

Enhancing supercurrent-based inertial sensing via interactions in atomtronic angular accelerometers

S. Carmona-López¹, A. Matos-Abiague², F. Isaule¹, and L. Morales-Molina¹

¹*Facultad de Física, Pontificia Universidad Católica de Chile, Casilla 306, Santiago 22, Chile*

²*Department of Physics and Astronomy, Wayne State University, Detroit, Michigan 48201, USA*



(Received 2 May 2026; accepted 26 August 2026; published 15 September 2026)

We theoretically investigate supercurrents of ultracold atoms in angularly ac-shaken ring lattices subjected to external rotation. Our results demonstrate how these supercurrents can be harnessed for the development of high-precision atomtronic angular accelerometers. Using both analytical and numerical approaches within the Bose-Hubbard model framework, we demonstrate that a significant net atomic current arises when the lattice driving frequency is tuned to an integer fraction of the Bloch frequency, while the current averages to nearly zero away from such a resonance. In the single-particle regime, the resonance width scales inversely with the averaging time, thereby setting a fundamental Fourier-limited bound on the measurement's sensitivity. Strikingly, our numerical simulations demonstrate that this Fourier limit—a fundamental barrier in the noninteracting system—can be surpassed by introducing weak interactions between atoms. In the interacting regime, the sensitivity surpasses the Fourier-limited scaling with the averaging time achieving an improvement of at least 2 orders of magnitude over the single-particle scenario, and exceeding the performance of previously proposed ultracold-atom-based angular accelerometers. These findings pave the way for developing new atomic-current-based inertial sensors with interaction-enhanced sensitivity.

DOI: [10.1103/zzx2-tttb](https://doi.org/10.1103/zzx2-tttb)

I. INTRODUCTION

Quantum sensors can offer significant advantages over classical devices, providing enhanced sensitivity and precision [1–6]. Among them, cold-atom sensors stand out for their ability to measure physical quantities with remarkable stability and accuracy, making them a cutting-edge area of research in ultracold atomic physics [1,3].

Among cold-atom sensors, those designed to measure inertial and gravitational signals have been the subject of intensive investigation [2,3]. A particularly notable class within this category relies on the dynamic modulation of optical lattices, such as lattice shaking, in which atoms are confined. Lattice shaking has emerged as a powerful technique for manipulating cold atoms, enabling applications that range from quantum momentum-state engineering to condensate control [7]. Its implementation has led to significant advancements in the performance of atom interferometry-based sensors [8–12]. Moreover, ac-driven optical lattices can give rise to resonant processes that either enhance the sensitivity of existing inertial sensors or enable new types of sensing mechanisms. A well-known example involves tilted periodic optical lattices, where gravity induces Bloch oscillations of cold atoms. In this context, tuning the driving frequency to match the Bloch

frequency has been shown to significantly improve the precision of the gravity acceleration measurements [13,14].

Extending this framework to rotational configurations, angular Bloch oscillations in ring-shaped optical lattices have recently been investigated theoretically [15], demonstrating that angular acceleration can be accurately estimated from the angular Bloch frequency. Motivated by these results, in this work, we present and analyze a theoretical proposal for an angular accelerometer based on an angularly ac-shaken ring optical lattice. In contrast to prior studies, we propose to leverage measurements of atomic supercurrents as a promising method for detecting angular acceleration arising from external rotation. Specifically, when the driving frequency of the shaken lattice is a subharmonic of the angular Bloch frequency, a nonzero average atomic current emerges. Such a net transport of atoms has been observed in one-dimensional lattices under similar resonance conditions [16]. This effect provides the foundation for an *atomtronic* angular accelerometer, which operates by measuring supercurrents, a core phenomenon in the field of Atomtronics [17–22].

A key advantage of our method proposed here is its reliance on the emergence of a net average current, which eliminates the need to track the spatial evolution of the atomic cloud, thereby avoiding uncertainties associated with its spreading during time evolution. However, in the noninteracting regime, the measurement sensitivity is fundamentally limited by the associated Fourier-limited bandwidth [13], a constraint common to resonant two-level systems that ultimately bounds the precision of the angular-acceleration estimate.

To overcome this limitation, we propose leveraging atomic interactions to enhance the supercurrent's sensitivity to

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

angular acceleration. Although typically detrimental to measurement precision in atom interferometry, such interactions can boost the sensitivity of some ultracold atomic sensors [23], as demonstrated in Bose-Einstein condensates, where interatomic collisions have been shown to improve the sensitivity of cold-atom gravimeters beyond the shot-noise limit [23].

In this work, we show that introducing weak atomic interactions can not only improve the sensitivity by several orders of magnitude compared to the single-particle case, but also surpass values previously predicted for angular accelerometers [15]. This interaction-enhanced sensitivity establishes our atomtronic accelerometer and its potential for developing a new class of inertial sensors based on supercurrents of weakly interacting ultracold atoms.

The paper is organized as follows. Section II introduces the theoretical framework of an angularly ac-shaken ring lattice and formulates the atomtronic angular accelerometer. In Sec. III, we present the sensing protocol based on the detection of time-averaged supercurrents. We first analyze the single-particle regime, where analytical and numerical calculations reveal the emergence of a finite average current at resonant drive frequencies determined by submultiples of the angular Bloch frequency, while off-resonant responses vanish upon long-time averaging. We then extend the analysis to the nonequilibrium dynamics following a phase quench. The section further addresses the weakly interacting regime, demonstrating that atomic interactions significantly enhance sensitivity by inducing a pronounced narrowing of the resonance. This leads to a sensitivity that surpasses the standard Fourier scaling with averaging time. We attribute this behavior to interaction-induced dephasing and interference among Floquet modes, which is further elucidated by an effective reduced Hamiltonian description near resonance. Finally, Sec. IV summarizes the main conclusions and outlines possible directions for future research.

II. THEORETICAL MODEL

We consider an effective one-dimensional system of ultracold atoms in a time-periodic, angularly shaken ring lattice, subjected to an external angular acceleration. One-dimensional lattices without periodic driving can be realized by combining strong confinement in a toroidal trap with a ring lattice potential generated by two Laguerre-Gaussian beams [15]. The full system (i.e., the lattice with periodic driving) is represented in Fig. 1.

The Hamiltonian of the single-particle system can be written as [15]

$$\hat{H}_0 = \frac{\hat{p}_\varphi^2}{2I_{\text{rot}}} + V[\hat{\varphi} + \varphi_0(t)] - \Omega(t)\hat{p}_\varphi, \quad (1)$$

where $\hat{\varphi}$ and $\hat{p}_\varphi$ are the angular position and conjugate angular momentum operators, respectively. The moment of inertia is denoted by I_{rot} , and $\Omega(t)$ is the external time-dependent angular velocity around the ring's symmetry axis. The lattice periodic potential is represented by $V[\hat{\varphi} + \varphi_0(t)]$, where the time-dependent phase $\varphi_0(t)$ accounts for lattice shaking.

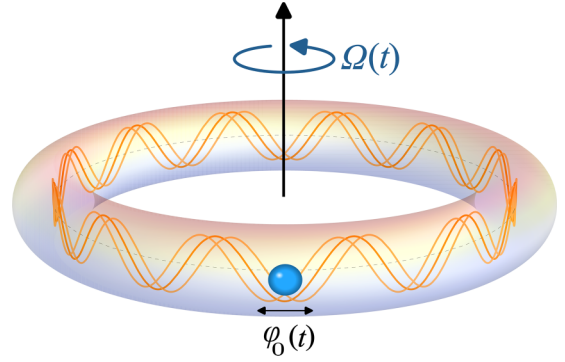


FIG. 1. Schematic of the proposed setup, consisting of atoms confined in a toroidal trap, effectively restricting their motion to a shaken ring optical lattice with a time-dependent angular displacement $\varphi_0(t)$. In addition, the trapped atoms experience an external rotation about the ring's symmetry axis with angular velocity $\Omega(t)$.

In this work, we consider a periodically shaken ring lattice, described by an angular displacement

$$\varphi_0(t) = A \cos(\omega t + \theta), \quad (2)$$

where A , ω , and θ , denote the amplitude, frequency, and phase, respectively.

To date, a ring lattice with a periodically modulated angular position has not been experimentally realized. Nevertheless, its key ingredients have been achieved independently. In particular, rotation sensing using shaken two-dimensional optical lattices has recently been demonstrated [12], and ring lattice configurations with optical potentials rotating at constant angular velocity have also been realized experimentally [24].

After an appropriate gauge transformation [25], the system's Hamiltonian takes the form

$$\hat{H}_0 = \frac{[\hat{p}_\varphi - \mathcal{A}(t)]^2}{2I_{\text{rot}}} + V(\hat{\varphi}), \quad (3)$$

where the potential satisfies $V(\varphi + \frac{2\pi}{N_s}) = V(\varphi)$ and N_s denotes the number of lattice sites. The physical meaning of the gauge vector potential, $\mathcal{A}(t) = I_{\text{rot}}[\Omega(t) - \dot{\varphi}_0(t)]$, is discussed below.

In the presence of atomic interactions and for a lattice potential depth $V_0 \gtrsim 5E_R$, where E_R is the recoil energy, the system is well described by the tight-binding approximation. In this regime, the Hamiltonian reduces to the Bose-Hubbard model [20],

$$\hat{H}_{\text{BH}} = -J \sum_{l=1}^{N_s} (e^{i\frac{\phi(t)}{N_s}} \hat{a}_l^\dagger \hat{a}_{l+1} + \text{H.c.}) + \frac{U}{2} \sum_{l=1}^{N_s} \hat{n}_l(\hat{n}_l - 1), \quad (4)$$

where J denotes the hopping energy and periodic boundary conditions are assumed for a lattice with N_s sites. The operator $\hat{a}_l^\dagger$ ($\hat{a}_l$) creates (annihilates) a particle at site l , and $\hat{n}_l = \hat{a}_l^\dagger \hat{a}_l$ is the corresponding particle number operator. The total number of particles, N , is then determined by the expectation value of the operator $\sum_{l=1}^{N_s} \hat{n}_l$. The phase $\phi(t) = 2\pi \mathcal{A}(t)/\hbar$, known as the Peierls phase [26], represents a fundamental tool for studying superfluidity in Bose-Einstein condensates confined to optical lattices [27,28]. Consequently, several methods to

engineer the Peierls phase in ultracold systems (including those driven by ac fields) have been proposed as a way to create artificial gauge fields [29–31]. The last term in Eq. (4) describes the on-site atomic interaction whose strength U can be experimentally tuned via Feshbach resonances [32].

