

Quantum Sensing of Time-Dependent Electromagnetic Fields with Single-Electron Excitations

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In this study, we investigate the potential of electronic interferometers for probing the quantum state of electromagnetic radiation on a chip at subnanosecond timescales. We propose to use single-electron excitations propagating within an electronic Mach-Zehnder interferometer in the Aharonov-Bohm-dominated regime. We discuss how information about the quantum state of the electromagnetic radiation is encoded into the interference contribution to the average outgoing electrical current. By investigating squeezed radiation and single-edge magnetoplasmons probed by leviton pulses in a realistic setup, we show that single-electron interferometers have the potential to probe quantum radiation in the time domain with subnanosecond to picosecond time resolution. Our research could have significant implications for probing the fundamental properties of light in the microwave to terahertz domains at extremely short timescales.

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I. INTRODUCTION

Initiated by the demonstration of on-demand single-electron sources in quantum Hall edge channels [1,2], electron quantum optics (EQO) [3] has seen the demonstration of several important milestones such as electronic Hanbury Brown and Twiss [4] and Hong-Ou-Mandel (HOM) [5] experiments, quantitative studies of electronic decoherence using HOM [6] and Mach-Zehnder interferometry in the stationary regime [7–11], and, more recently, with single-electron current pulses [12] called levitons [2,13]. Moreover, a full characterization of the quantum state and coherences of single-electron and hole excitations within a quantum Hall edge channel has been performed [14–17]. By demonstrating the ability to access electronic quantum states in a quantum conductor at an unprecedented level, these achievements strongly suggest that EQO is now mature for exploring its applications. The extreme sensitivity of individual electronic excitations to their electromagnetic environment suggests to explore their potential for the sensing of electromagnetic fields at the micrometer spatial scale and down to a few picosecond timescale [18].

This would be of great interest in the current effort to extend the paradigms of quantum coherent nanoelectronics to higher frequencies [19], possibly up to the terahertz range [20] since the electromagnetic wavelength (millimeters) is still much larger than the size of the device (from 1 to few tens of micrometers). For example, interferometric detection of a single electron at subnanosecond timescale is indeed instrumental in a recent proposal for single-shot detection of an electronic flying qubit [21,22]. In a broader perspective, solid-state systems envisioned for quantum technologies [23–25] involve quantum electromagnetic fields in the giga- to terahertz range. Quantum optics in the terahertz domain also offers interesting perspectives for studying quantum materials [26]. This calls for sensors that can probe the properties of these fields on the chip and at such short timescales. In a different context, the concepts and ideas presented here may be relevant for quantum coherent electronic microscopy [27–31].

However, sensing and analyzing local quantum electric fields on very short timescales is notoriously difficult. Quantum electrometers based on Rydberg atoms [32] have reached record sensitivities, but for static fields. Fast electric modulation of optical systems enables measuring the electric field with a terahertz bandwidth [33]. Unfortunately, such optical systems are not convenient for quantum mesoscopic devices due to the difficulty of combining optics and microwave electronics within the same cryostat and the large size of the sensing area.

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However, changes may arise from new approaches on efficient terahertz generation of voltage pulses compatible with cryogenic environment presently developed [34]. On-chip systems based on nanomechanical resonators [35,36], rf-capacitive gate-based sensing [37], and nitrogen-vacancy centers in diamond [38], as well as quantum dots [39,40] have been demonstrated, often as charge sensors able to detect a single-electron charge at a few tens of nanometers. However, their bandwidth is still limited to 1–10 MHz at best, obtained with rf single-electron transistor [41], quantum point contacts (QPCs) [42], rf superconducting quantum interference devices [43], or quantum dots [44]. Moreover, they are not designed to detect quantum features of the electromagnetic field such as nonclassical fluctuations.

In this paper, we discuss the “electron quantum radar” (EQR) as a way to probe the quantum state of an electromagnetic radiation using a single-electron interferometer. This idea is dual to the one underlying radars and coherent LIDARs that are electromagnetic interferometers probing a material target. Here, the EQR is an electronic Mach-Zehnder interferometer (MZI) [45], where one branch is capacitively coupled to a radiation channel in which the external electromagnetic radiation propagates as depicted in Fig. 1 (see also Ref. [46]). It obtains information on the quantum state of the radiation by comparing ballistic propagation within the “reference branch” of the MZI and propagation in the “target branch” coupled to the radiation. This is different from the proposal of Ref. [47] in which nonclassical radiation is probed via photoassisted tunneling. It is closer to an interferometric probe of a quantum electromagnetic field by matter [48], although,

here, it is not a quantum nondemolition measurement since the coupling operator is a quadrature of the field [49].

As shown in the present work, single-electron excitations are the most promising probes of quantum radiation involving one to a few photons because the associated quantum electric current generates a very small backaction on this radiation compared to electric currents involving more electronic excitations. The generation of wave packets in the tens to few picosecond range in a variety of systems [19,50–55] suggests that the versatility of single-electron sources may enable us to probe properties of quantum radiation both in time and frequency. Moreover, the interferometric method proposed here relies on the measurement of a dc current and therefore bypasses the need for a wide-bandwidth measurement chain.

Motivated by this perspective, a full theory of the single-electron quantum radar is presented here. The central result of this paper is the single-electron radar equation that gives the interference contribution to the average electric current in terms of the single-electron wave packet and the quantum state of the electromagnetic radiation. All the dynamics are contained in the linear response properties of the “radiation coupler” coupling the target branch to the radiation.

The single-electron radar equation accounts for the interferometer’s backaction on the incident radiation, which leads to electronic decoherence [56,57]. We show that all the information about the state of the incident quantum radiation is contained in a Franck-Condon recoil factor [58] describing the change of the incident radiation upon the propagation of a time-resolved single-electron excitation across the interferometer. This result is established using the plasmon scattering matrix related to finite-frequency admittances [59,60] to describe the coupling between the external radiation and the plasmon modes within the target branch of the MZI.

In the case of a classical radiation, this recoil factor includes a pure phase corresponding to the classical electric potential seen by the electron during its propagation. When the quantum state of the incident quantum radiation is Gaussian, the recoil factor captures the time-dependent fluctuations in the mode selected by the capacitive coupling between the radiation and the electrons. This enables us to give an operational criterion to test whether the fluctuations are subvacuum or not. This result may be of interest in light of recent theoretical [61,62] and experimental [63] studies of squeezing in quantum Hall edge channels as well as in tunnel junctions [64,65]. As a last example, we discuss the electron radar signature of single to few edge magnetoplasmons (EMPs) propagating in a nearby quantum Hall edge channel and show that, when the radiation coupler provides a broadband filtering of the incoming radiation, the recoil factor in the time domain is proportional to the instantaneous average heat current [66] carried by a single EMP.

This paper is structured as follows: Section II presents the basic ideas for sensing classical or quantum

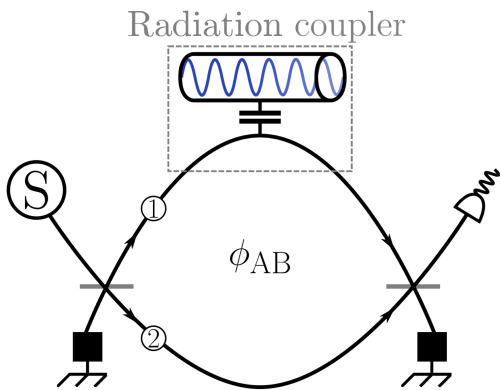


FIG. 1. Scheme of principle for sensing electromagnetic fields with single-electron excitations. An electronic MZI is formed by combining two quantum point contacts A and B acting as ideal electronic beam splitters. It encloses a magnetic flux Φ_B , inducing an Aharonov-Bohm phase $\phi_{AB} = e\Phi_B/\hbar$. The interferometer is fed by a single-electron source S located right before the first beam splitter A . One measures the Aharonov-Bohm flux-dependent part of the average outgoing electrical current $\langle i_{\text{1out}}(t) \rangle$ or $\langle i_{\text{1out}}(\omega) \rangle$. Branch 1 is capacitively coupled to external electromagnetic radiation.

electromagnetic fields with an electronic interferometer. In particular, we show the importance of the backaction of the interferometer when probing quantum electromagnetic radiation. The theory of a single-electron Mach-Zehnder interferometer with one of its branches irradiated by a quantum electromagnetic field is then presented in Sec. III and its core result—the single-electron radar equation—is derived. Explicit predictions for classical and quantum radiation (squeezed and Fock states) probed by levitons [2,13] are presented in Sec. IV, as well as a comparison with other state-of-the-art techniques to measure time-dependent quantum fluctuations of a voltage [63,64,67,68].

II. SENSING ELECTROMAGNETIC FIELDS WITH ELECTRONS

In this section, we discuss the physics of the electronic MZI as a probe of an external radiation at a qualitative level. Sections II A and II B are devoted to estimating the classical phase shift induced on the probing electron in the radiation coupler by a nearby propagating single-electron current and then a classical time-dependent voltage. In Sec. II C, we consider an incident quantum radiation and we show that, to avoid decoherence effects, a compromise must be found on the strength of the electron-radiation coupling. The discussion of backaction effects also explains why single-electron interferometry is appropriate for probing the quantum state of mesoscopic quantum radiation involving a low average number of photons.

A. Electromagnetic phase shifts

Before modeling the MZI, let us estimate the order of magnitude of the phase shift induced by a nearby electron on the electron propagating within the MZI. In Ref. [18], a “small electron collider” depicted in Fig. 2 has been considered. The phase shift $\delta\phi_{\text{coll}}$ associated with Coulomb potential for electrons flying nearby each other at velocity v_F is, up to some geometric factor, on the order of the effective fine-structure constant within the material

$$\delta\phi_{\text{coll}} = \alpha_{\text{eff}} \operatorname{arcsinh}(l/d), \quad (1a)$$

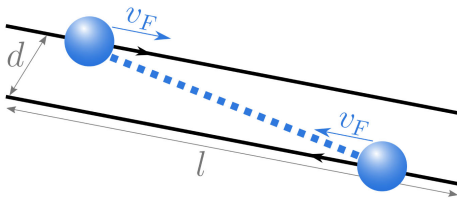


FIG. 2. Collision of two electrons propagating within two counterpropagating one-dimensional (1D) channels at velocity v_F , separated by a distance d , and feeling Coulomb potential within a region of length l in a dielectric material of relative permittivity ϵ_r .

$$\alpha_{\text{eff}} = \frac{e^2}{4\pi\epsilon_0\epsilon_r\hbar v_F} = \frac{\alpha_{\text{QED}}}{\epsilon_r} \frac{c}{v_F}, \quad (1b)$$

in which α_{QED} is the fine-structure constant of QED. For AlGaAs/AsGa with relative permittivity $\epsilon_r = 12.9$ and a typical Fermi velocity $v_F = 10^5 \text{ ms}^{-1}$, $\alpha_{\text{eff}} \sim 1.7$ and, therefore, for $l = 1 \mu\text{m}$ and $d = 100 \text{ nm}$, $\delta\phi_{\text{coll}}/2\pi \simeq 0.81$, which is not small compared to unity. Similar phase estimates have also been discussed in two-electron collision within an HOM interferometer [69–71].

This estimate suggests that, using the typical radiation coupler geometry depicted in Fig. 1, a single-electron excitation propagating within the MZI may be able to detect the time-dependent electromagnetic field associated with a single electron passing by in the radiation channel.

B. Sensing a classical voltage drive

To understand how the shape of the probing electron’s wave packet influences its detection capabilities, we now discuss more precisely the behavior of the single-electron MZI depicted in Fig. 1 using single-particle physics.

Two QPCs behave as ideal electronic beam splitters with scattering matrices

$$S_\alpha = \begin{pmatrix} \sqrt{T_\alpha} & i\sqrt{R_\alpha} \\ i\sqrt{R_\alpha} & \sqrt{T_\alpha} \end{pmatrix} \quad (2)$$

for $\alpha = A$ or $\alpha = B$ where T_α and R_α are the transmission and reflection probabilities ($T_\alpha + R_\alpha = 1$). A static perpendicular magnetic field B is applied, generating an Aharonov-Bohm phase ϕ_{AB} enclosed between the arms 1 and 2.

We assume that electrons feel a time-dependent classical potential $U(t)$ along branch 1 and propagate ballistically along branch 2 with time of flight τ_2 . The amplitude for a single electron to enter into branch 1 at time t' and arrive at the second beam splitter at time t is then given by

$$R(t, t') = e^{\frac{ie}{\hbar} \int_{t'}^t U(\tau) d\tau} \delta(t - t' - \tau_1), \quad (3)$$

in which τ_1 denotes the ballistic time of flight across branch 1. The prefactor is the electric phase accumulated by the electron during its propagation. We can then compute the probability for an electron emitted by the source S in a single-particle state φ_e to be detected in the outgoing branch 1,

$$p(1_{\text{out}}) = R_A R_B + T_A T_B + \mathcal{P}_q, \quad (4)$$

in which the quantum interference contribution $\mathcal{P}_q$ is given by

$$\mathcal{P}_q = \mathcal{K} \operatorname{Re} \left(e^{i\phi_{AB}} \int_{\mathbb{R}} \varphi_e(t - \tau_1) e^{\frac{ie}{\hbar} \int_{t-\tau_1}^t U(\tau) d\tau} \varphi_e(t - \tau_2)^* dt \right), \quad (5)$$

where $\mathcal{K} = \sqrt{R_A R_B T_A T_B}$. In the limit of a very short electronic wave packet emitted at time t_e and for $\tau_1 = \tau_2$, we obtain

$$\mathcal{P}_q \propto \operatorname{Re} \left(e^{i\phi_{AB}} e^{i\delta\phi_U(t_e)} \right), \quad (6)$$

where the phase

$$\delta\phi_U(t_e) = \frac{e}{\hbar} \int_{t_e}^{t_e + \tau_1} U(\tau) d\tau \quad (7)$$

is the electric phase accumulated during the time interval $[t_e, t_e + \tau_1]$. This result corresponds to our intuition from optics: The interferometer provides a way to access the phase difference between the two propagation paths, which, in the present case, is directly related to the classical voltage experienced by the electrons during their propagation along branch 1.

In order to discuss how an electronic MZI can be used to probe the quantum state of a radiation, a quantum description of the radiation-electron coupling is needed.

C. Interferometric sensing of a quantum system

The simplest way to understand how a quantum interferometer can probe the state of a quantum system coupled to one of its branches is to discuss a generalization of the Elitzur-Vaidman bomb detector [72].

In this work, a quantum interferometer is used to detect the presence of a bomb without necessarily triggering its explosion. The trigger is activated as soon as a particle travels across branch 1 of the interferometer (see Fig. 3). This assumes that the trigger is a perfectly efficient particle detector. The idea is then to tune the optical paths of the

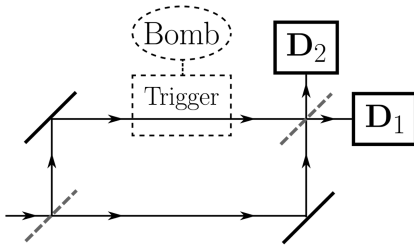


FIG. 3. The Elitzur-Vaidman interferometer: In the absence of the bomb, this single-particle (photon) Mach-Zehnder interferometer is calibrated so that there is no click in $\mathbf{D}_2$. In the presence of the bomb, the trigger absorbs an incident photon with 100% efficiency and detonates the bomb. Interferences are destroyed and there is a probability 1/4 for the traveling particles to be detected by $\mathbf{D}_2$.

MZI so that, in the absence of the bomb, the particle exits on one of its outgoing branches, but not in the other. In the presence of the bomb, the particle exits with probability 1/4 in the branch where it would never exit in the absence of the bomb. This provides a sure diagnostic of its presence without interacting with its trigger; hence the commonly used term “interaction-free measurement” to describe this process.

However, in the present context, we are interested in exploiting the interaction between the particle and the bomb’s trigger to gain information about not only its presence or absence, but also about its quantum state.

The bomb initially being in the $|\text{Idle}\rangle$ state, interaction with the incident particle leads to an entangled state involving two alternatives: First, as in Elitzur and Vaidman’s work, when the particle is absorbed, the bomb is detonated. However, we also account for the case where the bomb is not detonated and the particle is not absorbed: We speak of “fizzling out.” Since this is the result of an interaction between the particle and the bomb, their states are altered: the particle one being $|\Psi'\rangle$ and the bomb one being $|\text{Fizzled}\rangle$. The full interaction between the particle and the bomb is then described by the quantum coherent process

$$|\Psi\rangle \otimes |\text{Idle}\rangle \longrightarrow \mathcal{A}_0 |\emptyset\rangle \otimes |\text{Detonated}\rangle + \mathcal{A}_1 |\Psi'\rangle \otimes |\text{Fizzled}\rangle, \quad (8)$$

in which $|\emptyset\rangle$ denotes the vacuum state (the particle has been absorbed). The Elitzur-Vaidman interaction-free measurement corresponds to $|\mathcal{A}_0|^2 = 1$, but here both alternatives are allowed and thus $|\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 = 1$. For balanced beam splitters, the conditional output probabilities for the particle are then given by the following expressions:

$$p(\text{Absorbed}) = \frac{1}{2} (1 - |\mathcal{A}_1|^2), \quad (9a)$$

$$p(1_{\text{out}}) = \frac{1}{4} (1 + |\mathcal{A}_1|^2) - \mathcal{P}_q, \quad (9b)$$

$$p(2_{\text{out}}) = \frac{1}{4} (1 + |\mathcal{A}_1|^2) + \mathcal{P}_q \quad (9c)$$

in which quantum interference effects are contained in

$$\mathcal{P}_q = \frac{1}{2} \operatorname{Re} (\mathcal{A}_1 e^{i\phi_{AB}} \langle \text{Idle} | \text{Fizzled} \rangle \langle \Psi | \Psi' \rangle), \quad (10)$$

which is sensitive to the (Aharonov-Bohm) phase difference ϕ_{AB} associated with free propagation along the two branches of the MZI. Information on the quantum state of the bomb is contained in the product of the two overlaps $\langle \Psi | \Psi' \rangle$ and $\langle \text{Idle} | \text{Fizzled} \rangle$.

From our perspective, the bomb plays the role of the incoming electromagnetic radiation and the particle is the quantum electrical current propagating within the MZI.

The process in which the particle is absorbed and the bomb is detonated corresponds to a full electronic decoherence within the MZI. It occurs whenever Coulomb interactions lead to the generation of any extra electron-hole pair within channel 1 of the MZI compared to ballistic propagation along channel 2. The resulting many-body state will then have a vanishing overlap with the ballistic propagation of a single-electron excitation within branch 2 of the MZI. However, this is not the case of interest for the EQR since, in this case, the interference contribution to the average current vanishes and, as in the Elitzur-Vaidman case, nothing can be learned on the quantum state of the incoming radiation.

By contrast, in the absence of generation of extra electron-hole pairs, the state of the electron propagating within branch 1 is altered by its coupling to the incoming radiation: this corresponds to the change $|\Psi\rangle \rightarrow |\Psi'\rangle$ in the above discussion. For a classical radiation, this is the phase shift associated with the voltage experienced by the electrons (see Sec. II B).

The alteration of the bomb's state $|\text{Idle}\rangle \rightarrow |\text{Fizzled}\rangle$ in the above discussion corresponds to the effect of the propagating electron on the incident radiation in the situation where no extra electron-hole pairs are created. This is the backaction of the interferometer, seen as a measurement device, on the radiation. The amplitude $\langle \text{Idle} | \text{Fizzled} \rangle$ measures the “quantum recoil” of the radiation upon propagation of a single electron without generating extra electron-hole pairs along branch 1 of the MZI. As we shall see in Sec. III, this is precisely the part that will contain information on the quantum fluctuations of the incoming radiation.

When the backaction is too important, the overlap $\langle \text{Idle} | \text{Fizzled} \rangle$ may vanish and the interference signal is then lost. This point explains why quantum electrical currents carrying a single electronic excitation are relevant for probing mesoscopic quantum electromagnetic fields that involve a low average number of photons: In addition to decoherence, which is generically less important for them, single-electron currents lead to a smaller backaction than currents carrying more excitations.

In the end, this qualitative discussion suggests that extracting information on the incoming radiation thus requires a compromise: Electronic decoherence as well as backaction on the quantum radiation has to be moderate to ensure an experimentally accessible signal, but strong enough to ensure sensitivity to the incoming quantum radiation.

