

Fast Scrambling at the Boundary

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Many-body systems that saturate the quantum bound on chaos are attracting interest across a wide range of fields. Notable examples include the Sachdev-Ye-Kitaev model and its variations, all characterized by some form of randomness and all-to-all couplings. Here, we study many-body quantum chaos in a quantum impurity model showing non-Fermi-liquid physics, the overscreened multichannel $SU(N)$ Kondo model. We exactly compute the low-temperature behavior of the out-of-time order correlator in the limit of large N and large number of channels, K , at a fixed ratio $\gamma = K/N$. Because of strong correlations at the impurity site, the spin fractionalizes in auxiliary fermions and bosons. We show that all the degrees of freedom of our theory acquire a Lyapunov exponent that is linear in temperature as $T \rightarrow 0$, with a prefactor that depends on γ . Remarkably, for $N = K$, the impurity spin displays maximal chaos, while bosons and fermions only reach half of the maximal Lyapunov exponent. Our results highlight two key features: a nondisordered model that is maximally chaotic due to strong correlations at its boundary, with the maximal chaos appearing in a composite gauge-invariant operator, the impurity spin, and not in the auxiliary single-particle degrees of freedom.

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I. INTRODUCTION

Unitarity of quantum mechanics implies that information initially encoded in local degrees of freedom cannot be lost under time evolution but only scrambled into nonlocal ones, inaccessible to external probes [1–4]. The ability of quantum many-body systems to act as scramblers has emerged in recent years as a new lens to classify phases of matter. A powerful proxy to quantify quantum scrambling is provided by out-of-time ordered correlators (OTOCs) [5–8] originally introduced in the quasiclassical theory of superconductivity and more recently extended to a variety of different settings, from field theories [9–11] to random circuits [12–16] to models with local bounded Hilbert space [17]. Whenever the early-time growth of the

OTOC is exponential in time, one can define a quantum Lyapunov exponent λ_L , which generalizes the butterfly effect to the quantum domain and is often taken as a measure of many-body quantum chaos. Quantum mechanics fundamentally influences this exponent by setting an upper bound on chaos [18–20], set essentially by the temperature of the system, $\lambda_L \leq 2\pi k_B T / \hbar$. Systems that saturate this bound, so called fast scramblers, such as the celebrated Sachdev-Ye-Kitaev (SYK) model [21–25], are therefore particularly interesting for a broad range of communities [26–30].

Examples of fast scramblers and maximal chaos can be found in quantum phases of matter without well-defined quasiparticles. These phases violate Landau’s Fermi liquid paradigm and display unusual transport properties, which have attracted continuous interest over the past few decades [31]. Recently, a fresh new view on these non-Fermi-liquids has emerged, which is rooted in their ability to thermalize in the fastest possible way [32–34] and to efficiently (maximally) scramble quantum information [2,3]. Therefore, understanding the chaotic properties of

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non-Fermi liquids has become an urgent problem, and recent literature has been devoted to the study of OTOCs for different types of models, including variations of the SYK model [35–38] or interacting fermions coupled to critical bosons [39–41]. A common thread in the search for non-Fermi liquids and maximally chaotic quantum matter is the presence of quenched randomness in the form of all-to-all couplings, which are solvable [42]. However, whether randomness is a fundamental requirement is not understood, which raises the following question: What is the simplest model displaying maximal chaos? In this work, we show that, quite surprisingly, randomness is not needed for maximal chaos. Indeed, we will be able to display a model for maximal chaos that only describes a quantum spin strongly coupled to a bath, i.e., a quantum impurity.

Quantum impurity models have long been known to display exotic non-Fermi liquid behavior [43,44] whenever, for example, a single magnetic impurity—possibly with nontrivial orbital character—interacts with multiple channels of conduction electrons [45,46] or in the presence of magnetic interactions between impurities [47–50]. The two-channel Kondo model, in this respect, is among the most studied example of NFL behavior and is also experimentally relevant for mesoscopic physics [51–55]. Here, the breakdown of Fermi liquid theory occurs because two symmetrically coupled conduction-electron channels compete to screen the impurity, which is left partially unscreened, leading to a residual entropy at zero temperature and a NFL ground state [56–60].

A rich NFL physics is known to emerge upon enlarging the symmetry of the Kondo problem to a $SU(N)$ spin and $SU(K)$ channels (flavors of conduction electrons), leading to an overscreening of the impurity. In this regime, a controlled theory can be obtained by sending $N, K \rightarrow \infty$ at fixed ratio $\gamma = K/N$. The universal thermodynamic, spectral, and transport properties of this multichannel Kondo (MCK) model have been computed, and they demonstrated emergent conformal invariance and residual entropy [61,62], features known to occur in the SYK model. However, quantum chaos properties of this class of NFL are not known. Here, we fill this gap, specifically by computing the leading low-temperature behavior of the OTOC for the MCK, and extract the associated Lyapunov exponent. In the MCK model, the impurity spin fractionalizes in auxiliary fermions and bosons, which are coupled to conduction electrons. Using the ladder diagrammatics [63], we compute the OTOC of bulk fermions, auxiliary fermions, and bosons. We show that all the degrees of freedom have Lyapunov exponent λ_L linear in temperature as $T \rightarrow 0$, with a prefactor that is tunable with γ . For $\gamma = 1$, corresponding to a number of channels equal to the size of the spin, the Lyapunov exponent reaches its maximum value, equal to half of the bound value. We develop a diagrammatic technique to compute the OTOC of the physical impurity spin, which is a composite object.

Remarkably, we show that the spin OTOC saturates the bound on chaos. This result is quite striking since the MCK is essentially a model of free, nonchaotic fermions strongly coupled to a boundary quantum spin [64]. It therefore lacks the all-to-all random interactions that are usually associated with fast scramblers. Together with the structure of the large- N, K theory and its similarity with the SYK model, our result hints towards a gravity analog for the MCK.

The paper is organized as follows. In Sec. II, we summarize the main concepts and results of this work, which will be presented in more detail in the rest of the paper. In Sec. III, we introduce the multichannel Kondo model and review its large- N limit solution leading to non-Fermi liquid scaling. In Sec. IV, we discuss the chaos properties of the model and compute, in particular, the leading low-temperature behavior of the OTOC for the fundamental degrees of freedom of the theory. In Sec. V, we extend the diagrammatic calculation of the OTOC to the physical spin and obtain the low-temperature behavior of its Lyapunov exponent. In Sec. VI, we discuss our result in the broader perspective of integrability and chaos of quantum impurity models. Finally, Sec. VII is devoted to conclusions. In the appendixes, we provide technical details on our calculations.

II. SUMMARY OF MAIN RESULTS

In this section, we present a summary of the main results of this work, which will be discussed in more detail in the rest of the paper. The class of models that we consider in this work are quantum impurity models where a single quantum spin is coupled to a bath of noninteracting fermions. We consider an $SU(N)$ spin coupled to a conduction-electron bath featuring K channels of spinful electrons, leading to the MCK. The Hamiltonian of the model is

$$\hat{H} = \sum_{\vec{k}, i, \alpha} \epsilon_{\vec{k}} \hat{c}_{\vec{k}i\alpha}^\dagger \hat{c}_{\vec{k}i\alpha} + \frac{J}{N} \sum_{\vec{k}, \vec{k}', i, \alpha, \beta} \hat{S}_{\alpha\beta} \hat{c}_{\vec{k}i\alpha}^\dagger \hat{c}_{\vec{k}'i\beta}, \quad (1)$$

where $\hat{c}_{\vec{k}i\alpha}^\dagger$ creates a conduction electron with moment $\vec{k}$, channel index $i = 1, \dots, K$, and $SU(N)$ spin index $\alpha = 1, \dots, N$. We use a fermionic (antisymmetric) representation for the impurity spin with N auxiliary fermions $\hat{f}_\alpha$, with a constraint such that

$$\hat{S}_{\alpha\beta} = \hat{f}_\alpha^\dagger \hat{f}_\beta - \frac{Q}{N} \delta_{\alpha\beta}, \quad \sum_{\alpha=1}^N \hat{f}_\alpha^\dagger \hat{f}_\alpha = Q. \quad (2)$$

We consider the limit where both N and K are large, keeping the ratio $\gamma = K/N$ fixed. To ensure that the large- N limit is well defined, we also need to take a large spin size $Q \equiv q_0 N$ [62]. When written in terms of Abrikosov fermions, the Kondo coupling describes the interaction

between impurity fermions and conduction electrons, which can be described by the exchange of a bosonic excitation, leading to a simple diagrammatics in the large- N , K limit, which we review in Sec. III. We are interested in

the many-body chaos as encoded in the OTOC of our composite degrees of freedom. Specifically, for computational purposes, we consider a regularized version of the OTOC, defined as [10,18]

$$C_O(t_1, t_2) \sim \theta(t_1)\theta(t_2) \sum_{\mu, \nu} \text{Tr}[\sqrt{\rho}[\hat{O}_\mu(t_1), \hat{O}_\nu^\dagger(0)]_\pm \sqrt{\rho}[\hat{O}_\mu(t_2), \hat{O}_\nu^\dagger(0)]_\pm^\dagger], \quad (3)$$

where the thermal density matrix $\rho = e^{-\beta H}/Z$ is split into two contributions. In the above equation, $\hat{O}$ indicates an operator of our theory, which can be auxiliary fermions, bosons, or conduction electrons, as well as the physical spin, where μ, ν represent the quantum number label of the corresponding degrees of freedom. Equation (E3) is generic for both fermionic (impurity and conduction electrons) and bosonic degrees of freedom. As such, there are different prefactors in $1/N$ or $1/K$ that enter these definitions to guarantee a well-defined limit; see, for example, Sec. IV. Finally, $[\cdot]_\pm$ denotes a commutator or anticommutator whether we consider bosonic or fermionic operators, and we have included Heaviside functions $\theta(t_{1,2})$, required to express the OTOC as a correlator on the double Keldysh contour, as discussed in Appendix A. We are interested in the regime where the OTOC grows exponentially in time,

$$C_O(t_1, t_2) \sim \left(\frac{1}{N}\right) e^{\lambda_L t},$$

with $t = (t_1 + t_2)/2$ the central time and where we can identify a quantum Lyapunov exponent λ_L that we want to compute in the low-temperature limit.

From the point of view of auxiliary fermions, the large- N , K theory turns out to be structurally similar to the complex SYK model [31], and we show in Sec. IV that the OTOC of the auxiliary fermions can be computed exactly at low temperature by resumming ladder diagrams. We find that the Lyapunov exponent is linear in temperature with a prefactor that depends on $\gamma = K/N$ and that is maximal at $\gamma = 1$. Intriguingly, at this optimal value, the Lyapunov exponent is half of the maximal bound. Similarly, we compute the OTOC of the auxiliary bosons and show that the ladder structure gives rise to the same equations as the fermionic auxiliary degrees of freedom. As a result, we conclude that the Lyapunov exponent of bosons and fermions is identical and reaches its maximum at $\gamma = 1$, where we find $\lambda_L = \pi T$, only a half-value away from the bound. However, auxiliary fermions and bosons are not physical (gauge-invariant) degrees of freedom in our theory. It is therefore interesting to compute the chaos properties of the physical degrees of freedom. In Sec. IV, we compute the OTOC of conduction electrons. We show that the OTOC of the latter can be obtained directly from the OTOC of fermions and bosons, and it also leads to a

linear-in-temperature Lyapunov exponent, with the same prefactor as auxiliary fermions and bosons. This result is remarkable: Conduction electrons in our model are essentially free degrees of freedom in the bulk that interact only with the impurity at their boundary. Such a strong interaction is, however, sufficient to generate a linear-in-temperature quantum Lyapunov exponent, at least close to the impurity site. Finally, in Sec. IV, we compute the OTOC of the spin, which is the composite object in our theory of fermions coupled to bosons. We show that the OTOC of the impurity spin can still be computed in closed form and that, strikingly, its Lyapunov exponent saturates the bound on chaos for $\gamma = K/N = 1$. Strong correlations at the boundary of our otherwise noninteracting system therefore lead to maximal quantum chaos. This result appears to be equally shared by strongly coupled auxiliary degrees of freedom that our quantum spin fractionalizes into, a sort of equipartition of many-body quantum chaos. We present a detailed discussion of this remarkable result in Sec. VI and present our conclusions in Sec. VII.

A. Relation to prior works

Here, we connect our work to previous literature on many-body chaos in non-Fermi liquids and quantum many-body systems. Models of coupled bosons and fermions have been considered before, in the context of Yukawa models for random Dirac Fermions [38,40], antiferromagnetic quantum critical points [65], or fermions coupled to gauge fields [39,41]. We note that these works have typically focused only on OTOCs of single-particle operators. Models of fractionalized degrees of freedom have displayed similar large- N theories [66–68], but their quantum chaos content is not known. In this respect, our work goes beyond those studies in that we explicitly compute the OTOC of a fractionalized state of matter. This approach opens up the possibilities for nonperturbative treatments of those couplings in lattice models. Recently, the information scrambling of the 2CK model has been investigated within the Emery-Kivelson solution [69], leading to an OTOC that reaches a constant at long times. Similarly, the OTOC of a Bose-Fermi Kondo model was evaluated at intermediate temperatures [70]. With respect to these works, here we consider the large- N , K limit, as well as the low-temperature limit, and extract the leading low-temperature behavior of the Lyapunov

exponent. Similarly, our work differs from the recent concept of boundary chaos introduced in Ref. [71], where a quantum circuit with an impurity was studied from the point of view of correlation functions and entanglement.

