

Exploring Light-Induced Phases of 2D Materials in a Modulated 1D Quasicrystal

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(Received 13 August 2025; accepted 9 January 2026; published 25 February 2026)

Light-induced quantum phases offer the potential for simple and powerful tuning of material properties. For example, simply illuminating 2D materials in the integer quantum Hall regime with polarized light is predicted to drive quantum phase transitions. Such phenomena are largely beyond the current frontier of solid state experiments due to technical limitations on laser intensity and material purity. However, the Harper-Hofstadter mapping which relates a two-dimensional integer quantum Hall system to a 1D quasicrystal enables the same polarization-dependent light-induced phase transitions to be observed using a quantum gas in a driven quasiperiodic optical lattice. We report results of such an experiment. We observe an interlaced phase diagram of localization-delocalization phase transitions as a function of drive polarization and amplitude. Elliptically polarized driving can stabilize an extended critical phase featuring multifractal wave functions; we observe signatures of this phenomenon in anomalous polarization-dependent subdiffusive transport. In this regime, increasing the strength of the quasiperiodic potential can enhance rather than suppress transport. These experiments demonstrate a simple method for synthesizing exotic multifractal states and exploring light-induced quantum phases across different dimensionalities.

DOI: [10.1103/37vk-qz71](https://doi.org/10.1103/37vk-qz71)

Subject Areas: Atomic and Molecular Physics,
 Condensed Matter Physics

Tuning quantum matter by adjusting the strength, frequency, and polarization of external irradiation is a powerful and flexible method for creating novel on-demand properties, as demonstrated by rapid developments in the field of strongly driven solids [1,1–5]. In such experiments the polarization of the driving waveform is a crucial degree of freedom; for example, it can control time-reversal symmetry, stabilizing phases such as Floquet topological insulators [4,6]. Realizing the full promise of driven condensed matter is a serious challenge; in addition to their intrinsic complexity, these experiments require optical intensities and pulse times at the edge of feasibility with current materials and laser technology [3]. Complementary input from experiments on cold atom platforms, at very different energy scales but realizing the same Hamiltonians [7,8], can play a key role in exploring the frontiers of driven matter and guiding future experiments on quantum materials under intense arbitrarily polarized irradiation.

In this manuscript we describe and demonstrate an experiment along these lines, using cold atoms in a doubly driven 1D quasicrystal to explore the phase diagram of 2D Bloch electrons in a high magnetic field illuminated with

intense light of tunable polarization [9]. This depends on a cross-dimensional correspondence mapping a 1D quantum quasicrystal to and from a 2D electron gas in the integer quantum Hall regime [10–13], in which the quasicrystal encodes the additional dimension in its phasonic degree of freedom [12,14]. The Aubry-André localization phase transition in one dimension [15] corresponds to a 90° rotation of directional conductivity in the 2D model [Fig. 1(a)] [11,16–20], which was recently proposed to be potentially observable in strained Moiré materials even in the presence of interactions [21,22]. This mapping, which is extended here to a time-dependent context, also enables the counterintuitive realization of polarized driving in one dimension. A time-dependent dipolar electric field in the 1D quasicrystal maps to an electric field along one dimension in the 2D Hofstadter model, and a time-dependent phasonic mode in the quasicrystal maps to an electric field along the other dimension in two dimensions [Fig. 1(b)]. The combination of both modulations with variable amplitude and relative phase in 1D then directly corresponds to tunably polarized laser illumination of the 2D Hofstadter model. The experiment is capable of exploring polarization-dependent light-induced quantum phase transitions at extremely strong effective drive amplitudes.

We experimentally measure the effects of strong polarized driving by observing the expansion of a quantum gas in a doubly driven 1D quasiperiodic optical lattice. The results reveal an intricate tessellated metal-insulator quantum phase diagram controlled by the relative strengths and

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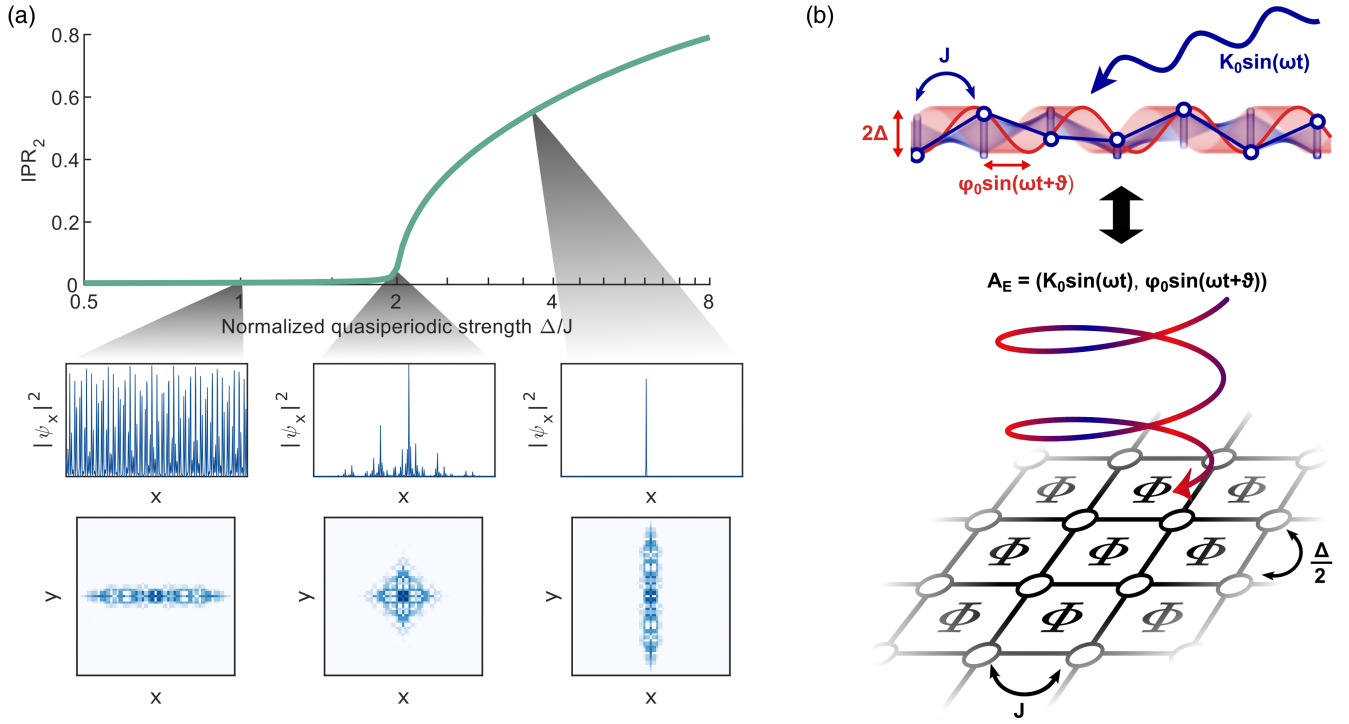


FIG. 1. Correspondence between modulated 1D quasicrystal and optically driven 2D crystal. (a) The undriven 1D AAH system undergoes a localization phase transition as the quasiperiodic strength Δ increases, diagnosed by the inverse participation ratio (IPR) and the density profiles of a representative eigenstate. The localization phase transition in one dimension is mapped to a 90° rotation of directional localization in two dimensions, as shown by the calculated time-evolved 2D density profile. (b) The 1D quasicrystal is subjected to both dipolar and phasonic modulation. In two dimensions, the quasiperiodic strength Δ becomes the tunneling along the additional dimension and thus controls the tunneling anisotropy (see also Appendix E). The dipolar and phasonic modulations map to orthogonal components of the vector potential of the irradiation in two dimensions.

phases of the two driving fields, which can be understood from the 2D perspective as a result of competition between drive-induced dynamic localization [7,23] along different dimensions [9,23], leading to coherent switching of directional localization by light. The polarization of the driving field in the higher-dimensional space plays a crucial and nontrivial role: for example, we observe that an elliptically polarized field produces anomalous transport attributable to eigenstate multifractality, whereas a linearly polarized field of the same intensity does not. This anomalous transport is counterintuitively enhanced rather than suppressed by the quasiperiodic potential. While noninteracting multifractal quantum states have recently attracted intense theoretical interest [9,24–33], experimental observations have been limited to date by the need for fine-tuned Hamiltonians [25,30,32]. The prospect of Floquet-engineering multifractal phases has been discussed and debated for decades [9,27,29,34–37]. Our results constitute experimental evidence that multifractality can arise simply and without any fine-tuning, under a range of amplitudes of elliptically polarized illumination.