III. THE SINGLE-ELECTRON RADAR THEORY

In this section, we derive the central result of this work, which is the single-electron radar equation expressing the interference contribution to the outgoing average current in terms of two distinct quantities: first, the excess single-electron coherence of the injected wave packet that depends

only on the electronic source S , and second, an effective single-particle scattering amplitude describing both the effects of electronic decoherence and the incoming radiation. The latter quantity reflects the dynamics of the interferometer coupled to its electromagnetic environment.

Although this result is formally derived in Appendix E using the bosonization framework, which allowed us to recover the results in the case of chiral quantum Hall edge channels, a more intuitive derivation is presented in Secs. III A and III B. For technical details, we refer to the Appendixes.

A. Radiation coupler modeling

The radiation coupler involves a capacitive coupling between a portion of the upper branch of the electronic MZI and the external radiation channel that is fed by the incoming electromagnetic radiation we want to study. The effect of Coulomb interactions within this region will be described within the framework of edge-magnetoplasmon scattering. Originally introduced in the context of finite-frequency quantum transport in 1D [59,60,73], it enables us to describe the scattering between the EMP modes propagating along the MZI edge channel and the bosonic modes within the external radiation channel [74].

For simplicity, we assume that the whole upper branch of the MZI is included in the radiation coupler so that the EMP scattering matrix will account for the detailed geometry of the sample. We also assume that only the EMP modes associated with the $b(\omega)$ and $b^\dagger(\omega)$ destruction and creation operators and the external radiation modes associated with the $a(\omega)$ and $a^\dagger(\omega)$ operators appear in the scattering matrix $S(\omega)$,

$$\begin{pmatrix} a_{\text{out}}(\omega) \\ b_{\text{out}}(\omega) \end{pmatrix} = \begin{pmatrix} S_{aa}(\omega) & S_{ab}(\omega) \\ S_{ba}(\omega) & S_{bb}(\omega) \end{pmatrix} \begin{pmatrix} a_{\text{in}}(\omega) \\ b_{\text{in}}(\omega) \end{pmatrix}. \quad (11)$$

Depending on the design of the radiation coupler, the $(a, a^\dagger)$ modes may be photonic [case of Fig. 4(b)] or edge

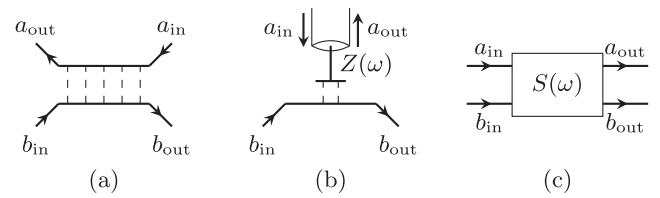


FIG. 4. The edge-magnetoplasmon scattering approach describes many situations, such as, for example, (a) two counter-propagating edge channels capacitively coupled over a distance l , (b) a chiral-edge channel capacitively coupled to a linear external circuit described by a frequency-dependent impedance $Z(\omega)$. (c) Solving the equation of motion leads to a frequency-dependent scattering matrix $S(\omega)$ between the channel's edge-magnetoplasmon modes and the bosonic modes of the other system.

magnetoplasmonic [see Fig. 4(a)]. These matrix elements are related to finite-frequency admittances [59,60,75]. In principle, they could be inferred from experimental measurements [76]. Quantitative predictions can also be made from theoretical models of the radiation coupler. Experimentally relevant examples include the case of two counterpropagating edge channels [see Fig. 4(a)] as well as the case of a capacitive coupling to a transmission line [see Fig. 4(b)]. Computations for the case of two counterpropagating edge channels in total mutual influence [see Fig. 4(a)] are detailed in Appendix D. This specific radiation coupler model will be used throughout the present manuscript to illustrate explicit examples.

B. The single-electron radar equation

1. Single-particle scattering approach

We now derive the radar equation assuming that electronic propagation inside the radiation coupler is described in terms of time-dependent single-particle scattering. We are neglecting electron-hole pair creation by Coulomb interactions within the MZI. We also assume that the two QPCs are ideal electronic beam splitters with energy-independent scattering matrix. Denoting by $R(t, t')$ the amplitude for an electron to enter branch 1 at time t' and exit it at time $t \geq t'$, we can compute the Aharonov-Bohm flux-dependent contribution to the outgoing time-dependent average electrical current $\langle i_{1\text{out}}(t) \rangle$. In a MZI, it reduces to

$$\langle i_{1\text{out}}(t) \rangle = -e(I_0(t) + e^{i\phi_{AB}} I_+(t) + e^{-i\phi_{AB}} I_+(t)^*), \quad (12)$$

where $\phi_{AB} = 2\pi\Phi_B/\Phi_0$ is the Aharonov-Bohm phase associated with the magnetic flux Φ_B ($\Phi_0 = h/e$ being the flux quantum). Denoting by $\mathcal{A}(S \xrightarrow{j} 1_{\text{out}})$ the amplitude for an electron to be emitted by the source S , propagating along branch j between the two QPCs and detected in 1_{out} , we have (see Fig. 5)

$$I_+(t) = v_F \mathcal{A}(S \xrightarrow{1} 1_{\text{out}}) \mathcal{A}(S \xrightarrow{2} 1_{\text{out}})^* \quad (13)$$

with

$$\mathcal{A}(S \xrightarrow{1} 1_{\text{out}}) = \sqrt{T_A T_B} \int_{\mathbb{R}} R(t, t') \varphi_e(t') dt', \quad (14a)$$

$$\mathcal{A}(S \xrightarrow{2} 1_{\text{out}}) = -\sqrt{R_A R_B} \varphi_e(t - \tau_2). \quad (14b)$$

Note that, in the present context, $2\text{Re}[I_+(t)e^{i\phi_{AB}}]$ plays the role of the interference term $\mathcal{P}_q$ in Sec. II. The quantity $X_+(t)$ defined by $I_+(t) = -\sqrt{R_A T_A R_B T_B} X_+(t)$ does not depend on the properties of the electronic beam splitters and completely determines the interference contribution to the average electrical current. It is equal to

$$X_+(t) = v_F \int_{\mathbb{R}} R(t, t') \varphi_e(t') \varphi_e(t - \tau_2)^* dt'. \quad (15)$$

This result is called the electronic radar equation by analogy with the existing signal processing literature [77]. Its integral over time

$$X_+^{(\text{dc})} = \int_{\mathbb{R}} X_+(t) dt \quad (16)$$

represents the interference contribution to the total charge detected on output 1 of the MZI. For a balanced MZI ($T_A = T_B = 1/2$), the average dc current measured when the experiment is repeated at measurement frequency f_m is given by

$$\langle i_{1\text{out}}^{(\text{dc})} \rangle = -\frac{ef_m}{2} \left[1 + \text{Re} \left(e^{i\phi_{AB}} X_+^{(\text{dc})} \right) \right]. \quad (17)$$

2. Coulomb interaction effects

The discussion of Sec. II C suggests that the result in the presence of Coulomb interactions and of an incoming quantum radiation is of the same form as Eq. (15) with an effective single-electron scattering amplitude $R_{\text{eff}}(t, t')$ that takes into account decoherence effects as well as the backaction of the electron on the incoming quantum radiation. The formal derivation of this fact is given in Appendix E, but here we will proceed with obtaining the form of the effective scattering amplitude using semi-qualitative arguments to emphasize its physical meaning.

In the absence of incoming electromagnetic radiation, the amplitude $R_{\text{eff}}(t, t')$ is the amplitude for a single electron to propagate elastically across branch 1, taking into account the capacitive coupling to the radiation channel. Any process leading to the generation of an extra electron-hole pair within the edge channel or of an excitation within the electromagnetic environment would lead to decoherence in the MZI. This elastic scattering amplitude has already appeared in studies of electronic decoherence in the MZI [78–82]. Electronic decoherence in MZI has been experimentally simulated using a voltage probe in Ref. [8], thereby showing the reduction of the interferometer's contrast by the amplitude (square root of the probability) for the electron to be transmitted across the probe.

In the absence of external radiation, $R_{\text{eff}}(t, t')$ should then be the elastic scattering amplitude for an incoming single-electron excitation injected at time t' in the presence of the Fermi sea to exit at time t without experiencing any inelastic scattering: $R_{\text{eff}}(t, t') = \Theta(t - t') \mathcal{Z}_1(t - t')$, where

$$\mathcal{Z}_1(\tau) = \int_0^{+\infty} \tilde{\mathcal{Z}}_1(\omega) e^{-i\omega\tau} \frac{d\omega}{2\pi} \quad (18)$$

is the Fourier transform of the elastic scattering amplitude $\tilde{Z}_1(\omega)$ for a single electron of energy $\hbar\omega > 0$ across branch 1 of the MZI. This amplitude, computed in Refs. [57,74,83], contains all the information about the electronic decoherence at zero temperature (induced by vacuum fluctuations) and has recently been considered in light of experimental studies of electronic relaxation [9,10,84]. Going back to the discussion of Sec. II C, $Z_1(\tau)$ corresponds to the product $\mathcal{A}_1\langle\Psi|\Psi'\rangle\langle\text{Idle}|\text{Fizzled}\rangle$ when the radiation channel is fed with the vacuum state.

We now have to discuss the backaction of the electron in the presence of incoming radiation. The backaction on the vacuum is already taken into account by the elastic scattering amplitude $Z_1(\tau)$ or $\tilde{Z}_1(\omega)$, but here we consider the effect of the single-electron current on the incoming photonic excitations.

An outgoing electron at time t corresponds, in terms of EMP, to a localized current pulse coming out of branch 1. Such a current pulse comes from an incoming pulse in edge channel 1 as well as some coherent pulse in the radiation channel whose amplitude can be inferred from the scattering matrix $S(\omega)$. Denoting by $\Lambda_t(\omega) = -e^{i\omega t}/\sqrt{\omega}$ the corresponding amplitude of the mode $b_{\text{out}}(\omega)$, the amplitudes of the incoming pulses are given by

$$\langle b_{\text{in}}(\omega) \rangle = S_{bb}^*(\omega)\Lambda_t(\omega), \quad (19a)$$

$$\langle a_{\text{in}}(\omega) \rangle = S_{ba}^*(\omega)\Lambda_t(\omega). \quad (19b)$$

Equation (19b) gives the amplitude of the “backaction kick” on the incoming radiation associated with the detection of an electron localized at time t at the end of the upper MZI branch. Consequently, the unitary operator representing the backaction is the infinite-dimensional displacement operator $D_{a_{\text{in}}}[S_{ba}^*\Lambda_t]$ [see Eq. (B7) for its definition].

The full backaction factor is then the average value of $D_{a_{\text{in}}}[S_{ba}^*\Lambda_t]$ in the reduced density operator ρ_{em} describing the state of the incoming quantum radiation fed into the radiation coupler. Since the contribution in the vacuum state $|0\rangle$ for the $a_{\text{in}}(\omega)$ modes is already included in $Z_1(t)$, the excess backaction on the incoming radiation is obtained by dividing $\langle D_{a_{\text{in}}}[S_{ba}^*\Lambda_t] \rangle_{\rho_{\text{em}}}$ by $\langle D_{a_{\text{in}}}[S_{ba}^*\Lambda_t] \rangle_{|0\rangle}$. This leaves us with the average value of the bosonic normal-ordered backaction displacement operator as the prefactor representing the recoil of the incident radiation induced by a single-electron excitation propagating across the upper branch of the MZI. By analogy with the Franck-Condon factor that appears in the spectroscopy of complex molecules [58] or in the Mössbauer effect [85,86], where it is called the Lamb-Mössbauer factor, we thus define the Franck-Condon factor

$$\mathcal{F}_{\rho_{\text{em}}}(t) = \langle :D_{a_{\text{in}}}[S_{ba}^*\Lambda_t]: \rangle_{\rho_{\text{em}}}, \quad (20)$$

in which we have introduced the bosonic normal ordering $:\cdots:$. As noticed in Ref. [87] and explicitly derived here in Appendix F, the Franck-Condon factor contains information on the full counting statistics of the charge propagating within the radiation channel. More specifically, when the radiation channel is a chiral quantum Hall edge channel at $\nu = 1$,

$$\mathcal{F}_{\rho}(t) = \left\langle :e^{2\pi i N(t)}: \right\rangle_{\rho_{\text{em}}}, \quad (21)$$

in which the $N(t)$ operator represents a filtering of the charge operator flowing across the interaction region within the radiation channel.

The quantum interference contribution to the average outgoing electrical current thus has the form

$$X_+(t) = v_F \int_{-\infty}^t R_{\text{eff}}(t, t') \varphi_e(t') \varphi_e(t - \tau_2)^* dt', \quad (22)$$

in which the effective single-particle scattering amplitude is given by

$$R_{\text{eff}}(t, t') = Z_1(t - t') \mathcal{F}_{\rho_{\text{em}}}(t). \quad (23)$$

In the case where independent repeated single-electron experiments are performed, one can replace the product $\varphi_e(t') \varphi_e(t - \tau_2)^*$ by the excess single-electron coherence $\Delta\mathcal{G}_S^{(e)}(t'|t - \tau_2)$ emitted by the imperfect single-electron sources. This takes into account the statistical fluctuations in the imperfect emission of single-electron excitations by the source S . Equation (22) can then be interpreted as described in Fig. 5. Appendix G discusses the frequency domain form of the single-electron radar equation and its interpretation.

Expression (23) takes into account electronic decoherence and the effect of the incoming quantum radiation. Together with the expressions for $Z_1(\tau)$ and the Franck-Condon factor, Eqs. (22) and (23) form the central result of the

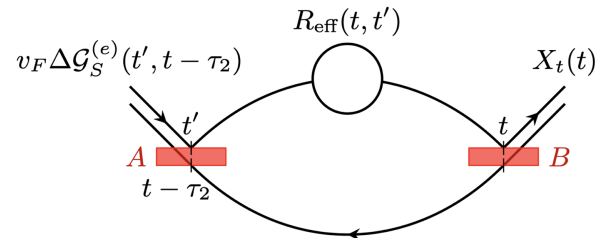


FIG. 5. Physical interpretation of the linear radar equation (22) in the time domain showing the contribution of incoming single-electron coherence to the temporal signal $X_+(t)$. The figure presents the product of two quantum amplitudes: The one with an arrow oriented away from the source corresponds for a direct amplitude, whereas the one with the arrow arriving to the source corresponds to the complex-conjugated amplitudes contributing to $X_+(t)$.

single-electron radar theory. The factorized form of the rhs of Eq. (23) has an important consequence: The Franck-Condon factor appears as the ratio of the time-resolved EQR signal in the presence of the external radiation to the one in the absence of it,

$$\frac{[X_+(t)]_{\rho_{\text{em}}}}{[X_+(t)]_{|0\rangle}} = \mathcal{F}_{\rho_{\text{em}}}(t). \quad (24)$$

However, high-precision broadband time-domain measurements of the average current are difficult to perform. We thus have to discuss how Eqs. (22) and (23) can be used to interpret the experimental data obtained by recording interference fringes on the dc average current, which is the usual quantity measured with high precision.

3. The dc-current interference contrast

Assuming that the source could inject a very short electronic wave packet of duration τ_e at time t_e , the EQR radar signal in the presence of external radiation would (naively) be given by (see Appendix H) $[X_+^{(\text{dc})}]_{\rho_{\text{em}}} \simeq \tau_e \mathcal{Z}(\tau_2) \mathcal{F}_{\rho_{\text{em}}}(t_e + \tau_2)$. Comparing the result in the presence and in the absence of electronic radiation $[\mathcal{F}_{|0\rangle}(t) = 1]$ injected into the radiation channel then leads to

$$[X_+^{(\text{dc})}]_{\rho_{\text{em}}} \simeq [X_+^{(\text{dc})}]_{|0\rangle} \mathcal{F}_{\rho_{\text{em}}}(t_e + \tau_2). \quad (25)$$

Consequently, sweeping the emission time t_e of the infinitely short probe wave packet would sample $\mathcal{F}_{\rho_{\text{em}}}(t_e + \tau_2)$, thereby providing us with a time-resolved probe of the electromagnetic radiation. Note that the τ_2 time delay follows from ballistic propagation along the lower branch of the MZI. Unfortunately, electronic wave packets with only positive energy components cannot be arbitrarily localized. Nevertheless, short duration levitons, which are purely electronic excitations generated by a Lorentzian voltage pulse [13,88], provide us with a close analog to a perfectly localized electronic excitation. However, as will be discussed in the next subsection, the issue of electronic decoherence must be addressed carefully.

C. Leviton excitations

We now discuss the dc-average current interference signal $X_+^{(\text{dc})}$ defined by Eq. (16) when the MZI is fed with a single leviton of duration τ_e injected at time t_e .

1. General result

As shown in Appendix I, the simple expression for the leviton wave packet in the frequency domain enables us to derive a convenient form for $X_+^{(\text{dc})}$ suitable for numerical evaluations,

$$X_+^{(\text{dc})} = \int_{\mathbb{R}} \tilde{\mathcal{F}}_{\rho_{\text{em}}}(\Omega) e^{-i\Omega(t_e + \tau_2)} f_{\tau_e, \tau_2}(\Omega) \frac{d\Omega}{2\pi}, \quad (26)$$

in which $\tilde{\mathcal{F}}_{\rho_{\text{em}}}(\Omega)$ denotes the Fourier transform of the Franck-Condon factor defined by Eq. (20). The filter

$$f_{\tau_e, \tau_2}(\Omega) = 4\pi\tau_e \int_{|\Omega|/2}^{+\infty} \tilde{\mathcal{Z}}_1\left(\omega - \frac{\Omega}{2}\right) e^{-2\omega\tau_e} e^{-i(\omega - \frac{\Omega}{2})\tau_2} \frac{d\omega}{2\pi} \quad (27)$$

contains the effects of electronic decoherence along branch 1 via the elastic scattering amplitude $\mathcal{Z}_1(\omega)$ and ballistic propagation along branch 2 of the interferometer for a leviton of duration τ_e .

2. The vacuum baseline

Let us start by the vacuum baseline, defined as the dc contrast in the absence of external radiation ($\rho_{\text{em}} = |0\rangle\langle 0|$),

$$[X_+^{(\text{dc})}]_{|0\rangle} = 4\pi\tau_e \int_0^{+\infty} \tilde{\mathcal{Z}}_1(\omega) e^{-2\omega\tau_e} e^{-i\omega\tau_2} \frac{d\omega}{2\pi}. \quad (28)$$

This quantity, which represents the interference contrast, is reduced because of two different physical effects.

First of all, even in the absence of electronic decoherence, the difference of times of flight along the target and the reference branch reduces the interference signal. For $\tilde{\mathcal{Z}}_1(\omega) = e^{i\omega\tau_1}$, which corresponds to ballistic propagation along the target branch with time of flight τ_1 , we find

$$[X_+^{(\text{dc})}]_{|0\rangle} = \frac{2\tau_e}{2\tau_e - i\tau_{12}}, \quad (29)$$

where $\tau_{12} = \tau_1 - \tau_2$ characterizes the interferometer's imbalance. As expected, the interference signal significantly decays as soon as $|\tau_{12}|$ becomes larger than $2\tau_e$.

In the perspective of the EQR radar, by using time-localized wave packets such as the levitons, the goal would be to achieve such time-of-flight synchronization in order to obtain the strongest information signal. Depending on the experimental situation, the geometry of the device may be constrained so that a lower bound $\tau_e \gtrsim \min |\tau_{12}|/2 > 0$ is imposed, thereby intuitively limiting the time resolution of the interferometer. In other devices, we may be able to tune τ_2 in order to maximize the contrast in the absence of external radiation. This is what we assume in the rest of this paper, but the general expressions given by Eqs. (26) and (27) enable us to discuss any situation.

