

Precision polarization tuning for light shift mitigation in trapped-ion qubits

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Trapped-ion qubits are among the most promising candidates for quantum computing, quantum information processing, and quantum simulation. In general, trapped ions are considered to have sufficiently long coherence times, which are mainly characterized under laser-free conditions. However, in reality, essential laser fields for quantum manipulation introduce residual light shift, which seriously degrades the coherence due to power fluctuations. Here, we present a comprehensive study of ac Stark shifts in the hyperfine energy levels of the $^{171}\text{Yb}^+$ ion, revealing an asymmetric light shift between two circular polarizations in the clock qubit and pronounced vector light shifts in the Zeeman qubits. By precisely tuning these polarizations, a remarkable enhancement in coherence time is observed, reaching over a 100-fold for the clock qubit and more than 10-fold for the Zeeman qubits, when comparing conditions of maximum and minimum shifts. These findings advance the practical realization of scalable trapped-ion quantum processors, enabling deep quantum circuit execution and long-duration adiabatic operations.

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I. INTRODUCTION

Trapped ions represent one of the most promising platforms for quantum computation and simulation, characterized by exceptional coherence times [1,2] and high-fidelity single- and two-qubit gate operations [3–5]. Although such remarkable coherence has predominantly been demonstrated under laser-free conditions, practical entangling operations typically necessitate laser-mediated coupling to collective vibrational modes [6–9]. This laser-induced coupling inevitably introduces residual light shifts if imperfectly compensated. These uncompensated shifts render the coherence times vulnerable to laser power fluctuations, significantly restricting the achievable circuit depth in quantum computations, limiting the feasibility of prolonged adiabatic operations in quantum simulations [10–14] and the long interrogation time for precision quantum metrology [15].

To mitigate these issues, two primary techniques have been developed: magic wavelength [16–19] and magic polarization [20,21]. Magic wavelength eliminates differential light shifts in optical transitions, significantly enhancing the precision and stability of optical lattice clocks [22,23] and dipole traps [24]. Similarly, magic polarization employs a specific laser polarization to cancel the differential light shift,

thereby mitigating Stark-induced dephasing effects in optical trapping [25,26], and enabling precise manipulation of alkali-metal atoms [27]. While magic wavelength and magic polarization techniques are now widely employed in neutral atoms, their implementation and development in trapped ions have received comparatively little attention. With the profound progress in trapped-ion systems for quantum simulation and quantum computing, the significance of mitigating ac Stark shifts has started to gain recognition. Recently, measuring the magic wavelength of $^{40}\text{Ca}^+$ ions has been proposed as a promising strategy for developing high-precision optical clocks [22]. Furthermore, magic polarization has been successfully employed to protect quantum information encoded in $^{133}\text{Ba}^+$ ground-state qubit within the *omg* (optical-metastable-ground state) protocol [28].

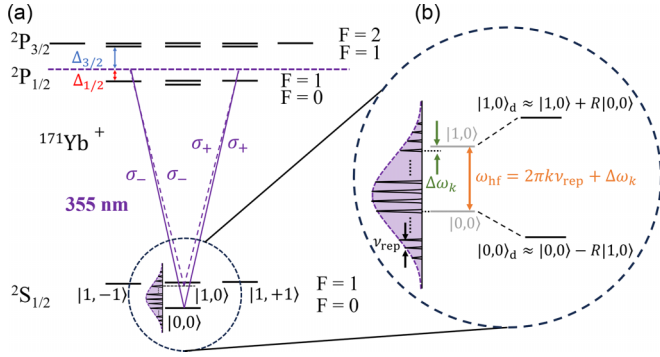
Clock and Zeeman states are widely used as qubits in trapped-ion quantum computing and quantum simulation. Particularly the clock qubits, insensitive to first-order Zeeman effects, have demonstrated extended coherence times for high-precision operations [1,2]. In this study, we investigate the ac Stark shift in the clock and Zeeman states within the $^2S_{1/2}$ manifold of the $^{171}\text{Yb}^+$ ion induced by various polarizations of a 355-nm picosecond pulsed laser [29–31]. We observe an unexpected asymmetric shift between σ_+ and σ_- polarizations in the clock qubit, a phenomenon recently reported only in the context of the *omg* protocol [28], along with vector shifts in the two Zeeman qubits. Based on a detailed characterization of these polarization-dependent ac Stark shift, we identify specific polarization conditions that minimize the differential shifts, which leads to a substantial improvement in coherence time—approximately a 100-fold enhancement for the clock qubit and a 10-fold enhancement for the Zeeman qubit when comparing conditions of maximum and minimum shifts, as

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the qubits become less sensitive to power fluctuations once the Stark shifts are effectively canceled.

II. THEORY FOR POLARIZATION-DEPENDENT STARK SHIFTS

A. Qubit encoding and manipulation

In the $^2S_{1/2}$ ground-state manifold of the $^{171}\text{Yb}^+$ ion, the clock qubit is encoded in the hyperfine states $|F=0, m_F=0\rangle$ and $|F=1, m_F=0\rangle$, which are separated by a hyperfine splitting of $\omega_{\text{hf}} = (2\pi) 12.642\,812$ GHz in the absence of a magnetic field and Zeeman qubits, characterized by transitions with $\Delta m_F = \pm 1$, are typically defined between the states $|F=1, m_F = \pm 1\rangle$ and $|F=0, m_F=0\rangle$, as illustrated in Fig. 1(a). These qubits are coherently manipulated by a 355-nm picosecond pulsed laser, where the frequency resides between the $^2P_{1/2}$ and $^2P_{3/2}$ states, with the detunings of $\Delta_{1/2} = +(2\pi) 33$ THz and $\Delta_{3/2} = -(2\pi) 67$ THz, respectively. We note that these detunings allow the laser to couple both states with comparable strengths. The picosecond pulsed laser generates a frequency comb with a repetition rate ν_{rep} , whose spectral components span the hyperfine splitting, thereby enabling the implementation of both single-qubit and multi-qubit gates via a two-photon Raman process [30–34].