Each experiment begins with a ^{84}Sr Bose-Einstein condensate adiabatically loaded into a combined crossed optical dipole trap and 1D quasiperiodic bichromatic lattice. The dipole trap is suddenly switched off and

sinusoidal modulation of the two optical lattices comprising the quasiperiodic lattice is simultaneously switched on, allowing the condensate to evolve in the modulated quasicrystal potential. The deeper primary lattice of depth V_P provides lateral confinement against gravity and admits a tight-binding description, while the secondary lattice with depth $V_S \ll V_P$ creates spatial quasiperiodicity [38]. Each lattice is formed by two counterpropagating beams which can be individually frequency modulated. Frequency modulation of one beam in the primary lattice creates an inertial force on the atoms which mimics the dipolar force on charges in an oscillating electric field. Relative motion between the two lattices introduces a phasonic modulation [9,20,23]. The resulting Hamiltonian in the velocity gauge is [20]

$$\hat{H}(t) = \sum_j -J e^{-iK_0 \sin(\omega t)} \hat{c}_{j+1}^\dagger \hat{c}_j + \text{H.c.} + \Delta \cos[2\pi\beta j + \kappa + \varphi_0 \sin(\omega t + \vartheta)] \hat{n}_j, \quad (1)$$

where J is the tunneling strength, Δ is the quasiperiodic strength, and $\beta \approx 1.2165$ is the incommensurate ratio set by the ratio of lattice constant of the two optical lattices. This Hamiltonian neglects interparticle interactions, and we

have verified experimentally that atom-atom interactions play a negligible role in the phenomena we report [20]. We set $V_P = 9E_{R,P}$ with $E_{R,P} \approx \hbar \times 2.1$ kHz the recoil energy from the primary lattice, leading to a tunneling time $T_J = \hbar/J = 3.1$ ms and a tunneling strength $J \approx \hbar \times 50.9$ Hz. K_0 is the dipolar modulation amplitude and φ_0 is the phasonic modulation amplitude; $\omega = 2\pi \times 500$ Hz $\approx 9.8J/\hbar$ is the modulation frequency, which is within the first gap of the unperturbed primary lattice and above the total bandwidth of the model [39]. At zero drive amplitude $K_0 = \varphi_0 = 0$ the Hamiltonian reduces to the well-known Aubry-André model [15], also corresponding to one Fourier slice of the 2D Harper-Hofstadter model [10,11,16] with the 2D annihilation (creation) operators defined through the Fourier transform $\hat{c}_{j,k}^{(\dagger)} = \sum_l e^{-ikl} \hat{a}_{j,l}^{(\dagger)}$. It features a localization quantum phase transition at $|\Delta/J| = 2$. Finite-strength dipolar and phasonic driving effectively rescale J and Δ , respectively, and can thereby move the quantum phase transition in either direction [9,23,40].

A crucial and unique feature of this experiment is the ability to control the temporal phase difference ϑ between the dipolar and phasonic modulations. From the perspective of the cross-dimensional mapping, this temporal phase difference maps exactly to the orientation angle of the polarization ellipse in 2D [9,20]. Thus, we can fully control the state of the polarization of the irradiation: applying linear, circular, or elliptical polarization in the 2D space is simply a matter of selecting the appropriate amplitudes K_0 and φ_0 and the phase ϑ .

Our main observable is the expansion of the atoms as measured by the time-dependent width $\sigma_x(t)$, extracted by fitting the density distribution to a bimodal Gaussian (see Appendix A). We define the fractional expansion $\sigma := (\sigma_x - \sigma_0)/\sigma_0$ where $\sigma_0 \approx 10$ μm is the initial width of the condensate, to account for the small but measurable dependence of the initial width on the quasiperiodic strength Δ [20]. In a localized phase, $\sigma \approx 0$, while σ increases with time in a delocalized phase. To characterize the phase diagram of the Hamiltonian with experiments with fixed evolution time, we further define an effective fractional expansion $\sigma_{\text{eff}} = \sigma / \max(|\mathcal{J}_0(K_0)|, \langle\delta\sigma\rangle)$ which normalizes with respect to the Bessel-modified hopping rate [7]. Here $\langle\delta\sigma\rangle \approx 0.12$ is the standard deviation of repeated measurements, averaged over the entire phase diagram.

The first main result of this work is the experimental measurement of the phase diagram of Hamiltonian (1) as a function of both the dipolar and phasonic modulation amplitude. Figure 2 shows predicted and measured phase diagrams at different quasiperiodic potential strengths Δ after the same evolution time $t_{\text{hold}} = 1.25$ s $\approx 400T_J$. Note that the color axes represent distinct but complementary quantities for the theoretical and experimental diagrams: inverse participation ratio (IPR) of the Floquet eigenstates in the position basis $\sum_j |\psi_j|^4$ [20] is the numerically accessible spectral diagnostic for localization, while σ_{eff}

is the experimentally measurable dynamical one. The IPR and the real-space expansion provide complementary probes of the localization phase diagram, and Fig. 2 demonstrates the agreement between them. The competition between the dipolar and phasonic modulations in one dimension, and thus between the x and y components of the drive field in two dimensions, leads to tessellated phase diagrams featuring interlaced regions of localized and delocalized phases. From the 2D perspective, these results demonstrate that intense polarized irradiation rescales the tunneling anisotropy, coherently switching the axis of conductivity.

The main features of the measured phase diagram can be understood analytically. Averaged over one drive cycle, the dipolar drive rescales the effective tunneling strength by a zeroth-order Bessel function: $J_{\text{eff}} = \langle J e^{-iK_0 \sin(\omega t)} \rangle_T = J \mathcal{J}_0(K_0)$. At the Bessel zeros where $J_{\text{eff}} = 0$, the resulting dynamic localization [7] freezes transport along the physical dimension and thus favors a localized phase [40]. Conversely, the phasonic drive rescales the effective quasiperiodic strength according to $\Delta_{\text{eff}} = \Delta \mathcal{J}_0(\varphi_0)$, and thus favors a delocalized phase [23]. The cross-dimensional mapping makes an exact analogy to dynamic localization apparent: in the 2D space, the quasiperiodic strength Δ is the tunneling and $\varphi(t)$ is the field component along the additional dimension [Fig. 1(b)]. The position of the quantum phase transition in the $K_0 - \varphi_0$ plane thus satisfies $|\mathcal{J}_0(\varphi_0)/\mathcal{J}_0(K_0)| = 2J$; these are the boundary lines drawn on Fig. 2.

Tuning the polarization of the driving radiation reveals additional rich physics [9]. In the second main result of this work, we experimentally measure and theoretically characterize the nontrivial effects of varying the drive polarization. The polarization is straightforwardly tuned by adjusting the phase difference ϑ between dipolar and phasonic modulations. In practice, this is implemented by delaying the modulation of the secondary lattice by an amount of time τ_{delay} . For elliptically polarized irradiation in 2D ($\vartheta = \pm\pi/2$), $\tau_{\text{delay}} = 0$, while for linearly polarized irradiation ($\vartheta = 0$), $\tau_{\text{delay}} > 0$ is completely determined by the desired dipolar and phasonic modulation amplitudes (K_0, φ_0) and the driving frequency ω [20].