The second cause of interference contrast reduction is electronic decoherence, which occurs when no radiation is sent into the radiation coupler. Being associated with the imprint left in its electromagnetic environment at zero temperature by the electronic wave packet, it is the

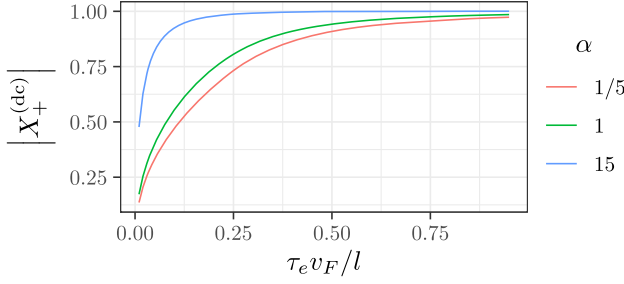


FIG. 6. Optimized contrast $\max_{\tau_2} |X_+^{(\text{dc})}|$ of dc-current interference fringes for a leviton of duration τ_e as function of $\tau_e v_F / l$ in the absence of external radiation. The radiation coupler involves two counterpropagating edge channels in total mutual electrostatic influence over a distance l (see Appendix D). Curves have been plotted for $\alpha = 1/5$ (red), 1 (green), and 15 (blue).

irreducible part of contrast reduction. In the present formalism, electronic decoherence is associated with the decay of the electronic quasiparticle which is measured by the inelastic scattering probability $1 - |\tilde{Z}_1(\omega)|^2$.

Figure 6 depicts the vacuum baseline $\max_{\tau_2} |X_+^{(\text{dc})}|$ optimized on τ_2 as a function of the leviton's width τ_e . These graphs, obtained for a radiation coupler built from two counterpropagating edge channels in total electrostatic influence over a distance l via a geometric capacitance C_g [see Fig. 4(a)], are plotted for different values of the dimensionless coupling constant

$$\alpha = \frac{e^2 l}{\hbar v_F C_g}, \quad (30)$$

encoding the importance of Coulomb interaction effects within this radiation coupler (see Appendix D). As expected, at fixed α , electronic decoherence is stronger for shorter leviton pulses due to their high-energy components. For a given τ_e , it is also stronger at small α compared to the Coulomb-dominated regime (large α), an effect already predicted [82] and observed [89] when an electron of energy $\hbar\omega_e$ propagates across a metallic island with Coulomb energy $E_C \gg \hbar\omega_e$. Note that a 50% contrast can be achieved when considering levitons of durations $\tau_e \gtrsim l/10v_F$, which, in the case of $l = 10 \mu\text{m}$ and $v_F = 10^5 \text{ m s}^{-1}$ corresponds to pulses of duration down to 10 ps.

As of today, established state-of-the-art techniques achieve leviton pulses of around 30 ps. However, recent advancements in this field [54,55] suggest that significant progress toward 10–20 ps durations are within reach.

3. Filtering of the Franck-Condon factor

We now consider the effect of the leviton's duration on the Franck-Condon prefactor. Without electronic decoherence, one expects the duration of the leviton to be the limiting time-resolution factor for accessing $\mathcal{F}_{\rho_{\text{em}}}(t)$. In the absence of electronic decoherence, using $\tilde{Z}_1(\omega) = e^{i\omega\tau_1}$ leads to

$$[X_+^{(\text{dc})}]_{\rho_{\text{em}}} = [X_+^{(\text{dc})}]_{|0\rangle} (K_{\tau_e, \tau_{12}} * \mathcal{F}_{\rho_{\text{em}}})(t_e + \tau_2), \quad (31)$$

where $[X_+^{(\text{dc})}]_{|0\rangle}$ is given by Eq. (29) and the kernel

$$K_{\tau_e, \tau_{12}}(\tau) = \frac{1}{\pi} \frac{\tau_e - i\frac{\tau_{12}}{2}}{(\tau_e - i\frac{\tau_{12}}{2})^2 + (\tau + \frac{\tau_{12}}{2})^2} \quad (32)$$

encodes the way the leviton's duration and the MZI's imbalance blur the temporal resolution on the Franck-Condon factor $\mathcal{F}_{\rho_{\text{em}}}(t)$. Equations (31) and (32) show that the duration τ_e of the leviton limits the time resolution for accessing $\mathcal{F}_{\rho_{\text{em}}}(t)$.

Electronic decoherence alters the expression of the kernel in Eq. (31). In this context, we define τ_1 as the Wigner-Smith time delay associated with the elastic scattering amplitude $\tilde{Z}_1(\omega)$: $\tau_1 = -i(d\{\ln[\tilde{Z}_1(\omega)]\}/d\omega)$ at $\omega = 0$. Note that τ_1 corresponds to the effective time of flight of electronic wave packets of low energy, so that they do not experience strong electronic decoherence [57]. We have to resort to the general filtering expressions given by Eqs. (26) and (27). As explained in Appendix I, the filtering function $f_{\tau_e, \tau_2}(\Omega)$ always satisfies

$$f_{\tau_e, \tau_2}(\Omega \geq 0) = e^{-\Omega\tau_e} f_{\tau_e, \tau_2}(0) = [X_+^{(\text{dc})}]_{|0\rangle} \times e^{-\Omega\tau_e}. \quad (33)$$

However, for $\Omega < 0$, one has

$$f_{\tau_e, \tau_2}(\Omega < 0) = e^{-|\Omega|\tau_e} e^{-i\Omega\tau_{12}} \times 4\pi\tau_e \int_0^{+\infty} e^{-2\omega\tau_e} e^{i\omega\tau_{12}} \tilde{Z}_{1*}(|\Omega| + \omega) \frac{d\omega}{2\pi}, \quad (34)$$

in which, for convenience, we have isolated the τ_1 dependence by introducing $\tilde{Z}_{1*}(\omega) = e^{-i\omega\tau_1} \tilde{Z}_1(\omega)$. Its Ω dependence is *a priori* more involved than in Eq. (33) and must be evaluated numerically.

Nevertheless, whenever the typical timescale associated with the radiation is much slower than τ_e and τ_D , defined as the inverse of the energy scale over which the inelastic scattering probability $\sigma_{\text{in}}(\omega)$ starts to increase significantly, one may neglect the Ω dependence in $\tilde{Z}_{1*}(|\Omega| + \omega)$, thereby leading to

$$f_{\tau_e, \tau_2}(\Omega < 0) \simeq e^{-|\Omega|\tau_e} e^{i|\Omega|\tau_{12}} f_{\tau_e, \tau_2}(0). \quad (35)$$

Equations (33) and (35) then lead to Eq. (31) with the same kernel $K_{\tau_e, \tau_{12}}$ as in Eq. (32).

Equation (35) is an “adiabatic radiation approximation,” assuming that electronic decoherence can be neglected for the frequencies involved in the Franck-Condon factor. Note that this approximation does not assume that leviton wave packets, which involve much higher frequencies, experience

a weak electronic decoherence. Electronic decoherence is accounted for by the vacuum baseline $[X_+^{(\text{dc})}]_{|0\rangle}$ whose modulus can be much lower than the modulus of the rhs of Eq. (29), which is only valid for ballistic propagation along the target branch.

Interestingly, for very short levitons and a balanced interferometer ($\tau_{12} = 0$), the kernel converges to the $\delta(\tau)$ distribution and the heuristic result of Eq. (25) is recovered. The point is that we have an exact expression for the vacuum baseline given by Eq. (28). Moreover, we know that this is the result of an approximation that breaks down when electronic decoherence effects manifest themselves at frequencies similar to the inverse timescale of variation of $\mathcal{F}_{\rho_{\text{em}}}(t)$. More importantly, the results presented here enable us to go beyond this approximation and to consider an imbalanced ($\tau_{12} \neq 0$) interferometer.

IV. PREDICTIONS FOR CLASSICAL AND QUANTUM RADIATION

We now consider various types of radiation directly relevant for forthcoming experiments. This includes classical radiation for which the physics of a time-dependent phase seen by independent electrons of Sec. II B will be recovered in the voltage-locked regime of the radiation coupler.

We then discuss the ability of the electron radar to probe nonclassical radiation by considering squeezed states as well as single EMPs. Such quantum states of radiation cannot be described in terms of a classically fluctuating voltage and, therefore, the full quantum approach of the previous section will be necessary to obtain quantitative predictions.

Finally, we discuss how our single-electron interferometry proposal compares to other state-of-the-art techniques to measure time-dependent quantum fluctuations.

A. Classical radiation

We consider a classical drive $V_g(t)$ applied to a top gate capacitively coupled to the edge channel along the target branch $|x| \leq l/2$ of the MZI via a geometric capacitance C_g as depicted in Fig. 7. This corresponds to Fig. 4(b) without dynamical degrees of freedom coupled to the top gate [$Z(\omega) = 0$]. This is the limit of an infinite number of electronic channels in the top gate's lead in Ref. [90].

Because there are no external dynamical degrees of freedom in this top-gate model, its dynamics can be described by an input-output relation for the edge channel's EMP modes of the form

$$i_{\text{out}}(\omega) = t(\omega)i_{\text{in}}(\omega) + Y(\omega)V_g(\omega), \quad (36)$$

where $t(\omega)$ denotes the EMP transmission amplitude across the region $|x| \leq l/2$ and the admittance $Y(\omega)$ describes the response of the outgoing current to the top-gate potential

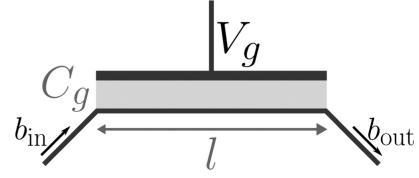


FIG. 7. A top gate is capacitively coupled to the $|x| \leq l/2$ region of a chiral-edge channel. It is driven by a time-dependent gate voltage $V_g(t)$. C_g denotes the geometric capacitance between the two conductors.

$V_g(\omega)$. Assuming total mutual influence between the top gate and target branch of the MZI, $Y(\omega)$ is the finite-frequency admittance of the electrical dipole formed by the top gate and the edge channel. It is related to $t(\omega)$ by [57,60,75]

$$Y(\omega) = \frac{e^2}{h} [1 - t(\omega)]. \quad (37)$$

Predictions for these coefficients can be obtained from the discrete element model corresponding to Fig. 7 presented in Appendix C.

Since the gate voltage translates the EMP operator $b_{\text{in}}(\omega)$ by $-Y(\omega)V_g(\omega)/e\sqrt{\omega}$, it introduces a phase factor

$$\mathcal{F}_{V_g}(t) = e^{\frac{ie}{h} \int_{\mathbb{R}} \Gamma(t-\tau) V_g(\tau) d\tau} \quad (38)$$

in front of the fermionic field coming out of the radiation coupler. The kernel $\Gamma(\tau)$ is the inverse Fourier transform of $-R_K Y(\omega)/i\omega$. This shows that the effective single-particle scattering amplitude retains the form given by Eq. (23) with $\mathcal{Z}_1(\tau)$ given by Eq. (18) with $t(\omega) = 1 - R_K Y(\omega)$ and $\mathcal{F}_{V_g}(t)$ playing the role of $\mathcal{F}_{\rho_{\text{em}}}(t)$ defined by Eq. (20) in the full quantum theory. In particular, because the only effect of the external drive $V_g(t)$ is to introduce a phase, it does not contribute to electronic decoherence in the time domain.

The importance of Coulomb interactions within the $|x| \leq l/2$ region depends on the dimensionless coupling constant $\alpha = C_q/C_g$ defined by Eq. (30). For $\alpha \ll 1$, the voltage drop at the capacitance vanishes and therefore the electrons directly see the gate voltage $V_g(t)$. This is the voltage-locked regime. On the other hand, for $\alpha \gtrsim 1$, Coulomb interactions are so strong that they tend to block charge accumulation below the top gate. This is the Coulomb-blockade regime.

1. The voltage-locked regime

At very small α , the electrochemical capacitance $C_\mu = C_g C_q / (C_g + C_q) \simeq C_q$ is dominated by the quantum capacitance $C_q \ll C_g$. The voltage drop at the capacitance is negligible and electrons within the $|x| \leq l/2$ region experience the gate potential $V_g(t)$. Moreover, in this limit,

$t(\omega) \simeq e^{i\omega l/v_F}$, the elastic scattering amplitude is $\mathcal{Z}_1(\omega) \sim e^{i\omega l/v_F}$ and the electronic decoherence can thus be neglected [57]. In other terms, there is no backaction of the incoming electron on the edge channel mediated by the top gate which could lead to electronic decoherence. The effective scattering amplitude can be approximated by

$$R_{\text{eff}}(t, t') \simeq \mathcal{F}_{V_g}(t) \delta\left(t - t' - \frac{l}{v_F}\right). \quad (39)$$

At very small α , $\Gamma(\tau) = \mathbf{1}_{[0, l/v_F]}(\tau)$ and, therefore, we recover Eq. (3) with $\tau_1 = l/v_F$ and $U(t) = V_g(t)$ as expected.

2. Intermediate regime

As α is increased, electronic decoherence starts to appear, but as long as we keep the energy of the electronic excitation low enough, it can be neglected [57]. Nevertheless, increasing α modifies the voltage seen by the electrons below the top gate, which becomes a filtering of $V_g(\omega)$ [see Eq. (C19)]. Consequently, as long as electronic decoherence can be neglected, the MZI detects the accumulated electric phase associated with this filtered effective time-dependent potential.

For leviton pulses, the discussion of Sec. III C shows that, as long as the external voltage involves only low frequencies compared to the ones for which electronic decoherence starts to be significative, the adiabatic approximation of Eq. (35) can still be applied. The ratio of the contrast with $V_g(t)$ and for $V_g(t) = 0$ can thus be obtained as

$$\frac{[X_+^{(\text{dc})}]_{V_g(t)}}{[X_+^{(\text{dc})}]_0} = \sum_{n \in \mathbb{Z}} \mathcal{F}_n[V_g(t)] e^{-2\pi n f \tau_e} \times e^{-2\pi i n f (t_e + \tau_2)} e^{-2\pi i n \Theta(-n) f \tau_{21}}, \quad (40)$$

in which $\mathcal{F}_n[V_g(t)]$ are the photoassisted amplitudes associated with $V_g(t)$ mediated through the top gate, i.e., the Fourier coefficients of the Franck-Condon factor given by Eq. (38). As explained in Sec. III C 2, the vacuum baseline $[X_+^{(\text{dc})}]_0$ takes into account electronic decoherence effects.

Therefore, for a slow enough drive, even when electronic decoherence effects are strong, quantitative predictions for the relative dc contrast $[X_+^{(\text{dc})}]_{V_g(t)}/[X_+^{(\text{dc})}]_0$ for leviton pulses can still be obtained from the simple formalism discussed in Sec. II B, provided one properly takes into account the filtering of $V_g(t)$ by the top gate.

3. The Coulomb-blockade regime

The limit of large α corresponds to the regime dominated by Coulomb interactions: they are so strong that no charge can accumulate on the $|x| \leq l/2$ interaction region of the edge channel, nor on the top gate. As explained in Appendix C 2, the outgoing EMP mode $b_{\text{out}}(\omega)$ has a very

small response to $V_g(t)$. Consequently, this is not the proper regime of operation for the EQR.

Note that electronic decoherence is not a problem in this regime. This echoes the results obtained recently in Ref. [91] in which a similar phenomenon occurs in a MZI where the top gate in the strong coupling regime is replaced by a metallic island with small enough capacitance.

B. Squeezed radiation

Let us now discuss the ability of the EQR to detect squeezing by considering a Gaussian squeezed state. Our proposal is based on comparing the interference contrast in the presence and in the absence of external squeezed radiations. Let us recall that vacuum fluctuations induce electronic decoherence and thus set the vacuum baseline for the interferometer's contrast (see Sec. III C 2). For sufficiently short electronic wave packets sent at the proper time, we expect transient subvacuum voltage fluctuations to reduce electronic decoherence. Such wave packets will thus interfere with a higher contrast than in the absence of external radiation. In this section, we discuss this idea quantitatively and show that the signature of squeezing is within reach of the experimental state of the art.

1. General squeezing criterion

First of all, the radiation coupler is sensitive only to the radiation in a specific filtered mode. The Franck-Condon factor can then be computed using a Gaussian approximation for this mode. This leads to

$$\mathcal{F}_{\rho_{\text{em}}}(t) = e^{i\phi(t)} e^{-\langle (\Delta Y_t)^2 \rangle_{\rho_{\text{em}}} - \langle (\Delta Y_t)^2 \rangle_{|0\rangle}}, \quad (41)$$

where the phase $\phi(t)$, due to the average value, can be viewed as arising from the classical voltage felt by the electrons within the upper branch of the MZI. The other contribution comes from the Gaussian fluctuations of the quadrature

$$Y_t = \frac{i}{\sqrt{2}} \int_0^{\omega_c} \left(\frac{S_{ba}(\omega)}{\sqrt{\omega}} e^{-i\omega t} a_\omega - \text{H.c.} \right) d\omega \quad (42)$$

of the mode filtered by the radiation coupler. Here, ω_c is any UV cutoff below which all the incoming radiation is emitted.

Because this second contribution changes $|\mathcal{F}_{\rho_{\text{em}}}(t)|$, it provides a sufficient criterion for squeezing: As soon as $|\mathcal{F}_{\rho_{\text{em}}}(t)| > 1$, there is squeezing in the filtered mode since this is a signature of the fact that, for these values of t , fluctuations of the quadrature Y_t are smaller than in the vacuum state: $\langle (\Delta Y_t)^2 \rangle_{\rho_{\text{em}}} < \langle (\Delta Y_t)^2 \rangle_{|0\rangle}$.

2. Squeezing around a given frequency

Because of its relevance for experiments, let us discuss the ability of the EQR to detect squeezed radiation in an

electromagnetic mode around $\omega \simeq \omega_0$ within a bandwidth γ_0 such that $Q_0 = \omega_0/\gamma_0$ is significantly larger than 1 so that $S_{ba}(\omega)$ can be taken as constant for $|\omega - \omega_0| \leq \gamma_0/2$. In Appendix J, we show that, for a time-periodic squeezed quantum noise whose power is concentrated in the narrow band $|\omega - \omega_0| \leq \gamma_0/2$, the Franck-Condon factor takes the form

$$\mathcal{F}_{\text{Sq}_z}(t) \simeq e^{\frac{|S_{ba}(\omega_0)|^2}{2Q_0} \sinh(2|z|) [\cosh(2|z|) \cos(2\omega_0 t - \phi_0) - \sinh(2|z|)]}, \quad (43)$$

where z is a squeezing parameter and $\phi_0 = 2\text{Arg}[S_{ba}(\omega_0)] + \text{Arg}(z)$. Its minimum value is

$$\min_t |\mathcal{F}_{\text{Sq}_z}(t)| = e^{-\frac{|S_{ba}(\omega_0)|^2}{2Q_0} (e^{4|z|} - 1)} < 1. \quad (44)$$

Let us recall that the compression factor for quadrature fluctuations in a squeezed vacuum $|\text{Sq}_z\rangle$ is given by $e^{-4|z|}$. As of today, a compression of 18% (0.86 dB) with respect to vacuum fluctuations has been achieved in quantum Hall edge channels [63]. Assuming a quality factor $Q_0 = 5$ and no losses $[|S_{ba}(\omega_0)|^2 = 1]$ for the radiation coupler leads to a maximal increase of $|\mathcal{F}_{\text{Sq}_z}(t)|$ by 1.8%, whereas a 3 dB squeezing would lead to an increase by 5.1%.

3. Numerical results and discussion

Let us now discuss whether or not this is observable using levitons. We have done it by evaluating numerically the filtering function $f_{\tau_e, \tau_2}(\Omega)$ as well as the exact Fourier series of the Franck-Condon factor given by Eq. (43).

To have an intuition of the results, we have obtained analytical expressions within the adiabatic radiation approximation [Eq. (35)] by retaining only the first harmonics in the Fourier series of the Franck-Condon factor [92]. Using this approximation, an analytical expression for the maximum over t_e of the relative contrast with respect to the vacuum baseline as a function of $|z|$ can be derived at the lowest order in $\Lambda = |S_{ba}(\omega_0)|^2/Q_0$,

$$\max_{t_e} \left| \frac{[X_+^{(\text{dc})}]_{\text{Sq}_z}}{[X_+^{(\text{dc})}]_{|0\rangle}} \right| \simeq 1 + \Lambda F_\eta(z) + \mathcal{O}(\Lambda^2), \quad (45)$$

where

$$F_\eta(z) = \eta \cosh(2|z|) \sinh(2|z|) - \sinh^2(2|z|), \quad (46)$$

with $\eta = e^{-2\omega_0 \tau_e} |\cos(2\omega_0 \tau_2)| < 1$. The behavior of $F_\eta(z)$ governs the observability of squeezing as an increase of the interference contrast. It starts from 0 for $z = 0$ and increases to a maximum positive value

$$\max_{|z|} \max_{t_e} \left| \frac{[X_+^{(\text{dc})}]_{\text{Sq}_z}}{[X_+^{(\text{dc})}]_{|0\rangle}} \right| \simeq 1 + \frac{\Lambda}{2} (1 - \sqrt{1 - \eta^2}) + \mathcal{O}(\Lambda^2), \quad (47)$$

reached for $|z|_{\text{opt}} = \text{artanh}(\eta)/4$. It is the maximum experimental signal expected for squeezing since $F_\eta(z)$ decreases for $|z| \geq |z|_{\text{opt}}$. It becomes negative for $|z| \geq 2|z|_{\text{opt}}$, meaning that, above $2|z|_{\text{opt}}$, the interference contrast for levitons appears smaller than the vacuum baseline. This follows from the increase of the average photon number in $|\text{Sq}_z\rangle$ with increasing z : As shown in Appendix J [see Eq. (J17)], randomly injected levitons of width τ_e will experience, on average, an increasing noise within the radiation coupler. For $|z| \geq 2|z|_{\text{opt}}$, this effect compensates the gain in contrast associated with subvacuum fluctuations experienced by the leviton, thereby leading to a decrease of the maximum relative contrast over t_e below unity.

