

Persistent Spin Currents in Superconducting Altermagnets

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Superconductors are famously capable of supporting persistent electrical currents, that is, currents that flow without any measurable decay as long as the material is kept in the superconducting state. We introduce here a class of materials—superconducting altermagnets—that can both generate and carry persistent *spin* currents. This includes spin-polarized electrical supercurrent as well as pure spin supercurrent that facilitates spin transport in the absence of any charge transport. A key to this remarkable property is the realization that the leading superconducting instability of altermagnetic metals consists of two independent condensates formed of spin-up and spin-down electrons. In the nonrelativistic limit, the two condensates are decoupled and can, thus, naturally support persistent currents with any spin polarization, including pure spin supercurrents realized in the charge counterflow regime. We describe a novel “spin-current dynamo effect” that can be used to generate pure spin supercurrent in such systems by driving a charge current along certain crystallographic directions. Away from the nonrelativistic limit, when spin-orbit interactions and magnetic disorder are present, we find that the spin current generically develops spatial oscillations but, importantly, no dissipation or decay. This is in stark contrast to spin currents in normal diffusive metals which tend to decay on relatively short length scales. We illustrate the above properties by performing model calculations relevant to two distinct classes of altermagnets and various device geometries.

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I. INTRODUCTION

The magic of supercurrent lies in the fact that it flows in the ground state of the system. In some geometries, such as a superconducting (SC) ring threaded by magnetic flux that is some fraction of the SC flux quantum $\Phi_0 = hc/2e$, the current cannot decay even as a matter of principle, simply because it flows in the quantum state with the lowest energy [1,2]. As a more practical matter, the fact that persistent currents are not subject to dissipation has been widely exploited in devices, e.g., to generate magnetic fields for a broad range of applications. Quantum coherence of the SC condensate also forms a basis for an important class of qubits and quantum sensors based on the ubiquitous Josephson effect.

Unlike electric charge, spin is generically not conserved in solids. Transporting spin over long enough distances so that it can be successfully manipulated and read out is one of

the key requirements of spintronics [3,4]. In diffusive normal metals and semiconductors, spin lifetimes are typically quite short owing to the spin-orbit coupling (SOC) and spin-flip scattering. The quest in the spintronic community to remedy this situation has led to the advent of superconducting spintronics [5,6], which seeks to exploit dissipationless currents carried by superconductors to achieve long spin transport lifetimes. This effort has produced a number of notable advances [7–10] and has culminated in the recent demonstration of a tunable pure spin supercurrent in heterostructures composed of a ferromagnet, spin-orbit coupled metal, and a conventional superconductor such as Pt/Co/Pt/Nb/Ni₈Fe₂ [11]. Additionally, a number of theoretical proposals have been advanced aimed at improving performance and attaining new functionalities in superconducting spintronic devices [12–17].

Despite this progress, platforms that combine superconductors with ferromagnets have some inescapable drawbacks: Ferromagnets produce stray fields that can lead to undesirable coupling between distant circuit elements. Also, conventional superconducting order tends to be strongly suppressed by the magnetic field through both orbital and Zeeman depairing. Likewise, magnetic exchange coupling is pair breaking for conventional spin-singlet Cooper pairs.

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As a result, it would be vastly preferable if one could generate and sustain spin-polarized persistent currents in structures with zero net magnetization. In what follows, we discuss one such platform based on the recently discovered family of altermagnetic metals [18–23]. Similar to antiferromagnets, altermagnets exhibit zero net magnetization enforced by their spin group symmetry [24–29]. Nevertheless, they possess spin-split Fermi surfaces which, as we demonstrate, are naturally suited to form a basis for exotic equal-spin triplet SC order capable of both generating and supporting persistent spin currents.

The reason for this is quite simple and can be readily understood from Fig. 1, which depicts spin-split Fermi surfaces typical of two families of altermagnets with fourfold and sixfold rotation symmetry, also known as *d*- and *g*-wave altermagnets, respectively. We observe that these types of Fermi surfaces are fundamentally incompatible with the conventional spin-singlet superconducting instability: For a spin-up electron at crystal momentum $+\mathbf{k}$, there is no requisite spin-down electron available at $-\mathbf{k}$. Instead, a zero-momentum Cooper pair must form from two electrons with equal spin, which also necessitates odd-parity spatial symmetry of the pair wave function $g_{\mathbf{k}} = -g_{-\mathbf{k}}$. This intuition has recently been formalized by a general group-theoretic classification of symmetry-allowed SC phases in altermagnets [30] which highlighted spin-triplet order parameters as most natural SC instabilities. It is to be noted that these arguments are most useful in the limit of weak coupling where the standard mean-field BCS theory and its extensions to magnetic systems provide a reliable explanatory framework. At strong coupling, the space of possible ground states becomes larger and must be considered separately.

Importantly for the task at hand, in the strictly non-relativistic limit (that is, in the absence of SOC), the two spin species are decoupled and so are the two superconducting condensates. It is this feature that ultimately enables all the interesting spintronic properties of such condensates discussed below. For example, one could

imagine setting up superflow in spin-up condensate only: The resulting persistent current would then be fully spin polarized. Likewise, a state with the two condensates flowing in the opposite direction realizes a pure spin supercurrent, transporting spin with no concomitant charge transport. We note that this type of behavior has been discussed in the context of superfluid ^3He which in its B phase can support spin-current vortices [31].

In normal metals, inclusion of SOC and spin-flip scattering is known to lead to short spin lifetimes, which are often the main limiting factors in spintronics devices [3]. An important question, therefore, arises as to the effect of such perturbations on the efficacy of the proposed new platform. To address this question, we perform an extensive analysis in the framework of microscopic models as well phenomenological Ginzburg-Landau (GL) theory and show that SOC does not destroy these behaviors unless its strength exceeds a critical value. Specifically, we find that inclusion of SOC causes spatial oscillations in spin supercurrent amplitude along the flow direction but, crucially, no decay or dissipation. Hence, superconducting altermagnets can carry persistent spin currents over long distances even when SOC is present. We also explore the effect of spin-flip scattering caused by magnetic disorder and find that, unless it is very strong (so as to significantly suppress the SC order), it also does not substantially degrade the spin supercurrent.

As argued previously [32–36], the leading SC instability of a *d*-wave altermagnet in the presence of a weak attractive interaction occurs in the equal-spin triplet channel with a fully gapped $p_x \pm ip_y$ order parameter. This is relevant to altermagnets with the square-lattice symmetry including the recently discovered quasi-2D oxychalcogenide metals $\text{KV}_2\text{Se}_2\text{O}$ [23] and $\text{Rb}_{1-\delta}\text{V}_2\text{Te}_2\text{O}$ [37]. In the following, we perform the same type of analysis for *g*-wave altermagnets and show that the leading SC instability also occurs in the equal-spin triplet $p_x \pm ip_y$ channel. This is potentially relevant to a number of confirmed altermagnets with the hexagonal symmetry including MnTe [38,39], RuO_2 [40], and CrSb [41–43].

While it is true that none of the above altermagnets have been yet reported to show superconductivity, research in this area is still in very early stages, and many more candidate altermagnet families have been identified by the recent theoretical work, both in naturally occurring crystals [44–49] and in artificially engineered structures [50]. Since many of these are good metals, it would be surprising if at least some did not show superconducting instabilities when cooled down to low temperatures. It is also worth emphasizing that the predicted unconventional equal-spin pairing does not require any exotic pairing mechanism—it emerges naturally as the leading SC instability of the altermagnetic normal metal in the presence of conventional weak attraction mediated by phonons [36] or any other bosonic mode.

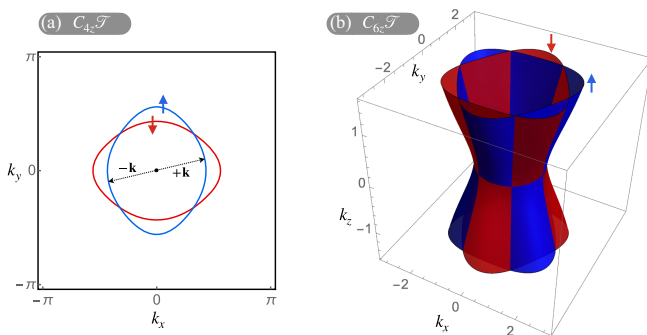


FIG. 1. Spin-split Fermi surfaces of (a) *d*-wave and (b) *g*-wave altermagnetic metals described by Hamiltonians Eqs. (1) and (3) for altermagnetic band splitting parameter $\eta = 0.2$. Blue and red color distinguishes up and down spin projections.

We remark that *proximity-induced* superconductivity in altermagnets has been studied by a number of authors [51–58]; see also the recent review in Ref. [59] for additional references. Structures formed of conventional superconductors and altermagnets have been predicted to show a multitude of interesting and potentially useful behaviors including supercurrent-induced edge magnetization, Cooper pair spin-splitter and filtering effect, controllable $0 - \pi$ transitions in the Josephson supercurrent, spin-polarized Andreev levels, and Josephson diode effect. The focus of this work, by contrast, is *intrinsic* SC order in altermagnets, which remains relatively unexplored.

II. MICROSCOPIC MODELS

In this section, we introduce minimal microscopic models for the two most common classes of altermagnets with d - and g -wave symmetry. We discuss their pairing instabilities in the presence of a weak attractive interaction and show that the leading instability is toward the equal-spin triplet chiral p -wave state. We remark that, except when explicitly noted, our results for persistent spin currents derived in subsequent sections apply to both models as well as systems in two and three spatial dimensions.

A. d -wave altermagnet

We consider a minimal model of a “ d -wave” altermagnetic metal [20,21] in two dimensions defined by the Hamiltonian $\mathcal{H}_0 = \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger h_0(\mathbf{k}) \psi_{\mathbf{k}}$ with $\psi_{\mathbf{k}} = (c_{\mathbf{k}\uparrow}, c_{\mathbf{k}\downarrow})^T$ and

$$h_0(\mathbf{k}) = -2t(\cos k_x + \cos k_y) - 2\eta\sigma_z(\cos k_x - \cos k_y). \quad (1)$$

Here, σ_μ are Pauli matrices in spin space, and t denotes the nearest-neighbor hopping amplitude on the square lattice. The η term gives the altermagnetic band splitting and, hence, breaks time-reversal symmetry $\mathcal{T}$. The model, however, remains invariant under combined C_4 rotation and $\mathcal{T}$, a hallmark feature of altermagnets which guarantees vanishing total magnetization. Unlike conventional antiferromagnetism, altermagnetic order does not enlarge the nonmagnetic unit cell. Thus, the crystallographic basis of altermagnets is, in principle, required to contain at least two sites. While interband effects have been shown to be an important factor in the microscopic formation of altermagnetic order [60,61], the corresponding Néel temperatures tend to be orders of magnitude larger than the superconducting energy scales considered here. Consequently, we can safely neglect coupling between different sublattices and consider an effective single-band model (1) with altermagnetic splitting introduced through spin-dependent tunneling amplitudes.

When the inversion symmetry of $\mathcal{H}_0$ is broken (e.g., by placing the system on a substrate), then a Rashba SOC term

$$h_R(\mathbf{k}) = 2\lambda_R(\sigma_x \sin k_y - \sigma_y \sin k_x) \quad (2)$$

becomes symmetry allowed. The main effect of nonzero λ_R is to resolve the remaining Fermi surface degeneracies along the Brillouin zone diagonals visible in Fig. 1(a). In the vicinity of these points, the Rashba term causes the spins to rotate into the plane.

B. g -wave altermagnet

Although a purely 2D model of a g -wave altermagnet can be defined [62], it requires a rather large unit cell which does not correspond to any known naturally occurring material. We, therefore, take inspiration from the crystal structure of CrSb, which is a confirmed altermagnetic metal [41,42], and introduce an effective 3D model on a layered triangular lattice which produces a qualitatively correct spin-split Fermi surface. The Hamiltonian is given as

$$h_0(\mathbf{k}) = -2t \sum_j \cos \delta_j \cdot \mathbf{k} - 2t_z \cos k_z - 2\eta \sum_j \cos [\sigma_z k_z + (\delta_j - \delta_{j-1}) \cdot \mathbf{k}], \quad (3)$$

where $\delta_1 = \hat{x}$ and $\delta_{2,3} = -\frac{1}{2}(\hat{x} \mp \sqrt{3}\hat{y})$ are nearest-neighbor vectors on the triangular lattice. t_z denotes the interlayer tunneling amplitude, and the η term produces the characteristic g -wave altermagnetic spin splitting depicted in Fig. 1. This term breaks $\mathcal{T}$, but the model remains invariant under combined C_{6z} rotation and $\mathcal{T}$, which qualifies it as a g -wave altermagnet.

To account for the relativistic effects, a Dresselhaus-type SOC of the form

$$h_D(\mathbf{k}) = 2\lambda_D \sum_{j,\alpha=\pm} \sigma^{(j)} \sin [\alpha k_z + (\delta_j - \delta_{j-1}) \cdot \mathbf{k}], \quad (4)$$

where $\sigma^{(j)} = \boldsymbol{\sigma} \cdot [\hat{z} \times (\delta_j - \delta_{j-1})]$, can be added. This term respects $\mathcal{T}$ but is odd under inversion $\mathcal{P}$. Similar to h_R , its chief effect is to split the remaining spin degeneracies located on high-symmetry planes that follow from the nonrelativistic Hamiltonian Eq. (3). In addition, SOC terms Eq. (2) and (4) break the $U(1)_\uparrow \times U(1)_\downarrow$ symmetry associated with the separate number conservation of spin-up and spin-down electrons down to a single $U(1)$ charge conservation symmetry.

