

Quantum Advantage for Sensing Properties of Classical Fields

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Modern precision experiments often probe unknown classical fields with bosonic sensors in quantum-noise-limited regimes where vacuum fluctuations limit conventional readout. We introduce quantum signal learning, a sensing framework that extends metrology to a broader property-learning setting, and propose a quantum-enhanced protocol that simultaneously estimates many properties of a classical signal with shot noise suppressed below the vacuum level. Our scheme requires only two-mode squeezing, passive optics, and static homodyne measurements, and enables *post hoc* classical estimation of many properties from the same experimental dataset. We prove that our protocol enables a quantum speedup for common classical sensing tasks, including measuring electromagnetic correlations, real-time feedback control of interferometric cavities, and Fourier-domain matched filtering. To establish these separations, we introduce an optimal-transport conditioning method and show both worst-case exponential separations from all entanglement-free strategies and practical speedups over homodyne and heterodyne baselines. We further show that when squeezing is treated as a resource, a protocol with squeezed light can sense a structured classical background exponentially faster than any coherent classical probe.

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I. INTRODUCTION

From gravitational-wave interferometers and microwave cavities to optical and circuit-QED transducers, many leading precision platforms now operate in regimes limited not by instrumental noise but by quantum fluctuations of the electromagnetic (EM) field [1–7]. In such experiments the signal is often a classical field, for example, ambient radiation, a fluctuating force, or a time-dependent control imperfection, coupled linearly to a bosonic sensor. The resulting inference tasks are rarely captured by the textbook single-parameter model $U_\theta(T) = e^{-i\theta GT}$ with known generator G . Instead, one typically cares about structured properties of an uncertain environment, such as quadrature correlations in an electromagnetic transducer [8–11], phase-space mixing in an interferometric cavity [12–15], or the ability to score many candidate waveforms using *post hoc* template-bank queries [16–23]. These are distributional

questions, since uncertainty in the field and in prior information induces a nontrivial distribution over sensor responses, and the object of interest is a functional of that distribution rather than a single parameter.

A basic obstacle is that the standard measurement primitives do not simultaneously deliver quantum-limited precision and broad learnability. Heterodyne detection, and more generally entanglement-free joint quadrature readout, is informationally complete and supports *post hoc* analysis, but it carries an irreducible vacuum noise penalty that cannot be circumvented with probe squeezing [24]. Homodyne measurement can resolve subvacuum fluctuations along one quadrature and is therefore well matched to carefully aligned tasks, but it reveals only one-dimensional marginals of the underlying phase-space response. In practice, rapidly and reliably preparing squeezed probes aligned with many homodyne quadrature angles is technically costly, so the available baseline is often a small fixed set of angles. This creates a dilemma in quantum-noise-limited sensing. One can either collect flexible data at the vacuum noise floor or sharper, quantum-limited data that cannot access intricate phase-space structure.

We show that entanglement resolves this dilemma and yields a quantum advantage for sensing classical fields. Our protocol prepares a two-mode squeezed vacuum (TMSV) between a sensing mode and an idler, applies the unknown field-induced channel to the sensing mode, and performs a

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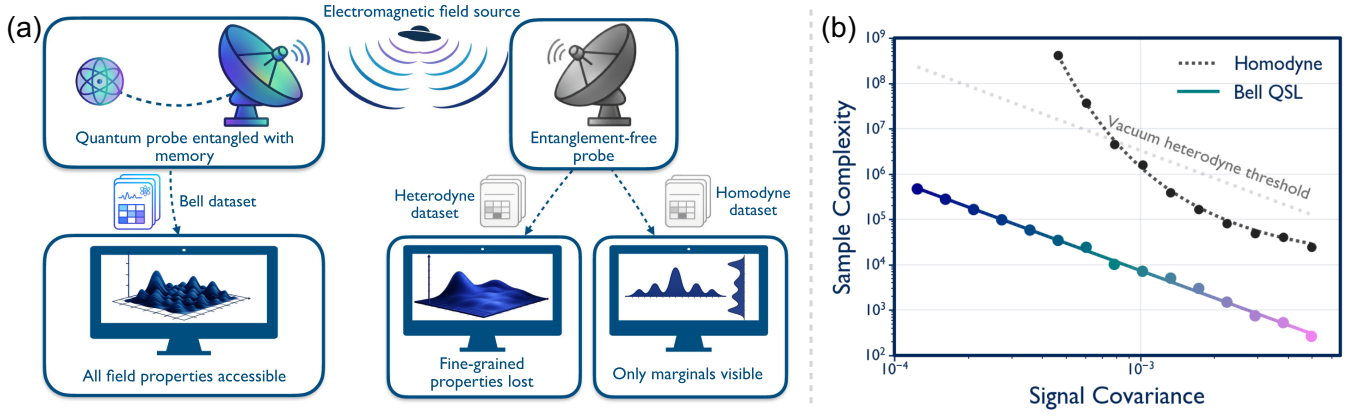


FIG. 1. (a) Quantum-enhanced signal learning vs conventional electromagnetic (EM) sensing. Bell QSL leverages probes entangled with quantum memory to collect a classical record that is used for quantum-limited, informationally complete estimation of the EM field distribution. Heterodyne measurements suffer a vacuum noise penalty, blurring fine-grained features, while homodyne measurements only resolve marginals that may not contain higher-dimensional properties. (b) Quantum advantage for estimating correlations in an electromagnetic field. Minimum sample complexity required to estimate a Gaussian EM field covariance c in the shot-noise-limited regime, as in Eq. (1). At $r = 2$ and $|\sigma_x^2 - \sigma_p^2| \ll c$, in the presence of white noise with amplitude $0.2c$, Bell QSL scales polynomially and orders of magnitude below the vacuum noise limit, achieving quantum advantage over both homodyne and heterodyne tomography.

standard continuous-variable (CV) Bell measurement of commuting Einstein-Podolsky-Rosen (EPR) quadratures [24–28]. Operationally, the classical field induces an unknown phase-space displacement α on the sensing mode, and the Bell measurement produces a complex outcome ζ that is a blurred snapshot of this displacement. ζ satisfies an additive-noise model $\zeta = \alpha + Z$, where Z is an independent complex Gaussian with law $\mathcal{N}_C(0, e^{-2r})$ set by the finite EPR correlations of the two-mode squeezed state. Thus each channel use yields simultaneous estimates of both quadratures, and the noise in both estimates is suppressed below the vacuum level by a factor e^{-2r} . This should be contrasted with heterodyne, which yields the same two-quadrature structure but with an irreducible vacuum variance. The resulting dataset is informationally complete and supports *post hoc* learning, since many properties of the underlying signal can be estimated from the same measurement record without rerunning the experiment.