III. MODEL, LARGE- N , K SOLUTION, AND NON-FERMI LIQUID BEHAVIOR

We start reviewing the equilibrium properties of the MCK model, Eq. (1), in the large- N , K limit [62]. The partition function for the model, written as a path integral, reads

$$Z = \int \mathcal{D}[\bar{f}, f, \bar{c}, c, \bar{B}, B, \mu] e^{-S},$$

where the action $S = S_0 + S_{\text{int}}$ is

$$\begin{aligned} S_0 = & \int_0^\beta d\tau d\tau' \sum_{i\alpha} \bar{c}_{i\alpha}(\tau) (G_c^{(0)})^{-1}(\tau, \tau') c_{i\alpha}(\tau') \\ & + \int_0^\beta d\tau \left\{ \sum_\alpha \bar{f}_\alpha(\tau) [\partial_\tau + i\mu(\tau)] f_\alpha(\tau) - iq_0 N \mu(\tau) \right\} \\ & + \frac{1}{J} \int_0^\beta d\tau \sum_i \bar{B}_i(\tau) B_i(\tau) \end{aligned} \quad (4)$$

$$\begin{aligned} S_{\text{int}} = & -\frac{1}{\sqrt{N}} \int_0^\beta d\tau \sum_{i\alpha} (\bar{B}_i(\tau) \bar{c}_{i\alpha}(\tau) f_\alpha(\tau) \\ & + B_i(\tau) \bar{f}_\alpha(\tau) c_{i\alpha}(\tau)). \end{aligned} \quad (5)$$

In this expression, we have introduced auxiliary fermions $f_\alpha, \bar{f}_\alpha$ to represent the spin, as in Eq. (2), and auxiliary bosons $B_i, \bar{B}_i$ to decouple the Kondo interaction in Eq. (1); we have denoted with $\mu(\tau)$ the Lagrange multiplier used to impose the constraint on the number of fermions. Furthermore, we have integrated out the conduction electrons in the bulk so that $c_{i\alpha} \equiv \sum_{\vec{k}} c_{\vec{k}i\alpha}$ is the fermionic field at the impurity site and $G_c^0 = \sum_{\vec{k}} 1/(i\omega_n - \epsilon_{\vec{k}})$ the noninteracting on-site Green's function. In the large- N limit, the Green's functions

$$G_f(\tau, \tau') = -\frac{1}{N} \sum_{\alpha=1}^N \langle f_\alpha(\tau) \bar{f}_\alpha(\tau') \rangle, \quad (6)$$

$$G_B(\tau, \tau') = +\frac{1}{K} \sum_{i=1}^K \langle B_i(\tau) \bar{B}_i(\tau') \rangle \quad (7)$$

satisfy the Schwinger-Dyson equations [62]

$$G_f^{-1}(i\omega_n) = i\omega_n + \lambda - \Sigma_f(i\omega_n), \quad (8)$$

$$G_B^{-1}(i\nu_n) = \frac{1}{J} - \Sigma_B(i\nu_n), \quad (9)$$

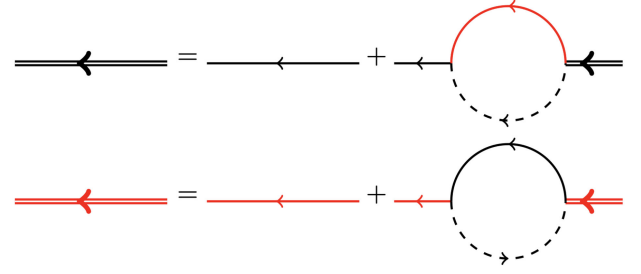


FIG. 1. Feynman diagrams for the Dyson equations (8) and (9) in the large- N limit. Double lines represent dressed propagators, and single lines are noninteracting propagators. Black solid lines correspond to the impurity fermion f , dashed lines to the conduction electron at the impurity site c , and red lines to the auxiliary boson B . In these diagrams, each interaction vertex is of order $1/\sqrt{N}$, and each loop contributes a factor of N . Thus, overall, the bubbles are of order $\mathcal{O}(1)$ in N and survive in the large N limit.

with Σ_f and Σ_B the self-energies,

$$\Sigma_f(\tau) = \gamma G_c^0(\tau) G_B(\tau), \quad \Sigma_B = G_c^0(\tau) G_f(\tau). \quad (10)$$

Here, $\omega_n = (2n+1)\pi/\beta$ and $\nu_n = 2n\pi/\beta$ are, respectively, the fermionic and bosonic frequencies. Equations (8)–(10) can be obtained by the saddle-point method after introducing the self-energies as Lagrange multipliers to enforce the definitions of the Green's functions G_f and G_B and integrating out the fields f, B , and c to obtain an effective action written only in terms of the bilocal fields G and Σ . Here, λ is the saddle-point value of the field $\mu(\tau)$, and it is fixed by the condition $G_f(\tau = 0^-) = q_0$. Another way of deriving the Schwinger-Dyson equations (8)–(10) is to expand the two-point functions (6) and (7) in powers of the interaction vertex (5) and keep only the diagrams of order $\mathcal{O}(1)$ in the $1/N$ expansion. The corresponding Feynman diagrams are shown in Fig. 1. Notice that if one expands the conduction-electron propagator $G_c(\tau, \tau') = -\langle c(\tau) \bar{c}(\tau') \rangle$, one finds that it is equal to the bare propagator G_c^0 in the large- N limit [since, in each bubble correction, the loop is of order $\mathcal{O}(1)$ and cannot cancel the $1/N$ factor coming from the two interaction vertices].

At low energies $\omega, T \ll T_K$, where T_K is the Kondo temperature, the Schwinger-Dyson equations (8)–(10) have emergent conformal and $U(1)$ gauge symmetries, and one can derive an explicit expression for the Green's functions [62,72],

$$G_{f,B}(\tau) = \begin{cases} -A_{f,B} \frac{e^{\alpha(\frac{\tau}{\beta} - \frac{1}{2})}}{\cosh \frac{\alpha}{2}} \left(\frac{\pi}{\beta \sin(\frac{\alpha\tau}{\beta})} \right)^{2\Delta_{f,B}} & 0 < \tau < \beta \\ \xi A_{f,B} \frac{e^{\alpha(\frac{\tau}{\beta} + \frac{1}{2})}}{\cosh \frac{\alpha}{2}} \left(\frac{\pi}{\beta \sin(-\frac{\alpha\tau}{\beta})} \right)^{2\Delta_{f,B}} & -\beta < \tau < 0, \end{cases} \quad (11)$$

$\xi = 1$ (respectively, -1) for fermions (respectively, bosons). We emphasize that these expressions account for the low-energy behavior of the Green's functions, and as such, they do not depend on microscopic details of the bath, such as its band dispersion, as long as it is metallic. The only information on the bath enters in the local noninteracting Green's function of the conduction electrons, which takes the scaling form $G_c^0(\tau) = -\rho_0\pi/(\beta \sin(\pi\tau/\beta))$, where ρ_0 is the density of states at the Fermi level. In Eq. (11), the scaling dimensions of the fermions and bosons $\Delta_{f,B}$ are given by

$$2\Delta_f = \frac{1}{1+\gamma}, \quad 2\Delta_B = \frac{\gamma}{1+\gamma}. \quad (12)$$

Note that α is an asymmetry parameter that is linked to the spin representation parameter q_0 by

$$\alpha = \ln \frac{\sin\left(\frac{\pi q_0}{1+\gamma}\right)}{\sin\left(\frac{\pi(1-q_0)}{1+\gamma}\right)}. \quad (13)$$

Finally, the product of amplitudes $A_f A_B$ satisfies the constraint

$$2\Delta_B = -2\gamma A_f A_B \rho_0 \Gamma(1-2\Delta_B) \Gamma(2\Delta_B) \frac{|\sin(\pi\Delta_B - i\frac{\alpha}{2})|^2}{\cosh^2(\frac{\alpha}{2})}. \quad (14)$$

Details about the origin of these formulas can be found in Ref. [62]. The saddle-point large- N , M solution of the MCK model therefore describes a non-Fermi liquid associated with the overscreening of the impurity spin by the K channels of the bath. In the next section, we will compute the OTOC properties of this model.

IV. OUT-OF-TIME ORDER CORRELATOR AND LYAPUNOV EXPONENT

In this section, we compute the OTOC for the different degrees of freedom entering our theory, namely, auxiliary fermions and bosons, as well as conduction electrons. In particular, we derive Bethe-Salpeter equations for these objects and extract the leading low-temperature behavior of the Lyapunov exponent λ_L , or scrambling rate, describing the short-time exponential growth of the OTOC.

A. The f -fermion and B -boson OTOCs

We start with the regularized retarded OTOC of the auxiliary fermions,

$$C_f(t_1, t_2) = \frac{\theta(t_1)\theta(t_2)}{N^2} \sum_{\alpha,\beta=1}^N \text{Tr}[\sqrt{\rho}\{\hat{f}_\alpha(t_1), \hat{f}_\beta^\dagger(0)\} \times \sqrt{\rho}\{\hat{f}_\alpha(t_2), \hat{f}_\beta^\dagger(0)\}^\dagger]. \quad (15)$$

This correlator can be computed using the twofold Keldysh contour shown in Fig. 2. This complex time contour



FIG. 2. Twofold Keldysh contour used to define the regularized OTOC that we consider in this work. The imaginary-time evolution is broken down into two time intervals, each with an evolution for an interval $-i\beta/2$, and the real-time evolution can occur on the up (u) or down (d) double contour.

consists of two real-time folds separated by an imaginary-time segment of length $-i\beta/2$. The position of the fields on the contour is specified by a fold index $\sigma = u, d$ and a branch index $s = +, -$ for forward and backward time evolution, respectively. To proceed, it is convenient to perform a Keldysh rotation on each fold and define the classical and quantum components

$$f_\alpha^{\sigma,cl/q}(t) = \frac{f_\alpha^{\sigma,+}(t) \pm f_\alpha^{\sigma,-}(t)}{\sqrt{2}}, \quad (16)$$

$$\bar{f}_\alpha^{\sigma,cl/q}(t) = \frac{\bar{f}_\alpha^{\sigma,+}(t) \mp \bar{f}_\alpha^{\sigma,-}(t)}{\sqrt{2}}. \quad (17)$$

A similar transformation is performed for the auxiliary bosons and the conduction electrons. Within the twofold Keldysh formalism, the regularized retarded OTOC can be rewritten as the four-point function (see Appendix A)

$$C_f(t_1, t_2) = \frac{1}{N^2} \sum_{\alpha,\beta=1}^N \langle f_\alpha^{d,cl}(t_1) \bar{f}_\beta^{d,cl}(0) \bar{f}_\alpha^{u,q}(t_2) f_\beta^{u,q}(0) \rangle, \quad (18)$$

where $\langle \dots \rangle$ denotes the average over the twofold Keldysh contour. For convenience, we define $C_f(t_1, t_2) \equiv F_f(t_1, t_2)/N$. We now expand this correlator in powers of the interaction vertex and keep only the diagrams at leading order $1/N$, as we discuss in detail in Appendix C. The $1/N$ diagrams form an infinite series of ladder diagrams, which, after resummation, are represented in Fig. 3. In these Feynman diagrams, horizontal lines represent retarded or advanced Green's functions $G^{R/A}$, and vertical lines are Wightman Green's functions G_{ud}^K or G_{du}^K (see Appendix B). The resummation of all the ladder diagrams shown in Fig. 3 corresponds to the BS equation,

$$F_f(t_1, t_2) = F_f^{(0)}(t_1, t_2) + \frac{\gamma}{4} \int dt_3 dt_4 K_f(t_1, t_2, t_3, t_4) \times F_f(t_3, t_4), \quad (19)$$

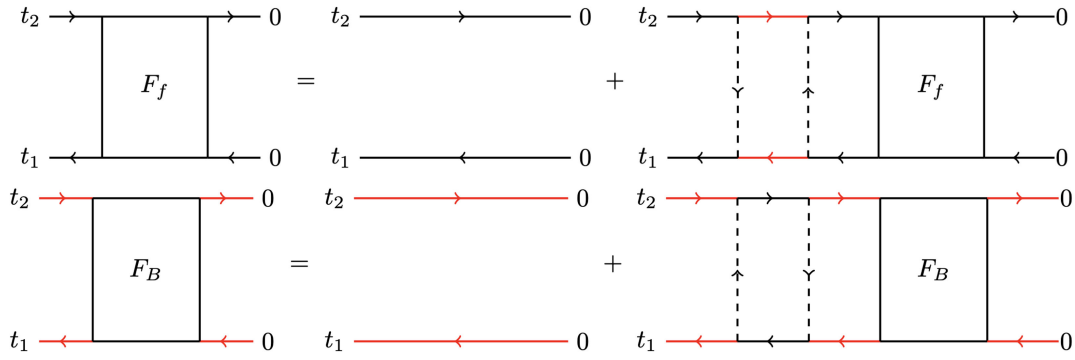


FIG. 3. Top diagrams: resummation of ladder diagrams for the OTOC of the auxiliary fermions, leading to a Bethe-Salpeter equation for $F_f(t_1, t_2)$. Dashed lines are conduction-electron propagators, while solid red lines are bosonic ones. The resulting expression is given in Eq. (19). Bottom diagrams: resummation of ladder diagrams for the OTOC of the auxiliary bosons, leading to a Bethe-Salpeter equation for $F_B(t_1, t_2)$. Dashed lines are conduction-electron propagators, while solid red lines are bosonic ones. The resulting expression is given in Eq. (36).

where $F_f^{(0)}(t_1, t_2) = G_f^R(t_1)G_f^A(-t_2)$ and the kernel K_f is given by

$$K_f(t_1, t_2, t_3, t_4) = \int dt dt' K_f^{(1)}(t_1, t_2, t, t') \times K_f^{(2)}(t, t', t_3, t_4), \quad (20)$$

$$K_f^{(1)}(t_1, t_2, t, t') = G_f^R(t_1, t)G_f^A(t', t_2)G_{c,du}^K(t, t'), \quad (21)$$

$$K_f^{(2)}(t, t', t_3, t_4) = G_B^R(t, t_3)G_B^A(t_4, t')G_{c,ud}^K(t_4, t_3). \quad (22)$$

Explicit expressions for the Green's functions entering the BS equation are given in Appendix B. With respect to the standard ladder diagrammatics of the OTOC for the SYK model [25], we note that, here, the kernel K_f is itself expressed as a convolution over the product of fermionic, bosonic, and conduction-electron Green's function. This result is a direct consequence of the structure of the Feynman vertex of the theory. A similar structure appears in other theories with fermions coupled to bosons [40,41,65,73].