B. Hyperfine clock qubit Stark shift

Our report focuses on characterizing the ac Stark shifts induced by a single 355-nm picosecond pulsed laser, which is critical for achieving high-precision frequency control in quantum gate operations while maintaining qubit coherence. For the clock qubit, second-order differential shifts, denoted as $\delta E_{10,00}^{(2)}$, arise from off-resonant coupling to excited electronic states. Additionally, the frequency comb nature of the

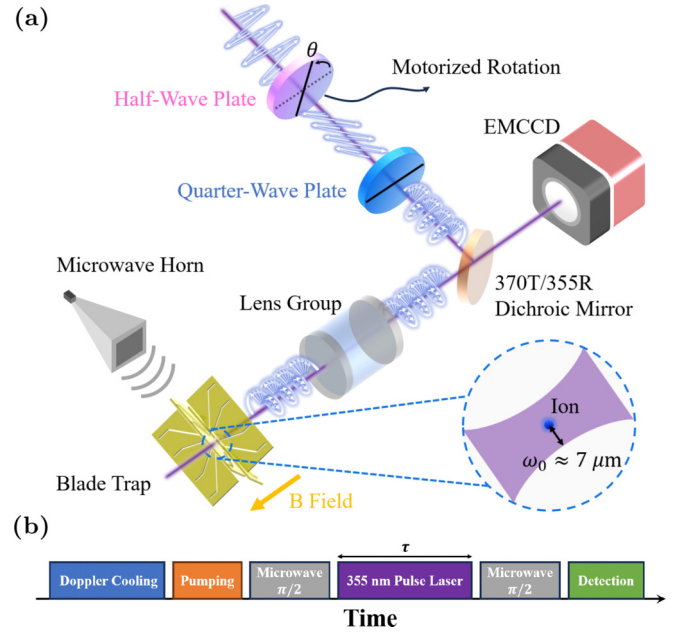


FIG. 2. (a) Diagram of the optical path. Half-wave plate (HWP) and quarter-wave plate (QWP) are used to adjust the polarization of the laser via the motorized rotation with precision of 0.03° , and θ is the relative angle between the HWP and QWP. The 355-nm laser is focused onto the ion with a waist radius (ω_0) of approximately $7\,\mu\text{m}$. (b) The Ramsey measurement sequence for the ac Stark shift measurement is realized by scanning the time interval τ between two $\pi/2$ microwave pulses.

pulsed laser gives rise to a fourth-order ac Stark shift, $\delta E_{10,00}^{(4)}$, determined by the detunings of $\Delta\omega_k = \omega_{\text{hf}} - 2\pi k\nu_{\text{rep}}$, where k indexes the modes of the frequency comb [35], as illustrated in Fig. 1(b). As a result, the total differential shift is the sum of both the second- and fourth-order contributions:

$$\delta E_{10,00}^{\text{total}} = \delta E_{10,00}^{(2)} + \delta E_{10,00}^{(4)}. \quad (1)$$

An external magnetic field $\mathbf{B}$ is typically applied to define the quantization axis. In our setup, $\mathbf{B}$ is aligned along the propagation direction of the 355-nm pulsed laser, thereby suppressing π -polarized transitions as shown in Fig. 2(a). The bare clock states are modified into dressed states with $\mathbf{B}$ [36], as written by

$$\begin{aligned} |1, 0\rangle_d &\approx |1, 0\rangle + \frac{\mu_B B}{w_{\text{hf}}} |0, 0\rangle, \\ |0, 0\rangle_d &\approx |0, 0\rangle - \frac{\mu_B B}{w_{\text{hf}}} |1, 0\rangle, \end{aligned} \quad (2)$$

where $|F, m_F\rangle$ denotes the bare states and $|F, m_F\rangle_d$ represents the corresponding dressed states as illustrated in Fig. 1(b). Magnetic-field-induced hyperfine mixing modifies the second-order differential shift (see the Appendix C for further details), which can be expressed as [31,35,37]

$$\begin{aligned} \delta E_{10,00}^{(2)} &\equiv \Delta E_{10}^{(2)} - \Delta E_{00}^{(2)} = (\epsilon_+^2 + \epsilon_-^2) \frac{\omega_{\text{hf}}}{12} \left(\frac{g_{1/2}^2}{\Delta_{1/2}^2} + \frac{2g_{3/2}^2}{\Delta_{3/2}^2} \right) I \\ &\quad - (\epsilon_+^2 - \epsilon_-^2) \frac{4\mu_B B}{12\omega_{\text{hf}}} \left(\frac{g_{1/2}^2}{\Delta_{1/2}} - \frac{g_{3/2}^2}{\Delta_{3/2}} \right) I, \end{aligned} \quad (3)$$

where $\Delta E_{10}^{(2)}$ and $\Delta E_{00}^{(2)}$ are the second-order ac Stark shifts for the $|1, 0\rangle$ and $|0, 0\rangle$ states, respectively. The laser polarization vector is parameterized as $\hat{\epsilon} = \epsilon_+ \hat{\sigma}_+ + \epsilon_- \hat{\sigma}_- + \epsilon_0 \hat{\pi}$, subject to the normalization condition $|\epsilon_+|^2 + |\epsilon_-|^2 + |\epsilon_0|^2 = 1$. Due to the laser propagating parallel to $\mathbf{B}$, the π -polarized component vanishes, i.e., $\epsilon_0 = 0$. Here, g_J^2 ($J = 1/2, 3/2$) quantifies the coupling strengths to the ^2P_J excited states, μ_B is the Bohr magneton, $I = P/(\pi\omega_0^2)$ denotes the laser intensity, and ω_0 is the beam waist. This result demonstrates that $\delta E_{10,00}^{(2)}$ exhibits a polarization-dependent light shift that is proportional to the magnetic field strength B .

The fourth-order ac Stark shift arises from multiple detuning components $\Delta\omega_k = \omega_{\text{hf}} - 2\pi k\nu_{\text{rep}}$ due to the presence of a frequency comb. The resulting fourth-order differential shift can be expressed as [29,35]

$$\delta E_{10,00}^{(4)} = (\epsilon_+^2 - \epsilon_-^2)^2 \frac{C_{10,00}}{72\Delta\omega_{\min}} \left(\frac{g_{1/2}^2}{\Delta_{1/2}} - \frac{g_{3/2}^2}{\Delta_{3/2}} \right)^2 I^2. \quad (4)$$

Here, $\Delta\omega_{\min}$ is the minimum detuning, defined as $\Delta\omega_{\min} = \min|\omega_{\text{hf}} - 2\pi k\nu_{\text{rep}}| = \omega_{\text{hf}} - 2\pi n\nu_{\text{rep}}$, with n denoting the index corresponding to the nearest comb mode. And the frequency-comb factor $C_{10,00}$ is given by

$$C_{00,10} = \sum_{k=-\infty}^{\infty} \frac{\text{sech}^2((n+k)\pi\nu_{\text{rep}}\tau_p)}{1 - \frac{2\pi k\nu_{\text{rep}}}{\Delta\omega_{\min}}}, \quad (5)$$

where τ_p denotes the pulse duration. In the fourth-order ac Stark shift, the influence of dressed-state corrections becomes the second order of $\mu_B B/\omega_{\text{hf}}$, which results in a negligible amount. In short, the total differential shift of the clock qubit consists of both second- and fourth-order contributions, which are significant under different laser polarizations.