We formalize these tasks via a learning framework we call quantum signal learning (QSL), which models classical-field sensing as estimating properties of the phase-space response induced by a linear bosonic coupling. Within this framework we give estimator families for a wide class of physical properties and prove sample-complexity guarantees that improve with squeezing. We then establish quantum advantage in two complementary senses. First, in worst-case regimes relevant to template-bank scoring, our Bell QSL protocol enables exponential improvements for Fourier-domain matched-filter queries over any entanglement-free data-collection strategy. Second, motivated by realistic laboratory constraints, we introduce an optimal-transport-(OT) based conditioning framework that quantifies when restricted-angle homodyne becomes ill conditioned for

learning genuinely two-dimensional phase-space features, while Bell QSL remains efficient. We illustrate these advantages in experimentally commonplace sensing problems motivated by correlated classical electromagnetic fields and phase rotation noise in interferometric cavities.

These separations follow the bosonic-metrology premise that probe energy governs sensing precision, placing squeezing and displacement on equal footing [29] and treating entanglement as the advantage-generating resource. While we argue for the importance of this interpretation, we also demonstrate that unentangled, squeezed-probe strategies can sense a fluctuating classical field exponentially faster than any coherent-state classical probe. In complementary work, Ref. [30] studies the unique power of squeezed light for sensing time-varying signals, achieving exponential gains over any squeezing-free classical protocol. Squeezing and entanglement thus emerge as distinct sources of quantum advantage in classical-field sensing.

II. QUANTUM SIGNAL LEARNING

An n -mode bosonic signal Hamiltonian consists of linear terms in the quadratures. This Hamiltonian circumscribes most classical-field sensing tasks, where quantum-classical interactions usually enter as c -number couplings to the canonical quadratures.

$$H(t) = \sum_{j=1}^n f_{x,j}(t) \hat{x}_j + f_{p,j}(t) \hat{p}_j.$$

QSL begins with the observation that the time-evolution channel induced by evolving n bosonic probes under $H(t)$ for total time T can be described by the following quantum

channel, where $\hat{D}(\alpha)$ is the displacement operator:

$$\mathcal{E}_H^T(\rho) = \int d^{2n} \alpha P_H^{(T)}(\alpha) \hat{D}(\alpha) \rho \hat{D}(\alpha)^\dagger.$$

The linear structure of H is realized by the channel consisting of only displacement terms. With this representation, any property Ψ of the signal is encoded in the classical density $P_H^{(T)}$, which can be extracted by selecting a function $\psi(\alpha)$, called the *property kernel*, and integrating it against $P_H^{(T)}$:

$$\Psi(P_H^{(T)}) = \int d^{2n} \alpha P_H^{(T)}(\alpha) \psi(\alpha).$$

In this manner, classical field properties are linear functionals of the displacement distribution induced by the signal. The QSL problem follows from this observation.

Definition 1 (Quantum signal learning). Given a linear Hamiltonian H and an evolution time T , the quantum signal learning problem $\text{QSL}(H, T, \Psi, \epsilon, \delta)$ is to estimate the selected linear functional $\Psi(P_H^{(T)})$ to within absolute error ϵ , with success probability at least $1 - \delta$, given query access to $\mathcal{E}_H^{(T)}$.

Several natural properties of classical fields can be viewed neatly in this framework. For instance, for a single mode, the kernel $\psi_{\text{cov}} = xp = \text{Re}(\alpha) \text{Im}(\alpha)$ specifies a property Ψ_{cov} which is exactly the covariance of a (mean zero) classical field. Alternatively, $\psi_k(\alpha) = e^{ika}$ induces that $\Psi_k(\alpha)$ is exactly the signal's Fourier coefficient of frequency $k \in \mathbb{C}^n$. QSL thus subsumes the canonical single-parameter sensing setting, where distributions are delta functions, and encompasses a wide array of multiparameter fields, stochastic signals, and noise sources. Moreover, beyond linearity, which holds for all relevant classical sources, QSL does not assume prior knowledge of the signal's structure as is otherwise necessary in the canonical metrology setting. As we will see, the learning-theoretic QSL reformulation of quantum-enhanced sensing opens up the possibility for quantum speedups not present in the usual metrological picture.

A. Bell QSL algorithm

Now we provide an efficient, entanglement-enabled algorithm for the QSL problem. Our algorithm, called Bell quantum signal learning, has several practically useful properties. The algorithm collects a classical dataset that can be reused for arbitrary *post hoc* property estimation as it is informationally complete. While our estimators provide rigorous guarantees for property estimation, the dataset is more feature rich than entanglement-free counterparts, and can be used as input to machine learning pipelines (see Fig. 3). Experimentally, the algorithm requires only constant-depth quantum information processing in the form of two-mode entangled state preparation and passive beam

splitters. Moreover, it utilizes only measurements in a static homodyne basis, requiring no real-time control or relocking of squeezing axes, and achieves optimal uncertainties in all quadrature readouts.

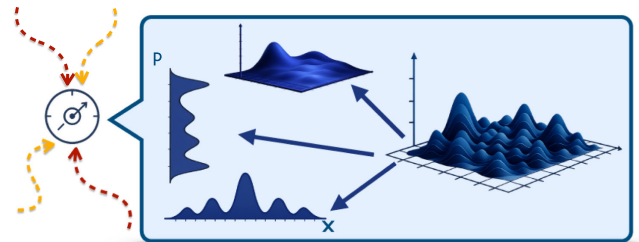
Let T_r denote the operator that maps any $f: \mathbb{C}^n \rightarrow \mathbb{C}$ to its squeezed-vacuum-blurred measurement distribution by convolution against a mean-zero Gaussian density with variance e^{-2r} . The algorithm, depicted in Fig. 2, proceeds in the following steps.

Algorithm 1 (Bell QSL). Given access to an n -mode linear Hamiltonian H for time T , perform the following steps.

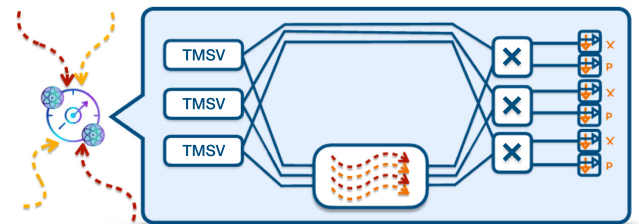
- (1) Prepare n two-mode squeezed vacuum states, and allow one bipartition to evolve under H for time T while storing the other half in quantum memory. Then, interfere each pair on a 50:50 beam splitter, and for each two-mode pair j , measure quadratures $\hat{x}_1^{(j)}, \hat{p}_2^{(j)}$.
- (2) Collect outcome $\zeta = \{\zeta_1, \dots, \zeta_n\}$, with $\zeta^{(j)} = \hat{x}_1^{(j)} + i\hat{p}_2^{(j)}$. Repeat N times.
- (3) Given property kernel ψ , return the estimator,

$$\hat{\Psi} = \frac{1}{N} \sum_{i=1}^N g_\psi(\zeta^{(i)}),$$

where g_ψ is the inverse of ψ under T_r .