We note that a similar equation was written for the Bose-Fermi Kondo model [70] and was solved numerically to extract the Lyapunov exponent at high temperature. Here, instead, we derive the scrambling rate λ_L in the $T \rightarrow 0$ limit, by making use of the conformal expressions of the Green's functions given in Appendix B. Since we expect F_f to grow exponentially, we neglect the first term $F_f^{(0)}$ in the ladder equation. The Bethe-Salpeter equation becomes

$$F(t_1, t_2) = C e^{i\frac{\pi}{\beta} t_{12}} \int dt_3 dt_4 dt_5 dt_6 \tilde{G}_f^R(t_{13}) \tilde{G}_f^R(t_{24}) \tilde{G}_c^W(t_{34}) \times \tilde{G}_B^R(t_{35}) \tilde{G}_B^R(t_{46}) \tilde{G}_c^W(t_{56}) e^{-i\frac{\pi}{\beta} t_{56}} F(t_5, t_6), \quad (23)$$

where we have used the notation $t_{ij} \equiv t_i - t_j$ and where $\tilde{G}_{f,B}^R(t)$ and $\tilde{G}_c^W(t)$ are defined by

$$\tilde{G}_{f,B}^R(t) = \theta(t) \left(\frac{\pi}{\beta \sinh(\frac{\pi t}{\beta})} \right)^{2\Delta_{f,B}}, \quad \tilde{G}_c^W(t) = \frac{\pi}{\beta \cosh(\frac{\pi t}{\beta})}, \quad (24)$$

with

$$C = 16 \frac{\Delta_f \Delta_B}{\Gamma(1 - 2\Delta_f)^2 \Gamma(1 - 2\Delta_B)^2}. \quad (25)$$

To eliminate the exponential phase factors, we define $\tilde{F}(t_1, t_2) = e^{-i\frac{\pi}{\beta} t_{12}} F(t_1, t_2)$. The BS equation takes the form of an eigenvalue problem $\tilde{K} \cdot \tilde{F} = k \tilde{F}$, with $k = 1$. To find the Lyapunov exponent, we assume that $\tilde{F}$ has an exponential growth

$$\tilde{F}(t_1, t_2) = e^{\lambda_L \frac{t_1 + t_2}{2}} \tilde{f}(t_1 - t_2), \quad (26)$$

and we determine the eigenvalues and eigenvectors of the kernel $\tilde{K}$ in this subspace. Note that λ_L should be seen as a free parameter, which is fixed at the end of the computation by setting $k(\lambda_L) = 1$. We note that the uniqueness of the solution of the Bethe-Salpeter equation, which is a singular integral equation, is a highly nontrivial mathematical problem, which we disregard here and focus our attention on finding one solution of the form given in Eq. (26).

Plugging the exponential ansatz into the BS equation and taking the Fourier transform with respect to $t_1 - t_2$, we obtain

$$k(\lambda_L) \tilde{f}(\omega) = C \tilde{G}_f^R \left(\omega + i \frac{\lambda_L}{2} \right) \tilde{G}_f^R \left(-\omega + i \frac{\lambda_L}{2} \right) \times \int \frac{d\omega'}{2\pi} \tilde{K}(\omega, \omega') \tilde{f}(\omega'), \quad (27)$$

with

$$\tilde{K}(\omega, \omega') = \int \frac{d\Omega}{2\pi} \tilde{G}_B^R\left(\Omega + i\frac{\lambda_L}{2}\right) \tilde{G}_B^R\left(-\Omega + i\frac{\lambda_L}{2}\right) \times \tilde{G}_c^W(\omega - \Omega) \tilde{G}_c^W(\Omega - \omega'). \quad (28)$$

Using the Fourier transforms of Eq. (24), we can eventually rewrite the BS equation as

$$k(h)\tilde{f}(u) = 16 \frac{\Delta_f \Delta_B}{(2\pi)^2} \left| \frac{\Gamma(\Delta_f + \frac{h}{2} - iu)}{\Gamma(1 - \Delta_f + \frac{h}{2} - iu)} \right|^2 \times \int du' dv \left[\left| \frac{\Gamma(\Delta_B + \frac{h}{2} - iv)}{\Gamma(1 - \Delta_B + \frac{h}{2} - iv)} \right|^2 \times \left| \Gamma\left(\frac{1}{2} + i(u - v)\right) \right|^2 \times \left| \Gamma\left(\frac{1}{2} + i(u' - v)\right) \right|^2 \tilde{f}(u') \right]. \quad (29)$$

Here, we have defined $h = \lambda_L/(2\pi T)$ and made the change of variable $u = \omega/(2\pi T)$. Using the identity on gamma functions,

$$\int_{-\infty}^{+\infty} du' \left| \Gamma\left(\frac{1}{2} + i(u - u')\right) \right|^2 \left| \Gamma\left(\frac{a}{2} + iu'\right) \right|^2 = \frac{2\pi}{a} \left| \Gamma\left(\frac{1}{2} + \frac{a}{2} + iu\right) \right|^2, \quad (30)$$

we see that the function $\tilde{f}(u)$,

$$\tilde{f}(u) = \left| \Gamma\left(\Delta_f + \frac{h}{2} + iu\right) \right|^2, \quad (31)$$

is the solution of the eigenproblem with eigenvalue

$$k(h) = 16 \frac{\Delta_f \Delta_B}{(2\Delta_f + h)(2\Delta_B + h)}. \quad (32)$$

Setting $k(h) = 1$ and keeping only the positive solution, we finally obtain

$$\lambda_L = \varphi(\gamma)2\pi T, \quad \varphi(\gamma) = \frac{-1 + \sqrt{1 + 12 \frac{\gamma}{(1+\gamma)^2}}}{2}. \quad (33)$$

We find that the scrambling rate is linear in temperature, with a prefactor that depends on the ratio γ . The function $\varphi(\gamma)$ is plotted in Fig. 4. It vanishes for $\gamma = 0$ and $\gamma = \infty$, which corresponds to the scaling dimensions of the fields $(\Delta_f, \Delta_B) = (\frac{1}{2}, 0)$ and $(\Delta_f, \Delta_B) = (0, \frac{1}{2})$, respectively. It is maximal when $\gamma = 1$ for which $\Delta_f = \Delta_B = \frac{1}{4}$, reaching the value $\varphi(1) = \frac{1}{2}$. Notice that at the maximum $\gamma = 1$, both fields f and B have the same scaling dimension as fermions described by the SYK Hamiltonian. However, differently from the SYK case, here the Lyapunov exponent does not saturate the chaos bound but only reaches a value that is

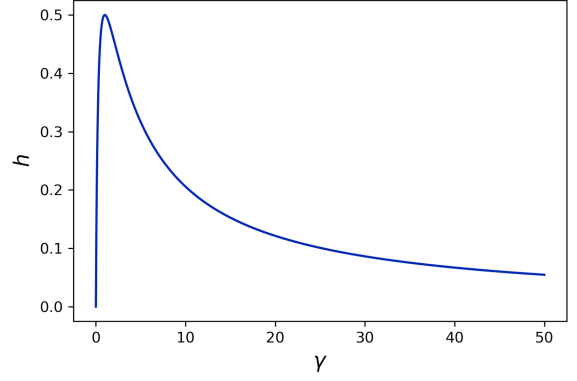


FIG. 4. Normalized Lyapunov exponent $h = \lambda_L/(2\pi T)$ of auxiliary fermions and bosons in the MCK model, as a function of $\gamma = K/N$, the ratio between the number of channels and spin indices. We see that the Lyapunov exponent vanishes for both small and large γ and has a maximum for $\gamma = 1$. The maximal value obtained is $h = 0.5$, corresponding to $\lambda_L = \pi T$.

exactly half of the maximal chaos, $\lambda_L = \lambda_L^{\max}/2 = \pi T$. We now turn to the computation of the OTOC of the auxiliary bosons B . As done previously, we define the regularized retarded OTOC,

$$C_B(t_1, t_2) = \frac{\theta(t_1)\theta(t_2)}{K^2} \sum_{i,j=1}^K \text{Tr}[\sqrt{\rho}[\hat{B}_i(t_1), \hat{B}_j^\dagger(0)] \times \sqrt{\rho}[\hat{B}_i(t_2), \hat{B}_j^\dagger(0)]^\dagger]. \quad (34)$$

The OTOC can be rewritten as a four-point function on the twofold Keldysh contour introduced above,

$$C_B(t_1, t_2) = \frac{-1}{K^2} \sum_{i,j=1}^K \langle B_i^{d,cl}(t_1) \bar{B}_j^{d,q}(0) \bar{B}_i^{u,cl}(t_2) B_j^{u,q}(0) \rangle. \quad (35)$$

Performing a similar diagrammatic expansion as for the auxiliary f fermions, we find that the $1/N$ part F_B of C_B satisfies the following ladder equation in the limit $N \rightarrow \infty$,

$$F_B(t_1, t_2) = F_B^{(0)}(t_1, t_2) + \frac{\gamma}{4} \int dt_3 dt_4 K_B(t_1, t_2, t_3, t_4) \times F_B(t_3, t_4), \quad (36)$$

where

$$K_B(t_1, t_2, t_3, t_4) = \int dt dt' K_B^{(1)}(t_1, t_2, t, t') \quad (37)$$

$$\times K_B^{(2)}(t, t', t_3, t_4), \quad (38)$$

$$K_B^{(1)}(t_1, t_2, t, t') = G_B^R(t_1, t) G_B^A(t', t_2) G_{c,ud}^K(t', t), \quad (39)$$

$$K_B^{(2)}(t, t', t_3, t_4) = G_f^R(t, t_3) G_f^A(t_4, t') G_{c,du}^K(t_3, t_4). \quad (40)$$

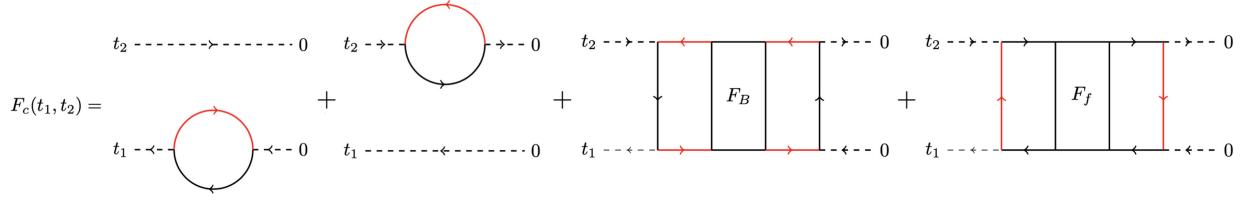


FIG. 5. Resummation of ladder diagrams for the OTOC of the conduction electrons, $F_c(t_1, t_2)$. Dashed lines are conduction-electron propagators, while solid red lines are bosonic ones. The resulting expression is given in Eqs. (44) and (45).

The corresponding Feynman diagrams are shown in Fig. 3. We note that the kernel K_B entering the bosonic OTOC has a similar structure with respect to the fermionic one, namely, two loops made with a fermionic (bosonic) Green's function and a vertical (Wightman) conduction electrons one. In the low-temperature limit, using the conformal expressions for the Green's function, we conclude that the two kernels K_f and K_B are equal upon changing the scaling dimensions of auxiliary fermions and bosons, i.e., upon sending $\Delta_f \leftrightarrow \Delta_B$. Then, the computation of the Lyapunov exponent proceeds in exactly the same way as above. The eigenvector (31) has the same form, with Δ_f replaced by Δ_B . Remarkably, the eigenvalue $k(h)$ is symmetric under the exchange of Δ_f and Δ_B . As a result, the scrambling rate for the auxiliary bosons B is the same as for impurity fermions f ; again, it is given by Eq. (33) and plotted in Fig. 4. To summarize, in our MCK, the physical spins fractionalize in strongly coupled auxiliary fermions and bosons. Their scrambling properties at low temperature turn out to be identical, a Lyapunov exponent λ_L linear in temperature with a prefactor that depends on γ . Intriguingly, for $\gamma = K/N = 1$, both modes acquire their maximal scrambling rate, which is, however, only half of the maximal value allowed by the bound on chaos. This result is therefore different from other models of fermions coupled to bosons in which the two sectors acquire a Lyapunov exponent parametrically smaller one from the other, as well as from the maximum allowed value [65].

At this stage, we should remark that these auxiliary degrees of freedom are not physical, gauge-invariant objects but only serve to represent the quantum spin. In order to fully elucidate the many-body chaos content of our MCK model, it is therefore crucial to evaluate the OTOC of physical degrees of freedom, such as conduction electrons and the impurity spin, as we discuss in the next sections.

B. Conduction electrons

We now turn to the computation of the OTOC of the conduction electrons at the impurity site,

$$C_c(t_1, t_2) = \frac{\theta(t_1)\theta(t_2)}{N^2 K^2} \sum_{i,j=1}^K \sum_{\alpha,\beta=1}^N \text{Tr}[\sqrt{\rho}\{\hat{c}_{i\alpha}(t_1), \hat{c}_{j\beta}^\dagger(0)\} \times \sqrt{\rho}\{\hat{c}_{i\alpha}(t_2), \hat{c}_{j\beta}^\dagger(0)\}^\dagger], \quad (41)$$

which can be represented as the four-point function on the twofold Keldysh contour as

$$C_c(t_1, t_2) = \frac{1}{N^2 K^2} \sum_{i,j=1}^K \sum_{\alpha,\beta=1}^N \langle c_{i\alpha}^{d,cl}(t_1) \bar{c}_{j\beta}^{d,cl}(0) \times \bar{c}_{i\alpha}^{u,q}(t_2) c_{j\beta}^{u,q}(0) \rangle. \quad (42)$$

We expand this correlator in powers of the interaction vertex including $1/N$ corrections,

$$C_c(t_1, t_2) = \frac{1}{NK} G_c^{R,0}(t_1) G_c^{A,0}(-t_2) + \frac{1}{N^2 K} F_c(t_1, t_2) + \dots \quad (43)$$

In the large- N limit, $F_c(t_1, t_2)$ satisfies the ladder equation shown in Fig. 5,

$$F_c(t_1, t_2) = F_c^{(0)}(t_1, t_2) + F_c^{(B)}(t_1, t_2) + F_c^{(f)}(t_1, t_2). \quad (44)$$

The first two diagrams, which correspond to $F_c^{(0)}$, are the $1/N$ bubble corrections to the bare electronic propagator. The exponential growth will come from the last two diagrams $F_c^{(B)}$ and $F_c^{(f)}$, and we will focus on these two. Notice that this ladder equation is not a self-consistent equation like the previous BS equations for fermions and bosons; i.e., F_c only appears on the left-hand side at the leading order. This is ultimately due to the structure of the diagrammatics in our MCK model, with a vertex coupling fermions, bosons, and conduction electrons. As such, we can evaluate the OTOC of conduction electrons directly since we already know F_f and F_B from the previous section.

