Relativistic quantum mechanics of charged vortex particles accelerated in a uniform electric field

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The relativistic quantum-mechanical description of a charged Laguerre-Gauss beam accelerated in a uniform electric field has been fulfilled. Stationary wave eigenfunctions are rigorously derived. The evolution of the beam parameters during acceleration is considered in detail. The practically important effect of extraordinary suppression of transverse spreading of the beam is discovered, carefully analyzed, and properly explained. Our results provide direct evidence that vortex particle beams can be accelerated without destroying their intrinsic vortex properties, paving the way for high-energy vortex beam applications.

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I. INTRODUCTION

Vortex (twisted) electrons carrying the orbital angular momentum (OAM) have numerous potential applications in fundamental and applied physics (see Refs. [1–10] and references therein). However, a potential of application of vortex electrons and other vortex particles significantly depends on their energy ranges. Therefore, a successful acceleration of vortex beams would be a very promising achievement. The simplest acceleration can be performed by a uniform electric field [11]. For this purpose, an axisymmetric electromagnetic lens can also be used [12]. The injection and acceleration of electrons can also be carried out in a twisted laser beam [13–15]. Production of high-energy particle beams carrying the OAM is now one of the most actual problems of physics of charged vortex particles [16–21]. However, the acceleration of *relativistic* particles is mostly studied with the Schrödinger equation [11,12,14] and the Dirac equation has been used only in Ref. [15]. Unfortunately, the last investigation is based on the non-normalizable Volkov-Bessel wave function. In contrast, we use the well-known normalizable Laguerre-Gauss (LG) wave function and fulfill a high-precision relativistic quantum-mechanical description of the acceleration of charged vortex particles needed for designing and implementing present and future [13–15,22] high-energy physics experiments.

We use the system of units $\hbar = 1,\ c = 1$, with $\hbar$ and c explicitly included when needed for clarity.

To perform a rigorous quantum-mechanical analysis, we use the initial Dirac-Pauli Hamiltonian for a charged particle in a uniform electric field collinear to the z axis:

$$\mathcal{H} = \beta m + \mathcal{E} + \mathcal{O}, \quad \mathcal{E} = e\Phi,$$
$$\mathcal{O} = c\boldsymbol{\alpha}\cdot\boldsymbol{p} + i\mu'\boldsymbol{\gamma}\cdot\boldsymbol{E}, \quad \Phi = \Phi_0 - E_z z. \tag{1}$$

Here, m is the particle rest mass, μ' is an anomalous magnetic moment, and standard denotations of the Dirac matrices (see Ref. [23]) are applied. A particle is accelerated along the z axis when $eE_z = |eE_z| > 0$.

Our analysis not only is applicable for an infinite space with a uniform electric field but also covers the transition of a charged particle beam from the free space or a solenoid with a uniform magnetic field to the electric field region. In the latter case, we use the "hard-edge" approximation and suppose that there is the sharp boundary between two media at $z = 0$. As follows from quantum mechanics, the wave function and its first derivative should be continuous on the boundary. In particular, the beam width and its first derivative, $w(0)$ and $w'(0)$, should be continuous.

To obtain a clear Schrödinger picture of relativistic quantum mechanics, it is convenient to fulfill the relativistic Foldy-Wouthuysen (FW) transformation [24–28]. The weak-field approximation is applicable when all terms in the FW Hamiltonian containing commutators of $\mathcal{E}$ are much less than mc^2. When $[\mathcal{O},\mathcal{E}] = 0$, the FW transformation is exact [24]. Therefore, the term $\mathcal{E}$ is not covered by the weak-field approximation. When this approximation is applied, the relativistic FW transformation for a particle in a uniform electric field results in [24,28]

$$\frac{\partial\psi_{\rm FW}}{\partial t} = \mathcal{H}_{\rm FW}\psi_{\rm FW},$$
$$\mathcal{H}_{\rm FW} = \beta\epsilon + e\Phi + \left(\frac{\mu_0 m}{\epsilon + m} + \mu'\right)\frac{1}{\epsilon}[\boldsymbol{\Sigma}\cdot(\boldsymbol{p}\times\boldsymbol{E})], \tag{2}$$

where $\mu_0 = e\hbar/(2m)$ is the Dirac magnetic moment and $\epsilon = \sqrt{m^2 + \boldsymbol{p}^2}$.

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In the present study, we consider stationary states ($\mathcal{H}_{\rm FW}\psi_{\rm FW} = \mathbb{E}_0\psi_{\rm FW}$). Such states can be realized only in stationary external fields. Any vortex beam is a continuum of coherent partial de Broglie waves with the same total energy. In stationary states, the total energy is constant and the same for any partial wave. Any additional nonuniform static electric and magnetic fields (e.g., fringe fields) do not change the *total* particle energy, and therefore the coherence of the beam is not violated. As a result, stationary LG beams remain coherent in any stationary electric, magnetic, and gravitational fields. Certainly, even weak additional fields can influence the wave function and the transverse shape of the beam. We do not consider nonstationary LG beams studied in Refs. [29,30].