For a system subject to a constant angular acceleration α , such that $\Omega(t) = \alpha t$, Eq. (2) yields the following expression for the Peierls phase:

$$\frac{\phi(t)}{N_s} = \omega_B t + \tilde{A} \sin(\omega t + \theta), \quad (5)$$

where $\tilde{A} \equiv A(2\pi I_{\text{rot}}\omega)/(N_s\hbar)$ is the dimensionless driving amplitude, and

$$\omega_B = \frac{2\pi I_{\text{rot}}}{\hbar N_s} \alpha \quad (6)$$

is the angular Bloch frequency, whose precise measurement enables the estimation of the angular acceleration [15]. Alternatively, the angular acceleration can be determined with higher precision by measuring average atomic supercurrents, as explained in the following section, where we discuss the working principle of the proposed atomtronic angular accelerometer.

III. SUPERCURRENTS FOR INERTIAL SENSING

To understand the operation of an angular accelerometer based on the measurement of supercurrents in an angularly shaken ring lattice, it is essential to characterize the atomic current induced by a time-dependent Peierls phase. To this end, we quantify the atomic current by introducing the current operator, which characterizes the flow of atoms between neighboring sites of the ring lattice. It is defined as

$$\hat{J} = \frac{1}{\hbar} \frac{\partial \hat{H}_{\text{BH}}}{\partial \phi} = -i \frac{\omega_J}{N_s} \sum_{l=1}^{N_s} (e^{i \frac{\phi(t)}{N_s}} \hat{a}_l^\dagger \hat{a}_{l+1} - \text{H.c.}), \quad (7)$$

where $\omega_J = J/\hbar$ is the hopping frequency. The observable represented by $\hat{J}$ can be measured experimentally by probing the phase gradient between atoms trapped at neighboring sites [19,33].

In this section, we analyze the response of this current in both the noninteracting and weakly interacting regimes. We begin by deriving an analytical expression for the atomic current in the single-particle limit, which provides physical insight into the sensing mechanism and serves as a benchmark for the many-body simulations discussed later. We then investigate, in both regimes, the nonequilibrium dynamics induced by a time-dependent Peierls phase in the Bose-Hubbard Hamiltonian [Eq. (4)]. This protocol allows us to start from a well-defined initial state with no imprinted phase. Specifically, the system is initially prepared in the many-body ground state, corresponding to a vanishing $\theta = 0$, in the Peierls phase. Then, at $t = 0$, the phase is quenched and subsequently evolves according to

$$\frac{\phi(t)}{N_s} = \begin{cases} 0, & \text{if } t = 0, \\ \omega_B t + \tilde{A} \sin(\omega t + \theta), & \text{if } t > 0. \end{cases} \quad (8)$$

The Bose-Hubbard Hamiltonian is solved using exact diagonalization [34,35], ensuring that the many-body dynamics and

observables are computed directly from the full Hamiltonian without relying on approximate many-body treatments. Time evolution is calculated using an adaptive time-step integration algorithm [36], and the results are independently verified using a Trotter-Suzuki unitary propagation scheme [37].

We first analyze the noninteracting regime, where the current response admits a simple analytical description, as discussed in the following subsection.

A. Noninteracting regime

In the single-particle limit ($U = 0$), the expectation value of the current is found to be

$$I(t) \equiv \langle \hat{J} \rangle = I_0 \sin \left[\frac{\phi(t)}{N_s} \right], \quad (9)$$

where $I_0 = \frac{2\omega_J}{N_s}$ is the current amplitude. Here, we assume that the particle occupies the system's ground state with zero momentum.

Combining Eqs. (5) and (9) and applying the Jacobi-Anger expansion, we obtain

$$\frac{I(t)}{I_0} = \sum_{n=-\infty}^{\infty} (-1)^n J_n(\tilde{A}) \sin(\Delta\omega_n t - n\theta), \quad (10)$$

where $J_n(A)$ is the n th Bessel function of the first kind and $\Delta\omega_n = \omega_B - n\omega$. Since the current contains multiple oscillatory contributions, an experimentally relevant sensing signal is obtained from the current averaged over a finite measurement time τ ,

$$\frac{\langle I \rangle_\tau}{I_0} = \sum_{n=-\infty}^{\infty} (-1)^n J_n(\tilde{A}) \frac{1}{\tau} \int_0^\tau \sin(\Delta\omega_n t - n\theta) dt, \quad (11)$$

where $\langle \cdots \rangle_\tau$ denotes the time average over the interval τ . When τ is sufficiently large, only the resonant term with $\Delta\omega_n = 0$ contributes significantly to the summation in Eq. (11), leading to peaks in the time-averaged current whenever the drive frequency satisfies

$$\omega = \frac{\omega_B}{n} \equiv \omega_n. \quad (12)$$

Therefore, when the drive frequency exactly matches the m th subharmonic, the time-averaged current is approximately

$$\langle I \rangle_{\tau,m} \approx I_0 (-1)^{m+1} J_m(\tilde{A}) \sin(m\theta). \quad (13)$$

This expression reveals that by tuning the drive amplitude $\tilde{A}$, one can selectively enhance or suppress the time-averaged current at specific subharmonics. Both the sign and magnitude of the current are governed by the Bessel function $J_m(\tilde{A})$, enabling precise control over its direction and strength. Consequently, by sweeping the drive frequency until $\langle I \rangle_\tau$ becomes appreciable, one can directly determine the angular Bloch frequency within the resonance width, and hence the angular acceleration experienced by the atoms.

To gain further insight into the accuracy of measuring the Bloch frequency, we analyze the behavior of the time-averaged current for drive frequencies close to resonance with the angular Bloch frequency. Specifically, we consider frequencies near a subharmonic, i.e., $\omega \approx \omega_m$. In this regime, the harmonic component with small detuning $\Delta\omega_m$ varies slowly

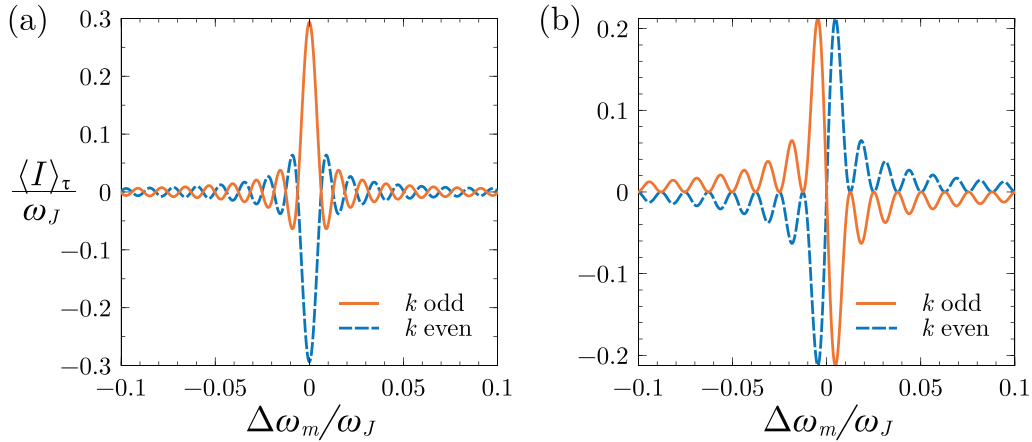


FIG. 2. Time-averaged current as a function of $\Delta\omega_m/\omega_J$ near the resonance condition for (a) $m\theta = (2k+1)\frac{\pi}{2}$ and (b) $m\theta = k\pi$. The remaining parameters are $\tau = 500/\omega_J$, $N = 1$, $N_s = 3$, and $\tilde{A} = 1$.

in time, whereas the remaining harmonics, with detunings $\Delta\omega_n$ for $n \neq m$, oscillate rapidly. As a result, the dominant contribution to the time-averaged current arises from the $n = m$ term in Eq. (11). Thus, near the resonant subharmonic ω_m , the time-averaged current is approximately

$$\frac{\langle I \rangle_\tau(\Delta\omega_m)}{I_0} \approx (-1)^m J_m(\tilde{A}) \operatorname{sinc}\left(\frac{\Delta\omega_m \tau}{2}\right) \sin\left(\frac{\Delta\omega_m \tau}{2} - m\theta\right), \quad (14)$$

which depends explicitly on the detuning $\Delta\omega_m$.

Equation (14) reveals that the qualitative behavior of the time-averaged current depends on the drive phase θ . Of particular relevance for sensing is the case in which the phase satisfies $m\theta = (2k+1)\frac{\pi}{2}$, with $k \in \mathbb{Z}$. In this configuration, the averaged current reduces to a sinclike resonance profile

$$\frac{\langle I \rangle_\tau(\Delta\omega_m)}{I_0} \approx (-1)^{m+k} J_m(\tilde{A}) \operatorname{sinc}(\Delta\omega_m \tau), \quad (15)$$

which exhibits a maximum at the resonance condition and is therefore particularly well suited for precise frequency (and hence acceleration) estimation.

The time-averaged current as a function $\Delta\omega_m$ is shown in Fig. 2(a). As expected from Eq. (15), the time-averaged current is modulated by the sinc function, with the width of the peak around resonance inversely proportional to the measurement time τ . Consequently, an efficient measurement strategy is to start with a relatively short τ , producing a broader resonance peak that facilitates detection during a frequency sweep. Increasing τ subsequently narrows the peak, enhancing the precision in determining the angular Bloch frequency and, in turn, the angular acceleration. However, the maximum reachable τ is limited by the condensate coherence time, which in turn constrains the precision achievable in the noninteracting regime.