As a starting point for these investigations, we consider the dynamically localized strip where $K_0 = K_0^{\text{DL}} = 2.405$. Here $J_{\text{eff}} \approx 0$: kinetic energy is effectively eliminated to zeroth order, so naive theory predicts a localized phase for essentially all values of the phasonic drive amplitude φ_0 since $\Delta_{\text{eff}}/J_{\text{eff}} \rightarrow \infty$. In accordance with this expectation, for linearly polarized modulation we measure barely any expansion at any φ_0 even after $t_{\text{hold}} = 10$ s $\approx 3200T_J$ [diamonds in Fig. 3(a)]. However, when we apply elliptically polarized modulation with exactly the same dipolar and phasonic amplitudes, the results differ drastically, in a way which disagrees sharply with the naive expectation of featureless localization. In particular, we observe peaks in

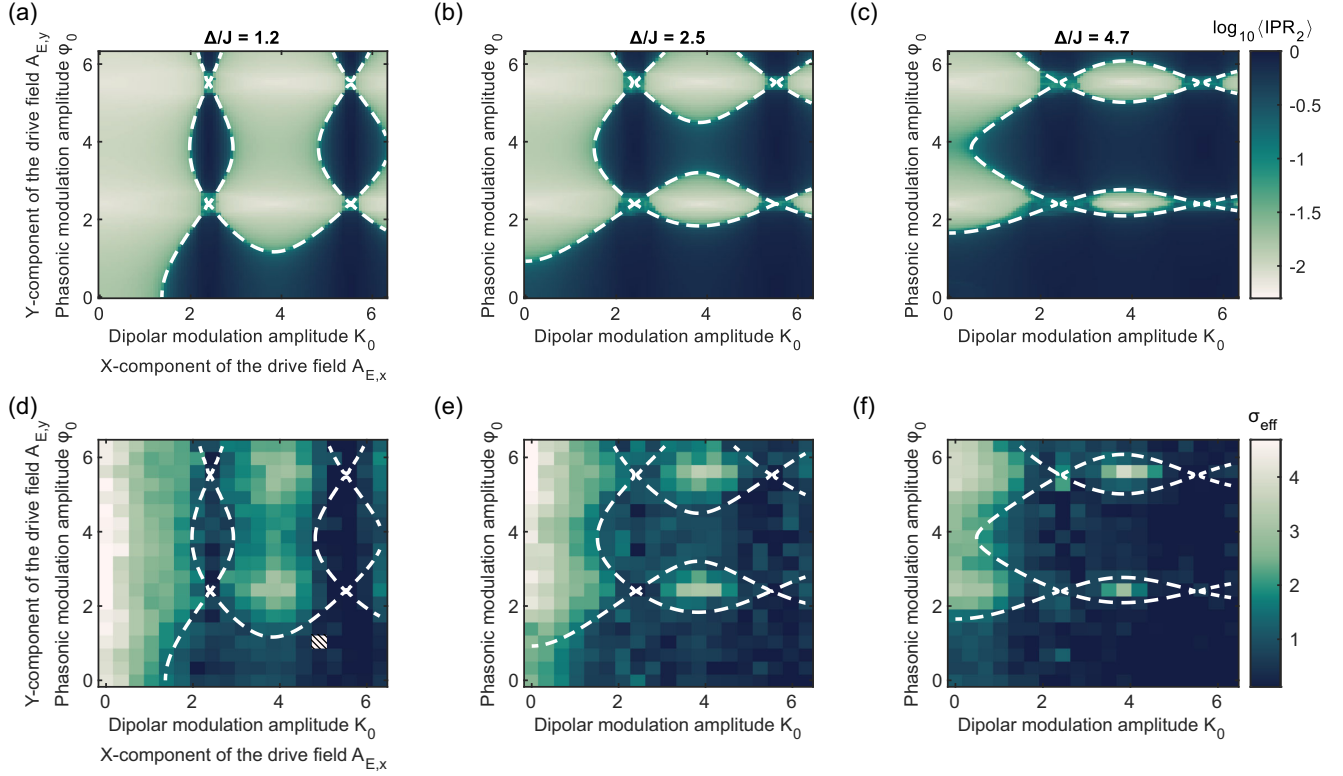


FIG. 2. Interlaced localization-delocalization quantum phase transitions controlled by polarized driving. Figure shows theoretically predicted and experimentally measured phase diagrams at quasiperiodic strengths $\Delta/J \approx 1.2, 2.5$, and 4.7 ; below, near, and above the critical strength without driving. Axes represent dipolar and phasonic components of the drive in one dimension, corresponding to x and y components of the driving electric field in two dimensions. (a)–(c) Theoretically calculated inverse participation ratio (IPR) averaged over all Floquet eigenstates in the central Brillouin zone. System size is 200 lattice sites. (d)–(f) Measured effective fractional width σ_{eff} after $t_{\text{hold}} = 1.25 \text{ s} \approx 400T_J$. The patched data point in (d) is cut due to significant atom loss. All data, theoretical or experimental, are taken with $\vartheta = \pi/2$ (elliptical polarization).

the expansion rate when φ_0 is near the zeros of $\mathcal{J}_0(\varphi_0)$, at $\varphi_0 \approx 2.405$ and 5.520 where the time-averaged Hamiltonian vanishes completely.

What drives this clear difference in transport between the two drive polarizations? To probe this question experimentally, we focus on the first delocalized region around $\varphi_0 \approx 2.405$ and measure the expansion exponent γ that quantifies and connects the transport dynamics to the fractal dimension [42]. This is extracted by fitting the width $\sigma_x(t)$ to the phenomenological relation $\sigma_x(t)/\sigma_0 = (1 + t/t_0)^\gamma$ where t_0 represents an “activation time” [43]. We use t_{hold} values up to $20 \text{ s} (\approx 6400T_J)$ to minimize the effects of initial state dependence. In a localized phase, $\gamma \approx 0$; in a ballistic phase, $\gamma \approx 1$; for diffusive dynamics, $\gamma = 0.5$. As shown in Fig. 3(c), the measured exponents γ are always close to zero for the linearly polarized driving case, corresponding to a localized phase. In the case of elliptically polarized driving, however, the exponents are non-vanishing within a finite window of phasonic amplitudes φ_0 near 2.4 . For φ_0 outside this region, the exponent drops close to zero. The fact that both measurements are taken with identical drive amplitudes K_0 and φ_0 suggests that the

observed subdiffusive expansion is not due to mean-field interaction [43] or transverse heating [44,45], but rather originates from the spectral structure [42]. We interpret these results as evidence for the presence of a subdiffusive delocalized phase which emerges only in the presence of elliptically polarized driving.

This interpretation is also supported by further theoretical analysis. Motivated by the connection between the expansion exponent and eigenstate fractal dimension [42], we compute the fractal dimension D_2 of the Floquet eigenstates of Hamiltonian (1) under the condition of elliptically polarized modulation. The results are shown in Fig. 3(b). For ballistic (localized) phases, $D_2 = 1(0)$ for all eigenstates and there is no fractality. In contrast, we calculate that $D_2 \neq 0, 1$ for some eigenstates in a window of φ_0 corresponding exactly to the window where we observe nontrivial expansion exponents. These nontrivial values of D_2 indicate eigenstate multifractality without fine-tuning to a critical point. Our calculation suggests the presence of a multifractal phase: a nonvanishing fraction of eigenstates are multifractal over a range of parameters rather than just at one critical point [41,46,47].

