

Robustness of multipartite entangled states in passive $\mathcal{PT}$ -symmetric qubits

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(Received 17 December 2024; accepted 30 June 2025; published 14 July 2025)

Non-Hermitian quantum systems have attracted significant interest in recent years due to the presence of unique spectral singularities in which eigenvalues and eigenvectors coalesce. Those singularities are known as exceptional points (EPs). The drastic changes of the non-Hermitian systems around their EPs have led to unique entanglement dynamics, which remained elusive until quite recently. In this work, we theoretically investigate the robustness of multipartite entanglement induced by EPs of the passive $\mathcal{PT}$ -symmetric non-Hermitian superconducting qubits, both in stand-alone configurations and hybrid setups with Hermitian qubits. In particular, we consider the qubits with both all-to-all and nearest-neighbor couplings under uniform and nonuniform coupling strengths. Our results reveal that non-Hermitian qubits with all-to-all coupling generate GHZ states, while those with nearest-neighbor interactions produce W states. These entangled states are resilient to nonuniform couplings, off-resonant driving fields, and decoherence effects. Moreover, the hybrid configurations combining Hermitian and non-Hermitian qubits indicate the necessity of EPs for rapidly generating and maintaining genuine multipartite entanglement. Additionally, driving the $\mathcal{PT}$ -symmetric qubits with a strong Rabi frequency can help sustain the entanglement over time by countering losses, while strong interqubit coupling favors the entangled states in the low dissipation regime. We also demonstrate that increasing the number of non-Hermitian qubits speeds up multipartite entanglement generation due to enhanced collective interactions. These findings reveal that exploiting non-Hermitian systems and their associated EPs is beneficial for generating robust and genuine entangled states which are useful for both fundamental studies and quantum technologies.

DOI: [10.1103/ypd8-r9gq](https://doi.org/10.1103/ypd8-r9gq)

I. INTRODUCTION

Non-Hermitian quantum systems have recently attracted significant attention due to the existence of spectral singularities called exceptional points (EPs) [1–15]. These systems possess Liouvillian EPs when described by the Liouvillian superoperators or non-Hermitian EPs when governed by the non-Hermitian Hamiltonians [16,17]. The difference between these EPs depends on whether the considered system dynamics includes or excludes the effect of quantum jumps, respectively [16,18]. An exclusive consideration of the non-Hermitian Hamiltonian is sufficient for open quantum systems upon post selection. In fact, EPs of such non-Hermitian systems are different from the conventional diabolic points (DPs), where the eigenvalues coalesce but the corresponding eigenstates remain orthogonal [19,20]. In contrast, the loss of the conventional orthogonality condition at the EPs of the

non-Hermitian systems leads to a real-complex spectral transition under both parity ($\mathcal{P}$) and time reversal ($\mathcal{T}$) transformations [10,11,21]. $\mathcal{PT}$ -symmetric systems [22], characterized by real spectra, have been implemented by simultaneously controlling gain and loss [23–25] or just by controlling loss alone via Hamiltonian dilations [26–28]. In the former approach, balancing gain and loss creates active $\mathcal{PT}$ -symmetric systems, while the latter, comprising only lossy components, results in passive $\mathcal{PT}$ -symmetric systems, which can broaden the scope of feasible experimental setups by relaxing the strict requirement for balanced gain and loss.

Beyond their role in fundamental studies of physics in the classical and semiclassical regimes [29–31], $\mathcal{PT}$ -symmetric systems [22] have recently been shown to exhibit unique quantum entanglement dynamics owing to their EPs [32–36]. Entanglement of strongly coupled non-Hermitian qubits undergoes a spectral transition between the passive $\mathcal{PT}$ -symmetric and $\mathcal{PT}$ -broken phases [35,36]. In the $\mathcal{PT}$ -symmetric phase, it exhibits oscillatory behavior, whereas in the $\mathcal{PT}$ -broken phase, the entanglement remains stationary with finite strength. Moreover, externally driven and weakly coupled non-Hermitian qubits generate a Bell state that is faster than their Hermitian counterparts [32]. This result has been extended to multi-qubit systems [33], in which three qubits exhibit the same timescaling as two qubits while the four-qubit system further speeds up entanglement generation,

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showing an advantage of the higher-order EPs of many-body systems.

Furthermore, involving an additional gain to the passive $\mathcal{PT}$ -symmetric qubits has shown to boost the success rate at the cost of large individual qubit driving [34]. Such rapid entanglement generation and control are useful in practical settings, particularly to harness quantum effects within the coherence time [37,38]. Moreover, fast entangled state generation ensures the reliability and accuracy of quantum computations and communications by reducing errors from qubit decoherence, which is fundamentally important in quantum information processing [39,40].

In this article, we investigate the robustness of EP-induced multipartite entanglement in non-Hermitian qubits and hybrid setups consisting of both Hermitian and non-Hermitian qubits. We consider different coupling regimes, where the qubits are connected either through all-to-all or through nearest-neighbor interactions, with both uniform and nonuniform coupling strengths. Our findings show that all-to-all coupled non-Hermitian qubits generate GHZ classes [41,42], whereas those with nearest-neighbor coupling produce W classes [43,44]. These entangled states develop over the same timescale despite the fundamental differences between the GHZ and W states [44]. Moreover, these states persist across a broad range of nonuniform interqubit coupling strengths and exhibit robustness against off-resonant driving effects.

We further demonstrate that switching one of the non-Hermitian qubits in the tripartite system to a Hermitian qubit results in biseparable entangled states due to mismatched evolution times between the Hermitian and non-Hermitian qubits. Additionally, increasing the Rabi frequency drives the system from the $\mathcal{PT}$ -broken phase to the $\mathcal{PT}$ -symmetric phase, where genuine tripartite-entangled states can be generated rapidly and sustained for a long time under strong driving fields. We also show fine-tuning the coupling constant from weak to strong regimes benefits non-Hermitian qubits only in the small dissipation regime, which indicates an entanglement growth in the Hermitian limit with negligible dissipation as the coupling strengths increase.

Besides, our results show that fast-entangled states generated by the non-Hermitian Hamiltonian are robust to decoherence effects that stem from quantum jumps. In addition, multipartite entanglement captured through monitored non-Hermitian dynamics using quantum trajectories is stronger than that inferred from ensemble-averaged density matrices. We also demonstrate multipartite entanglement generation in large non-Hermitian qubit systems requires weaker coupling due to enhanced collective interactions; however, at fixed coupling, this collective effect demands a slightly stronger drive to accelerate entanglement generation.

The manuscript is organized as follows. In Sec. II A, we present the system Hamiltonian, introduce the biorthogonal formalism, outline the perturbative approach, describe the dynamics of non-Hermitian qubits, and define the entanglement measure used throughout. Section III analyzes entanglement in systems with all-to-all and nearest-neighbor couplings, and investigates its robustness under nonuniform coupling conditions, off-resonant driving, quantum jumps, and strong-coupling effects. In Sec. IV, we study entanglement generation in hybrid systems combining Hermitian and

non-Hermitian qubits. Section V extends our analysis to larger system sizes to demonstrate multipartite entanglement in non-Hermitian qubits. We discuss possible experimental proposals in Sec. VI and summarize our findings in Sec. VII.

II. THEORETICAL MODEL

A. Hamiltonian and non-Hermitian properties

We consider a transmon circuit consisting of a capacitor and an inductor connected by a superconducting wire [4,7,32]. Dynamics of the system are captured by the Hamiltonian written in the interaction picture as [4,7,32]

$$\hat{H} = \sum_{j=1}^n \left[\left(\Delta_j - \frac{i\gamma_j}{2} \right) \hat{\sigma}_j \hat{\sigma}_j^\dagger + \Omega_j \hat{\sigma}_j \hat{\sigma}_j^x \right] + \sum_{j \neq k} \sum_{k=1}^n J_{jk} (\hat{\sigma}_j^\dagger \hat{\sigma}_k + \hat{\sigma}_j \hat{\sigma}_k^\dagger), \quad (1)$$

where $\hat{\sigma}_j^x = \hat{\sigma}_j + \hat{\sigma}_j^\dagger$ represents the Pauli-X matrix, in which $\hat{\sigma}_j = |e\rangle_j \langle f|$ and $\hat{\sigma}_j^\dagger = |f\rangle_j \langle e|$ are the ladder operators. Here, $|e\rangle_j$ and $|f\rangle_j$ denote the first and second excited states of the j th qubit, respectively [please see Eq. (9)]. Besides, each qubit experiences dissipation from the state $|e\rangle_j$ at a rate γ_j and is driven with an external field Ω_j that is detuned from resonance by Δ_j . The coupling strength J_{jk} between qubits j and k is nonzero for all pairs (j, k) in the all-to-all coupling scheme, whereas in the nearest-neighbor scheme, $J_{jk} = 0$ unless $k = j \pm 1$. Crucially, when $J_{jk} \ll \gamma_j$, this interqubit coupling serves as a perturbation to the non-Hermitian Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_I$, where $\hat{H}_0$ accounts for the local non-Hermitian terms and $\hat{H}_I$ is a Hermitian contribution from interacting qubits.