(a) Conventional Field Sensing



(b) Quantum-Enhanced Bell QSL

FIG. 2. (a) Conventional bosonic sensing of classical fields. Homodyne measurements project high-dimensional field distributions onto chosen marginal axes, losing information, while heterodyne measurements blur sharp features with vacuum noise. (b) Bell QSL. Entangled two-mode squeezed vacuum probes are prepared, with half the modes acting as probes and the other half stored in quantum memory. Beam splitters and homodyne measurements constitute CV Bell-basis measurement, enabling informationally complete, sharp property recovery.

In practice, $\hat{\Psi}$ can always be computed using the Fourier transform of ψ when it is well defined, or by a particular polynomial transformation of measurement shots; these estimators are discussed in detail, with fully general sample complexity analyses, in Supplemental Material (SM) [31].

The data-collection strategy of Bell QSL is a well-studied primitive across CV quantum optics and information. Originally established for CV teleportation and dense coding [32,33], recent works have used Bell-basis measurements to establish quantum speedups in certain sensing and learning settings [28,34], with an experimental demonstration of multimode displacement channel learning in Ref. [35]. Our central contributions lie in applying this experimental primitive within the QSL framework to extract utility for a wide array of practical sensing tasks, and in developing novel theoretical methods to establish quantum speedups in these regimes. In the following sections, we analyze the performance of Bell QSL in multiple regimes, establishing that our algorithm can achieve practical quantum advantages over conventional benchmarks.

III. APPLICATIONS OF BELL QSL

Here we demonstrate the utility of Bell QSL for shot-noise-limited field sensing, including characterizing correlated electromagnetic fields and real-time feedback control in interferometers.

A. Electromagnetic field sensing

A broad class of electromagnetic sensors, from microwave and circuit-QED resonators to narrow band optical cavities, can be modeled as a single bosonic mode driven by an incident classical field. An LC mode with canonical charge and flux operators $[\hat{\Phi}, \hat{Q}] = i\hbar$ has the free Hamiltonian $H_0 = \hat{Q}^2/(2C) + \hat{\Phi}^2/(2L)$. An incident classical electromagnetic field induces an effective electromotive force and flux bias determined by the sensor geometry,

$$V_{\text{eff}}(t) \equiv \oint_C \mathbf{E}(\mathbf{r}, t) \cdot d\boldsymbol{\ell}, \quad \Phi_{\text{ext}}(t) \equiv \int_S \mathbf{B}(\mathbf{r}, t) \cdot d\mathbf{A},$$

so that (up to an irrelevant c number) the driven Hamiltonian can be written as

$$H(t) = H_0 - V_{\text{eff}}(t)\hat{Q} - I_{\text{eff}}(t)\hat{\Phi}, \quad I_{\text{eff}}(t) := \Phi_{\text{ext}}(t)/L.$$

An unknown EM environment enters through two linear couplings to conjugate quadratures, one “electriclike” ($V_{\text{eff}}\hat{Q}$) and one “magneticlike” ($I_{\text{eff}}\hat{\Phi}$). Treating $[V_{\text{eff}}(t), I_{\text{eff}}(t)]$ as stochastic processes, natural when modeling uncertain signals, clutter, or fluctuating backgrounds, bounded-time evolution induces a displacement channel with end point distribution $P_H^{(T)}(\alpha)$. (The reduction to this

channel, including the rotating-frame drive and explicit displacement amplitude, is given in SM [31].)

In many common operating regimes (e.g., narrow band sensing, aggregation over many microscopic field contributions, or approximately Gaussian priors), the induced end point displacement α is well modeled as a centered Gaussian on phase space,

$$\alpha \sim \mathcal{N}_{\mathbb{C}}(0, \Sigma(c)), \quad \Sigma(c) = \begin{pmatrix} \sigma_x^2 & c \\ c & \sigma_p^2 \end{pmatrix}, \quad (1)$$

where c encodes a *joint* quadrature covariance of the incident field as seen by the transducer [36–39]. In general, electromagnetic dynamics and boundary conditions tie electriclike and magneticlike degrees of freedom, and after transduction they appear precisely as correlations between conjugate quadratures.

In the shot-noise-limited regime, we are interested in estimating values of $c \ll 1$, below the vacuum-limited heterodyne variance. Moreover, squeezed homodyne tomography can be executed only along fixed measurement axes, because of the challenge of rellocking squeezing axes during a short sensing window. Even in this simple Gaussian-covariance estimation problem, one can see that, e.g., a pair of hypothesis covariances $P_{\pm} = \mathcal{N}(0, \Sigma(\pm c))$ share identical x and p marginals, differing in the sign of the correlation. Common homodyne measurements at angles $\Theta = \{0, \pi/2\}$, which image these marginals up to squeezed-Gaussian smoothing, cannot distinguish these hypotheses.

This “hard instance” can be extended in a practically meaningful way by moving slightly away from exact isotropy. Concretely, consider a near-isotropic family in which the marginal variances are almost equal (e.g., $\sigma_x^2 = \sigma^2 + \Delta/2$ and $\sigma_p^2 = \sigma^2 - \Delta/2$ with $|\Delta| \ll \sigma^2$), so that the x and p marginals are nearly insensitive to the correlation parameter even though the physically relevant information resides in the joint quadrature correlation c . The key observation is that while instances which are strictly *impossible* for homodyne learners are knife-edge phenomena, common learning tasks *near* this isotropic worst case will be very hard for entanglement-free homodyne protocols, which access c only through small symmetry-breaking effects, resulting in an ill-conditioned inference problem. Meanwhile, Bell QSL will extract c with optimal estimator variance, regardless of the instance, simply by using the empirical covariance $\hat{c} = (1/N) \sum_{i=1}^N \zeta_x^{(i)} \zeta_p^{(i)}$.

Theorem 1 (Practical quantum advantage in learning an EM correlation, informal). Consider the near-isotropic EM family characterized by Eq. (1) with $\sigma_x^2, \sigma_p^2, c \ll O(1)$ and $\Delta = |\sigma_x^2 - \sigma_p^2| \ll |c|$. Then Bell QSL distinguishes the pair using $O(e^{-4r}/c^2)$ shots, where r is the squeezing parameter, while any adaptive $\{x, p\}$ homodyne strategy requires at least $\Omega(e^{-4r}/\Delta^2)$ shots to do so with high probability.

Theorem 1 implies a separation on the sample complexity of learning c , because homodyne strategies can often fail by a sign error. The performance gap between Bell QSL and entanglement-free benchmarks, which improves with squeezing, is demonstrated in Fig. 1. In Sec. IV B (and in SM [31]) we formalize this notion of ill conditioning and a “distance to marginal blindness” using an optimal-transport stability framework.

