Ref. [47], it is instructive to compare the performance of the various basic building blocks available in the two frequency domains to learn about their respective physical limitations, which can help assess the prospects of applying large-scale CV entanglement schemes in the microwave domain.

1. Squeezers

Single-mode and two-mode squeezers capable of generating high-quality CV entanglement are pivotal components for building CV-based quantum teleportation schemes. Two main figures of merit that characterize such EPR-like sources are the achievable level of squeezing and bandwidth, which loosely speaking, are measures of the quality of the emitted EPR pairs and the rate at which they are generated. While the former measure mainly impacts the teleportation fidelity, the latter determines the maximum rate at which the teleportation apparatus can operate.

With regard to the first measure, the record for squeezing in the optical domain, to the best of our knowledge, is 15 dB [103] achieved at wavelength 1064 nm (near-infrared) in a degenerate single-mode squeezer, i.e., an optical parametric amplifier (OPA) that uses three-wave mixing. In the microwave domain, single-mode and two-mode squeezing have

been demonstrated in resonator-based Josephson parametric amplifiers (JPAs) as well as in Josephson TWPAs. Single-mode squeezing up to 10–11 dB has been demonstrated in a JPA [104] and a TWPA device [85].

Similarly, we achieve two-mode squeezing up to 5 dB in our JM device. Other notable demonstrations of two-mode squeezing with resonator-based JPAs achieve up to 12 dB with a single-port device [105] and 3.4 dB with a two-port JM [58]. Likewise, two-mode squeezing of up to 9.5 dB is achieved in a TWPA device [85], though in our case, unlike the latter, the JM possesses spatial separation between the two entangled modes, which we exploit to form hardware-efficient quantum teleportation and entanglement swapping schemes.

Based on these squeezing figures, microwave devices do not fall far behind their optical counterparts. They also have some room for improvement, such as reducing loss in TWPAs [85] and decreasing the JRM nonlinearity in JM devices [69].

As to the second measure, the bandwidth of squeezers in the optical domain can range from tens of megahertz obtained using cavity-based OPAs with more than 10 dB of squeezing [103,106] to an estimated 2.5 THz at 6 dB of squeezing using single-pass traveling-wave OPAs [107]. In the microwave domain, on the other hand, the bandwidth of mode squeezers can range from a few megahertz to a few gigahertz. Typically, narrow bandwidths in the range 5–50 MHz are obtained using resonant-based JPAs [55,105,108], which include the nondegenerate JMs measured in this work, while large bandwidths in the range 1–5 GHz are attained using TWPAs.

Notably, two-mode squeezing was measured recently in multiple TWPA variations [64,84,85], with Refs. [64,85] reporting large signal-idler frequency separations up to 400 and 1500 MHz, respectively. It is worth noting, though, that these broadband devices lack spatial separation between the signal and idler modes.

Recently, a third group of Josephson parametric devices has emerged that aim to maintain the advantages of resonator-based JPAs (i.e., low loss and low added noise) while significantly boosting their bandwidths (200–900 MHz) and saturation powers (−110 to −90 dBm) [86,109,110]. Such enhancements are achieved by incorporating impedance matching networks between the Josephson mixing elements (such as the dc or rf superconducting quantum interference devices) and the external feedlines, which define the device ports. So far, all impedance-matched JPAs are based on degenerate devices, since they are easier to engineer and implement [110]. However, applying similar design strategies to boost the bandwidth and saturation power of JM devices should be feasible as well. More specifically, it should be possible, in principle, to make nondegenerate impedance-matched JM devices, which have bandwidths of about 600 MHz at 20 dB of gain.

2. Supporting components

By this term, we mean the microwave or optical components that take part in the entanglement generation by the CV entangler, the “Bell detection” performed by Alice, and the “local operations” applied by Bob.

Examples of such components in the optical domain include 50:50 beam splitters, polarizing optical beam splitters, and mirrors. Analogously, in the microwave domain, such components include magnetic-based circulators, hybrids, and directional couplers. While beam splitters and mirrors can be very low loss in the optical domain, normal-metal microwave components and particularly magnetic-based circulators can introduce losses of up to a few decibels in the path of EPR beams (depending on their number and their operating frequency range).