In the following, we consider superconducting instabilities of $\mathcal{H}_0$ for both the d - and g -wave model altermagnets in the presence of weak attractive interaction. When specified, we study the effect of weak SOC with magnitude small compared to the SC gap as well as magnetic disorder described by a Hamiltonian specified below. We take $t = 1$ and express all energies in units of t .

C. Pairing instabilities

To study SC instabilities of the altermagnetic normal metals introduced above, we consider Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I$, where

$$\mathcal{H}_0 = \sum_k \psi_k^\dagger [h_k - \mu] \psi_k \quad (5)$$

describes the underlying normal metal with h_k defined as a sum of $h_0(\mathbf{k})$ and the corresponding SOC terms given in Eqs. (1)–(4) above. For both d - and g -wave cases, we consider in-plane nearest-neighbor attraction defined in real space as

$$\mathcal{H}_I = -V_1 \sum_{\langle i,j \rangle} n_i n_j, \quad (6)$$

where V_1 is a positive interaction parameter and $n_j = \sum_\sigma c_{j\sigma}^\dagger c_{j\sigma}$ denotes the electron number operator on site j . We remark that the nearest-neighbor attraction adopted in Eq. (6) represents a minimal assumption that allows one to study the competition between singlet and triplet pairing in a lattice model setting. Note that spin-triplet pairing with an inversion-odd spatial wave function cannot arise from a purely on-site attraction [63]. Physically, the form in Eq. (6) can be viewed as arising from an interplay between screened Coulomb repulsion and a finite-range attraction mediated by phonons or another bosonic degree of freedom. We also note that adding an attractive or repulsive on-site term to the model does not qualitatively alter the conclusions described below.

We next perform standard mean-field decoupling of $\mathcal{H}_I$ in spin-singlet and equal-spin triplet channels and define the corresponding SC order parameters

$$\Delta_{ij}^0 = V_1 \langle c_{i\uparrow} c_{j\downarrow} \rangle, \quad \Delta_{ij}^\sigma = V_1 \langle c_{i\sigma} c_{j\sigma} \rangle, \quad (7)$$

associated with a bond between nearest-neighbor sites i and j . The singlet component is even under inversion, $\Delta_{ij}^0 = \Delta_{ji}^0$, and can be thought of as an extended s -wave order parameter. The triplet component must be inversion odd, $\Delta_{ij}^\sigma = -\Delta_{ji}^\sigma$, and can be realized as a p -wave order parameter.

Our discussion thus far has been completely general in that it applies to both d - and g -wave altermagnets and, after recasting h_0 in real space, the formalism can describe spatially nonuniform situations such as in the presence of boundaries or currents. We now wish to study the competition between singlet and triplet instabilities. To this end, we specialize to spatially uniform configurations of the order parameters by assuming

$$\Delta_{ij}^0 = \Delta_0, \quad \Delta_{ij}^\sigma = e^{\pm i\theta_{ij}} \Delta_\sigma, \quad (8)$$

where θ_{ij} denotes the angle between the bond and the positive x axis. Δ_{ij}^σ in Eq. (8) describes the chiral p -wave order parameter (with $\pm$ sign reflecting the two possible chiralities). We expect such a chiral $p_x \pm ip_y$ order parameter to be energetically favored over pure p_x or p_y , because it generally leads to a fully gapped Fermi surface. In the language of Ref. [30], these phases belong among the family “spin chiral” superconductors.

We also expect the chiral triplet to be energetically favored over the finite-momentum Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) -type phases with nonzero total momentum $\mathbf{q}$. This is because, for altermagnets in the nonrelativistic limit, the normal state energy dispersion for each spin retains the inversion symmetry, $\epsilon_\sigma(\mathbf{k}) = \epsilon_\sigma(-\mathbf{k})$. In this situation, formation of Cooper pairs with zero total momentum between electrons $(\mathbf{k}, \sigma)$ and $(-\mathbf{k}, \sigma)$ cleanly gaps out the entire Fermi surface, thus gaining maximum condensation energy. This conclusion is supported by an explicit evaluation of pair susceptibilities (Appendix A) for the models defined above which shows clear dominance of spin-triplet zero-momentum pairing over FFLO-type states. We find that, for generic Fermi surfaces without a special nesting condition, the triplet susceptibility exhibits the usual BCS logarithmic divergence as $\mathbf{q} \rightarrow 0$, whereas the singlet susceptibility remains finite for all $\mathbf{q}$.

In momentum space, the decoupled interaction Hamiltonians take the following form:

$$\mathcal{H}_{\text{MF}}^0 = 2\Delta_0 \sum_k C_k (c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger + \text{H.c.}) + \frac{4N}{V_1} \Delta_0^2, \quad (9a)$$

$$\mathcal{H}_{\text{MF}}^\sigma = \Delta_\sigma \sum_k (S_{k\sigma} c_{k\sigma}^\dagger c_{-k\sigma}^\dagger + \text{H.c.}) + \frac{2N}{V_1} \Delta_\sigma^2, \quad (9b)$$

where on the square lattice

$$C_k = \cos k_x + \cos k_y, \quad S_{k\sigma} = \sin k_x \pm i \sin k_y, \quad (10)$$

and analogous expressions for the triangular lattice. This leads to a description in terms of the Bogoliubov–de Gennes (BdG) Hamiltonian

$$\mathcal{H}_{\text{BdG}} = \sum_k \Psi_k^\dagger \begin{pmatrix} h_k - \mu & \hat{\Delta}_k \\ \hat{\Delta}_k^\dagger & -h_{-k}^* + \mu \end{pmatrix} \Psi_k + E_0, \quad (11)$$

where $\Psi_k = (c_{k\uparrow}, c_{k\downarrow}, c_{-k\uparrow}^\dagger, c_{-k\downarrow}^\dagger)^T$ is the Nambu spinor,

$$\hat{\Delta}_k = \begin{pmatrix} \Delta_\uparrow S_{k\uparrow} & \Delta_0 C_k \\ -\Delta_0 C_k & \Delta_\downarrow S_{k\downarrow} \end{pmatrix}, \quad (12)$$

and $E_0 = \sum_k \text{Tr}(h_k) + [(2N)/V_1](2\Delta_0^2 + \Delta_\uparrow^2 + \Delta_\downarrow^2)$.

To study the competition between singlet and triplet channels, we now focus on the nonrelativistic case where

SOC terms are absent. The system then respects the inversion symmetry, and the two order parameters belong to different irreducible representations of the point group. In this situation, when only one order parameter is nonzero, the 4×4 BdG matrix in Eq. (11) can be recast as a 2×2 matrix in the relevant subspace and can be easily diagonalized. The corresponding BdG energy eigenvalues are found as

$$E_{k0} = \xi_{k-} \pm \sqrt{\xi_{k+}^2 + \Delta_0^2 C_k^2} \quad (\text{singlet}), \quad (13a)$$

$$E_{k\sigma} = \pm \sqrt{\xi_{k\sigma}^2 + \Delta_\sigma^2 |S_{k\sigma}|^2} \quad (\text{triplet}). \quad (13b)$$

Here, $\xi_{k\sigma}$ are the eigenvalues of $h_0(\mathbf{k})$ referenced to the chemical potential μ and $\xi_{k\pm} = (\xi_{k\uparrow} \pm \xi_{k\downarrow})/2$.

From the knowledge of the energy eigenvalues, it is straightforward to construct the free energy $F = -(1/\beta) \ln Z$, where Z is the partition function of the system and $\beta = 1/k_B T$ denotes the inverse temperature. We obtain

$$F_0 = \frac{4N}{V_1} \Delta_0^2 + \sum_k \xi_{k-} - \frac{1}{\beta} \sum_k \ln (2 \cosh \beta \xi_{k-} + 2 \cosh \beta \epsilon_k) \quad (14)$$

for singlet, where $\epsilon_k = \sqrt{\xi_{k+}^2 + \Delta_0^2 C_k^2}$, and

$$F_\sigma = \frac{2N}{V_1} \Delta_\sigma^2 - \frac{1}{\beta} \sum_k \ln \left(4 \cosh \frac{1}{2} \beta E_{k\sigma} \right) \quad (15)$$

for triplet. Gap equations are obtained by minimizing these expressions with respect to the order parameters. We thus find

$$\Delta_0 = \frac{V_1}{2N} \sum_k \frac{\Delta_0 C_k^2}{\epsilon_k} \frac{\sinh \beta \epsilon_k}{\cosh \beta \xi_{k-} + \cosh \beta \epsilon_k}, \quad (16a)$$

$$\Delta_\sigma = \frac{V_1}{2N} \sum_k \frac{\Delta_\sigma |S_{k\sigma}|^2}{E_{k\sigma}} \tanh \frac{1}{2} \beta E_{k\sigma}, \quad (16b)$$

where the momentum summations extend over the first Brillouin zone. We observe that, while the gap equation for the triplet order parameter (16b) assumes the standard BCS form, Eq. (16a) is nonstandard in that it shows explicit dependence on the altermagnetic spin splitting through $\xi_{k-} \sim \eta$. This is a consequence of the nonstandard energy spectrum Eq. (13a) and reflects the expectation that spin-singlet pairing will be sensitive to the $\mathcal{T}$ breaking present in the altermagnet while triplet pairing will not.

We solve Eqs. (16a) and (16b) numerically and show the behavior of superconducting critical temperatures for singlet s -wave and triplet $p_x \pm ip_y$ order parameters as a

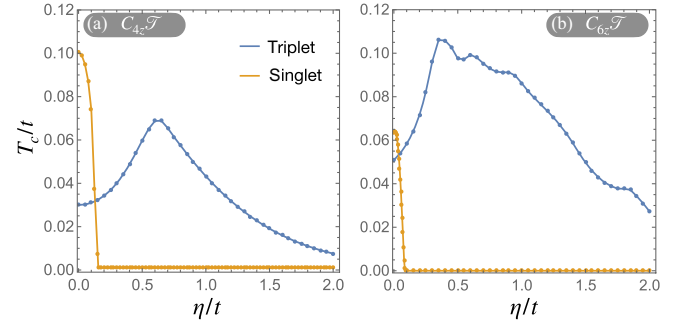


FIG. 2. Superconducting critical temperatures in singlet and triplet channels extracted from numerical solutions to gap equations (13a) and (13b) for (a) d -wave and (b) g -wave model altermagnetic metals. We use $(\mu, V_1) = (-2.6, 1.93)t$ for d -wave and $(-2.0, 1.95)t$ for g -wave as representative parameters.

function of spin splitting η in Fig. 2. The behavior supports the general argument made in Sec. I. As expected, the leading SC instability for a spin-degenerate Fermi surface at $\eta = 0$ occurs in the singlet channel which is signaled by higher singlet T_c . However, even small values of η cause rapid suppression of the singlet T_c —altermagnetic spin splitting is strongly pair breaking for singlet Cooper pairs. By contrast, the triplet T_c is largely unaffected and even grows with increasing η . (This initial rise of T_c is attributed to the increase with η of the Fermi surface size and, hence, the density of states N_F . At a constant interaction strength V_1 , the standard BCS expression $T_c \propto e^{-1/N_F V_1}$ then implies increasing T_c .)

Crucially, when $\eta \gtrsim \Delta_0$, the equal-spin triplet $p_x \pm ip_y$ channel becomes the leading SC instability. In altermagnetic metals, η is typically of the order of hundreds of meV and can be as large as ~ 1 eV. Typical values of Δ_0 will be 1–10 meV. This leads to a firm conclusion that superconductivity in any altermagnet will almost certainly be in the equal-spin triplet channel.

We note that the behavior depicted in Fig. 2 is quite generic, in that the spin-singlet order parameter is always strongly suppressed by altermagnetic splitting whereas spin-triplet is not. The detailed dependence of critical temperatures is sensitive to the model details, but the above basic feature is a robust property of altermagnetic metals at weak coupling. One can, for instance, modify the model by adding an on-site attraction V_0 to $\mathcal{H}_I$ defined in Eq. (6). This has the effect of boosting T_c of the singlet channel for small η . Nevertheless, the singlet T_c is still strongly suppressed by spin splitting, and the triplet order robustly prevails when $\eta > \Delta_0$. One can also modify the normal-state dispersion by adding further-neighbor hopping terms. In Appendix B, we provide additional information on solving gap equations (16) and discuss in more depth the stability of the spin-triplet pairing and reasons behind its dominance over spin-singlet in altermagnetic metals as well as cover broader range of band structures and coupling strengths.

TABLE I. Four degenerate ground states of the equal-spin triplet chiral p -wave superconductor in the nonrelativistic limit.