We detail the derivation for the diagram involving F_f ; it is then straightforward to adapt the computation to the other one. The fermionic contribution $F_c^{(f)}(t_1, t_2)$ can be read off from Fig. 5, and it is given by

$$F_c^{(f)}(t_1, t_2) = \int \prod_{i=3}^{i=6} dt_i G_c^R(t_1, t_3) G_c^A(t_4, t_2) G_{B,ud}^K(t_4, t_3) \times F_f(t_3, t_4, t_5, t_6) G_c^R(t_5) G_c^A(-t_6) G_{B,du}^K(t_5, t_6). \quad (45)$$

In this expression, $F_f(t_1, t_2, t_3, t_4)$ is a generalization of $F_f(t_1, t_2)$, where we replace the times 0 by t_3 and t_4 on the right end of the ladders in Eq. (4). To compute this quantity, we replace the exponential ansatz (26) by $\tilde{F}(t_1, t_2, t_3, t_4) = e^{\lambda_L \frac{t_1+t_2-t_3-t_4}{2}} f_1(t_1-t_2) f_2(t_3-t_4)$. After Fourier transforming with respect to t_1-t_2 and t_3-t_4 , f_2 drops out of the ladder equation and f_1 satisfies Eq. (27) and so is equal to Eq. (31). Taking the inverse Fourier transform, we obtain

$$F_f(t_1, t_2, t_3, t_4) = e^{i\frac{\omega}{\beta}(t_1-t_2)} e^{\lambda_L \frac{t_1+t_2-t_3-t_4}{2}} \frac{\Gamma(\Delta_f + \frac{h}{2})}{(2\pi T)^{2\Delta_f+h-1}} \times \left(\frac{\pi}{\beta \cosh(\frac{\pi t_{12}}{\beta})} \right)^{2\Delta_f+h} f_2(t_3-t_4). \quad (46)$$

Plugging this expression into Eq. (45), the integral over (t_5, t_6) factorizes and gives a constant. If we replace $G_{B,ud}^K$ by its conformal expression given in Appendix B, we finally obtain

$$F_c^{(f)}(t_1, t_2) \sim e^{\lambda_L \frac{t_1+t_2}{2}} H(t_1-t_2), \quad (47)$$

with

$$H(t) = \int \frac{d\omega}{2\pi} e^{-i\omega(t_1-t_2)} G_c^{R,0} \left(\omega + i\frac{\lambda_L}{2} \right) \times G_c^{A,0} \left(-\omega + i\frac{\lambda_L}{2} \right) f_c(\omega), \quad (48)$$

with $f_c(\omega) \equiv \{\pi/[\beta \cosh(\frac{\pi t}{\beta})]\}^{1+h}$. In Eq. (47), we omit an unimportant multiplicative prefactor. One would find a similar expression for $F_c^{(B)}$, so we can conclude that F_c grows exponentially, $F_c(t_1, t_2) \sim e^{\lambda_L \frac{t_1+t_2}{2}}$, with the same Lyapunov exponent λ_L as the impurity fermions and the auxiliary bosons. This result is particularly remarkable since the conduction electrons are essentially noninteracting degrees of freedom in their bulk and hence nonchaotic (their Lyapunov exponent far away from the impurity should vanish). However, because of the strong coupling at the boundary with the impurity spin, they develop many-body chaos with a linear-in-temperature Lyapunov exponent. This finding suggests that the OTOC of the conduction electrons should develop a nontrivial spatial dependence, analogous to the Kondo screening cloud. We leave the investigation of this interesting feature to future work. Next, we focus on the OTOC and scrambling properties of the impurity spin.

V. IMPURITY-SPIN OTOC

In the previous sections, we have computed the OTOC of auxiliary fermions, bosons, and conduction electrons.

These types of correlators can be written as two-particle Green's functions in the doubled Keldysh contour, and they can be evaluated, to a large extent, following similar calculations for the SYK model. In this section, we tackle a more challenging OTOC of the impurity spin, which, as discussed, is a composite object in our theory of the MCK model. As we are going to show, the result is a different diagrammatic structure; summing simple ladder diagrams will not be enough. Remarkably, we are able to extract the leading low-temperature behavior of the Lyapunov exponent and show that it saturates the bound on chaos.

Let us consider the regularized retarded OTOC for the impurity spin, defined as

$$C_S(t_1, t_2) = \frac{\theta(t_1)\theta(t_2)}{N^4} \sum_{\alpha, \beta, \gamma, \delta=1}^N \text{Tr}[\sqrt{\rho}[\hat{S}_{\alpha\beta}(t_1), \hat{S}_{\delta\gamma}(0)] \times \sqrt{\rho}[\hat{S}_{\alpha\beta}(t_2), \hat{S}_{\delta\gamma}(0)]^\dagger]. \quad (49)$$

First, we need to rewrite this correlator as an average over the twofold Keldysh action. We perform the Keldysh rotation for the spin degrees of freedom and define

$$S_{\alpha\beta}^{\sigma,cl} = \frac{S_{\alpha\beta}^{\sigma,+} + S_{\alpha\beta}^{\sigma,-}}{\sqrt{2}}, \quad S_{\alpha\beta}^{\sigma,q} = \frac{S_{\alpha\beta}^{\sigma,+} - S_{\alpha\beta}^{\sigma,-}}{\sqrt{2}}, \quad (50)$$

with $S_{\alpha\beta}^{\sigma,\pm} = \bar{f}_\alpha^{\sigma,\pm} f_\beta^{\sigma,\pm}$. Then, we can rewrite C_S as

$$C_S(t_1, t_2) = -\frac{1}{N^4} \sum_{\alpha, \beta, \gamma, \delta} \langle S_{\alpha\beta}^{d,cl}(t_1) S_{\delta\gamma}^{d,q}(0) S_{\beta\alpha}^{u,cl}(t_2) S_{\gamma\delta}^{u,q}(0) \rangle. \quad (51)$$

It will be convenient to express $S^{\sigma,cl}$ and $S^{\sigma,q}$ in terms of $f^{\sigma,cl}$ and $f^{\sigma,q}$:

$$S_{\alpha\beta}^{\sigma,cl} = \bar{\Psi}_\alpha^\sigma \gamma^q \Psi_\beta^\sigma, \quad S_{\alpha\beta}^{\sigma,q} = \bar{\Psi}_\alpha^\sigma \gamma^{cl} \Psi_\beta^\sigma, \quad (52)$$

where

$$\Psi^\sigma = \begin{pmatrix} f^{\sigma,cl} \\ f^{\sigma,q} \end{pmatrix}, \quad \bar{\Psi}^\sigma = \begin{pmatrix} \bar{f}^{\sigma,cl} & \bar{f}^{\sigma,q} \end{pmatrix}, \quad (53)$$

and where

$$\gamma^{cl} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \gamma^q = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (54)$$

The impurity-spin OTOC now takes the form

$$\begin{aligned}
C_S(t_1, t_2) &= -\frac{1}{N^4} \sum_{\alpha, \beta, \gamma, \delta=1}^N \langle \bar{\Psi}_\alpha^d(t_1) \gamma^q \Psi_\beta^d(t_1) \bar{\Psi}_\delta^d(0) \gamma^{cl} \Psi_\gamma^d(0) \bar{\Psi}_\beta^u(t_2) \gamma^q \Psi_\alpha^u(t_2) \bar{\Psi}_\gamma^u(0) \gamma^{cl} \Psi_\delta^u(0) \rangle \\
&= -\sum_{\substack{\mu_1 \dots \mu_4 \\ \nu_1 \dots \nu_4}} \gamma_{\mu_1 \mu_2}^q \gamma_{\nu_2 \nu_1}^{cl} \gamma_{\mu_4 \mu_3}^q \gamma_{\nu_3 \nu_4}^{cl} C_{\mu_1 \nu_1; \mu_2 \nu_2}^{\mu_3 \nu_3; \mu_4 \nu_4}(t_1, t_2),
\end{aligned} \tag{55}$$

with

$$C_{\mu_1 \nu_1; \mu_2 \nu_2}^{\mu_3 \nu_3; \mu_4 \nu_4}(t_1, t_2) = \frac{1}{N^4} \sum_{\alpha, \beta, \gamma, \delta=1}^N \langle \bar{f}_\alpha^{d, \mu_1}(t_1) f_\beta^{d, \mu_2}(t_1) \bar{f}_\delta^{d, \nu_2}(0) f_\gamma^{d, \nu_1}(0) \bar{f}_\beta^{u, \mu_4}(t_2) f_\alpha^{u, \mu_3}(t_2) \bar{f}_\gamma^{u, \nu_3}(0) f_\delta^{u, \nu_4}(0) \rangle. \tag{56}$$

We see that the impurity-spin OTOC, when written in terms of the auxiliary fermion, takes the form of a correlator involving eight fermionic fields with a specific structure of Keldysh index. We now proceed diagrammatically, as previously done for the OTOC of fermions and bosons. Specifically, we expand the correlator $C_{\mu_1 \nu_1; \mu_2 \nu_2}^{\mu_3 \nu_3; \mu_4 \nu_4}(t_1, t_2)$ in powers of the interaction vertex and keep only the leading diagrams in the large- N and K limit. As we discuss in detail in Appendix E, in this limit, the diagrammatic structure greatly simplifies. In particular, we can show that the only diagrams surviving are again ladder noncrossing diagrams between different parts of the spin OTOC, as we show in Fig. 6 for the lowest orders. Remarkably, these diagrams can be factorized into the product of auxiliary fermion OTOCs,

$$C_{\mu_1 \nu_1; \mu_2 \nu_2}^{\mu_3 \nu_3; \mu_4 \nu_4}(t_1, t_2) = \bar{F}_{\mu_1 \nu_1}^{\mu_3 \nu_3}(t_1, t_2) F_{\mu_2 \nu_2}^{\mu_4 \nu_4}(t_1, t_2), \tag{57}$$

where

$$F_{\mu_2 \nu_2}^{\mu_4 \nu_4}(t_1, t_2) = \frac{1}{N^2} \sum_{\beta, \delta} \langle f_\beta^{\mu_2, d}(t_1) \bar{f}_\delta^{\nu_2, d}(0) \bar{f}_\beta^{\mu_4, u}(t_2) f_\delta^{\nu_4, u}(0) \rangle, \tag{58}$$

$$\bar{F}_{\mu_1 \nu_1}^{\mu_3 \nu_3}(t_1, t_2) = \frac{1}{N^2} \sum_{\alpha, \gamma} \langle \bar{f}_\alpha^{\mu_1, d}(t_1) f_\gamma^{\nu_1, d}(0) f_\alpha^{\mu_3, u}(t_2) \bar{f}_\gamma^{\nu_3, u}(0) \rangle. \tag{59}$$

We emphasize that the result in Eq. (57) is highly nontrivial since it involves a factorization into a product of two-particle Green's functions (the fermionic OTOC), which themselves involve nontrivial vertex corrections. Furthermore, it implies the vanishing of higher-order vertex corrections for the spin OTOC. Using the expressions of the matrices γ^{cl} and γ^q , the fact that the correlator $\langle f^q \bar{f}^{cl} \rangle$ is zero, and symmetries between the different OTOCs, we can simplify the sum into

$$\begin{aligned}
C_S(t_1, t_2) &= -[F_{++}(t_1, t_2) F_{--}(t_1, t_2)^* \\
&\quad - F_{+-}(t_1, t_2) F_{-+}(t_1, t_2)^* + \text{c.c.}],
\end{aligned} \tag{60}$$

with

$$\begin{aligned}
F_{ss'}(t_1, t_2) &= \frac{1}{N^2} \sum_{\alpha, \beta=1}^N \text{Tr}[\sqrt{\rho} [\hat{f}_\alpha(t_1), \hat{f}_\beta^\dagger(0)]_s \\
&\quad \times \sqrt{\rho} [\hat{f}_\alpha(t_2), \hat{f}_\beta^\dagger(0)]_s^\dagger],
\end{aligned} \tag{61}$$

with $s, s' = \pm$. The four possible combinations of the product of the commutator and anticommutator $F_{ss'}$ appear in the definition of the OTOC. We have already computed the first one, F_{++} , and one could be tempted to write down a ladder equation for the other three and try to find the scrambling rate by the same technique that we have used above. However, the Keldysh structure of the resulting Bethe-Salpeter equations makes it hard to find a closed analytic solution. Since we are only interested in computing the scrambling rate of the spin, i.e., the combination of correlators in Eq. (60), we will take a different approach. First, we single out the OTOC part of $C_S(t_1, t_2)$ for equal-time arguments $t_1 = t_2 = t$, which is responsible for the exponential chaotic growth. As we show in Appendix E, starting from Eq. (60), we can write

$$C_S(t, t) = 8\text{Re}[(F_1^{\text{otoc}}(t, t))^2] + \tag{62}$$

$$-2[(F_+^{\text{toc}}(t, t))^2 - (F_-^{\text{toc}}(t, t))^2], \tag{63}$$

where the second line only includes the time-ordered contributions (TOCs) to $C_S(t)$, $F_\pm^{\text{toc}}(t, t)$, which do not grow at long times. In the first line, we have introduced the correlator

$$F_1^{\text{otoc}}(t, t) = \frac{1}{N^2} \sum_{\alpha\beta} \text{Tr}[\sqrt{\rho} \hat{f}_\alpha(t_1) \hat{f}_\beta^\dagger(0) \sqrt{\rho} \hat{f}_\alpha^\dagger(t_2) \hat{f}_\beta(0)], \tag{64}$$

which is explicitly out-of-time ordered. To proceed, we notice that the correlator in Eq. (64) can be obtained via analytic continuation, i.e.,

$$F_1^{\text{otoc}}(t, t) = \mathcal{F}_{1/4}\left(t + i\frac{\beta}{4}, t + i\frac{\beta}{4}\right), \tag{65}$$