II. RELATIVISTIC QUANTUM MECHANICS OF LG BEAMS IN A UNIFORM ELECTRIC FIELD

For any partial beam in a uniform electric field, the spin is quantized along the axis normal to the vectors $\boldsymbol{E}$ and $\boldsymbol{p}$. For a vortex beam, the spin quantization is only approximate and only in the case of $L_z \gg \hbar$. In this case, the spin quantization axis is radial ($s_r \approx \pm 1/2$). The states with $s_r \approx \pm 1/2$ are mixed due to a nonzero radial particle momentum.

As a rule, practically used vortex beams satisfy the paraxial approximation ($|\boldsymbol{p}_\perp| \ll |p_z|$). In this case, a transition to the second-order paraxial equation is rather convenient. We can mention that the transition from the first-order to the second-order relativistic quantum-mechanical equation can be fulfilled with any required precision [28,31,32]. In the considered case, we obtain [28]

$$\left[\left(i\frac{\partial}{\partial t} - \mathfrak{E}\right)^2 - \boldsymbol{p}^2 - m^2\right]\psi = 0,$$
$$\mathfrak{E} = e\Phi + \left(\frac{\mu_0 m}{\epsilon + m} + \mu'\right)\frac{1}{\epsilon}[\boldsymbol{\Sigma}\cdot(\boldsymbol{p}\times\boldsymbol{E})]. \tag{3}$$

For stationary solutions,

$$[(\mathbb{E}_0 - \mathfrak{E})^2 - \mathbf{p}^2 - m^2]\psi = 0,$$
$$\mathbb{E}_0 = \epsilon_0 + e\Phi_0 = \sqrt{m^2 + p_0^2} + e\Phi_0, \tag{4}$$

where $\mathbb{E}_0$ is the conserved total energy. The spin-dependent term in the formula for $\mathfrak{E}$ is rather small compared with $e\Phi$ and can be neglected ($\mathfrak{E} = e\Phi$). In the paraxial approximation, $\boldsymbol{p}^2 \approx pp_z + \boldsymbol{p}_\perp^2/2$ and Eq. (4) takes the form

$$[2(\mathbb{E}_0 - e\Phi)^2 - (2pp_z + \boldsymbol{p}_\perp^2) - 2m^2]\psi = 0. \tag{5}$$

Evidently,

$$\boldsymbol{p}^2 = (\mathbb{E}_0 - e\Phi)^2 - m^2 = p_0^2 + 2\epsilon_0|eE_z|z + e^2E_z^2z^2. \tag{6}$$

To proceed to the paraxial equation, one needs to introduce the wave numbers $k \equiv k(z) = p/\hbar$ and $k_0 = p_0/\hbar$. Here,

$$k(z) = k_0\sqrt{1 + 2K_1z + K_2^2z^2} \tag{7}$$

and

$$K_1 = \frac{\epsilon_0|eE_z|}{c^2p_0^2}, \quad K_2 = \frac{|eE_z|}{cp_0}. \tag{8}$$

$k(z)$ should not be confused with k_z. $K_1 > K_2$ because $\epsilon_0 = \sqrt{m^2 + p_0^2} > p_0$. The substitution $\psi = \exp[i\int k(z)\,dz]\Psi$ (cf. Refs. [33,34]) into the equation

$$\left(2\boldsymbol{p}^2 + 2i\hbar p\frac{\partial}{\partial z} - \boldsymbol{p}_\perp^2\right)\psi = 0$$

results in

$$\left[\boldsymbol{\nabla}_\perp^2 + 2ik(z)\frac{\partial}{\partial z}\right]\Psi = 0. \tag{9}$$

Unlike the conventional paraxial wave equation in free space, this equation features the wave number k that varies with z. The paraxial equation in free space,

$$\left(\boldsymbol{\nabla}_\perp^2 + 2ik\frac{\partial}{\partial z}\right)\Psi = 0 \tag{10}$$

($k = \text{const}$), has the following solution for LG beams [35–37]:

$$\Psi = \mathbb{A}\exp(i\Phi), \quad \int \Psi^\dagger\Psi r dr d\phi = 1,$$
$$\mathbb{A} = \frac{C_{n\ell}}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|\ell|}L_n^{|\ell|}\left(\frac{2r^2}{w^2(z)}\right)\exp\left(-\frac{r^2}{w^2(z)}\right)\eta,$$
$$\Phi = l\phi + \frac{kr^2}{2R(z)} - \Phi_G(z), \quad C_{n\ell} = \sqrt{\frac{2n!}{\pi(n+|\ell|)!}}, \tag{11}$$