Ground state	Abbreviation	Chern number	Property
$ \uparrow\uparrow(p_x + ip_y)\rangle \otimes \downarrow\downarrow(p_x + ip_y)\rangle$	$p_+^\uparrow \otimes p_+^\downarrow$	+2	Chiral
$ \uparrow\uparrow(p_x - ip_y)\rangle \otimes \downarrow\downarrow(p_x + ip_y)\rangle$	$p_-^\uparrow \otimes p_+^\downarrow$	0	Helical
$ \uparrow\uparrow(p_x + ip_y)\rangle \otimes \downarrow\downarrow(p_x - ip_y)\rangle$	$p_+^\uparrow \otimes p_-^\downarrow$	0	Helical
$ \uparrow\uparrow(p_x - ip_y)\rangle \otimes \downarrow\downarrow(p_x - ip_y)\rangle$	$p_-^\uparrow \otimes p_-^\downarrow$	-2	Chiral

In the rest of this paper, we focus on the limit of moderate to large spin-splitting parameter η relevant to existing altermagnetic metals and weak attractive interaction V_1 , where the ground state is equal-spin triplet chiral p -wave superconductor. In the nonrelativistic limit, such a ground state exhibits a fourfold degeneracy due to two possible chiralities for each spin. Two of the ground states are helical and two are chiral as listed in Table I. Helical phases exhibit two counterpropagating spin-filtered edge modes, whereas in chiral phases the edge modes are copropagating. The assignment of the Chern number follows from the fact that a spinless $p_x \pm ip_y$ state has Chern number $C = \pm 1$, respectively [64]. Correspondingly, when placed on a geometry with open edges, such as a long strip, the four ground states will show protected edge modes that are either chiral for $C = \pm 2$ states or helical for $C = 0$ states. Because the chiral edge modes carry substantial electrical currents [65], one expects, for small samples, the helical states to be lower in energy when one includes the inductive effects, that is, the energy cost of the magnetic field B produced by the edge current. As we see, helical edge states carry an aggregate spin current which, however, produces no magnetic field; hence, we expect helical states to be lower in energy than chiral states even in the strictly nonrelativistic limit.

In the presence of various perturbations, the ground state degeneracy will be lifted even in large samples. For the $C_{2z}\mathcal{T}$ model, this has been investigated in Refs. [32,34]. According to these works, Rashba SOC selects helical $p_+^\uparrow \otimes p_+^\downarrow$ as the unique ground state. In-plane magnetic field, on the other hand, selects the two chiral states which remain degenerate. For the sake of concreteness, we in the following assume weak SOC and focus on the helical $p_+^\uparrow \otimes p_+^\downarrow$ state in our calculations. However, we note that a majority of the results pertaining to spin currents presented in the rest of this paper are insensitive to the choice of the ground state, as they principally rely on the existence of two separate condensates for spin-up and spin-down electrons. Only when we include perturbations that permit coupling between the two condensates (generally detrimental to spin currents) do some differences between various ground states show up. These are discussed more fully in Sec. IV A below and in Appendix D.

III. SPIN-POLARIZED SUPERCURRENT GENERATION IN d -WAVE ALTERMAGNETS

A. General considerations

As our first example of useful spintronic properties of SC altermagnets, we show that they can be used to generate persistent spin currents. This property is most pronounced in d -wave altermagnets with equal-spin triplet superconducting order and relies on the fact that the underlying Fermi surfaces possess low C_{2z} symmetry for each spin projection; cf. Fig. 1(a). In this situation, it is easy to see that the superfluid density tensor $\hat{\rho}_\sigma$, which governs the supercurrent response of the material according to

$$\mathbf{j}_\sigma = \hat{\rho}_\sigma \left(\hbar \nabla \varphi_\sigma - \frac{2e}{c} \mathbf{A} \right), \quad (17)$$

will be anisotropic with *opposite* anisotropies for the two spin projections. Here, $\mathbf{j}_\sigma$ is the supercurrent density carried by condensate Δ_σ , and φ_σ is its phase. Because Cooper pairs with either spin carry the same charge $2e$, they couple identically to the vector potential $\mathbf{A}$. However, phases φ_σ can, in general, be different.

The C_{2z} symmetry of the band dispersion for each spin constrains the form of $\hat{\rho}_\sigma$ such that

$$\hat{\rho}_\sigma = \begin{pmatrix} \rho_0 & 0 \\ 0 & \rho_0 \end{pmatrix} + \sigma \begin{pmatrix} \rho_\eta & 0 \\ 0 & -\rho_\eta \end{pmatrix}, \quad (18)$$

where ρ_0 denotes the conventional isotropic component and ρ_η is the anisotropy related to the altermagnetic spin splitting, which is, crucially, opposite for two spin projections labeled by $\sigma = \pm$. The form of the superfluid density tensor Eq. (18) is clearly compatible with the $C_{4z}\mathcal{T}$ symmetry of a d -wave altermagnet. Microscopically, it follows from the fact that the superfluid density tensor is proportional to the inverse effective mass tensor, $[\hat{\rho}_\sigma]_{\alpha\beta} \propto \partial^2 \epsilon_{k\sigma} / \partial k_\alpha \partial k_\beta |_{k=0}$. For completeness, we review the relevant arguments in Appendix C, where we also find, for the d -wave altermagnet model defined in Sec. II,

$$\rho_0 \simeq n_e, \quad \rho_\eta \simeq (\eta/t) n_e, \quad (19)$$

where n_e denotes the electron density.

To explore the consequences of Eq. (18), let us first assume, for the sake of simplicity, that the two condensates experience the same superfluid velocities $\mathbf{v}_\sigma = \{\hbar \nabla \varphi_\sigma - [(2e)/c] \mathbf{A}\}$. This occurs, e.g., when the current is driven principally by $\mathbf{A}$ or when the phases $\varphi_\uparrow$ and $\varphi_\downarrow$ are locked together energetically by SOC. (We explore the effects of unequal velocities in later sections.) In this situation, it is easy to deduce from Eqs. (17) and (18) that the electrical current $\mathbf{j}^e = \mathbf{j}_\uparrow + \mathbf{j}_\downarrow$ will be governed by ρ_0 , while the spin current $\mathbf{j}^s = \mathbf{j}_\uparrow - \mathbf{j}_\downarrow$ by ρ_η , namely,

$$\mathbf{j}^e = \rho_0 \mathbf{v}, \quad \mathbf{j}^s = \rho_\eta \tau^z \mathbf{v}. \quad (20)$$

Here, $\mathbf{v}$ is the common superfluid velocity of the two condensates, and τ^z is the third Pauli matrix. From Eq. (20), one can also express the spin current in terms of the electrical current as

$$\mathbf{j}^s = (\rho_\eta / \rho_0) \tau^z \mathbf{j}^e. \quad (21)$$

The simple-looking Eq. (21) has some interesting consequences illustrated in Fig. 3(a). It implies that a persistent electrical current $\mathbf{j}^e$ directed along x or y principal axes will be accompanied by a persistent spin current $\mathbf{j}^s = (\rho_\eta / \rho_0) \mathbf{j}^e$ along the same direction. Even more remarkably, when $\mathbf{j}^e$ is directed along the diagonal, e.g., $(1, 1)$, direction, spin current will flow along the orthogonal $(1, -1)$ direction. As depicted in Fig. 3(b), this property can be exploited

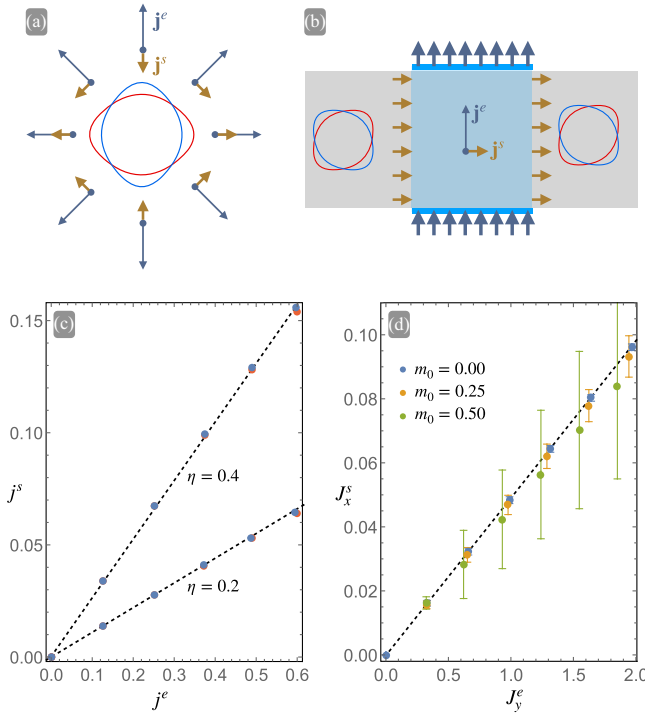


FIG. 3. Spin-polarized supercurrent generation in a d -wave alternomagnet. (a) visualizes the relation between $\mathbf{j}^s$ and $\mathbf{j}^e$ expressed by Eq. (21). (b) illustrates the spin-current dynamo effect which may be used to generate pure spin supercurrent by applying ordinary charge current to a strip oriented along the $(1, 1)$ direction relative to the underlying alternomagnet principal axes. In (c), we show spin-current density j^s as a function of j^e (both in the units of t) for a 20×20 cluster with periodic boundary conditions. Blue (orange) symbols correspond to $\mathbf{j}^e$ flowing along the $\hat{x}$ ($\hat{x} + \hat{y}$) direction. (d) shows spin current J_x^s as a function of J_y^e in the dynamo geometry for the system studied in Fig. 4. Data points represent the total spin current averaged over 40 positions along $\hat{x}$, while the error bar quantifies its fluctuation computed as a standard deviation of these values. Parameters used are $\mu = -2.1$ and $V_1 = 2.0$ with $\eta = 0.2$ in (d).

to generate pure spin supercurrent with the magnitude controllable by the applied charge current in the transverse direction. This pure spin supercurrent then can be injected from the SC alternomagnet into another material; we call this the *spin-current dynamo* effect and explore it in more detail below. It can be thought of as a superconducting version of the electrical spin splitter effect discussed in the context of organic antiferromagnets and RuO_2 in Refs. [44,66], respectively. More broadly, the spin-current dynamo effect belongs to the family of spin Hall effects that play an important role in the field of spintronics [4].

B. Spin-current dynamo effect

To confirm the validity of general arguments given in the previous subsection, we now perform numerical calculations using the BdG Hamiltonian Eq. (11) formulated on the real-space square lattice. It takes the form

$$\mathcal{H}_{\text{BdG}} = \sum_{\mathbf{r}, \mathbf{r}'} \Psi_{\mathbf{r}}^\dagger \begin{pmatrix} h_{\mathbf{r}\mathbf{r}'} - \mu & \hat{\Delta}_{\mathbf{r}\mathbf{r}'} \\ \hat{\Delta}_{\mathbf{r}\mathbf{r}'}^\dagger & -h_{\mathbf{r}\mathbf{r}'}^* + \mu \end{pmatrix} \Psi_{\mathbf{r}'} + E_0, \quad (22)$$

where $\Psi_{\mathbf{r}} = (c_{\mathbf{r}\uparrow}, c_{\mathbf{r}\downarrow}, c_{\mathbf{r}\uparrow}^\dagger, c_{\mathbf{r}\downarrow}^\dagger)^T$ is the Nambu spinor on the lattice site $\mathbf{r}$. In addition, $h_{\mathbf{r}\mathbf{r}'}$ and $\hat{\Delta}_{\mathbf{r}\mathbf{r}'}$ are real-space representations of the tight-binding Hamiltonian Eq. (1) and the pair field matrix Eq. (12), respectively.

We carry out two types of calculations addressing different geometries. Specifically, we diagonalize the real-space Hamiltonian (22) for (i) a spatially uniform order parameter Δ_σ on an $L \times L$ cluster with periodic boundary conditions and (ii) on a $L_x \times L_y$ strip in the “dynamo” geometry [Fig. 3(b)] with periodic boundary conditions along x and current injected through ten sites at the bottom open edge and extracted through top sites. In both cases, we probe currents by computing expectation values of the spin-resolved bond current operator

$$j_{ij}^\sigma = -it_{ij}^\sigma c_{j\sigma}^\dagger c_{i\sigma} + \text{H.c.}, \quad (23)$$

where t_{ij}^σ denotes the tunneling amplitude between sites i and j for electrons with spin σ .

Case (i) geometry corresponds to a torus, and we drive electrical current $\mathbf{j}^e$ by inserting magnetic fluxes Φ_x and Φ_y through the two holes of the torus. This is implemented through a Peierls substitution $t_{ij}^\sigma \rightarrow t_{ij}^\sigma \exp[-i(\mathbf{r}_i - \mathbf{r}_j) \cdot \mathbf{A}]$, where $\mathbf{A} = \pi(\Phi_x, \Phi_y)/L\Phi_0$ is the vector potential. By adjusting the fluxes, we can drive charge current $\mathbf{j}^e$ in any desirable direction. Subsequent measurements of $\mathbf{j}^s$ are summarized in Fig. 3(c) and give results that are in good qualitative agreement with the prediction of Eq. (21). The relation between j^s and j^e is indeed linear with the coefficient proportional to η/t and about factor of 2 smaller than the estimate based on the quadratic dispersion.