When our system is driven exactly on resonance ($\Delta_j = 0$), the non-Hermitian qubits exhibit passive $\mathcal{PT}$ -symmetry, in which one could control the system via the loss rate without requiring gain [7,45]. In this case, Eq. (1) simplifies to $\hat{H}(\Delta_j = 0) = \hat{H}_{\mathcal{PT}} - \sum_j (i\gamma_j/4) \hat{I}$, where $\hat{I}$ denotes an identity matrix, and the $\mathcal{PT}$ -symmetric Hamiltonian $\hat{H}_{\mathcal{PT}}$ satisfies $\mathcal{PT} \hat{H}_{\mathcal{PT}} (\mathcal{PT})^{-1} = \hat{H}_{\mathcal{PT}}$. Here, the parity operator is $\mathcal{P} = \hat{\sigma}_1^x \hat{\sigma}_2^x \cdots \hat{\sigma}_n^x$, and the time-reversal operator $\mathcal{T}$ corresponds to complex conjugation.

Furthermore, the system can enter either the $\mathcal{PT}$ -unbroken or $\mathcal{PT}$ -broken phase depending on the relative strength between the driving amplitude Ω_j and the dissipation rate γ_j . These phases are separated by a higher-order EP located at $\Omega_j = \gamma_j/4$, where both the eigenvalues and eigenvectors of the non-Hermitian Hamiltonian coalesce. The $\mathcal{PT}$ -unbroken regime ($\Omega_j > \gamma_j/4$) is characterized by distinct real eigenvalues with identical imaginary parts, leading to coherent dynamics under post-selection. In contrast, the $\mathcal{PT}$ -broken phase ($\Omega_j < \gamma_j/4$) features distinct imaginary eigenvalues with vanishing real parts, and the system asymptotically evolves toward the longest-lived eigenstate [46].

B. Biorthogonal basis of noninteracting qubits

Biorthogonal bases are essential to describe the dynamics of non-Hermitian systems [1,11,47]. In the absence of

coupling ($J_{jk} = 0$), the Hamiltonian in Eq. (1) reduces to the local non-Hermitian term $\hat{H}_0$. Diagonalizing this Hamiltonian yields the corresponding right and left eigenvectors, which are given by

$$\hat{H}_0|\varphi_m\rangle = E_m|\varphi_m\rangle, \quad (2)$$

where $|\varphi_m\rangle$ represents the eigenvector and $E_m \in \mathbb{C}$ is the corresponding complex eigenvalue and $m = 1, 2, \dots, 2^n$.

Although the right eigenvectors can help identify order of EP along with degeneracy of E_m , they do not generally form an orthogonal set since $\langle\varphi_{m'}|\varphi_m\rangle \neq 0$. Therefore, the conventional inner product structure breaks down in non-Hermitian systems, and a biorthogonal formalism is required to construct a complete basis of the non-Hermitian Hamiltonian $\hat{H}_0$ [11,47]. To calculate this basis, each right eigenvector $|\varphi_m\rangle$ is paired with a corresponding left eigenvector $\langle\tilde{\varphi}_m|$ ($\langle\tilde{\varphi}_m|\hat{H}_0^\dagger = (E_m)^*\langle\tilde{\varphi}_m|$) to give normalized biorthogonal states defined as [22,47,48]

$$|\phi_m\rangle = \frac{|\varphi_m\rangle}{\sqrt{\langle\tilde{\varphi}_m|\varphi_m\rangle}}, \quad |\tilde{\phi}_m\rangle = \frac{|\tilde{\varphi}_m\rangle}{\sqrt{\langle\tilde{\varphi}_m|\varphi_m\rangle}}, \quad (3)$$

which now satisfy $\langle\tilde{\phi}_{m'}|\phi_m\rangle = \delta_{m'm}$. This biorthogonal basis can be used for spectral decompositions of operators, and for the evaluation of physical quantities of noninteracting qubits.

C. Perturbation theory in biorthogonal basis

In the weak coupling regime ($J_{jk} \ll \gamma$), the Hermitian interaction Hamiltonian $\hat{H}_I$ acts as a perturbation to the non-Hermitian qubits. To accurately describe its effect, the perturbation must be treated in the biorthogonal basis [47,49,50]. The first-order correction to a nondegenerate eigenvalue E_m is then given by $E_m^{(1)} = \langle\tilde{\phi}_m|\hat{H}_I|\phi_m\rangle$. In addition, the nondegenerate biorthogonal states are updated to [47,50]

$$\begin{aligned} |\Phi_m\rangle &= |\phi_m\rangle + \sum_{m' \neq m} \frac{\langle\tilde{\phi}_{m'}|\hat{H}_I|\phi_m\rangle}{E_m - E_{m'}} |\phi_{m'}\rangle, \\ |\tilde{\Phi}_m\rangle &= |\tilde{\phi}_m\rangle + \sum_{m' \neq m} \frac{\langle\phi_{m'}|\hat{H}_I|\tilde{\phi}_m\rangle}{E_m - E_{m'}} |\tilde{\phi}_{m'}\rangle, \end{aligned} \quad (4)$$

with $\langle\tilde{\Phi}_m|\Phi_m\rangle = \delta_{m,m} + \mathcal{O}(J_{jk}^2)$.

If $\hat{H}_0$ has a degenerate eigenvalue E_d of multiplicity r , the degenerate subspace $\mathcal{D}$ is spanned by biorthogonal bases $\{|\phi_m^\beta\rangle\}_{\beta=1}^r$ and $\{|\tilde{\phi}_m^\alpha\rangle\}_{\alpha=1}^r$. The perturbation is projected onto this subspace to form an effective Hamiltonian $(\hat{H}_I^{(\mathcal{D})})_{\alpha\beta} = \langle\tilde{\phi}_m^\alpha|\hat{H}_I|\phi_m^\beta\rangle$, which is a diagonalizable Hermitian matrix of dimension $r \times r$ as long as $\hat{H}_I$ is Hermitian [32,34]. Diagonalizing this matrix yields a biorthogonal set of eigenstates within the degenerate subspace. Let $\{|\chi_k\rangle\}$ and $\{|\tilde{\chi}_k\rangle\}$ be the resulting right and left eigenvectors. Up to first-order correction in J_{jk} , the eigenstates incorporating all contributions from outside the degenerate subspace are

given by [47,50]

$$\begin{aligned} |\Psi_k\rangle &= |\chi_k\rangle + \sum_{m \notin \mathcal{D}} \frac{\langle\tilde{\phi}_m|\hat{H}_I|\chi_k\rangle}{E_d - E_m} |\phi_m\rangle, \\ |\tilde{\Psi}_k\rangle &= |\tilde{\chi}_k\rangle + \sum_{m \notin \mathcal{D}} \frac{\langle\phi_m|\hat{H}_I|\tilde{\chi}_k\rangle}{E_d - E_m} |\tilde{\phi}_m\rangle. \end{aligned} \quad (5)$$

The corresponding first-order energy corrections $\delta E_k^{(1)}$ that lift degeneracy of the eigenvalue E_d are obtained from the diagonalizable matrix $(\hat{H}_I^{(\mathcal{D})})_{\alpha\beta}$.

Equations (4) and (5) provide a complete set of perturbed right and left eigenstates of $\hat{H}$, which form a unified biorthogonal basis spanning both nondegenerate and degenerate eigenspaces. We denote these bases by $\{|\Psi_\mu\rangle, |\tilde{\Psi}_\mu\rangle\}$, where each index μ refers either to a nondegenerate eigenstate pair $(|\Phi_m\rangle, |\tilde{\Phi}_m\rangle)$ or to a degenerate pair $(|\Psi_k\rangle, |\tilde{\Psi}_k\rangle)$. These states satisfy $\hat{H}|\Psi_\mu\rangle = E_\mu|\Psi_\mu\rangle$ and $\langle\tilde{\Psi}_\mu|\hat{H}^\dagger = E_\mu^*\langle\tilde{\Psi}_\mu|$, and the biorthonormality condition $\langle\tilde{\Psi}_\nu|\Psi_\mu\rangle = \delta_{\mu\nu}$ up to first-order in J_{jk} .