C. Hyperfine Zeeman qubits Stark shifts

For Zeeman qubits, the total differential shift is dominated by the second-order contribution. Since no π -polarized component is present in our configuration, the fourth-order ac Stark shift vanishes. Therefore, the total shift can be written as (see the Appendix C for further details)

$$\delta E_{1\pm 1,00}^{\text{total}} \equiv \Delta E_{1\pm 1}^{(2)} - \Delta E_{00}^{(2)} = \pm(\epsilon_-^2 - \epsilon_+^2) \frac{1}{12} \left(\frac{g_{1/2}^2}{\Delta_{1/2}} - \frac{g_{3/2}^2}{\Delta_{3/2}} \right) I, \quad (6)$$

where $\Delta E_{1\pm 1}$ denote the second-order ac Stark shifts for the $|1, \pm 1\rangle$ states. The terms of $\Delta E_{1\pm 1}$ are usually written in the spherical-tensor formalism [21,38,39]:

$$\Delta E_{1m_F}^{(2)} = \frac{1}{4}|E|^2 \left[\alpha_{nJF}^s + (\epsilon_-^2 - \epsilon_+^2) \alpha_{nJF}^v \frac{m_F}{F} + (3\epsilon_-^2 - 1) \alpha_{nJF}^T \frac{3m_F^2 - F(F+1)}{2F(2F-1)} \right], \quad (7)$$

where α_{nJF}^s , α_{nJF}^v , and α_{nJF}^T are the scalar, vector, and tensor polarizability, respectively. E is the electric-field amplitude and $m_F = \pm 1$. For differential shifts $\delta E_{1\pm 1,00}^{\text{total}}$, the scalar polarizability α_{nJF}^s is negligible, where $\Delta_{1/2}$ and $\Delta_{3/2}$ are much larger than hyperfine splitting ω_{hf} as for 355-nm laser with the $^{171}\text{Yb}^+$ ion. The tensor polarizability α_{nJF}^T is zero for $^2S_{1/2}$ manifold of the $^{171}\text{Yb}^+$ ion [39]. Therefore, the term of

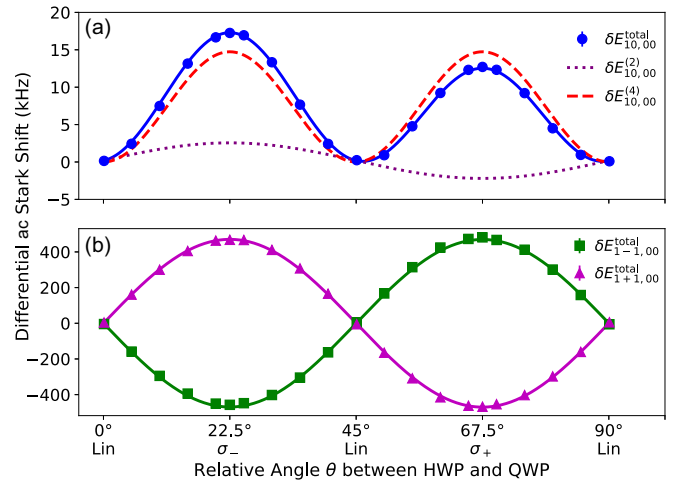


FIG. 3. Differential ac Stark shift as a function of relative angle θ between the HWP and QWP. (a) The clock qubit $\Delta m_F = 0$ (filled circles) exhibits a pronounced asymmetry in ac Stark shifts between σ_+ and σ_- polarizations, with the shift induced by σ_- being larger than that of σ_+ . The solid, dotted, and dashed lines represent total, second-order, and fourth-order differential shifts, respectively, calculated using Eqs. (1), (3), and (4). (b) The Zeeman qubits $\Delta m_F = +1$ (triangles) and $\Delta m_F = -1$ (squares) exhibit shifts over an order of magnitude larger than those observed in the clock states. These shifts demonstrate a clear vector-Stark-shift behavior: $\Delta m_F = +1$ (-1) states experience positive (negative) shifts under σ_- polarization and negative (positive) shifts under σ_+ polarization. The solid curves show theoretical predictions from Eq. (6), with beam width ω_0 fitted to experimental data. Each data point was extracted via Ramsey interferometry. Error bars (standard error) were calculated from 2000 bootstrap resamples, which are smaller than the marker size and thus not visible.

vector polarizability α_{nJF}^v , which shows clearly polarization dependence, is in agreement with Eq. (6).

III. EXPERIMENTAL RESULTS AND ANALYSIS

A. Experimental setup

We experimentally study the polarization-dependent ac Stark shift of a clock qubit and two Zeeman qubits of trapped $^{171}\text{Yb}^+$ ion, induced by 355-nm laser with a pulse duration τ_p of 12.883 ps and a repetition rate ν_{rep} of 118.993 MHz. As shown in Fig. 2(a), the polarization can be controlled using a quarter-wave plate and a half-wave plate. The external magnetic field has a magnitude of 11.343 G, oriented parallel to the propagation direction of the laser. The ac Stark shift is measured using the microwave Ramsey sequence, as illustrated in Fig. 2(b).

B. Polarization-dependent Stark shifts

The measured differential shifts depending on the polarization of the laser are presented in Fig. 3. The polarization is tuned continuously from linear, through circular, and back to linear by rotating the HWP, while keeping the QWP fixed, as illustrated in Fig. 2(a). All data are acquired with a 355-nm beam of fixed power 52.1 mW and a waist radius $\omega_0 = 7 \mu\text{m}$, which is obtained from a fit to experimental results. For the

clock qubit shown in Fig. 3(a), the measured shifts (filled circles) exhibit a clear asymmetry between σ_- and σ_+ polarizations, where σ_- produces a substantially larger shift. The solid, dotted, and dashed lines are mathematical curves based on Eqs. (1), (3), and (4), respectively. This asymmetry arises from the magnetic-field-induced second-order ac Stark shift in Eq. (3). Consequently, the total differential shift cannot be minimized simply by employing an equal superposition of σ_- and σ_+ , which is linear polarization [28]. For the Zeeman qubits ($\Delta m_F = \pm 1$) [shown in Fig. 3(b)], the data display the characteristic vector-Stark-shift behavior, which are in good agreement with theoretical expectation of Eq. (6), where two qubits experience equal-magnitude shifts of opposite sign for a given polarization. Moreover, their maximum shifts are nearly an order of magnitude larger than those of the clock qubit.