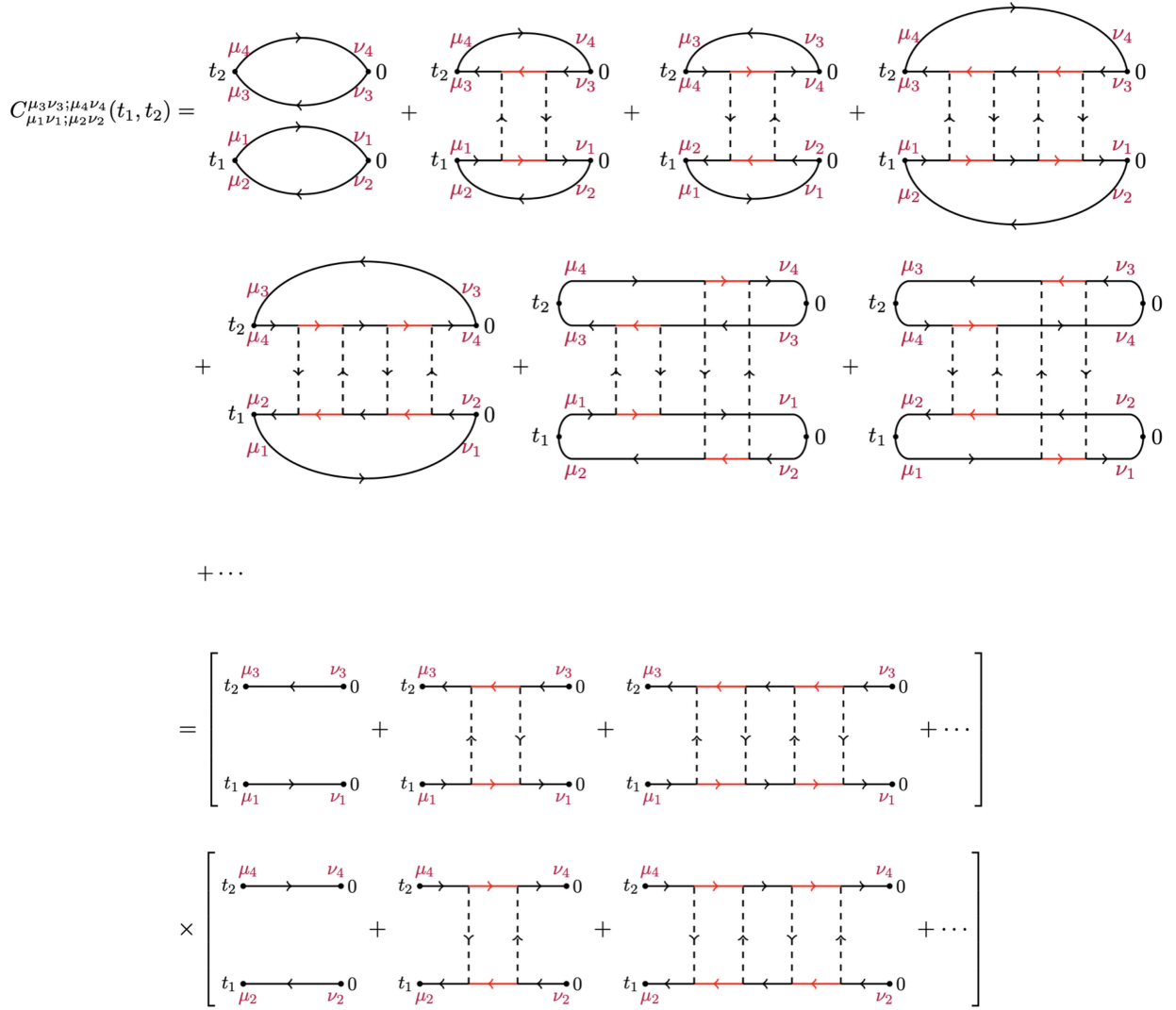


FIG. 6. Feynman diagrams for the impurity-spin OTOC and resummation of ladder diagrams for the OTOC of the impurity-spin fermions. Dashed lines are conduction-electron propagators, while red solid lines are bosonic ones. The resulting expression is given in Eq. (57).

with

$$\mathcal{F}_{1/4}(t_1, t_2) \equiv \frac{1}{N^2} \sum_{\alpha\beta} \text{Tr}[\rho^{\frac{1}{4}} \hat{f}_\alpha(t_1) \rho^{\frac{1}{4}} \hat{f}_\beta^\dagger(0) \rho^{\frac{1}{4}} \hat{f}_\alpha^\dagger(t_2) \rho^{\frac{1}{4}} \hat{f}_\beta(0)]. \quad (66)$$

This function corresponds to a different OTOC regularization [19,74], where the fermionic operators are split by a quarter of the thermal circle, i.e., $\rho^{1/4}$. This regularization was originally introduced for the SYK model in Refs. [18,26] since it provides advantages in the evaluation of the OTOC in the chaos regime. For our purposes, it is sufficient to note that $\mathcal{F}_{1/4}(t_1, t_2)$ can be obtained by analytical continuation of the imaginary four-point function (see Appendix D), and it is therefore also obtained by resumming ladder diagrams [26,30,36,75] leading to an integral equation of the form

$$\mathcal{F}_{1/4}(t_1, t_2) = \mathcal{F}_{1/4}^0(t_1, t_2) + \int dt_3 dt_4 \mathcal{K}(t_1, t_2, t_3, t_4) \times \mathcal{F}_{1/4}(t_3, t_4), \quad (67)$$

with $\mathcal{K}(t_1, t_2, t_3, t_4)$ the same kernel defined in Sec. IV for the OTOC $F_f(t_1, t_2)$. We can thus borrow the results of Sec. IV and write the exponentially growing solution for $\mathcal{F}_{1/4}(t_1, t_2)$ as

$$\mathcal{F}_{1/4}(t_1, t_2) = e^{i\frac{\beta}{4}(t_1 - t_2)} f(t_1 - t_2) e^{\lambda_L \frac{t_1 + t_2}{2}}, \quad (68)$$

with f a *real* function given in Eq. (31). Then, we conclude that $\mathcal{F}_{1/4}$ grows exponentially with a Lyapunov exponent λ_L equal to the one of the retarded OTOC correlator $F_f(t_1, t_2)$,

$$F_1^{\text{otoc}}(t, t) = \mathcal{F}_{1/4}\left(t + i\frac{\beta}{4}, t + i\frac{\beta}{4}\right) = f(0) e^{i\frac{\beta\lambda_L}{4}} e^{\lambda_L t}. \quad (69)$$

We emphasize that changing the regularization of the OTOC correlator, as we have done in Eq. (66), is not expected to affect the Lyapunov exponent obtained from the Bethe-Salpether equation but only the prefactor of the OTOC correlator, which is nonuniversal and depends explicitly on the regularization [74]. As such, one should not attach physical meaning to this prefactor and should only interpret this result regarding the estimate of the Lyapunov exponent. Nevertheless, we mention that there are examples of fully unregularized OTOC correlators for which the Lyapunov exponent exceeds the chaos bound [76], indicating that the issue of regularization for OTOCs is subtle and not yet fully understood. Using this result and the expression for the spin OTOC in Eq. (62), we obtain

$$C_S(t) \sim f(0) \cos\left(\frac{\beta\lambda_L}{2}\right) e^{2\lambda_L t}. \quad (70)$$

Therefore, we conclude that the impurity-spin OTOC has a Lyapunov exponent given by *twice* the value of the fermionic and bosonic one. This factor of 2 is crucial since it allows the impurity spin—a physical and gauge-invariant degree of freedom of our theory—to become maximally chaotic and to saturate the bound on chaos for $\gamma = 1$, i.e., when the size of the impurity spin is equal to the number of channels. Right at this point bosons and fermions, the composite degrees of freedom of the impurity spin, are also chaotic, but they only acquire half of the maximal Lyapunov exponent, a sort of equipartition of maximal chaos. It is worth highlighting how this result arises from the special structure of the MCK model and the double limit $N, K \rightarrow \infty$ at a fixed ratio, which indeed implies that the spin OTOC exactly factorizes in the product of the auxiliary fermion OTOC, Eq. (57), as we discuss in Appendix E. This result is not at all trivial, and it might not hold in other contexts, such as the Sachdev-Ye model or even the SYK model when considering different composite operators. Indeed, in these models, the fermionic OTOC already saturates the bound on chaos, which suggests vertex corrections intervene to suppress the OTOC of composite operators. Understanding this structure is an exciting perspective for future work.

VI. DISCUSSION

The results obtained in the previous section show that the MCK model in the infinite N, K limit at fixed K/N is maximally chaotic and saturates the bound on chaos. This result is striking from several standpoints, which we briefly highlight here. First, maximal chaos occurs in a model that, quite differently from the SYK and its variations, does not present any random all-to-all coupling but only boundary interactions between a large quantum spin, strongly coupled to a large number of free fermionic channels. As we will argue, this feature has direct consequences on the physical origin of the fast scrambling dynamics of the

MCK model that sets it apart from other well-known examples.

A second reason for which this result is remarkable can be seen from the point of view of the properties of the MCK model. In fact, these models are integrable [77,78] at any finite N, K . For example, for $N = K = 2$, the model reduces to the two-channel Kondo model (2CK), which is known to be integrable at low energy. In particular, it is known from CFT that the spectrum is made of a finite number of towers of states, where, within each tower, the levels are equally spaced by $\pi v_F/L$. Hence, although the system is a non-Fermi liquid, there is a quasiparticle description that is revealed by CFT [46,49,58]. Furthermore, in this case at zero temperature, there is a residual entropy coming from a decoupled Majorana mode [56]. Recent work investigating the OTOC in the 2CK model [69] did not find any trace of exponential growth or Lyapunov chaos, a result which is compatible with the fact that models with finite local Hilbert space (i.e., spin chains) do not have an exponential regime for the growth of OTOC.

To reconcile the known results at finite N, K with our results in the limit $N, K \rightarrow \infty$, it is crucial to understand how the finite-size spectrum, residual entropy, and OTOC evolve in the MCK as the large- N, K limit is approached. The case of the SYK model, in this respect, is well understood: The finite-size spectrum contains exponentially many states above the ground state, with exponentially small level spacing, leading to an extensive residual zero-temperature entropy [31] if the large volume limit is taken first with respect to the zero-temperature one. However, the structure of the spectrum for the finite N, K MCK model is very different from the case of SYK. In the 2CK model, the residual entropy at $T = 0$ arises from states with high degeneracy, whose level spacing is still regular and set by the inverse system size. In the thermodynamic limit, as the level spacing becomes small, the states with high degeneracy yield a residual entropy when the temperature is sent to zero after taking the thermodynamic limit [46]. A similar mechanism is at play for any finite K, N , with levels described by $SU_K(N)$. We can now qualitatively understand how the structure of the spectrum of the MCK model evolves as N, K are increased: While the CFT tower with the $1/L$ spacing is always there, the key is the behavior of the spectrum at large quantum numbers and whether exponentially many levels contribute. This interpretation is indeed confirmed by looking at the residual entropy of the MCK for generic N, K , computed in Ref. [62], which gives a result of order N in the large- N limit, where N is the size of the spin. We conjecture that the fast scrambling we find in our calculation is associated with this manifold of states with high degeneracies.

What are the consequences for the OTOC at finite but large N, K ? The exponential growth of the OTOC is expected to hold on timescales shorter than the Ehrenfest time [18], which, in the present case, should be identified with $t_E \sim \log N$. In the case of the SYK, the long-time

dynamics of the OTOC is controlled by Goldstone-mode fluctuations associated with time reparametrization. These fluctuations act as a restoring force, giving rise to a timescale above which a power-law decay of the OTOC is observed [79,80]. We can speculate that a mechanism similar to this one will be at play in controlling the long-time behavior of the OTOC for $t \gg t_E$ at finite N, K .

Finally, it is worth commenting on the relation between our MCK model and the SYK model. While microscopically very different—the former describing a spin-impurity coupled to a bath while the latter describes all-to-all disordered fermions—we can indeed show a deep connection between the two models. As we discuss in Appendix F, one can integrate out the bath in the MCK model and write down an effective action for the fermionic degrees of freedom $f_\alpha, f_\beta^\dagger$ representing the $SU(N)$ spin. This effective action, to lowest order in the Kondo coupling, takes the form of a sparse SYK-like model with all-to-all fermion interactions. Rather than being quench disordered, the couplings here are fluctuating in time since the interaction is mediated by the bath, which suggests that our MCK model can be seen as a dissipative SYK model in the large $SU(N)$ space. Crucially, the chaotic nature of the MCK model arises because these interactions decay slowly in time, as a power law, due to the critical nature of the bath. This feature and the strong interaction within the $SU(N)$ manifold ultimately lead to fermionic correlators with nontrivial power-law decays in time. These correlators are the building block of the Bethe-Salpeter calculation of the OTOC, leading to maximal chaos. We remark that this scenario is rather opposite to other dissipative versions of the SYK model, where fast baths such as white noise [81,82] or Markovian dissipation [83–85] cut off the long-range nature of the dynamics and suppress the quantum Lyapunov exponent.

VII. CONCLUSIONS

Models that saturate the quantum bound on chaos are appealing for a wide range of communities, from high-energy theory to condensed matter and statistical mechanics. While few examples displaying maximal chaos exist, beyond the celebrated SYK model, they typically involve extended systems with random all-to-all interactions or conformal symmetries. In this work, we exhibit a simple model of a fast scrambler, which only requires a quantum $SU(N)$ spin coupled to a metallic bath with K channels, in the large- N and K limit, with fixed $\gamma = K/N$. In this limit, the quantum spin fractionalizes due to strong correlations into auxiliary fermionic and bosonic degrees of freedom, coupled to the conduction electrons of the metallic bath. Using the generalized twofold Keldysh time contour,

we have diagrammatically computed the OTOC for auxiliary fermions and bosons as well as the conduction electrons. We have demonstrated that, for all these degrees of freedom, the Lyapunov exponent is linear in temperature as $T \rightarrow 0$, with a tunable prefactor maximum at $\gamma = 1$. Finally, we have shown that the OTOC of the physical impurity spin factorizes in the large- N limit and can be deduced from the OTOC of the auxiliary fermions. In particular, for the value $\gamma = K/N = 1$, the spin OTOC saturates the bound on chaos.

The onset of maximal chaos raises the intriguing question of whether there is a gravity analog of the MCK model, i.e., whether the effective theory of the soft mode associated with a broken reparametrization symmetry would be related to some model of gravity. Remarkably, holographic realizations of multichannel Kondo impurity models have been proposed in the context of the AdS/CFT correspondence [86–88], even though their chaos properties on the gravity side have not been investigated, to the best of our knowledge.

The field theory technique used in this work to evaluate the OTOC and Lyapunov exponent of composite operators such as the impurity spin could find immediate applications in the investigation of scrambling and chaos in models for doped quantum spin glasses, which display similar fractionalization [66–68]. Another interesting direction would be to compute a refined version of the Lyapunov exponent via the replica trick, used in the context of semiclassical systems [89], and compare with our results. Finally, a natural extension of the present work would be to investigate the nonequilibrium properties of this model [90] and compare them to the known results on the SYK model. In particular, it would be interesting to see if the relaxation towards thermal equilibrium in this model also displays a Planckian behavior, with a thermalization rate proportional to the temperature.