where

$$w(z) = w_0\sqrt{1 + \frac{z^2}{z_R^2}}, \quad R(z) = z + \frac{z_R^2}{z}, \quad z_R = \frac{kw_0^2}{2},$$
$$\Phi_G(z) = N\arctan\left(\frac{z}{z_R}\right), \quad N = 2n + |\ell| + 1, \tag{12}$$

the real functions $\mathbb{A}$ and Φ define the amplitude and phase, k is the beam wave number, w_0 is the beam waist (minimum beam width), $R(z)$ is the radius of curvature of the wave front, $\Phi_G(z)$ is the Gouy phase, z_R is the Rayleigh diffraction length, $L_n^{|\ell|}$ is the generalized Laguerre polynomial, and $n = 0, 1, 2, \ldots$ is the radial quantum number. The spin function η is an eigenfunction of the Pauli operator σ_z: $\sigma_z\eta^\pm = \pm\eta^\pm$, $\eta^+ = \binom{1}{0}$, $\eta^- = \binom{0}{1}$. Ψ is a spinor and, evidently, is not an eigenfunction of the operator p_z. Therefore, the free-space wave function Eqs. [(11) and (12)] characterizes a beam formed by partial waves with different p_z.

The free-space LG beam is perfectly symmetric. When the beam waist is shifted relative to the point $z = 0$ and $w(0) = w_0$, $w'(0) = w_0'$, the focus position z_f and the beam waist W_0 are defined by

$$X = \frac{z_f}{z_R} = \frac{kw_0w_0'}{2}, \quad W_0 = \frac{w_0}{\sqrt{1+X^2}}, \quad z_R = \frac{kW_0^2}{2}. \tag{13}$$

In the considered case, the quantities X, z_f, and w_0' are positive and negative for real and imaginary focuses, respectively.

Our derivation shows that the eigenfunction of the LG beam in the uniform electric field has the form (11) by replacing $k \to k_0$ in the phase Φ. However, the beam parameters are not defined by Eq. (12). It is obtained in Sec. I of the

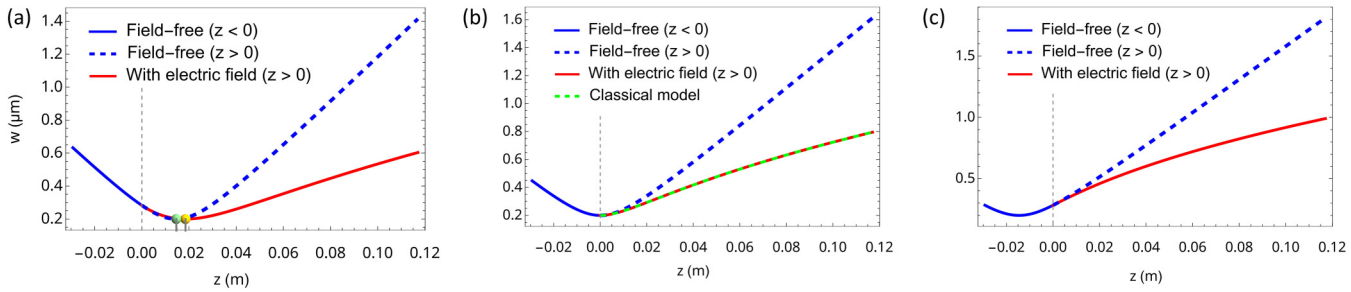


FIG. 1. Evolution of the beam width $w(z)$ under a uniform electric field (1.4 MV/m) for initial conditions (a) $w_0' = -2/k_0 w_0$, (b) $w_0' = 0$, and (c) $w_0' = 2/k_0 w_0$. The initial kinetic energy is 20 keV and the free-space LG beam waist is $W_0 = 200$ nm. In panel (a), the yellow sphere marks the analytical minimum predicted by Eqs. (22) and (23), and the green sphere indicates the free-space imaginary focus. The two short gray bars indicate the corresponding minima extracted numerically from the data, which are in good agreement with the analytical prediction.

Supplemental Material [38] that they satisfy the relations

$$\frac{k_0}{R(z)} = \frac{k(z)w'(z)}{w(z)}, \tag{14a}$$

$$\left[\frac{ik_0}{R(z)} - \frac{2}{w(z)^2}\right]^2 + ik(z)\left[\frac{ik_0}{R(z)} - \frac{2}{w(z)^2}\right]' = 0, \tag{14b}$$

$$\Phi_G'(z) = \frac{2(2n + |\ell| + 1)}{k(z)w(z)^2}. \tag{14c}$$

Setting $f(z) = \frac{ik_0}{R(z)} - \frac{2}{w(z)^2}$ leads Eq. (14b) to the form

$$f(z)^2 + ik(z)f'(z) = 0. \tag{15}$$

Imposing the boundary conditions $w(0) = w_0$, $w'(0) = w_0'$, and using Eq. (14a), we obtain the corresponding expression for $f(0)$:

$$f(0) = \frac{ik_0 w_0'}{w_0} - \frac{2}{w_0^2}.$$

The solution to Eq. (15) for $f(z)$ is given by

$$f(z) = \frac{k_0 K_2 (ik_0 w_0 w_0' - 2)}{k_0 K_2 w_0^2 + 2(2i + k_0 w_0 w_0')A(z)}, \tag{16}$$

where

$$A(z) = \operatorname{arctanh}\left[\frac{K_2}{2K_1 + K_2^2 z}\left(\frac{k(z)}{k_0} - 1\right)\right] \tag{17}$$

and $k(z)$ is given by Eq. (7).