For completeness, we note that when the phase satisfies $m\theta = k\pi$, with $k \in \mathbb{Z}$, the time-averaged current takes the form

$$\frac{\langle I \rangle_\tau(\Delta\omega_m)}{I_0} \approx (-1)^{m+k} J_m(\tilde{A}) \frac{1}{\tau \Delta\omega_m} \sin^2\left(\frac{\Delta\omega_m \tau}{2}\right). \quad (16)$$

This expression is antisymmetric about the resonance and vanishes exactly at $\Delta\omega_m = 0$, as shown in Fig. 2(b). Since

this configuration does not yield a detectable supercurrent at resonance, it is not suitable for sensing applications.

Overall, the dependence of the time-averaged current in Eq. (14) on the static phase θ , the drive amplitude, and the resonant subharmonic index allows for fine control over the current at different resonant frequencies. This enables selective enhancement of the peak associated with a given subharmonic, while providing detailed insight into the net atomic current in the vicinity of resonance under well-controlled conditions.

Nonequilibrium dynamics induced by a phase quench

While the above analysis assumes a system with an initially imprinted phase, realistic experimental preparations start from a nontransporting ground state. Consequently, when the phase quench defined in Eq. (8) is applied, the system is driven into nonequilibrium dynamics. To capture the essential features of the current response induced by such a phase quench, it is sufficient to consider a three-site ring lattice [38,39], which constitutes the minimal system with periodic boundary conditions. The resulting trimer therefore provides the most elementary model on which our atomtronic angular accelerometer could be tested.

The numerical results for the time-averaged current as a function of the driving frequency for $\tilde{A} = 1.0$ and $\theta = \pi/2$ are shown in Fig. 3. The time-averaged current diminishes and becomes negligible as the frequency moves away from integer divisors of ω_B , as illustrated in Fig. 3(a). Near the resonance frequencies, the behavior associated with the subharmonics $m = 1, 3$ and $m = 2$ closely follows the predictions of Eqs. (15) and (16), respectively. A close-up of the time-averaged current near the main resonance ($\omega/\omega_B = 1$) is shown in Fig. 3(b) for two measurement times $\tau = 1000/\omega_J$ and $\tau = 2000/\omega_J$, where numerical simulations and analytical predictions show excellent agreement. This agreement persists despite the fact that the numerical simulations include a phase quench. Such a result is not surprising, since in the single-particle limit the resonance current response is governed solely by frequency matching and is therefore

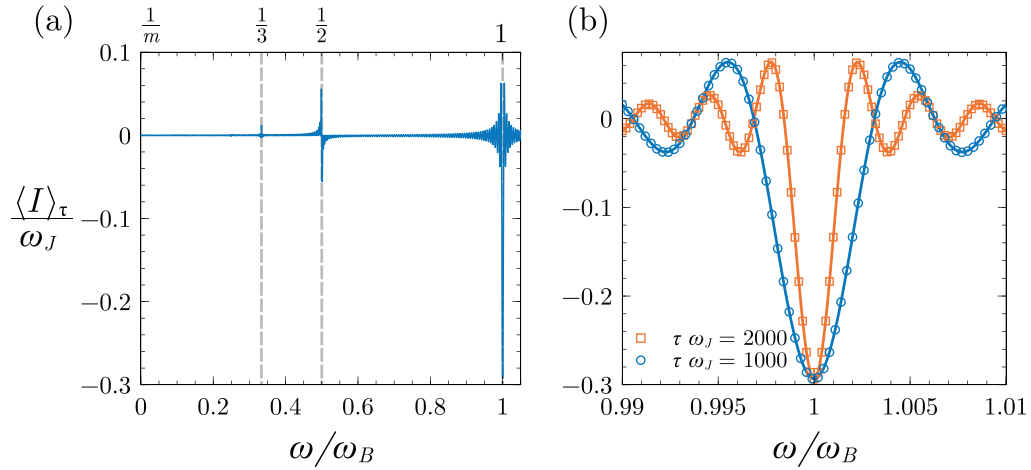


FIG. 3. (a) Time-averaged current as a function of ω/ω_B , for $\tau = 1000/\omega_J$. (b) Time-averaged current near the resonance at $\omega/\omega_B = 1$ for measurement times $\tau = 1000/\omega_J$ and $\tau = 2000/\omega_J$. The numerical simulation results are shown as blue empty circles for $\tau = 1000/\omega_J$ and orange empty squares for $\tau = 2000/\omega_J$, while the corresponding theoretical predictions from Eq. (15) are shown as solid lines with matching colors. The analytical predictions are in excellent agreement with the numerical simulations for $U/J = 0$. The remaining parameters are $N = 1$, $N_s = 3$, $\tilde{A} = 1$, and $\theta = \pi/2$.

insensitive to whether the Peierls phase is introduced adiabatically or through a sudden quench. As expected from Eq. (15), the results display a sinelike profile, featuring a pronounced net atomic current near the resonance. Moreover, Fig. 3(b) clearly illustrates that the resonance width—and, consequently, the sensitivity of the time-averaged current measurement—depends on the measurement time τ .

To provide a more comprehensive view of the resonance behavior across the parameter space, Fig. 4(a) presents the time-averaged current as a function of both the driving frequency and amplitude. The figure shows that tuning $\tilde{A}$ enables both suppression and enhancement of the time-averaged current at different subharmonics and even allows control over the direction of the net atomic current. When combined with the choice of θ , this enables precise control over which subharmonics are suppressed or activated, thereby facilitating

their selective detection as the driving frequency is varied. Furthermore, Figs. 4(b) and 4(c) clearly highlight the resonance peak amplitudes at $\omega/\omega_B = 1$ and $\omega/\omega_B = 0.2$, demonstrating an excellent agreement between the numerical simulations (empty dots) and the predictions of Eq. (13) (dashed lines).

Despite the high degree of control and tunability, the dependence of the resonance width on the measurement time remains a fundamental limitation on the precision of angular-acceleration measurements. This Fourier-limited sensitivity is well known in Bloch-oscillation-based linear accelerometers [13,14], and it also arises in theoretical proposals for ultracold-atom-based angular accelerometers [15]. In the following subsection, we demonstrate that this limitation can be overcome by incorporating weak interatomic interactions into the system.

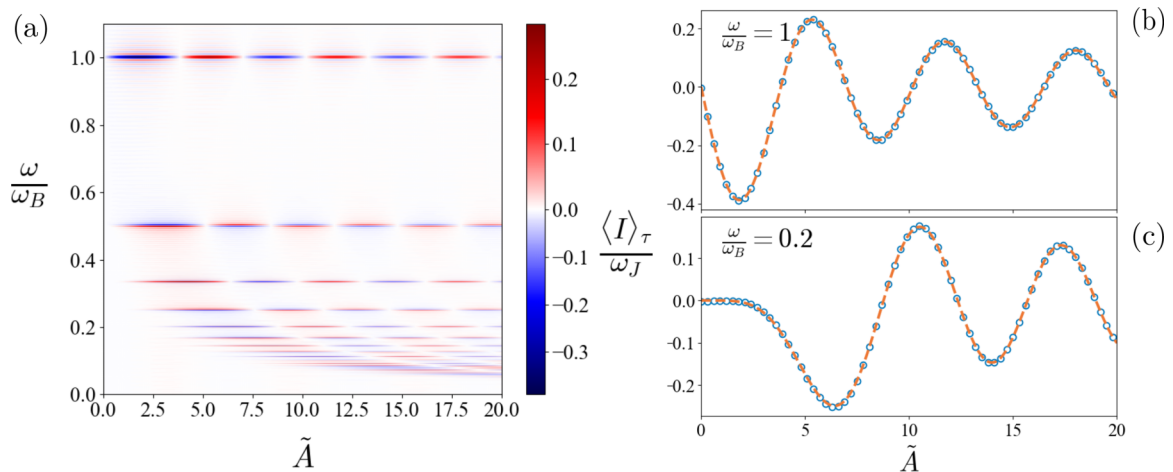


FIG. 4. (a) Numerical time-averaged current as a function of ω/ω_B and $\tilde{A}$ for $N = 1$, $N_s = 3$, $\theta = \pi/2$, $U = 0$, and $\tau = 500/\omega_J$. (b), (c) Numerically computed time-averaged current at resonance as a function of $\tilde{A}$ for $\omega/\omega_B = 1$ and $\omega/\omega_B = 0.2$ (empty dots), together with the corresponding theoretical prediction from Eq. (13) (dashed orange line), showing excellent agreement.

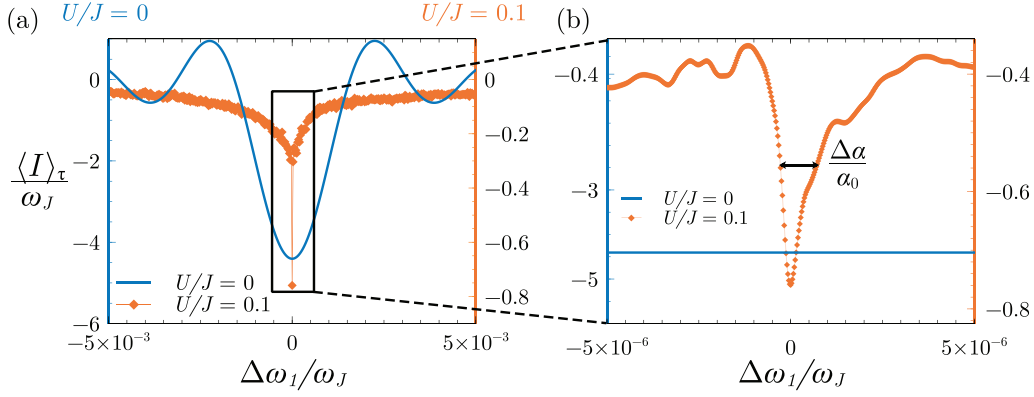


FIG. 5. (a) Time-averaged current as a function of $\Delta\omega_1/\omega_J$ for $\theta = \pi/2$ and $\tau = 2000/\omega_J$. The noninteracting case (solid blue line) is compared with the weakly interacting case (orange diamonds) for $N_s = 5$, $N = 15$, and $U/J = 0.1$. The left (right) y axis corresponds to the noninteracting (interacting) data, with colors matching their respective curves. The resonance width is significantly narrower in the interacting case. (b) Enlargement of the resonance region. The peak corresponding to $U/J = 0.1$ has an appreciable width, while the peak for $U = 0$ appears nearly flat on this scale. The double-headed arrow marks the full width at half maximum (FWHM), $\Delta\alpha/\alpha_0$, which we use as a measure of the device sensitivity. As in panel (a), the left (right) y axis corresponds to the noninteracting (interacting) data.