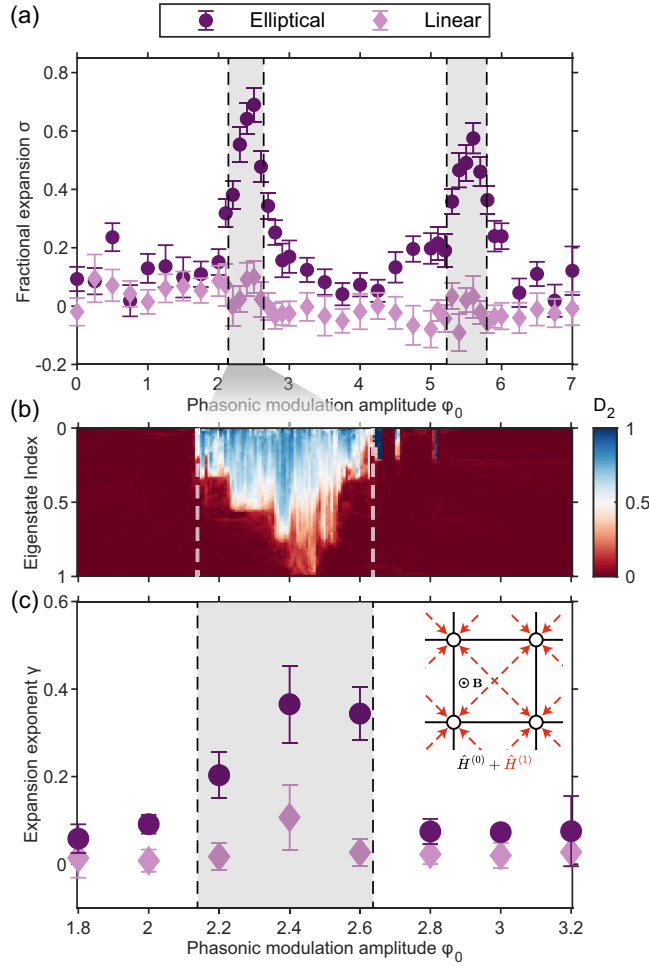


FIG. 3. Elliptically polarized driving induces exotic transport. Phasonic driving modifies the dynamically localized state ($K_0 \approx 2.405$) in a polarization-dependent way. (a) Fractional expansion σ as a function of phasonic modulation amplitude ϕ_0 , for elliptical (circles) or linear (diamonds) polarization. For these data $t_{\text{hold}} = 10$ s and $\hbar\omega = h \times 500$ Hz $\approx 9.8J$. Dashed lines and shaded areas represent the theoretically predicted phase transition into a multifractal phase $|\Delta_{\text{eff}}| = 2|J_{\text{EP}}|$ [see Eq. (2) and [41], and discussion in the text] for elliptically polarized driving. Error bars represent standard error of the mean (s.e.m.) of repeated measurements. (b) Theoretically calculated fractal dimension D_2 of all eigenstates, zoomed in to the region $\phi_0 \in [1.8, 3.2]$. (c) Measured expansion exponents for elliptically and linearly polarized driving, using the same experimental parameters. Error bars represent 95% confidence interval of the fit. Inset is a visualization of the effective Hamiltonian under elliptical polarization in two dimensions, where red dashed arrows represent NNN hopping [Eq. (2)].

An especially clear picture of this observation emerges in the 2D perspective. As has been known for more than a decade from the study of Floquet topological insulators [6], elliptically polarized irradiation in two dimensions can drive effective next-nearest-neighbor (NNN) hopping in real space due to time-reversal symmetry breaking, whereas linearly polarized irradiation cannot. For our rectangular

Hofstadter model, only elliptically polarized radiation can generate an effective NNN hopping across the diagonal of a plaquette. It has long been known that such a beyond NN hopping can lead to a broad multifractal phase [41,46,47]. This theoretical picture supports the conclusion that we can engineer multifractal phases by applying elliptically polarized irradiation in the presence of irrational flux per plaquette [9]. Quantitatively, these predictions can be derived from a high-frequency expansion [48,49] that leads to the effective Hamiltonian $\hat{H}_{\text{eff}} \approx \hat{H}^{(0)} + \hat{H}^{(1)}$, whose 2D correspondence is schematically shown in the inset of Fig. 3(c). For $K_0 = K_0^{\text{DL}}$, $\hat{H}^{(0)}$, the time-averaged term, is just the quasiperiodic potential dressed by the phasonic modulation, since $J_{\text{eff}} = 0$. For linearly polarized driving, $\hat{H}^{(1)} = 0$. For elliptically polarized driving corresponding to Fig. 3,

$$\hat{H}^{(1)} = J_{\text{EP}}(\phi_0) \sum_j \cos(2\pi\beta j + \pi\beta + \kappa) \hat{c}_{j+1}^\dagger \hat{c}_j + \text{H.c.}, \quad (2)$$

where the tunneling energy scale is

$$J_{\text{EP}}(\phi_0) = \frac{4J\Delta}{\hbar\omega} \sin(\pi\beta) \times \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2n+1} \mathcal{J}_{2n+1}(K_0^{\text{DL}}) \mathcal{J}_{2n+1}(\phi_0). \quad (3)$$

The full expression of $\hat{H}^{(1)}$ in a general setting can be found in [9] and Supplemental Material [20]. The elliptically polarized drive thus leads to spatially quasiperiodic tunneling which is projected from the NNN tunneling in two dimensions [41] and which creates a multifractal delocalized phase. This expression allows us to quantitatively predict the phase boundary between a localized and a critical phase at $|\Delta_{\text{eff}}| = 2|J_{\text{EP}}(\phi_0)|$ [41]. The resulting phase boundaries shown in Figs. 3 and 4 demonstrate excellent agreement with the experimental measurements of width σ and exponent γ , and with the predicted multifractal regime of noninteger D_2 . Taken together, these results thus provide strong evidence for multifractality-driven anomalous transport due solely to the polarization of the applied drive. We note that the experimental conditions and physical mechanism of the observed phasonically driven delocalization are completely different from those observed previously in [23]. The similarity in the functional shape is simply because the delocalized phases, ballistic or multifractal, are around the zeros of the Bessel function $\mathcal{J}_0(\phi_0)$. The delocalization observed in [23], corresponding to $K_0 = 0$ and thus linearly polarized modulation, is driven by the rescaling of Δ_{eff} and does not involve any underlying eigenstate multifractality.

A striking further prediction of the theory described in Eqs. (2) and (3) is that the anomalous expansion driven by elliptically polarized irradiation should be enhanced, rather

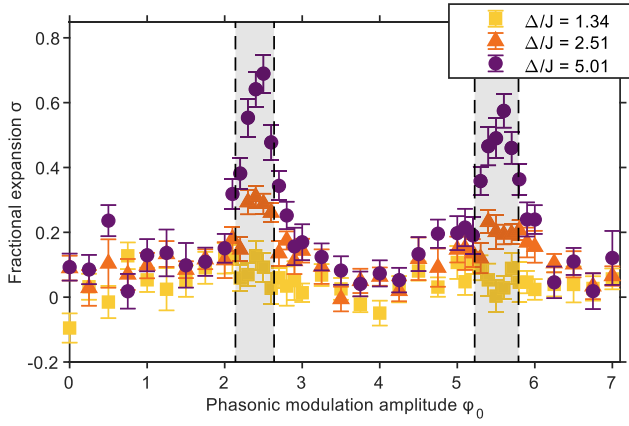


FIG. 4. Anomalous expansion under elliptically polarized modulation is enhanced by the quasiperiodic strength. As the quasiperiodic strength Δ is increased, the expansion increases in the shaded region representing a multifractal phase, but remains localized otherwise. Here the condensate expands for $t_{\text{hold}} = 10$ s for all datasets. Error bars represent s.e.m. of repeated measurements.

than suppressed, for larger values of the quasiperiodic potential strength Δ . This counterintuitive prediction, while not obvious from the time-dependent Hamiltonian itself, naturally emerges from the effective Hamiltonian description, since $\hat{H}^{(1)}$ resembles a quasiperiodically modulated kinetic energy term whose strength is proportional to Δ . The third and final main result of this work is the experimental confirmation of this prediction, shown in Fig. 4. The two transport peaks corresponding to the anomalous phase become much more pronounced as quasiperiodic strength increases, from being barely visible at $\Delta/J \approx 1.34$ to very prominent at $\Delta/J \approx 5.01$. This demonstrates the predicted enhancement of anomalous transport by the quasiperiodic potential. From the 2D perspective, increasing Δ , while suppressing the transport along the physical dimension, enhances the strength of NNN hopping that supports the anomalous multifractal dynamics [see inset of Fig. 3(c)].