As we have shown in the paper, the main limitation for the distribution of high-quality CV entanglement in the microwave domain is insertion loss, which can be due to four sources: (i) nonsuperconducting microwave components in the path of the EPR states, such as magnetic-based circulators, hybrids, and directional couplers. This loss can be eliminated or significantly reduced by realizing on-chip superconducting hybrids and directional couplers [111] and doing away with magnetic-based circulators by, for example, implementing nondegenerate Josephson mixers that work in transmission rather than reflection (see Fig. 17) [83]. (ii) The contact interface between the superconducting coax cables and the different components at Alice and Bob and the CV entangler. This loss can be on the order of 0.1 dB per connector if commercial normal-metal connectors such as SMA connectors are used. However, such contact loss can be significantly reduced if the contact is established directly between the superconducting center conductor of the coax cable and the superconducting transmission line on the chip [11,15,112]. (iii) Reflections due to mismatches in the impedance of the different components. This effective loss can be significantly mitigated by reducing the number of connectors and components used in the setup as much as possible and improving the impedance matching between

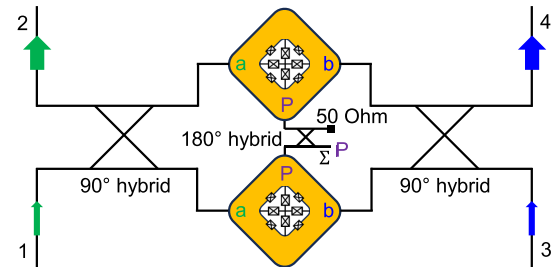


FIG. 17. Circuit diagram of a nondegenerate through Josephson mixer [83]. The device operation and performance are identical to that of a single nondegenerate Josephson mixer but works in transmission instead of reflection. Note that the device response is reciprocal between ports 1 and 2 and ports 3 and 4.

the various components. (iv) The superconducting coax cables carrying the entangled states, which we discuss as part of the third building block referred to as CV channels.

3. CV channels

By this term, we mean the communication medium through which the CV entangled state is transmitted from the CV entangler, i.e., the EPR source, to Alice and Bob. Examples of such channels in the optical domain include free space [43–45] and optical fibers [46,102], whereas in the microwave domain they are mainly superconducting coax cables made of NbTi or aluminum [11,15,112]. Some of the key advantages of using optical fibers stem from their extremely low attenuation per unit length and their wide use in commercial communication applications. Interestingly, superconducting coax cables possess similar advantages. They are commercially available, and coaxial lines made of aluminum exhibit remarkably low loss per unit length, in the range 0.32–0.39 dB/km [15,112] around 5 GHz, which is on par with the typical attenuation attained in optical fibers of about 0.2 dB/km at 1550 nm [102].

Thus, if attenuation per unit length were the sole determining factor, the maximum feasible teleportation distance of microwave radiation would most likely match the kilometer scale achieved via optical fibers. However, in the microwave domain there are other factors at play that can significantly limit the maximum distance over which quantum teleportation can be realized. Chief among them is the availability of large and distributed cryogenic infrastructure that can cover long distances while applying enough cooling power to keep the long coax cables thermalized at low ambient temperatures in the range 20–100 mK [17,113]. The reason for this stringent cooling requirement is twofold, namely, to keep the coaxial cables in the superconducting state (way below their critical temperature) to prevent temperature-related losses, such as quasiparticle generation and flux penetration, and to minimize the thermal photon population in the coaxial lines, which can be detrimental for microwave signals due to their very low energy.

For example, for a signal frequency of about 7.5 GHz passing through a transmission line at around 2 K, the average thermal photon population in the line is about 5 [114]. It is important to point out here that, although thermal photon population in optical fibers at room temperature is generally negligible, CV entanglement in the optical domain can be degraded by excess noise induced by the effect of depolarized guided acoustic wave Brillouin scattering in the fibers [115], generated, for example, by the local oscillator signals needed for processing at Alice and Bob and which copropagate alongside the CV entangled states.

One possible approach for relaxing the strict requirement on the cooling power and thereby allowing us to extend the lengths of the microwave channels between different

parties is using low-loss superconducting NbTi coax lines that pass through elevated temperature stages, such as 1 or 4 K (which are easier to cool down and maintain). However, for this approach to work, certain cooling techniques must be applied to maintain low thermal population in the coax line, such as radiative cooling of the channel ahead of any quantum signal transmission [114].

In conclusion, despite the numerous outstanding challenges, the prospects of applying CV entanglement or CV-based quantum teleportation in the microwave domain are quite promising. Based on the discussion above, the main roadblock that we foresee for extending the teleportation distances in the microwave domain beyond tens of meters, assuming all other challenges outlined above are solved, is the practicality and viability of realizing long cryogenic links between distant entities.

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