ACKNOWLEDGMENTS

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APPENDIX A: DETAILS ON KELDYSH DOUBLE CONTOUR

In this appendix, we provide additional details on the twofold Keldysh technique used in the main text to compute the retarded regularized OTOCs, like the f -OTOC,

$$C_f(t_1, t_2) = \frac{\theta(t_1)\theta(t_2)}{N^2} \sum_{\alpha, \beta=1}^N \text{Tr}[\sqrt{\rho}\{\hat{f}_\alpha(t_1), \hat{f}_\beta^\dagger(0)\}\sqrt{\rho}\{\hat{f}_\alpha(t_2), \hat{f}_\beta^\dagger(0)\}^\dagger], \quad (\text{A1})$$

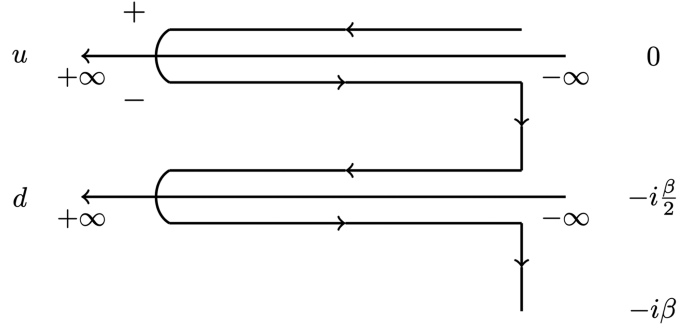


FIG. 7. Twofold Keldysh contour.

where $\sqrt{\rho} = e^{-\beta\hat{H}/2}$ and $\hat{f}_\alpha(t) = e^{i\hat{H}t}\hat{f}_\alpha e^{-i\hat{H}t}$. By rearranging the exponential factors, $C_f(t_1, t_2)$ can be rewritten as

$$C_f(t_1, t_2) = \frac{\theta(t_1)\theta(t_2)}{N^2} \sum_{\alpha, \beta=1}^N \left\langle \left\{ \hat{f}_\alpha \left(t_1 - i\frac{\beta}{2} \right), \hat{f}_\beta^\dagger \left(-i\frac{\beta}{2} \right) \right\} \left\{ \hat{f}_\alpha(t_2), \hat{f}_\beta^\dagger(0) \right\}^\dagger \right\rangle_\beta, \quad (\text{A2})$$

with $\langle \cdots \rangle_\beta = \text{Tr}[\cdots e^{-\beta\hat{H}}]/Z$ the thermal average. The partition function of the model Z along the twofold time contour of Fig. 7 is given by

$$Z = \int \mathcal{D}[\bar{f}, f, \bar{c}, c, \bar{B}, B, \mu] e^{i(S_0 + S_{\text{int}})}, \quad (\text{A3})$$

with S_0 the noninteracting action and S_{int} the interacting part,

$$S_{\text{int}} = -\frac{i}{\sqrt{N}} \int_{-\infty}^{+\infty} dt \sum_{\sigma=u,d} \sum_{s=+,-} \sum_{i\alpha} (\bar{B}_i^{\sigma,s}(t) \bar{c}_{i\alpha}^{\sigma,s}(t) f_\alpha^{\sigma,s}(t) + B_i^{\sigma,s}(t) \bar{f}_\alpha^{\sigma,s}(t) c_{i\alpha}^{\sigma,s}(t)). \quad (\text{A4})$$

Here, $\sigma = u, d$ denotes the upper and lower folds of the contour, and $+, -$ are the forward and backward branches on the corresponding fold. Thermal expectation values $\langle \cdots \rangle_\beta$ can then be represented as averages over the action S on the twofold contour $\langle \cdots \rangle_S$, for instance,

$$\left\langle \hat{f} \left(t_1 - i\frac{\beta}{2} \right) \hat{f}^\dagger \left(-i\frac{\beta}{2} \right) \hat{f}^\dagger(t_2) \hat{f}(0) \right\rangle_\beta = \langle f^{d,-}(t_1) \bar{f}^{d,+}(0) \bar{f}^{u,-}(t_2) f^{u,+}(0) \rangle_S. \quad (\text{A5})$$

The retarded OTOCs, like Eq. (A2), are more easily represented by using the Keldysh rotated fields on each time fold σ ,

$$f_\alpha^{\sigma,cl/q}(t) = \frac{f_\alpha^{\sigma,+}(t) \pm f_\alpha^{\sigma,-}(t)}{\sqrt{2}}, \quad \bar{f}_\alpha^{\sigma,cl/q}(t) = \frac{\bar{f}_\alpha^{\sigma,+}(t) \mp \bar{f}_\alpha^{\sigma,-}(t)}{\sqrt{2}}, \quad (\text{A6})$$

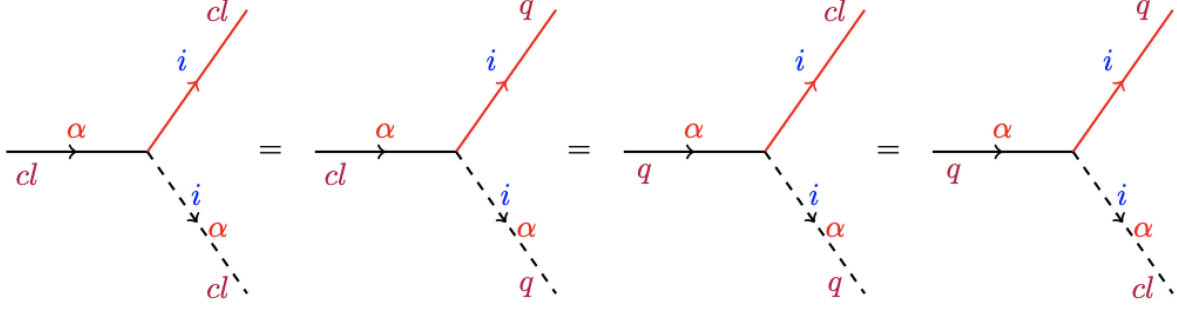
$$B_i^{\sigma,cl/q}(t) = \frac{B_i^{\sigma,+}(t) \pm B_i^{\sigma,-}(t)}{\sqrt{2}}, \quad \bar{B}_i^{\sigma,cl/q}(t) = \frac{\bar{B}_i^{\sigma,+}(t) \pm \bar{B}_i^{\sigma,-}(t)}{\sqrt{2}}, \quad (\text{A7})$$

$$c_{i\alpha}^{\sigma,cl/q}(t) = \frac{c_{i\alpha}^{\sigma,+}(t) \pm c_{i\alpha}^{\sigma,-}(t)}{\sqrt{2}}, \quad \bar{c}_{i\alpha}^{\sigma,cl/q}(t) = \frac{\bar{c}_{i\alpha}^{\sigma,+}(t) \mp \bar{c}_{i\alpha}^{\sigma,-}(t)}{\sqrt{2}}. \quad (\text{A8})$$

In this basis, the interacting part of the action on the twofold Keldysh contour is expressed as

$$S_{\text{int}} = -\frac{i}{\sqrt{2N}} \int_{-\infty}^{+\infty} dt \sum_{i,\alpha,\sigma} [\bar{B}_i^{\sigma,cl} (\bar{c}_{i\alpha}^{\sigma,cl} f_\alpha^{\sigma,cl} + \bar{c}_{i\alpha}^{\sigma,q} f_\alpha^{\sigma,q}) + \bar{B}_i^{\sigma,q} (\bar{c}_{i\alpha}^{\sigma,cl} f_\alpha^{\sigma,q} + \bar{c}_{i\alpha}^{\sigma,q} f_\alpha^{\sigma,cl}) + B_i^{\sigma,cl} (\bar{f}_\alpha^{\sigma,cl} c_{i\alpha}^{\sigma,cl} + \bar{f}_\alpha^{\sigma,q} c_{i\alpha}^{\sigma,q}) + B_i^{\sigma,q} (\bar{f}_\alpha^{\sigma,cl} c_{i\alpha}^{\sigma,q} + \bar{f}_\alpha^{\sigma,q} c_{i\alpha}^{\sigma,cl})]. \quad (\text{A9})$$

These vertices will be represented by



together with their complex conjugates, where all the arrows are reversed.

To write the OTOC (A2) as a four-point function on the twofold contour 1, one can develop the product of the two anticommutators and use the identity

$$\theta(t)(\hat{f}(t)\hat{f}^\dagger(0) + \hat{f}^\dagger(0)\hat{f}(t)) = \frac{1}{2}[T(\hat{f}(t)\hat{f}^\dagger(0)) - \tilde{T}(\hat{f}(t)\hat{f}^\dagger(0)) + \hat{f}(t)\hat{f}^\dagger(0) + \hat{f}^\dagger(0)\hat{f}(t)], \quad (\text{A10})$$

where T and $\tilde{T}$ are the time-ordering and anti-time-ordering operations. Using identities like Eq. (A5) and the definition of the rotated fields in Eq. (A6), after some rearrangements, we finally obtain

$$C_f(t_1, t_2) = \frac{1}{N^2} \sum_{\alpha, \beta=1}^N \langle f_\alpha^{d,cl}(t_1) \bar{f}_\beta^{d,cl}(0) \bar{f}_\alpha^{u,q}(t_2) f_\beta^{u,q}(0) \rangle. \quad (\text{A11})$$

The intrafold two-point functions are the same as in the usual single-fold Keldysh formalism and are identical on the upper and lower folds:

$$\mathbf{G}_f^{\sigma\sigma} = \begin{pmatrix} G_f^R & G_f^K \\ 0 & G_f^A \end{pmatrix}, \quad \mathbf{G}_c^{\sigma\sigma} = \begin{pmatrix} G_c^R & G_c^K \\ 0 & G_c^A \end{pmatrix}, \quad \mathbf{G}_B^{\sigma\sigma} = \begin{pmatrix} G_B^K & G_B^R \\ G_B^A & 0 \end{pmatrix}, \quad \sigma = u, d. \quad (\text{A12})$$

In these notations, the matrix elements for a given field ψ are $[\mathbf{G}_\psi^{\sigma\sigma'}]^{ss'}(t, t') = -i\xi \langle \psi^{\sigma,s}(t) \bar{\psi}^{\sigma',s'}(t') \rangle$, with $\sigma, \sigma' = u, d$ and $s, s' = cl, q$ and where $\xi = 1$ (respectively, -1) if ψ is fermionic (respectively, bosonic). We have omitted the spin and channel indices to simplify notations. The retarded, advanced, and Keldysh Green's functions are defined by

$$iG_\psi^R(t, t') = \xi \theta(t - t') \langle [\hat{\psi}, \hat{\psi}^\dagger(t')]_\xi \rangle, \quad (\text{A13})$$

$$iG_\psi^A(t, t') = -\xi \theta(t' - t) \langle [\hat{\psi}, \hat{\psi}^\dagger(t')]_\xi \rangle, \quad (\text{A14})$$

$$iG_\psi^K(t, t') = \xi \langle [\hat{\psi}, \hat{\psi}^\dagger(t')]_{-\xi} \rangle, \quad (\text{A15})$$

where the average is over the thermal density matrix and $[A, B]_\xi = AB + \xi BA$.

The two-point functions in this basis are the retarded, advanced, and Keldysh Green's functions. The noninteracting fermionic Green's function is represented by a simple line,

$$t \xrightarrow[\alpha]{(cl, q) \quad (cl, q)} t' = \begin{pmatrix} \langle f_\alpha^{cl}(t) \bar{f}_\beta^{cl}(t') \rangle_0 & \langle f_\alpha^{cl}(t) \bar{f}_\beta^q(t') \rangle_0 \\ \langle f_\alpha^q(t) \bar{f}_\beta^{cl}(t') \rangle_0 & \langle f_\alpha^q(t) \bar{f}_\beta^q(t') \rangle_0 \end{pmatrix} = \begin{pmatrix} iG_{f,0}^R(t, t') & iG_{f,0}^K(t, t') \\ 0 & iG_{f,0}^A(t, t') \end{pmatrix} \delta_{\alpha\beta} \quad (\text{A16})$$

$$t \xrightarrow[\alpha]{(cl, q) \quad (cl, q)} t' = \begin{pmatrix} \langle c_{i\alpha}^{cl}(t) \bar{c}_{j\beta}^{cl}(t') \rangle_0 & \langle c_{i\alpha}^{cl}(t) \bar{c}_{j\beta}^q(t') \rangle_0 \\ \langle c_{i\alpha}^q(t) \bar{c}_{j\beta}^{cl}(t') \rangle_0 & \langle c_{i\alpha}^q(t) \bar{c}_{j\beta}^q(t') \rangle_0 \end{pmatrix} = \begin{pmatrix} iG_{c,0}^R(t, t') & iG_{c,0}^K(t, t') \\ 0 & iG_{c,0}^A(t, t') \end{pmatrix} \delta_{ij} \delta_{\alpha\beta} \quad (\text{A17})$$

$$t \xrightarrow[\alpha]{(cl, q)} \xleftarrow[\beta]{(cl, q)} t' = \begin{pmatrix} \langle B_i^{cl}(t) \bar{B}_j^{cl}(t') \rangle_0 & \langle B_i^{cl}(t) \bar{B}_{ij}^q(t') \rangle_0 \\ \langle B_i^q(t) \bar{B}_j^{cl}(t') \rangle_0 & \langle B_i^q(t) \bar{B}_j^q(t') \rangle_0 \end{pmatrix} = \begin{pmatrix} -iG_{B,0}^K(t, t') & -iG_{B,0}^R(t, t') \\ -iG_{B,0}^A(t, t') & 0 \end{pmatrix} \delta_{ij} \quad (\text{A18})$$

For the interfold Green's functions, only the Keldysh component is nonzero,

$$\mathbf{G}_f^{\sigma\bar{\sigma}} = \begin{pmatrix} 0 & G_{f,\sigma\bar{\sigma}}^K(t, t') \\ 0 & 0 \end{pmatrix}, \quad \mathbf{G}_c^{\sigma\bar{\sigma}} = \begin{pmatrix} 0 & G_{c,\sigma\bar{\sigma}}^K(t, t') \\ 0 & 0 \end{pmatrix}, \quad \mathbf{G}_B^{\sigma\bar{\sigma}} = \begin{pmatrix} G_{B,\sigma\bar{\sigma}}^K(t, t') & 0 \\ 0 & 0 \end{pmatrix}, \quad (\text{A19})$$

where we used the notation $\bar{u} = d$ and vice versa. These components are the so called Wightman correlators, which are defined as the thermal averages

$$iG_{\psi,ud}^K(t, t') = 2\xi \left\langle \hat{\psi} \left(t - i\frac{\beta}{2} \right) \hat{\psi}^\dagger(t') \right\rangle, \quad iG_{\psi,du}^K(t, t') = -2\xi \left\langle \hat{\psi}^\dagger \left(t' - i\frac{\beta}{2} \right) \hat{\psi}(t) \right\rangle. \quad (\text{A20})$$