B. Weakly interacting regime

Thus far, our analysis has focused on the single-particle limit, which captures the basic sensing mechanism and allows for a transparent analytical treatment. In realistic Bose-Einstein condensates, however, interatomic interactions are inevitably present and can significantly influence the precision of time-averaged current measurements. Understanding how weak interactions modify the resonance structure of the current is therefore essential for assessing the performance of current-based inertial sensing schemes. To systematically explore the effects of interatomic interaction, we study the nonequilibrium dynamics following the phase quench [Eq. (8)] by performing numerical simulations of a ring lattice with multiple particles in the weakly interacting regime $U/J \ll 1$. Experimentally, this regime is readily accessible, as the interaction strength U can be tuned via Feshbach resonances [32], while the tunneling amplitude J can be controlled by adjusting the lattice depth. The numerical simulations were carried out for systems with $N_s = 3, 4$, and 5 sites, and filling factors $\nu = N/N_s = 1, 2$, and 3, enabling a systematic analysis of weak interaction effects and allowing us to benchmark the results against the noninteracting predictions discussed in the previous subsection.

Figure 5(a) shows the time-averaged currents as a function of the normalized driving frequency, ω/ω_B , for five-site rings with $N = 15$ for $U/J = 0$ (solid blue line) and $U/J = 0.1$ (orange diamonds), both evaluated at $\tau = 2000/\omega_J$. Complementarily, an enlargement of the resonant region is shown in Fig. 5(b).

According to Eq. (6), the sensitivity for measuring the angular acceleration is given by $\Delta\alpha = \Delta\omega_B(\hbar N_s)/(2\pi I_{\text{rot}})$ and is therefore ultimately limited by the uncertainty in the angular Bloch frequency, $\Delta\omega_B$. The relative uncertainty $\Delta\omega_B/\omega_J$ can be extracted from the FWHM of the time-averaged supercurrent resonance, as illustrated in Fig. 5(b). One thus obtains

$$\text{FWHM} = \frac{\Delta\omega_B}{\omega_J} = \frac{\Delta\alpha}{\alpha_0}, \quad (17)$$

where $\alpha_0 = N_s \hbar \omega_J / (2\pi I_{\text{rot}})$. Consequently, the FWHM of the time-averaged supercurrent provides a direct measure of the accelerometer sensitivity ($\Delta\alpha = \alpha_0$ FWHM), with narrower resonance peaks allowing for a more precise determination of the angular Bloch frequency and, hence, of the angular acceleration. Note, however, that smaller values of $\Delta\alpha/\alpha_0$ correspond to higher sensitivity, as they reflect improved measurement precision.

The results shown in Fig. 5 indicate that, for weakly interacting atoms, the resonance peak becomes significantly narrower than in the noninteracting case, thereby enhancing the precision of angular-acceleration measurements. However, the presence of weak interactions can also reduce the peak amplitude of the time-averaged current, as shown in Fig. 5. Thus, while interactions enhance sensitivity, the reduced current may pose experimental challenges, requiring a trade-off that defines the interaction range for optimal operation, as discussed later in this article.

1. Resonance peak narrowing via dephasing-induced interference

The narrowing of the resonance peak observed in the net atomic current can be understood by analyzing the effective dynamics derived from the ac-driven Bose-Hubbard Hamiltonian (see Appendix A for details). For illustration, we consider the case $\tilde{A} = 1$ and small detuning, $\Delta\omega_1 \ll \omega_B$. Under these conditions, the time evolution is dominated by the $n = 0, 1$ modes, and the full Hamiltonian can effectively be reduced to

$$\begin{aligned} H_{\text{red}} = & -JJ_0(\tilde{A}) \sum_{l=1}^{N_s} (e^{i\omega_B t} \hat{a}_{l+1}^\dagger \hat{a}_l + \text{H.c.}) \\ & + JJ_1(\tilde{A}) \sum_{l=1}^{N_s} (e^{i\tilde{\theta}} \hat{a}_{l+1}^\dagger \hat{a}_l + \text{H.c.}) \\ & + \frac{U}{2} \sum_{l=1}^{N_s} \hat{n}_l (\hat{n}_l - 1), \end{aligned} \quad (18)$$

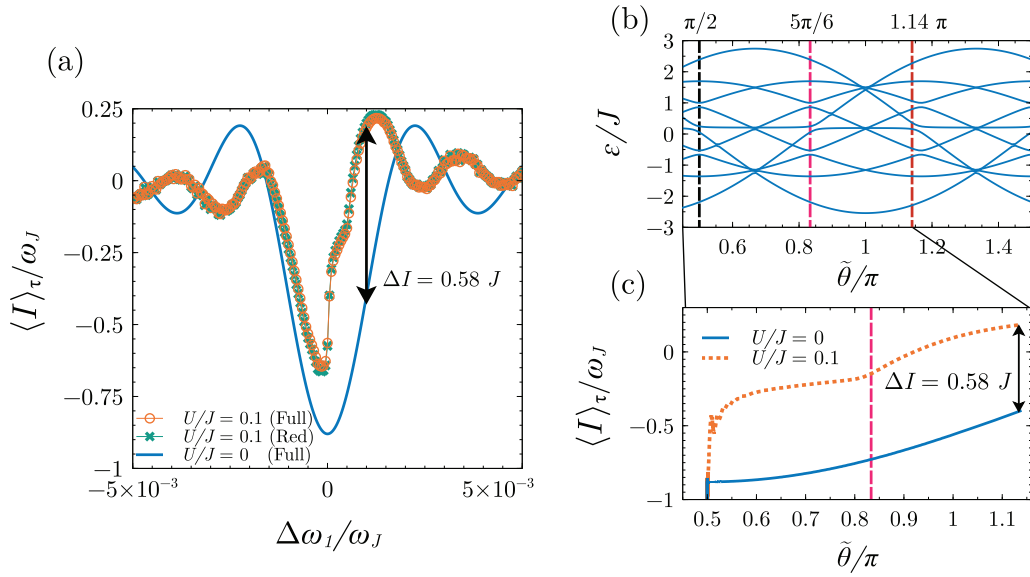


FIG. 6. (a) Time-averaged current as a function of the detuning $\Delta\omega_1/\omega_J$ for $\theta = \pi/2$ and $\tau = 2000/\omega_J$. The weakly interacting case is shown both from full simulations (orange empty circles) and from the reduced Hamiltonian (green crosses) for $N = 3$ and $N_s = 3$, together with the noninteracting result (solid blue line). The vertical double-headed arrow indicates the difference $\Delta I = 0.58 J$ between the interacting and noninteracting time-averaged currents, evaluated at $\Delta\omega_1/\omega_J = 10^{-3}$. (b) Quasienergy spectrum of the reduced Hamiltonian [Eq. (18)] for $U/J = 0.1$. The magenta dashed vertical line marks the avoided crossing at $\tilde{\theta} = 5\pi/6$. (c) Time-averaged current as a function of $\tilde{\theta}$ for $U/J = 0$ (solid blue line) and $U/J = 0.1$ (orange dotted line), computed at fixed detuning $\Delta\omega_1/\omega_J = 10^{-3}$. The magenta dashed vertical line indicates the same value $\tilde{\theta} = 5\pi/6$, corresponding to the avoided crossing shown in panel (b). The same difference ΔI is highlighted by a vertical double-headed arrow.

where $\tilde{\theta} = \Delta\omega_1 t - \theta$. To verify the validity of this reduced Hamiltonian, we consider a ring with $N_s = N = 3$. Figure 6(a) shows the averaged current near resonance for both the full system and the reduced model, revealing excellent agreement between the two descriptions. In this near-resonant regime, the system can be averaged over the period $T_B = 2\pi/\omega_B$, namely, the micromotion timescale [40–42], allowing us to compute the quasienergies as functions of the parameter $\tilde{\theta}$ shown in Fig. 6(b). The resulting quasienergy spectrum, shown in Fig. 6(b), exhibits multiple avoided crossings, with gap sizes proportional to the interaction strength U .

The presence of a finite detuning introduces a “rate” of change of $\tilde{\theta}$, corresponding to a displacement along the quasienergy bands, so that adiabatic following is repeatedly interrupted at the avoided crossings. This leads to interference among different Floquet modes, each associated with a distinct mean momentum. Consequently, destructive interference accumulates over time, suppressing the net current. This interpretation is supported by the cumulant of the average current shown in Fig. 6(c), where pronounced changes in the current response occur at the avoided crossings at $\tilde{\theta} = \pi/2$ (vertical black dashed line) and $\tilde{\theta} = 5\pi/6$ (vertical magenta dashed line).

Similar interference mechanisms, previously studied in tilted lattices with interacting atoms [43–45], have been shown to cause dispersion, dephasing, and delocalization due to interaction-induced coupling between Floquet modes. Although those studies did not explicitly analyze the current, our results demonstrate that the same interference processes directly account for the observed narrowing of the resonance peak. Moreover, the pattern of fluctuating current values near

resonance is consistent with this interpretation, as shown in Fig. 5.