To sum up, we have experimentally and theoretically investigated a doubly driven 1D quasiperiodic lattice that maps to an anisotropic Hofstadter model illuminated by light of tunable polarization. Measurement of the phase diagram demonstrates that the amplitude and polarization of applied radiation coherently control the localization quantum phase transition in one dimension [7,23,40], a phenomenon which maps to switching the direction of conductivity by intense irradiation in the 2D Hofstadter model. These results can be interpreted as observation in a 1D quantum system of a driven 2D Hamiltonian important in condensed matter, or equivalently as introducing the concept of variable light polarization to 1D. We observe that linearly and elliptically polarized drives generate strikingly different dynamics for identical drive intensities. We experimentally and theoretically demonstrate that elliptically polarized illumination can generate without

any fine-tuned parameters an anomalous critical phase featuring multifractal eigenstates and subdiffusive transport, and confirm experimentally that transport in the anomalous phase is enhanced, rather than suppressed, by the strength of quasiperiodicity. Our protocol challenges the traditional understanding of Floquet multifractality in which localized and ballistic phases are mixed with a low-frequency drive [27,29], and where the multifractality is predicted to disappear in the thermodynamic limit. In contrast, our protocol is in the high-frequency regime without any spectral mixing.

These results provide a novel and straightforward roadmap for engineering multifractal matter on demand, potentially implementable in a variety of other platforms. While interactions are not important in the dynamics we discuss here, straightforward extensions of the approach could study the same physics in a strongly interacting regime, probing the existence and stability of a Floquet many-body critical phase [50] or quasifractal charge density wave [33]. Extensions of techniques demonstrated here could also be used to realize chiral control of magnetism [51], and probe higher-dimensional quantum geometry using lower-dimensional circular dichroism [52–54]. Driving with more exotic electric fields, for example with Lissajous or unpolarized character, could create novel topological properties [55–57]. The demonstration of spatially inhomogeneous effective tunneling from a global modulation may open up new and simpler avenues to the study of synthetic event horizons [58,59]. A naturally complementary direction of investigation is towards solid-state realizations of related light-induced phenomena. The approach presented here can provide both path finding and motivation for such work and a complementary toolset for joint investigation of non-equilibrium quantum matter [8,21,54].

We acknowledge experimental assistance from Petros Kousis and Peter Dotti and helpful discussions with Xiao Chai and Jeremy Tanlimco. We acknowledge research support from the Air Force Office of Scientific Research (FA9550-20-1-0240), the NSF QLCI program (OMA-2016245), the Noyce Foundation, and the UC Santa Barbara NSF Quantum Foundry funded via the Q-AMASE-i program under Grant No. DMR1906325. A. R. D. acknowledges support from the NSF NRT program under Grant No. 2152201. The development of the bichromatic optical lattice techniques which are the basis for this work was supported by the U.S. Department of Energy, Office of Science, National Quantum Information Science Research Centers, Quantum Science Center.

Y. B., A. R. D., and T. S. prepared the experimental setup and performed the experiments. Y. B. conceptualized the project and performed theoretical calculations and data analysis. D. M. W. supervised the project. All authors contributed substantially to the work presented in this

paper, including discussions of data and preparation of the manuscript.

All authors declare no competing interests.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: METHODS

The experiments described in the main text commence by adiabatically loading a Bose-Einstein condensate of about 2×10^5 ^{84}Sr atoms into the combination of a 1D bichromatic optical lattice along an axis perpendicular to gravity and a weak crossed dipole trap. The experiments begin when the crossed dipole trap is suddenly switched off and the lattice modulation simultaneously starts.

The primary lattice is generated by two counterpropagating laser beams with wavelength $\lambda_P = 1064$ nm. One of the beams composing the primary lattice has a greater intensity to provide transverse confinement and support the atoms against gravity. The combined potential including gravity has a trap depth of about 1.4 μK . We estimate the radial and longitudinal trap frequencies of to be $\omega_r = 2\pi \times 117.8$ and $\omega_{\text{long}} = 2\pi \times 0.36$ Hz. The amplitudes and frequencies of the two beams comprising the primary lattice are independently controlled using acousto-optic modulators (AOMs). In all experiments, V_P is maintained at $9E_{\text{rec},P}$, where $E_{\text{rec},P} = \hbar^2 k_P^2 / 2m$, calibrated first with Kapitza-Dirac diffraction and then with dipolar modulation spectroscopy [60].

The two laser beams that compose the secondary lattice have wavelength $\lambda_S = 874.61$ nm and are derived from a continuous-wave titanium-sapphire laser whose wavelength is stabilized against drift using a wave meter.

The two beams have roughly equal intensity and are adjusted in tandem to control the depth of the secondary lattice V_S . Similarly to the case of the primary lattice, the amplitude and frequency of each secondary lattice beam is independently controlled using AOMs.

Expansion of the atoms is quantified by fitting the density distribution to a narrow Gaussian atop a broad shallow pedestal which due to a background of thermally excited atoms.

The radiofrequency drives of the AOMs employed in the position modulation of the lattices are generated by 2-channel arbitrary waveform generators (Keysight 33600A) such that the first channel modulates the frequency of the second channel which is applied to the AOMs. All sources share a clock signal derived from the same Rubidium reference (FS725, Stanford Research Systems).

APPENDIX B: PHASE DIAGRAMS WITHOUT THE BESSEL FUNCTION SCALING

We report the drive-controlled phase diagram probed by an effective width σ_{eff} , defined as the fractional expansion scaled by the norm of the zeroth Bessel function of the dipolar amplitude, $|\mathcal{J}_0(K_0)|$. This is motivated by the fact that the expansion speed is proportional to the bandwidth in a clean lattice. In Fig. 5, we include the unscaled phase diagrams with the fractional expansion. While the contrast is lower, these phase diagrams still clearly demonstrates the phase boundary.

APPENDIX C: ANOMALOUS EXPANSION DEPENDENCE ON DRIVING FREQUENCY

The tunneling strength J_{EP} in Eq. (3), characterizing the strength of the anomalous expansion, decreases as the driving frequency increases. Thus the anomalous expansion should be suppressed with higher driving frequency. Results shown in Fig. 6 directly test this prediction. Indeed, we observed a slower anomalous expansion with increasing driving frequency. These results provide another support of our theoretical modeling through effective Hamiltonian.