In the Feynman diagrams, they will be represented as vertical lines connecting the upper and lower folds,

$$\begin{array}{ll} \begin{array}{c} u \quad t' \quad q \\ | \\ \beta \\ | \\ \alpha \\ d \quad t \quad cl \end{array} = \left\langle f_\alpha^{d,cl}(t) \bar{f}_\beta^{u,q}(t') \right\rangle = iG_{f,du}^K(t, t') \delta_{\alpha\beta} & \begin{array}{c} u \quad t \quad cl \\ | \\ \alpha \\ | \\ \beta \\ d \quad t' \quad q \end{array} = \left\langle f_\alpha^{u,cl}(t) \bar{f}_\beta^{d,q}(t') \right\rangle = iG_{f,ud}^K(t, t') \delta_{\alpha\beta} \\ \\ \begin{array}{c} u \quad t' \quad q \\ | \\ i \quad \beta \\ | \\ j \quad \alpha \\ d \quad t \quad cl \end{array} = \left\langle c_{i\alpha}^{d,cl}(t) \bar{c}_{j\beta}^{u,q}(t') \right\rangle = iG_{c,du}^K(t, t') \delta_{ij} \delta_{\alpha\beta} & \begin{array}{c} u \quad t \quad cl \\ | \\ i \quad \alpha \\ | \\ j \quad \beta \\ d \quad t' \quad q \end{array} = \left\langle c_{i\alpha}^{u,cl}(t) \bar{c}_{j\beta}^{d,q}(t') \right\rangle = iG_{c,ud}^K(t, t') \delta_{ij} \delta_{\alpha\beta} \\ \\ \begin{array}{c} u \quad t' \quad q \\ | \\ j \\ | \\ i \\ d \quad t \quad cl \end{array} = \left\langle B_i^{d,cl}(t) \bar{B}_j^{u,q}(t') \right\rangle = -iG_{B,du}^K(t, t') \delta_{ij} & \begin{array}{c} u \quad t \quad cl \\ | \\ i \\ | \\ j \\ d \quad t' \quad q \end{array} = \left\langle B_i^{u,cl}(t) \bar{B}_j^{d,q}(t') \right\rangle = -iG_{B,ud}^K(t, t') \delta_{ij} \end{array}$$

One can obtain explicit expressions for all these real-time Green's function in the low-energy limit $\omega, T \ll T_K$ by analytic continuation of the imaginary-time Green's function (11), as discussed in Appendix B.

APPENDIX B: REAL-TIME GREEN'S FUNCTION

For bosons and fermions, the real-time retarded, advanced, and Wightman two-point functions are related to the imaginary-time Matsubara Green's function by analytic continuation,

$$G^R(t) = i\theta(t)(G(\tau = it + \epsilon) - G(\tau = it - \epsilon)), \quad (\text{B1})$$

$$G^A(t) = -i\theta(-t)(G(\tau = it + \epsilon) - G(\tau = it - \epsilon)), \quad (\text{B2})$$

$$G_{du}^K(t) = 2iG\left(\tau = it + \frac{\beta}{2}\right), \quad (\text{B3})$$

$$G_{ud}^K(t) = 2iG\left(\tau = it - \frac{\beta}{2}\right). \quad (\text{B4})$$

By using the conformal expression of the Matsubara Green's functions (11), after some simple manipulations, one finds, for the auxiliary fermions f ,

$$G_f^R(t) = -2iA_f\theta(t)\cos\left(\pi\Delta_f - i\frac{\alpha}{2}\right)\frac{e^{\frac{i\alpha}{\beta}t}}{\cosh(\frac{\alpha}{2})}\left(\frac{\pi}{\beta\sinh(\frac{\pi t}{\beta})}\right)^{2\Delta_f}, \quad (\text{B5})$$

$$G_f^A(t) = 2iA_f\theta(-t)\cos\left(\pi\Delta_f + i\frac{\alpha}{2}\right)\frac{e^{\frac{i\alpha}{\beta}t}}{\cosh(\frac{\alpha}{2})}\left(\frac{\pi}{\beta\sinh(-\frac{\pi t}{\beta})}\right)^{2\Delta_f}, \quad (\text{B6})$$

$$G_{f,du}^K(t) = -2iA_f\frac{e^{\frac{i\alpha}{\beta}t}}{\cosh(\frac{\alpha}{2})}\left(\frac{\pi}{\beta\cosh(\frac{\pi t}{\beta})}\right)^{2\Delta_f}, \quad (\text{B7})$$

$$G_{f,ud}^K(t) = 2iA_f\frac{e^{\frac{i\alpha}{\beta}t}}{\cosh(\frac{\alpha}{2})}\left(\frac{\pi}{\beta\cosh(\frac{\pi t}{\beta})}\right)^{2\Delta_f}. \quad (\text{B8})$$

For the auxiliary bosons B , we have

$$G_B^R(t) = -2A_B\theta(t)\sin\left(\pi\Delta_B - i\frac{\alpha}{2}\right)\frac{e^{\frac{i\alpha}{\beta}t}}{\cosh(\frac{\alpha}{2})}\left(\frac{\pi}{\beta\sinh(\frac{\pi t}{\beta})}\right)^{2\Delta_B}, \quad (\text{B9})$$

$$G_B^A(t) = -2A_B\theta(-t)\sin\left(\pi\Delta_B + i\frac{\alpha}{2}\right)\frac{e^{\frac{i\alpha}{\beta}t}}{\cosh(\frac{\alpha}{2})}\left(\frac{\pi}{\beta\sinh(-\frac{\pi t}{\beta})}\right)^{2\Delta_B}, \quad (\text{B10})$$

$$G_{B,du}^K(t) = -2iA_B\frac{e^{\frac{i\alpha}{\beta}t}}{\cosh(\frac{\alpha}{2})}\left(\frac{\pi}{\beta\cosh(\frac{\pi t}{\beta})}\right)^{2\Delta_B}, \quad (\text{B11})$$

$$G_{B,ud}^K(t) = -2iA_B\frac{e^{\frac{i\alpha}{\beta}t}}{\cosh(\frac{\alpha}{2})}\left(\frac{\pi}{\beta\cosh(\frac{\pi t}{\beta})}\right)^{2\Delta_B}. \quad (\text{B12})$$

Finally, for the conduction electrons, we obtain

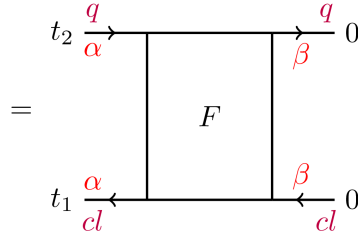
$$G_{c,du}^K(t) = -2i\rho_0\frac{\pi}{\beta\cosh(\frac{\pi t}{\beta})}, \quad (\text{B13})$$

$$G_{c,ud}^K(t) = 2i\rho_0\frac{\pi}{\beta\cosh(\frac{\pi t}{\beta})}. \quad (\text{B14})$$

APPENDIX C: FERMIONIC OTOC

In this appendix, we provide explicit details for the calculation of the fermionic OTOC defined in the main text as

$$\begin{aligned}
 C(t_1, t_2) &= \frac{\theta(t_1)\theta(t_2)}{N^2} \sum_{\alpha, \beta=1}^N \text{Tr} \left[\sqrt{\rho} \{ \hat{f}_\alpha(t_1), \hat{f}_\beta^\dagger(0) \} \sqrt{\rho} \{ \hat{f}_\alpha(t_2), \hat{f}_\beta^\dagger(0) \}^\dagger \right] \\
 &= \frac{1}{N^2} \sum_{\alpha, \beta=1}^N \langle f_\alpha^{cl, d}(t_1) \bar{f}_\beta^{cl, d}(0) \bar{f}_\alpha^{q, u}(t_2) f_\beta^{q, u}(0) \rangle \\
 &= \frac{1}{N} F(t_1, t_2)
 \end{aligned} \tag{C1}$$



For convenience, we have defined $C_f(t_1, t_2) \equiv (1/N)F_f(t_1, t_2)$. We now expand this correlator in powers of the interaction vertex using Wick's theorem. We want to identify the diagrams of order $1/N$, which are the dominant contributions to F_f in the large- N , K limit. Factors of N will come from each vertex, which amounts to $1/\sqrt{N}$, and from the sum over the internal spin and channel indices. Since we are performing an expansion around the noninteracting action, the lines that we draw are the *bare* propagators, but we anticipate that we can resum the bubble diagrams to obtain the interacting propagators for f and B (see Fig. 1), without additional factors of N .

We expand $C(t_1, t_2)$ in powers of the coupling. The first Feynman diagrams are

$$\begin{aligned}
 & \text{Diagram 1} = \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} + \dots \\
 & \text{Diagram 1: Square loop } F(t_1, t_2) \\
 & \text{Diagram 2: Horizontal line with two vertices} \\
 & \text{Diagram 3: Horizontal line with a bubble} \\
 & \text{Diagram 4: Horizontal line with a bubble} \\
 & \text{Diagram 5: Horizontal line with two vertices} \\
 & \text{Diagram 6: Horizontal line with a bubble}
 \end{aligned} \tag{C2}$$

Let us look at how the first diagrams scale with N . For the zeroth-order one,

$$\begin{array}{c}
t_2 \xrightarrow[\alpha]{q} \xrightarrow{\beta}{q} 0 \\
t_1 \xleftarrow[cl]{\alpha} \xleftarrow{cl}{\beta} 0
\end{array}
\sim \frac{1}{N^2} \sum_{\alpha, \beta=1}^N \delta_{\alpha\beta} \sim \frac{1}{N}. \quad (C3)$$

At second order in the vertex, we have the first bubble correction to the bare propagators. At fourth order in the interaction, apart from higher corrections to the bare propagators, we have two ladder diagrams connecting the upper and lower folds. The first ladder diagram scales as $1/N$ just like the zeroth-order term:

$$\begin{array}{c}
t_2 \xrightarrow[\alpha]{q} \xrightarrow[i]{q} 0 \\
t_1 \xleftarrow[cl]{\alpha} \xleftarrow[i]{cl} 0
\end{array}
\sim \frac{1}{N^2} \sum_{\alpha, \beta=1}^N \sum_{i=1}^K \frac{1}{N^2} \sim \frac{K}{N^2} \sim \frac{\gamma}{N} \quad (C4)$$

where we have explicitly used the fact that $K = \gamma N$. However, the second ladder diagram scales like $1/N^2$,

$$\begin{array}{c}
t_2 \xrightarrow[\alpha]{q} \xrightarrow[i]{q} \xrightarrow[\alpha]{q} 0 \\
t_1 \xleftarrow[cl]{\alpha} \xleftarrow[i]{cl} \xleftarrow[\alpha]{cl} 0
\end{array}
\sim \frac{1}{N^2} \sum_{\alpha, \beta=1}^N \sum_{i=1}^K \frac{\delta_{\alpha\beta}}{N^2} \sim \frac{K}{N^3} \sim \frac{\gamma}{N^2} \quad (C5)$$

so these kinds of diagrams are subleading. By keeping only the diagrams of order $1/N$, we can write the self-consistent equation

$$\begin{array}{c}
t_2 \xrightarrow[\alpha]{q} \xrightarrow{\beta}{q} 0 \\
t_1 \xleftarrow[cl]{\alpha} \xleftarrow{\beta}{cl} 0
\end{array}
= \begin{array}{c}
t_2 \xrightarrow[\alpha]{q} \xrightarrow{\beta}{q} 0 \\
t_1 \xleftarrow[cl]{\alpha} \xleftarrow{\beta}{cl} 0
\end{array}
+ \begin{array}{c}
t_2 \xrightarrow[\alpha]{q} \xrightarrow[i]{q} \xrightarrow{\beta}{q} 0 \\
t_1 \xleftarrow[cl]{\alpha} \xleftarrow[i]{cl} \xleftarrow{\beta}{cl} 0
\end{array} \quad (C6)$$

which recovers the one provided in the main text as an integral equation.

APPENDIX D: FOUR-POINT FUNCTION IN IMAGINARY-TIME AND LADDER DIAGRAMS

In this appendix, we provide additional details on the imaginary-time four-point function for the fermion f , which is needed to compute the OTOC with a different regularization. We define the four-point function as

$$\mathcal{C}(\tau_1, \tau_2, \tau_3, \tau_4) = \frac{1}{N^2} \sum_{\alpha\beta} \langle T_{\tau} f_{\alpha}(\tau_1) f_{\alpha}^{\dagger}(\tau_2) f_{\beta}^{\dagger}(\tau_3) f_{\beta}(\tau_4) \rangle, \quad (D1)$$

which has a disconnected term plus terms of order $1/N$, i.e.,

$$\mathcal{C}(\tau_1, \tau_2, \tau_3, \tau_4) = G_f(\tau_1, \tau_2) G_f(\tau_3, \tau_4) + \frac{1}{N} \mathcal{F}(\tau_1, \tau_2, \tau_3, \tau_4). \quad (D2)$$

Diagrammatically expanding the four-point function, using the vertex in the imaginary-time action given in Sec. III of the main text, we obtain a series of diagrams that closely resemble what we discussed for the fermionic OTOC in Appendix C. In particular, one can see that the leading-order diagrams in the double limit $N, K \rightarrow \infty$ are ladder diagrams of the form

$$\mathcal{F}(\tau_1, \tau_2, \tau_3, \tau_4) = \mathcal{F}_0(\tau_1, \tau_2, \tau_3, \tau_4) + \int dt_1 dt_3 \mathcal{K}(\tau_1, \tau_2, \tau, \tau') \mathcal{F}_0(\tau, \tau', \tau_3, \tau_4) + \dots, \quad (\text{D3})$$

where $\mathcal{F}_0(\tau_1, \tau_2, \tau_3, \tau_4) = G_f(\tau_1, \tau_3)G_f(\tau_2, \tau_4)$, while the kernel reads

$$\mathcal{K}(\tau_1, \tau_2, \tau, \tau') = \int dt dt_2 [G_f(\tau_1, t)G_f(t_2, \tau_2)G_c(t, \tau')][G_b(t, \tau)G_b(t_2, \tau')G_c(\tau, \tau')], \quad (\text{D4})$$

where $G_{b,c}$ are bosonic and conduction-electron Green's functions, and all the integrals are performed on the Matsubara imaginary-time axis. We can resum all the ladder diagrams by writing the Bethe-Salpeter equation in imaginary time,

$$\mathcal{F}(\tau_1, \tau_2, \tau_3, \tau_4) = \mathcal{F}_0(\tau_1, \tau_2, \tau_3, \tau_4) + \int d\tau d\tau' \mathcal{K}(\tau_1, \tau_2, \tau, \tau') \mathcal{F}(\tau, \tau', \tau_3, \tau_4). \quad (\text{D5})$$

We note that the kernel entering this BS equation has the same structure as the kernel of the real-time fermionic OTOC, with the only difference being that all functions here are in imaginary time. As discussed in Ref. [26] for the case of the SYK model, one can obtain the OTOC $F_{1/4}$ with the regularization of Eq. (66) in the main text by analytical continuation of the imaginary-time kernel, leading to the BS equation reported in Sec. V.