For $z > 0$, the beam is accelerated and the argument of the arctanh function in Eq. (17) lies within the interval (0,1), ensuring that $A(z)$ remains real. The beam width and radius of curvature are then determined by separating the real and imaginary parts of $f(z)$ in Eq. (16):

$$w(z) = w_0\sqrt{\left(1 + \frac{2A(z)w_0'}{K_2 w_0}\right)^2 + \frac{16A(z)^2}{k_0^2 K_2^2 w_0^4}}, \tag{18}$$

$$R(z) = \frac{2A(z)}{K_2} + \frac{k_0^2 w_0^3 [2A(z)w_0' + K_2 w_0]}{k_0^2 w_0^2 w_0' [2A(z)w_0' + K_2 w_0] + 8A(z)}. \tag{19}$$

As shown in Sec. II of the Supplemental Material [38], Eq. (18) can be simplified for $w_0' = 0$:

$$w(z) = w_0\sqrt{1 + \frac{4}{k_0^2 w_0^4 K_2^2}\ln^2\left|\frac{K_1 + K_2^2 z + K_2\sqrt{1 + 2K_1 z + K_2^2 z^2}}{K_1 + K_2}\right|}. \tag{20}$$

Using Eqs. (14c) and (18), the Gouy phase is given by

$$\Phi_G(z) = \Phi_G(0) + N\operatorname{arccot}\left[\frac{k_0 w_0 w_0'}{2} + \frac{k_0 K_2 w_0^2}{4A(z)}\right]. \tag{21}$$

The beam width $w(z)$ and its evolution for various initial conditions are shown in Fig. 1, with the corresponding beam shapes in the xz plane displayed in Fig. 2. In the short-distance limit $|eE_z|z \ll p_0$, corresponding to $K_1 z \ll 1$, $K_2 z \ll 1$, the solution in the presence of the electric field can be reduced to the familiar free-space LG solution, as demonstrated in Sec. III of the Supplemental Material [38]. Beyond this regime, however, the longitudinal field significantly alters the beam dynamics, extraordinarily suppressing its natural spreading and giving rise to distinct focusing behavior that depends sensitively on the initial conditions.

It is proven in Sec. IV of the Supplemental Material [38] that the beam undergoes initial focusing for $w_0' < 0$ [see Fig. 1(b)], reaching a minimum width of

$$w_{\min} = \frac{w_0}{\sqrt{1 + \frac{1}{4}k_0^2 w_0^2 w_0'^2}}. \tag{22}$$

The corresponding focal position is given by

$$z_f = \frac{K_1}{K_2^2}\left[\cosh\left(\frac{k_0^2 K_2 w_0^3 w_0'}{4 + k_0^2 w_0^2 w_0'^2}\right) - 1\right] - \frac{1}{K_2}\sinh\left(\frac{k_0^2 K_2 w_0^3 w_0'}{4 + k_0^2 w_0^2 w_0'^2}\right). \tag{23}$$

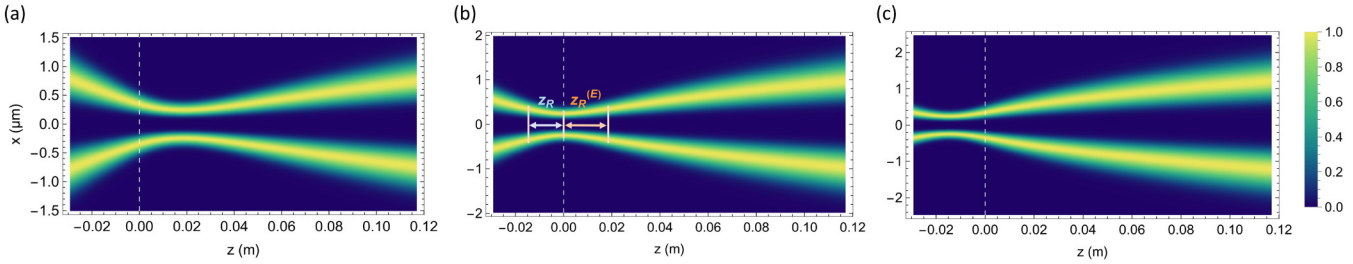


FIG. 2. Beam shapes in the xz plane at $y = 0$, with parameters identical to those in Fig. 1. The example corresponds to $n = 0$, $\ell = -3$. The brightness of color scale is proportional to the probability density $|\Psi(x, y = 0, z)|^2$, normalized to unity at each z plane. In panel (b), the Rayleigh distance in free space, z_R, and the modified Rayleigh distance in the presence of the electric field, $z_R^{(E)}$ [see Eq. (24)], are indicated.