Figure 8 displays the results of the numerical evaluation of the contrast in the voltage-locked regime with 15 ps levitons. Filled black lines correspond to the full numerical evaluation of the contrast. Dashed black horizontal lines correspond to the vacuum baseline $[X_+^{(\text{dc})}]_{|0\rangle}$, which takes into account the effects of vacuum fluctuations. Results are plotted for $z \simeq 0.0496$ corresponding to 18% (0.86 dB) noise reduction demonstrated in Ref. [63], $z \simeq 0.0719$

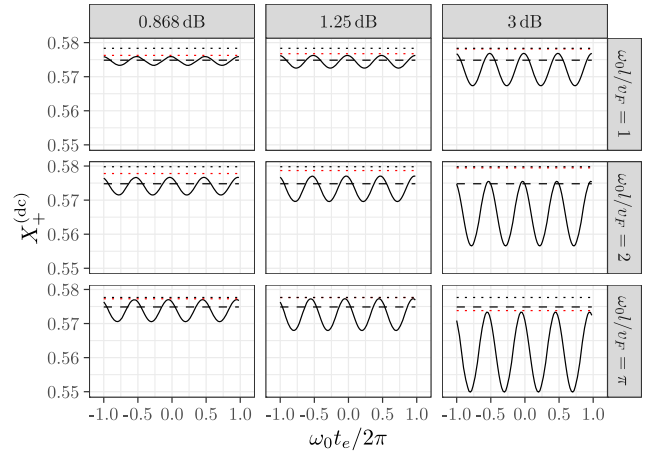


FIG. 8. The various plots depict the interference contrast $[X_+^{(\text{dc})}]_{\text{Sq}_z}$ for leviton excitations of width $\tau_e = 15$ ps assuming a 10- μm -long radiation coupler with $v_F = 10^5$ m s $^{-1}$ with coupling strength $\alpha = 1/5$. The plots show the absolute contrast as a function of the dimensionless injection time $v_F t_e / l$ for different values of the squeezing (expressed in decibels) and $\omega_0 l / v_F = 1, 2$, and π . The dashed black horizontal lines correspond to the vacuum baseline $[X_+^{(\text{dc})}]_{|0\rangle}$. The dotted black lines correspond to the maximally optimized maximum contrast given by Eq. (47), whereas the dotted red lines correspond to the evaluation of Eq. (45) for the actual value of the squeezing parameter considered in the example.

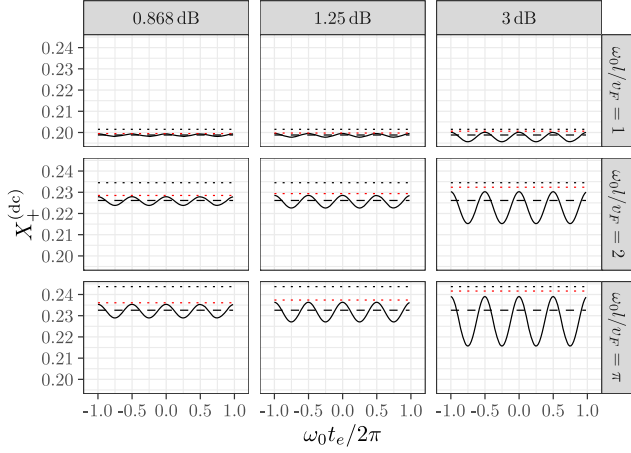


FIG. 9. The various plots depict the interference contrast $[X_+^{(dc)}]_{\text{sq}(z)}$ for leviton excitations of width $\tau_e = 2.5$ ps assuming the same parameters and legend as in Fig. 8.

corresponding to 1.25 dB (25%) noise reduction, and $z \simeq 0.1733$ corresponding to 3 dB (50%) noise reduction.

At some operating points, the contrast in the presence of the squeezed vacuum does exceed the vacuum baseline. It is a positive signature of squeezing but the overshoot is small: 0.3% on a baseline of 57.5%. The best numbers are obtained in the voltage-locked regime for 1.25 dB squeezing and $\omega_0 l / v_F = \pi$. This frequency choice corresponds to the optimal coupling between the two edge channels. As expected from the analytics, increasing too much $|z|$ leads to a decrease of the maximum relative contrast and even brings it below the vacuum baseline. Note also the good agreement between Eq. (45) and the numerical evaluation of the maximum absolute contrast. Note also that the best operating point ($\omega_0 l / v_F = \pi$ and 1.25 dB squeezing) is close to the optimal maximum contrast given by Eq. (47). Shorter levitons are expected to lead to a lower vacuum baseline (see Fig. 6) and also to shift the detrimental effect of the average noise toward higher values of $|z|$. This can be seen on Fig. 9 where 2.5 ps levitons are considered, all other parameters being the same as for Fig. 8. We see that the maximum absolute contrast still increases with increasing $|z|$ up to 3 dB squeezing instead of decreasing at 1.25 dB for 15 ps levitons. Even if the absolute contrast overshoot over the vacuum baseline is not greater than for 5 ps levitons, it comes over a 0.225 vacuum baseline instead of 0.57 and thus represents a 2 times increase in terms of the relative contrast increase. This clearly shows the pros and cons of shorter leviton pulses: the filtering associated with their duration is better, but they also lead to a lower vacuum baseline. We emphasize that, for the experimental parameters considered here, the pulse duration does not significantly change the absolute contrast overshoot over the vacuum baseline. Thus, the experimental feasibility of detecting such enhanced oscillations

remains well within reach of contemporary technology, which currently achieves pulse durations of 30 ps.

C. Fock states

We now consider the problem of detecting Fock states in a specific EMP mode. This state being non-Gaussian, the Gaussian result given by Eq. (41) breaks down.

1. The Franck-Condon factor

We consider $|N; \chi\rangle$, the N photon Fock state in the normalized mode χ . An explicit computation detailed in Appendix B 2 shows that the Franck-Condon factor $\mathcal{F}_{|N; \chi\rangle}(t)$ is then given by

$$\mathcal{F}_{|N; \chi\rangle}(t) = L_N \left(2\pi |\langle \chi | S_{ba}^* \Lambda_t | \rangle|^2 \right), \quad (48)$$

in which L_N denotes the N th Laguerre polynomial and

$$\langle S_{ba}^* \Lambda_t | \chi \rangle = - \int_0^{+\infty} \frac{S_{ba}(\omega)}{\sqrt{\omega}} e^{-i\omega t} \chi(\omega) \frac{d\omega}{2\pi}. \quad (49)$$

We now focus on the specific example of an EMP mode whose wave function in the frequency domain $\chi(\omega)$ is centered at ω_0 with Lorentzian line shape of width γ_0 . We assume that ω_0 / γ_0 is significantly larger than unity, as expected from spontaneous emission by a two-level emitter.

2. Narrow bandwidth approximation

Denoting by $x(t) = 2\pi |\langle \chi | S_{ba}^* \Lambda_t | \rangle|^2$ the argument of the Laguerre polynomial in Eq. (48), and assuming that $\chi(\omega)$ is concentrated around ω_0 with a small bandwidth $\gamma_0 \ll \omega_0$, $x(t)$ can be rewritten as (see Appendix K)

$$x(t) \simeq 2\pi \frac{\gamma_0}{\omega_0} |S_{ba}(\omega_0)|^2 \frac{\langle \mathcal{J}_Q(t) \rangle_{|1; \chi\rangle}}{\gamma_0 \hbar \omega_0}, \quad (50)$$

in which

$$\mathcal{J}_Q(t) = \frac{R_K}{2} : i(t)^2 : \quad (51)$$

denotes the instantaneous heat current operator injected in the radiation channel expressed in terms of the electrical current $i(t)$. Consequently, for $\gamma_0 / \omega_0 \lesssim 1$, the expansion $L_N(x) \simeq 1 - Nx + \mathcal{O}(x^2)$ for the Laguerre polynomial leads to

$$\mathcal{F}_{|N; \chi\rangle}(t) \simeq 1 - 2\pi N \frac{\omega_0}{\gamma_0} |S_{ba}(\omega_0)|^2 \frac{\langle \mathcal{J}_Q(t) \rangle_{|1; \chi\rangle}}{\gamma_0 \hbar \omega_0}. \quad (52)$$

Therefore, within this approximation [93], the Franck-Condon factor provides a direct measurement of the average heat current carried by the single EMP. Provided

$2\pi N\omega_0|S_{ba}(\omega_0)|^2/\gamma_0$ does not take large values for the values of N with non-negligible p_N and noticing that $\langle \mathcal{J}_Q(t) \rangle_{|N;\chi\rangle} = N\langle \mathcal{J}_Q(t) \rangle_{|1;\chi\rangle}$ enables us to average over N and show that

$$\mathcal{F}_{\rho_{\text{mix}}}(t) \simeq 1 - 2\pi \frac{\omega_0}{\gamma_0} |S_{ba}(\omega_0)|^2 \frac{\langle \mathcal{J}_Q(t) \rangle_{\rho_{\text{mix}}}}{\gamma_0 \hbar \omega_0} \quad (53)$$

for $\rho_{\text{mix}} = \sum_{N=0}^{+\infty} p_N |N;\chi\rangle \langle N;\chi|$. For such a radiation, the EQR acts as a time-resolved “quantum bolometer” by converting the incident energy flux within the radiation channel into the Franck-Condon factor. As before, assessing the potential for single-EMP detection requires discussing experimentally realistic signals in experimentally realistic situations.

3. Results and discussion

We consider Lorentzian EMP modes of duration $\gamma_0^{-1} = 1 \text{ ns} = 10l/v_F$ centered on ω_0 such that $\omega_0 l/v_F = 2, 5.5$, and 10 corresponding to respective frequencies $3.2, 8.75$, and 15.9 GHz . Levitons of duration $\tau_e = 10 \text{ ps}$ have a time resolution on the order of 1% of the EMP wave packet duration. With the parameters considered here, the vacuum baseline is expected to be above 50% (see Sec. III B 3).

Figure 10 depicts the various numerical estimates for the relative contrast decrease in the presence of a single EMP with respect to the vacuum baseline. The results, presented in Fig. 10, confirm that an observable relative contrast decrease of a few percent can be expected. Considering the dependance on ω_0 shows that the transparency of the radiation coupler around ω_0 is the dominant effect in the amplitude of the effect.

We have also compared the numerical evaluation of the full exact analytical expression to Eq. (53) (dotted line) which expresses the Franck-Condon factor in terms of the average instantaneous heat current. The full expression clearly appears as a filtered version of the heat current except on short timescales where higher-frequency contributions are expected to matter. We also present the numerical evaluation of $x(t)$ (dashed line) and this also shows the effect of the filtering of high frequencies associated with the finite duration of the leviton.

D. Comparison with the state of the art

Let us now compare single-electron interferometric measurement of time-dependent electromagnetic fluctuations with other techniques to classical or quantum electromagnetic radiation within a quantum circuit.

1. Proof of principle for classical radiation

A recent experiment [54] demonstrates the feasibility of using electron interferometry to sense a classic electromagnetic field with microvolt voltage resolution and picosecond temporal resolution.

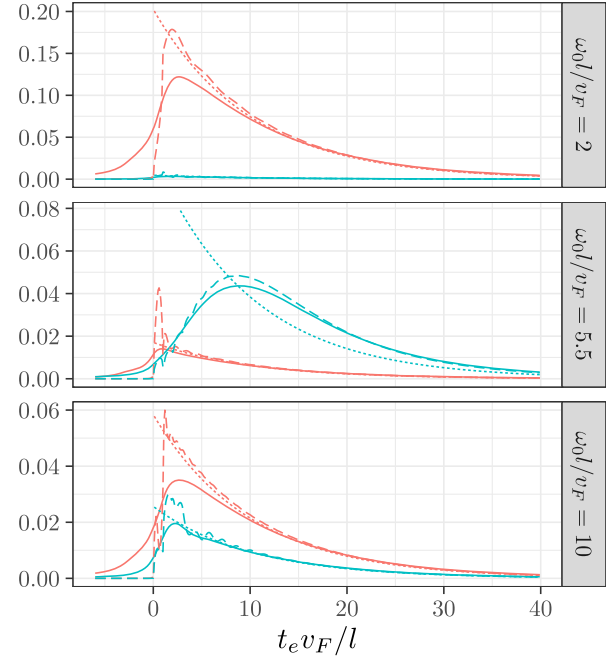


FIG. 10. Plot of the relative contrast decrease $1 - |[X_+]^{(\text{dc})}]_{|1;\chi\rangle} / [X_+]^{(\text{dc})}]_{|0\rangle}|$ (full line) associated with the detection of a single EMP with energy $\hbar\omega_0$ and Lorentzian line shape of width $\gamma_0 = 10^9 \text{ s}^{-1}$ when using leviton pulses of width $\tau_e = 10 \text{ ps}$ as a function of the leviton's injected time t_e . The radiation coupler is described in Appendix D. Red curves correspond to the voltage-locked regime ($\alpha = 1/10$), whereas the blue curves correspond to the Coulomb-blockade regime ($\alpha = 15$). Three different values of $\omega_0 l/v_F$ have been considered, showing the importance of the filtering by the radiation coupler. The dotted line depicts $\mathcal{F}_{|1;\chi\rangle}(t)$ using the analytic expression given by Eq. (52), whereas the dashed line represents $x(t) = 2\pi |\langle \chi | S_{ba} \Lambda_t \rangle|^2$.

Although in a different setup (Fabry-Perot interferometer instead of Mach-Zehnder), this experiment can be viewed as a proof of principle for the ideas discussed in Sec. IV A. Because electronic decoherence in this experiment is well tamed, it can be understood in terms of imprinting a time-dependent phase onto a short wave packet. Without optimizing the radiation coupler, its sensitivity is equivalent to detecting ten photons. This suggests that achieving single-photon resolution for quantum fields is within reach, aligning closely with the precision of our proposed setup and paving the way for detecting quantum states of incoming radiation.

2. Toward probing quantum radiation

As discussed in Secs. IV B and IV C, detecting quantum radiation requires accessing quantum electromagnetic fluctuations and, potentially, their time dependence. Accessing time-dependent current noise is also crucial to the measurement of the instantaneous average heat current discussed in Ref. [66]. The above discussion of single-electron interferometric detection of squeezing as well as of probing

of incoherent mixtures of Fock states suggests that the interferometric method can bring useful information on these classes of quantum states. Here, we comment on the pros and cons of single interferometry compared to alternative, more standard, methods for accessing time-dependent quantum current noise.

In high-impedance systems such as quantum Hall edge channels, detection of time-dependent subvacuum fluctuations have relied on high-sensitivity current noise measurements at finite and low frequency, performed through homodyne protocols as in Ref. [63]. This technique has also been used in lower-impedance systems with a tunnel junction [64]. These measurements are notoriously difficult to perform, especially in high-impedance systems where impedance matching problems tend to degrade the signal-to-noise ratio of the measurement chain.

By contrast, our interferometric method requires only dc-average current measurements, which are much easier to perform at the level of sensitivity required to observe subvacuum fluctuations. This reduces not only the complexity of the measurement chain but also offers a significant time gain, enabling quicker and more straightforward characterization of quantum current noise. In high-impedance systems, we expect the measurement time to drop from days to a few hours.

For low-impedance systems, time-dependent measurements benefit from a much better impedance matching, which leads to a better collection of the signal enabling the use of high-end analog-to-digital converters that can operate at a pace of 3.2×10^{10} samplings/s. Even with an amplification chain with a broader bandwidth (an already challenging task in itself), this would set a time resolution of 30 ps [67,68] comparable to the resolution obtained using state-of-the-art Lorentzian pulses. Nevertheless, the forthcoming generation of pulses in the 10 ps range and, in the longer term, even lower may give our technique a significant advantage in terms of time resolution.

V. CONCLUSIONS AND PERSPECTIVES

In this work, we have discussed single-electron interferometric sensing of classical and quantum electromagnetic fields. The idea is to probe a time-dependent, possibly quantum, electromagnetic field by coupling it to one of the branches of an electronic MZI fed by single-electron excitations. They provide a very sensitive, although minimally invasive, probe. Information about the field is then obtained by measuring the interference contribution to the average outgoing dc current from the interferometer.

We have developed a general framework describing such a single-electron interferometer in the Aharonov-Bohm regime for any type of radiation coupler realizing the capacitive coupling between the radiation to be probed and the electrons propagating within the MZI. In the limit where the radiation coupler is strongly dominated by its quantum capacitance, the electrons directly experience the

voltage imposed by the external radiation. A coherent state of the incoming radiation just imprints a time-dependent phase on the single-electron excitations injected into the MZI, as expected when interaction effects between electrons can be neglected. However, our framework goes beyond this simple picture of time-dependent single-particle scattering and fully incorporates electronic decoherence effects associated with Coulomb interactions.

We have explored the potential for sensing quantum features of the probed radiation. Although negativity of the multimode Wigner function is a sufficient criterion for nonclassicality, performing a full tomography of a broadband radiation requires a measurement overhead exponentially growing with the number of modes considered [94]. As in quantum photonics [95,96], one has to look for simple ways to probe a quantum radiation for specific families of quantum states.

Following this philosophy, we have shown that, for Gaussian incident radiation, short enough ($\lesssim 30$ ps) electronic wave packets would lead to a measurable increase of the contrast of Aharonov-Bohm interference fringes when experiencing subvacuum fluctuations of a 1.15 dB squeezed microwave mode propagating within a nearby quantum Hall edge channel. Motivated by the perspective of generating Fock states of edge magnetoplasmons in quantum Hall devices, we have shown that short leviton pulses enable measuring the average heat current carried by such quantum EMP states as a function of time, a quantity never measured before.

These examples suggest that the single-electron interferometer fed by suitably short levitons may be used as a time-resolved quantum noise sensor on subnanosecond, possibly down to few picosecond, timescales. Importantly, these examples show that one can obtain relevant information on time-dependent noise via a dc-current measurement, thereby bypassing the notoriously difficult high-sensitivity measurements of time-dependent current noise [63].

Several perspectives are opened by the present work. First of all, obtaining more quantitative predictions for forthcoming experiments may require completing the present study to account for finite temperature and charging effects as in Refs. [97–99]. Second, recent progress in the shaping [90] and characterization [15,17] of electronic excitations emitted by single-electron sources [100] suggest investigating the potential of well-known techniques from classical radar engineering to improve the electron radar's bandwidth [101]. An important related question is to design a set of single-electron excitations enabling a full reconstruction of the effective single-particle amplitude induced by the coupling to the external radiation, thereby mimicking the use of infinite chirps [102] in radars and sonars to reconstruct the target's scattering amplitude by inverse Radon transform, a technique commonly used in computed tomography scans [103] and quantum optics [104,105] and for reconstructing the quantum state of

solitary electrons [16]. However, in the presence of a Fermi sea, infinite chirps cannot be generated because of the Fermi sea, thereby leaving the problem of tomographic reconstruction of the effective single-electron scattering amplitude open for further investigation.

Another important issue consists of studying the signal-to-noise ratio in such an electronic interferometer. Quantitative predictions for the current noise would open the way to discussing the basic question of quantum metrology in the present context: how to choose the source and the measured quantity in order to discriminate between various quantum states of the incoming radiation in an optimal way. This is analogous to the problem of super-resolution encountered in optics and astronomy [106–109]. The standard methods used in this field can certainly be transposed but this requires an evaluation of the outgoing current noise. Motivated by the recent work in Ref. [110], one could also investigate whether or not injecting a few electron states into the MZI would lead to some quantum advantage as entangled states do in quantum metrology [111].