In many precision experiments where signals are fleeting and optical probes operate at low noise temperatures, Bell QSL (Fig. 2) only requires storage of entangled idler modes for short times. However, in lower-frequency applications such as radar, longer integration times require the prepared entanglement to be stored coherently for longer times, which can be practically challenging. In such settings, photon loss is often the dominant error source. Assuming a loss rate of $0 < p < 1$ during the storage process, one finds that the e^{-4r}/ϵ^2 idealized sample complexity is replaced by $(1-p)e^{-4r}/\epsilon^2 + O(p)$ [28]. Operationally, under a finite number of samples, the loss rate creates a *precision floor*, such that for values of ϵ above this floor, the entangled protocol retains its full e^{-4r} speedup, whereas below the floor, the provable speedup is lost if one compares against a noiseless benchmark. This effect is identified in Ref. [40] as an unavoidable consequence of local noise models which entirely decohere the given state.

B. Feedback control in interferometric cavities

Beyond direct sensing of external fields, a natural application is learning internal cavity or interferometer noise parameters that appear as Gaussian distortions of optical quadratures. In high-sensitivity interferometers, including gravitational-wave detectors such as LIGO, many relevant technical and quantum noise processes are well approximated as Gaussian, so their net effect on a probe field is captured by a small number of covariance parameters: loss, squeezing strength, and quadrature rotation [41–43]. However, these parameters can be difficult to track in real-time sensing from homodyne marginals alone.

A common effective model for phase or control imperfections is an unknown quadrature rotation,

$$\begin{pmatrix} \hat{x} \\ \hat{p} \end{pmatrix} \mapsto R(\theta) \begin{pmatrix} \hat{x} \\ \hat{p} \end{pmatrix}, \quad R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix},$$

where θ may represent squeezing-angle drift, detuning-induced quadrature mixing, or more general phase-space misalignment. Bell QSL provides a natural real-time monitoring strategy that can be used to learn θ with a practical speedup.

Concretely, one can apply a known displacement β onto the probe arm of a TMSV, inject it into a sideband of the interferometer near the primary sensing band, and run the remainder of Bell QSL as usual on the returned probe and stored idler. As shown in SM, the resulting data $\zeta^{(i)}$ then

have mean $\mathbb{E}[\zeta] = e^{i\theta}\beta$ while their per-quadrature measurement noise is $\nu_r = \frac{1}{2}e^{-2r}$ [31]. Thus the sample mean $\bar{\zeta} = (1/N) \sum_{i=1}^N \zeta^{(i)}$ yields the signed phase estimate $\hat{\theta} = \arg(e^{-i\arg\beta} \bar{\zeta}/|\beta|)$, with sample complexity $N = O(\nu_r/(|\beta|^2\epsilon^2))$ for ϵ -accurate estimation, in direct analogy with the moment estimators used for Gaussian covariance learning. Both quadrature components are recoverable from every measurement.

Meanwhile, heterodyne measurements incur the standard multiplicative overhead of e^{2r} . Because θ lies along an unknown axis, homodyne measurements must be performed adaptively to identify both its magnitude and sign, and each measurement shot may carry little information about one of the two quadratures. In real-time measurements, this adaptive scanning may be experimentally intractable in conjunction with axis-aligned squeezing, preventing entanglement-free error mitigation beyond the vacuum noise limit.

Operationally, the two-mode squeezed light may be injected into optical sidebands at each analysis frequency, preventing interaction with the primary signal, while the idler can be kept anywhere outside the main readout band. This enables noninvasive quantum-enhanced noise monitoring, and data can be fed back to stabilize operating parameters such as squeezing angle or detuning [9,13,44]. In this way, Bell QSL interfaces naturally with the control layer that often determines whether shot-noise-limited interferometry can be sustained over long integration times.

IV. QUANTUM ADVANTAGE FOR QSL

In Sec. III, we proposed applications of Bell QSL to learning electromagnetic signals and noise and demonstrated a simple example of practical quantum advantage in learning field correlations. Here, we establish much more general results on quantum advantage in QSL.

A. Matched filtering

The quantum advantage in Theorem 1 was established against a practically motivated restricted-angle homodyne baseline. Bell QSL also achieves a worst-case exponential advantage against *any* entanglement-free data-collection strategy, including adaptive protocols with arbitrarily complex measurement bases. This separation arises in matched filtering and beamforming, ubiquitous sensing primitives in which one scores a large bank of candidate templates against a signal measurement record and selects the hypotheses most consistent with the data. Variants of this procedure appear throughout signal processing, from communications to radar, and are central to modern high-precision searches such as template-bank pipelines in gravitational-wave detectors, where the number of candidate waveforms can be enormous [16–23].

Concretely, consider an n -mode resonant waveform,

$$F(t) = \sum_{k=1}^n \sqrt{\omega_k} \left(A_k \cos(\omega_k t) + B_k \sin(\omega_k t) \right),$$

which induces a random displacement $\alpha \in \mathbb{C}^n$ with law $P(\alpha)$. A template $w \in \mathbb{C}^n$, corresponding to a hypothesis for the $2n$ values $\{A_k, B_k\}$, defines the matched-filter output $Y_w = \text{Re}(\alpha^\dagger w)$. For Fourier-domain scoring we use the bounded feature $\varphi_{Y_w}(t) = \mathbb{E}[e^{itY_w}]$, which is a numerically stable surrogate for maximum-likelihood-based scores and is especially natural when the signal modes are independent in the Fourier basis. Crucially, a query (w, t) is equivalent to a characteristic-function evaluation via the relation $\varphi_{Y_w}(t) = \chi_P(itw/2)$. Scoring a bank of M templates thus reduces to estimating χ_P at M points. With this observation, we find that Bell QSL achieves an exponential worst-case advantage in estimating sinusoidal waveforms via matched filtering.

Theorem 2 (Worst-case quantum advantage for matched filtering, informal). There exist priors over the sinusoidal coefficients $\{A_k, B_k\}$ for which any entanglement-free protocol (including those with squeezed probe states) requires $N \geq \Omega(3^n)$ channel uses to score any single template to constant accuracy. In contrast, Bell QSL with two-mode squeezing $r = \Theta(\log n)$ can estimate all M scores to accuracy ϵ using only $N = O(\epsilon^{-2} \log M)$ shots, independent of n .

While the sinusoidal-waveform sensing task considered in Theorem 2 is commonplace, this worst-case advantage occupies only a narrow region of the full problem space, since it relies on a highly structured prior over the coefficients $\{A_k, B_k\}$ that hides the relevant information from entanglement-free readout. Next, we return to experimentally motivated baselines and develop a framework that quantifies the trade-off between the magnitude of quantum advantage and the fraction of physically relevant instances for which that advantage persists.