Two focuses can exist for $w_0' \neq 0$. When $w_0' > 0$, the beam has a real focus for its free-space part and can have an imaginary one for an electric-field part, and vice versa for $w_0' < 0$.

An analog of the Rayleigh range in the electric field, defined as the propagation length over which the beam's cross-sectional area doubles from the focus, is derived from Eq. (18). Assuming the focus at $z = 0$, with $w_0' = 0$ and $\pi w(z_R^{(E)})^2 = 2\pi w_0^2$, the Rayleigh range is given by

$$z_R^{(E)} = \frac{2K_1}{K_2^2} \sinh\left(\frac{1}{4}k_0 K_2 w_0^2\right)^2 + \frac{\sinh\left(\frac{1}{2}k_0 K_2 w_0^2\right)}{K_2}. \quad (24)$$

This extended Rayleigh range, as compared to the free-space case, is illustrated in Fig. 2(b).

III. THE CLASSICAL MODEL FOR LG BEAMS IN A UNIFORM ELECTRIC FIELD

Calculated beam behavior can be properly explained in the framework of classical particle physics. A paraxial partial de Broglie wave characterized by transverse momentum $\boldsymbol{p}_\perp$ and total momentum $p \approx p_z$ is equivalent to a particle with the same parameters. At the waist, an LG beam moving in free space can be modeled by a continuum of coherent particle states with identical p_ϕ and p and various ϕ. The probability of particle position azimuth should be uniformly distributed on ϕ. Let r_0 be the distance between the particle and the axis of symmetry of the beam. In the transverse plane, any particle moves freely with constant velocity. At the waist, $v_r = 0$ and the direction x of its movement is orthogonal to $\boldsymbol{r}$: $dx = v_\phi dt$. This direction remains unchanged all the time. After time t, the azimuth of the particle position changes, the radial velocity becomes nonzero, and the distance to the axis of symmetry increases: $r(t) = \sqrt{r_0^2 + v_\phi^2 t^2}$ (see Fig. 3). To check the compatibility with wave theory, it is convenient to determine the connection between r and z. Since $dz = v_z dt \approx v dt$ and $v_\phi/v = p_\phi/p$, we obtain

$$r(z) = \sqrt{r_0^2 + \frac{p_\phi^2 z^2}{p^2}}. \quad (25)$$

Comparison with Eq. (12) shows that the two approaches become equivalent at $r_0 = w_0$, $p_\phi = 2\hbar/w_0$.

If the beam waist is at $z = 0$ and the beam is accelerated at $z \geqslant 0$, then $p = \hbar k(z)$ and p_ϕ remains unchanged. However, v_ϕ decreases in classical and quantum-mechanical pictures because $v_\phi = p_\phi/\epsilon(z)$. In this case, $dx = [p_\phi/\epsilon(z)]\, dt = [p_\phi/p(z)]dz$. After integration, we obtain

$$x = \frac{p_\phi}{p_0}\int_0^z \frac{dz}{\sqrt{1 + 2K_1 z + K_2^2 z^2}} = \frac{p_\phi}{p_0 K_2} \ln\left|\frac{K_1 + K_2^2 z + K_2\sqrt{1 + 2K_1 z + K_2^2 z^2}}{K_1 + K_2}\right|. \quad (26)$$

It is amazing that the same substitution as before, $r_0 = w_0$, $p_\phi = 2\hbar/w_0$, results in Eq. (20) for the beam width.

This simple model perfectly explains a fundamental suppression of transverse spreading of the beam in an electric field, but it is approximate. It does not take into account the real quantum-mechanical structure of the LG state. In particular, there is a radial motion even at the waist and the transverse and longitudinal momenta are not definite. Nevertheless, the model leads to the same Eq. (20), which describes an evolution of the beam width.