C. Intensity dependence of Stark shifts

To identify the characteristics of differential shifts, we measure the intensity dependence of light shift for σ_- ($\theta = 22.50^\circ$) and σ_+ ($\theta = 67.50^\circ$) polarizations. For the clock qubit, where fourth-order terms dominate, the shifts display the expected quadratic scaling with intensity as shown in Fig. 4(a). When subtracting the differential shifts of σ_- and σ_+ polarizations, denoted as $\mathcal{D}$, we observe a linear growth of $\mathcal{D}$ with intensity. Since it equals twice the second-order contribution predicted by Eq. (3), it is direct evidence that the residual asymmetry originates from the second-order ac Stark shift. In contrast, the Zeeman qubits are governed entirely by second-order effects, yielding shifts that vary linearly with intensity, as shown in Figs. 4(b) and 4(c) and in full agreement with Eq. (6).

D. Coherence time measurements

To quantify the influence of polarization control on qubit coherence, we measure the coherence times of the clock qubit ($\Delta m_F = 0$) and the Zeeman qubit ($\Delta m_F = +1$) under three conditions: (1) 355 nm laser-free, (2) the minimum shift, and (3) the maximum shift. As shown in Fig. 5(a), the clock-qubit coherence times are 331 ms without the laser, 55.6 ms at the minimum shift ($\theta = 45.57^\circ$), and 0.478 ms at maximum shift ($\theta = 22.40^\circ$). The drastic decoherence observed in the maximum-shift case is primarily attributed to power fluctuations. By carefully controlling the polarization to minimize the light shift, the qubit becomes less sensitive to such fluctuations, leading to over two orders of magnitude improvement in coherence time between the maximum- and minimum-shift conditions. In our experiment, the minimum shift exhibits a reduced coherence time compared to the “laser-free” case, because the configuration that yields the minimum shift still results in a residual light shift. Complete elimination of the residual shift would require a stronger magnetic field. Moreover, as the laser power increases, a correspondingly higher magnetic field strength is needed [28] (see the Appendix F for further details).

The Zeeman qubit results are shown in Fig. 5(b). The coherence time without the laser is measured to be 853 μ s, decreasing to 467 μ s at the minimal-shift point ($\theta = 45.03^\circ$)

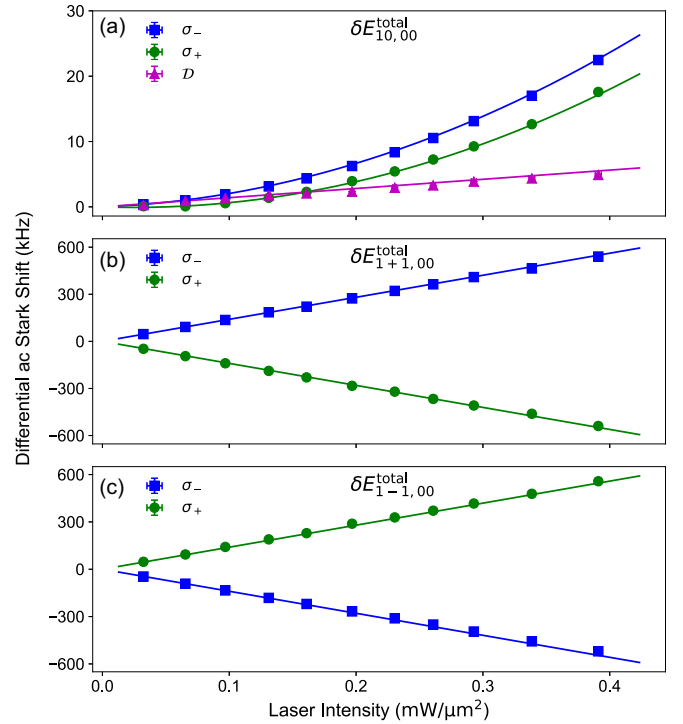


FIG. 4. Differential shift as a function of laser intensity. (a) The clock qubit exhibits polarization-dependent behavior in its intensity-dependent ac Stark shift. For both σ_- (blue) and σ_+ (green), the shift is primarily determined by fourth-order light shifts, resulting in a characteristic quartic dependence on intensity. The $\mathcal{D}$ (pink), defined as the shift difference between the two polarizations, displays a linear dependence on intensity, originating from the magnetic-field-dependent term in Eq. (3). (b) The Zeeman qubits $\Delta m_F = +1$ and (c) $\Delta m_F = -1$ exhibit a linear dependence of the ac Stark shift on laser intensity under both σ_- (blue) and σ_+ (green) polarizations, consistent with second-order light shift behavior in Eq. (6). Solid lines represent theoretical predictions using beam size parameters obtained from the fit in Fig. 3. Squares, filled circles, and triangles denote experimental data points. Error bars (standard error) were calculated from 2000 bootstrap resamples, and most error bars are smaller than the marker size and thus not visible.

and to 32.6 μ s at the maximum-shift point ($\theta = 22.50^\circ$), demonstrating more than a tenfold enhancement in coherence with optimal polarization. The “laser-free” coherence time remains longer than that at the minimal-shift point, as the Zeeman qubit is highly sensitive to polarization imperfections, which degrade coherence. We also note that the θ for the minimum differential shifts of the clock ($\theta = 45.57^\circ$) and Zeeman ($\theta = 45.03^\circ$) qubits are different, which indicates that the optimal polarization for minimizing Stark shifts in the clock qubit is not purely linear.

IV. CONCLUSION

In conclusion, we have experimentally characterized the polarization- and intensity-dependent ac Stark shifts for both clock and Zeeman qubits in a 355-nm laser-driven $^{171}\text{Yb}^+$ ion system. Our results reveal polarization asymmetries arising from magnetic-field-induced hyperfine state mixing and