Figure 4 shows our results for the dynamo geometry case (ii) in a cluster of 40×10 sites. In order to accommodate this geometry naturally on a square lattice, we modify the normal state Hamiltonian by rotating the spin-dependent term in Eq. (1) by 45° , which gives

$$-2\eta\sigma_z[\cos(k_x + k_y) - \cos(k_x - k_y)]. \quad (24)$$

This produces a d_{xy} altermagnet with Fermi surface crossings along x and y axes and allows us to straightforwardly implement the geometry indicated in Fig. 3(b). To probe the dynamo effect, we then drive charge current in the vertical direction through the central portion of the strip. This is achieved by connecting sites at the bottom edge of the strip with their partners at the same position x at the top edge through a spin-independent hopping amplitude $t_J e^{-i\pi\Phi_x/\Phi_0}$. This creates a Josephson junction between a portion of the bottom and top edge biased by flux Φ_x . When Φ_x is nonzero, current flows through the strip as indicated in Figs. 4(a) and 4(b), which show the corresponding spin-resolved current densities. We remark that in this constrained geometry it is important to use self-consistently determined order parameter amplitudes (shown as color scale background) in order to ensure current conservation.

Figure 4(c) displays current densities resolved into their charge and spin components indicated by blue and orange arrows, respectively. The spin-current density is dominated by prominent edge contributions flowing along the open edges of the strip. These are indeed expected due to the presence of topologically protected edge modes in the $p_\perp^\uparrow \otimes p_\perp^\downarrow$ superconductor which are known to carry substantial currents [65]. Because of their spin-filtered helical nature, the corresponding charge current density vanishes, but significant spin current is seen to remain. The modes are, however, exactly counterbalanced between the two edges and carry, therefore, no net spin (or charge) current along the strip. In order to visualize the spin-current dynamo effect, we display in Fig. 4(d) spin-current density referenced to the situation with $\Phi_x = 0$ where no charge current is driven across the strip. This subtraction eliminates the edge currents and allows us to clearly see that bulk spin current is flowing along the strip.

In order to ascertain the robustness of the spin-current dynamo effect, we now consider the effect of magnetic disorder. To this end, we include a term

$$\mathcal{H}_{\text{dis}} = \sum_j \mathbf{m}_j \cdot \boldsymbol{\sigma}_{\alpha\beta} c_{j\alpha}^\dagger c_{j\beta} \quad (25)$$

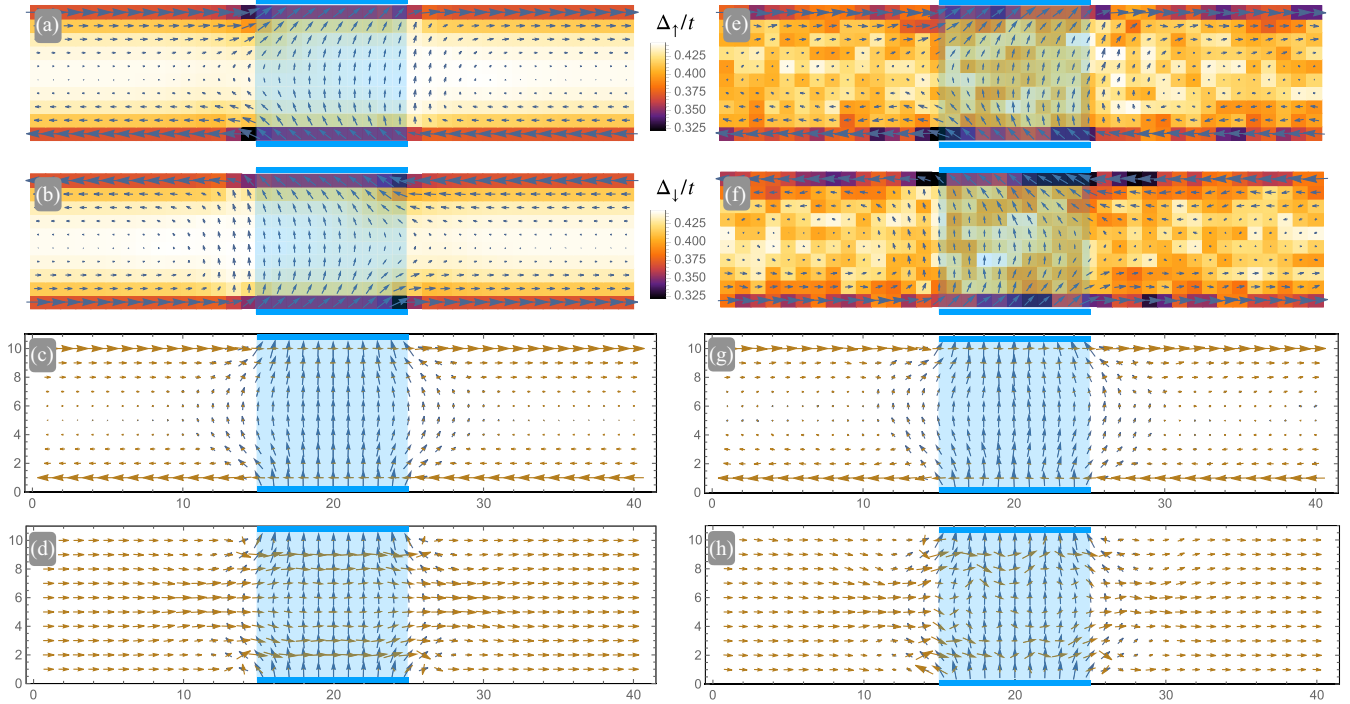


FIG. 4. Spin-current dynamo effect: generation of pure spin supercurrent from perpendicular charge current. (a) and (b) show current densities $\mathbf{j}_\uparrow$ and $\mathbf{j}_\downarrow$ calculated from the BdG theory for a 40×10 cluster of sites with periodic boundary conditions along x . The current is injected through the bottom blue electrode and collected from the top blue electrode. The color scale indicates the amplitude of the self-consistently determined SC order parameter Δ_σ . (c) shows the current densities resolved into the electrical component $\mathbf{j}^e$ and spin component $\mathbf{j}^s$, represented by blue and orange arrows, respectively. (d) shows those same currents relative to the reference state when no external current is injected. (e)–(h) show the same quantities for the system with magnetic disorder with $m_0 = 0.25t$. Spin currents in (d) and (h) have been scaled up by a factor of 6 for better visibility. Parameters are the same as in Fig. 3 with $t_J = 1.0$ and $\Phi_x = 0.4\Phi_0$.

in the normal-state electron Hamiltonian, where $\mathbf{m}_j$ represents a static local magnetic moment on site j . We take Cartesian components of $\mathbf{m}_j$ as random independent variables drawn from a uniform distribution between $(-m_0, m_0)$. This corresponds to a random magnetic moment with mean-square amplitude $\langle \mathbf{m}_j^2 \rangle^{1/2} = m_0$ on every site of the lattice coupled to electron spin. Figures 4(e)–4(h) show the effect of moderately strong disorder $m_0 = 0.25t$, which corresponds to about half of the SC gap amplitude Δ_σ . We observe clear effects of disorder in local fluctuations of the gap amplitude, although its average value is only mildly suppressed. The current flow is also affected and shows visible local fluctuations in current densities when compared to the clean case.

It is to be noted that spin current is no longer conserved in the presence of $\mathcal{H}_{\text{dis}}$, because nonzero in-plane components of $\mathbf{m}_j$ generate spin-flip scattering. Nevertheless, we observe in Fig. 4(h) clear evidence of the dynamo effect with a magnitude that is comparable to the clean case. This is quantified in Fig. 3(d), where we show the total spin current J_x^s flowing along the strip in response to charge current J_y^e for three representative disorder strengths m_0 . Because of the nonconservation mentioned above, J_x^s fluctuates as a function of position x along the strip when $m_0 \neq 0$. To provide a sense for these fluctuations, we display in Fig. 3(d) both the average value of J_x^s and its standard deviation. We observe that fluctuations are modest for $m_0 = 0.25$ but become large for $m_0 = 0.50$, even though the average value is suppressed only slightly. We remark that $m_0 = 0.50$ corresponds to the limit of very strong disorder with the magnetic impurity strength on each site exceeding the size of the SC gap $\Delta_\sigma \simeq 0.42$. It is remarkable that the spin-current dynamo effect survives essentially intact even in this highly disordered limit. For even stronger disorder, it becomes increasingly difficult to obtain fully converged self-consistent solutions. The SC gap collapses at $m_0 \simeq 0.7$, which we identify as the critical disorder strength.

For the sake of completeness, we also investigated the effect of nonmagnetic disorder. Because such disorder does not generate spin-flip scattering, its impact on the spin-current dynamo effect is expected to be much weaker, which is indeed borne out in our simulations. We remark that, in the presence of purely nonmagnetic disorder, spin current is strictly conserved, and, hence, it cannot decay even as a matter of principle.

Some remarks are in order before we conclude this section. First, we note that the $C_{6v}\mathcal{T}$ symmetry of g -wave altermagnets mandates that $\rho_\eta = 0$, and, hence, they cannot be used to generate spin currents in this fashion; however, the effect might reemerge when strain is applied [67]. Interesting properties besides spin-current generation are explored below. Second, our general analysis leading to Eq. (21) relied crucially on the simplifying assumption of equal superfluid velocities $\mathbf{v}_\sigma$ for the two spin species. In

reality, the two phases φ_σ are independent degrees of freedom that assume spatial profiles so as to minimize the system free energy under the specific conditions imposed by the experimental setup. Hence, generically, $\mathbf{v}_\uparrow$ need not equal $\mathbf{v}_\downarrow$. Nevertheless, for the geometries considered in this section, we found Eq. (21) to be in good agreement with the numerical simulations in which φ_σ are treated as independent variables and are computed self-consistently from the gap equation.

Finally, we note that the spin current generated by the dynamo effect is odd under $\eta \rightarrow -\eta$. This implies that, if multiple magnetic domains are present in the sample, their contributions to j^s will tend to average out. To achieve a significant effect, one, thus, ideally requires a single-domain sample or a sample dominated by one magnetic domain. In the recent experiment on altermagnet MnTe, domain sizes greater than 10 μm have been reported with careful annealing [68].

IV. PERSISTENT SPIN CURRENT IN VARIOUS GEOMETRIES

A key conclusion reached in Sec. II was that the leading SC instability in an altermagnet with large enough spin splitting η occurs in the equal-spin triplet channel. In the absence of SOC, this leads to two decoupled condensates, one for spin-up and one for spin-down electrons represented by $\Delta_\uparrow$ and $\Delta_\downarrow$ order parameters, respectively. At a high level, it is intuitively clear that such decoupled condensates are capable of supporting persistent currents with an arbitrary spin polarization, including pure spin supercurrent $\mathbf{j}^s \neq 0$ with $\mathbf{j}^e = 0$, achieved when spin-up charge current $\mathbf{j}_\uparrow$ is exactly counterbalanced by an opposite $\mathbf{j}_\downarrow$.

To better understand various properties of these persistent currents in different situations and also to understand the effect of SOC, we now construct the GL theory for the equal-spin triplet superconductor and analyze its solutions in various relevant geometries.

A. Ginzburg-Landau theory

The GL free energy density for the equal-spin triplet superconductor can be written as

$$f[\psi_\uparrow, \psi_\downarrow] = \sum_\sigma f_\sigma[\psi_\sigma] - \frac{1}{2} D(\psi_\uparrow^* \psi_\downarrow + \text{c.c.}) + \frac{B^2}{8\pi}, \quad (26)$$

where ψ_σ are complex scalar order parameters describing the two condensates and f_σ is the standard GL free energy density for the individual condensate,

$$f_\sigma[\psi] = \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4 + \gamma_\sigma \left(i\nabla + \frac{2e}{\hbar c} \mathbf{A} \right) \psi|^2. \quad (27)$$

By symmetry, the α and β parameters must be the same for the two condensates. For a d -wave altermagnet, the γ

parameter becomes a tensor which reflects the superfluid density anisotropy discussed in Sec. III. In the following, for the sake of maximum clarity and simplicity, we focus on the isotropic case where γ_σ is taken as a constant equal to γ for both condensates. Such a theory is directly applicable to g -wave altermagnets and can also describe multidomain d -wave altermagnets on long length scales where the anisotropy has averaged out.

The D term in Eq. (26) describes coupling between the two condensates and becomes symmetry allowed in the presence of SOC for the two helical states listed in Table I. In this situation, we expect $D \propto \lambda^2$, where λ stands for either Rashba λ_R in 2D or Dresselhaus λ_D in 3D. We note that for the two chiral states the D term is not allowed. As explained in Appendix D, in this case the leading allowed coupling is a fourth-order term ($\psi_\uparrow^* \psi_\downarrow^2 + \text{c.c.}$) and has a prefactor proportional to λ^4 .

Since we are mostly interested in the behavior of persistent currents on long distances compared to the SC coherence length ξ , we adopt the London approximation $\psi_\sigma = \psi_0 e^{i\varphi_\sigma}$ with spatially constant ψ_0 , whose value we take as real and positive. This leads to a Josephson free energy density of the form

$$f[\varphi_\uparrow, \varphi_\downarrow] = f_0 + \sum_\sigma \gamma \psi_0^2 \left(\nabla \varphi_\sigma - \frac{2e}{\hbar c} \mathbf{A} \right)^2 - J \cos(\varphi_\uparrow - \varphi_\downarrow), \quad (28)$$

where f_0 is the component independent of the phases (which we suppress from now on) and $J = \psi_0^2 D$. The last term expresses the energy cost for configurations where two phases are not aligned and can, therefore, be thought of as describing an internal Josephson effect. A direct calculation of J from the microscopic d -wave model is outlined in Appendix D and gives

$$J \simeq \lambda_R^2 k_F^4 / 4\pi t, \quad (29)$$

where $k_F = \sqrt{(\mu + 4t)/t}$ is the dimensionless Fermi momentum. Equation (29) assumes the $p_\perp^\uparrow \otimes p_\perp^\downarrow$ helical phase and is valid for $\lambda_R \lesssim \Delta_\sigma$ and μ near the bottom of the band, that is, $k_F \lesssim 1$.