Finally, the present work discusses the information extracted from repeated experiments in which a precise synchronization between the source and the external radiation can be established. The potential emergence of one-shot single-electron detection [21,22,112] calls for an adaptation of the present work to these forthcoming detection methods. These very interesting questions, which go well beyond the scope of the present work, are left for future investigations.

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APPENDIX A: NOTATIONS AND NORMALIZATIONS

This appendix recalls the basic conventions used for fermionic modes and electronic single-particle states within the present work.

1. Electronic modes and wave packets

The mode decomposition for fermionic fields is defined by

$$\psi(t) = \int_{\mathbb{R}} c(\omega) e^{-i\omega t} \frac{d\omega}{\sqrt{2\pi v_F}} \quad (\text{A1})$$

so that these modes obey the canonical anticommutation relations

$$\{c(\omega), c^\dagger(\omega')\} = \delta(\omega - \omega'). \quad (\text{A2})$$

Equivalently, we have

$$c(\omega) = \sqrt{\frac{v_F}{2\pi}} \int_{\mathbb{R}} \psi(t) e^{i\omega t} dt, \quad (\text{A3})$$

given an electronic wave packet described by a normalized wave function $\varphi_e(x)$ such that

$$\int_{\mathbb{R}} |\varphi_e(x)|^2 dx = 1. \quad (\text{A4})$$

Throughout this paper, we will use the notation $\varphi_e(t)$ for $\varphi_e(-v_F t)$ so that

$$v_F \int_{\mathbb{R}} |\varphi_e(t)|^2 dt = 1. \quad (\text{A5})$$

We define

$$\tilde{\varphi}_e(\omega) = v_F \int_{\mathbb{R}} \varphi_e(t) e^{i\omega t} dt \quad (\text{A6})$$

so that

$$\frac{1}{v_F} \int_{\mathbb{R}} |\tilde{\varphi}_e(\omega)|^2 \frac{d\omega}{2\pi} = 1. \quad (\text{A7})$$

The creation operator for the electronic wave packet φ_e is then defined as

$$\psi^\dagger[\varphi_e] = v_F \int_{\mathbb{R}} \varphi_e(t) \psi^\dagger(t) dt \quad (\text{A8a})$$

$$= \int_{\mathbb{R}} \tilde{\varphi}_e(\omega) c^\dagger(\omega) \frac{d\omega}{\sqrt{2\pi v_F}}. \quad (\text{A8b})$$

APPENDIX B: BOSONIZATION AND BOSONIC MODES

Here, we briefly recall useful formulas in the bosonization of quantum Hall edge channels as well as for dealing with bosonic excitations.

1. Bosonization of a quantum Hall edge channel

For a right-moving quantum Hall edge channel at filling fraction $\nu = 1$, the charge and current densities are related to the free chiral right-moving field $\phi_R(x, t)$ by

$$\rho_R(x, t) = -\frac{e}{\sqrt{\pi}}(\partial_x \phi_R)(x, t), \quad (\text{B1a})$$

$$i_R(x, t) = \frac{e}{\sqrt{\pi}}(\partial_t \phi_R)(x, t), \quad (\text{B1b})$$

and the right-moving field is decomposed in terms of bosonic destruction and creation operators $b(\omega)$ and $b^\dagger(\omega)$ for $\omega > 0$ satisfying

$$[b(\omega), b^\dagger(\omega')] = \delta(\omega - \omega') \quad (\text{B2})$$

by

$$\phi_R(x, t) = \frac{-i}{\sqrt{4\pi}} \int_{\mathbb{R}^+} \left(b(\omega) e^{i\omega(x/v_F - t)} - \text{H.c.} \right) \frac{d\omega}{\sqrt{\omega}}. \quad (\text{B3})$$

This leads to the following mode decompositions for the charge and current densities:

$$\rho_R(x, t) = -e \int_{\mathbb{R}^+} \sqrt{\omega} \left(b(\omega) e^{i\omega x/v} + b^\dagger(\omega) e^{-i\omega x/v} \right) \frac{d\omega}{2\pi v}, \quad (\text{B4a})$$

$$i_R(x, t) = -\frac{e}{2\pi} \int_{\mathbb{R}^+} \sqrt{\omega} \left(b(\omega) e^{i\omega x/v} - b^\dagger(\omega) e^{-i\omega x/v} \right) d\omega, \quad (\text{B4b})$$

thereby connecting the finite-frequency current to the EMP creation and destruction operators: $i_R(\omega > 0) = -e\sqrt{\omega}b(\omega)$.

The right-moving electronic field is then expressed in terms of the chiral bosonic field by

$$\psi_R(x, t) = \frac{\hat{U}}{\sqrt{2\pi a}} e^{i\sqrt{4\pi}\phi_R(x, t)}, \quad (\text{B5})$$

in which a , which has the dimension of a length, is an UV cutoff and $\hat{U}$ is the operator lowering the total fermion number in the edge channel. Using the mode expansion Eq. (B3), one arrives at the expression for $\psi_R(x = 0, t)$ in terms of an infinite-dimensional displacement operator,

$$\psi_R(0, t) = \frac{\hat{U}}{\sqrt{2\pi a}} D_b[\Lambda_t], \quad (\text{B6})$$

in which $\Lambda_t(\omega) = -e^{i\omega t}/\sqrt{\omega}$. The infinite-dimensional displacement operator with complex-valued functional parameter $\Lambda: \omega \mapsto \Lambda(\omega)$ is defined as

$$D_b[\Lambda] = \exp \left[\int_0^{+\infty} (\Lambda(\omega) b^\dagger(\omega) - \text{H.c.}) d\omega \right]. \quad (\text{B7})$$

2. Conventions for bosonic excitations

A normalized excitation is described by $\chi(\omega)$ for $\omega > 0$ such that

$$\int_0^{+\infty} |\chi(\omega)|^2 \frac{d\omega}{2\pi} = 1 \quad (\text{B8})$$

so that the single-particle state

$$|\chi\rangle = \int_0^{+\infty} \chi(\omega) b^\dagger(\omega) |\emptyset\rangle \frac{d\omega}{\sqrt{2\pi}} \quad (\text{B9})$$

is normalized. If we define the corresponding creation operator

$$b^\dagger[\chi] = \int_0^{+\infty} \chi(\omega) b^\dagger(\omega) \frac{d\omega}{\sqrt{2\pi}} \quad (\text{B10})$$

and its adjoint $b[\chi]$, these operators obey the commutation relations

$$[b[\chi_1], b^\dagger[\chi_2]] = \langle \chi_1 | \chi_2 \rangle \mathbf{1}, \quad (\text{B11})$$

where $\langle \chi_1 | \chi_2 \rangle$ denotes the scalar product

$$\langle \chi_1 | \chi_2 \rangle = \int_0^{+\infty} \chi_1(\omega)^* \chi_2(\omega) \frac{d\omega}{2\pi} \quad (\text{B12})$$

on the space $\mathcal{L}_2(\mathbb{R}^+)$ of square-summable functions on $\mathbb{R}^+$. Note that with these conventions $\chi(\omega) = \sqrt{2\pi} \langle \omega | \chi \rangle$, where $|\omega\rangle = b^\dagger(\omega) |\emptyset\rangle$ is the single-photon state resolved in energy.

Finally, when given an orthonormal basis of normalized single-particle states $|\chi_n\rangle$ indexed by n , the mode operators $b(\omega)$ can be expressed in terms of the $b_n = b[\chi_n]$ as

$$b(\omega) = \frac{1}{\sqrt{2\pi}} \sum_n \chi_n(\omega) b_n. \quad (\text{B13})$$

Starting from a normalized single-particle state $|\chi\rangle$, we can express $b^\dagger(\omega)$ in terms of $b^\dagger[\chi]$ and of $b^\dagger[\chi_\perp]$ where $|\chi_\perp\rangle$ denotes the normalized projection of $|\omega\rangle$ on the space of single-particle excitations orthogonal to $|\chi\rangle$,

$$b^\dagger(\omega) = \langle \chi | \omega \rangle b^\dagger[\chi] + \sqrt{1 - |\langle \chi | \omega \rangle|^2} b^\dagger[\chi_\perp]. \quad (\text{B14})$$

APPENDIX C: THE CLASSICALLY DRIVEN EDGE CHANNEL

In this appendix, we discuss two models for a classically driven edge channel. In the first model, discussed in Sec. C 1, we consider that a classical drive $U(t)$ is directly applied to the electrons propagating within the edge channel. However, in experiments, the external drive is not directly applied to the edge channels but to a top gate. This leads us to the second model, discussed in Sec. C 2, which involves a capacitive coupling between a finite-length region of the edge channel and a top gate to which the time-dependent classical voltage is applied. We then discuss how this second model reduces to the first one in the regime of weak Coulomb interactions.

1. Direct coupling to the external voltage

Let us first consider electrons propagating within a chiral-edge channel with Fermi velocity v_F and experiencing in the $0 \leq x \leq l$ a time-dependent $U(x, t)$.

a. EMP scattering

The starting point is the equation of motion for the chiral bosonic field $\phi_R(x, t)$ built from the EMP modes of the chiral-edge channel,

$$(\partial_t + v_F \partial_x) \phi_R(x, t) = \frac{e\sqrt{\pi}}{h} U(x, t). \quad (\text{C1})$$

This equation can be solved using the method of characteristics,

$$\begin{aligned} \phi_R(x + v_F \tau, t + \tau) \\ = \phi_R(x, t) + \frac{e\sqrt{\pi}}{h} \int_0^\tau U(x + v_F \tau', t + \tau') d\tau', \end{aligned} \quad (\text{C2})$$

which gives the outgoing field $\phi_{R,\text{out}}(t) = \phi_R(l, t)$ in terms of the incoming field $\phi_{R,\text{in}}(t) = \phi_R(0, t)$ and of the time- and space-dependent potential $U(x, t)$,

$$\begin{aligned} \phi_{R,\text{out}}(t) = \phi_{R,\text{in}}(t - l/v_F) \\ + \frac{e\sqrt{\pi}}{h} \int_0^{l/v_F} U(v_F \tau', t + \tau' - l/v_F) d\tau'. \end{aligned} \quad (\text{C3})$$

The field and therefore the EMP modes propagate ballistically at velocity v_F and the time- and space-dependent voltage adds a source term.

For comparison with subsequent models, let us consider the case where $U(x, t)$ is uniform within the $|x| \leq l/2$ region and equal to the time-dependent potential $U(t)$ and compute how the $b(\omega)$ EMP annihilation operator is scattered. Specializing Eq. (C3), reexpressing it in the Fourier domain, and using the $b(\omega)$ operators leads to

$$b_{\text{out}}(\omega) = e^{i\omega\tau_l} b_{\text{in}}(\omega) + \frac{iel\sqrt{\omega}}{\hbar v_F} f\left(\frac{\omega l}{v_F}\right) U(\omega), \quad (\text{C4})$$

where $\tau_l = l/v_F$ is the electronic time of flight across the region of length l to which the potential is applied and $f(X) = (e^{iX} - 1)/iX$. Rewriting this scattering formula in terms of incoming and outgoing currents leads to

$$i_{\text{out}}(\omega) = e^{i\omega\tau_l} i_{\text{in}}(\omega) - i\omega C_q(\omega) U(\omega), \quad (\text{C5})$$

in which we recognize the ω -dependent dynamical quantum capacitance

$$C_q(\omega) = \frac{e^2 l}{\hbar v_F} f\left(\frac{\omega l}{v_F}\right), \quad (\text{C6})$$

which, in the present case, reduces to the expected $C_q = e^2 l / \hbar v_F$ at low frequencies.

b. Electronic phase

Equation (C3) can then be used to compute how the electronic field is propagated using Eq. (B6),

$$\psi_{\text{out}}(t) = e^{i\vartheta_U(t)} \psi_{\text{in}}(t - l/v_F), \quad (\text{C7a})$$

$$\vartheta_U(t) = \frac{e}{\hbar} \int_0^{l/v_F} U(v_F \tau', t + \tau' - l/v_F) d\tau'. \quad (\text{C7b})$$

This corresponds to a time-dependent single-particle scattering matrix for the electrons,

$$S(t, t') = \delta(t - t' - l/v_F) e^{i\vartheta_U(t)}, \quad (\text{C8})$$

which precisely describes the propagation at constant velocity v_F of a particle of charge $-e$, traveling across the region where the potential $U(x, t)$ is present between space-time coordinates $(0, t - l/v_F)$ and (l, t) . For such a particle, the accumulated phase $\vartheta_U(t)$ is the potential seen along a trajectory that connects $(0, t - l/v_F)$ to (l, t) .

2. The classically driven top gate

a. EMP scattering

We now consider a top gate classically driven by a time-dependent voltage $V_g(t)$ and capacitively coupled to a chiral-edge channel. This region is modeled as a capacitor with geometric capacitance C_g involving the top gate and part of a chiral edge channel of length l (see Fig. 7) in a discrete element description. The two conductors are in total influence. We want to get the expression of the outgoing EMP modes b_{out} in terms of the incoming ones b_{in} and of V_g .

The basic equations for this system are the equation of motion for the chiral bosonic field $\phi_R(x, t)$, built from the EMP modes of the chiral-edge channel,

$$(\partial_t + v_F \partial_x) \phi_R(x, t) = \frac{e\sqrt{\pi}}{h} U(x, t). \quad (\text{C9})$$

$U(x, t)$ is the potential seen by the electrons. If we assume that $U(x, t)$ is uniform in the $|x| \leq l/2$ region, it can be calculated by adding the potential drop across the capacitor to $V_g(t)$,

$$U(t) - V_g(t) = \frac{Q(t)}{C_g}, \quad (\text{C10})$$

where $Q(t)$ denotes the excess charge in the $|x| \leq l/2$ of the chiral-edge channel. Using the bosonization expression

$$Q(t) = \frac{e}{\sqrt{\pi}} (\phi_R(-l/2, t) - \phi_R(l/2, t)) \quad (\text{C11})$$

in addition to Eqs. (C9) and (C10), we can obtain a closed differential equation for $\phi_R(x, t)$ within the $|x| \leq l/2$ region. This method is identical to the one used to discuss the Coulomb interaction effects within a chiral-edge channel in Ref. [57].

The final result can be expressed as connecting the outgoing EMP modes $b_{\text{out}}(\omega)$ to the incoming EMP modes $b_{\text{in}}(\omega)$ and $V_g(\omega)$. This relation is best expressed in terms of the incoming and outgoing currents using $i(\omega) = -e\sqrt{\omega}b(\omega)$,

$$i_{\text{out}}(\omega) = t(\omega)i_{\text{in}}(\omega) + \frac{e^2}{h}(1 - t(\omega))V_g(\omega), \quad (\text{C12})$$

in which, using $X = \omega l/v_F$,

$$t(\omega) = e^{iX} \frac{1 + \alpha f^*(X)}{1 + \alpha f(X)}, \quad (\text{C13})$$

where as before $f(X) = (e^{iX} - 1)/iX$ and

$$\alpha = \frac{e^2 l}{h v_F C_g}, \quad (\text{C14})$$

a dimensionless coupling constant whose physical meaning is discussed in Sec. III C.

b. Impedance and discussion

Folding the edge channel as shown in Fig. 11(a) enables us to consider the quantum RC circuit formed by the top gate and the folded-edge channel. Because of the total mutual influence, its admittance matrix defined by

$$\mathbf{Y}_{\alpha\beta}(\omega) = \frac{\partial \langle i_\alpha(\omega) \rangle}{\partial V_\beta(\omega)}, \quad (\text{C15})$$

where $(\alpha, \beta) \in \{u, d\}^2$ is both gauge invariant and charge conserving [113] and therefore determined by a single finite-frequency admittance

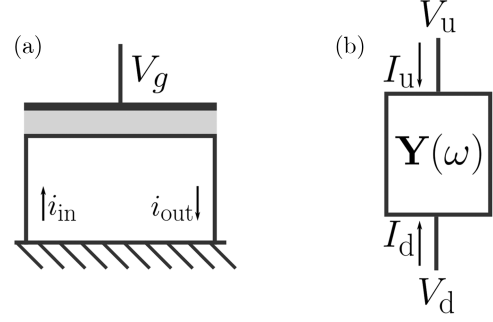


FIG. 11. (a) Folding of the edge channel capacitively coupled to the classical top gate to obtain an electrical dipole. (b) Drawing of an electrical dipole with $\mathbf{Y}(\omega)$ its admittance matrix, V_u , V_d , I_u , and I_d , respectively, denote the voltage and the average electrical current in the two connecting leads. (a) $V_u = V_g$, $V_d = 0$, and $I_d = i_{\text{in}} - i_{\text{out}}$.

$$\mathbf{Y}(\omega) = Y(\omega) \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad (\text{C16})$$

with

$$Y(\omega) = \frac{e^2}{h}(1 - t(\omega)) = \frac{e^2}{h} \frac{1 - e^{iX}}{1 + \alpha f(X)}. \quad (\text{C17})$$

Since $t(\omega) = e^{i\theta(\omega)}$ because of energy conservation and the absence of dynamical degrees of freedom in the classically driven top gate, $R_K Y(\omega) = 1 - t(\omega)$ spans a circle of radius 1 centered on 1 in the complex plane [114]. Its low-frequency expansion

$$R_K Y(\omega) \simeq -i \frac{\omega l/v_F}{1 + \alpha} + \frac{1}{2} \frac{(\omega l/v_F)^2}{(1 + \alpha)^2} + \dots \quad (\text{C18})$$

up to second order corresponds to an RC circuit with resistance $R_K/2$ and electrochemical capacitance C_μ such that $R_K C_\mu = l/(1 + \alpha)v_F$.

Figure 12 shows the behavior of $\text{Arg}(1 - R_K Y(\omega)) = \text{Arg}(t(\omega))$ as well as of $|R_K Y(\omega)| = |1 - t(\omega)|$ as a function of $X = \omega l/v_F$. The former describes how the phase of the transmission varies with ω , whereas the latter describes the filtering of the applied voltage $V_g(\omega)$ by the top gate. As can be seen on the bottom panel of Fig. 12, the filtering exhibits a texture even at low α . However, in this case, it is very similar to the filtering that would arise from applying $V_g(t)$ to the electrons in the $|x| \leq l/2$ region as in Sec. C 1, whereas at large α , this behavior is recovered only for $\omega \gtrsim 1/R_K C_\mu \gg v_F/l$, whereas for $\omega R_K C_\mu \lesssim 1$, filtering selects frequencies close to $\omega \simeq 2\pi n v_F/l$ with n being a positive integer.

In general, the gate voltage $V_g(t)$ is filtered into an effective time-dependent voltage $U_{\text{eff}}(t)$ via

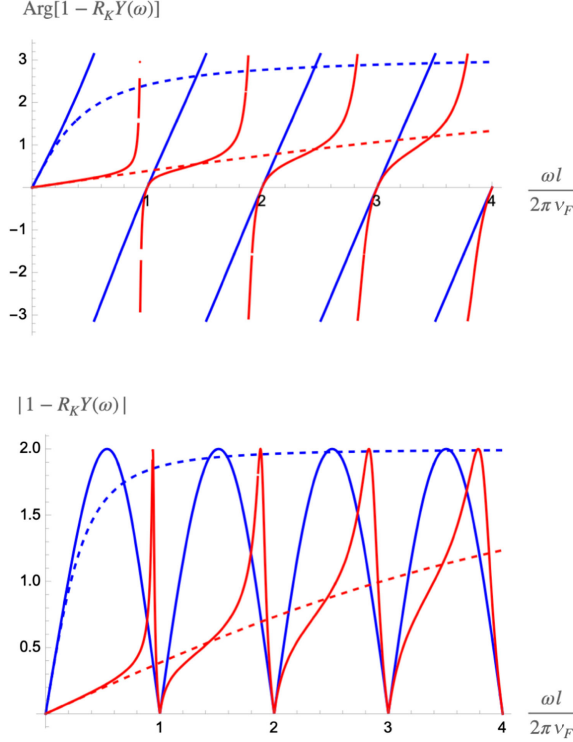


FIG. 12. Top: plot of the phase $\text{Arg}[1 - R_K Y(\omega)]$ as a function of $\omega l / 2\pi v_F$ in the voltage-locked ($\alpha = 1/5$, blue full line) and Coulomb-blockade ($\alpha = 15$, red full line) regimes. In each case, the angle $\text{Arg}[1 - R_K Y(\omega)]$ for the corresponding $R_q C_\mu$ circuit is plotted in dashed lines with the same colors. Bottom: plots of $|1 - R_K Y(\omega)|$ for the same examples as in the top panel (coloring identical to the top panel).

$$U_{\text{eff}}(\omega) = \frac{V_g(\omega)}{1 + \alpha f(\omega l / v_F)}. \quad (\text{C19})$$

The other effect of the top gate is to induce a nonlinear EMP transmission phase $\theta(\omega)$ that is responsible for electronic decoherence beneath the top gate [57]. On the other hand, the single-particle model discussed in Sec. C 1 does not lead to electronic decoherence since, in this case, Eq. (C5) corresponds to a dispersionless $\theta(\omega) = \omega l / v_F$.