D. Dynamics of non-Hermitian qubits and multipartite entanglement measures

The state of the non-Hermitian qubits then evolves in time under the total Hamiltonian $\hat{H}$, according to

$$\frac{d}{dt}|\psi(t)\rangle = -i\hat{H}|\psi(t)\rangle. \quad (6)$$

The solution can be expressed in terms of the perturbed biorthogonal bases $\{|\Psi_\mu\rangle, |\tilde{\Psi}_\mu\rangle\}$ as

$$|\psi(t)\rangle = \sum_{\mu} \langle\tilde{\Psi}_\mu|\psi(0)\rangle e^{-iE_\mu t} |\Psi_\mu\rangle, \quad (7)$$

where the expansion coefficients $\langle\tilde{\Psi}_\mu|\psi(0)\rangle$ project the initial state $|\psi(0)\rangle$ onto the corresponding left eigenstates, and the exponential factor $e^{-iE_\mu t}$ encodes both coherent oscillations, via $\text{Re}(E_\mu)$, and exponential amplification or decay, through $\text{Im}(E_\mu)$. Realization of such a system requires post-selecting non-Hermitian evolution, where the dynamics are conditioned on the absence of quantum jumps [4,7]. In this setting, the state evolves under a non-Hermitian Hamiltonian, and normalization is applied at each time step to maintain physical interpretation.

While the biorthogonal energy basis is useful for solving non-Hermitian dynamics, physical observables and reduced density matrices are defined in the computational basis. Therefore, transforming the state back to this basis is necessary to evaluate measurable quantities. Accordingly, the state of an n -qubit system can be expanded as

$$|\psi(t)\rangle = \sum_{k=1}^{2^n} \tilde{a}_k(t) |\alpha_k\rangle, \quad (8)$$

where each $|\alpha_k\rangle$ represents the computational bases arranged from $|e\rangle^{\otimes n}$ for $k = 1$ to $|f\rangle^{\otimes n}$ for $k = 2^n$, and the complex probability amplitudes $\tilde{a}_k(t) \in \mathbb{C}$ encode both the coherent and dissipative effects introduced by the non-Hermitian Hamiltonian. Derivations of these amplitudes involve detailed algebraic steps, and a rigorous treatment has been presented

for both passive $\mathcal{PT}$ -symmetric two-qubit system [32] and active $\mathcal{PT}$ -symmetric two- and three-qubit systems [34].

In this work, we study multipartite entanglement dynamics in n passive $\mathcal{PT}$ -symmetric qubits evolving under the total Hamiltonian $\hat{H}$ as described by Eq. (6), starting from the initial state $|\psi(0)\rangle$. We numerically solve the time evolution using the Quantum Toolbox in Python (QuTiP) package [51,52], and we normalize the state at each time step to obtain Eq. (8). The resulting complex probability amplitudes capture the dynamics of quantum entanglement in this nonunitary evolution. Then, we investigate whether, and under what conditions, GHZ- and W-type multipartite entanglement can be generated from an initial separable state.

The GHZ state is a maximally entangled state shared across the global system and is defined for an n -qubit system as [42,53]

$$|\text{GHZ}_{n \geq 3}\rangle = \frac{1}{\sqrt{2}}(|e\rangle^{\otimes n} + |f\rangle^{\otimes n}). \quad (9)$$

By contrast, an n -qubit W state is constructed from a single excitation symmetrically distributed among the qubits, and it can be written in the form [54]

$$|\text{W}_{n \geq 3}\rangle = \frac{1}{\sqrt{n}} \sum_{k=1}^n |e\rangle^{\otimes(k-1)} \otimes |f\rangle \otimes |e\rangle^{\otimes(n-k)}. \quad (10)$$

These two classes of genuine multipartite entanglement represent fundamentally different quantum correlations [44]: GHZ-type states exhibit strong nonlocality and maximal violation of Bell inequalities [55], while W-type states retain pairwise entanglement even after qubit loss [44].

The three-qubit system constitutes the minimal configuration that can support these entangled states. Specifically, three-qubit pure states derived from Eq. (7) for $n = 3$ can be locally transformed into a canonical form involving five entangled basis states, and are classified as either GHZ-type or W-type under local operations and classical communication (LOCC) [56]. This classification motivates several entanglement measures, including pairwise concurrences [57], the residual three-tangle [44,58], and von Neumann entropies of reduced subsystems [33,59].

Concurrence was originally introduced as a measure of bipartite entanglement in both pure and mixed two-qubit systems. For a pure two-qubit state $|\psi(t)\rangle = \sum_{k=1}^4 \tilde{a}_k(t) |\alpha_k\rangle$, the concurrence is given by [57]

$$\mathcal{C}_{12} = 2|\tilde{a}_1(t)\tilde{a}_4(t) - \tilde{a}_2(t)\tilde{a}_3(t)|, \quad (11)$$

and reaches unity for maximally entangled Bell states, while vanishing for separable states.

Beyond bipartite systems, concurrence is a valuable tool to measure and distinguish multipartite entanglement through the evaluation of pairwise entanglement across all qubit pairs. In a three-qubit system, the bipartite entanglement between any two subsystems can be evaluated by tracing out the remaining subsystem, i.e., $\hat{\rho}_{jk}(t) = \text{Tr}_l[\hat{\rho}(t)]$, where $\hat{\rho}(t)$ is the full state density matrix and $l \neq j, k$. The pairwise concurrence between qubits j and k is then defined as [57,60]

$$\mathcal{C}_{jk} = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (12)$$

where λ_m denotes the eigenvalues of the Hermitian matrix $\sqrt{\sqrt{\hat{\rho}_{jk}(t)}\tilde{\hat{\rho}}_{jk}(t)\sqrt{\hat{\rho}_{jk}(t)}}$, in which $\tilde{\hat{\rho}}_{jk}(t) = (\hat{\sigma}_y \otimes \hat{\sigma}_y)\hat{\rho}_{jk}^*(t)(\hat{\sigma}_y \otimes \hat{\sigma}_y)$, and $\hat{\sigma}_y$ is the Pauli y matrix.

Vanishing pairwise concurrences suggest that there is no bipartite entanglement among qubit pairs. In fact, this is an intrinsic property of GHZ-type entangled states [44]. On the other hand, pairwise concurrences remain finite for W states, reflecting that only the GHZ state has entanglement fully distributed among all three qubits with vanishing bipartite entanglement. Therefore, pairwise concurrences are valuable to identify multipartite GHZ and W states generated from the dynamics of our non-Hermitian qubits.

However, pairwise concurrences do not capture genuine multipartite entanglement, particularly in asymmetric systems with nonuniform couplings, where entanglement is not symmetrically distributed across qubit pairs. To address this, we first consider the residual three-tangle, which measures the amount of genuine entanglement shared among three parties [44,58]. For a three-qubit pure state, the three-tangle is defined in terms of the pairwise concurrences: [44,58] $\tau = \mathcal{C}_{j(kl)}^2 - \mathcal{C}_{jk}^2 - \mathcal{C}_{jl}^2$, where $\mathcal{C}_{j(kl)} = \sqrt{2(1 - \text{Tr}[\hat{\rho}_j^2(t)])}$ signifies a bipartition concurrence between qubit j and the subsystem kl , and $\rho_j(t)$ denotes the reduced density matrix of qubit j . Furthermore, by expressing the three-qubit state as $|\psi(t)\rangle = \sum_{k=1}^8 \tilde{a}_k(t) |\alpha_k\rangle$, the residual three-tangle can be simplified to [44,58]

$$\tau = 4|d_1 - 2d_2 + 4d_3|, \quad (13)$$

where $d_1 = (\tilde{a}_1\tilde{a}_8)^2 + (\tilde{a}_2\tilde{a}_7)^2 + (\tilde{a}_3\tilde{a}_6)^2 + (\tilde{a}_4\tilde{a}_5)^2$, $d_2 = \tilde{a}_1\tilde{a}_8(\tilde{a}_4\tilde{a}_5 + \tilde{a}_3\tilde{a}_6 + \tilde{a}_2\tilde{a}_7) + \tilde{a}_3\tilde{a}_4\tilde{a}_5\tilde{a}_6 + \tilde{a}_4\tilde{a}_5\tilde{a}_2\tilde{a}_7 + \tilde{a}_2\tilde{a}_7\tilde{a}_3\tilde{a}_6$, and $d_3 = \tilde{a}_1\tilde{a}_7\tilde{a}_4\tilde{a}_6 + \tilde{a}_2\tilde{a}_8\tilde{a}_3\tilde{a}_5$. This quantifier, i.e., Eq. (13), satisfies the following properties: (1) it is symmetric under qubit permutations, (2) it becomes unity for GHZ states, and (3) it vanishes for both W and biseparable states. Thus, together with pairwise concurrences, the three-tangle help us distinguish GHZ and W states in the dynamics of our non-Hermitian qubits.