IV. DISCUSSION

Vortex beams can enter an electric field not only from free space but also from a solenoid. In this case, $w_0' = 0$ for

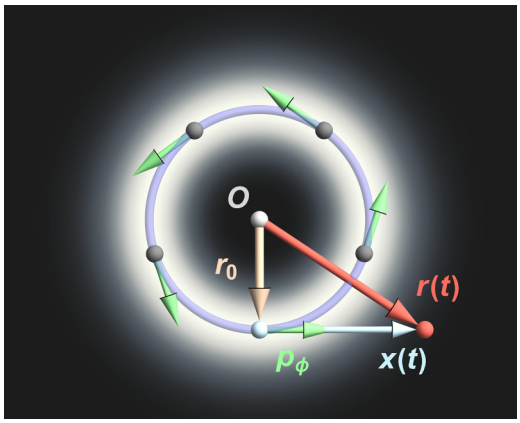


FIG. 3. Simple classical model for an LG beam in a uniform electric field. In the transverse plane, this field does not influence the particle momentum p_ϕ but decreases the particle velocity. The increasing distance $r(t)$ between the particle and the beam symmetry axis O defines transverse spreading of the beam.

a Landau state, but w_0' can be positive and negative for an LG beam with a spatially oscillating width [9,10]. While an electric field extraordinarily suppresses transverse spreading of the beam, the beam focusing can be important. We suppose that it can be performed with magnetic lenses.

Our results can be directly applied to electrostatic linear accelerators (linacs) [39], accelerating electrons by a static and approximately uniform electric field. They also cover rf linacs with a time-varying rf field [14,22]. When the beam motion is synchronized with the rf oscillations, the beam is governed by an approximately static and uniform electric field acting on moving electrons. Further exploration of this regime can be considered as an outlook.

It is known that particles can lose OAMs at their helical motion in a magnetic field followed by the radiation of vortex photons [40–48]. Certainly, the decrease of the particle OAM is equal to the total OAM of radiated photons [40,46]. This decrease is defined by

$$\frac{dL_z}{dt} = (\dot{\boldsymbol{r}} \times \boldsymbol{p} + \boldsymbol{r} \times \dot{\boldsymbol{p}})_z, \quad \dot{\boldsymbol{r}} = \frac{\partial \mathcal{H}}{\partial \boldsymbol{p}} = \frac{\boldsymbol{p}}{\sqrt{m^2 + \boldsymbol{p}^2}},$$
$$\mathcal{H} = \sqrt{m^2 + \boldsymbol{p}^2} + e\Phi, \tag{27}$$

where $\mathcal{H}$ is the quantum-mechanical (and classical) Hamiltonian. Here, $\boldsymbol{p}_\perp = \text{const}$ and $\dot{\boldsymbol{p}} = \boldsymbol{F}_C + \boldsymbol{f}$, where $\boldsymbol{F}_C = eE_z\boldsymbol{e}_z$ is the Coulomb force and $\boldsymbol{f} = \frac{2e^2}{3c^3}\ddot{\boldsymbol{v}}$ [40,48] is the radiative reaction one. Since both the forces are collinear to the z axis, $\frac{dL_z}{dt} = 0$ and the particle OAM is not decreased (see the similar example in Ref. [40]). Thus, radiative transitions changing the longitudinal OAM are forbidden in classical theory. Forbidden transitions are possible in quantum theory of radiation, but, as a rule, their probabilities are rather small. In addition, even radiative transitions in a longitudinal magnetic field that are allowed do not lead to the OAM loss. The calculated effective time of the OAM loss in this field is many orders of magnitude larger than a typical acceleration time [49,50]. Therefore, it can be concluded that a change of the OAM in the accelerating electric field can be neglected.

The radiation of nonvortex photons cannot change the particle OAM and destroy the beam coherence, but it can influence the wave function. The last effect does not occur within the framework of the classical theory of radiation but can appear when the energies of the photon and particle are comparable.

V. CONCLUSIONS

In summary, the practically important problem of acceleration of vortex beams is studied. It is shown that charged particle beams keep their coherence and OAMs in a uniform electric field, despite the emission of photons. Stationary LG wave eigenfunctions of relativistic twisted Dirac fermions accelerated in such a field are rigorously derived in the "hard-edge" approximation. The evolution of the beam in the field is considered in detail. The important effect of extraordinary suppression of transverse spreading of the beam is discovered, carefully analyzed, and appropriately explained. Our results can be successfully used for the acceleration of vortex beams in linacs.

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DATA AVAILABILITY

No data were created or analyzed in this study.

[1] K. Y. Bliokh, I. P. Ivanov, G. Guzzinati, L. Clark, R. Van Boxem, A. Béché, R. Juchtmans, M. A. Alonso, P. Schattschneider, F. Nori, and J. Verbeeck, Theory and applications of free-electron vortex states, Phys. Rep. **690**, 1 (2017).

[2] S. M. Lloyd, M. Babiker, G. Thirunavukkarasu, and J. Yuan, Electron vortices: Beams with orbital angular momentum, Rev. Mod. Phys. **89**, 035004 (2017).

[3] H. Larocque, I. Kaminer, V. Grillo, G. Leuchs, M. J. Padgett, R. W. Boyd, M. Segev, and E. Karimi, "Twisted" electrons, Contemp. Phys. **59**, 126 (2018).

[4] I. P. Ivanov, Promises and challenges of high-energy vortex states collisions, Prog. Part. Nucl. Phys. **127**, 103987 (2022).