B. Optimal transport and practical advantage

We now introduce a general conditioning methodology that turns measurement constraints into sample-complexity lower bounds. Any single-shot measurement strategy can be described as a map $\mathcal{M}$ that sends the underlying displacement law P to a one-round outcome distribution $\mathcal{M}(P)$. For instance, homodyne measurements made along a fixed set of axes in one round consist of projection to finitely many quadrature marginals followed by known Gaussian blurring, and $\mathcal{M}$ is simply the resulting product distribution. Let $\mathcal{C}$ denote the hypothesis class of possible phase-space distributions (e.g., the family of centered Gaussians earlier). Now consider the following optimal-transport stability modulus,

$$\omega_{\mathcal{M}, \mathcal{C}}(\eta)$$

$$:= \sup \{ W_1(P, Q) : P, Q \in \mathcal{C}, \text{TV}[\mathcal{M}(P), \mathcal{M}(Q)] \leq \eta \},$$

where W_1 is the Wasserstein-1 (earthmover) distance. Intuitively, $\omega_{\mathcal{M}, \mathcal{C}}$ quantifies how far apart two distributions in $\mathcal{C}$ can be while remaining nearly indistinguishable after measurement. Equivalently, it measures how much phase-space structure is hidden from $\mathcal{M}$, and how ill conditioned the inverse problem is. This intuition can be directly tied to sample lower bounds.

Theorem 3 (OT ambiguity implies minimax lower bounds, informal). Fix an experiment $\mathcal{M}$ and a class $\mathcal{C}$ with finite first moment. Then there exists a 1-Lipschitz function f such that any estimator based on N i.i.d. samples from $\mathcal{M}(P)$ has worst-case error at least $\Omega[\omega_{\mathcal{M}, \mathcal{C}}(1/N)]$ in estimating $\mathbb{E}_P[f]$.

Note that when $\omega_{\mathcal{M}, \mathcal{C}}(0) > 0$, no number of measurements can drive the error below a constant, meaning some property is completely nonidentifiable. This optimal-transport viewpoint also supports an inverse approach to finding a quantum advantage: Given a sensing constraint, one identifies nearly indistinguishable instance pairs by maximizing ω , and then relaxes around them to quantify how much entanglement improves sample complexity.

The following toy example illustrates a geometric intuition for Theorem 3: Quantum advantage depends on how much of phase space the problem class explores. Consider the distributions

$$P = \frac{\delta_{(a,b)} + \delta_{(-a,-b)}}{2}, \quad Q = \frac{\delta_{(a,-b-\epsilon)} + \delta_{(-a,b+\epsilon)}}{2}. \quad (2)$$

When $\epsilon = 0$, P and Q have identical $\{x, p\}$ marginals despite living in disjoint quadrants of phase space, so $\{x, p\}$ -limited homodyne can never distinguish them. This example is used to demonstrate the utility of Bell measurement in collecting a robust dataset for classical learning algorithms beyond our estimators in Fig. 3. We therefore interpret ϵ as a symmetry-breaking parameter that controls how strongly the hidden two-dimensional structure becomes visible to restricted homodyne. We call the corresponding restricted instance family a *problem wedge*: a subset of the full problem space whose scale is determined by ϵ , viewed as a function of the intrinsic scale b .

For $\epsilon > 0$, the homodyne marginals differ only at $O(\epsilon^2)$ (in, e.g., Kullback-Leibler (KL) divergence), so $\{x, p\}$ -limited homodyne incurs sample complexity $\Theta(e^{-2r}/\epsilon^2)$, while Bell QSL distinguishes using only $O(e^{-2r}/b^2)$ shots. The advantage factor is thus controlled by $(b/\epsilon)^2$: In the red wedge of Fig. 4, where ϵ is exponentially small in b , the advantage is exponential, while for polynomially small ϵ it is polynomial. Although the delta mixture makes this geometry particularly transparent, the same ϵ^{-2} conditioning governs the advantage found in Theorem 1 for EM field sensing, where the role of ϵ is played by the

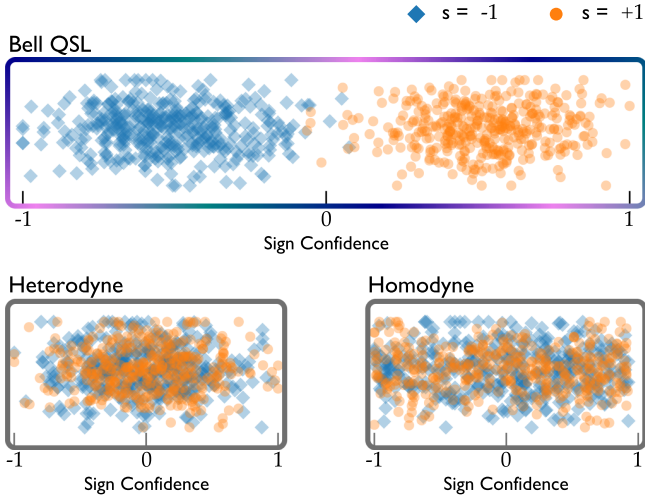


FIG. 3. Classical unsupervised learning employing Bell QSL data can learn signal properties beyond the scope of heterodyne and homodyne tomography. Each measurement method collects many draws from the underlying distributions P, Q from Eq. (2), with squeezing $r = 1.2$, $a = 0.35$, $b = 0.06$, and $\varepsilon = 0$. Each dataset is fed into unsupervised machine learning (ML) (Gaussian kernel principal component analysis) to learn whether the data were drawn from P (sign $+1$) or Q (sign -1). Even at modest $b < 1$ and squeezing, and with appreciably large ε , the classical Bell dataset constitutes a far more feature-rich training dataset for ML algorithms than conventional strategies.

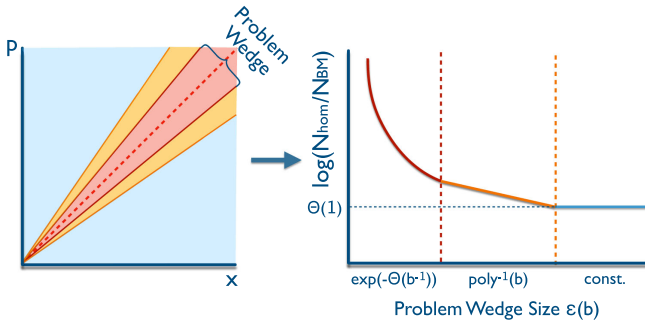


FIG. 4. The *problem wedge* of quantum advantage, illustrated for the delta-mixture toy example. As the symmetry-breaking parameter $\varepsilon \rightarrow 0$, homodyne shots become nonidentifiable and sample complexity diverges. In the red region, ε is exponentially small relative to b , yielding exponential advantage, and in the orange region, it is polynomially small. The same scaling controls the advantage in Theorem 1, with $\varepsilon \rightarrow \Delta$.

symmetry-breaking Δ that leaks c into the accessible homodyne statistics. In our optimal transport language, the W_1 separation is $\Theta(b)$ while $\{x, p\}$ -restricted homodyne only sees the symmetry breaking through $O(\varepsilon^2)$ leakage, yielding the corresponding $1/\varepsilon^2$ lower bound. Using Theorem 3, these simple, illustrative examples of problem-wedge trade-off can be generalized to less structured, multimodal sensing tasks, and against homodyne learners with different measurement settings.