We also apply entanglement entropy, which is one of the most widely used measure of quantum entanglement in large systems [59]. Entanglement entropy characterizes the full convertibility between entangled states, analogous to entropy in thermodynamics [61]. It measures the entanglement between a chosen subsystem and the rest of the system. Here, we partition the total n -qubit system into a single-qubit subsystem and an $(n - 1)$ -qubit complement. The reduced density matrix for qubit j is obtained by tracing out the other qubits from the global density matrix $\hat{\rho}(t) = |\psi(t)\rangle\langle\psi(t)|$

$$\hat{\rho}_j(t) = \text{Tr}_{\{1,\dots,n\}\setminus\{j\}}[\hat{\rho}(t)] = \begin{pmatrix} \mathcal{E}_j & \mathcal{F}_j \\ \mathcal{F}_j^* & \mathcal{G}_j \end{pmatrix}, \quad (14)$$

with eigenvalues $\Lambda_{\pm}^j = [1 \pm \sqrt{(\mathcal{E}_j - \mathcal{G}_j)^2 + 4|\mathcal{F}_j|^2}]/2$. The single-qubit entanglement entropy, which quantifies the quantum correlation between an individual qubit and the rest of the system, is then given by

$$S_j = -\text{Tr}[\hat{\rho}_j(t) \ln \hat{\rho}_j(t)] = -\Lambda_-^j \ln \Lambda_-^j - \Lambda_+^j \ln \Lambda_+^j, \quad (15)$$

which becomes maximal, $S_j = \ln 2$, for each qubit subsystem in a multipartite GHZ state, and attains $S_j = \ln 3 - \frac{2}{3} \ln 2 \approx 0.637$ for a multipartite W state [62].

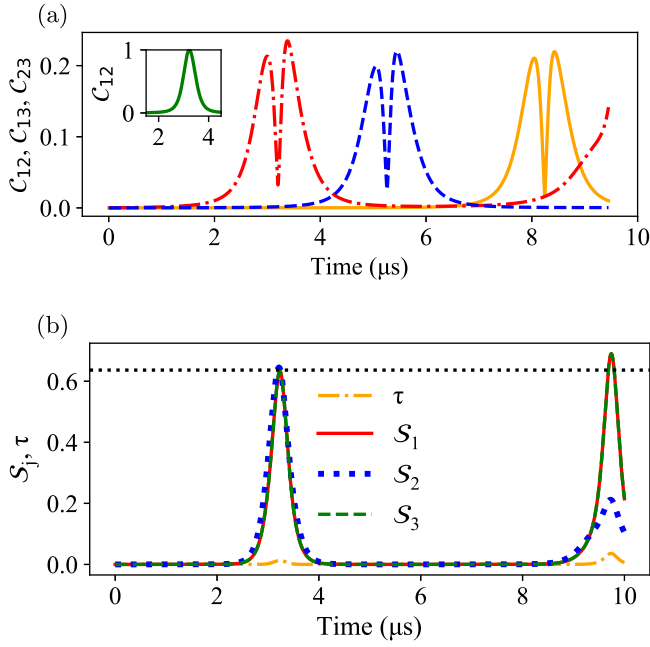


FIG. 1. Dynamics of three non-Hermitian superconducting qubits from an initial state $|\psi(0)\rangle = 2^{-3/2}(|f\rangle - i|e\rangle)^{\otimes 3}$. (a) Pairwise concurrences for $\Delta = 0$, $\gamma = 6 \text{ rad}/\mu\text{s}$, $\{\Omega, J\} = \{1.512 \text{ rad}/\mu\text{s}, 10^{-5} \text{ rad}/\mu\text{s}\}$ (solid orange), $\{\Omega, J\} = \{1.529 \text{ rad}/\mu\text{s}, 10^{-4} \text{ rad}/\mu\text{s}\}$ (dotted blue), and $\{\Omega, J\} = \{1.576 \text{ rad}/\mu\text{s}, 10^{-3} \text{ rad}/\mu\text{s}\}$ (dash-dotted red). The inset shows bipartite entanglement (solid green) when the third qubit is disconnected from the composite system as a comparison. (b) Entanglement entropies [first qubit (solid red), second qubit (dotted blue), and third qubit (dashed green)], and residual three tangle (dash-dotted orange) of non-Hermitian qubits coupled with nearest neighbor coupling for $\{\Omega, J\} = \{1.576 \text{ rad}/\mu\text{s}, 10^{-3} \text{ rad}/\mu\text{s}\}$. Dotted-black line in (b) marks $S_1 = S_2 = S_3 = \ln 3 - (2/3)\ln 2$ (see text) which is the case for maximally entangled three-qubit W state.

We next present our main results, starting with weakly coupled non-Hermitian qubits in the $\mathcal{PT}$ -symmetric regime. We assume identical system parameters for all qubits, i.e., $\Omega_j = \Omega$, $\gamma_j = \gamma$, $\Delta_j = 0$, and uniform coupling $J_{jk} = J$, unless stated otherwise.

III. TRIPARTITE ENTANGLEMENT IN NON-HERMITIAN QUBITS

A. All-to-all and nearest-neighbor couplings

In this section, we first summarize entanglement of identical non-Hermitian qubits with all-to-all coupling as shown in Fig. 1(a). We then extend the discussion to the case with nearest-neighbor coupling in Fig. 1(b).

In Fig. 1(a), we present pairwise concurrences C_{jk} of cyclically coupled non-Hermitian qubits for different optimal driving frequencies Ω and coupling strengths J . In these parameter regimes, pairwise concurrences C_{ij} immediately form narrow dips as a signature of highly entangled states produced at specific times. When one qubit decouples from the three-qubit system, the remaining non-Hermitian qubits become maximally entangled with each other. This is illustrated in the inset of Fig. 1(a), where we show maximal entanglement between the first two qubits with $C_{12} \approx 1$ at $t \approx 3.23 \mu\text{s}$.

On the other hand, the bipartite entanglement between any qubit pairs vanishes for cyclically coupled three qubits, and entanglement among the three-qubit system emerges with entanglement entropies $S_j \approx \ln 2$ and $\tau \approx 1$, which is the property of three-qubit GHZ state [44,58]. Since the resulting state occupies all computational basis with non-negligible amplitudes, the state does not exactly match the standard GHZ form. Nevertheless, the local unitary (LU)-invariant properties of the entanglement measures considered in this work, i.e., $C_{jk} \approx 0$, $S_j \approx \ln 2$, and $\tau \approx 1$, indicate that the state belongs to the GHZ class [53,56,63].

We next demonstrate three-qubit system with nearest-neighbor coupling as shown in Fig. 1(b). In this setup, entanglement entropies S_j of the reduced qubits remain nearly identical for over $t = 8 \mu\text{s}$ that surpasses the evolution period $T_{\text{NHQ}} = 4\pi/\sqrt{16\Omega^2 - \gamma^2} = 6.5 \mu\text{s}$ of the qubits. At $t = 3.23 \mu\text{s}$, the qubits achieve entanglement entropies $S_1 \approx S_2 \approx S_3 \approx \ln 3 - \frac{2}{3}\ln 2$, a vanishing residual three-tangle $\tau \approx 0$ (see dash-dotted orange line), and pairwise concurrences $C_{jk} \approx \frac{2}{3}$. These values are consistent with the characteristic entanglement properties of a three-qubit W state and its equivalence class [44,58]. This indicates that the fast entangled states generated under nearest-neighbor coupling can be mapped to a standard W state under LOCC [54].

However, this property of W state disappears in the next entanglement revival phase around $T_{\text{NHQ}} + 3.23 \mu\text{s}$ due to the build up of strong bipartite quantum correlation between the first and third qubits with $S_1 = S_3 \approx \ln 2$, while the middle qubit acting as a high-purity channel with smaller entanglement entropy.

The three-qubit entangled states discussed above are induced by the higher-order EP as the system transitions into the $\mathcal{PT}$ -symmetric regime, where the qubits exhibit increased sensitivity to their interqubit coupling. This spectral response, combined with the interplay among coupling configurations, coherent drive, and non-Hermitian effects, determines the evolution of the system toward either GHZ- or W-type entangled states. Without coupling, the qubits exhibit Rabi-like oscillations due to coherent energy exchange with the drive. The introduction of all-to-all coupling enhances populations of the nondegenerate states $|fff\rangle$ and $|eee\rangle$, suppresses those of the degenerate states, and establishes smooth and continuous phase evolution [33]. Under these conditions, the system evolves toward a GHZ state at $t = 3.23 \mu\text{s}$. In contrast, under nearest-neighbor coupling, the partial lifting of degeneracy among singly and doubly excited states creates asymmetry in populations and relative phases, which favors the formation of a W state.