It is convenient for our subsequent considerations to define average and relative phases

$$\Omega = \frac{1}{2}(\varphi_\uparrow + \varphi_\downarrow), \quad \varphi = \varphi_\uparrow - \varphi_\downarrow,$$

and rewrite Eq. (28) as

$$f[\Omega, \varphi] = \gamma \psi_0^2 \left[2 \left(\nabla \Omega - \frac{2e}{\hbar c} \mathbf{A} \right)^2 + \frac{1}{2} (\nabla \varphi)^2 \right] - J \cos \varphi. \quad (30)$$

Charge current density then follows from varying the free energy $F = \int f dV$ with respect to $\mathbf{A}$, which gives

$$\mathbf{j}^e = \mathbf{j}_\uparrow + \mathbf{j}_\downarrow = 8e \frac{\gamma \psi_0^2}{\hbar} \left(\nabla \Omega - \frac{2e}{\hbar c} \mathbf{A} \right). \quad (31)$$

Similarly, we define the spin-current density as

$$\mathbf{j}^s = \mathbf{j}_\uparrow - \mathbf{j}_\downarrow = 4e \frac{\gamma \psi_0^2}{\hbar} \nabla \varphi. \quad (32)$$

GL equations of motion follow from varying F with respect to the two phases. $\delta F / \delta \Omega = 0$ gives $\nabla \cdot \mathbf{j}^e = 0$ expressing the electrical current conservation. $\delta F / \delta \varphi = 0$ yields an equation of motion

$$\gamma \psi_0^2 \nabla^2 \varphi + J \sin \varphi = 0, \quad (33)$$

which must be solved subject to the appropriate boundary conditions in order to find the spin current. Below, we discuss some specific examples.

B. Spin-polarized supercurrent in a long wire

A question of practical interest for any spintronic application is this: If we inject spin-polarized current into a wire made of a SC altermagnet, how far from the injection point will the spin polarization persist? In the following, we address this question by studying the GL theory derived above for a long wire geometry with the spin-polarized current injected from the left. We assume a homogeneous wire with the current injected at $x = 0$ flowing along the positive x direction. We consider a charge current with an arbitrary spin polarization expressed by specifying values of j^e and j^s .

We first consider the nonrelativistic limit. In this case, there is no SOC which translates to $J = 0$. Equation (33) can then be recast as $\nabla \cdot \mathbf{j}^s = 0$ expressing spin-current conservation exactly as one would expect when the two condensates are decoupled. Physically, this implies that *any* spin polarization present in the injected current will persist to arbitrarily long length scales.

Away from the nonrelativistic limit, we have to include the effect of nonvanishing J . Qualitatively, it is clear from Eq. (32) that nonzero spin current requires nonzero gradient of the relative phase φ . This also means that φ cannot be uniformly zero to minimize the $-J \cos \varphi$ term in the free energy Eq. (30): The spin current will incur an energy cost. To see how this impacts the spin-current amplitude along the wire, we recast Eq. (33) as

$$\frac{d^2 \varphi}{dx^2} + k_J^2 \sin \varphi = 0, \quad (34)$$

where $k_J = \sqrt{J / \gamma \psi_0^2}$ is a constant with the dimension of inverse length. This is a second-order differential equation

which we want to solve subject to boundary conditions at $x = 0$ for φ and its first derivative φ' . The latter is determined by the amplitude of the injected spin current through Eq. (32), while $\varphi_0 = \varphi(x = 0)$ can be fixed by physical conditions at the boundary, e.g., when the current is injected from another superconductor.

To gain some intuition for the solutions of Eq. (34), it is useful to note that it is mathematically equivalent to the equation of motion for the angle of a mechanical pendulum:

$$\frac{d^2\varphi}{dt^2} + \omega^2 \sin \varphi = 0, \quad (35)$$

where $\omega = \sqrt{g/l}$ is the frequency of small oscillations. (g and l are the gravitational constant and the pendulum length, respectively.) Based on this analogy, an important observation can be made right away. Because Eq. (34) corresponds to an *undamped* pendulum motion, we may conclude immediately that the spin current injected into the wire will not decay; instead, it will exhibit oscillations as a function of x with a constant amplitude. In analogy with the frictionless mechanical pendulum, one expects two regimes for these oscillations: (i) The current can oscillate around zero, which is analogous to small oscillations of the pendulum about the stable equilibrium at $\varphi = 0$, or (ii) it can oscillate around a nonzero value j_{avg}^s , which corresponds to the pendulum rotating around the pivot with a steadily increasing angle. The two regimes are separated by a critical point where the pendulum has just enough energy to reach the unstable equilibrium at the top of its trajectory.

Solutions of Eq. (34) displayed in Fig. 5 indeed confirm these expectations. When large enough spin current $j^s > j_c^s$ is injected, Fig. 5(a) shows amplitude oscillations about a nonzero average value j_{avg}^s which persist without any decay to arbitrary distances. When smaller current $j^s < j_c^s$ is injected, then the amplitude still oscillates without any decay but now around zero [Fig. 5(c)]. Only when the injected current is exactly tuned to the critical value $j^s = j_c^s$ does it decay [Fig. 5(b)]. We note, however, that because this is an unstable fixed point (corresponding to a precisely tuned total energy in the pendulum analogy) an oscillatory

behavior of one or the other type will generically occur. We conclude that, even in the presence of SOC, spin-polarized current injected into the wire will propagate to arbitrary distances without any decay. For $j^s < j_c^s$ one must detect it away from the nodes of the oscillatory profile depicted in Fig. 5(c), whereas for $j^s > j_c^s$ it can be detected everywhere along x .

To connect with experiment, it is useful to estimate the period of spin-current oscillations and the critical spin current j_c^s which separates the two regimes. The period, given by $2\pi/k_J$, follows from recalling the well-known GL relations [69] for $\psi_0^2 = -\alpha/\beta$, the coherence length $\xi^2 = \gamma/|\alpha|^2$, and condensation energy $\frac{1}{2}N_F\Delta_0^2 = \alpha^2/2\beta$, where $N_F \simeq 1/4\pi t$ is the density of states at the Fermi level. Combining these with Eq. (29), one obtains

$$\frac{2\pi}{k_J} \simeq 2\pi\xi \left(\frac{\Delta_0}{\lambda_R} \right) k_F^{-2}, \quad (36)$$

valid when $\lambda_R \lesssim \Delta_0$. We observe that the period diverges when $\lambda_R \rightarrow 0$ and $\mu \rightarrow -4t$, in accord with expectations: In these limits, coupling J between the two condensates vanishes. Even away from these extreme limits, Eq. (36) predicts the period of oscillations to exceed the GL coherence length ξ , which justifies our usage of the London approximation to derive these results.

The critical spin-current density corresponds to $\varphi'(0) = 2k_J$, which together with Eq. (32) gives $j_c^s = 8e(\gamma\psi_0^2/\hbar)k_J$. Using the relations listed above Eq. (36), one can express it more usefully as

$$j_c^s \simeq j_c^e \left(\frac{\lambda_R}{\Delta_0} \right) \sqrt{3}k_F^2, \quad (37)$$

where $j_c^e = 8e(\psi_0^2/3\hbar)\sqrt{\gamma|\alpha|/3}$ is the GL critical current density [69]. We observe that j_c^s will be small in comparison with j_c^e either when SOC is weak or when μ is near the bottom of the band.

A natural question arises when pondering the results in Fig. 5, namely, to what extent is the oscillatory behavior dependent on the perfect translational invariance of the

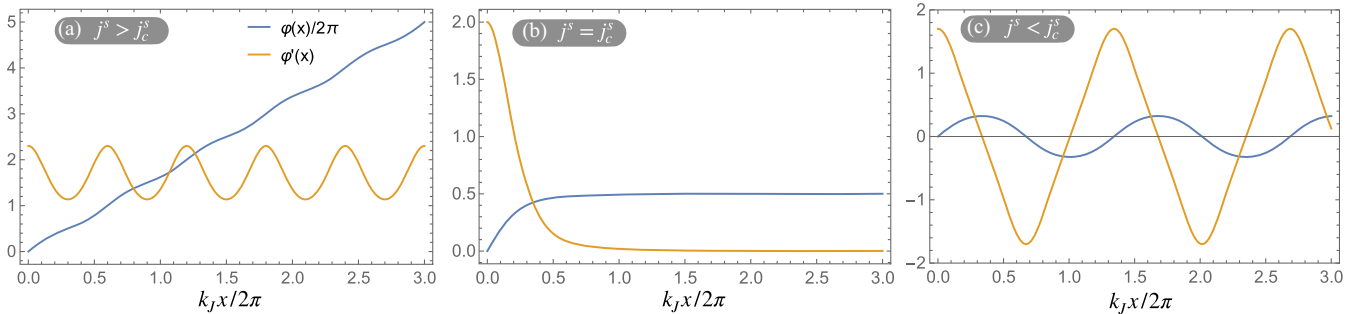


FIG. 5. Evolution of a spin current $j^s(x) \propto \varphi'(x)$ injected into a long wire at $x = 0$ in the presence of SOC. The current is obtained by numerically solving Eq. (34) with a boundary condition $\varphi(0) = 0$ and $\varphi'(0) = (2.3, 2.0, 1.7)k_J$ in (a), (b), and (c), respectively.

system? In other words, is this behavior stable with respect to disorder and inhomogeneity? We address this question in Appendix E and find that moderate levels of disorder, modeled as a random spatially varying component of the Josephson coupling J , do not substantially modify our conclusions. Specifically, the spin current injected into a long wire still propagates without dissipation; the main effect of disorder is to add a degree of irregularity to oscillations as illustrated in Fig. 10.

To further support the above predictions of the GL theory, we performed analogous calculations within the microscopic model defined by the BdG Hamiltonian Eq. (10) with Rashba SOC implemented via Eq. (2). We employ here the same long strip geometry with d -wave nodes aligned with the principal axes as in Sec. III B and with periodic boundary conditions along x , open along y , forming topologically a ring. To generate spin current along x , we initialize the order parameters according to

$$\Delta_\sigma = \Delta_0 e^{2\pi i \sigma n_v x / L_x}, \quad (38)$$

with n_v denoting the integer vorticity, opposite for two spin projections. For nonzero n_v , this choice describes spin-up and -down condensates whose relative phase φ winds by $4\pi n_v$ around the ring. When $\lambda_R = 0$, the two condensates are decoupled, and one trivially obtains the charge counterflow regime with uniform pure spin current circulating the ring.

The effect of nonzero λ_R on the spin current is illustrated in Fig. 6, where we display fully self-consistent solutions of the BdG theory starting from the ansatz Eq. (38). We remark that achieving full self-consistency between the BdG Hamiltonian Eq. (10), suitably recast in the real space, and the corresponding gap equations (7) is essential for obtaining physical, charge-current conserving solutions. We also note that under this iterative procedure the ansatz Eq. (38) leads to stable solutions with nonzero phase winding, even though the global ground state is in the zero-winding sector.

In agreement with the results of the GL theory, we find that SOC brings about spatial oscillations in the spin current J_s^x along the ring [Fig. 6(b)]. The amplitude of oscillations increases with λ_R , but, importantly, there is no decay in the sense that solutions of the BdG equations are always spatially periodic. This remains true even for very long strips; we were able to simulate systems up to 160×10 in size and various values of n_v , all with similar outcomes.