[5] I. P. Ivanov, Measuring the phase of the scattering amplitude with vortex beams, Phys. Rev. D **85**, 076001 (2012).

[6] D. V. Karlovets, Electron with orbital angular momentum in a strong laser wave, Phys. Rev. A **86**, 062102 (2012).

[7] I. P. Ivanov, N. Korchagin, A. Pimikov, and P. Zhang, Doing spin physics with unpolarized particles, Phys. Rev. Lett. **124**, 192001 (2020).

[8] L. Zou, P. Zhang, A. J. Silenko, and L. Lu, Recent progress in the physics of twisted particles, Innovation **4**, 100432 (2023).

[9] L. Zou, P. Zhang, and A. J. Silenko, General quantum-mechanical solution for twisted electrons in a uniform magnetic field, Phys. Rev. A **103**, L010201 (2021).

[10] Q. Meng, X. Liu, W. Ma, Z. Yang, L. Lu, A. J. Silenko, P. Zhang, and L. Zou, Generalized Gouy rotation of electron vortex beams in uniform magnetic fields, Phys. Rev. Res. **7**, 023306 (2025).

[11] D. Karlovets, Vortex particles in axially symmetric fields and applications of the quantum Busch theorem, New J. Phys. **23**, 033048 (2021).

[12] S. S. Baturin, D. V. Grosman, G. K. Sizykh, and D. V. Karlovets, Evolution of an accelerated charged vortex particle in an inhomogeneous magnetic lens, Phys. Rev. A **106**, 042211 (2022).

[13] X. Tang, J. Hao, and Y. Shi, Electron injection and acceleration in a twisted laser driven by the light fan, High Power Laser Sci. Eng. **12**, e86 (2024).

[14] A. S. Dyatlov, V. V. Bleko, K. V. Cherepanov, V. V. Kobets, M. A. Martyanov, M. A. Nozdrin, A. N. Sergeev, N. E. Sheremet, A. S. Zhemchugov, and D. V. Karlovets, Status report on modification of JINR UV laser for generating relativistic

vortex electrons, in *Proceedings of the 2024 International Conference Laser Optics (ICLO)* (IEEE, Saint Petersburg, 2024), p. 438.

[15] M. Ababekri, Y. Wang, R.-T. Guo, Z.-P. Li, and J.-X. Li, Dynamics of relativistic vortex electrons in external laser fields, Phys. Rev. A **110**, 052207 (2024).

[16] Z. Li, S. Liu, B. Liu, L. Ji, and I. P. Ivanov, Unambiguous detection of high-energy vortex states via the superkick effect, Phys. Rev. Lett. **133**, 265001 (2024).

[17] Y. Wu, S. Gargiulo, F. Carbone, C. H. Keitel, and A. Pálffy, Dynamical control of nuclear isomer depletion via electron vortex beams, Phys. Rev. Lett. **128**, 162501 (2022).

[18] M. Ababekri, J.-L. Zhou, R.-T. Guo, Y.-Z. Ren, Y.-H. Kou, Q. Zhao, Z.-P. Li, and J.-X. Li, Generation of ultrarelativistic vortex leptons with large orbital angular momenta, Phys. Rev. D **110**, 076024 (2024).

[19] Z.-W. Lu, L. Guo, M. Ababekri, J.-L. Zhang, X.-F. Weng, Y. Wu, Y.-F. Niu, and J.-X. Li, Angular momentum resolved inelastic electron scattering for nuclear giant resonances, Phys. Rev. Lett. **134**, 052501 (2025).

[20] P. Zhao, I. P. Ivanov, and P. Zhang, Decay of the vortex muon, Phys. Rev. D **104**, 036003 (2021).

[21] L. Zou, P. Zhang, and A. J. Silenko, Production of twisted particles in magnetic fields, J. Phys. B: At. Mol. Opt. Phys. **57**, 045401 (2024).

[22] A. S. Dyatlov, A. V. Afanasyev, V. V. Kobets, A. E. Levichev, D. A. Nikiforov, M. A. Nozdrin, K. A. Sibiryakova, K. E. Yunenko, and D. V. Karlovets, Classical and quantum beam dynamics simulation of the RF photoinjector test bench at JINR, arXiv:2509.00732.

[23] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics*, 2nd ed. (Pergamon Press, Oxford, 1982).

[24] A. J. Silenko, Foldy–Wouthuysen transformation for relativistic particles in external fields, J. Math. Phys. **44**, 2952 (2003).

[25] A. J. Silenko, Foldy-Wouthyusen transformation and semiclassical limit for relativistic particles in strong external fields, Phys. Rev. A **77**, 012116 (2008).

[26] A. J. Silenko, Energy expectation values of a particle in nonstationary fields, Phys. Rev. A **91**, 012111 (2015).

[27] A. J. Silenko, General method of the relativistic Foldy-Wouthuysen transformation and proof of validity of the Foldy-Wouthuysen Hamiltonian, Phys. Rev. A **91**, 022103 (2015).