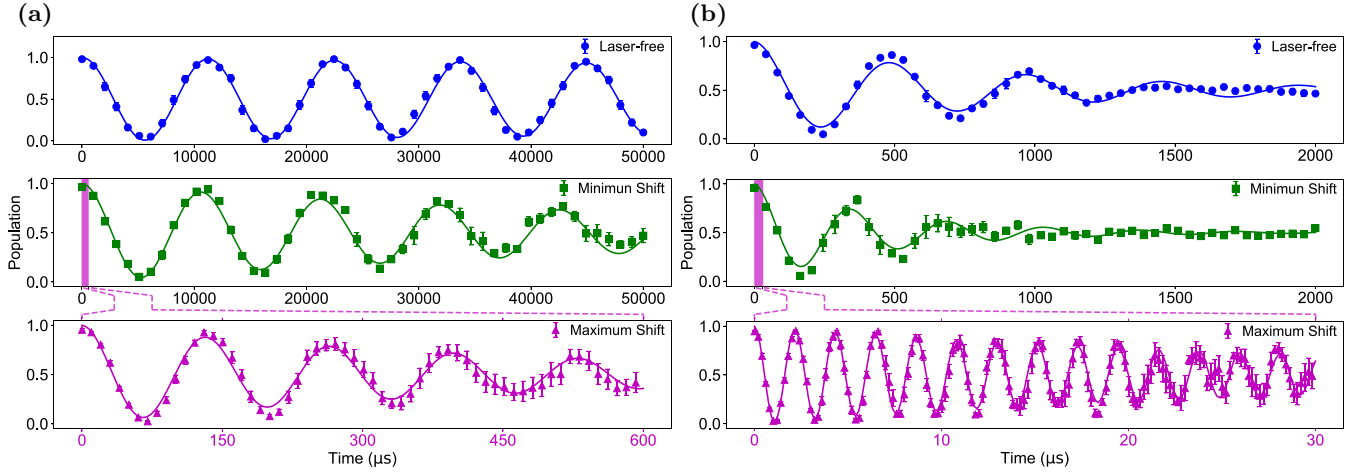


FIG. 5. (a) Detuned microwave Ramsey measurement of clock qubit for the cases of 355 nm laser-free (top), the minimum ac Stark shift (middle), and the maximum ac Stark shift (bottom), where the corresponding coherence times are 331, 55.6, and 0.478 ms. Microwave detunings of +100 Hz, +100 Hz, and +10 kHz were applied in the three cases to enhance fringe visibility and facilitate envelope fitting. The minimum and maximum ac Stark shifts are achieved by setting the θ to 45.57° and 22.40° , respectively. (b) Detuned microwave Ramsey measurement of Zeeman qubit for the cases of 355 nm laser-free (top), the minimum ac Stark shift (middle), and the maximum ac Stark shift (bottom), where the corresponding coherence times are 853, 467, and $32.6 \mu\text{s}$, respectively. Microwave detunings of +2, +4, and 0 kHz were applied in these three cases. The minimum and maximum ac Stark shifts are achieved by setting the θ to 45.03° and 22.50° , respectively. Solid lines represent fitted curves. Filled circles denote experimental data points. Each data point is the mean of five independent sets of 100 realizations (total $N = 500$); error bars denote standard error across sets.

demonstrate that the optimal polarization condition deviates from the purely linear case previously assumed. This precise polarization tuning significantly suppresses power-fluctuation-induced decoherence, approaching the “laser-free” coherence time. These findings not only deepen the understanding of light shifts in $^{171}\text{Yb}^+$ ion systems but also offer practical guidance for mitigating decoherence in high-precision quantum control of other ion species. Beyond trapped-ion platforms, the insights gained here can be applied to neutral atom systems, where optical trapping and control similarly induce polarization-sensitive Stark shifts. Our results underscore the importance of polarization engineering for extending qubit coherence, enabling deeper quantum circuits and longer adiabatic evolutions essential for scalable quantum simulation and computation.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: DRESSED STATE DERIVATION

The formation of the dressed state arises primarily from the interaction between these two states, leading to the presence of off-diagonal elements in the matrix. Typically, since $g_J \ll g_I$ and $L = 0$ for the $^2S_{1/2}$ state, the interaction can be simplified as

$$H_{\text{hf}} = w_{\text{hf}} \mathbf{I} \cdot \mathbf{J} + 2\mu_B \mathbf{J} \cdot \mathbf{B} \\ = w_{\text{hf}} I_z J_z + \frac{w_{\text{hf}}}{2} (I_+ J_- + I_- J_+) + 2\mu_B J_z B_z. \quad (\text{A1})$$

Expanding the Hamiltonian in the four states $|m_I, m_J\rangle$, the following result was obtained, where $m_I = \pm\frac{1}{2}$, $m_J = \pm\frac{1}{2}$, $M = w_{\text{hf}}$, and $N = \mu_B B_z$:

(m_I, m_J)	$ \frac{1}{2}, +\frac{1}{2}\rangle$	$ \frac{1}{2}, -\frac{1}{2}\rangle$	$ \frac{1}{2}, +\frac{1}{2}\rangle$	$ \frac{1}{2}, -\frac{1}{2}\rangle$
$ \frac{1}{2}, +\frac{1}{2}\rangle$	$\frac{1}{4}M + N$			
$ \frac{1}{2}, -\frac{1}{2}\rangle$		$-\frac{1}{4}M - N$	$\frac{1}{2}M$	
$ \frac{1}{2}, +\frac{1}{2}\rangle$		$\frac{1}{2}M$	$-\frac{1}{4}M + N$	
$ \frac{1}{2}, -\frac{1}{2}\rangle$				$\frac{1}{4}M - N$

(A2)

In the absence of an external magnetic field, the four eigenstates of the matrix are

$$|F = 0, m = 0\rangle = \frac{1}{\sqrt{2}} \left(\left| -\frac{1}{2}, +\frac{1}{2} \right\rangle - \left| +\frac{1}{2}, -\frac{1}{2} \right\rangle \right), \\ |F = 1, m = 0\rangle = \frac{1}{\sqrt{2}} \left(\left| -\frac{1}{2}, +\frac{1}{2} \right\rangle + \left| +\frac{1}{2}, -\frac{1}{2} \right\rangle \right), \\ |F = 1, m = +1\rangle = \left| +\frac{1}{2}, +\frac{1}{2} \right\rangle, \\ |F = 1, m = -1\rangle = \left| -\frac{1}{2}, -\frac{1}{2} \right\rangle, \quad (\text{A3})$$

TABLE I. Dipole moment perturbed by the external magnetic field.

$\sigma_{\pm}$	$ ^2P_{1/2}, 1, \pm 1\rangle$	$ ^2P_{3/2}, 1, \pm 1\rangle$	$ ^2P_{3/2}, 2, \pm 1\rangle$
$ 0, 0\rangle_d$	$\sqrt{\frac{2}{6}}(1 \pm R)$	$\sqrt{\frac{4}{6}}(1 \mp \frac{R}{2})$	$-\sqrt{\frac{3}{6}}R$
$ 1, 0\rangle_d$	$\mp\sqrt{\frac{2}{6}}(1 \mp R)$	$\pm\sqrt{\frac{1}{6}}(1 \pm 2R)$	$\sqrt{\frac{3}{6}}$

when involving the external magnetic field, the above eigenstates change to

$$\begin{aligned}
|F=0, m=0\rangle_d &= A \left(\left| -\frac{1}{2}, +\frac{1}{2} \right\rangle - (2R + \sqrt{1+4R^2}) \left| +\frac{1}{2}, -\frac{1}{2} \right\rangle \right) \\
&= |F=0, m=0\rangle \\
&\quad - R |F=1, m=0\rangle + o(R^2), \\
|F=1, m=0\rangle_d &= A \left(\left| -\frac{1}{2}, +\frac{1}{2} \right\rangle - (2R - \sqrt{1+4R^2}) \left| +\frac{1}{2}, -\frac{1}{2} \right\rangle \right) \\
&= |F=1, m=0\rangle \\
&\quad + R |F=0, m=0\rangle + o(R^2),
\end{aligned}$$

$$\begin{aligned}
|F=1, m=+1\rangle_d &= \left| +\frac{1}{2}, +\frac{1}{2} \right\rangle = |F=1, m=+1\rangle, \\
|F=1, m=-1\rangle_d &= \left| -\frac{1}{2}, -\frac{1}{2} \right\rangle \\
&= |F=1, m=-1\rangle,
\end{aligned} \tag{A4}$$

which leads to the result of Eq. (2) in the main manuscript and $R = N/M$, A is the normalized coefficient.