Prior exponential quantum learning advantages often arise only under worst-case benchmarks against all entanglement-free learners [28,45–48], a class broad enough to include exponentially deep, computationally intractable measurement strategies. Quantum advantage is then forced into vanishingly small corners of the instance space that do not coincide with the set of physically interesting instances. By instead benchmarking against weaker, but experimentally feasible conventional strategies, our perspective reveals sizable regimes where quantum information processing yields genuine, usable advantages. Going forward, benchmarking against experimentally feasible strategies will be essential to avoid artificially narrowing the search for quantum-enhanced experiments in ways that obscure such useful advantages.

C. Advantage without entanglement

We have shown that Bell QSL, leveraging entanglement, can operate at equal probe-state energy to homodyne and heterodyne tomography while resolving features of classical fields inaccessible to entanglement-free protocols. Here we show that it is also possible to demonstrate exponential separations that arise from squeezing alone.

We consider the following problem. Place a sensor in a background where it experiences many random kicks of fixed amplitude. In each sensing time step, the sensor is thus evolved by the displacement operator $\hat{D}(X)$, with $X = \sum_{i=1}^n z_i$, and $z_i = \pm 1$; however, we impose that the z_i 's are either random or satisfy a parity constraint. This is a toy instantiation of a more practical physical problem: estimating *moments* of classical signals.

Theorem 4 (Exponential quantum advantage with only squeezed probes). Any strategy with arbitrary energy and arbitrary quantum measurements, but which uses classical light to probe the signal, requires $\exp[\Omega(n)]$ homodyne measurement shots to distinguish random and parity-constrained classical backgrounds, while there is a strategy using a single squeezed probe of $r = \Theta(\log n)$ that requires $O(1)$ shots.

By embedding the effective instance size n into a single-mode task, Theorem 4 (proven in SM [31]) shows that exponential-in- n advantages are possible purely from using squeezed light vs classical (coherent) light.

We remark that this result falls under a different operational definition of quantum advantage than is assumed in our previous separations. Note that classical coherent-state probes can differ in energy based on mean photon number (i.e., phase-space displacement), while squeezed light is generated from coherent states by non-energy-conserving symplectic unitaries. Because both displacements and squeezing can enhance probe sensitivity, the canonical bosonic Heisenberg limit treats the two transformations on equal footing and pertains only to probe state preparation with a bounded energy budget [29]. However, in this Fisher-information picture, optimizing the trade-off

between displacement and squeezing energy yields at most polynomial improvements. Theorem 4 shows that a learning-theoretic QSL analysis can be much more illustrative: For certain problems, squeezed probe states can be exponentially more informative than *any* displaced classical probe, independent of energy. This establishes *nonclassical* energy budget as a unique sensing resource whose effect cannot be replicated by any amount of classical resources.

V. DISCUSSION

Our work establishes that quantum advantage for sensing classical fields becomes both sharper and more useful once we formulate the task in terms of the practical limitations of real-world experiments. Having introduced the quantum signal learning framework to capture these features, we established that quantum sensors enhanced by quadrature squeezing and entangled quantum memory can extract properties of classical signals far more efficiently than their conventional counterparts. Our work makes such quantum learning advantage statements precise, demonstrating utility for ubiquitous physical tasks such as electromagnetic field sensing and interferometric feedback control. Moreover, our algorithm requires only a single layer of entanglement-generating operations and conventional homodyne measurements, while enabling *post hoc* estimation of many properties from the collected classical record. We introduce an optimal transport-based method to prove quantum learning speedups over practical, experimentally feasible conventional benchmarks, going beyond previous worst-case advantage statements.

Our work extends and generalizes the literature on bosonic learning theory and metrology. While Ref. [28] exhaustively studies the task of learning a complete description of a displacement channel with *post hoc* queries (a theoretically motivated task akin to full process tomography), QSL takes a more general and experimentally motivated perspective: Properties can be any desired functional of the signal. Requiring *post hoc* learning of the full channel can make the task artificially hard as it is equivalent to learning exponentially many predetermined observables, so QSL only requires learning the few relevant signal properties efficiently. This allows for nontrivial and experimentally useful separations in less specialized settings than worst-case multimode channel learning, including with even a single sensor mode. Moreover, the query-complexity reformulation of sensing efficiency makes new kinds of quantum speedups possible that are not present from the canonical Fisher-information perspective. QSL, being simple yet extremely expressive, allows for a unification of the learning-theoretic and metrological perspectives in an experimentally minded fashion.

We have demonstrated preliminary applications of Bell QSL to tasks of electromagnetic field sensing and noise characterization. Expanding on these concepts may be fruitful for radar sensing and detectors in high-precision fundamental

physics experiments and astronomy [2,4,8,49–52]. Entanglement-enabled real-time noise monitoring could enable real-time error mitigation in interferometers for gravitational-wave astronomy, and meaningfully improve large-hypothesis matched-filter testing in, e.g., extreme-high-bandwidth multiplexing.

Theoretically, our work establishes a preliminary framework for proving *practical* quantum advantage in quantum sensing by associating a limited measurement model with an optimal-transport stability modulus. Interfacing these concepts with the learning tree formalism [45], which is already equipped with sophisticated lower-bound proof tools, may allow the quantum advantage statements in this work to generalize to other quantum-enhanced experimental settings.