For larger values of $\lambda_R \gtrsim 0.22$, the spin current is seen to vanish in segments of the strip. We associate this behavior with reaching and exceeding the critical strength of SOC for a given value of the spin current. According to the GL results in a semi-infinite geometry, we expect in this regime behaviors like those shown in Figs. 5(b) and 5(c), respectively. In our present ring geometry, the relative phase φ is constrained to wind by $4\pi n_v$, and, hence, behaviors of this type cannot be reached. Instead, the system shows a

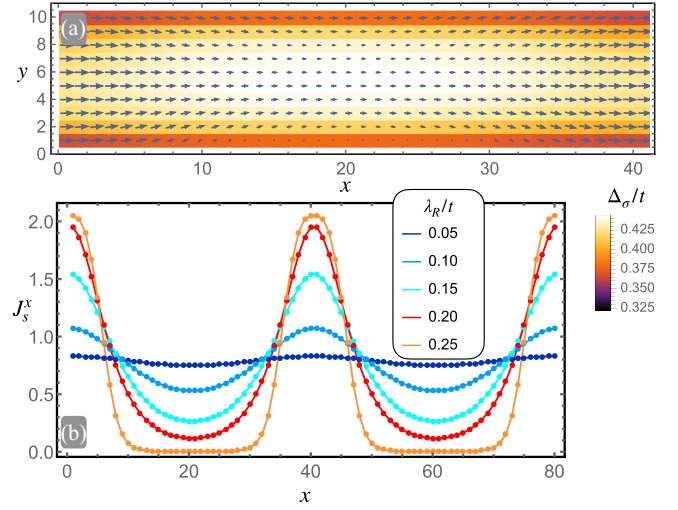


FIG. 6. Effect of Rashba SOC on pure spin supercurrent in a ring geometry modeled as an 80×10 strip with periodic boundary conditions along x , open along y . (a) Order parameter amplitude $|\Delta_\sigma|$ (identical for both spins) shown as the color scale background and spin-current density distribution $\mathbf{j}_s$ indicated by arrows. Both quantities are shown for the left half of the strip; the right half is identical. In order to filter out the edge current, we subtracted $\mathbf{j}_s$ calculated for the same parameters with $n_v = 0$. (b) Total spin current J_s^x as a function of the position x along the strip for several representative values of Rashba SOC. The same values of BdG model parameters are used as in Fig. 3 with $n_v = 1$ and $\lambda_R = 0.15$ in (a).

transition between the regime with nonzero J_s^x everywhere along the ring to the regime with $J_s^x = 0$ along parts of the ring. Physically, the phases of two condensates are locked together in these regions and, hence, $J_s^x = 0$. The relative phase varies rapidly elsewhere, resulting in large J_s^x , in accord with Eq. (32).

C. Persistent spin current in a thin ring

We now consider a ring threaded by magnetic flux Φ , illustrated in Fig. 7. We focus here on the limit where the ring thickness h and width w are both smaller than the magnetic penetration depth. This can be achieved if the ring is made of a thin film such that $h \ll \lambda_L$, the bulk London penetration depth. In this situation, the effective in-plane penetration depth will be given by the Pearl length, $\lambda_{\text{Pearl}} = 2\lambda_L^2/h \gg \lambda_L$, making the condition $w \ll \lambda_{\text{Pearl}}$ easy to satisfy.

In the situation described above, the ring material will not quantize the total enclosed flux and the calculations become straightforward, as we do not need to consider screening effects. In the nonrelativistic limit ($J = 0$), the current densities contributed by the two decoupled condensates follow from Eq. (28) and read

$$\mathbf{j}_\sigma = 4e \frac{\gamma \psi_0^2}{\hbar} \left(\nabla \varphi_\sigma - \frac{2e}{\hbar c} \mathbf{A} \right). \quad (39)$$

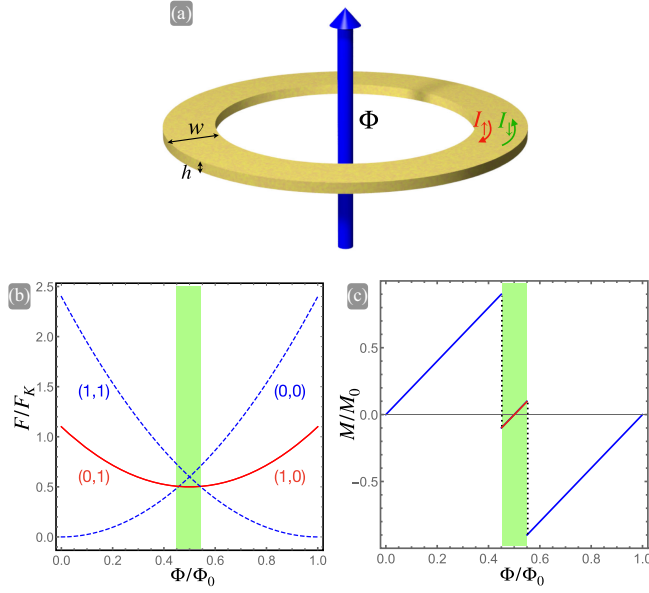


FIG. 7. (a) A SC ring threaded by magnetic flux Φ . Red and green arrows illustrate the charge counterflow regime that becomes energetically favorable when Φ is close to a half-integer multiple of the SC flux quantum at weak SOC. (b) Free energy Eq. (41) as a function of the inserted flux Φ/Φ_0 . Various branches correspond to winding numbers $(n_\uparrow, n_\downarrow)$ as indicated and $F_K = \frac{1}{2}L_K I_0^2$. The green-shaded region marks the range of flux values where charge counterflow solutions minimize the free energy and are, thus, stable. (c) The corresponding magnetization M normalized to its maximum value M_0 .

We represent the flux by $\mathbf{A} = \hat{\theta}\Phi/\ell$, where $\hat{\theta}$ is a unit vector in the azimuthal direction and ℓ denotes the ring circumference. Phases φ_σ are constrained by single-valuedness of the corresponding order parameters ψ_σ . This permits the phase to wind by an integer multiple of 2π around the ring, $\varphi_\sigma(\ell) = \varphi_\sigma(0) + 2\pi n_\sigma$, where n_σ is the integer winding number and we parametrized φ_σ by its position along the ring. Assuming, furthermore, that phases vary uniformly along the ring, we have $\nabla\varphi = 2\pi n_\sigma \hat{\theta}/\ell$. Taken together, this allows us to define the ring current

$$I_\sigma = S\hat{\theta} \cdot \mathbf{j}_\sigma = I_0(n_\sigma - \Phi/\Phi_0), \quad (40)$$

for each spin, where $I_0 = 4eS[(\gamma\psi_0^2)/\hbar][(2\pi)/\ell]$ and $S = wh$ is the ring cross section.

To understand the energetics (working still in the $J = 0$ limit), it is useful to write the free energy associated with these currents as

$$F = \frac{1}{2}L_K(I_\uparrow^2 + I_\downarrow^2) + \frac{1}{2}L_G(I_\uparrow + I_\downarrow)^2. \quad (41)$$

The first term represents the energy cost of the superflow and follows directly from the free energy density Eq. (28) with the kinetic inductance defined as $L_K = \hbar^2\ell/8e^2S\gamma\psi_0^2$.

The second term expresses the energy stored in the magnetic field B generated by the total electric current $I = I_\uparrow + I_\downarrow$ flowing in the ring with L_G denoting its geometric inductance and follows from the last term in Eq. (26). The precise value of L_G depends on the details of the ring geometry. For illustration purposes, we can estimate it by the well-known Kirchhoff formula $L_G \simeq (8\pi\ell/c^2)[\ln(\ell/\rho) - 1.508]$ for the self-inductance of a ring with circular cross section of radius ρ and circumference ℓ , accurate in the limit $\ell \gg \rho$ [70].

A simple but important observation follows from Eq. (41): In situations when the magnitudes of $I_\uparrow$ and $I_\downarrow$ are equal, the free energy is minimized when $I_\uparrow = -I_\downarrow$, because then the second term vanishes. This situation corresponds to the *charge counterflow* regime in which there is a pure spin current flowing in the wire. The physical reason behind this interesting conclusion is simply that, while charge current incurs an energy cost due to the induced magnetic field, spin current does not.

A practical route toward the realization of the charge counterflow regime offers itself based on the above insight. Figure 7(b) shows the behavior of the free energy Eq. (41) as a function of the flux Φ with various branches representing different vorticities $(n_\uparrow, n_\downarrow)$. In equilibrium, the system will be on the branch that minimizes F for the given flux. We observe that close to $\Phi = \Phi_0/2$ the global minimum occurs when $(n_\uparrow, n_\downarrow) = (1, 0)$ or $(0, 1)$. In view of Eq. (40), these solutions correspond to spin-up and -down currents flowing in the opposite directions. At exact half-flux quantum, this implies pure spin-persistent currents flowing in the clockwise and counterclockwise direction for $(n_\uparrow, n_\downarrow) = (1, 0)$ and $(0, 1)$, respectively. Solutions with winding numbers $(1, 1)$ and $(0, 0)$ correspond to conventional (unpolarized) persistent currents but are higher in energy by $\frac{1}{2}L_G I_0^2$.

We, thus, conclude that in the nonrelativistic limit one can realize pure spin supercurrent in a ring threaded by half-integer number of magnetic flux quanta. The two states with this property minimize the global free energy and are, thus, truly persistent in the sense that spin currents cannot decay even as a matter of principle.

We note, finally, that the pure spin supercurrent predicted to occur near the half-flux quantum can be detected by a simple magnetization measurement; i.e., no direct spin-current detection is required. Consider increasing Φ from zero. Initially, the system will be on the $(0, 0)$ branch in Fig. 7(b) and will, hence, support charge current proportional to Φ according to Eq. (40), accompanied with the corresponding magnetization M , also proportional to Φ . Sufficiently close to $\Phi_0/2$, the system will switch to the $(1, 0)$ or $(0, 1)$ branch. Here, the charge current is near zero and so is the magnetization illustrated in Fig. 7(c). The signature of a pure spin supercurrent is, therefore, given by a range of near-vanishing magnetization around

$\Phi = \Phi_0/2$ —this is in contrast to the normal case where magnetization is maximal in this region.

A natural question arises: How does a departure from the nonrelativistic limit affect this result? Persistent spin currents discussed above involve solutions with different winding numbers n_σ for the two spin projections. Therefore, the phase difference $\varphi_\uparrow - \varphi_\downarrow$ in the argument of the J term in the free energy Eq. (28) will also wind around the ring. This implies an energy cost $\Delta F_J \simeq JS\ell$ compared to the state with $\varphi_\uparrow = \varphi_\downarrow$. On this basis, we expect the persistent spin-current solutions to be energetically favored as long as $\Delta F_J \lesssim \frac{1}{2}L_G I_0^2$. This places an upper bound on the strength of SOC that can be tolerated, expressed as $J < J_c \simeq L_G I_0^2 / 2S\ell$. Using Eq. (29) and various relations listed above Eq. (36), one can translate this into a condition $\lambda_R < \lambda_{Rc}$ with

$$\lambda_{Rc}^2 \simeq \frac{4\pi^2 \Delta_0^2}{k_F^4} \left(\frac{\xi}{\lambda_L} \right)^2 \frac{S}{\ell^2} \left[\ln \frac{\ell}{\rho} - 1.508 \right]. \quad (42)$$

Here, $\lambda_L^2 = \hbar^2 c^2 / 8\pi(2e)^2 \gamma \psi_0^2$ is the London penetration depth, and $\rho \simeq \sqrt{S/\pi}$ is the effective radius of the ring cross section area.

We, thus, find that persistent spin-current solutions can be favored even away from the nonrelativistic limit as long as the SOC strength satisfies $\lambda_R < \lambda_{Rc}$ with λ_{Rc} given in Eq. (42). To get a sense for the relevant magnitudes, we consider a ring with radius $R = 1 \mu\text{m}$ and a rectangular cross section with $(w, h) = (0.1, 0.01) \mu\text{m}$. If we, furthermore, assume representative parameter values $\xi/\lambda_L \approx 1$ and $k_F = 0.5$, then Eq. (42) gives $\lambda_{Rc} \simeq 0.3\Delta_0$. Hence, we conclude that under rather generic conditions there will exist a reasonable window of permissible SOC strength in which pure persistent spin current can be induced in a ring by half-integer flux. This window, according to Eq. (42), becomes wider for smaller samples, a useful property that can be used detect and tune the effect experimentally.

V. OUTLOOK

Efficient generation, manipulation, and detection of spin currents is one of the key goals of the field of spintronics. Our work establishes superconducting altermagnets as a new viable platform for achieving some of these goals in a setting where spin dissipation, a key challenge that must be faced in normal metal and semiconductor devices, is not an issue. We have demonstrated that SC altermagnets are capable of carrying spin-polarized currents without dissipation over arbitrarily long distances, even in the presence of SOC and magnetic disorder which generally lead to short spin lifetimes in nonsuperconducting materials. Compared to the existing devices based on proximity-coupled heterostructures [11], superconducting altermagnets may offer at least two potential advantages. First is the absence of stray fields; altermagnets are compensated magnets with

strictly vanishing magnetization and, hence, no possibility of magnetic coupling between nearby elements. Second is the inherent simplicity: Proximity-coupled devices reported in Ref. [11] require 12 distinct layers with precisely tuned thicknesses and controlled interfaces to function. By contrast, our theoretical work suggests that a single layer of superconducting altermagnet could potentially offer a comparable functionality.

The above advances are rooted in two key insights: (i) The characteristic spin-split band structure of metallic altermagnets makes them uniquely susceptible to an exotic form of spin-triplet p -wave superconductivity, perfectly suited for generation and transport of spin currents; and (ii) the resulting spin currents are persistent in that they flow in the equilibrium state of the system, and, hence, once established, they cannot decay. This last property is well documented in the context of conventional charge supercurrents. Here, we showed that with small modifications it extends to spin supercurrents. We found that while perturbations away from the strictly nonrelativistic limit (e.g., SOC and magnetic disorder) can cause spin current to undergo spatial oscillations and fluctuations, crucially, it still propagates over arbitrary distances without any overall decay. The difference is rooted in the fact that, unlike charge, spin is not a conserved quantity in real materials, and, hence, spin polarization is allowed to fluctuate. Nevertheless, the structure of the theory still does not permit dissipation to occur simply because the system with spin supercurrent is already in the state of the lowest free energy compatible with its boundary conditions.