APPENDIX B: DIPOLE MOMENT PERTURBATION OF DRESSED STATES

To obtain the dependence on the external magnetic field, the dipole moment, perturbed by the dressed state, needs to be calculated. As an example, we choose $|0, 0\rangle_d \rightarrow |^2P_{1/2}, 1, 1\rangle$ in the σ_+ polarization, which is given by

$$\begin{aligned}
\langle ^2P_{1/2}, 1, 1 | \hat{\mathbf{d}} | 0, 0 \rangle_d &= \langle ^2P_{1/2}, 1, 1 | \hat{\mathbf{d}} | 0, 0 \rangle \\
&\quad - R \langle ^2P_{1/2}, 1, 1 | \hat{\mathbf{d}} | 1, 0 \rangle \\
&= \sqrt{\frac{2}{6}} d_1 + R \sqrt{\frac{2}{6}} d_1 = \sqrt{\frac{2}{6}} (1 + R) d_1.
\end{aligned} \tag{B1}$$

Thus, the dipole moment changes from $\sqrt{\frac{2}{6}}$ to $\sqrt{\frac{2}{6}}(1 + R)$. Similarly, the dipole moment for the two dressed states can also be revised, with the results summarized in Table I.

APPENDIX C: SECOND-ORDER AC STARK SHIFT

The ac Stark shift induced by the laser can be expressed as

$$\Delta E_g^{(2)} = - \sum_e \frac{\langle g | \mathcal{E}^* \cdot \mathbf{d}^* | e \rangle \langle e | \mathbf{d} \cdot \mathcal{E} | g \rangle}{4(\omega_e - \omega_g - \omega_L)} - \sum_e \frac{\langle e | \mathcal{E}^* \cdot \mathbf{d}^* | g \rangle \langle g | \mathbf{d} \cdot \mathcal{E} | e \rangle}{4(\omega_e - \omega_g + \omega_L)}, \tag{C1}$$

where $|e\rangle$ is the state inducing the ac Stark shift to $|g\rangle$, $\mathbf{d}$ is the electronic dipole moment, $\mathcal{E}$ is the laser electronic field, and ω_e , ω_g , and ω_L are the energies of $|e\rangle$, $|g\rangle$, and laser, respectively.

When considering the coupling of the four states of the $^2S_{1/2}$ manifold to the excited state, the ac Stark shift of these four states can be calculated as follows. Under the condition that the laser is aligned along the magnetic field $\mathbf{B}$, the ion will only experience left- and right-circularly polarized light. This polarization constraint simplifies the calculation, as only these two components contribute to the Stark shift. The resulting ac Stark shifts for the four states can then be expressed as

$$\begin{aligned}
\Delta E_{00}^{(2)} &= (\epsilon_+^2 + \epsilon_-^2) \left(\frac{g_{1/2}^2 I(t)}{12\Delta_{1/2}} + \frac{2g_{3/2}^2 I(t)}{12\Delta_{3/2}} \right), \\
\Delta E_{10}^{(2)} &= (\epsilon_+^2 + \epsilon_-^2) \left(\frac{g_{1/2}^2 I(t)}{12(\Delta_{1/2} - \omega_{\text{hf}})} + \frac{2g_{3/2}^2 I(t)}{12(\Delta_{3/2} - \omega_{\text{hf}})} \right), \\
\Delta E_{1+1}^{(2)} &= \epsilon_+^2 \left(\frac{3g_{3/2}^2 I(t)}{12(\Delta_{3/2} - \omega_{\text{hf}})} \right) + \epsilon_-^2 \left(\frac{2g_{1/2}^2 I(t)}{12(\Delta_{1/2} - \omega_{\text{hf}})} + \frac{g_{3/2}^2 I(t)}{12(\Delta_{3/2} - \omega_{\text{hf}})} \right), \\
\Delta E_{1-1}^{(2)} &= \epsilon_+^2 \left(\frac{2g_{1/2}^2 I(t)}{12(\Delta_{1/2} - \omega_{\text{hf}})} + \frac{g_{3/2}^2 I(t)}{12(\Delta_{3/2} - \omega_{\text{hf}})} \right) + \epsilon_-^2 \left(\frac{3g_{3/2}^2 I(t)}{12(\Delta_{3/2} - \omega_{\text{hf}})} \right),
\end{aligned} \tag{C2}$$

where $g_J^2 = \frac{\gamma_J^2}{2I}$, where $J = 1/2$ or $3/2$, γ is the spontaneous emission rate, $\bar{I}$ is the saturation intensity, and $I(t)$ is the laser power. In the calculation, we explicitly neglect contributions from hyperfine splitting in the excited $^2P_{1/2}$ and $^2P_{3/2}$ states.

When using the above dipole moment to Eq. (C2), the ac Stark shifts of clock states are corrected to

$$\begin{aligned}\Delta E_{00}^{(2)} &= \left[\epsilon_+^2 \left(\frac{g_{1/2}^2}{12\Delta_{1/2}} (1+R)^2 + \frac{2g_{3/2}^2}{12\Delta_{3/2}} \left(1 - \frac{R}{2}\right)^2 + \frac{3g_{3/2}^2}{24\Delta_{3/2}} R^2 \right) \right. \\ &\quad \left. + \epsilon_-^2 \left(\frac{g_{1/2}^2}{12\Delta_{1/2}} (1-R)^2 + \frac{2g_{3/2}^2}{12\Delta_{3/2}} \left(1 + \frac{R}{2}\right)^2 + \frac{3g_{3/2}^2}{24\Delta_{3/2}} R^2 \right) \right] I(t) \\ &= \left[(\epsilon_+^2 + \epsilon_-^2) \frac{1}{12} \left(\frac{g_{1/2}^2}{\Delta_{1/2}} + \frac{2g_{3/2}^2}{\Delta_{3/2}} \right) + (\epsilon_+^2 - \epsilon_-^2) \frac{2R}{12} \left(\frac{g_{1/2}^2}{\Delta_{1/2}} - \frac{g_{3/2}^2}{\Delta_{3/2}} \right) + o(R^2) \right] I(t),\end{aligned}\quad (C3)$$