Searching for meaningful quantum-enhanced experiments requires keeping experimental constraints firmly in view. Taking this approach, our work reveals quantum speedups across a wide range of important sensing tasks, establishing that practical quantum advantage extends naturally to the sensing of classical fields.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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- [1] J. Aasi *et al.* (LIGO Scientific Collaboration), *Advanced LIGO*, *Classical Quantum Gravity* **32**, 074001 (2015).
 - [2] M. Tse *et al.* (LIGO Scientific Collaboration), *Quantum-enhanced Advanced LIGO detectors in the era of gravitational-wave astronomy*, *Phys. Rev. Lett.* **123**, 231107 (2019).
 - [3] B. M. Brubaker *et al.*, *First results from a microwave cavity axion search at 24 μ eV*, *Phys. Rev. Lett.* **118**, 061302 (2017).
 - [4] K. M. Backes *et al.*, *A quantum enhanced search for dark matter axions*, *Nature (London)* **590**, 238 (2021).
 - [5] C. Macklin, K. O’Brien, D. Hover, M. E. Schwartz, V. Bulkhovsky, X. Zhang, W. D. Oliver, and I. Siddiqi,

- A near-quantum-limited Josephson traveling-wave parametric amplifier*, *Science* **350**, 307 (2015).
- [6] T. P. Purdy, R. W. Peterson, and C. A. Regal, *Observation of radiation pressure shot noise on a macroscopic object*, *Science* **339**, 801 (2013).
- [7] Ishaan Kannan, Sridhar Prabhu, Saeed A. Khan, Mandar M. Sohoni, Xingrui Song, Saswata Roy, Alen Senanian, Valla Fatemi, Peter L. McMahon, and Jordan Cotler, *Exponential quantum advantage for learning signals with a single qubit*, [arXiv:2608.13521](https://arxiv.org/abs/2608.13521).
- [8] M. Malnou, D. A. Palken, B. M. Brubaker, Leila R. Vale, Gene C. Hilton, and K. W. Lehnert, *Squeezed vacuum used to accelerate the search for a weak classical signal*, *Phys. Rev. X* **9**, 021023 (2019).
- [9] Daniel W. Gould, Min Jet Yap, Vaishali B. Adya, Bram J. J. Slagmolen, Robert L. Ward, and David E. McClelland, *Optimal quantum noise cancellation with an entangled witness channel*, *Phys. Rev. Res.* **3**, 043079 (2021).
- [10] Jack Y. Qiu, Arne Grimsmo, Kaidong Peng, Bharath Kannan, Benjamin Lienhard, Youngkyu Sung, Philip Krantz, Vladimir Bolkhovskiy, Greg Calusine, David Kim, Alex Melville, Bethany M. Niedzielski, Jonilyn Yoder, Mollie E. Schwartz, Terry P. Orlando, Irfan Siddiqi, Simon Gustavsson, Kevin P. O'Brien, and William D. Oliver, *Broadband squeezed microwaves and amplification with a Josephson travelling-wave parametric amplifier*, *Nat. Phys.* **19**, 706 (2023).
- [11] Arjen Vaartjes, Anders Kringhøj, Wyatt Vine, Tom Day, Andrea Morello, and Jarryd J. Pla, *Strong microwave squeezing above 1 Tesla and 1 Kelvin*, *Nat. Commun.* **15**, 4229 (2024).
- [12] Eric Oelker, Tomoki Isogai, John Miller, Maggie Tse, Lisa Barsotti, Nergis Mavalvala, and Matthew Evans, *Audio-band frequency-dependent squeezing for gravitational-wave detectors*, *Phys. Rev. Lett.* **116**, 041102 (2016).
- [13] L. McCuller, C. Whittle, D. Ganapathy, K. Komori, M. Tse, A. Fernandez-Galiana, L. Barsotti, P. Fritschel, M. MacInnis, F. Matichard, K. Mason, N. Mavalvala, R. Mittleman, Haocun Yu, M. E. Zucker, and M. Evans, *Frequency-dependent squeezing for advanced LIGO*, *Phys. Rev. Lett.* **124**, 171102 (2020).
- [14] D. Ganapathy, W. Jia, M. Nakano, V. Xu, N. Aritomi, T. Cullen, N. Kijbunchoo, S. E. Dwyer, A. Mullavey, L. McCuller *et al.*, *Broadband quantum enhancement of the LIGO detectors with frequency-dependent squeezing*, *Phys. Rev. X* **13**, 041021 (2023).
- [15] Wenxuan Jia, Victoria Xu, Kevin Kuns, Masayuki Nakano, Lisa Barsotti, Matthew Evans, Nergis Mavalvala *et al.*, *Squeezing the quantum noise of a gravitational-wave detector below the standard quantum limit*, *Science* **385**, 1318 (2024).
- [16] Bruce Allen, χ^2 time-frequency discriminator for gravitational wave detection, *Phys. Rev. D* **71**, 062001 (2005).
- [17] Bruce Allen, Warren G. Anderson, Patrick R. Brady, Duncan A. Brown, and Jolien D. E. Creighton, *FIND-CHIRP: An algorithm for detection of gravitational waves from inspiraling compact binaries*, *Phys. Rev. D* **85**, 122006 (2012).
- [18] S. Babak, R. Biswas, P. R. Brady, D. A. Brown, K. Cannon *et al.*, *Searching for gravitational waves from binary coalescence*, *Phys. Rev. D* **87**, 024033 (2013).
- [19] Samantha A. Usman, Alexander H. Nitz, Ian W. Harry, Christopher M. Biwer, Duncan A. Brown *et al.*, *The PyCBC search for gravitational waves from compact binary coalescence*, *Classical Quantum Gravity* **33**, 215004 (2016).
- [20] Cody Messick, Kent Blackburn, Patrick Brady, Patrick Brockill, Kipp Cannon *et al.*, *Analysis framework for the prompt discovery of compact binary mergers in gravitational-wave data*, *Phys. Rev. D* **95**, 042001 (2017).
- [21] Tito Dal Canton and Ian W. Harry, *Designing a template bank to observe compact binary coalescences in advanced LIGO's second observing run*, [arXiv:1705.01845](https://arxiv.org/abs/1705.01845).
- [22] Javier Roulet, Liang Dai, Tejaswi Venumadhav, Barak Zackay, and Matias Zaldarriaga, *Template bank for compact binary coalescence searches in gravitational wave data: A general geometric placement algorithm*, *Phys. Rev. D* **99**, 123022 (2019).
- [23] Debnandini Mukherjee, Sarah Caudill, Ryan Magee, Cody Messick, Stephen Privitera, Surabhi Sachdev, Kent Blackburn, Patrick Brady, Patrick Brockill, Kipp Cannon *et al.*, *Template bank for spinning compact binary mergers in the second observation run of Advanced LIGO and the first observation run of Advanced Virgo*, *Phys. Rev. D* **103**, 084047 (2021).
- [24] Alessio Serafini, *Quantum Continuous Variables: A Primer of Theoretical Methods* (CRC Press, Boca Raton, FL, 2017), p. 368.
- [25] Samuel L. Braunstein and H. Jeff Kimble, *Teleportation of continuous quantum variables*, *Phys. Rev. Lett.* **80**, 869 (1998).
- [26] Akira Furusawa, J. L. Sørensen, Samuel L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, *Unconditional quantum teleportation*, *Science* **282**, 706 (1998).
- [27] Samuel L. Braunstein and H. J. Kimble, *Dense coding for continuous variables*, *Phys. Rev. A* **61**, 042302 (2000).
- [28] Changhun Oh, Senrui Chen, Yat Wong, Sisi Zhou, Hsin-Yuan Huang, Jens A. H. Nielsen, Zheng-Hao Liu, Jonas S. Neergaard-Nielsen, Ulrik L. Andersen, Liang Jiang, and John Preskill, *Entanglement-enabled advantage for learning a bosonic random displacement channel*, *Phys. Rev. Lett.* **133**, 230604 (2024).
- [29] Dominik Šafránek and Ivette Fuentes, *Optimal probe states for the estimation of Gaussian unitary channels*, *Phys. Rev. A* **94** (2016), [10.1103/PhysRevA.94.062313](https://doi.org/10.1103/PhysRevA.94.062313).
- [30] Francesco Anna Mele, Nadine Meister, Haimeng Zhao, Senrui Chen, and Hsin-Yuan Huang, *Exponential quantum speedup in sensing time-varying classical fields via squeezed light* (to be published).
- [31] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/6ps1-mw9l> for a survey of related literature, formal statement of the QSL problem and Bell QSL algorithm, general-purpose query-complexity guarantees, and detailed statements and proofs of all main Theorems.
- [32] Charles H. Bennett and Stephen J. Wiesner, *Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states*, *Phys. Rev. Lett.* **69**, 2881 (1992).
- [33] Charles H. Bennett, Gilles Brassard, Claude Crépeau, Richard Jozsa, Asher Peres, and William K. Wootters, *Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels*, *Phys. Rev. Lett.* **70**, 1895 (1993).