2. Sensitivity enhancement via interactions

To quantify the impact of weak interactions on the precision of angular-acceleration measurements, we analyze the resonance peak width as a measure of the sensitivity. Specifically, we extract the FWHM of the time-averaged current $\Delta\alpha/\alpha_0$, as illustrated in Fig. 5. This allows us to systematically compare how the resonance sharpens with increasing interaction strength.

Figure 7(a) shows the sensitivity as a function of the interaction parameter U/J in the weakly interacting regime ($U/J \ll 1$). We consider various system sizes, all with a filling factor of 3, which enables high sensitivity values. In all cases, angular-acceleration sensitivity is substantially enhanced, with the resonance peak narrowing by up to 2 orders of magnitude compared to the noninteracting (single-particle) scenario. This narrowing corresponds to a dramatic theoretical improvement in the sensitivity of the proposed atomtronic angular accelerometer.

However, increasing the interaction strength also leads to a suppression of the current. Figure 7(b) shows the current at the resonance point as a function of U/J , revealing that the current begins to decay around $U/J = 0.1$, and eventually drops by 2 orders of magnitude for larger U/J values. To qualitatively understand this behavior, we derive in Appendix B an approximate expression for the time-averaged current near the

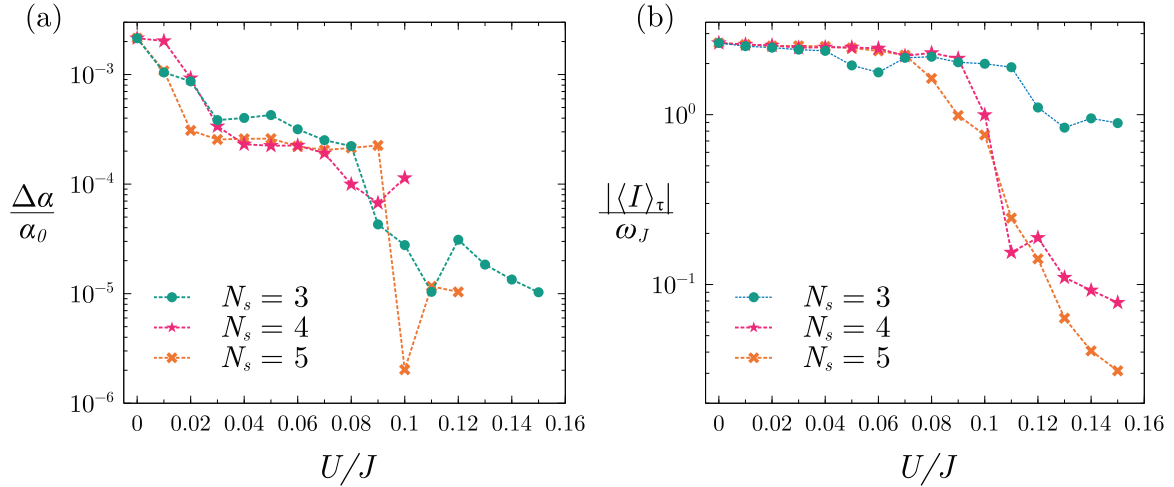


FIG. 7. (a) Estimated angular-acceleration sensitivity (in units of α_0) as a function of interaction strength U/J . (b) Absolute value of the time-averaged current (in units of ω_J) at the main resonant peak, shown as a function of U/J . In both panels, results are presented for three system sizes: $N_s = 3$ (green circles), $N_s = 4$ (magenta stars), and $N_s = 5$ (orange crosses). Simulations are performed at the main resonance, $\omega/\omega_B = 1$, with $\tau = 2000/\omega_J$, $\theta = \pi/2$, $\tilde{A} = 1$, and a filling factor $\nu = 3$.

m th resonance,

$$\begin{aligned} \frac{\langle I \rangle_\tau^U}{I_0} &= \frac{\langle I \rangle_\tau}{I_0} \left(N - \frac{1}{2} \sum_{k \neq 0} \frac{\omega_U^2}{\gamma_{km}^2} \right) \\ &+ \frac{(-1)^m}{4} J_m(\tilde{A}) \sum_{k \neq 0} \sum_{s=\pm 1} \left\{ \frac{\omega_U^2}{\gamma_{km}^2} \text{sinc} \left[\frac{(\Delta\omega_m + 2s\gamma_{km})\tau}{2} \right] \right. \\ &\times \left. \sin \left[\frac{(\Delta\omega_m + 2s\gamma_{km})\tau}{2} - m\theta \right] \right\}, \end{aligned} \quad (19)$$

where $\gamma_{km} = \sqrt{(\Omega_{km} + \omega_U)^2 - \omega_U^2}$, with $\Omega_{km} = 2\omega_J[1 - (-1)^m J_m(\tilde{A}) \cos(k) \cos(m\theta)]$ and $\omega_U = UN/(\hbar N_s)$.

To gain insight into the predictions of Eq. (19), we first examine the resonant case $\Delta\omega_m = 0$. In the noninteracting limit, only the leading contribution survives, yielding a current that scales linearly with N . In the presence of interactions, however, additional terms with $k \neq 0$ appear and give rise to a negative correction proportional to ω_U^2 , thereby reducing the total current. Physically, this suppression originates from interaction-induced quasiparticle excitations, which deplete the condensate population. This provides a qualitative explanation for the monotonic decrease in the current observed in Fig. 7(b).

Away from resonance ($\Delta\omega_m \neq 0$), the interaction-induced sideband terms become oscillatory components with frequencies shifted by $\pm 2\gamma_{km}$. These sidebands are generally out of phase with the primary resonant term, resulting in partial cancellation of the current. This additional destructive interference further suppresses the net signal. We emphasize that the expression above includes only the dominant sideband contributions; higher-order terms, not captured in this expression, would introduce further oscillatory components that can reinforce the overall interference pattern.

From an experimental perspective, the suppression of the current poses a challenge for signal detection, placing a practical limit on the range of interaction strengths suitable for high-sensitivity measurements of angular acceleration. This

indicates the existence of an optimal operational regime, where a measurable current is maintained while preserving high sensitivity. Outside this regime, either the current is too small to detect, or the sensitivity approaches that of the non-interacting case.

C. Maximal sensitivity values and experimental realization

In this section, we analyze the sensitivity of angular-acceleration measurements as a function of the averaging time, accounting for atomic interactions.

Figure 8 compares the sensitivity in the weakly interacting regime for filling factors $\nu = 1, 2$, and 3 with that of the single-particle case. In the noninteracting limit (blue solid line), the resonance width scales inversely with the averaging time, reflecting the Fourier limit. This constraint is common to ac-driven analog sensors, including Bloch-oscillation-based accelerometers [13,14].

When interactions are included, the sensitivity exhibits a pronounced enhancement of the scaling, due to interaction-induced resonance narrowing. Figure 8 shows that, as the averaging time increases—allowing sufficient time for inter-atomic correlations to develop—the sensitivity progressively deviates from the Fourier-limited scaling $\propto \tau^{-1}$. This trend is illustrated in panels (a), (b), and (c), corresponding to filling factors $\nu = 1, 2$, and 3, respectively. The deviation from Fourier scaling occurs at shorter averaging times for larger filling factors, indicating a stronger interaction-induced enhancement associated with increasing many-body contributions. This behavior is expected, since the narrowing of the resonance width originates from the interference of sideband harmonics in the current response [Eq. (19)], whose cumulative contribution becomes increasingly pronounced at longer averaging times.

A further distinctive feature across the three panels is that, beyond a filling-dependent crossover time, the sensitivity approaches an asymptotic power-law scaling of the form

$$\Delta\alpha/\alpha_0 \propto \tau^{-p}, \quad (20)$$

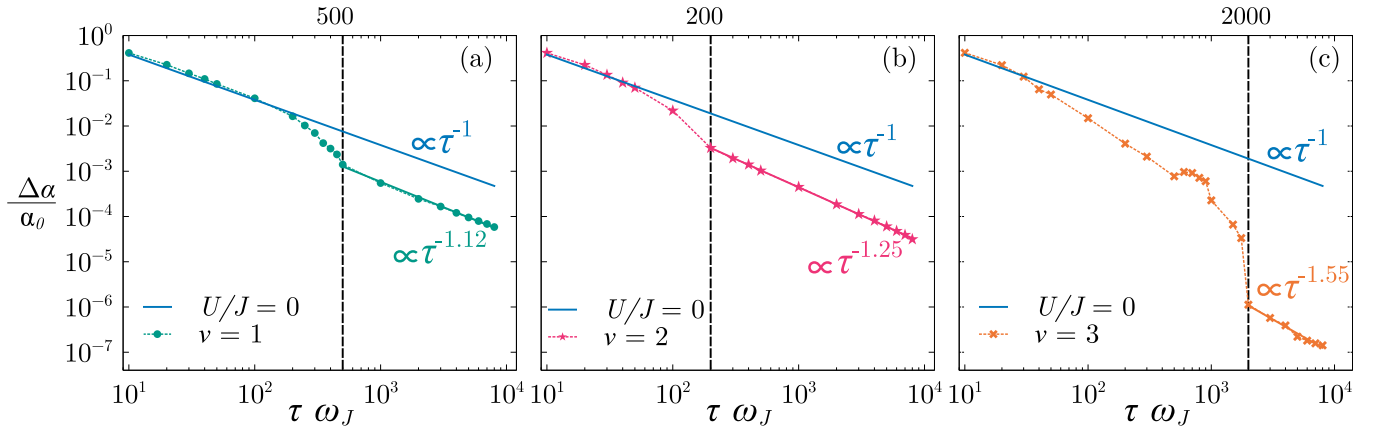


FIG. 8. Estimated angular-acceleration sensitivity as a function of the measurement time (in units of $1/\omega_J$) on a log-log scale. Panels (a), (b), and (c) correspond to integer filling factors $\nu = 1$, $\nu = 2$, and $\nu = 3$, respectively, and share the same vertical axis. In each panel, the sensitivity obtained within the analytical noninteracting approximation is shown as a solid blue line, while numerical results for $N_s = 5$ and $U/J = 0.1$, are shown as green dots in panel (a), magenta stars in panel (b), and orange crosses in panel (c). Power-law fits in the long-time regime, where a linear trend emerges in the log-log representation, are shown as solid lines.

with exponents $p \simeq 1.12, 1.25$, and 1.55 for $\nu = 1, 2, 3$, respectively, and $U/J = 0.1$. This scaling identifies a regime dominated by interaction-driven many-body dynamics, where the resonant processes accumulated during the time evolution govern the ultimate sensitivity. Remarkably, by the end of the crossover time (marked by the vertical dashed lines in Fig. 8), the system has fully transitioned from the initial regime with approximate Fourier-limited scaling ($p = 1$) to the interaction-driven regime characterized by $p > 1$. By this point, the sensitivity is already substantially enhanced (by roughly 3 orders of magnitude for $\nu = 3$). Beyond the crossover, the interaction-induced scaling ($p > 1$) further improves the sensitivity with increasing averaging time.