$$\begin{aligned}\Delta E_{10}^{(2)} &= \left[\epsilon_+^2 \left(\frac{g_{1/2}^2}{12(\Delta_{1/2} - \omega_{\text{hf}})} (1-R)^2 + \frac{g_{3/2}^2}{24(\Delta_{3/2} - \omega_{\text{hf}})} (1+2R)^2 + \frac{3g_{3/2}^2}{24(\Delta_{3/2} - \omega_{\text{hf}})} \right) \right. \\ &\quad \left. + \epsilon_-^2 \left(\frac{g_{1/2}^2}{12(\Delta_{1/2} - \omega_{\text{hf}})} (1+R)^2 + \frac{g_{3/2}^2}{24(\Delta_{3/2} - \omega_{\text{hf}})} (1-2R)^2 + \frac{3g_{3/2}^2}{24(\Delta_{3/2} - \omega_{\text{hf}})} \right) \right] I(t) \\ &= \left[(\epsilon_+^2 + \epsilon_-^2) \frac{1}{12} \left(\frac{g_{1/2}^2}{(\Delta_{1/2} - \omega_{\text{hf}})} + \frac{2g_{3/2}^2}{(\Delta_{3/2} - \omega_{\text{hf}})} \right) - (\epsilon_+^2 - \epsilon_-^2) \frac{2R}{12} \left(\frac{g_{1/2}^2}{(\Delta_{1/2} - \omega_{\text{hf}})} - \frac{g_{3/2}^2}{(\Delta_{3/2} - \omega_{\text{hf}})} \right) + o(R^2) \right] I(t).\end{aligned}\quad (C4)$$

Considering the differential shift of the clock qubit and ignoring the higher orders, the result is

$$\delta E_{10,00}^{(2)} \equiv \Delta E_{10}^{(2)} - \Delta E_{00}^{(2)} = \left[(\epsilon_+^2 + \epsilon_-^2) \frac{\omega_{\text{hf}}}{12} \left(\frac{g_{1/2}^2}{\Delta_{1/2}^2} + \frac{2g_{3/2}^2}{\Delta_{3/2}^2} \right) - (\epsilon_+^2 - \epsilon_-^2) \frac{4R}{12} \left(\frac{g_{1/2}^2}{\Delta_{1/2}} - \frac{g_{3/2}^2}{\Delta_{3/2}} \right) + o(R^2) \right] I(t). \quad (C5)$$

For the Zeeman qubits, the result is

$$\begin{aligned}\delta E_{1\pm 1,00}^{(2)} &\equiv \Delta E_{1\pm 1,00}^{(2)} - \Delta E_{00}^{(2)} = \left[\epsilon_{\pm}^2 \left(-\frac{g_{1/2}^2}{12\Delta_{1/2}} + \frac{g_{3/2}^2}{12(\Delta_{3/2} - \omega_{\text{hf}})} + \frac{2\omega_{\text{hf}}}{\Delta_{3/2}} \frac{g_{3/2}^2}{12(\Delta_{3/2} - \omega_{\text{hf}})} \mp \frac{2R}{12} \left(\frac{g_{1/2}^2}{\Delta_{1/2}} - \frac{g_{3/2}^2}{\Delta_{3/2}} \right) \right) \right. \\ &\quad \left. + \epsilon_{\mp}^2 \left(\frac{g_{1/2}^2}{12(\Delta_{1/2} - \omega_{\text{hf}})} - \frac{g_{3/2}^2}{12(\Delta_{3/2} - \omega_{\text{hf}})} + \frac{2\omega_{\text{hf}}}{\Delta_{3/2}} \frac{g_{3/2}^2}{12(\Delta_{3/2} - \omega_{\text{hf}})} + \frac{\omega_{\text{hf}}}{\Delta_{1/2}} \frac{g_{1/2}^2}{12(\Delta_{1/2} - \omega_{\text{hf}})} \right) \right. \\ &\quad \left. \pm \frac{2R}{12} \left(\frac{g_{1/2}^2}{\Delta_{1/2}} - \frac{g_{3/2}^2}{\Delta_{3/2}} \right) \right] I(t) \\ &= \pm (\epsilon_-^2 - \epsilon_+^2) \frac{1}{12} \left(\frac{g_{1/2}^2}{\Delta_{1/2}} - \frac{g_{3/2}^2}{\Delta_{3/2}} \right) I(t) + o(R) + o\left(\frac{\omega_{\text{hf}}}{\Delta_J}\right).\end{aligned}\quad (C6)$$

In the main text, we neglect the small value terms in Zeeman qubits associated with the factor R and the ratio $\frac{\omega_{\text{hf}}}{\Delta_J}$, where $J = 1/2$ or $1/3$. These contributions are approximately three orders of magnitude smaller than the primary energy shift and thus have a negligible effect.

APPENDIX D: NUMERICAL ESTIMATION OF AC STARK SHIFT

To get the concrete number of the shift, the numerical values are calculated from physical values in Table II.

TABLE II. The value of physical parameter for the $^{171}\text{Yb}^+$ ion.

ω_{hf}	12.624812 GHz	$C_{10,00}$	0.971
$\gamma_{1/2}$	19.703 MHz [40]	$\gamma_{3/2}$	25.895 MHz [40]
$\bar{I}_{1/2}$	510.3 W/m ² [41]	$\bar{I}_{3/2}$	950.6 W/m ² [41]
$g_{1/2}^2$	0.380×10^{12} s/kg	$g_{3/2}^2$	0.353×10^{12} s/kg
$\Delta_{1/2}$	$+(2\pi) 33$ THz	$\Delta_{3/2}$	$-(2\pi) 67$ THz

Thus, the final ac Stark shift can be calculated as follows:
(1) For the clock qubit $|m=0\rangle$,

$$\begin{aligned}\delta E_{10,00} &= \Delta E_{1,0} - \Delta E_{0,0} = \delta E_{\text{hf}}^{(2)} + \delta E_{\text{hf}}^{(4)} \\ &= (\epsilon_+^2 + \epsilon_-^2) 5.337 \times 10^{-7} \frac{I(t)}{\text{W/m}^2} \text{Hz} \\ &\quad - (\epsilon_+^2 - \epsilon_-^2) 7.066 \times 10^{-6} \frac{I(t)}{\text{W/m}^2} \text{Hz} \\ &\quad + (\epsilon_+^2 - \epsilon_-^2)^2 1.327 \times 10^{-13} \left(\frac{I(t)}{\text{W/m}^2} \right)^2 \text{Hz}.\end{aligned}\quad (D1)$$