- [34] Sridhar Prabhu, Vladimir Kremenetski, Saeed A. Khan, Ryotatsu Yanagimoto, and Peter L. McMahon, *Exponential advantage in quantum sensing of correlated parameters*, *Quantum* **10**, 1963 (2026).
- [35] Zheng-Hao Liu, Romain Brunel, Emil E. B. Østergaard, Oscar Cordero, Senrui Chen, Yat Wong, Jens A. H. Nielsen, Axel B. Bregnsbo, Sisi Zhou, Hsin-Yuan Huang, Changhun Oh, Liang Jiang, John Preskill, Jonas S. Neergaard-Nielsen, and Ulrik L. Andersen, *Quantum learning advantage on a scalable photonic platform*, *Science* **389**, 1332 (2025).
- [36] E. P. Menzel, F. Deppe, M. Mariani, M. A. Araque Caballero, A. Baust, T. Niemczyk, E. Hoffmann, A. Marx, E. Solano, and R. Gross, *Dual-path state reconstruction scheme for propagating quantum microwaves and detector noise tomography*, *Phys. Rev. Lett.* **105**, 100401 (2010).
- [37] A. A. Clerk, M. H. Devoret, S. M. Girvin, Florian Marquardt, and R. J. Schoelkopf, *Introduction to quantum noise, measurement, and amplification*, *Rev. Mod. Phys.* **82**, 1155 (2010).
- [38] Mankei Tsang, Howard M. Wiseman, and Carlton M. Caves, *Fundamental quantum limit to waveform estimation*, *Phys. Rev. Lett.* **106**, 090401 (2011).
- [39] F. Mallet, M. A. Castellanos-Beltran, H. S. Ku, S. Glancy, E. Knill, K. D. Irwin, G. C. Hilton, L. R. Vale, and K. W. Lehnert, *Quantum state tomography of an itinerant squeezed microwave field*, *Phys. Rev. Lett.* **106**, 220502 (2011).
- [40] Jordan Cotler, Weiyuan Gong, and Ishaan Kannan, *Noisy quantum learning theory*, *Nat. Commun.* **17**, 6979 (2026).
- [41] Carlton M. Caves and Bonny L. Schumaker, *New formalism for two-photon quantum optics. I. Quadrature phases and squeezed states*, *Phys. Rev. A* **31**, 3068 (1985).
- [42] Alessandra Buonanno and Yanbei Chen, *Quantum noise in second generation, signal-recycled laser interferometric gravitational-wave detectors*, *Phys. Rev. D* **64**, 042006 (2001).
- [43] Stefan L. Danilishin and Farid Ya. Khalili, *Quantum measurement theory in gravitational-wave detectors*, *Living Rev. Relativity* **15**, 5 (2012).
- [44] Min Jet Yap, Paul Altin, Terry G. McRae, Bram J. J. Slagmolen, Robert L. Ward, and David E. McClelland, *Generation and control of frequency-dependent squeezing via Einstein-Podolsky-Rosen entanglement*, *Nat. Photonics* **14**, 223 (2020).
- [45] Sitan Chen, Jordan Cotler, Hsin-Yuan Huang, and Jerry Li, *Exponential separations between learning with and without quantum memory*, in *Proceedings of the 2021 IEEE 62nd Annual Symposium on Foundations of Computer Science (FOCS)*, arXiv:2111.05881.
- [46] Dorit Aharonov, Jordan Cotler, and Xiao-Liang Qi, *Quantum algorithmic measurement*, *Nat. Commun.* **13**, 887 (2022).
- [47] Hsin-Yuan Huang, Michael Broughton, Jordan Cotler, Sitan Chen, Jerry Li, Masoud Mohseni, Hartmut Neven, Ryan Babbush, Richard Kueng, John Preskill, and Jarrod R. McClean, *Quantum advantage in learning from experiments*, *Science* **376**, 1182 (2022).
- [48] Sitan Chen, Weiyuan Gong, and Qi Ye, *Optimal tradeoffs for estimating Pauli observables*, arXiv:2404.19105.
- [49] Maximilien Silva-Feaver et al., *Design overview of the DM radio pathfinder experiment*, *IEEE Trans. Appl. Supercond.* **27**, 1400204 (2017).
- [50] C. W. Sandbo Chang, A. M. Vadiraj, J. Bourassa, B. Balaji, and C. M. Wilson, *Quantum-enhanced noise radar*, *Appl. Phys. Lett.* **114**, 112601 (2019).
- [51] Jonathan L. Ouellet, Chiara P. Salemi, Joshua W. Foster, Reyco Henning, Zachary Bogorad, Janet M. Conrad, Joseph A. Formaggio, Yonatan Kahn, Joe Minervini, Alexey Radovinsky, Nicholas L. Rodd, Benjamin R. Safdi, Jesse Thaler, Daniel Winklehner, and Lindley Winslow, *First results from ABRACADABRA-10 cm: A search for sub- μ eV axion dark matter*, *Phys. Rev. Lett.* **122**, 121802 (2019).
- [52] A. Romanenko, R. Harnik, A. Grassellino, R. Pilipenko, Y. Pischalnikov, Z. Liu, O. S. Melnychuk, B. Giaccone, O. Pronitchev, T. Khabiboulline, D. Frolov, S. Posen, A. Berlin, and A. Hook, *Search for dark photons with superconducting radio-frequency cavities*, *Phys. Rev. Lett.* **130**, 261801 (2023).