Further approaching the higher-order EP from the $\mathcal{PT}$ -symmetric regime requires smaller coupling strengths and longer timescales to generate two- and three-qubit entangled states [32,33]. This can be attributed to the fact that the response of the qubits to their interqubit coupling weakens for smaller coupling strengths, and the qubits slowly evolve near the higher-order EPs.

Although entanglement generation experiences delays near the EPs [solid orange line in Fig. 1(a)], it remains significantly faster compared to the timescales of Hermitian qubits. The timescale for the latter is inversely proportional to the

interqubit coupling strengths [64] and can be related to the timescale of the all-to-all coupled non-Hermitian qubits as $t_{\text{NHQs}}^{\text{Opt}} \propto J \times t_{\text{HQs}}^{\text{Opt}}$, where $t_{\text{NHQs}}^{\text{Opt}}$ and $t_{\text{HQs}}^{\text{Opt}}$ represent the optimal entanglement generation times for the non-Hermitian and Hermitian qubits, respectively. Furthermore, our numerical results indicate that the timescale $t_{\text{NHQs}}^{\text{Opt}}$ is approximately 50% of the complete evolution period T_{NHQ} when the non-Hermitian qubits are initialized in coherent superpositions $|\psi(0)\rangle = 2^{-3/2}(|f\rangle - i|e\rangle)^{\otimes 3}$ and approaches T_{NHQ} when initialized in the state $|\psi(0)\rangle = |fff\rangle$ [33].

Moreover, our system can achieve both bipartite and tripartite entangled states within the same timescale (see Figs. 1(a) and 1(b), including the inset), regardless of the differences in EP orders between two and three non-Hermitian qubits. However, this situation may change for multiple-qubit configurations. For example, higher-order EPs of four qubits can further reduce timescale of entanglement generation below that of two and three non-Hermitian qubits [33].

B. Robustness to nonuniform couplings and off-resonant drivings

Next we discuss the robustness of entanglement in non-Hermitian qubits against nonuniform couplings and off-resonant drivings. We demonstrate the residual three-tangle for all-to-all coupled non-Hermitian qubits in Fig. 2(a) at $t \approx 3.23 \mu\text{s}$. We find that finite entangled states persist over a broad range of nonuniform interqubit couplings but vanish with larger coupling variations. As shown earlier in Sec. III A, this setup produces a GHZ state with $\tau \approx 1$ when $J_{jk} \approx 10^{-3} \text{ rad}/\mu\text{s}$. Under this condition, we present robustness of the state to off-resonant driving effects in Fig. 2(b) where we show consistency entanglement with the resonant case as long as $\Delta \lesssim J^{\text{Opt}}$ with an optimal coupling $J^{\text{Opt}} = J_{jk}^{\text{Opt}} = 10^{-3} \text{ rad}/\mu\text{s}$. However, large detuning frequencies significantly exceeding an optimal interqubit coupling J^{Opt} hampers the entanglement generation.

Furthermore, in Fig. 2(c), we illustrate entanglement entropies of the reduced qubits at $t = 3.23 \mu\text{s}$ for the nearest-coupling ($J_{13} = 0$) and for finite J_{23} as we vary J_{12} . When $J_{12} = 0$, this three-qubit configuration only supports bipartite entanglement between the last two qubits, as the first qubit is decoupled from the composite system (see Sec. III A also). Coupling the first qubit to the second qubit and increasing J_{12} slightly reduces this bipartite entanglement, while the purity of the first qubit decreases rapidly, suggesting possible three-qubit entanglement generation. Interestingly, at $J_{12} = J_{23} = 10^{-3} \text{ rad}/\mu\text{s}$, the entanglement entropies of the reduced qubits converge to nearly equal values $S_j \approx \ln 3 - (2/3)\ln 2$ and the residual three tangle almost vanishes there [see Fig. 1(b)]. This indicates that optimal W state generation also requires uniform nearest-neighbor coupling, and the robustness of entanglement to nonuniform coupling is evident from Fig. 2(c) where the decreased entanglement entropies of the second and third qubits are compensated by an increased entanglement entropy of the first qubit over a wide range of J_{12} especially near $S_j \approx \ln 3 - (2/3)\ln 2$. Meanwhile, further increasing J_{12} does not benefit entanglement generation.

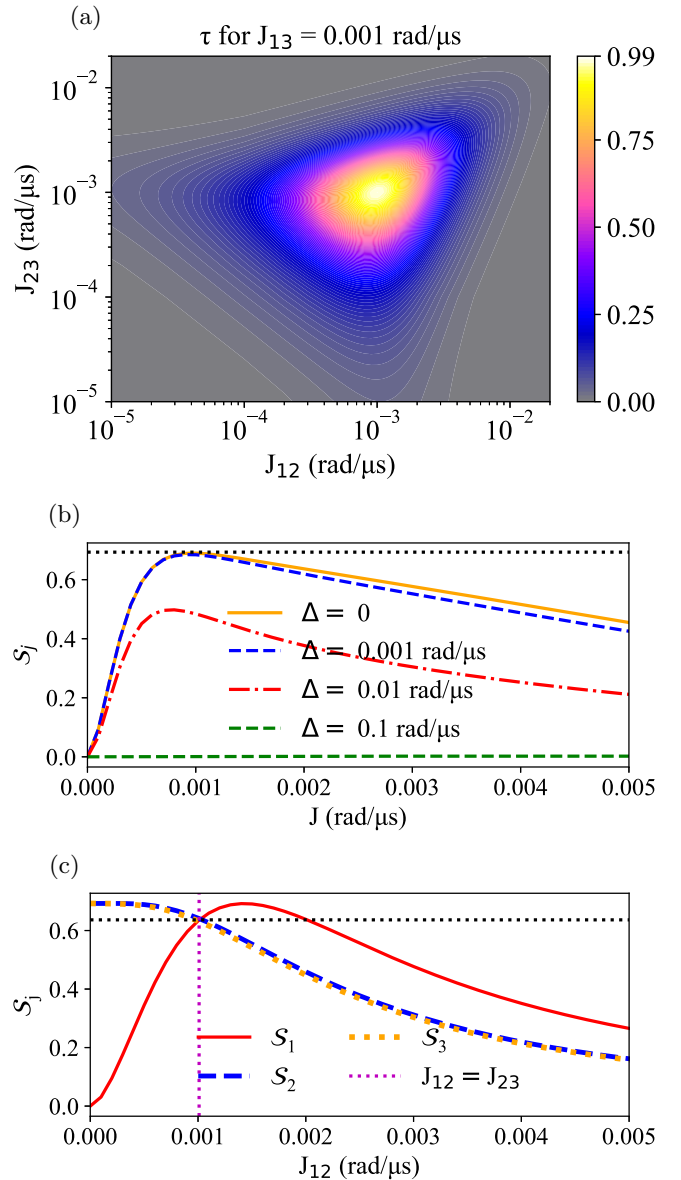


FIG. 2. Effect of nonuniform coupling and off-resonant driving on the entanglement of non-Hermitian qubits. (a) Residual three-tangle of nonuniformly coupled non-Hermitian qubits with all-to-all configuration for $\{\Omega, J_{13}, t\} = \{1.576 \text{ rad}/\mu\text{s}, 10^{-3} \text{ rad}/\mu\text{s}, 3.23 \mu\text{s}\}$. (b) Robustness of optimal entanglement [bright region in (a)] under off-resonant driving for $\Delta = 10^{-3}$ (dashed blue), 10^{-2} (dash-dotted red), and $10^{-1} \text{ rad}/\mu\text{s}$ (dotted green). The resonant case $\Delta = 0$ (solid orange) is shown for comparison. (c) Entanglement with nonuniform nearest-neighbor coupling, $\{\Omega, J_{23}, J_{13}, t\} = \{1.576 \text{ rad}/\mu\text{s}, 10^{-3} \text{ rad}/\mu\text{s}, 0, 3.23 \mu\text{s}\}$. Dotted dark lines indicate upper bounds (b) $S_j = \ln 2$ for a GHZ state and (c) $S_j = \ln 3 - (2/3)\ln 2$ for a W state (see text), while the dotted magenta line in (c) represents uniform nearest-neighbor coupling ($J_{12} = J_{23}$) for optimal W state generation. The initial state is the same as in Fig. 1, and rest of the parameters are set to $\Delta = 0$, $\Omega = 1.576 \text{ rad}/\mu\text{s}$, $\gamma = 6 \text{ rad}/\mu\text{s}$ unless otherwise specified.