(2) For the Zeeman qubit $|m = +1\rangle$,

$$\begin{aligned}\delta E_{1+1,00} &= \Delta E_{1+1} - \Delta E_{00} \\ &= -(\epsilon_+^2 - \epsilon_-^2) 1.40 \times 10^{-3} \frac{I(t)}{W/m^2} \text{ Hz.}\end{aligned}\quad (\text{D2})$$

(3) For the Zeeman qubit $|m = -1\rangle$,

$$\begin{aligned}\delta E_{1-1,00} &= \Delta E_{1-1} - \Delta E_{00} \\ &= (\epsilon_+^2 - \epsilon_-^2) 1.40 \times 10^{-3} \frac{I(t)}{W/m^2} \text{ Hz.}\end{aligned}\quad (\text{D3})$$

APPENDIX E: THE RELATION BETWEEN THE POLARIZATION AND RELATIVE WAVE PLATE ANGLE θ

The σ_+ and σ_- circular polarizations can be expressed as

$$E_{\sigma_+} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}, \quad (\text{E1})$$

$$E_{\sigma_-} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}. \quad (\text{E2})$$

The Jones matrix of the wave plate can be expressed as

$$J = \begin{bmatrix} \cos^2 \theta + \sin^2 \theta e^{i\delta} & \sin \theta \cos \theta (1 - e^{i\delta}) \\ \sin \theta \cos \theta (1 - e^{i\delta}) & \sin^2 \theta + \cos^2 \theta e^{i\delta} \end{bmatrix}, \quad (\text{E3})$$

where δ represents the phase delay between the fast and slow axes of the wave plate and θ is the angle between the fast axis and the horizontal x axis. In this case, the QWP has a matrix corresponding to $\phi = \pi/2$ and the HWP corresponds to $\phi = \pi$.

When the QWP is fixed, the polarization changes as the relative angle θ of the HWP. Starting with an initial horizontal polarization, the polarization after rotation can be expressed as

$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}_{\text{QWP}} \cdot \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}_{\text{HWP}} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{\text{initial}} = \begin{bmatrix} \cos 2\theta \\ i \sin 2\theta \end{bmatrix}. \quad (\text{E4})$$

Therefore, the coefficient can be expressed as

$$\epsilon_+ = \frac{1}{\sqrt{2}} [1, i] \cdot \begin{bmatrix} \cos 2\theta \\ i \sin 2\theta \end{bmatrix} = \cos(2\theta + \pi/4), \quad (\text{E5})$$

$$\epsilon_- = \frac{1}{\sqrt{2}} [1, -i] \cdot \begin{bmatrix} \cos 2\theta \\ i \sin 2\theta \end{bmatrix} = \sin(2\theta + \pi/4). \quad (\text{E6})$$

In experiment, we employed a Thorlabs PRM1Z8 precision motorized rotation stage, which has an angular precision of 0.03° according to the specification. We calibrate the relative angle between the QWP and the HWP with polarimeter (Thorlabs PAX1000VIS).

APPENDIX F: POLARIZATION OF MINIMUM DIFFERENTIAL SHIFT FOR CLOCK QUBIT

In this sequence, we illustrate how the minimum ac Stark shift of the clock qubit is affected by the presence of an external magnetic field. The curves in the figure represent simulated ac Stark shifts under a laser power of 52.1 mW and a beam waist radius of $7 \mu\text{m}$. As shown in Fig. 6, the blue star

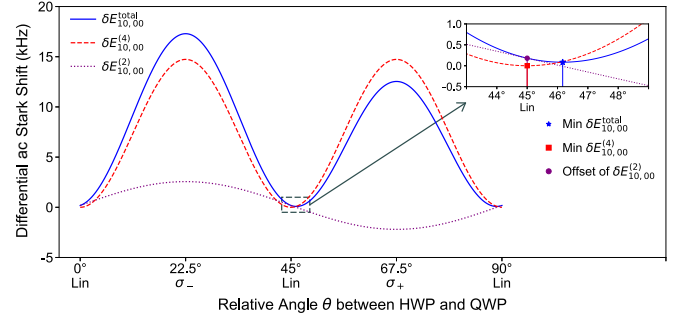


FIG. 6. Polarization-dependent differential shift of the clock qubit. The asymmetry induced by the external magnetic field shifts the position of the overall minimum (blue star) away from the fourth-order minimum (red square). The purple filled circle indicates the offset of the second-order ac Stark shift in the absence of the magnetic field. The inset highlights the displacement of the minimal point due to magnetic-field-induced asymmetry.

marks the overall minimum of the total ac Stark shift, while the red square indicates the minimum of the fourth-order contribution. The purple filled circle represents the offset of the second-order ac Stark shift, corresponding to the shift in the absence of an external magnetic field ($B = 0$), as defined in Eq. (3). Due to the magnetic field, the ac Stark shift exhibits asymmetric behavior for polarization, causing a displacement of the overall minimum. As illustrated in the inset, this shifts the optimal point from the red square to the blue star.

Our simulations indicate that residual shifts persist under the current experimental conditions. As shown in Fig. 7, the magnitude of these shifts is related to the applied magnetic field strength. When the field strength exceeds 15.557 G, the shift function crosses zero. By tuning the polarization to operate at these zero-crossing points, the ac Stark shift can be completely canceled, a condition known as magic polarization.

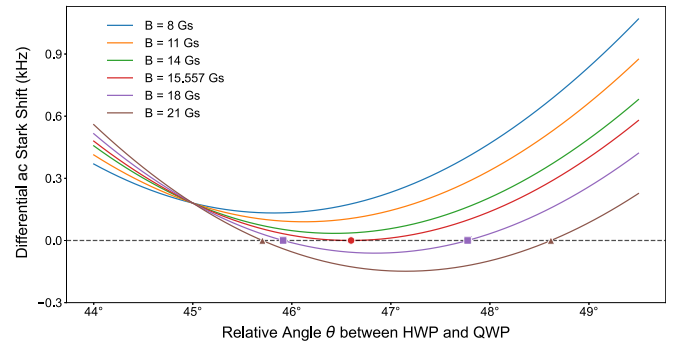


FIG. 7. Calculated ac Stark shift as a function of the relative angle under different magnetic field strengths. The calculation assumes a wavelength of 355 nm, an optical power of 52.1 mW, and a beam waist of $7 \mu\text{m}$. The solid lines represent the theoretical shifts for various magnetic fields, while the filled circles, squares, and triangles mark the corresponding zero-crossing points. No zero crossing is observed when the magnetic field is below 15.557 G, indicating that the light shift cannot be fully canceled. In contrast, for fields exceeding 15.557 G, zero-crossing points emerge, allowing for perfect cancellation of the ac Stark shift via magic polarization.

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