A key challenge going forward lies in discovery of a suitable family of altermagnets with an intrinsic superconducting instability occurring at temperatures that are not too low to prevent practical applications. Although no SC altermagnets have yet been identified, it is to be noted that many existing and predicted altermagnets are good metals. With zero net magnetization enforced by spin group symmetry, such metals are generically susceptible to Cooper pair formation in the presence of a weak attractive interaction. Importantly, our work shows that the highly coveted spin-triplet SC order is expected to occur due to the underlying spin-split band structure and does not require any exotic pairing mechanism; the conventional retarded phonon-induced attraction is perfectly adequate for the task. In addition, it is known that the presence of magnetism in a material is not necessarily inimical to superconductivity: High- T_c cuprates and some iron-based superconductors famously emerge from doped antiferromagnets [71]. Likewise, several ferromagnetic superconductors have been reported in the literature, including a number of f -electron compounds such as CeCu_2Si_2 [72], as well as UGe_2 , URhGe , and UCoGe [73].

It should be noted that Cooper pairing at nonzero center-of-mass momentum [74,75] has been discussed as a possible alternative to the zero-momentum spin-triplet

p -wave SC order in altermagnets. Although we find it to be energetically disfavored in minimal models defined in Sec. II, it is possible that other models can naturally support finite-momentum pairing instabilities. A discovery of such a pair density or FFLO state would be perhaps as exciting as the spin-triplet p -wave order, although its utility in spintronics or other practical applications is yet to be understood. In either case, it is clear that search for superconducting altermagnets holds promise of exciting future discoveries.

A possibility of proximity-induced SC order in altermagnets, of the type useful for spintronic applications [58], is an interesting alternative to the intrinsic superconductivity studied in this work. Instabilities at strong coupling that lie beyond the reach of the standard mean-field BCS theory employed in this work are also of great interest but are left for future studies.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: FINITE-MOMENTUM PAIRING

To examine the stability of ordinary zero-momentum pairing against possible instabilities at nonzero momentum, we compute pair susceptibilities for singlet and triplet channels in the d -wave altermagnet model introduced in Sec. II. These are defined by [76]

$$\chi_{\Delta}^{\alpha\beta}(\omega, \mathbf{q}) = \int_{-\infty}^{\infty} dt e^{-i\omega t} \langle \hat{\Delta}_{\mathbf{q}}^{\alpha\beta}(t)^{\dagger} \hat{\Delta}_{\mathbf{q}}^{\alpha\beta}(0) \rangle, \quad (\text{A1})$$

where $\hat{\Delta}_{\mathbf{q}}^{\alpha\beta} = \sum_{\mathbf{k}} g_{\mathbf{k}} c_{\mathbf{k}+\mathbf{q}\alpha}^{\dagger} c_{-\mathbf{k}\beta}^{\dagger}$ is the Cooper pair operator with $g_{\mathbf{k}}$ the structure factor equal to $C_{\mathbf{k}}$ for singlet and $S_{\mathbf{k}}$ for triplet. We are interested in the static limit $\omega \rightarrow 0$ where the standard evaluation of the correlator in Eq. (A1) gives

$$\chi_{\Delta}^{\alpha\beta}(\mathbf{q}) = \sum_{\mathbf{k}} |g_{\mathbf{k}} + g_{\mathbf{k}-\mathbf{q}}|^2 \frac{1 - n_F(\xi_{\mathbf{k}\alpha}) - n_F(\xi_{\mathbf{q}-\mathbf{k}\beta})}{\xi_{\mathbf{k}\alpha} + \xi_{\mathbf{q}-\mathbf{k}\beta}}. \quad (\text{A2})$$

Here, $n_F(\xi)$ is the Fermi function, and $\xi_{\mathbf{k}\alpha}$ denotes the band energy of an electron with crystal momentum $\mathbf{k}$ and spin $\alpha = \uparrow, \downarrow$, referenced to the Fermi level μ .

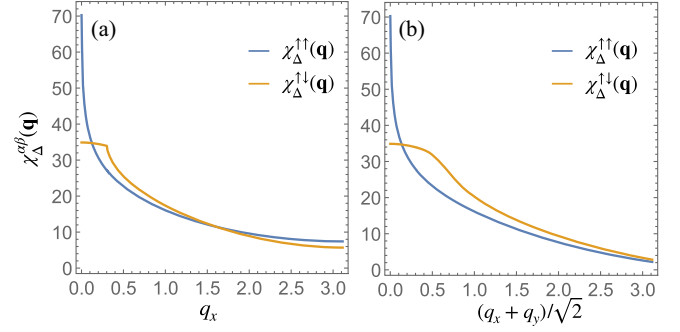


FIG. 8. Static pair susceptibilities $\chi_{\Delta}^{\alpha\beta}(\mathbf{q})$ in a d -wave altermagnetic metal evaluated from Eq. (A2) for singlet ($\uparrow\downarrow$) and triplet ($\uparrow\uparrow$) pairing. (a) shows the result for center-of-mass momentum $\mathbf{q}$ along x , while (b) along the diagonal direction. We used $\mu = -2.5$ and $\eta = 0.2$ for this calculation, but we find qualitatively similar results whenever there is a significant altermagnetic spin splitting. The singlet susceptibility develops a $q \rightarrow 0$ divergence only when $\eta = 0$ and can become the leading instability in that limit.

We evaluate Eq. (A2) numerically for the model d -wave altermagnet defined in Eq. (1) and show the results in Fig. 8. We observe that in accord with the intuitive argument given in Sec. I the triplet susceptibility shows a BCS-type logarithmic divergence in the $q \rightarrow 0$ limit, whereas the singlet susceptibility remains finite for all q . These results imply that equal-spin triplet chiral p -wave pairing at zero momentum is the leading superconducting instability in a metallic altermagnet, at least in the context of the minimal model considered in this work. We expect this to be true more generally except perhaps in special cases when spin-up and -down Fermi surfaces satisfy a nesting condition that would give rise to a competing divergence in the singlet channel.

APPENDIX B: DOMINANCE OF TRIPLET PAIRING

We discuss here in more detail the solution of gap equations for singlet and triplet pairing derived in Sec. II C. We consider a range of coupling strengths V_1 such that the SC gaps are small compared to the bandwidth and mean-field BCS treatment remains applicable and accurate. To cover a broader range of normal-state band structures, we also supplement the model energy dispersion defined in Eq. (1) with a second neighbor hopping t' resulting in a term

$$-2t'[\cos(k_x + k_y) + \cos(k_x - k_y)]. \quad (\text{B1})$$

To determine the leading SC instability of the model at weak coupling, we use gap equations (16) to extract the corresponding critical temperatures T_c . The leading instability is the one with higher T_c . To this end, we set gaps Δ_0 and Δ_{σ} to zero and obtain

$$\frac{1}{V_1} = \frac{1}{2N} \sum_k \frac{C_k^2}{\xi_{k+}} \frac{\sinh \beta \xi_{k+}}{\cosh \beta \xi_{k-} + \cosh \beta \xi_{k+}}, \quad (\text{B2a})$$

$$\frac{1}{V_1} = \frac{1}{2N} \sum_k \frac{|S_{k\sigma}|^2}{\xi_{k\sigma}} \tanh \frac{1}{2} \beta \xi_{k\sigma}. \quad (\text{B2b})$$

Solving these equations for β at a given interaction strength V_1 determines the critical temperature in the singlet and the triplet channels, respectively. Some typical solutions are displayed in Fig. 2. To gain additional insights into the nature of these solutions, it is instructive to examine the behavior of the rhs of these gap equations as a function of temperature for various values of the spin splitting parameter η . These can be thought of as static, zero-momentum pair susceptibilities in singlet $\chi_{\Delta}^{\uparrow\downarrow}(T)$ and triplet $\chi_{\Delta}^{\uparrow\uparrow}(T)$ channels, respectively.

Figure 9(a) shows the behavior in the nonmagnetic case $\eta = 0$. In this limit, both susceptibilities exhibit the characteristic BCS log divergence in the $T \rightarrow 0$ limit. This implies that the SC instability will occur at arbitrarily weak attractive interaction, a hallmark of the BCS theory. Mathematically, this follows because the singlet gap equation (B2a) becomes identical to Eq. (B2) in this limit, apart from distinct structure factors $S_{k\sigma}$ vs C_k . We also observe that, as one might expect, at weak coupling the singlet channel has higher critical temperature: The non-magnetic metal will become a conventional singlet *s*-wave superconductor at a sufficiently low temperature.

The situation changes dramatically when altermagnetic splitting is present. As illustrated in Fig. 9(b), the log divergence in the singlet susceptibility $\chi_{\Delta}^{\uparrow\downarrow}(T)$ is now quenched, while $\chi_{\Delta}^{\uparrow\uparrow}(T)$ still diverges in the $T \rightarrow 0$ limit. Mathematically, the absence of the log divergence is a direct consequence of the unconventional form of the singlet gap equation (B2a) in the presence of spin splitting. Intuitively, it reflects the fact that for a spin-split Fermi surface it is not possible to form a conventional spin-singlet Cooper pair from electrons at opposite crystal momenta, as per the argument given in the context of Fig. 1(a). As a result, the SC instability in the triplet channel will still occur at an arbitrarily weak coupling V_1 . By contrast, the singlet instability requires V_1 larger than a critical value V_{1c} . At sufficiently weak coupling, therefore, altermagnetic metal can become only a spin-triplet superconductor.

The above analysis can be usefully summarized by constructing a phase diagram of the system. We have seen that for any nonzero value of the spin-splitting parameter η there exists a critical value of V_1 below which only a triplet order parameter can condense. Conversely, for a given value of V_1 the system becomes a spin-triplet superconductor when the spin-splitting parameter η exceeds a critical value η_c . The latter can be determined from Eq. (B2a) as a solution of

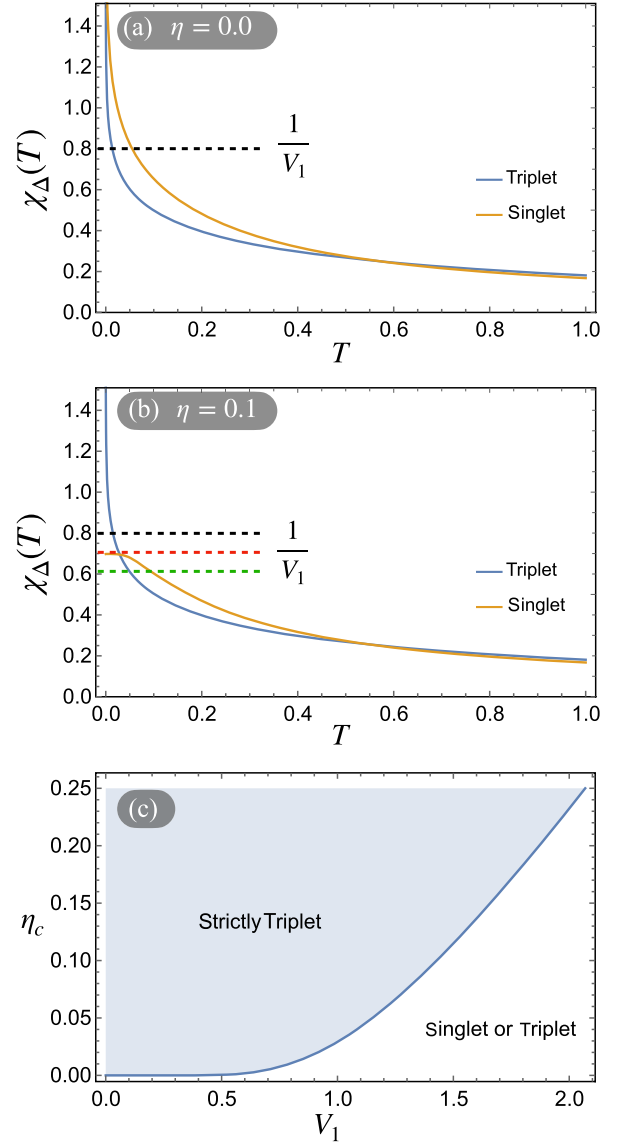


FIG. 9. Determination of critical temperatures in singlet and triplet channels in a *d*-wave altermagnet. (a) and (b) show static pair susceptibilities $\chi_{\Delta}(T)$ defined as the rhs of Eqs. (B2) for two representative values of spin splitting parameter η . (c) shows the critical value η_c extracted from numerically solving Eq. (B3). When $\eta > \eta_c$, the gap equation has no solution in the singlet channel and only the triplet solution exists. For $\eta < \eta_c$, solutions in both channels exist, and the singlet T_c is typically higher than the triplet T_c . All calculations are performed for a *d*-wave altermagnet model defined in Sec. II A with $\mu = -2.3$ and $t' = 0.2$, but the results do not change qualitatively when these parameters are varied.

$$\frac{1}{V_1} = \lim_{T \rightarrow 0} \chi_{\Delta}^{\uparrow\downarrow}(T), \quad (\text{B3})$$

realizing the condition illustrated by a red dashed line in Fig. 9(b). The corresponding values of η_c are shown in Fig. 9(c). This can be viewed as a phase diagram of the system in the $V_1 - \eta$ plane. In the blue-shaded region above

η_c , only the triplet solution exists and the material can become only a spin-triplet superconductor. This corresponds to the black dashed line in Fig. 9(b). In the unshaded region below η_c , both singlet and triplet solutions exist [green dashed line in Fig. 9(b)] and the system will adopt the state with higher T_c .