APPENDIX D: TWO COUNTERPROPAGATING EDGE CHANNELS

In this appendix, we consider the EMP scattering matrix for a model of radiation coupler involving two counter-propagating integer quantum Hall edge channels, capacitively coupled over a region of length l . The geometric capacitance of this length l capacitor will be denoted by C_g . We assume that these two length l conductors are in total mutual influence, a hypothesis ensuring maximal coupling between them.

This modeling has been used in Ref. [116] for a rectangular quantum Hall bar of length l at filling fraction $\nu = 1$. The EMP scattering matrix can thus be directly

extracted from supplemental material of Ref. [116] and we will discuss its various limiting regimes.

1. The EMP scattering matrix

Denoting by b the EMP modes of the MZI edge channels and a the EMP modes of the radiation-edge channel, the EMP scattering matrix $S(\omega)$ is given by the result of Sec. III of supplemental material of Ref. [116] specialized for $\nu = 1$,

$$S_{ba}(\omega) = S_{ab}(\omega) = \frac{-iXf(X)}{2 + \alpha f(X)}, \quad (\text{D1a})$$

$$S_{aa}(\omega) = S_{bb}(\omega) = 1 - S_{ba}(\omega), \quad (\text{D1b})$$

in which $X = \omega l / v_F$ and $f(X) = (e^{iX} - 1)/iX$.

The dimensionless coupling $\alpha = C_q / C_g$ is the inverse ratio of the geometric capacitance C_g to the quantum capacitance $C_q = e^2 l / h v_F$ of a single-edge channel of length l . It reflects the strength of Coulomb interaction effects: Up to a numerical factor, it is the ratio of the electrostatic energy $e^2 / 2C_g$ for a single-electron charge in a capacitance C_g to the kinetic energy scale $\hbar v_F / l$ for a single electron added in a length l closed-edge channel.

The regime of low α ($C_g \gg C_q$) is the regime where the voltage drop at the capacitor formed by the two length l facing counterpropagating regions of length l can be neglected: Both channels see the same potential. On the other hand, $\alpha \gg 1$ is the regime of strong coupling in which Coulomb energy is so large that charging these length l regions is almost impossible. This is the Coulomb-blockade regime.

We shall now discuss the limiting forms of the EMP scattering region in these two regimes. A key point in interpreting the results is the relation between the EMP scattering matrix and finite-frequency impedances. Under the hypothesis of total screening, there are no current leaks to any external grounded conductor. Considering the b_{in} and b_{out} as the incoming and outgoing modes for a B lead and the a_{in} and a_{out} modes as the corresponding modes for the A leads, the finite-frequency admittance of this conductor is

$$Y(\omega) = \frac{e^2}{h} S_{ba}(\omega). \quad (\text{D2})$$

At low enough frequency, this dipole can be viewed as an RC circuit with finite-frequency admittance

$$Y(\omega) = -iC_\mu \omega + RC_\mu^2 \omega + \mathcal{O}(\omega^3), \quad (\text{D3})$$

which leads to

$$C_\mu = \frac{C_q}{2 + \alpha} = \frac{C_g C_q / 2}{C_q / 2 + C_g} \quad \text{and} \quad R = R_K. \quad (\text{D4})$$

We recover the expression of the quantum capacitance as the series addition of two single-channel quantum capacitances (one for the a modes and the other for the b modes) with the geometric capacitance C_g . The total resistance R_K is the series addition of the two contact resistances $R_K/2$ for the two folded-edge channels.

The total screening hypothesis leads to $S_{bb}(\omega) = 1 - S_{ba}(\omega)$. On the other hand, energy conservation leads to $|S_{bb}(\omega)|^2 + |S_{ab}(\omega)|^2 = 1$. Parametrizing

$$S_{bb}(\omega) = \frac{1}{2}(1 + \xi(\omega)), \quad (\text{D5a})$$

$$S_{ba}(\omega) = \frac{1}{2}(1 - \xi(\omega)) \quad (\text{D5b})$$

thus leads to $|\xi(\omega)| = 1$. Consequently, the parametric plot of $S_{ba}(\omega)$ in the complex plane is a circle of radius $1/2$ centered on the point $z = 1/2$. Note that this is also the case for an RC circuit with resistance R_K and capacitance C_μ . In the end, the EMP scattering matrix can be fully characterized by the description of the angle $\vartheta(\omega) = \text{Arg}(\xi(\omega))$.

Figure 13 displays the phase ϑ as a function of $\omega l / v_F$ in units of 2π for both the voltage-locked and Coulomb-blockade regimes together with the graphs for the RC circuit reproducing the low-frequency behavior of $Y(\omega)$. The probability $|S_{ab}(\omega)|^2$ for an EMP to be transmitted from the radiation channel into the MZI upper arm is plotted in the lower panel.

2. The voltage-locked regime

The $\alpha \rightarrow 0$ limit of the EMP scattering matrix given by Eq. (D1) is

$$S_{ba}(\omega) = S_{ab}(\omega) = \frac{1}{2}(1 - e^{iX}), \quad (\text{D6a})$$

$$S_{aa}(\omega) = S_{bb}(\omega) = \frac{1}{2}(1 + e^{iX}), \quad (\text{D6b})$$

which therefore implies that $\vartheta(\omega) = \omega l / v_F$. This is consistent with the full blue line behavior displayed on the upper panel of Fig. 13 ($\alpha = 1/5$).

In this regime, the electrochemical capacitance is dominated by the quantum capacitance's contribution: $C_\mu \simeq C_q/2$, which means that the corresponding RC time-scale is $R_K C_\mu \simeq l/2v_F$. The fact that, even at vanishing α , $S_{ba}(\omega)$ is nonzero comes from the fact that this “weak coupling limit” is not a limit without interactions between the two counterpropagating edge channels: It only means that the voltage drop at the capacitance vanishes and that both edge channels see the same voltage. In this regime, the

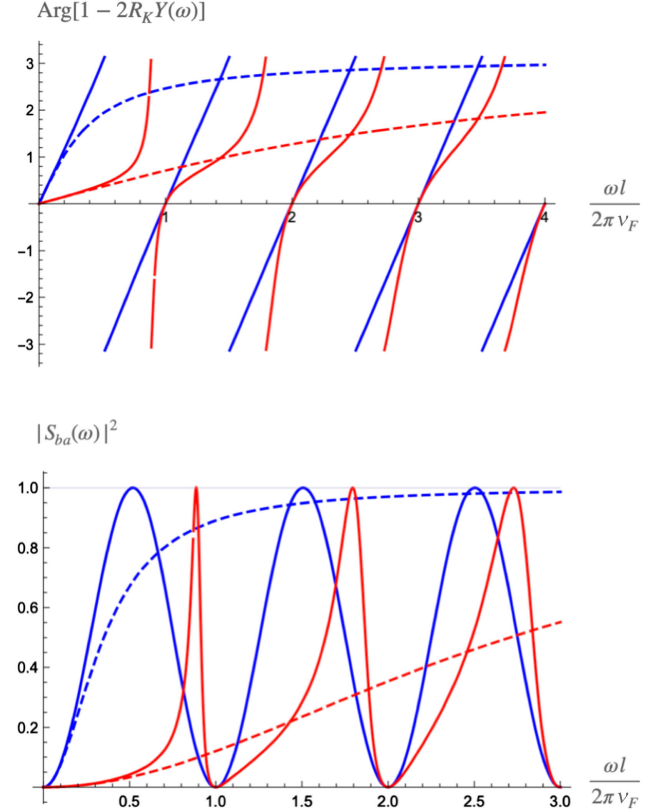


FIG. 13. Top: plot of the phase $\text{Arg}[1 - 2R_K Y(\omega)]$ as a function of $\omega l / 2\pi v_F$ for two different values of the coupling: $\alpha = 1/5$ (blue full line) and $\alpha = 15$ (red full line). For each of these values of α , the angle for the $R_K C_\mu$ circuit is plotted in dashed lines with the same colors. Bottom: plots of $|S_{ba}(\omega)|^2$ for the same examples as in the top panel (coloring identical to the top panel).

transmission probability exhibits sinusoidal oscillations as a function of $\omega l / v_F$ just like a microwave directional coupler or two strongly coupled copropagating edge channels with local interactions would do (see Ref. [117]).

Avoiding this physically counterintuitive behavior in the $\alpha \ll 1$ regime requires considering a model without the total screening hypothesis. However, note that such a model would ensure a less efficient coupling between the external quantum radiation and the electrons propagating within the MZI.

3. The Coulomb-blockade regime

Increasing α introduces nonlinearities in the phase $\vartheta(\omega)$ as shown on the upper panel of Fig. 13. The electrochemical capacitance increases since it is not dominated by the geometric capacitance $C_\mu \sim C_g$ ($C_g \ll C_q$). The $R_K C_\mu \sim R_K C_g$ timescale is then much shorter than the free electron time of flight $R_K C_q = l/v_F$. As a result $1/R_K C_\mu$ increases, thus explaining the flatter low-frequency behavior of $\vartheta(\omega)$ in terms of $\omega l / v_F$ seen in the upper panel of Fig. 13. The lower panel of this figure shows a strong distortion of the

oscillations in the transmission probability $|S_{ab}(\omega)|^2$ for $\omega R_K C_\mu \lesssim 1$. It still reaches unity for $\omega l/v_F \equiv 0 \bmod 2\pi$ but with narrow resonances for frequencies $\omega/2\pi$ close to integer multiples of v_F/l . These resonances become broader with increasing order and the typical oscillations of a microwave directional coupler are recovered for $\omega R_K C_\mu \gtrsim 1$.

Consequently, at low energy $\omega \lesssim 2\pi l/v_F$, there is almost no EMP scattering amplitude in the very large coupling limit and the EMP transmission amplitude corresponds to a vanishing time of flight across the length l interaction region. This is the result of the zero-charge constraint at infinite coupling: every charge density disturbance has to go out of the interaction region immediately and leads to no charge density change within the interaction region (hence the vanishing interchannel EMP scattering amplitude).

At higher ω , the modulus of decoherence amplitude goes down at each resonance, which becomes broader at higher ω . This comes from the frequency selectivity of the radiation coupler for a finite but large α : It can transmit photons only at the resonances shown in the lower panel of Fig. 13. This is an interesting feature for building a frequency-selective detector, the narrowest resonance being the first one for $\omega/2\pi = l/v_F$.

When $\omega R_K C_\mu \gtrsim 1$, the directional coupler behavior expected in the voltage-locked regime of the previous section is recovered. This is the regime where EMP modes have enough energy to overcome the Coulomb energy $e^2/2C_g$.

APPENDIX E: DERIVATION OF THE SINGLE-ELECTRON RADAR EQUATION

In this appendix, the single-electron radar equation for the outgoing average electrical current is derived. We will first derive it within the framework of time-dependent single-particle scattering theory (linear EQO) using a technique that can then be adapted to the presence of Coulomb interaction effects that belong to the nonlinear regime of EQO.

1. Time-dependent single-particle scattering approach

We consider a MZI as depicted in Fig. 1(a) and discussed in Sec. II B. Let us recall that the single-electron scattering matrices of the two QPC are assumed to be energy independent and given by

$$S_\alpha = \begin{pmatrix} \sqrt{T_\alpha} & i\sqrt{R_\alpha} \\ i\sqrt{R_\alpha} & \sqrt{T_\alpha} \end{pmatrix}, \quad (\text{E1})$$

in which T_α and R_α , respectively, denote the transmission and reflection probabilities at QPC $\alpha = A$ or B ($T_\alpha + R_\alpha = 1$).

The main idea is to express the outgoing electron field in branch 1 in terms of incoming fields by backtracing it from the output to the input of the MZI. Assuming free propagation along branch 2, with time of flight τ_2 leads to

$$\psi_{1\text{out}}(t) = \sqrt{T_B} \psi_{1,B-}(t) + i\sqrt{R_B} \psi_{2,B-}(t) \quad (\text{E2a})$$

$$= i\sqrt{R_B} e^{-i\phi_{AB}/2} \left(\sqrt{T_A} \psi_{2\text{in}}(t - \tau_2) + i\sqrt{R_A} \psi_{1\text{in}}(t - \tau_2) \right) \quad (\text{E2b})$$

$$+ \sqrt{T_B} e^{i\phi_{AB}/2} \psi_{1,B-}(t), \quad (\text{E2c})$$

in which $\psi_{\alpha,B-}(t)$ denotes the fermionic field right before the QPC B and $\psi_{\alpha\text{in}}(t)$ denotes the incoming fields right before QPC A . Equation (E2c) involves the outgoing fermionic field from the radiation coupler. In the case where propagation within this region can be described by a time-dependent linear scattering, we can relate it linearly to the incoming field $\psi_{1,A+}$ by

$$\psi_{1,B-}(t) = \int_{\mathbb{R}} R(t, t') \psi_{1,A+}(t') dt', \quad (\text{E3})$$

which assumes that no electron can be injected from any other channel than branch 1 of the MZI [118]. This enables us to write down the fully general expression for the outgoing electrical current $i_{1\text{out}}(t) = -ev_F : (\psi_{1\text{out}}^\dagger \psi_{1\text{out}}) : (t)$ in terms of the incoming electronic fields. We obtain the outgoing current operator $i_{1\text{out}}(t)$ as

$$i_{1\text{out}}(t) = \hat{I}_0(t) - e(e^{i\phi_{AB}} \hat{I}_+(t) + e^{-i\phi_{AB}} \hat{I}_-(t)), \quad (\text{E4})$$

in which, at the operator level,

$$\begin{aligned} \hat{I}_+(t) = & -i\sqrt{T_B R_B} \int_{\mathbb{R}} R(t, t') \left[\sqrt{T_A} \psi_{2\text{in}}^\dagger - i\sqrt{R_A} \psi_{1\text{in}} \right] \\ & \times (t - \tau_2) \left[\sqrt{T_A} \psi_{1\text{in}} + i\sqrt{R_A} \psi_{2\text{in}} \right] (t') dt'. \end{aligned} \quad (\text{E5})$$

When computing the average current, only terms that contain the same numbers of $\psi_{\alpha\text{in}}^\dagger$ and $\psi_{\alpha\text{in}}$ are retained since the MZI is fed by two independent electron sources. Consequently, the AB flux-dependent part of the average current is $\langle I_+(t) \rangle = -e\sqrt{R_A T_A R_B T_B} X_+(t)$, where

$$X_+(t) = \int_{\mathbb{R}} R(t, t') \left(\mathcal{G}_{1\text{in}}^{(e)} - \mathcal{G}_{2\text{in}}^{(e)} \right) (t' | t - \tau_2) dt'. \quad (\text{E6})$$

This is the time-domain electron radar equation given by Eq. (22). Note that, within the single-particle scattering formalism, this equation is valid for any electron source, not necessarily emitting a single-electron excitation.

2. The interacting case

a. The EMP scattering approach

Let us now consider the case where Coulomb interactions cannot be neglected within the radiation coupler. We shall model it using the EMP scattering formalism discussed in Sec. III A.

Note that the starting point of Eq. (E2) is still valid. However, contrary to the previous paragraph, the main challenge is now to backtrack the fermionic field along branch 1 of the MZI. Equivalently, we have to express $\psi_{1,B_-}(t)$ in terms of the incoming fields $\psi_{\alpha_{\text{in}}}$ for $\alpha = 1, 2$. In order to do so, we consider the fermionic field $\psi_{1,B_-}(t)$, expressed in terms of the outgoing EMP modes, and use the EMP scattering matrix to express it in terms of the incoming modes into the radiation coupler.

To simplify the notation, the EMP modes along branch 1 of the MZI will be denoted by $b_\alpha(\omega)$ with $\alpha = \text{in}$ or $\alpha = \text{out}$, depending whether they are incoming (position A_+) or outgoing (position B_-). In the same way, the electromagnetic modes within the radiation channel are denoted by $a_\alpha(\omega)$. The scattering matrix describing the coupling between the edge channel and the electromagnetic modes is

$$S(\omega) = \begin{pmatrix} S_{bb}(\omega) & S_{ba}(\omega) \\ S_{ab}(\omega) & S_{aa}(\omega) \end{pmatrix} \quad (\text{E7})$$

so that

$$\begin{pmatrix} b_{\text{out}}(\omega) \\ a_{\text{out}}(\omega) \end{pmatrix} = S(\omega) \begin{pmatrix} b_{\text{in}}(\omega) \\ a_{\text{in}}(\omega) \end{pmatrix}. \quad (\text{E8})$$

Using the bosonization formula for the fermionic field [see Eq. (B6)] and backpropagating the bosonic mode operators across the radiation coupler, $\psi_{1,\text{out}}(t)$ can be expressed as

$$\psi_{1,B_-}(t) = e^{i\Theta} \psi_{1,A_+}(t) D_{b_1}[(S_{bb}^* - 1)\Lambda(t)] \otimes D_a[S_{ba}^* \Lambda(t)], \quad (\text{E9})$$

where the phase Θ is independent from t . Therefore, we need to compute the correlator $\mathcal{G}_{\rho_i, B_-}^{(e)}(1, t|2, t)$, which is equal to

$$\langle \psi_{2,A_+}^\dagger(t - \tau_2) \psi_{1,A_+}(t) D_{b_1}[(S_{bb}^* - 1)\Lambda(t)] D_a[S_{ba}^* \Lambda(t)] \rangle, \quad (\text{E10})$$

in which the correlator is taken over the incoming many-body state $\rho_i = \rho_S \otimes \rho_{\text{em}}$, which is the tensor product of the many-body electronic state ρ_S injected by the source by the incoming radiation state ρ_{em} for the $a(\omega)$ modes.

b. The Franck-Condon factor

Let us now discuss how the correlator Eq. (E10) can be evaluated. First of all, the part that depends on the incident radiation state ρ_{em} can be singled out thanks to the identity

$$\begin{aligned} \langle D_a[S_{ba}^* \Lambda(t)] \rangle_{\rho_{\text{em}}} &= \langle D_a[S_{ba}^* \Lambda(t)] \rangle_{|0\rangle} \\ &\times \langle :D_a[S_{ba}^* \Lambda(t)]: \rangle_{\rho_{\text{em}}}, \end{aligned} \quad (\text{E11})$$

in which we have introduced the bosonic normal ordering $:\cdots:$. All the dependence in the incident radiation state ρ_{em} is thus contained in the average value of the normal-ordered displacement operator $:D_a[S_{ba}^* \Lambda(t)]:$ for the $a(\omega)$ modes. The average value $\langle D_a[S_{ba}^* \Lambda(t)] \rangle_{|0\rangle}$ is taken over the vacuum state for the $a(\omega)$ modes, which means that it can be reabsorbed into the correlator given by Eq. (E10) except that this time the quantum average is taken over the state $\rho_{S,0} = \rho_S \otimes |0\rangle\langle 0|$. This can be summarized by

$$\mathcal{G}_{\rho_i, B_-}^{(e)}(1, t|2, t) = \mathcal{G}_{\rho_{S,0}, B_-}^{(e)}(1, t|2, t) \quad (\text{E12a})$$

$$\times \langle :D_a[S_{ba}^* \Lambda(t)]: \rangle_{\rho_{\text{em}}}, \quad (\text{E12b})$$

since $\rho_{S,0} = \rho_S \otimes |0\rangle\langle 0|$ corresponds to a situation where no incident radiation is sent onto the MZI. Therefore, the correlator $\mathcal{G}_{\rho_{S,0}, B_-}^{(e)}(1, t|2, t)$ is exactly the one appearing when computing the average current flowing out of the MZI in the absence of electromagnetic radiation sent onto it via the $a(\omega)$ modes. This problem corresponds to the problem of electronic decoherence within the MZI.