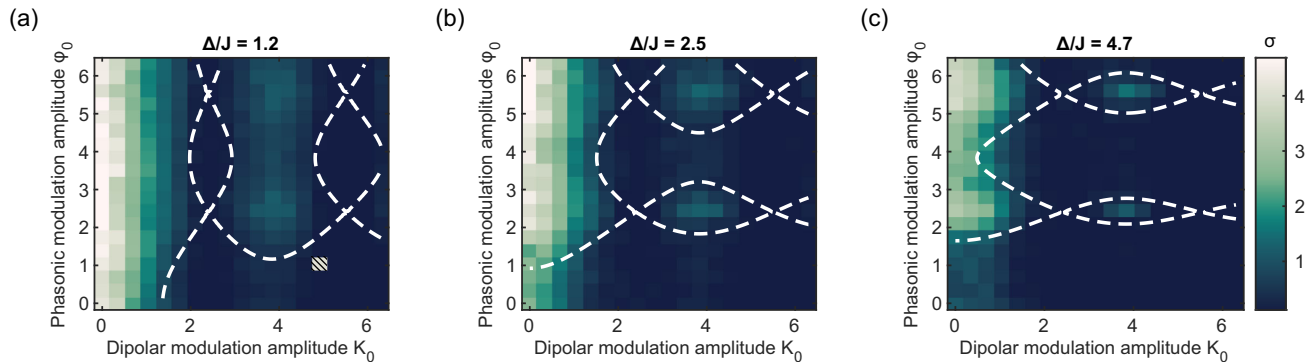


FIG. 5. Unscaled fractional expansion σ under dipolar and phasonic amplitudes. While the delocalized regions are still visible, their contrast is lower due to effectively reduced kinetic energy. Dashed white curves represent the theoretically predicted phase boundary.

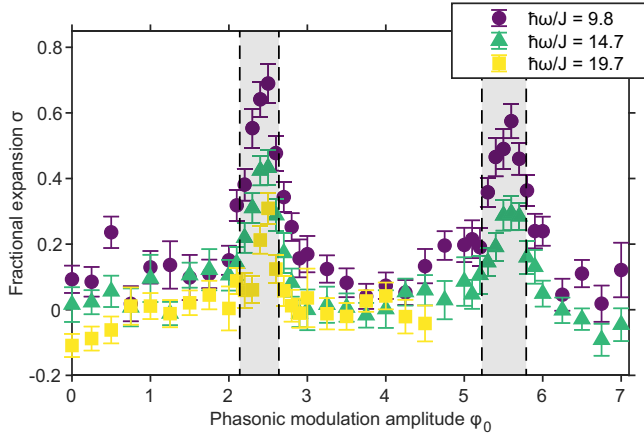


FIG. 6. Dependence of anomalous expansion under elliptically polarized modulation with respect to the driving frequency $\hbar\omega$. The expansion is suppressed as the driving frequency is increased, according to Eq. (3). The condensate expands for $t_{\text{hold}} = 10$ s for all cases here. Only part of the data for $\hbar\omega = 19.7J$ are displayed due to excessive atom loss from large phasonic amplitude and high driving frequency. Error bars are one s.e.m.

APPENDIX D: EXPERIMENTAL BENCHMARK OF THE EVOLUTION

We experimentally benchmark our expansion dynamics up to the longest possible evolution time of $t_{\text{hold}} = 20$ s $\approx 6400T_J$. Specifically, we measure the expansion of the condensate in the shaken primary lattice only (that is, $V_S = 0$), at $K_0 = 2.2$ and $K_0 = K_0^{\text{DL}} = 2.405$. Since there is no quasiperiodicity, the condensate expands ballistically when $K_0 = 2.2$ despite a greatly reduced kinetic energy [$\mathcal{J}_0(K_0) \approx 0.11$], and dynamically localizes when $K_0 = K_0^{\text{DL}}$. The resulting data are shown in Fig. 7,

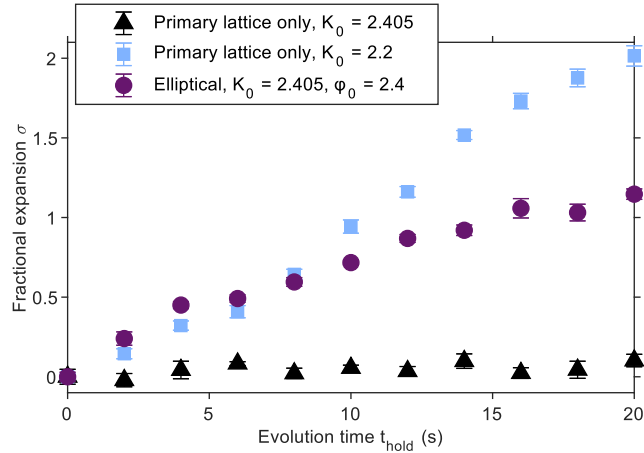


FIG. 7. Benchmarking condensate expansion. Measured width of the condensate during time evolution of up to $t_{\text{hold}} = 20$ s. For the case with primary lattice only, ballistic expansion and dynamic localization are maintained during the entirety of the time evolution. The case with critical subdiffusive expansion is also shown for comparison. Error bars are one s.e.m.

demonstrating that we observed the expected ballistic and dynamically localized dynamics for up to $t_{\text{hold}} = 20$ s, indicating that interaction effects did not qualitatively affect either state. We further include the measured expansion close to the first peak of the anomalous phase ($K_0 = 2.405$, $\varphi_0 = 2.4$, $\vartheta = \pi/2$), which shows the characteristic sub-ballistic expansion. Since its width is smaller than its ballistic counterpart during the time evolution, we conclude that the reduced expansion is not due to other experimental imperfections such as residual longitudinal trapping but rather reflects the physics discussed in the main text.

APPENDIX E: A SHORT REVIEW OF THE HARPER-HOFSTATDER MAPPING

Here we briefly review the cross-dimensional mapping from the 2D Harper-Hofstadter model to the 1D Aubry-André-Harper model, first described by Harper [10] and subsequently Hofstadter [11] and others. The 2D Harper-Hofstadter model describes a non-interacting electron in a 2D rectangular lattice, with nearest-neighbor hopping and a magnetic flux $\beta = \Phi/\Phi_0$ per plaquette, where Φ_0 is the flux quantum. In the Landau gauge, the Hamiltonian is

$$\hat{H}_{\text{Harper}} = \sum_{j,l} -J\hat{d}_{j+1,l}^\dagger \hat{d}_{j,l} + J_y e^{-i2\pi\beta j} \hat{d}_{j,l+1}^\dagger \hat{d}_{j,l} + \text{H.c.}, \quad (\text{E1})$$

where J is the tunneling strength along the x direction (site labelled by j) and $J_y := \Delta/2$ the tunneling strength along the y direction (labeled by l), and $d_{j,l}^{(\dagger)}$ is the annihilation (creation) operator in two dimensions. This 2D Hamiltonian is independent of the y direction, so we can employ a plane wave basis along that direction. We define the annihilation (creation) operators $\hat{c}_{j,\kappa}^{(\dagger)}$ with anticommutation relation $\{\hat{c}_{j,\kappa}, \hat{c}_{j',\kappa'}^\dagger\} = \delta_{j,j'}\delta_{\kappa,\kappa'}$. They are related to the 2D operators as $\hat{c}_{j,\kappa}^{(\dagger)} = \sum_l e^{-i\kappa l} \hat{d}_{j,l}^{(\dagger)}$. With these newly defined operators, we arrive at the 1D Hamiltonian H_{AAH} :

$$\begin{aligned} \hat{H}_{\text{Harper}} &= \int_0^{2\pi} \frac{d\kappa}{2\pi} \hat{H}_{\text{AAH}}(\kappa), \\ \hat{H}_{\text{AAH}}(\kappa) &= \sum_j -J\hat{c}_{j+1}^\dagger \hat{c}_j + \text{H.c.} + \Delta \cos(2\pi\beta j + \kappa) \hat{n}_j, \end{aligned} \quad (\text{E2})$$

where $\hat{n}_j = \hat{c}_j^\dagger \hat{c}_j$ and we drop the κ dependence in the operator. Accordingly, we refer to the x direction as the “physical” dimension and the y direction as the “additional” dimension.