[28] A. J. Silenko, Leading correction to the relativistic Foldy-Wouthuysen Hamiltonian, Phys. Rev. A **111**, 032210 (2025).

[29] G. K. Sizykh, A. D. Chaikovskaia, D. V. Grosman, I. I. Pavlov, and D. V. Karlovets, Nonstationary Laguerre–Gaussian states in a magnetic field, Prog. Theor. Exp. Phys. **2024**, 053A02 (2024).

[30] G. K. Sizykh, A. D. Chaikovskaia, D. V. Grosman, I. I. Pavlov, and D. V. Karlovets, Transmission of vortex electrons through a solenoid, Phys. Rev. A **109**, L040201 (2024).

[31] A. J. Silenko, Transformation of operator equations describing the interaction of relativistic particles with an electric field, Phys. Atom. Nucl. **64**, 977 (2001).

[32] A. J. Silenko, Exact quantum-mechanical equations for particle beams, Mod. Phys. Lett. A **37**, 2250097 (2022).

[33] A. J. Silenko, P. Zhang, and L. Zou, Relativistic quantum-mechanical description of twisted paraxial electron and photon beams, Phys. Rev. A **100**, 030101 (2019).

[34] L. Zou, P. Zhang, and A. J. Silenko, Paraxial wave function and Gouy phase for a relativistic electron in a uniform magnetic field, J. Phys. G: Nucl. Part. Phys. **47**, 055003 (2020).

[35] S. M. Barnett, M. Babiker, and M. J. Padgett, Optical orbital angular momentum, Philos. Trans. R. Soc. A. **375**, 20150444 (2017).

[36] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes, Phys. Rev. A **45**, 8185 (1992).

[37] W. N. Plick and M. Krenn, Physical meaning of the radial index of Laguerre-Gauss beams, Phys. Rev. A **92**, 063841 (2015).

[38] See Supplemental Material at http://link.aps.org/supplemental/10.1103/z6j7-grs2 for additional details of the calculations.

[39] C. S. Epstein, R. Johnston, S. Lee, J. C. Bernauer, R. Corliss, K. Dow, P. Fisher, I. Friščić, D. Hasell, R. G. Milner, P. Moran, S. G. Steadman, Y. Wang, J. Dodge, E. Ihloff, J. Kelsey, C. Vidal, and C. M. Cooke, Measurement of Møller scattering at 2.5 MeV, Phys. Rev. D **102**, 012006 (2020).

[40] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, 4th ed. (Butterworth-Heinemann, Oxford, 1980), pp. 209–210; This edition, unlike the Russian one, contains typos.

[41] M. Katoh, T. Fujimoto, H. Kawaguchi, T. Konomi, K. Miyamoto, N. Nakamura, Y. Takashima, T. Tanaka, and M. Hosaka, Angular momentum of twisted radiation from an electron in spiral motion, Phys. Rev. Lett. **118**, 094801 (2017).

[42] M. Katoh, H. Kawaguchi, T. Fujimoto, T. Konomi, K. Miyamoto, N. Nakamura, Y. Takashima, T. Tanaka, and M. Hosaka, Helical phase structure of radiation from an electron in circular motion, Sci. Rep. **7**, 6130 (2017).

[43] S. Sasaki and I. McNulty, Proposal for generating brilliant x-ray beams carrying orbital angular momentum, Phys. Rev. Lett. **100**, 124801 (2008).

[44] J. Bahrdt, K. Holldack, P. Kuske, R. Müller, M. Scheer, and P. Schmid, First observation of photons carrying orbital angular momentum in undulator radiation, Phys. Rev. Lett. **111**, 034801 (2013).

[45] E. Hemsing, M. Dunning, C. Hast, T. Raubenheimer, and Dao Xiang, First characterization of coherent optical vortices from harmonic undulator radiation, Phys. Rev. Lett. **113**, 134803 (2014).

[46] V. Epp and U. Guselnikova, Angular momentum of radiation from a charge in circular and spiral motion, Phys. Lett. A **383**, 2668 (2019).

[47] T. Kaneyasu, Y. Hikosaka, M. Fujimoto, H. Iwayama, M. Hosaka, E. Shigemasa, and M. Katoh, Observation of an optical vortex beam from a helical undulator in the XUV region, J. Synchrotron Radiat. **24**, 934 (2017).

[48] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, New York, 1998), p. 748.

[49] D. Karlovets, D. Grosman, and I. Pavlov, Angular momentum dynamics of vortex particles in accelerators, arXiv:2507.08763.

[50] G. V. Zmaga, G. K. Sizykh, D. V. Grosman, Q. Meng, L. Zou, P. Zhang, and D. V. Karlovets, Radiation of "breathing" vortex electron packets in magnetic field, arXiv:2509.21195.