C. Strong driving and strong coupling effects

In the previous section, we have discussed that three identical coupled non-Hermitian qubits are necessary to speed up

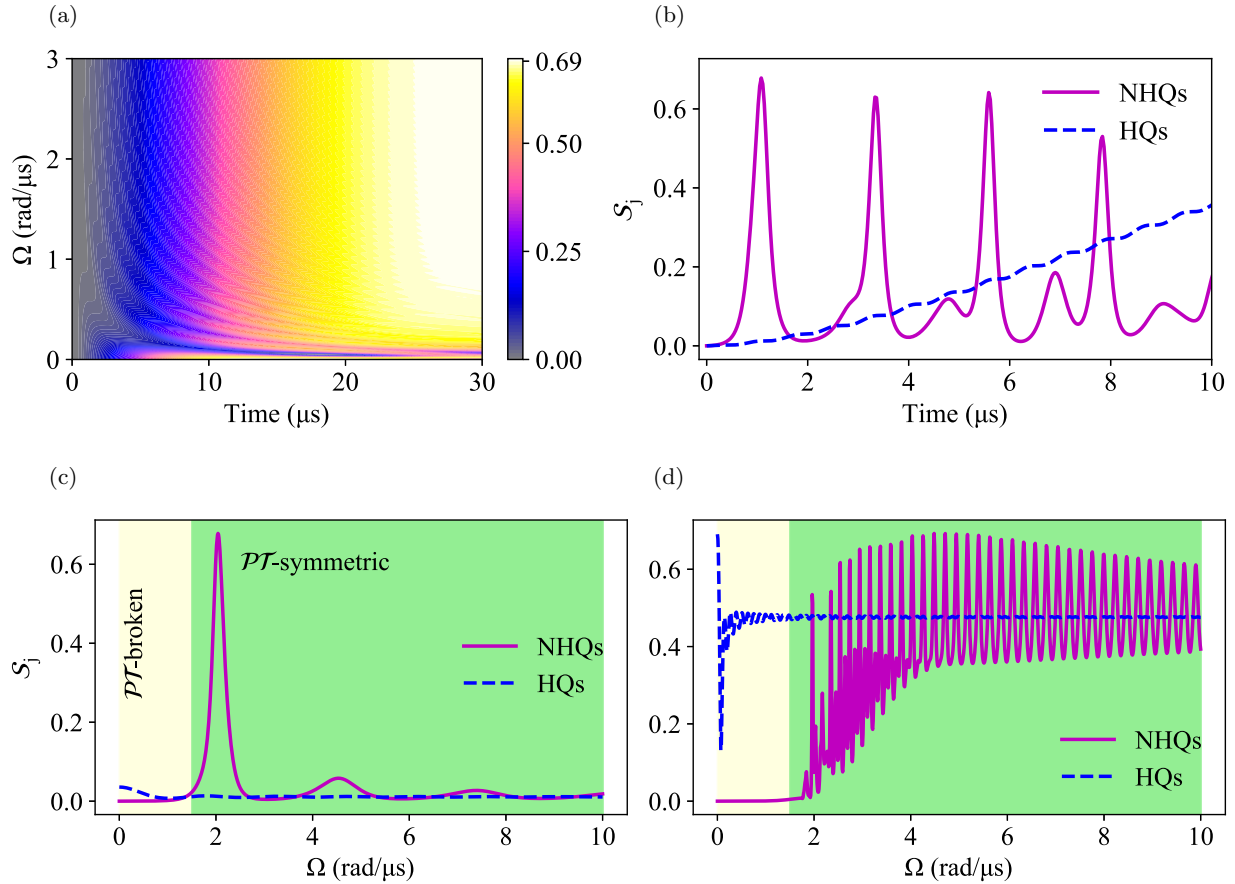


FIG. 3. Effect of strong coupling and strong driving on entanglement of the non-Hermitian qubits from the same initial state in Fig. 1. (a) Entanglement entropy of reduced Hermitian qubits with parameters $J = 0.1$ rad/ μ s, $\Delta = 0$, and $\gamma = 0$. (b) Comparison of entanglement entropies for reduced Hermitian qubits (dashed blue, $\gamma = 0$) and non-Hermitian qubits (solid magenta, $\gamma = 6$ rad/ μ s), under conditions $J = 0.1$ rad/ μ s, $\Delta = 0$, $\Omega = 2.04$ rad/ μ s, and $\gamma = 6$ rad/ μ s. (c), (d) Entanglement entropy of reduced qubits as a function of Rabi frequency: (c) at an early stage of the dynamics ($t = 1.08$ μ s) and (d) at a later stage ($t = 13$ μ s). The system undergoes $\mathcal{PT}$ protected spectral transition from broken phase (light yellow) to passive symmetric phase (light green) by tuning the Rabi frequency as shown (c) and (d).

tripartite entanglement generation. We next make a detailed analysis of this entanglement by considering a strong driving regime or strong coupling regime. We increase the coupling constant to $J = 0.1$ rad/ μ s for which the timescale of entanglement generation in the non-Hermitian qubits is still better than that of the Hermitian qubits. This result points out that EPs are essential for fast entanglement generation.

We first consider the entanglement of all-to-all coupled Hermitian qubits in Fig. 3(a) as a reference. These qubits produce GHZ states nearly at 13 μ s even without driving them. Exposing the individual qubits to the driving field is, indeed, helpful for generating a three-qubit GHZ state and its classes that remain stable for a long time. In Fig. 3(b), we compare the entanglement of Hermitian and non-Hermitian qubits under an optimal driving amplitude $\Omega = 2.04$ rad/ μ s associated with the coupling $J = 0.1$ rad/ μ s. Non-Hermitian qubits generate entangled states that periodically oscillate with time, and a stronger entangled state appears earlier around $t = 1.08$ μ s. This timescale is about 20 times faster than the rate of entanglement generation in the driven Hermitian qubits. Furthermore, Fig. 3(c) illustrates that further increasing the Rabi frequency drives the non-Hermitian qubits from the $\mathcal{PT}$ -broken (light yellow) to the $\mathcal{PT}$ -symmetric phase (light

green), causing rapid entanglement growth, while Hermitian qubits show negligible entanglement.

The entanglement entropy of qubits at a later stage of dynamics is shown in Fig. 3(d) with an example at $t = 13$ μ s. Hermitian qubits become entangled even without driving and form stable entangled states with finite strength when driven. In contrast, non-Hermitian qubits fail to generate entanglement in the $\mathcal{PT}$ -broken regime due to significant losses over time that require strong driving to counteract. This indicates that the effects of non-Hermiticity, including EPs, stem from the competition between driven and dissipative processes in our system. For strong driving, these entangled states oscillate rapidly and follow the Hermitian result for $\Omega \geq \gamma$. We have also checked that only specific coupling strengths can yield entanglement in non-Hermitian qubits, as increasing the coupling constant alone does not overcome the loss, unlike driving. In contrast, Hermitian qubits exhibit increased entanglement with higher coupling strengths.

D. Effect of quantum jump on entanglement dynamics

While our previous analysis is based on the effective non-Hermitian Hamiltonian, a complete description of an open

quantum system must also incorporate quantum jump processes resulting from spontaneous decay. These processes can be treated using the quantum trajectory approach [16,65,66], which describes the evolution of pure quantum states under stochastic quantum jumps. In this approach, the state vector $|\psi(t)\rangle$ is given, prior to normalization, by

$$d|\psi(t)\rangle = -i\hat{H}_{\text{eff}} dt + \sum_{j=1}^n \left(\frac{\hat{L}_j |\psi(t)\rangle}{\|\hat{L}_j |\psi(t)\rangle\|} - |\psi(t)\rangle \right) dN_j(t), \quad (16)$$

where $\hat{H}_{\text{eff}} = \hat{H} - \frac{i}{2} \sum_{j=1}^n \hat{L}_j^\dagger \hat{L}_j$ represents the effective non-Hermitian Hamiltonian, $\hat{L}_j = \sqrt{\gamma_j^s} \hat{\sigma}_j$ denotes the collapse operator corresponding to spontaneous decay rate $\gamma_j^s = \gamma^s$, and $dN_j(t)$ is a stochastic Poisson increment. We note that the engineered loss rate γ_j [see Eq. (1)] induces decay processes outside the qubit subspace and therefore contributes only to the non-Hermitian part of the evolution in Eq. (16).