In the end, the effect of the radiation injected into the radiation coupler is described by the factor

$$\mathcal{F}_{\rho_{\text{em}}}(t) = \langle :D_a[S_{ba}^* \Lambda(t)]: \rangle_{\rho_{\text{em}}}, \quad (\text{E13})$$

which is the exact analogue of the Franck-Condon factor that appears in the spectroscopy of complex molecules [58].

c. Electronic propagation contribution

The electronic coherence $\mathcal{G}_{\rho_{S,0}, B_-}^{(e)}(1, t|2, t)$ is more difficult to evaluate because it corresponds to the outgoing single-electron coherence after propagation across the MZI in the presence of Coulomb interactions within branch 1 of the MZI. It turns out that simple and physically transparent expressions can be found in the case where S is an ideal single-electron source. In this case, the state S is of the form

$$\rho_S = \psi_{\text{lin}}^\dagger[\varphi_e] |F\rangle \langle F| \psi_{\text{lin}}[\varphi_e], \quad (\text{E14})$$

where $|F\rangle$ corresponds to the Fermi sea with chemical potential $\mu = 0$ in the two incoming electronic channels of the MZI and $\psi_{\text{lin}}^\dagger[\varphi_e]$ creates a single-electron excitation with wave function φ_e injected into the MZI (position A_-).

Expressing $\psi_{\text{lin}}^\dagger[\varphi_e]$ in terms of $\psi_{1,A_+}^\dagger[\varphi_e]$ and $\psi_{2,A_+}^\dagger[\varphi_e]$ enables us to show that

$$\mathcal{G}_{\rho_{S,0},B_-}^{(e)}(1,t|2,t) = -i\sqrt{R_A T_A} \times \langle F, 0_a | \psi_{2,A_+}[\varphi_e] \psi_{2,A_+}^\dagger(t - \tau_2) \psi_{1,B_-}(t) \psi_{1,A_+}^\dagger[\varphi_e] | F, 0_a \rangle, \quad (\text{E15a})$$

$$= -i\sqrt{R_A T_A} \times \langle F_2 | \psi_{2,A_+}[\varphi_e] \psi_{2,A_+}^\dagger(t - \tau_2) | F_2 \rangle \times \langle F_1, 0_a | \psi_{1,B_-}(t) \psi_{1,A_+}^\dagger[\varphi_e] | F_1, 0_a \rangle, \quad (\text{E15b})$$

in which $|F, 0_a\rangle$ denotes the tensor product of the Fermi sea in both branches of the MZI and the ground state for the environmental modes. Since φ_e is an electronic excitation above the Fermi level, the contribution associated with propagation along branch 2 of the MZI can be readily evaluated,

$$\langle \psi_{2,A_+}[\varphi_e] \psi_{2,A_+}^\dagger(t - \tau_2) \rangle_{|F_2\rangle} = \varphi_e(t - \tau_2)^*. \quad (\text{E16})$$

We are thus left with evaluating

$$\begin{aligned} & \langle \psi_{1,B_-}(t) \psi_{1,A_+}^\dagger[\varphi_e] \rangle_{|F_1, 0_a\rangle} \\ &= \int_{\mathbb{R}} v_F \varphi_e(t') \langle \psi_{1,B_-}(t) \psi_{1,A_+}(t') \rangle_{|F_1, 0_a\rangle} dt'. \end{aligned} \quad (\text{E17})$$

The time-domain amplitude

$$\mathcal{Z}_1(\tau) = v_F \langle \psi_{1,B_-}(\tau) \psi_{1,A_+}^\dagger(0) \rangle_{|F_1, 0_a\rangle} \quad (\text{E18})$$

is the elastic single-electron scattering amplitude across branch 1 of the MZI. It is related to the elastic scattering amplitude $\tilde{\mathcal{Z}}_1(\omega)$ computed in Refs. [56,74] by a Fourier transform

$$\mathcal{Z}_1(\tau) = \int_0^{+\infty} \tilde{\mathcal{Z}}_1(\omega) e^{-i\omega\tau} \frac{d\omega}{2\pi}. \quad (\text{E19})$$

Evaluating $\tilde{\mathcal{Z}}_1(\omega)$ can be done using the expressions given in Refs. [57,119], which we recall here for completeness,

$$\tilde{\mathcal{Z}}_1(\omega) = 1 + \int_0^\omega B(\omega') d\omega', \quad (\text{E20})$$

in which $B(\omega)$ is the solution of the integral equation

$$\omega B(\omega) = S_{bb}(\omega) - 1 + \int_0^\omega B(\omega') (S_{bb}(\omega') - 1) d\omega', \quad (\text{E21})$$

with initial condition $B(0^+) = (dS_{bb}/d\omega)(\omega = 0^+)$. Finally, we obtain

$$\langle \psi_{1,B_-}(t) \psi_{1,A_+}^\dagger[\varphi_e] \rangle_{|F_1, 0_a\rangle} = \int_{\mathbb{R}} \varphi_e(t') \mathcal{Z}_1(t - t') dt'. \quad (\text{E22})$$

The correlator $\langle \psi_{2,A_+}[\varphi_e] \psi_{2,B_-}^\dagger(t - \tau_2) \rangle_{|F_2\rangle}$ could also be rewritten in a similar way,

$$\langle \psi_{2,A_+}[\varphi_e] \psi_{2,B_-}^\dagger(t) \rangle_{|F_2\rangle} = \int_{\mathbb{R}} \varphi_e(t')^* \mathcal{Z}_2(t - t')^* dt' \quad (\text{E23})$$

using the elastic scattering amplitude $\mathcal{Z}_2(\tau) = \delta(\tau - \tau_2)$ corresponding to ballistic propagation during time of flight τ_2 .

Note that this form of the correlator $\langle \psi_{2,A_+}[\varphi_e] \psi_{2,B_-}(t) \rangle$ remains valid in the presence of electronic decoherence along branch 2 of the MZI provided it is not caused by direct or indirect interactions (bath mediated) between the two branches of the MZI. This means that there must be no crosstalk between the two branches: Each of these branches do interact with their own environment, which are prepared in their ground states. In this case, we should use the elastic scattering amplitude $\tilde{\mathcal{Z}}_2(\omega)$ for $\omega > 0$ to define $\mathcal{Z}_2(\tau)$. The elastic scattering amplitude $\tilde{\mathcal{Z}}_2(\omega)$ can be computed in terms of the finite-frequency admittance of branch 2.

d. General result

Let us finally collect the general result for the interbranch coherence $\mathcal{G}_{\rho_{S,0},B_-}(1,t|2,t)$ right before the second QPC in the general situation where each of the branches involves Coulomb interactions and possibly a coupling to its own radiation channel fed by the vacuum state. In the absence of external radiation, the expression

$$\begin{aligned} & v_F \mathcal{G}_{\rho_{S,0},B_-}^{(e)}(1,t|2,t) \\ &= \int_{\mathbb{R}^2} v_F \varphi_e(t - \tau_1) \varphi_e(t - \tau_2)^* \mathcal{Z}_1(\tau_1) \mathcal{Z}_2(\tau_2) d\tau_1 d\tau_2 \end{aligned} \quad (\text{E24})$$

has a physically transparent interpretation: Each wave packet gets propagated according to the elastic scattering amplitude $\mathcal{Z}_\alpha(\tau)$ along the corresponding branch.

Considering an incoming radiation arriving on each radiation coupler of each branch α , whose quantum state is described by independent density operators $\rho_{\text{em},\alpha}$, Eq. (12) generalizes to

$$\begin{aligned} \mathcal{G}_{\rho_i,B_-}^{(e)}(1,t|2,t) &= \mathcal{G}_{\rho_{S,0},B_-}^{(e)}(1,t|2,t) \\ &\times \prod_{\alpha=1,2} \left\langle : D_{a_\alpha} \left[(S_{ba}^{(\alpha)})^* \Lambda(t) \right] : \right\rangle_{\rho_{\text{em},\alpha}}. \end{aligned} \quad (\text{E25})$$

In the end, the interference signal $X_+(t)$ is given by

$$X_+(t) = v_F \int_{\mathbb{R}^2} \varphi_e(t - \tau_1) \varphi_e(t - \tau_2)^* \times R_{\text{eff},1}(t, t - \tau_1) R_{\text{eff},2}(t, t - \tau_2) d\tau_1, d\tau_2, \quad (\text{E26})$$

in which the effective single-particle scattering amplitudes are given by the product of the elastic scattering amplitude by the Franck-Condon factor,

$$R_{\text{eff},\alpha}(t, t') = \mathcal{Z}_\alpha(t - t') \mathcal{F}_{\rho_{\text{em},\alpha}}(t), \quad (\text{E27})$$

thus leading to the single-electron radar equation (22) which reduces to Eq. (23) in the case of ballistic propagation in time τ_2 along branch 2 and no coupling to external radiation on this branch.

Last but not least, in the presence of Coulomb interactions, the radar equation is valid only for an incoming single-electron excitation injected into the electronic MZI.

APPENDIX F: CONNECTION TO FULL COUNTING STATISTICS

In the context of experiments performed on AlGaAs/AsGa systems in the quantum Hall regime, the radiation channel may also be a chiral quantum Hall edge channel, as depicted in Fig. 14. In this case, $\mathcal{F}_{\rho_{\text{em}}}(t)$ can also be interpreted in terms of a form of full counting statistics for electronic transport in the radiation channel as noticed in Ref. [120]. Since this work relied on a specific interaction model as well as on single-particle scattering, we present here a full many-body derivation independent of the details of the Coulomb interaction model.

When the radiation channel is a single-integer quantum Hall edge channel, the incoming electrical current arriving into the interaction region via this edge channel is, for $\omega > 0$,

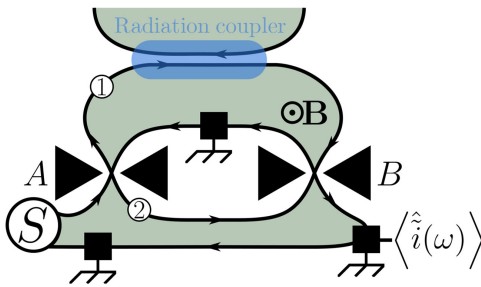


FIG. 14. Sample design of the EQR using chiral-edge channels in the integer quantum Hall regime (2DEG in green). The blue region where two counterpropagating quantum Hall edge channels are facing each other at short ($\lesssim 100$ nm) distance forms a radiation coupler between channel 1 of the MZI and the radiation channel in which the radiation to be analyzed is sent. The quantity of interest is the average outgoing current from the MZI and, more precisely, the first harmonic in the Aharonov-Bohm phase.

$$i_{\text{in}}(\omega) = -e\sqrt{\omega}a_{\text{in}}(\omega). \quad (\text{F1})$$

Introducing

$$\Gamma_{ba}(\tau) = \int_0^{+\infty} \frac{S_{ba}(\omega)}{-i\omega} e^{-i\omega\tau} \frac{d\omega}{2\pi} + \text{c.c.}, \quad (\text{F2})$$

the normalized filtered charge operator

$$N(t) = \int_{\mathbb{R}} \Gamma_{ba}(t - \tau) \frac{i_{\text{in}}(\tau)}{-e} d\tau \quad (\text{F3})$$

associated with the windowing function $\tau \mapsto \Gamma_{ba}(t - \tau)$ can be used to provide a compact expression for $\mathcal{F}_{\rho_{\text{em}}}(t)$,

$$\mathcal{F}_{\rho}(t) = \left\langle : e^{2\pi i N(t)} : \right\rangle_{\rho_{\text{em}}}. \quad (\text{F4})$$

The $2\pi N(t)$ operator represents the quantum phase kicks associated with the incoming electrical current felt by a localized electron propagating within the MZI. Note that the exponential of this quantum phase kick operator is normal ordered with respect to the bosonic modes $a_{\text{in}}(\omega)$. One can then connect this ordering to the time ordering in order to establish the precise connection to time-ordered electrical current correlators.

Finally, let us comment on the interpretation of the windowing function $\Gamma_{ba}(t - \tau)$. Since $\Gamma_{ba}(\omega) = iS_{ba}(\omega)/\omega$ for $\omega > 0$, the relation between the EMP scattering matrix and finite-frequency admittances [57,60,75,76] leads to

$$\Gamma_{ba}(\omega) = \frac{S_{ba}(\omega)}{-i\omega} = \frac{R_K}{-i\omega} \frac{\partial I_b^{(\text{out})}(\omega)}{\partial V_a^{(\text{in})}(\omega)}, \quad (\text{F5})$$

in which $I_b^{(\text{out})}(\omega)$ denotes the average total current going out from the interaction region within the MZI—the

propagation channel for the b EMP modes—and $V_a^{(\text{in})}(\omega)$ denotes a classical voltage drive applied to the incoming channel of the radiation channel in which the EMP a modes propagate. This equation thus connects $\Gamma_{ba}(\omega)$ to the finite-frequency admittance of the electrical dipole associated with the interaction region. Since the coupling is purely capacitive, one expects this finite-frequency response to be capacitive, that is, of the form

$$\frac{\partial I_b^{(\text{out})}(\omega)}{\partial V_a^{(\text{in})}(\omega)} = -i\omega C_{ba}^{(\text{eff})}(\omega), \quad (\text{F6})$$

in which the effective frequency dependence capacitance $C_{ba}^{(\text{eff})}(\omega)$ goes to a nonzero capacitance in the low-frequency limit. Consequently, $\Gamma_{ba}(\tau)$ appears as the effective response rate associated with this capacitance,

$$\Gamma_{ba}(\tau) = \int_0^{+\infty} R_K C_{ba}^{(\text{eff})}(\omega) e^{-i\omega\tau} \frac{d\omega}{2\pi} + \text{c.c.} \quad (\text{F7})$$

APPENDIX G: THE SINGLE-ELECTRON RADAR EQUATION IN THE FREQUENCY DOMAIN

Let us introduce

$$\tilde{R}(\omega_+, \omega_-) = \int_{\mathbb{R}^2} R(t, t') e^{i(\omega_+ t - \omega_- t')} dt dt', \quad (\text{G1})$$

which is directly proportional to the amplitude for scattering an incoming electron at energy $\hbar\omega_-$ to the energy $\hbar\omega_+$. Denoting by $\tilde{X}_+(\omega)$ the Fourier transform of the signal $X_+(t)$ at frequency $\omega/2\pi$, Eq. (22) then takes the form

$$\begin{aligned} \tilde{X}_+(\omega) = \int_{\mathbb{R}^2} v_F \Delta \tilde{\mathcal{G}}_S^{(e)}(\omega_-, \omega_+ - \omega) \\ \times e^{i(\omega - \omega_+) \tau_2} \tilde{R}(\omega_+, \omega_-) \frac{d\omega_+ d\omega_-}{(2\pi)^2}, \end{aligned} \quad (\text{G2})$$

in which

$$\Delta \tilde{\mathcal{G}}_S^{(e)}(\omega_+ | \omega_-) = \int_{\mathbb{R}^2} \Delta \mathcal{G}_S^{(e)}(t | t') e^{i(\omega_+ t - \omega_- t')} dt dt' \quad (\text{G3})$$

denotes the excess single-electron coherence in the frequency domain. Equation (G2) determines the interference contribution to the finite-frequency average electrical current in the 1_{out} branch of the MZI.

Its physical interpretation is quite clear: For $\omega > 0$, the average finite-frequency current $\langle i_{1_{\text{out}}}(\omega) \rangle$ probes the outgoing electronic coherence between ω_+ and $\omega_+ - \omega$ for all ω_+ . The latter propagates along branch 2 of the MZI with time of flight τ_2 , hence picking a phase $e^{i(\omega_+ - \omega)\tau_2}$. Since it appears in the complex-conjugated amplitude, this leads to the phase factor $e^{i(\omega - \omega_+)\tau_2}$ in Eq. (G2). On branch 1 of the MZI, the electron enters the radiation coupler with energy $\hbar\omega_-$ and exits with energy $\hbar\omega_+$ with an amplitude $\tilde{R}(\omega_+, \omega_-)$. Therefore, the contribution of the outgoing

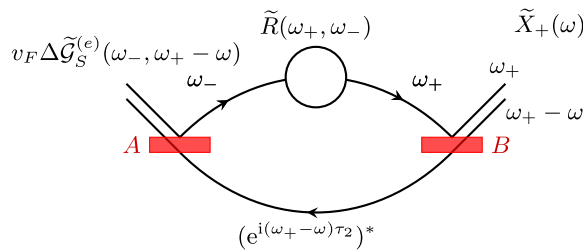


FIG. 15. Physical interpretation of the linear electron radar equation (G2) in the frequency domain showing the contribution of incoming single-electron coherence in the frequency domain to $\tilde{X}_+(\omega)$ for $\omega > 0$. Definition of the two paths is the same as in Fig. 5.

coherence between ω_+ and $\omega_+ - \omega$ comes from an incoming coherence between ω_- and $\omega_+ - \omega$ emitted by the source S as summarized in Fig. 15.

APPENDIX H: LIMITING REGIMES OF THE SINGLE-ELECTRON RADAR EQUATION

We now discuss the limiting regimes of a time- or frequency-resolved single-electron excitation. The resulting limiting form of the electron radar equation will be relevant whenever the time (respectively, frequency) extension of the probe is much smaller than the typical time (respectively, frequency) length scales of the effective scattering matrix. As expected, time-resolved wave packets are well suited to explore the time dependence of the scattering, whereas energy-resolved excitations give access to the scattering amplitude in the frequency domain.

1. Energy-resolved probes

We consider a resolved excitation with Gaussian line shape, centered at energy $\hbar\omega_e$ and with linewidth $\gamma_e \gg |\omega_e|$. Its wave function in the energy domain, defined by Eq. (A6), is

$$\tilde{\varphi}_e(\omega) = \mathcal{N} \Theta(\text{sign}(\omega_e) \omega) e^{-(\omega - \omega_e)^2 / 2\gamma_e^2}, \quad (\text{H1})$$

where the normalization condition (A7) gives [121] $\mathcal{N}^2 \gamma_e / v_F \sqrt{4\pi} \simeq 1$. For $\omega_e > 0$, we are dealing with an electronic excitation, whereas for $\omega_e < 0$ we are dealing with a hole excitation. In the limit where $\gamma_e \ll |\omega_e|$ is much smaller than the scales of variation of $\tilde{R}(\omega_+, \omega_-)$, the EQR signal in the frequency domain given by Eq. (G2) then gives

$$\tilde{X}_+(\omega; \tau_2) \simeq \frac{\gamma_e}{\sqrt{\pi}} e^{-i\omega_e \tau_2} \tilde{R}(\omega + \omega_e, \omega_e). \quad (\text{H2})$$

Whenever ω_e and $\omega + \omega_e$ have the same sign, we are accessing the electron-to-electron or hole-to-hole effective scattering, whereas whenever ω_e and $\omega + \omega_e$ do not have the same sign, we are accessing electron-to-hole or hole-to-electron effective scattering.

2. Time-resolved probes

We consider a normalized Gaussian single-electron wave packet

$$\varphi_{\text{tr}}(t) = \frac{1}{\sqrt{v_F \sigma_e \sqrt{\pi}}} e^{-(t - t_e)^2 / 2\tau_e^2}, \quad (\text{H3})$$

with t_e the emission time and τ_e the duration of the single-electron wave packet. Using Eq. (22) to compute the average time-dependent current and brutally taking the limit $\sigma_e \rightarrow 0$ leads to, at first order in σ_e ,

$$X_+(t) \simeq \tau_e R(t_e + \tau_2, t_e) \delta(t - \tau_2 - t_e). \quad (\text{H4})$$

The condition $t = t_e + \tau_2$ comes from ballistic propagation along the reference arm of the MZI, and $X_+(t)$ is then proportional to the amplitude for the electron to enter branch 1 at t_e and exit it at time t .

However, this naive computation does not take into account the constraint that the incoming single-electron excitation must be restricted to positive energies due to the presence of the Fermi sea, which is not the case for the wave packet considered in Eq. (H3). Introducing a finite-energy shift via the factor $e^{-i\omega_e(t-t_e)}$ in front of the rhs of Eq. (H3) with $\omega_e \gg 2\pi v_F/\tau_e$ leads to a wave packet with negligible weight at negative energies. However, this introduces a rapidly oscillating term $e^{-i\omega_e(t-t'-\tau_2)}$ in front of $R(t, t') = \mathcal{Z}_1(t-t')\mathcal{F}_{\rho_{\text{cm}}}(t)$ in the radar equation (22). This means that we are probing electronic decoherence at energies close to $\hbar\omega_e$. However, as shown in Ref. [119], electronic decoherence is expected to be worse at high energies and may indeed kill the interference signal.