This regime contrasts sharply with the noninteracting case, which strictly follows the Fourier-limited τ^{-1} scaling (blue line). The strong dependence of the scaling exponent on the filling factor indicates that interactions give rise to distinct dynamical regimes, which can be exploited to achieve substantially higher sensitivities at shorter measurement times. This feature is particularly advantageous in realistic cold-atom experiments, where finite coherence times and technical noise impose stringent constraints on measurement duration and stability.

To assess the performance of our proposed accelerometer relative to previous designs, we compare our sensitivity results with the theoretical predictions of Ref. [15] for a nondriven lattice. In our calculations, we use $\omega_J \approx 500$ Hz (corresponding to a lattice depth $V_0 \approx 7E_R$ [46], where E_R is the recoil energy) and $N_s = 5$, whereas Ref. [15] considers $N_s = 20$. For consistency in the comparison, we adopt a lattice radius of $90 \mu\text{m}$ and atoms of mass corresponding to ^{87}Rb as in Ref. [15]. For these parameters, Ref. [15] predicts a sensitivity of $\Delta\alpha \approx 10^{-4}$ rad/s² for a measurement time $\tau = 10$ s using $N = 10^5$ atoms. In contrast, our fully quantum, weakly interacting model achieves $\Delta\alpha \approx 8 \times 10^{-6}$ rad/s² with only $N = 15$ atoms for the same measurement time. This represents a significant improvement in sensitivity, while reducing the required particle number by 4 orders of magnitude, demonstrating a substantial enhancement in measurement precision.

Furthermore, the lattice-shaking amplitude in our model is given by

$$A^* = \frac{N_s \hbar}{\pi I_{\text{rot}}} \tilde{A}, \quad (21)$$

which can be experimentally tuned by adjusting the number of lattice sites N_s , providing a convenient control knob for optimizing the sensor's performance.

A factor that could, in principle, lead to resonance broadening and thereby affect the sensitivity is temperature. However, such broadening is expected to be negligible when the initial thermal population of excited states is sufficiently suppressed, i.e., when $\exp[-|E_1 - E_0|/(k_B T)] \ll 1$, where E_0 and E_1 denote the ground- and first-excited-state energies, respectively. In the weakly interacting limit considered here, $|E_1 - E_0| \sim \hbar\omega_J$, and temperature effects can therefore be neglected provided that $\exp[-\hbar\omega_J/(k_B T)] \ll 1$. For the value $\omega_J = 500$ Hz used in our sensitivity estimates, this condition is well satisfied for temperatures $T \lesssim 2$ nK. Such ultralow temperatures, reaching down to a few hundred picokelvins, have been achieved in ultracold-atom experiments with dilute Bose-Einstein condensates (see, e.g., Ref. [47]).

Overall, the results discussed above underscore the novelty and potential of our proposed atomtronic angular accelerometer, demonstrating that weak interactions can be harnessed to surpass Fourier-limited sensitivity and achieve high-precision inertial sensing with only a few atoms.

IV. CONCLUDING REMARKS

We have proposed and theoretically analyzed an atomtronic angular accelerometer based on an angularly ac-shaken ring lattice. The sensing signal is the time-averaged atomic current, which encodes the external angular acceleration. In the single-particle regime, we demonstrated—both analytically and numerically—that a nonzero average current emerges when the drive frequency matches a submultiple of the angular Bloch frequency; for other frequencies, the current remains negligibly small provided the averaging time

is sufficiently long. Near resonance, the response exhibits a sinc-like profile with a Fourier-limited width, while the drive phase controls the direction of the atomic current.

Our analysis further shows that weak atomic interactions can fundamentally enhance sensitivity beyond the conventional Fourier limit by significantly narrowing the resonance. In ring-lattice systems, even weak interactions reduce the resonance width by several orders of magnitude compared to the noninteracting case. In addition, the interacting regime displays a markedly steeper dependence of the sensitivity on the averaging time than in the Fourier-limited scenario. This pronounced dependence on particle number reveals distinct interaction-driven dynamical regimes that can be harnessed for quantum-enhanced metrological performance.

We identified the physical origin of the resonance narrowing as dephasing-induced interference among Floquet modes. Using a reduced Hamiltonian valid near resonance, we showed that the quasienergy spectrum develops multiple avoided crossings, with gap sizes set by the interaction strength. A finite detuning drives the system along the quasienergy bands, causing repeated breakdowns of adiabatic following and generating interference between modes with different mean momenta. The resulting destructive interference accumulates over time, suppressing the net current and thereby narrowing the resonance. This mechanism is consistent with the observed sideband structure and with the interaction-dependent corrections captured by our momentum-space expression for the averaged current.

Finally, we showed that the metrological advantage introduced by interactions is accompanied by a reduction of the resonant current signal. As the interaction strength increases within the weakly interacting regime, the current decreases and can be suppressed by up to 2 orders of magnitude. Despite this trade-off, our proposal achieves superior sensitivity compared to mean-field predictions [15]. Moreover, the scheme is experimentally feasible with current ultracold-atom platforms, where interactions can be tuned via Feshbach resonances and the lattice-shaking amplitude can be controlled through the lattice laser parameters.

In summary, we have introduced an atomic-current-based inertial sensor whose sensitivity is enhanced by weak atomic interactions. We demonstrated Fourier-beating scaling exponents and identified the interference mechanism responsible for the interaction-induced resonance narrowing, thereby establishing a route toward few-atom, high-precision inertial sensing. These results pave the way for next-generation atomtronic inertial sensors and may stimulate further theoretical and experimental exploration of interaction-enabled quantum metrology in lattice-based platforms.

ACKNOWLEDGMENTS

S.C.-L. acknowledges financial support from ANID through the National Master's Scholarship, ANID-Subdirección de Capital Humano/Magíster Nacional/2025-22260327. F.I. acknowledges funding from ANID through FONDECYT Postdoctorado Project No. 3230023. L.M.-M. acknowledges financial support from the Institute of Physics, PUC.

DATA AVAILABILITY

The data that support the findings of this article are available from the authors upon reasonable request.

APPENDIX A: REDUCED HAMILTONIAN

The Bose-Hubbard Hamiltonian (4) can be recast using the Jacobi-Anger expansion as

$$H = \sum_{n=-\infty}^{\infty} H_0^{(n)} + \frac{U}{2} \sum_{l=1}^{N_s} \hat{n}_l (\hat{n}_l - 1), \quad (\text{A1})$$

where

$$H_0^{(n)} = \Omega_n \sum_{l=1}^{N_s} (e^{i(\Delta\omega_n t - \chi_n)} \hat{a}_{l+1}^\dagger \hat{a}_l + \text{H.c.}), \quad (\text{A2})$$

with $\Omega_n = (-1)^{n+1} J J_n(\tilde{A})$ and $\chi_n = n\theta$. Hence, by tuning the amplitude of the driving $\tilde{A}$, the amplitude Ω_n of the Hamiltonian terms $H_0^{(n)}$ can be controlled.

Moreover, Hamiltonians $H_0^{(n)}$ with large detunings $\Delta\omega_n$ oscillate rapidly and therefore can be neglected for a long-term dynamics. Thus, for $\tilde{A} = 1$ and small detuning $\Delta\omega_1 \ll \omega_B$, the hopping term Hamiltonian can be reduced to two terms, i.e.,

$$\begin{aligned} \sum_{n=-\infty}^{\infty} H_0^{(n)} &\approx H_0^{(0)} + H_0^{(1)} \approx -J \sum_{l=1}^{N_s} [(J_0(\tilde{A}) e^{i\omega_B t} \\ &\quad - J_1(\tilde{A}) e^{i(\Delta\omega_1 t - \theta)}) \hat{a}_{l+1}^\dagger \hat{a}_l + \text{H.c.}]. \end{aligned} \quad (\text{A3})$$

Since $\Delta\omega_1 \ll \omega_B$, the evolution operator over one Bloch period T_B may be approximated by

$$U(T_B) \simeq \mathcal{T} \exp \left[-i \int_0^{T_B} (H_0^{(0)}(t) + H_0^{(1)}(t)) dt \right]. \quad (\text{A4})$$

APPENDIX B: BOGOLIUBOV APPROXIMATION

To obtain approximate analytical expressions in the weakly interacting regime, we assume that all particles initially occupy the zero-momentum condensate state and adopt a mean-field approximation. Although the mean-field approximation is strictly valid only in the large-particle-number limit, it can still provide qualitative insight into the resonant behavior of the supercurrent and the role of interatomic interactions. Therefore, the expressions derived in this Appendix are intended only as a qualitative description, complementary to the exact numerical calculations presented in the main text.