When $dN_j(t) = 1$, a quantum jump occurs at site j , and the state vector is updated as $\hat{L}_j |\psi(t)\rangle / \|\hat{L}_j |\psi(t)\rangle\|$. In contrast, when $dN_j(t) = 0$, no jump occurs, and the system is governed by the effective non-Hermitian Hamiltonian $\hat{H}_{\text{eff}}$. By averaging over many such pure-state quantum trajectories, one recovers the mixed-state evolution governed by the Lindblad master equation [16]

$$\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H}_{\text{eff}}, \hat{\rho}(t)] + \sum_{j=1}^n \hat{L}_j \hat{\rho}(t) \hat{L}_j^\dagger, \quad (17)$$

where the first term describes the non-Hermitian evolution, and the second term accounts for quantum jumps at the level of ensemble-average.

Under an effective non-Hermitian Hamiltonian where engineered loss γ dominates, spontaneous decay shifts the location of the higher-order EPs from $\Omega^{\text{EP}} = \gamma/4$ to $\tilde{\Omega}^{\text{EP}} = \gamma^{\text{eff}}/4$ with $\gamma^{\text{eff}} = \gamma - \gamma^s$. These new EPs now determine the optimal conditions for entanglement generation. Specifically, for each weak coupling constant J , one can still achieve maximal entanglement by tuning the driving amplitude, provided that the dissipation is adjusted to the effective value γ^{eff} . Therefore, spontaneous decay shifts the optimal parameter regimes for maximal entanglement in parameter space, but does not qualitatively alter the entanglement properties or their degree under post-selection.

To gain further insight into the effects of decoherence, we present the dynamics of tripartite entanglement in Fig. 4(a), using the same parameter regimes as in Secs. III A and III C, and considering different spontaneous decay rates. The inclusion of spontaneous emission introduces an extra non-Hermitian term $-\frac{i}{2} \sum_{j=1}^3 \gamma^s \hat{\sigma}_j^\dagger \hat{\sigma}_j$ in the effective Hamiltonian, and this term suppresses entanglement growth and shifts the peak of the entanglement entropy to earlier times [see solid lines in Fig. 4(a)]. Each nonzero γ^s lowers the effective decay rate γ^{eff} , thereby shifting the higher-order EPs to smaller driving amplitudes, i.e., $\Omega^{\text{EP}} > \tilde{\Omega}^{\text{EP}}$. Consequently, for finite spontaneous decay, the parameters optimal under the engineered decay rate γ_j are no longer optimal under the modified rate γ^{eff} , though they can still support finite entanglement. The timing of entanglement generation is determined

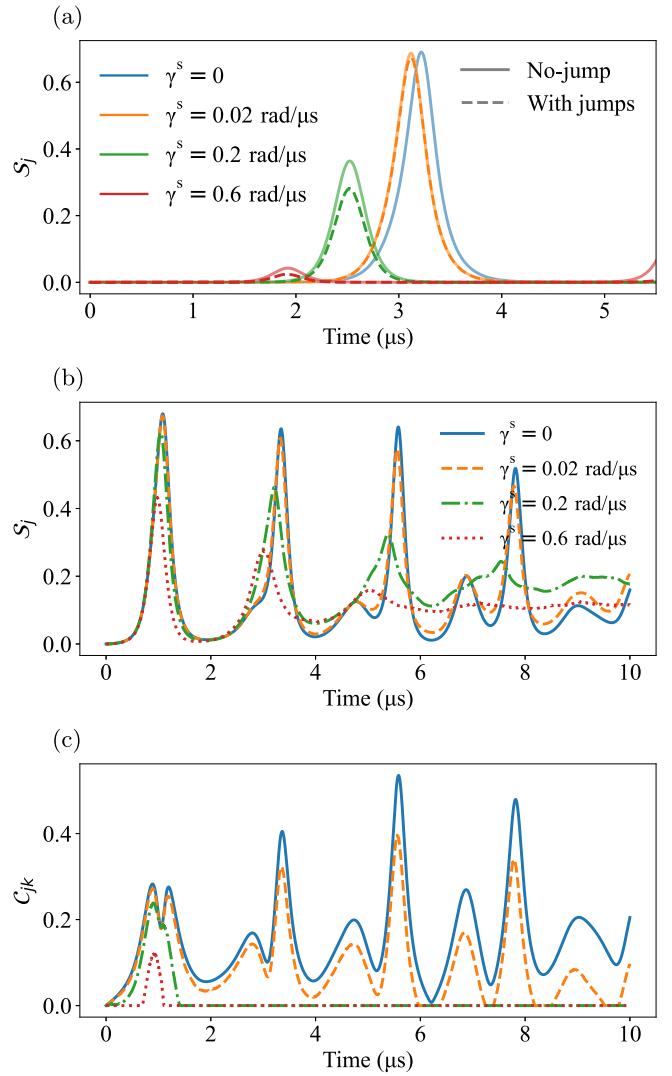


FIG. 4. Effect of quantum jumps on entanglement dynamics. (a) Trajectory-averaged entanglement entropy S_j for three non-Hermitian qubits with $\{\Omega, J, \gamma\} = \{1.576 \text{ rad}/\mu\text{s}, 10^{-3} \text{ rad}/\mu\text{s}, 6 \text{ rad}/\mu\text{s}\}$, and spontaneous decay rates $\gamma^s = 0$ (light blue), $0.02 \text{ rad}/\mu\text{s}$ (orange), $0.2 \text{ rad}/\mu\text{s}$ (green), and $0.6 \text{ rad}/\mu\text{s}$ (red). Solid lines represent deterministic non-Hermitian evolution without quantum jumps, and dashed lines involve quantum jumps due to the spontaneous decays. (b) Dynamics of S_j and (c) pairwise concurrences C_{jk} computed from ensemble-averaged state for the qubits with $\{\Omega, J, \gamma\} = \{2.04 \text{ rad}/\mu\text{s}, 0.1 \text{ rad}/\mu\text{s}, 6 \text{ rad}/\mu\text{s}\}$ and $\gamma^s = 0$ (solid light blue), $0.02 \text{ rad}/\mu\text{s}$ (dashed orange), $0.2 \text{ rad}/\mu\text{s}$ (dash-dotted green), and $0.6 \text{ rad}/\mu\text{s}$ (dotted red). All results are obtained using the Monte Carlo wave function method with 1000 quantum trajectories.

by the ratio $\Omega/\tilde{\Omega}^{\text{EP}}$. As γ^s increases, $\tilde{\Omega}^{\text{EP}}$ decreases, and the fixed driving amplitude $\Omega = 1.576 \text{ rad}/\mu\text{s}$, being stronger than $\tilde{\Omega}^{\text{EP}}$, accelerates finite entanglement generation.

Furthermore, we demonstrate the robustness of entanglement dynamics against quantum jump processes [see dashed lines in Fig. 4(a)]. In the quantum trajectory picture, the system evolves coherently under an effective non-Hermitian Hamiltonian between successive quantum jump events, and this allows entanglement to develop through non-Hermitian

dynamics under weak interqubit couplings. A quantum jump interrupts this evolution by collapsing the state of the qubits and erasing quantum correlations [67]. An increase in the spontaneous emission rate γ^s results in more frequent jump events, which reduces the peak of the trajectory-averaged entanglement entropy. Therefore, the non-Hermitian evolution governs the emergence of quantum entanglement, while quantum jump processes reduce its strength. Nevertheless, in the parameter regime of interest, $\gamma \gg \gamma^s$, these quantum jump events are rare compared to the dominant deterministic evolution governed by the effective non-Hermitian Hamiltonian. As a result, multipartite entangled states rapidly generated by the higher-order EPs remains robust even in the presence of decoherence effects.

To further elucidate the behavior of this noisy model, we compare the trajectory-averaged entanglement entropy with the pairwise concurrences computed from the ensemble-averaged state. The results are shown in Fig. 4(b) and Fig. 4(c), respectively, for moderate coupling $J = 0.1$ rad/ μ s. We find that the trajectory-averaged entanglement entropy exhibits greater resilience to decoherence, as can be seen in Fig. 4(b), particularly in the short-time dynamics. In contrast, the pairwise concurrences of the mixed state decay significantly with increasing spontaneous emission rate [see Fig. 4(c)]. This difference arises because the ensemble-averaged state is a statistical mixture of pure-state trajectories, leading to a loss of purity and relative phase information, and thus a suppression of quantum correlations [66,68]. This indicates that trajectory-specific features are not captured in ensemble-averaged state descriptions, and that entanglement quantifiers, being nonlinear functions of the density matrix, yield lower values when applied to $\hat{\rho}(t)$ than when averaged over individual quantum trajectories. Therefore, multipartite entangled states generated through the monitored dynamics of non-Hermitian qubits exhibit robustness against decoherence.