APPENDIX E: DETAILS ON IMPURITY-SPIN OTOC

1. Derivation of Eq. (62)

We start from the OTOC correlator in Eq. (61) of the main text,

$$F_{a_1 a_2}(t_1, t_2) = \frac{1}{N^2} \sum_{\alpha\beta} \text{Tr}(\sqrt{\rho}[\hat{f}_\alpha(t_1), \hat{f}_\beta^\dagger(0)]_{a_1} \sqrt{\rho}[\hat{f}_\alpha(t_2), \hat{f}_\beta^\dagger(0)]_{a_2}^\dagger), \quad (\text{E1})$$

where $a_{1,2} = \pm$ and $\rho = e^{-\beta H}/Z$ is the thermal weight. We can rewrite this correlator explicitly in terms of TOCs and OTOCs, namely,

$$F_{a_1 a_2}(t_1, t_2) = F_1^{\text{toc}}(t_1, t_2) + a_1 a_2 F_2^{\text{toc}}(t_1, t_2) + a_2 F_1^{\text{otoc}}(t_1, t_2) + a_1 F_2^{\text{otoc}}(t_1, t_2), \quad (\text{E2})$$

where we can read off the definitions of $F_{1,2}^{\text{toc}}$ and $F_{1,2}^{\text{otoc}}$ from the previous equation, namely,

$$F_1^{\text{otoc}}(t_1, t_2) = \frac{1}{N^2} \sum_{\alpha\beta} \text{Tr}[\sqrt{\rho} \hat{f}_\alpha(t_1) \hat{f}_\beta^\dagger(0) \sqrt{\rho} \hat{f}_\alpha^\dagger(t_2) \hat{f}_\beta(0)], \quad (\text{E3})$$

$$F_2^{\text{otoc}}(t_1, t_2) = \frac{1}{N^2} \sum_{\alpha\beta} \text{Tr}[\sqrt{\rho} \hat{f}_\beta^\dagger(0) \hat{f}_\alpha(t_1) \sqrt{\rho} \hat{f}_\beta(0) \hat{f}_\alpha^\dagger(t_2)], \quad (\text{E4})$$

as well as for the time-ordered ones,

$$F_1^{\text{toc}}(t_1, t_2) = \frac{1}{N^2} \sum_{\alpha\beta} \text{Tr}[\sqrt{\rho} \hat{f}_\alpha(t_1) \hat{f}_\beta^\dagger(0) \sqrt{\rho} \hat{f}_\beta(0) \hat{f}_\alpha^\dagger(t_2)], \quad (\text{E5})$$

$$F_2^{\text{toc}}(t_1, t_2) = \frac{1}{N^2} \sum_{\alpha\beta} \text{Tr}[\sqrt{\rho} \hat{f}_\beta^\dagger(0) \hat{f}_\alpha(t_1) \sqrt{\rho} \hat{f}_\alpha^\dagger(t_2) \hat{f}_\beta(0)]. \quad (\text{E6})$$

We can now perform a rotation and define

$$F_\pm^{\text{toc}}(t_1, t_2) = F_1^{\text{toc}}(t_1, t_2) \pm F_2^{\text{toc}}(t_1, t_2), \quad (\text{E7})$$

$$F_\pm^{\text{otoc}}(t_1, t_2) = F_1^{\text{otoc}}(t_1, t_2) \pm F_2^{\text{otoc}}(t_1, t_2). \quad (\text{E8})$$

In terms of these quantities, we can write the original $F_{a_1 a_2}$ correlator as

$$F_{\pm\pm}(t_1, t_2) = F_+^{\text{toc}}(t_1, t_2) \pm F_+^{\text{otoc}}(t_1, t_2), \quad (\text{E9})$$

$$F_{\pm\mp}(t_1, t_2) = F_-^{\text{toc}}(t_1, t_2) \mp F_-^{\text{otoc}}(t_1, t_2). \quad (\text{E10})$$

This result highlights how, in the definition of the original correlators entering the impurity-spin OTOC, there are both OTOC and TOC contributions. We can now use these results to rewrite the correlator $C_S(t, t)$ in the main text. We start from Eq. (60) of the main text evaluated at equal times $t_1 = t_2 = t$,

$$C_S(t, t) = -[F_{++}(t, t)F_{--}(t, t)^* - F_{+-}(t, t)F_{-+}(t, t)^* + \text{c.c.}], \quad (\text{E11})$$

and use the fact that $(F_{--}(t, t))^* = F_{--}(t, t)$ as well as $(F_{-+}(t, t))^* = F_{+-}(t, t)$ to rewrite

$$C_S(t, t) = -2F_{++}(t, t)F_{--}(t, t) + (F_{+-}^2(t, t) + F_{-+}^2(t, t)) \quad (\text{E12})$$

$$= -2[(F_+^{\text{toc}}(t, t))^2 - (F_-^{\text{toc}}(t, t))^2] + 2[(F_+^{\text{otoc}}(t, t))^2 + (F_-^{\text{otoc}}(t, t))^2] \quad (\text{E13})$$

$$= -2[(F_+^{\text{toc}}(t, t))^2 - (F_-^{\text{toc}}(t, t))^2] + 4[(F_1^{\text{otoc}}(t, t))^2 + (F_2^{\text{otoc}}(t, t))^2]. \quad (\text{E14})$$

By inspection of Eq. (E3), we see that the following identity holds,

$$F_1^{\text{otoc}}(t_2, t_1) = (F_2^{\text{otoc}}(t_1, t_2))^*. \quad (\text{E15})$$

Therefore, for equal-time arguments, we can use this identity to rewrite Eq. (E12) as

$$C_S(t, t) = 8(F_1^{\text{otoc}}(t, t))^2 - 2[(F_+^{\text{toc}}(t, t))^2 - (F_-^{\text{toc}}(t, t))^2], \quad (\text{E16})$$

which gives us the equation in the main text.

2. Diagrammatics for the impurity-spin OTOC

In this appendix, we show that, by expanding $C_{\mu_1 \nu_1; \mu_2 \nu_2}^{\mu_3 \nu_3; \mu_4 \nu_4}(t_1, t_2)$ diagrammatically, to leading order in the large- N , K limit, we have the factorization

$$C_{\mu_1 \nu_1; \mu_2 \nu_2}^{\mu_3 \nu_3; \mu_4 \nu_4}(t_1, t_2) = \bar{F}_{\mu_1 \nu_1}^{\mu_3 \nu_3}(t_1, t_2) F_{\mu_2 \nu_2}^{\mu_4 \nu_4}(t_1, t_2). \quad (\text{E17})$$

The zeroth-order diagram is of order $1/N^2$,

$$\sim \frac{1}{N^4} \sum_{\alpha\beta\gamma\delta} \delta_{\alpha\gamma} \delta_{\beta\delta} \sim \frac{1}{N^2} \quad (\text{E18})$$

Second-order diagrams are only the first bubble corrections to the propagators. At fourth order in the interaction, the only diagrams of order $1/N^2$ are the two ladder diagrams,

$$\sim \frac{1}{N^4} \sum_{\alpha\beta\gamma\delta} \sum_i \frac{\delta_{\beta\delta}}{N^2} \sim \frac{K}{N^3} \sim \frac{\gamma}{N^2} \quad (\text{E19})$$

Other diagrams are subleading, including the following graphs:

$$\sim \frac{1}{N^4} \sum_{\alpha\beta\gamma\delta} \sum_i \frac{\delta_{\alpha\gamma} \delta_{\beta\delta} \delta_{\alpha\beta}}{N^2} \sim \frac{K}{N^5} \sim \frac{\gamma}{N^4} \quad (\text{E20})$$

$$\sim \frac{1}{N^4} \sum_{\alpha\beta\gamma\delta} \sum_i \frac{\delta_{\beta\delta} \delta_{\alpha\gamma}}{N^2} \sim \frac{K}{N^4} \sim \frac{\gamma}{N^3} \quad (\text{E21})$$

At order eight in the interaction, the diagrams in $1/N^2$ are the following:

$$\sim \frac{\gamma^2}{N^2} \quad (\text{E22})$$

$$\sim \frac{\gamma^2}{N^2} \quad (\text{E23})$$

The other graphs that we can draw are of higher order in $1/N^2$, like

$$\sim \frac{\gamma}{N^4} \quad (\text{E24})$$

$$\sim \frac{\gamma^2}{N^4} \quad (\text{E25})$$

Thus, to the leading $1/N^2$ order, $C_{\mu_1\nu_1;\mu_2\nu_2}^{\mu_3\nu_3;\mu_4\nu_4}(t_1, t_2)$ is the sum of the diagrams

$$\begin{aligned}
 C_{\mu_1\nu_1;\mu_2\nu_2}^{\mu_3\nu_3;\mu_4\nu_4}(t_1, t_2) = & \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \\
 & + \text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} + \text{Diagram 8} + \dots
 \end{aligned}
 \tag{E26}$$

The diagrams represent various Feynman-like diagrams for the correlation function. They consist of two horizontal lines (top and bottom) with vertices labeled t_1, t_2 and 0 . The top line has indices μ_3, ν_3 and the bottom line has indices μ_1, ν_1 (for the first two diagrams) or μ_4, ν_4 and μ_2, ν_2 (for the last two diagrams). The diagrams show different ways of connecting these lines with internal lines and vertices, including loops and multiple internal lines.

which can be factorized as

$$\begin{aligned}
 C_{\mu_1\nu_1;\mu_2\nu_2}^{\mu_3\nu_3;\mu_4\nu_4}(t_1, t_2) = & \left[\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \right] \\
 & \times \left[\text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} + \dots \right] \\
 = & \bar{F}_{\mu_1\nu_1}^{\mu_3\nu_3} \times F_{\mu_2\nu_2}^{\mu_4\nu_4}
 \end{aligned}
 \tag{E27}$$

The factorization shows that the correlation function is the product of two separate diagrams, $\bar{F}_{\mu_1\nu_1}^{\mu_3\nu_3}$ and $F_{\mu_2\nu_2}^{\mu_4\nu_4}$, which represent the two-point functions for the top and bottom lines respectively.

giving the desired result.

APPENDIX F: MAPPING ONTO AN EFFECTIVE DISSIPATIVE SYK MODEL

In this appendix, we provide details on how to connect the MCK model to an effective SYK-like model in the $SU(N)$ space, in the presence of a dissipative interaction. We write the partition function of the MCK model in the main text, which reads

$$Z = \int D\bar{c}_{ika} Dc_{ika} D\bar{f}_{ia} Df_{ia} e^{-S}, \quad (\text{F1})$$

with the imaginary-time action given by

$$S = \int_0^\beta d\tau \sum_{ika} c_{ika} (\partial_\tau - \varepsilon_k) \bar{c}_{ika} + \int_0^\beta d\tau \sum_\alpha f_\alpha (\partial_\tau + i\mu) \bar{f}_\alpha + \frac{J}{N} \int_0^\beta d\tau \sum_{kp} \sum_{ia\beta} (\bar{f}_\alpha f_\beta - q_0 \delta_{a\beta}) \bar{c}_{ika} c_{ip\beta}. \quad (\text{F2})$$

We now proceed to formally integrate out the conduction electrons and obtain an effective action for the fermionized spin only, which reads

$$Z = \int D\bar{f}_{ia} Df_{ia} e^{-S_{\text{eff}}[\bar{f}, f]} = \int D\bar{f}_{ia} Df_{ia} e^{-S_0[\bar{f}, f]} \langle T_\tau e^{-\frac{J}{N} \int_0^\beta \sum_{kpia\beta} O_{a\beta} \bar{c}_{ika} c_{ip\beta}} \rangle, \quad (\text{F3})$$

where we have defined $O_{a\beta} = (\bar{f}_\alpha f_\beta - q_0 \delta_{a\beta})$, such that the effective action reads

$$S_{\text{eff}}[\bar{f}, f] = S_0[\bar{f}, f] - \log \langle T_\tau e^{-\frac{J}{N} \int_0^\beta \sum_{kpia\beta} O_{a\beta} \bar{c}_{ika} c_{ip\beta}} \rangle, \quad (\text{F4})$$

with $S_0[\bar{f}, f]$ the free part of the action. To lowest order in the Kondo coupling J , we obtain

$$S_{\text{eff}}[\bar{f}, f] = S_0[\bar{f}, f] + \int d\tau d\tau' \sum_{\alpha\beta\gamma\delta} (\bar{f}_\alpha(\tau) f_\beta(\tau) - q_0 \delta_{a\beta}) J_{\alpha\beta\gamma\delta}(\tau - \tau') (\bar{f}_\gamma(\tau') f_\delta(\tau') - q_0 \delta_{\gamma\delta}), \quad (\text{F5})$$

where the correlator $J_{\alpha\beta\gamma\delta}(\tau - \tau')$ is defined as the local spin-spin correlation function of the bath,

$$J_{\alpha\beta\gamma\delta}(\tau - \tau') = -\frac{J^2}{N^2} \sum_{kk'pp'} \sum_{ij} \langle T_\tau c_{kia}^\dagger(\tau) c_{k'i\beta}(\tau) c_{p'j\gamma}^\dagger(\tau') c_{p'j\delta}(\tau') \rangle. \quad (\text{F6})$$

Using Wick's theorem, we can rewrite this coupling as

$$J_{\alpha\beta\gamma\delta}(\tau - \tau') = \frac{J^2 K}{N} \delta_{a\delta} \delta_{\beta\gamma} G_c^{(0)}(\tau - \tau') G_c^{(0)}(\tau' - \tau), \quad (\text{F7})$$

where $G_c^{(0)}(\tau)$ is the noninteracting local Green's function of the bath fermions. Therefore, we see that the fermionic action takes the form of a SYK-like model in the $SU(N)$ space, with dissipative long-ranged-in-time interactions mediated by the bath.

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