Within the mean-field approximation, the Bose-Hubbard Hamiltonian in momentum space can be written as

$$\hat{H}_{\text{MF}} = E_{\text{MF}} + \hat{H}_B, \quad (\text{B1})$$

where E_{MF} denotes the condensate energy and

$$\begin{aligned} \hat{H}_B &= \sum_k \xi_k(t) \hat{b}_k^\dagger \hat{b}_k + \frac{UN}{2N_s} \sum_k (\hat{b}_k^\dagger \hat{b}_{-k}^\dagger + \text{H.c.}), \\ k &= \frac{2\pi m}{N_s} \quad (m = 1, \dots, N_s - 1), \end{aligned} \quad (\text{B2})$$

is the Bogoliubov Hamiltonian describing excitations above the condensate. The single-particle dispersion entering the

Bogoliubov spectrum is given by

$$\xi_k(t) = \epsilon_k(t) - \epsilon_0 + UN/N_s, \quad (\text{B3})$$

where N is the total particle number, $\epsilon_k(t) = -2J \cos[k + \phi(t)/N_s]$, and ϵ_0 denotes the zero-momentum single-particle energy prior to the onset of the driving.

The time-dependent supercurrent density is given by

$$I(t) = 2\omega_J \sin \left[\frac{\phi(t)}{N_s} \right] n_0(t), \quad (\text{B4})$$

where $\omega_J = J/\hbar$ and $n_0(t)$ denotes the time-dependent condensate density. Conservation of the total particle number N implies

$$n_0(t) + \sum_{k \neq 0} n_k(t) = \frac{N}{N_s}, \quad (\text{B5})$$

where $n_{k \neq 0}(t)$ represents the particle density in the excited momentum states. Moreover, assuming that all particles initially occupy the zero-momentum condensate state leads to the initial condition $n_k(0) = \delta_{k,0}$, N/N_s . Using these relations, Eq. (B4) can be recast as

$$I(t) = \frac{2\omega_J N}{N_s} \sin \left[\frac{\phi(t)}{N_s} \right] \left(1 - \frac{1}{N} \sum_{k \neq 0} N_k(t) \right), \quad (\text{B6})$$

where $N_k(t)$ denotes the time-dependent occupation of the momentum state k . In the following, we employ the nonequilibrium Green's function (GF) formalism to compute $N_k(t)$.

Using Eq. (B2), we find the equation of motion for the field operator $\hat{\Psi}_k = (\hat{b}_k, \hat{b}_k^\dagger)^T$,

$$i\hbar \frac{\partial \hat{\Psi}_k}{\partial t} = \mathcal{H}_k(t) \hat{\Psi}_k, \quad (\text{B7})$$

where

$$\mathcal{H}_k(t) = \begin{pmatrix} \xi_k(t) & U \frac{N}{N_s} \\ -U \frac{N}{N_s} & -\xi_{-k}(t) \end{pmatrix}. \quad (\text{B8})$$

We can use the field operators to, respectively, build the retarded and lesser GF matrices,

$$\begin{aligned} \check{G}^R(k, t, t') &= -i\Theta(t - t') \langle [\hat{\Psi}_k(t), \hat{\Psi}_k^\dagger(t')] \rangle, \\ \check{G}^<(k, t, t') &= -i \langle \hat{\Psi}_k^\dagger(t') \otimes \hat{\Psi}_k(t) \rangle, \end{aligned} \quad (\text{B9})$$

where the commutator is taken with respect to the tensor product, i.e., $[A, B]_\otimes = A \otimes B - B \otimes A$.

The equation of motion for the retarded GF at $t' = 0$ is found to be

$$\frac{\partial \check{G}^R(k, t, 0)}{\partial t} = -i\delta(t)\sigma_z - \frac{i}{\hbar} \mathcal{H}_k \check{G}^R(k, t, 0), \quad (\text{B10})$$

whose formal solution is

$$\check{G}^R(k, t, 0) = -i\Theta(t) \hat{T} \left\{ e^{-\frac{i}{\hbar} \int_0^t \mathcal{H}_k(s) ds} \right\} \sigma_z, \quad (\text{B11})$$

where $\hat{T}$ is the time-ordering operator and σ_z is the z -Pauli matrix. To obtain an approximate analytical expression for the

retarded GF, one can use Magnus expansion,

$$\begin{aligned} \hat{T} \left\{ e^{-\frac{i}{\hbar} \int_0^t \mathcal{H}_k(s) ds} \right\} &\approx \exp \left\{ -\frac{i}{\hbar} \int_0^t ds_1 \mathcal{H}_k(s_1) - \frac{1}{2\hbar^2} \int_0^t ds_1 \right. \\ &\quad \times \left. \int_0^{s_1} ds_2 [\mathcal{H}_k(s_1), \mathcal{H}_k(s_2)] + \dots \right\}. \end{aligned} \quad (\text{B12})$$

Keeping only the first term in the expansion and after some algebraic manipulations, we obtain

$$\check{G}^R(k, t, 0) \approx -i\Theta(t) e^{i\omega_k t} \left[\cos(\gamma_k t) \sigma_0 + \frac{1}{\gamma_k t} \sin(\gamma_k t) M \right], \quad (\text{B13})$$

where σ_0 is the identity matrix, $\gamma_k = \sqrt{(\Omega_k + \omega_U)^2 - \omega_U^2}$, and

$$M = -i(\Omega_k + \omega_U)t\sigma_z + (\omega_U t)\sigma_y. \quad (\text{B14})$$

The frequency associated with the interaction energy is $\omega_U = UN/(\hbar N_s)$, while

$$\begin{aligned} \omega_k &= 2\omega_J \sin(k) \sum_{n=-\infty}^{\infty} (-1)^n J_n(\tilde{A}) \\ &\quad \times \left[\frac{\cos(\Delta\omega_n t - n\theta) - \cos(n\theta)}{\Delta\omega_n t} \right], \end{aligned} \quad (\text{B15})$$

$$\begin{aligned} \Omega_k &= 2\omega_J \left\{ 1 - \cos(k) \sum_{n=-\infty}^{\infty} (-1)^n J_n(\tilde{A}) \right. \\ &\quad \times \left. \left[\frac{\sin(\Delta\omega_n t - n\theta) + \sin(n\theta)}{\Delta\omega_n t} \right] \right\}, \end{aligned} \quad (\text{B16})$$

and $\omega_J = J/\hbar$.

We can then use the retarded GF matrix to calculate the lesser GF from the relation,

$$\begin{aligned} \check{G}^<(k, t, t') &= \check{G}^R(k, t, 0) \sigma_z \check{G}^<(k, 0, 0) \sigma_z \check{G}^A(k, 0, t') \\ &= \check{G}^R(k, t, 0) \sigma_z \check{G}^<(k, 0, 0) \sigma_z [\check{G}^R(k, 0, t')]^\dagger, \end{aligned} \quad (\text{B17})$$

with the initial condition

$$\begin{aligned} \check{G}^<(k, 0, 0) &= -i \begin{pmatrix} \langle \hat{b}_k^\dagger(0) \hat{b}_k(0) \rangle & \langle \hat{b}_k(0) \hat{b}_k(0) \rangle \\ \langle \hat{b}_k^\dagger(0) \hat{b}_k^\dagger(0) \rangle & \langle \hat{b}_k(0) \hat{b}_k^\dagger(0) \rangle \end{pmatrix} \\ &= -i \begin{pmatrix} N\delta_{k,0} & N\delta_{k,0} \\ N\delta_{k,0} & 1 + N\delta_{k,0} \end{pmatrix}. \end{aligned} \quad (\text{B18})$$

The particle number in a state with momentum k can then be obtained as

$$N_k(t) = \langle \hat{b}_k^\dagger(t) \hat{b}_k(t) \rangle = i\check{G}_{11}^<(k, t, t), \quad (\text{B19})$$

which after using Eqs. (B13) and (B17) reduces to

$$N_{k \neq 0}(t) \approx \Theta(t) \frac{\omega_U^2}{2\Omega_k(\Omega_k + 2\omega_U)} [1 - \cos(2\gamma_k t)]. \quad (\text{B20})$$

Finally, the current can be computed by substituting Eq. (B20) into Eq. (B6).

In the near-resonant regime, one can neglect the rapid time oscillations of the contributions to Ω_k when evaluating the time-averaged current. Consequently, in the vicinity of the m th resonance ($\Delta\omega_m = \omega_B - m\omega \approx 0$), Ω_k and γ_k can be

replaced by their respective approximations,

$$\Omega_{km} = 2\omega_J[1 - (-1)^m J_m(\tilde{A}) \cos(k) \cos(m\theta)],$$

$$\gamma_{km} = \sqrt{(\Omega_{km} + \omega_U)^2 - \omega_U^2}. \quad (\text{B21})$$

Within this approximation, the total time-averaged supercurrent near the m th resonance becomes

$$\frac{\langle I \rangle_\tau^U}{I_0} = \frac{\langle I \rangle_\tau}{I_0} \left(N - \frac{1}{2} \sum_{k \neq 0} \frac{\omega_U^2}{\gamma_{km}^2} \right) + \frac{(-1)^m}{4} J_m(\tilde{A}) \sum_{k \neq 0} \sum_{s=\pm 1} \left\{ \frac{\omega_U^2}{\gamma_{km}^2} \text{sinc} \left[\frac{(\Delta\omega_m + 2s\gamma_{km})\tau}{2} \right] \sin \left[\frac{(\Delta\omega_m + 2s\gamma_{km})\tau}{2} - m\theta \right] \right\}. \quad (\text{B22})$$

Notably, in the limit $\omega_U \rightarrow 0$ (i.e., $U \rightarrow 0$), Eq. (B22) reduces to the time-averaged current of the noninteracting case [see Eq. (14)].