This 1D Hamiltonian is known as the Aubry-André-Harper (AAH) model [15] and can be thought of as representing a 1D lattice with a spatially modulated chemical potential. Here Δ becomes the strength of the

modulating potential and β , the incommensurate ratio, is the ratio between the period of the potential modulation and the lattice spacing. When β is irrational the system is quasiperiodic and exhibits a localization quantum phase transition from a delocalized ballistic phase ($\Delta < 2J$) to a localized phase ($\Delta > 2J$) with exponentially localized wave functions in position space.

The inclusion of external driving field $\mathbf{A}(t)$ is achieved by essentially the same procedure with the addition of Peierls substitution, replacing $J_{x,y} \rightarrow J_{x,y} \exp[-iA_{x,y}(t)]$, respectively. In Supplemental Material [20], we discuss the dynamical consequence of this mapping and its inclusion of the driving field in details.

APPENDIX F: FROM SHAKEN 1D BICHROMATIC LATTICE TO DOUBLY DRIVEN AAH MODEL

In this section, we detail the recipe to create the polarized driving protocol through combined sublattice modulation.

For ultracold atoms trapped in a doubly driven 1D bichromatic lattice potential, the Hamiltonian is

$$\hat{H}_{\text{bic}} = \frac{\hat{p}^2}{2m} + \frac{V_P}{2} \cos(2k_P[\hat{x} - x_P(t)]) + \frac{V_S}{2} \cos(2k_S[\hat{x} - x_S(t)] + \kappa), \quad (\text{F1})$$

where m is the mass of the trapped atoms, $k_{P,S}$ are the wave vectors and $V_{P,S}$ the depths of the primary and secondary lattice, respectively. The time-dependent terms $x_{P,S}(t)$ describe the position modulation (“shaking”) of the primary and secondary lattice, respectively, which are independently controlled. Only sinusoidal modulation is used in this work, so $x_{P,S}(t) = \alpha_{P,S} \sin(\omega t + \theta_{P,S})$ with amplitudes $\alpha_{P,S}$, phases $\theta_{P,S}$ and identical driving frequency ω . A unitary transformation to the co-moving frame of the moving primary lattice shows that the inertial force from $x_P(t)$ acts as a dipolar, spatially homogeneous ac field $F(t) := m\ddot{x}_P(t)$:

$$\hat{H}_{\text{bic}} \rightarrow \frac{\hat{p}^2}{2m} + V_P \cos(2k_P \hat{x}) + V_S \cos(2k_S[\hat{x} - x_{\text{phason}}(t)] + \kappa) + \hat{x}F(t). \quad (\text{F2})$$

In this frame, the modulation of the secondary lattice, $x_{\text{phason}}(t)$, depends on the interference between $x_P(t)$ and $x_S(t)$:

$$x_{\text{phason}} = x_S(t) - x_P(t) := \alpha_{\text{phason}} \sin(\omega t + \theta_{\text{phason}}), \quad (\text{F3})$$

$$\alpha_{\text{phason}} = \pm \sqrt{\alpha_P^2 + \alpha_S^2 - 2\alpha_P\alpha_S \cos(\theta_S - \theta_P)} \quad (\text{F4})$$

$$\theta_{\text{phason}} = \text{atan2}(u, v), \quad (\text{F5})$$

where we define $u := \alpha_S \sin \theta_S - \alpha_P \sin \theta_P$, $v := \alpha_S \cos \theta_S - \alpha_P \cos \theta_P$ and $\text{atan2}(u, v)$ is the two-argument arctangent function.

We keep $V_P \gg V_S$ such that the locations of potential minima of primary lattice are not significantly altered by the secondary lattice [38]. We further restrict the driving frequency to be within the first gap of the unperturbed primary lattice. Under these conditions, the single-band, tight-binding approximation applies and the proper Hamiltonian can be written as $\hat{H} = \sum_j \hat{H}_j$, where

$$\begin{aligned} \hat{H}_j = & -J\hat{c}_{j+1}^\dagger \hat{c}_j + \text{H.c.} \\ & + \Delta \cos[2\pi\beta j + \varphi_0 \sin(\omega t + \theta_{\text{phason}}) + \kappa] \hat{n}_j \\ & + jK \cos(\omega t + \theta_x) \hat{n}_j, \end{aligned} \quad (\text{F6})$$

where $\hat{c}_j^{(\dagger)}$ is the annihilation (creation) operator at site j , $\hat{n}_j$ is the density operator at site j , and K is the strength of the force. Since the external force exactly resembles the dipolar force on a charge exposed to a spatially homogeneous, linear field, it is referred to as the dipolar modulation. The phasonic modulation $\varphi_0 \sin \omega t$ refers to the time dependence of the phason in the incommensurate potential. A further frame transformation to the velocity gauge absorbs the linear oscillating force into an oscillating phase in the tunneling term

$$\begin{aligned} \hat{H}_j = & -J e^{-iK_0 \sin(\omega t + \theta_x)} \hat{c}_{j+1}^\dagger \hat{c}_j + \text{H.c.} \\ & + \Delta \cos[2\pi\beta j + \kappa + \varphi_0 \sin(\omega t + \theta_{\text{phason}})] \hat{n}_j, \end{aligned} \quad (\text{F7})$$

We thus recover the doubly driven AAH model from the experimentally relevant Hamiltonian Eq. (F2).

To connect this Hamiltonian to what we actually control in the experiment, we first review and describe frequency modulation of an optical lattice, often referred to as shaking, a well-established technique for generating artificial gauge fields. We then extend such modulation to a shaken bichromatic lattice and connect the experimental parameters to those in the DDAAH model. Our method enables “polarized” dipolar and phasonic driving through time-dependent spatial translations $x_{P(S)}(t)$ of each of the optical lattice potentials along the lattice direction. We employ sinusoidal frequency modulation on the primary and secondary lattices:

$$f_P(t) = f_{0,P} \sin(\omega t), \quad f_S(t) = f_{0,S} \sin(\omega t + \phi), \quad (\text{F8})$$

where $f_{0,P(S)}$ is the experimentally relevant amplitude of frequency deviation between the laser beams forming the primary (secondary) lattice and sets the amplitude of the frequency modulation waveform, ω is the angular modulation frequency, and ϕ is the phase difference between the modulation waveforms on the primary and secondary lattices. In particular, we note that the phase difference

ϕ does not equate to the polarization phase ϑ . Their relation will be clarified shortly. The range of dimensionless shaking amplitudes K_0 and φ_0 examined in the experiments described in this paper is thus realized through appropriately set amplitudes and phases of the two frequency modulation waveforms $f_{0,P}$ and $f_{0,S}$.