In realistic altermagnets, spin splitting is typically a significant fraction of the bandwidth. The phase diagram, therefore, indicates that at weak coupling most altermagnetic metals should become triplet superconductors at sufficiently low temperature. Only at strong coupling or small spin splitting is the singlet channel favored.

APPENDIX C: SUPERFLUID DENSITY TENSOR

Various equivalent expressions can be found in the literature for the superfluid density of a BCS superconductor. Here, we use an expression derived in two different ways in Refs. [77,78], valid for a 2D lattice superconductor with a normal state dispersion ξ_k and gap function Δ_k . Adapted to our model of a d -wave altermagnet, it reads

$$[\hat{\rho}_\sigma]_{\alpha\beta} = \frac{e}{2\hbar^2 S} \sum_k \left[\frac{\partial^2 \xi_{k\sigma}}{\partial k_\alpha \partial k_\beta} \left(1 - \frac{\xi_{k\sigma}}{E_{k\sigma}} \tanh \frac{1}{2} \beta E_{k\sigma} \right) - \frac{1}{2} \beta \frac{\partial \xi_{k\sigma}}{\partial k_\alpha} \frac{\partial \xi_{k\sigma}}{\partial k_\beta} \text{sech}^2 \frac{1}{2} \beta E_{k\sigma} \right]. \quad (\text{C1})$$

Here, the sum is over the Brillouin zone, $E_{k\sigma} = \sqrt{\xi_{k\sigma}^2 + \Delta_\sigma^2 |S_{k\sigma}|^2}$ is the quasiparticle excitation energy, and S denotes the area of the system.

For a fully gapped superconductor, the expression simplifies in the $T \rightarrow 0$ limit and gives

$$[\hat{\rho}_\sigma]_{\alpha\beta} = \frac{e}{2\hbar^2 S} \sum_k \frac{\partial^2 \xi_{k\sigma}}{\partial k_\alpha \partial k_\beta} \left(1 - \frac{\xi_{k\sigma}}{E_{k\sigma}} \right). \quad (\text{C2})$$

If we further assume that the gap is small compared to the Fermi energy and consider electron filling near the bottom of the band, Eq. (C2) can be approximated as

$$[\hat{\rho}_\sigma]_{\alpha\beta} \simeq [M_\sigma^{-1}]_{\alpha\beta} \frac{e}{2S} \sum_k \left(1 - \frac{\xi_{k\sigma}}{|\xi_{k\sigma}|} \right). \quad (\text{C3})$$

Here, $[M_\sigma^{-1}]_{\alpha\beta} = \hbar^{-2} \partial^2 \xi_{k\sigma} / \partial k_\alpha \partial k_\beta$ is the inverse effective mass tensor, and the sum represents the electron density n_e per spin.

For the altermagnetic energy dispersion that follows from Eq. (1), we have

$$M_\sigma^{-1} \simeq \frac{2}{\hbar^2} \begin{pmatrix} t + \sigma\eta & 0 \\ 0 & t - \sigma\eta \end{pmatrix}, \quad (\text{C4})$$

which then leads directly to Eq. (19).

APPENDIX D: PHYSICS OF THE $\psi_\uparrow^* \psi_\downarrow$ TERM

The $\psi_\uparrow^* \psi_\downarrow$ term in GL free energy Eq. (26) is symmetry allowed for the two helical phases listed in Table I but is disallowed for the chiral phases. To see this, we consider its behavior under C_{4z} rotation. It is useful to note that the z component of the total angular momentum of the equal-spin triplet order parameters is $J_z = (0, 0, 2, -2)$ for $(p_\uparrow^\uparrow, p_\uparrow^\downarrow, p_\downarrow^\uparrow, p_\downarrow^\downarrow)$, respectively. Under C_{4z} , such order parameters transform as $\psi \rightarrow e^{iJ_z \pi/2} \psi$.

It follows that the bilinear term $D(\psi_\uparrow^* \psi_\downarrow + \text{c.c.})$ in Eq. (26) is invariant under C_{4z} if the two order parameters represent $p_\uparrow^\uparrow \otimes p_\downarrow^\downarrow$ or $p_\uparrow^\downarrow \otimes p_\downarrow^\uparrow$ configurations, that is, the two helical phases. On the other hand, the bilinear term is odd under C_{4z} for the two chiral combinations $p_\uparrow^\uparrow \otimes p_\downarrow^\uparrow$ and $p_\uparrow^\downarrow \otimes p_\downarrow^\downarrow$. In this case, the lowest-order coupling involves a quartic term $D_4(\psi_\uparrow^2 \psi_\downarrow^2 + \text{c.c.})$.

In the presence of weak SOC, we expect $D \propto \lambda^2$ and $D_4 \propto \lambda^4$, because the two terms represent tunneling of one and two Cooper pairs between two spin sectors, respectively. In the following, we use the microscopic BdG model defined in Sec. II to explicitly evaluate the Josephson coupling $J = D\psi_0^2$ for the helical $p_\uparrow^\uparrow \otimes p_\downarrow^\downarrow$ phase of a d -wave altermagnet. Specifically, we consider the normal-state lattice Hamiltonian defined by Eqs. (1) and (2) in the presence of attractive interaction (6), decoupled in the triplet channel as shown in Eq. (9).

We obtain J by expanding the free energy of the BdG model

$$F(\varphi) = E_0 - \frac{2}{\beta} \sum_{k,a} \ln \left[2 \cosh \left(\frac{1}{2} \beta E_{ka}(\varphi) \right) \right] \quad (\text{D1})$$

to leading order in the relative phase φ between the two order parameters. Here, β denotes the inverse temperature, E_0 contains terms independent of φ , and $E_{ka}(\varphi)$ are two positive eigenvalues of the 4×4 BdG matrix Hamiltonian Eq. (11). For the helical $p_\uparrow^\uparrow \otimes p_\downarrow^\downarrow$ phase, these can be expressed as

$$E_{ka}^2(\varphi) = \xi_k^2 + \eta_k^2 + |\lambda_k|^2 + |\Delta_k|^2 + 2aD_k(\varphi), \quad (\text{D2})$$

with $a = \pm$,

$$D_k^2(\varphi) = \xi_k^2(\eta_k^2 + |\lambda_k|^2) + \text{Im}(\lambda_k \Delta_k)^2, \quad (\text{D3})$$

and

$$\begin{aligned}
\xi_k &= -2t(\cos k_x + \cos k_y) - \mu, \\
\eta_k &= -2\eta(\cos k_x - \cos k_y), \\
\lambda_k &= \lambda_R e^{i\varphi/2}(\sin k_y + i \sin k_x) \\
\Delta_k &= \Delta_0(\sin k_x + i \sin k_y).
\end{aligned} \tag{D4}$$

In order to make headway, we now focus on the chemical potential near the bottom of the band and expand ξ_k and η_k in Eq. (D4) to leading order in small momentum k :

$$\xi_k \simeq t(k^2 - k_F^2), \quad \eta_k \simeq \eta(k_x^2 - k_y^2), \tag{D5}$$

where $k_F = \sqrt{(\mu + 4t)/t}$ is the Fermi momentum. The remaining quantities in Eq. (D4) are similarly expanded near the Fermi level, e.g.,

$$\Delta_k \simeq \Delta_0(k_x + i k_y) \approx \Delta_0 k_F e^{i\alpha_k}, \tag{D6}$$

where α_k is the angle between vector $\mathbf{k}$ and the k_x axis.

We note that the entire φ dependence of the free energy is contained in the last term of Eq. (D3), which is simplified in this approximation as

$$\text{Im}(\lambda_k \Delta_k)^2 \simeq \lambda_R^2 \Delta_0^2 k_F^4 \cos^2(\varphi/2). \tag{D7}$$

We expect the leading behavior of the free energy to be of the form $F(\varphi) \propto \lambda_R^2 \cos \varphi$. To isolate this term, we now focus on the free energy derivative $F'(\varphi)$, which can be expressed as

$$F' = - \sum_{k,a} \frac{1}{2E_{ka}} \frac{\partial E_{ka}^2}{\partial \varphi} \tanh \frac{1}{2} \beta E_{ka}. \tag{D8}$$

The phase derivative can be written as

$$\frac{\partial E_{ka}^2}{\partial \varphi} = -\frac{1}{2} a \lambda_R^2 \Delta_0^2 k_F^4 \frac{\sin \varphi}{D_k(\varphi)}. \tag{D9}$$

Given that this term is proportional to λ_R^2 and we are interested in the result to this order, we can evaluate the sum in Eq. (D8) by taking $\lambda_R \rightarrow 0$ in the rest of the expression. This simplifies things considerably, and we obtain, in the limit of zero temperature T ,

$$F' \simeq -\frac{1}{2} \lambda_R^2 \Delta_0^2 k_F^4 \sum_{k,a} \frac{a \sin \varphi}{|\xi_k \eta_k| \sqrt{(\xi_k + a \eta_k)^2 + \Delta_0^2}}. \tag{D10}$$

Hence, we see that to leading order $F' \propto \lambda_R^2 \sin \varphi$, as expected.

To find the proportionality constant, we must evaluate the remaining sum over $\mathbf{k}$. The leading contribution can be estimated already in the limit of zero spin splitting η . Taking the limit $\eta \rightarrow 0$, we find

$$F' \simeq \lambda_R^2 \Delta_0^2 k_F^4 \sin \varphi \sum_k \frac{1}{(\xi_k^2 + \Delta_0^2)^{3/2}}. \tag{D11}$$

We evaluate the sum and arrive at

$$F' \simeq \frac{\lambda_R^2 k_F^4}{4\pi t} \sin \varphi. \tag{D12}$$

The expression is valid for $\lambda_R \lesssim \Delta_0$ and for the chemical potential near the bottom of the band such that the expansion of ξ_k in Eq. (D5) is accurate. The last condition is met when $k_F \lesssim 1$.

We, thus, conclude that to the leading order in the phase difference the free energy can be written as

$$F(\varphi) \simeq F_0 - J \cos \varphi, \quad J \simeq \frac{\lambda_R^2 k_F^4}{4\pi t}, \tag{D13}$$

where F_0 is independent of φ . We note that, because of the 4π factor in the denominator, even a fairly significant SOC strength $\lambda_R \sim t$ will tend to yield relatively small values of J . In addition, J can be further suppressed by going to the low electron density limit where $k_F \ll 1$. We conclude that generically one may expect the linear coupling between spin-up and spin-down order parameters to be parametrically small compared to other energy scales in the problem.

We note that a similar calculation can be performed for the $p_+^\uparrow \otimes p_-^\downarrow$ helical phase. In this case, $\text{Im}(\lambda_k \Delta_k)$ in Eq. (D3) is replaced by $\text{Im}(\lambda_k \Delta_k^*)$. One obtains the same expression (D13) for the free energy except with a modified $J \simeq -\lambda_R^2 k_F^6 / 192t$. Finally, the calculation can be performed for the two chiral phases listed in Table 1. In this case, the form of the BdG energy eigenvalues is different from Eq. (E2), and the calculation to the same order gives $J = 0$, consistent with the expectation that the leading contribution is fourth order in λ_R .

APPENDIX E: SPIN CURRENT WITH RANDOM J

To model disorder, we consider free energy density Eq. (29) with the internal Josephson coupling containing a spatially varying component:

$$J(x) = J_0 + \delta J(x). \tag{E1}$$

Here, $\delta J(x)$ is generated by smoothly interpolating M random numbers $w(x_j)$ drawn from a uniform distribution $[-r, r]$ at spatial positions $x_j = jL/M$ with $j = 1, \dots, M$. The corresponding Euler-Lagrange equation

$$\frac{d^2 \varphi}{dx^2} + k_J^2 [1 + \zeta(x)] \sin \varphi = 0, \tag{E2}$$

with $\zeta(x) = \delta J(x)/J_0$, is then solved numerically subject to boundary conditions at $x = 0$ just like in Sec. IV B. A representative solution is displayed in Fig. 10. We find that

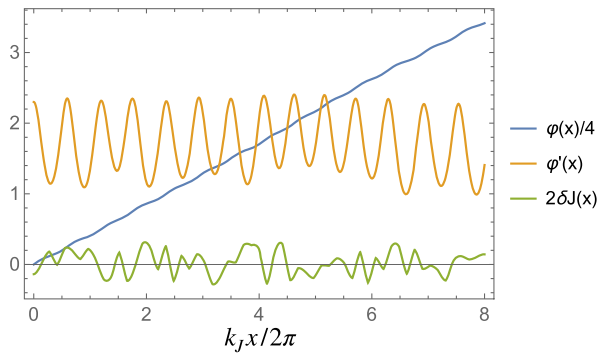


FIG. 10. Spin current in a long wire calculated from the GL theory with a spatially varying random internal Josephson coupling described in the text. We used $M = 42$ random values with $\langle \delta J^2 \rangle^{1/2} = 0.3J_0$ and $\varphi'(0) = 2.3$.

nondecaying oscillatory solutions of the type displayed generally persist for small-amplitude disorder. When the injected current density is close to the critical value j_c^s , we sometimes find that a large fluctuation in δJ can switch the behavior between two types displayed in Figs. 5(a) and 5(c). Nevertheless, the current remains undamped in all cases.

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