In order to mitigate this problem, the proper approach consists of introducing electronic wave packets that are close to the Fermi surface and well localized in space such as the leviton. This is discussed in Appendix I.

APPENDIX I: FRINGE CONTRAST FOR LEVITONS

Let us consider the case of a leviton wave packet of width τ_e . Its wave function is Lorentzian in the time domain and exponential in the frequency domain [122]. We are interested in the interference contribution to the outgoing electrical current for a leviton of duration τ_e injected at time t_e .

Because of the simple expression for the leviton wave packet in the frequency domain, the electron radar equation in the frequency domain [see Eq. (G2)] gives us a convenient form for $X_+^{(\text{dc})}$ suitable for numerical evaluations,

$$X_+^{(\text{dc})} = \int_{\mathbb{R}} \tilde{\mathcal{F}}_{\rho_{\text{cm}}}(\Omega) e^{-i\Omega(t_e+\tau_2)} f_{\tau_e, \tau_2}(\Omega) \frac{d\Omega}{2\pi}, \quad (\text{I1})$$

in which the filter

$$f_{\tau_e, \tau_2}(\Omega) = 4\pi\tau_e \int_{|\Omega|/2}^{+\infty} \tilde{\mathcal{Z}}_1\left(\omega - \frac{\Omega}{2}\right) e^{-2\omega\tau_e} e^{-i(\omega - \frac{\Omega}{2})\tau_2} \frac{d\omega}{2\pi} \quad (\text{I2})$$

contains all the effects of electronic decoherence along branch 1 and ballistic propagation along branch 2 of the interferometer.

Because the elastic scattering amplitude $\tilde{\mathcal{Z}}_1(\omega)$ tends to decrease in modulus with increasing ω , $f_{\tau_e, \tau_2}(\Omega)$ is expected to be a low-pass filter. The limit $\tau_e \rightarrow 0^+$ may

even lead to vanishing signal since, in the case of the two counterpropagating edge channel models considered in Appendix D, a very short leviton may experience fractionalization as it flies across the radiation coupler. For short wave packets, this would kill the electronic interference signal. In the absence of external radiation, since $\mathcal{F}_{|0\rangle}(t) = 1$, we obtain

$$\left[X_+^{(\text{dc})}\right]_{|0\rangle} = 4\pi\tau_e \int_0^{+\infty} \tilde{\mathcal{Z}}_1(\omega) e^{-2\omega\tau_e} e^{-i\omega\tau_2} \frac{d\omega}{2\pi}. \quad (\text{I3})$$

This quantity, which does not depend on t_e anymore, but does depend on τ_2 , represents the base interference contribution for the dc outgoing current from the MZI.

APPENDIX J: SQUEEZING DETECTION

This appendix presents the computations of the Franck-Condon factor in the case of a time-periodic noise, with specialization to narrow band squeezed noise. We first briefly recall the structure of a single-mode squeezed vacuum for pedagogy and then proceed to the discussion of time-periodic noise. In Sec. J2, we show that such a noise can exhibit subvacuum fluctuations coming from two-mode squeezing in a narrow band around a given frequency $\omega_0/2\pi$. The harmonic structure of the resulting Franck-Condon factor is then discussed in Sec. J3.

1. Squeezed vacuum in a single mode

In this appendix, we recall the basics needed to describe the squeezed vacuum for a single mode [123],

$$|\text{Sq}_z\rangle = e^{z(a^\dagger)^2 - z^* a^2} |0\rangle, \quad (\text{J1})$$

where a and $a^\dagger$ are the creation and destruction operators for a single mode and $z \in \mathbb{C}$. The squeezing operator $\mathbb{S}_z$ equal to $e^{z(a^\dagger)^2 - z^* a^2}$ performs a Bogoliubov transformation on the original mode operators,

$$\mathbb{S}_z^\dagger a \mathbb{S}_z = \cosh(2|z|)a + e^{i\varphi} \sinh(2|z|)a^\dagger. \quad (\text{J2})$$

This enables us to compute the expectation value of any products of a and $a^\dagger$ in the state $|\text{Sq}_z\rangle$ as the expectation value of the same expression in terms of the Bogoliubov transformed operators

$$a_z = \cosh(2|z|)a + e^{i\varphi} \sinh(2|z|)a^\dagger, \quad (\text{J3a})$$

$$a_z^\dagger = \cosh(2|z|)a^\dagger + e^{i\varphi} \sinh(2|z|)a, \quad (\text{J3b})$$

where $\varphi = \text{Arg}(z)$. More precisely,

$$\langle \mathcal{O}[a, a^\dagger] \rangle_{|\text{Sq}_z\rangle} = \langle \mathcal{O}[a_z, a_z^\dagger] \rangle_{|0\rangle}. \quad (\text{J4})$$

Applying this relation to the quadrature

$$X_\vartheta = \frac{1}{\sqrt{2}}(e^{i\vartheta}a + e^{-i\vartheta}a^\dagger) \quad (\text{J5})$$

leads to $\langle X_\vartheta \rangle_{|\text{Sq}_z\rangle} = 0$ and

$$\langle (\Delta X_\vartheta)^2 \rangle_{|\text{Sq}_z\rangle} = \frac{1}{2} + \sinh^2(2|z|) \quad (\text{J6a})$$

$$+ \cosh(2|z|) \sinh(2|z|) \cos(2\vartheta + \varphi), \quad (\text{J6b})$$

which shows that the fluctuations are anisotropic in the Fresnel plane and that $\text{Arg}(z)$ determines the principal axes of the ellipsoid of fluctuations. Its extrema are

$$\max_\vartheta \langle (\Delta X_\vartheta)^2 \rangle_{|\text{Sq}_z\rangle} = \frac{1}{2} e^{4|z|}, \quad (\text{J7a})$$

$$\min_\vartheta \langle (\Delta X_\vartheta)^2 \rangle_{|\text{Sq}_z\rangle} = \frac{1}{2} e^{-4|z|}. \quad (\text{J7b})$$

The vacuum fluctuations being given by 1/2, the state $|\text{Sq}_z\rangle$ appears as squeezed with subvacuum fluctuations compressed at most by a factor $e^{-4|z|} < 1$ as soon as $|z| \neq 0$. Note that, with $\vartheta = \omega_0 t$, we recover the $2\omega_0$ oscillations of the fluctuations of harmonic mode of energy $\hbar\omega_0$.

Since the isotropic part of the fluctuations corresponds to $\frac{1}{2} + \langle a^\dagger a \rangle_{|\text{Sq}_z\rangle}$, Eq. (J6) determines the average photon number in the squeezed vacuum $|\text{Sq}_z\rangle$,

$$\langle a^\dagger a \rangle_{|\text{Sq}_z\rangle} = \sinh^2(2|z|). \quad (\text{J8})$$

Thinking of this harmonic mode as an optical mode, the above discussion shows that increasing the squeezing parameter z increases the average number of photons. A squeezed vacuum is thus on average more noisy than the true vacuum even if increasing $|z|$ decreases its minimal fluctuations.

2. Narrow band time-periodic noise

Let us consider a time-periodic current noise with period π/ω_0 which, as discussed in the previous paragraph, is natural for squeezing around a frequency $\omega_0/2\pi$. This is more natural in the spirit of mesoscopic physics where the radiation source may be a periodically driven conductor at a given frequency that would, by time translation invariance, generate a radiation whose correlators have the same time periodicity.

We then consider the simplest correlation structure for the creation and destruction operators $a(\omega)$ and $a^\dagger(\omega)$ that satisfies both the requirement of π/ω_0 periodicity as well as the requirement that its time-averaged power spectrum is concentrated in a narrow band around ω_0 . This leads to ($\omega_\pm > 0$)

$$\langle a^\dagger(\omega_-)a(\omega_+) \rangle = \delta(\omega_+ - \omega_-)\bar{n}(\omega), \quad (\text{J9a})$$

$$\langle a(\omega_-)a(\omega_+) \rangle = \delta(\omega_+ + \omega_- - 2\omega_0)\xi\left(\frac{\omega_+ - \omega_-}{2}\right), \quad (\text{J9b})$$

where $\bar{n}(\omega) \geq 0$ denotes the average photon number in the mode at ω and $\xi(\Omega)$ denotes the average photon-pair correlation between the modes at frequencies $\omega_0 + \Omega$ and $\omega_0 - \Omega$. In the narrow band situation where the emitted power density $\hbar\omega\bar{n}(\omega)$ is concentrated near ω_0 , $\bar{n}(\omega) \neq 0$ only for $|\omega - \omega_0| \lesssim \gamma_0/2$ and $|\xi(\Omega)| \neq 0$ only for $|\Omega| \lesssim \gamma_0/2$. Note that the δ -function constraints in these equations arise from time periodicity as well as from Cauchy-Schwartz inequalities (see Ref. [17] for analogous considerations in the context of EQO).

Substituting these correlators in Eq. (41) leads to

$$|\mathcal{F}_{\rho_{\text{em}}}(t)| = e^{\text{Re}[\xi_{\text{eff}} e^{-2i\omega_0 t}] - N_{\text{eff}}}, \quad (\text{J10})$$

in which

$$N_{\text{eff}} = \int_0^{+\infty} \frac{|S_{ba}(\omega)|^2}{\omega} \bar{n}(\omega) d\omega, \quad (\text{J11a})$$

$$\xi_{\text{eff}} = \int_{-\omega_0}^{+\omega_0} \frac{S_{ba}(\omega_0 + \Omega)S_{ba}(\omega_0 - \Omega)}{\sqrt{\omega_0^2 - \Omega^2}} \xi(2\Omega) d\Omega. \quad (\text{J11b})$$

In the narrow band approximation in which $\bar{n}(\omega) = \bar{n}$ for $|\omega - \omega_0| \leq \gamma_0/2$, $\xi(\Omega) = \xi$ for $|\Omega| \leq \gamma_0/2$, and where the frequency dependence of $S_{ba}(\omega)$ around ω_0 is neglected, we obtain

$$N_{\text{eff}} = |S_{ba}(\omega_0)|^2 \frac{\gamma_0}{\omega_0} \bar{n}, \quad (\text{J12a})$$

$$\xi_{\text{eff}} = |S_{ba}(\omega_0)|^2 \frac{\gamma_0}{\omega_0} e^{2i\text{Arg}[S_{ba}(\omega_0)]} \xi. \quad (\text{J12b})$$

Two-mode squeezing between $\omega_0 + \Omega/2$ and $\omega_0 - \Omega/2$ for all $|\Omega| \leq \gamma_0$ leads to

$$\bar{n} = \sinh^2(2|z|), \quad (\text{J13a})$$

$$|\xi| = \sinh(4|z|)/2, \quad (\text{J13b})$$

where z is the squeezing parameter, assumed to be the same for all of these pairs of modes. Substituting this into Eqs. (J12) and the result into Eq. (J10) then leads to Eq. (43).

3. Harmonic decomposition

We now decompose the time-periodic Franck-Condon factor given by Eq. (43) in Fourier series,

$$\mathcal{F}_{\text{Sq}_z}(t) \simeq e^{-\Lambda \sinh^2(2|z|)} \times \sum_{n \in \mathbb{Z}} I_n(\Lambda \cosh(2|z|) \sinh(2|z|)) e^{-2in\omega_0(t-\tilde{t}_e)}, \quad (\text{J14})$$

where $\Lambda = |S_{ba}(\omega_0)|^2/Q_0$ and I_n denotes the modified Bessel function of order n . The time $\tilde{t}_e$ arises from the phase of $S_{ba}(\omega_0)$ as well as of the squeezing parameter. We will forget it in the following since it can be absorbed in a redefinition of the leviton's injection times.

Because $|S_{ba}(\omega_0)|^2 \leq 1$ and Q_0 is significantly larger than 1, $\Lambda \ll 1$. Remembering also that experimentally reachable squeezing factors are not very high ($|z| \simeq 0.1755$ for 3 dB squeezing) and, therefore, $\Lambda \cosh(2|z|) \sinh(2|z|) \lesssim 1$. Moreover, $I_n(x) \simeq (x/2)^n/n!$ for $n \geq 1$. Consequently, the most important contribution comes from the first harmonics: $n = 0$ and $n = \pm 1$ in Eq. (J14). Retaining only these leads to

$$\mathcal{F}_{\text{Sq}_z}(t) \simeq \mathcal{F}_0(z, \Lambda) + e^{-2i\omega_0 t} \mathcal{F}_1(z, \Lambda) + e^{2i\omega_0 t} \mathcal{F}_{-1}(z, \Lambda), \quad (\text{J15})$$

where, at the lowest nontrivial order in Λ ,

$$\begin{aligned} \mathcal{F}_0(z, \Lambda) &= e^{-\Lambda \sinh^2(2|z|)} I_0(\Lambda \cosh(2|z|) \sinh(2|z|)) \\ &\simeq 1 - \Lambda \sinh^2(2|z|) + \mathcal{O}(\Lambda^2), \end{aligned} \quad (\text{J16a})$$

$$\begin{aligned} \mathcal{F}_{\pm 1}(z, \Lambda) &= e^{-\Lambda \sinh^2(2|z|)} I_1(\Lambda \cosh(2|z|) \sinh(2|z|)) \\ &\simeq \frac{\Lambda}{2} \cosh(2|z|) \sinh(2|z|) + \mathcal{O}(\Lambda^2). \end{aligned} \quad (\text{J16b})$$

The $n = 0$ harmonic contains the contrast obtained when averaging over the emission time t_e ,

$$\overline{[X_+^{(\text{dc})}]_{\text{Sq}(z)}^{t_e}} = [X_+^{(\text{dc})}]_{|0\rangle} \mathcal{F}_0(z, \Lambda). \quad (\text{J17})$$

Since $|\mathcal{F}_0(z, \Lambda)| < 1$, this t_e -independent measurement is lower than the one when only vacuum injected in the radiation channel. It also represents the average contrast for levitons of width τ_e injected at random emission and it is lower than the vacuum baseline. This is the effect of the average number of photons in a squeezed vacuum given by Eq. (J8).

APPENDIX K: SINGLE-EMP DETECTION

We now discuss the evaluation of the parameter $x(t) = 2\pi|\langle\chi|S_{ba}^*\Lambda_t\rangle|^2$ that appears in the evaluation of the Franck-Condon factor for Fock states [see Eq. (48)]. The qualitative behavior of this quantity can be understood by using a time-frequency representation of the current noise. The case of a narrow band EMP then leads to analytical approximated expressions for $x(t)$.

1. Time-frequency analysis of current noise

Using $\hat{i}(\omega) = -e\sqrt{\omega}b(\omega)$ ($\omega > 0$) for a single chiral integer quantum Hall edge channel, $x(t) = 2\pi|\langle\chi|S_{ba}^*\Lambda_t\rangle|^2$ can be expressed in terms of the excess current noise $\Delta S_{|1;\chi\rangle}^{(i)}(t, t')$ of a single quantum EMP in the mode χ ,

$$x(t) = \frac{2\pi^2}{e^2} \int_{\mathbb{R}^2} \Gamma_{ba}(t-t_+) \Gamma_{ba}(t-t_-)^* \Delta S_{|1;\chi\rangle}^{(i)}(t_+, t_-) dt_+ dt_-, \quad (\text{K1})$$

in which Γ_{ba} denotes the function defined by Eq. (F2). The physical meaning of this expression is better understood in terms of the Wigner function $\Delta W_{\rho}^{(i)}(t, \omega)$ of the excess current noise

$$\Delta W_{\rho}^{(i)}(t, \omega) = \int_{\mathbb{R}} \left\langle : \hat{i}\left(t - \frac{\tau}{2}\right) \hat{i}\left(t + \frac{\tau}{2}\right) : \right\rangle_{\rho, c} e^{i\omega\tau} d\tau, \quad (\text{K2})$$

in which $\langle AB \rangle_c$ denotes the connected correlator $\langle AB \rangle - \langle A \rangle \langle B \rangle$. Introducing the Ville transform associated with $\Gamma_{ba}(t)$,

$$W_{\Gamma_{ba}}(t, \omega) = \int_{\mathbb{R}} \Gamma_{ba}\left(t + \frac{\tau}{2}\right) \Gamma_{ba}\left(t - \frac{\tau}{2}\right) e^{i\omega\tau} d\tau \quad (\text{K3})$$

leads to

$$x(t) = \frac{2\pi^2}{e^2} \int_{\mathbb{R}^2} W_{\Gamma_{ba}}(t - \tau, \omega) \Delta W_{|1;\chi\rangle}^{(i)}(\tau, \omega) \frac{d\omega d\tau}{2\pi}. \quad (\text{K4})$$

This shows that the radiation coupler's response function $\Gamma_{ba}(\tau)$ leads to time-frequency filtering of the excess quantum current noise of the single-EMP state $|1, \chi\rangle$. The time resolution for single-EMP detection is thus limited by the duration of the excess current noise associated with the single EMP, as well as by the response time of the radiation coupler, typically the RC timescale appearing in Γ_{ba} (see Appendix F).

2. Current noise of a single EMP

For the single plasmon with wave function χ , the average excess current $\langle \hat{i}(t) \rangle_{|1;\chi\rangle}$ is zero and the Wigner function of the excess current noise defined in Eq. (K2) is given by

$$\begin{aligned} \Delta W_{|1;\chi\rangle}^{(i)}(t, \omega) &= \frac{e^2}{2\pi} \int_{|\Omega| \leq 2|\omega|} e^{-i\Omega t} \sqrt{\omega^2 - \frac{\Omega^2}{4}} \\ &\times \chi\left(|\omega| + \frac{\Omega}{2}\right) \chi\left(|\omega| - \frac{\Omega}{2}\right)^* \frac{d\Omega}{2\pi}. \end{aligned} \quad (\text{K5})$$

In the case of a narrow band single plasmon centered at the energy $\hbar\omega_0$ with Lorentzian linewidth γ_0 ,

$$\chi(\omega) = \frac{\sqrt{\gamma_0}\Theta(\omega)}{\omega - \omega_0 + \frac{i\gamma_0}{2}}, \quad (\text{K6})$$

such that $\gamma_0 \ll \omega_0$; the Wigner function of the excess current noise can be approximated by the usual expression for such energy-resolved excitations,

$$\Delta W_{|1;\chi}^{(i)}(t, \omega) \simeq \Theta(t) \frac{e^2 \omega_0}{2\pi} 4\gamma_0 t \sin c(2(|\omega| - \omega_0)t) e^{-\gamma_0 t}. \quad (\text{K7})$$

The general expression of the heat current $\mathcal{J}_Q(t)$ in terms of the current $i(t)$ leads to

$$\langle \mathcal{J}_Q(t) \rangle_\rho = \frac{R_K}{2} \left(\langle i(t) \rangle_\rho^2 + \int_{\mathbb{R}} \Delta W_\rho^{(i)}(t, \omega) \frac{d\omega}{2\pi} \right). \quad (\text{K8})$$

For a single energy-resolved EMP with energy $\hbar\omega_0$ and Lorentzian line shape of width γ_0 , this enables us to compute the instantaneous heat current just from the excess noise since $\langle i(t) \rangle_{|1;\chi} = 0$. Using Eq. (K7), we find

$$\langle \mathcal{J}_Q(t) \rangle_{|1;\chi} = \hbar\omega_0 \Theta(t) \gamma_0 e^{-\gamma_0 t}. \quad (\text{K9})$$

This heat current carries an average energy $\int_{\mathbb{R}} \langle \mathcal{J}_Q(t) \rangle_{|1;\chi} dt = \hbar\omega_0$ as expected.

3. Filtering of current noise

If the EMP has a narrow band compared to the typical scale of variation of $\Gamma_{ba}(\omega)$, the variation of $S_{ba}(\omega)$ around ω_0 can be neglected in Eq. (49). One then recognizes in $x(t)$ the modulus square

$$\frac{e^2}{2\pi} \left| \int_0^{+\infty} \sqrt{\omega\chi(\omega)} e^{-i\omega t} \frac{d\omega}{\sqrt{2\pi}} \right|^2 = \int_{\mathbb{R}} \Delta W_{|1;\chi}^{(i)}(t, \omega) \frac{d\omega}{2\pi}, \quad (\text{K10})$$

where the rhs directly follows from Eq. (K5). Equation (K8) then shows that this is related to the average instantaneous heat current carried by the single EMP in state χ . This finally leads to Eq. (50).

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