-
- [1] C. L. Degen, F. Reinhard, and P. Cappellaro, Quantum sensing, *Rev. Mod. Phys.* **89**, 035002 (2017).
 - [2] R. Geiger, A. Landragin, S. Merlet, and F. Pereira Dos Santos, High-accuracy inertial measurements with cold-atom sensors, *AVS Quantum Sci.* **2**, 024702 (2020).
 - [3] C. Salducci, Y. Bidel, M. Cadoret, S. Darmon, N. Zahzam, A. Bonnin, S. Schwartz, C. Blanchard, and A. Bresson, Quantum sensing of acceleration and rotation by interfering magnetically launched atoms, *Sci. Adv.* **10**, eadq4498 (2024).
 - [4] G. Pelegrí, J. Mompart, and V. Ahufinger, Quantum sensing using imbalanced counter-rotating Bose–Einstein condensate modes, *New J. Phys.* **20**, 103001 (2018).
 - [5] C. Sabín, D. E. Bruschi, M. Ahmadi, and I. Fuentes, Phonon creation by gravitational waves, *New J. Phys.* **16**, 085003 (2014).
 - [6] L. Morales-Molina and E. Arévalo, Quantum sensing of matter waves using BEC oscillations, *Results Phys.* **41**, 105923 (2022).
 - [7] S. Pötting, M. Cramer, and P. Meystre, Momentum-state engineering and control in Bose-Einstein condensates, *Phys. Rev. A* **64**, 063613 (2001).
 - [8] C. A. Weidner, H. Yu, R. Kosloff, and D. Z. Anderson, Atom interferometry using a shaken optical lattice, *Phys. Rev. A* **95**, 043624 (2017).
 - [9] C. A. Weidner and D. Z. Anderson, Experimental demonstration of shaken-lattice interferometry, *Phys. Rev. Lett.* **120**, 263201 (2018).
 - [10] R. Karcher, A. Imanaliev, S. Merlet, and F. P. D. Santos, Improving the accuracy of atom interferometers with ultracold sources, *New J. Phys.* **20**, 113041 (2018).
 - [11] L.-Y. Chih and M. Holland, Reinforcement-learning-based matter-wave interferometer in a shaken optical lattice, *Phys. Rev. Res.* **3**, 033279 (2021).
 - [12] L.-Y. Chih and M. Holland, Reinforcement learning for rotation sensing with ultracold atoms in an optical lattice, *Phys. Rev. Res.* **6**, 043191 (2024).
 - [13] V. V. Ivanov, A. Alberti, M. Schioppo, G. Ferrari, M. Artoni, M. L. Chiofalo, and G. M. Tino, Coherent delocalization of atomic wave packets in driven lattice potentials, *Phys. Rev. Lett.* **100**, 043602 (2008).
 - [14] M. G. Tarallo, A. Alberti, N. Poli, M. L. Chiofalo, F.-Y. Wang, and G. M. Tino, Delocalization-enhanced Bloch oscillations and driven resonant tunneling in optical lattices for precision force measurements, *Phys. Rev. A* **86**, 033615 (2012).
 - [15] B. Konrad and M. Efremov, Angular Bloch oscillations and their applications, *Quantum Sci. Technol.* **10**, 015025 (2025).
 - [16] E. Haller, R. Hart, M. J. Mark, J. G. Danzl, L. Reichsöllner, and H.-C. Nägerl, Inducing transport in a dissipation-free lattice with super Bloch oscillations, *Phys. Rev. Lett.* **104**, 200403 (2010).
 - [17] L. Amico, M. Boshier, G. Birkel, A. Minguzzi, C. Miniatura, L.-C. Kwek, D. Aghamalyan, V. Ahufinger, D. Anderson, N. Andrei *et al.*, Roadmap on atomtronics: State of the art and perspective, *AVS Quantum Sci.* **3**, 039201 (2021).
 - [18] L. Amico, D. Anderson, M. Boshier, J.-P. Brantut, L.-C. Kwek, A. Minguzzi, and W. von Klitzing, *Colloquium: Atomtronic circuits: From many-body physics to quantum technologies*, *Rev. Mod. Phys.* **94**, 041001 (2022).
 - [19] L. Amico, A. Osterloh, and F. Cataliotti, Quantum many particle systems in ring-shaped optical lattices, *Phys. Rev. Lett.* **95**, 063201 (2005).
 - [20] J. Polo, W. Chetcuti, T. Haug, A. Minguzzi, K. Wright, and L. Amico, Persistent currents in ultracold gases, *Phys. Rep.* **1137**, 1 (2025).
 - [21] D. Spohner, L. Morales-Molina, and S. A. Reyes, Persistent currents in Bose–Bose mixtures after an interspecies interaction quench, *New J. Phys.* **23**, 063025 (2021).
 - [22] M. Cominotti, D. Rossini, M. Rizzi, F. Hekking, and A. Minguzzi, Optimal persistent currents for interacting bosons on a ring with a gauge field, *Phys. Rev. Lett.* **113**, 025301 (2014).
 - [23] S. S. Szigeti, S. P. Nolan, J. D. Close, and S. A. Haine, High-precision quantum-enhanced gravimetry with a Bose-Einstein condensate, *Phys. Rev. Lett.* **125**, 100402 (2020).
 - [24] S. Franke-Arnold, J. Leach, M. J. Padgett, V. E. Lembessis, D. Ellinas, A. J. Wright, J. M. Girkin, P. Öhberg, and A. S. Arnold, Optical Ferris wheel for ultracold atoms, *Opt. Express* **15**, 8619 (2007).
 - [25] K. Viebahn, Introduction to Floquet theory, Institute for Quantum Electronics, ETH Zurich 8093, 2020, http://utff.mff.cuni.cz/~cizek/Kvantovka_appl/floquetlecture_Konrad%20Viebahn.pdf.
 - [26] R. Peierls, Zur Theorie des Diamagnetismus von Leitungselektronen, *Z. Phys.* **80**, 763 (1933).

- [27] R. Roth and K. Burnett, Superfluidity and interference pattern of ultracold bosons in optical lattices, *Phys. Rev. A* **67**, 031602(R) (2003).
- [28] D. Jaksch and P. Zoller, Creation of effective magnetic fields in optical lattices: The Hofstadter butterfly for cold neutral atoms, *New J. Phys.* **5**, 56 (2003).
- [29] J. Struck, C. Ölschläger, M. Weinberg, P. Hauke, J. Simonet, A. Eckardt, M. Lewenstein, K. Sengstock, and P. Windpassinger, Tunable gauge potential for neutral and spinless particles in driven optical lattices, *Phys. Rev. Lett.* **108**, 225304 (2012).
- [30] Y.-J. Lin, R. L. Compton, K. Jiménez-García, J. V. Porto, and I. B. Spielman, Synthetic magnetic fields for ultracold neutral atoms, *Nature (London)* **462**, 628 (2009).
- [31] J. Dalibard, F. Gerbier, G. Juzeliūnas, and P. Öhberg, *Colloquium*: Artificial gauge potentials for neutral atoms, *Rev. Mod. Phys.* **83**, 1523 (2011).
- [32] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Feshbach resonances in ultracold gases, *Rev. Mod. Phys.* **82**, 1225 (2010).
- [33] L. Pezzè, K. Xhani, C. Daix, N. Grani, B. Donelli, F. Scazza, D. Hernandez-Rajkov, W. J. Kwon, G. Del Pace, and G. Roati, Stabilizing persistent currents in an atomtronic Josephson junction necklace, *Nat. Commun.* **15**, 4831 (2024).
- [34] J. Zhang and R. Dong, Exact diagonalization: The Bose–Hubbard model as an example, *Eur. J. Phys.* **31**, 591 (2010).
- [35] D. Raventós, T. Graß, M. Lewenstein, and B. Juliá-Díaz, Cold bosons in optical lattices: A tutorial for exact diagonalization, *J. Phys. B: At. Mol. Opt. Phys.* **50**, 113001 (2017).
- [36] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes: The Art of Scientific Computing*, 3rd ed. (Cambridge University Press, Cambridge, England, 2007).
- [37] M. Suzuki, General theory of higher-order decomposition of exponential operators and symplectic integrators, *Phys. Lett. A* **165**, 387 (1992).
- [38] L. Morales-Molina, S. A. Reyes, and M. Orszag, Current and entanglement in a three-site Bose-Hubbard ring, *Phys. Rev. A* **86**, 033629 (2012).
- [39] G. Arwas, A. Vardi, and D. Cohen, Triangular Bose-Hubbard trimer as a minimal model for a superfluid circuit, *Phys. Rev. A* **89**, 013601 (2014).
- [40] C. Weitenberg and J. Simonet, Tailoring quantum gases by Floquet engineering, *Nat. Phys.* **17**, 1342 (2021).
- [41] A. Eckardt, *Colloquium*: Atomic quantum gases in periodically driven optical lattices, *Rev. Mod. Phys.* **89**, 011004 (2017).
- [42] N. Goldman and J. Dalibard, Periodically driven quantum systems: Effective Hamiltonians and engineered gauge fields, *Phys. Rev. X* **4**, 031027 (2014).
- [43] D. O. Krimmer, R. Khomeriki, and S. Flach, Delocalization and spreading in a nonlinear Stark ladder, *Phys. Rev. E* **80**, 036201 (2009).
- [44] D. Witthaut, M. Werder, S. Mossmann, and H. J. Korsch, Bloch oscillations of Bose-Einstein condensates: Breakdown and revival, *Phys. Rev. E* **71**, 036625 (2005).
- [45] A. R. Kolovsky and H. J. Korsch, Bloch oscillations of cold atoms in optical lattices, *Int. J. Mod. Phys. B* **18**, 1235 (2004).
- [46] A. Rubio-Abadal, M. Ippoliti, S. Hollerith, D. Wei, J. Rui, S. L. Sondhi, V. Khemani, C. Gross, and I. Bloch, Floquet prethermalization in a Bose-Hubbard system, *Phys. Rev. X* **10**, 021044 (2020).
- [47] A. E. Leanhardt, T. A. Pasquini, M. Saba, A. Schirotzek, Y. Shin, D. Kielpinski, D. E. Pritchard, and W. Ketterle, Cooling Bose-Einstein condensates below 500 picokelvin, *Science* **301**, 1513 (2003).