The sinusoidal frequency modulation leads to time dependent position modulations

$$x_P(t) = \frac{d_P f_{0,P}}{\omega} - \frac{d_P f_{0,P}}{\omega} \cos(\omega t), \quad (\text{F9})$$

$$x_S(t) = \frac{d_S f_{0,S}}{\omega} \cos(\phi) - \frac{d_S f_{0,S}}{\omega} \cos(\omega t + \phi), \quad (\text{F10})$$

where $d_{P(S)} = \lambda_{P(S)}/2$ is the lattice spacing of the primary (secondary) lattice. Rewriting the position modulations as sinusoidal modulations $x_{P(S)} = \alpha_{P(S)} \sin(\omega t + \theta_{P(S)})$ leads to the expression of the shaking amplitudes $\alpha_{P(S)}$ and phases $\theta_{P(S)}$ in terms of the controllable parameters of the frequency modulation waveform:

$$\alpha_{P(S)} = \frac{d_{P(S)} f_{0,P(S)}}{\omega}, \quad \theta_P = -\frac{\pi}{2}, \quad \theta_S = \phi - \frac{\pi}{2}. \quad (\text{F11})$$

We now relate the experimentally accessible parameters to the parameters in the DDAAH Hamiltonian. The dimensionless dipolar shaking amplitude K_0 , which characterizes the amplitude of the field component parallel to the physical dimension, is determined only by the shaking amplitude α_P of the primary lattice:

$$K_0 = \frac{\pi}{\hbar k_P} m \omega \alpha_P = \frac{\pi}{4} \frac{f_{0,P}}{f_{\text{rec},P}}, \quad (\text{F12})$$

where $f_{\text{rec},P}$ is the recoil frequency of the primary lattice. The phasonic modulation $x_{\text{phason}}(t)$ is the time-dependent difference in position of the secondary and primary lattices, $x_{\text{phason}}(t) = x_S(t) - x_P(t)$. Thus, α_{phason} depends on both amplitudes α_P and α_S as well as the phase difference between the modulation waveforms as expressed in Eq. (F5). The dimensionless phasonic shaking amplitude φ_0 and the phase difference θ_{phason} are

$$\begin{aligned} \varphi_0 &= 2k_s \alpha_{\text{phason}} \\ &= \pm \frac{\beta}{f} \sqrt{f_{0,P}^2 + \frac{1}{\beta^2} f_{0,S}^2 - \frac{2}{\beta} f_{0,P} f_{0,S} \cos(\phi)}, \end{aligned} \quad (\text{F13})$$

$$\theta_{\text{phason}} = \text{atan2}(\beta f_{0,P} - f_{0,S} \cos \phi, f_{0,S} \sin \phi), \quad (\text{F14})$$

in terms of the modulation waveform parameters, where $\beta = \lambda_P/\lambda_S$ again is the incommensurate ratio of the bichromatic lattice and $\text{atan2}(y, x)$ is the two-argument arctangent. We find that choosing the expression of φ_0 with

TABLE I. Relations of drive parameters in the Hamiltonian [Eq. (1)] to directly experimentally controllable quantities.

Parameters	Expressions
Dipolar amplitude K_0	$(\pi/4) f_{0,P}/f_{\text{rec},P}$
Phasonic amplitude φ_0	$\pm \frac{\beta}{f} \sqrt{f_{0,P}^2 + \frac{1}{\beta^2} f_{0,S}^2 - \frac{2}{\beta} f_{0,P} f_{0,S} \cos(\phi)}$
Polarization ϑ	$\text{atan2}(\beta f_{0,P} - f_{0,S} \cos \phi, f_{0,S} \sin \phi)$

the negative sign offers us greater experimental tunability; this choice simply corresponds to a convention regarding the chirality of the polarization.

Finally, the polarization phases θ_x , θ_y and $\vartheta = \theta_y - \theta_x$ are simply

$$\theta_x = \theta_P + \frac{\pi}{2} = 0, \quad \theta_y = \theta_{\text{phason}} \longrightarrow \vartheta = \theta_{\text{phason}}.$$

All relevant relations between the driving parameters in the Hamiltonian [Eq. (1)] and the experimentally controlled quantities are summarized in Table I.

We now discuss how the polarization of the drive is controlled in the experiment. Operationally, we first choose the desired dimensionless parameters K_0 , φ_0 and the polarization ϑ , and then compute the required experimental parameters $f_{0,P}$, $f_{0,S}$ and ϕ which are sent to the AWG. Thus, in this section, we express these experimental parameters in terms of the dimensionless model parameters K_0 , φ_0 and ϑ . Here we explicitly restrict the term “elliptical polarization” to the case of $\vartheta = \pm\pi/2$ (with $K_0, \varphi_0 \neq 0$). That is, we restrict ourselves to the case where the axes of the polarization ellipse of the light are aligned to the lattice axes. Other cases of elliptical polarization are outside the scope of this work but can be achieved by generalizing the recipe given here.

The condition $\vartheta = \pm\pi/2$ is most simply accomplished experimentally by enforcing the condition

$$f_{0,S} \sin \phi = 0. \quad (\text{F15})$$

Either $f_{0,S} = 0$ [23] or $\phi = 0, \pm\pi$ would satisfy this condition. We found that choosing the two modulations to be in phase ($\phi = 0$) offers the greatest tunability of the phasonic modulation amplitude φ_0 . Under this condition, the position modulations $x_P(t)$ and $x_S(t)$ destructively interfere and the phasonic amplitude is given by

$$\varphi_0 = -\frac{1}{f} (\beta f_{0,P} - f_{0,S}) = \frac{4\beta f_{\text{rec}}}{\pi f} K_0 - \frac{f_{0,S}}{f}. \quad (\text{F16})$$

This gives the desired frequency modulation amplitude for the secondary lattice as

$$f_{0,S} = f\varphi_0 + \beta f_{0,P}. \quad (\text{F17})$$

For linearly polarized drive ($\vartheta = 0, \pm\pi$), we require that

$$\beta f_{0,P} = f_{0,S} \cos \phi \quad \text{and} \quad f_{0,S} \sin \phi \neq 0. \quad (\text{F18})$$

The first equality in the above condition implies that

$$f\varphi_0 = f_{0,S} \sin \phi.$$

We restrict $\phi \in [0, \pi/4]$ such that $\varphi_0 \geq 0$. The condition of linear polarization can then be reexpressed as

$$f_{0,S} \cos \phi = \beta f_{0,P}, \quad \text{and} \quad f_{0,S} \sin \phi = f\varphi_0. \quad (\text{F19})$$

Thus, under the condition of linear polarization, the frequency modulation amplitude of the secondary lattice $f_{0,S}$ and the relative phase between the lattice modulation ϕ trace out a circle, in the space of $f_{0,P}$ and φ_0 .

This geometric picture provides a simple recipe for realizing a linearly polarized drive. We choose the desired values of the dipolar amplitude controlled only by $f_{0,P}$, the phasonic amplitude φ_0 , the driving frequency f . Then to achieve a linearly polarized drive, we set

$$f_{0,S} = \sqrt{(\beta f_{0,P})^2 + (f\varphi_0)^2}, \quad (\text{F20})$$

$$\phi = \omega\tau_{\text{delay}} = \arctan\left(\frac{f\varphi_0}{\beta f_{0,P}}\right) = \arctan\left(\frac{\pi}{4\beta} \frac{f}{f_{\text{rec,P}}} \frac{\varphi_0}{K_0}\right). \quad (\text{F21})$$

An experiment using linearly polarized modulation begins first with modulation of the primary lattice only. The relative phase ϕ between the two lattice modulations is controlled by an initial delay of duration τ_{delay} , after which the modulation of the secondary lattice begins. This delay means that the electric field would first be elliptically polarized because only the primary lattice is shaking ($f_{0,S}(t < \tau_d) = 0$) until $\tau_{\text{delay}} < 0.25/f$, after which the field maintains linear polarization until the end of modulation. Table II summarizes the expressions used to control the drive polarization used in this work. We emphasize that

TABLE II. Expressions of the experimental parameters for the polarized modulations used in the experiments.

Parameters	Elliptical	Linear
$f_{0,P}$	$4K_0 f_{\text{rec,P}}/\pi$	$4K_0 f_{\text{rec,P}}/\pi$
$f_{0,S}$	$f\varphi_0 + \beta f_{0,P}$	$\sqrt{(\beta f_{0,P})^2 + (f\varphi_0)^2}$
τ_{delay}	0	$\arctan(f\varphi_0/\beta f_{0,P})/\omega$

these are not the only possible approaches; a desired polarization is achieved as long as the corresponding conditions [Eq. (F15) or Eq. (F18)] are satisfied.

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