IV. HYBRID SETUPS OF HERMITIAN AND NON-HERMITIAN QUBITS

In realistic quantum platforms such as circuit QED and photonic systems, it is now experimentally possible to engineer dissipation locally at the level of individual quantum systems through bath engineering [69,70]. Motivated by this experimental capability, we consider a hybrid quantum system composed of superconducting qubits, where some of the qubits, labeled by $j \in \mathcal{NH}$, are subject to non-Hermitian dynamics through continuous quantum monitoring [7], while the remaining qubits, labeled by $k \in \mathcal{H}$, are isolated from dissipation channels and evolve under a Hermitian dynamics. In particular, the non-Hermitian qubits are governed by the effective Hamiltonian

$$\hat{H}_j = \left(\Delta_j - \frac{i\gamma_j}{2} \right) \hat{\sigma}_j \hat{\sigma}_j^\dagger + \Omega_j \hat{\sigma}_j^x, \quad (18)$$

where Δ_j denotes the detuning, Ω_j represents the Rabi frequency, and $\gamma_j > 0$ is the engineered dissipation rate resulting from coupling to a lossy channel. Moreover, we assume that certain qubits are decoupled from the lossy channel and evolve

locally under the Hamiltonian

$$\hat{H}_k = \Delta_k \hat{\sigma}_k \hat{\sigma}_k^\dagger + \Omega_k \hat{\sigma}_k^x, \quad (19)$$

which describes a coherently driven dynamics of the Hermitian qubits $k \in \mathcal{H}$.

To enable coherent coupling across the hybrid system, we introduce an interaction Hamiltonian written as

$$\hat{H}_{\text{int}} = \sum_{l \neq m=1} J_{lm} (\hat{\sigma}_l^\dagger \hat{\sigma}_m + \hat{\sigma}_l \hat{\sigma}_m^\dagger), \quad (20)$$

where $l, m \in \mathcal{H} \cup \mathcal{NH}$ index all qubits in the system, and J_{lm} denotes the coupling strength between qubits l and m . This coupling Hamiltonian leads to direct excitation transfer between any pair of qubits including Hermitian to Hermitian, non-Hermitian to non-Hermitian, and Hermitian to non-Hermitian combinations, and provides coherent pathways through which the local dynamics of both Hermitian and non-Hermitian qubits can influence the global behavior of the hybrid system.

The total Hamiltonian that describes the hybrid system dynamics is then given by

$$\hat{H}_{\text{hbd}} = \sum_{k \in \mathcal{H}} \hat{H}_k + \sum_{j \in \mathcal{NH}} \hat{H}_j + \hat{H}_{\text{int}}. \quad (21)$$

This hybrid Hamiltonian captures the coherent, unitary evolution of Hermitian qubits while incorporating the nonunitary dynamics of non-Hermitian qubits at the same time. Such a configuration is experimentally feasible in circuit QED platforms using transmon qubits [4,7,69], and can help study dissipation-driven entanglement control and roles of higher-order EPs.

Previous studies have shown that coupling a single Hermitian qubit to a non-Hermitian one gives rise to EPs that can be exploited to generate maximal bipartite entanglement [71] and used to realize the quantum Hatano–Nelson model with asymmetric hopping [72]. In this work, we extend the investigation to hybrid configurations involving three qubits and analyze how non-Hermiticity, through its associated EPs, affects tripartite entangled states.

We first demonstrate in Fig. 5(a) that the three-non-Hermitian qubits generate the maximally tripartite entangled states around $t = 3.23 \mu$ s (see Ref. [33] for the details). Next we present entanglement entropies of one Hermitian and two non-Hermitian qubits in Fig. 5(b), in which two non-Hermitian qubits are maximally entangled while the Hermitian qubit remains separable ($S_1 \approx 0$). Without coupling, this qubit-arrangement possesses two symmetrical fourth-order EPs at $\Omega = 1.5$ rad/ μ s, and one of the EPs induces rapid entanglement build up between the non-Hermitian qubits when they are driven by $\Omega = 1.576$ rad/ μ s and uniformly coupled with $J_{jk} = 10^{-3}$ rad/ μ s, while the other EP is redundant and plays no role in the entanglement generation.

Moreover, the evolution times of the Hermitian and the two non-Hermitian qubits are incompatible due to the introduced non-Hermitian effects. Therefore, switching one of the qubits to be Hermitian will not generate genuine tripartite entanglement but only a bipartite entangled state. This approach can be extended to rapidly generate multiqubit biseparable

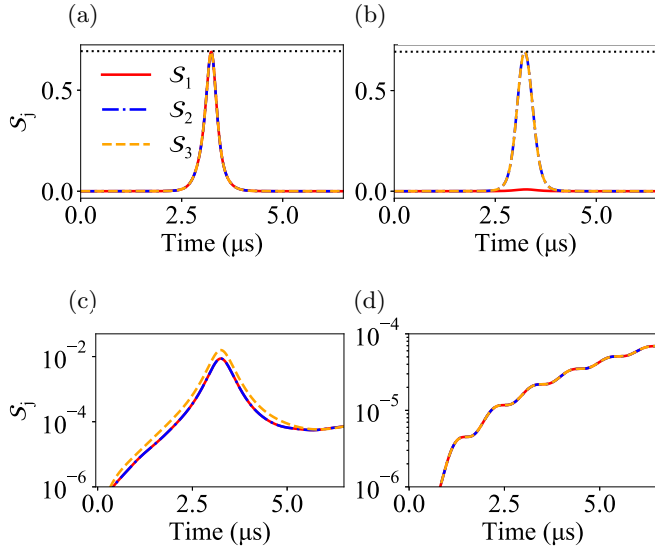


FIG. 5. Dynamics of quantum entanglement in the hybrid setups of non-Hermitian (NHQ) and Hermitian (HQ) qubits. (a)–(d) Entanglement entropies (a) for all non-Hermitian qubits, (b) for one Hermitian (solid red) and two non-Hermitian qubits (dash-dotted blue and dashed orange), (c) for two Hermitian qubits (solid red and dash-dotted blue) and one non-Hermitian qubits (dashed orange), and (d) for three identical Hermitian qubits. The initial state is the same as in Fig. 1, and the rest of the parameters are $\Delta = 0$, $J_{jk} = 10^{-3}$ rad/ μ s, $\Omega = 1.576$ rad/ μ s and $\gamma = 6$ rad/ μ s.

entangled states of GHZ classes, which can keep entanglement even if some qubits are detached similar to multiparty W states upon particle loss. Such biseparable entangled states can also be prepared by carefully controlling interqubit coupling, e.g., by setting $J_{23} = J_{13} = 0$ in three-qubit system while J_{12} is finite.

We also show in Fig. 5(c) that a setup of two Hermitian and one non-Hermitian qubits further reduces the strength of the entanglement, but we can still notice the signature of non-Hermitian qubit at $t = 3.23 \mu$ s and entanglement entropies can be larger than that of all three Hermitian qubits as shown in Fig. 5(d). Such combinations of Hermitian and non-Hermitian qubits highlight the applicability of higher-order EPs of the multiqubit system to achieve and maintain multipartite entangled states.

Genuine tripartite entanglement can be restored in the hybrid system by increasing the driving amplitude to $\Omega = 4$ rad/ μ s and the coupling strength to $J = 0.1$ rad/ μ s. The increased driving synchronizes the relative phase evolution among qubits, and the coupling redistributes populations to initiate the formation of genuine entangled states. This entanglement behavior occurs in both hybrid configurations; however, maximal entanglement develops more slowly in the two-Hermitian-qubit setup due to the weak influence of non-Hermiticity in this configuration. This indicates that higher-order EPs in non-Hermitian qubits are essential for achieving and sustaining genuine tripartite entanglement at rates significantly faster than those observed in both hybrid and Hermitian systems.

V. LARGE SYSTEM-SIZE EFFECT

We extend our study of three-qubit system to multipartite non-Hermitian qubit arrays to investigate how system-size affects entanglement dynamics. Specifically, we compute the entanglement entropy S_j as shown in Figs. 6(a) and 6(b) for $n = 3$ to 6 qubits by varying coupling strengths and resonant driving amplitudes. In both figures, we fix the dissipation rate at $\gamma = 6$ rad/ μ s across all system sizes to identify the unique effects of the exchange interactions and driving fields.

In Fig. 6(a), we show that each system size n exhibits a distinct optimal coupling strength J^{Opt} at which the entanglement entropy peaks near $S_j \approx \ln 2$. As n increases from 3 to 6, this optimal coupling shifts slightly to lower values, and new regimes exhibiting finite entanglement emerge. This shift originates from the total exchange interaction energy, which grows quadratically with system size as $\propto n^2 J$ in the all-to-all coupling scheme, while the local dissipation rate increases linearly as $\propto n\gamma$ [73]. As a result, larger systems experience relatively stronger collective interactions and require smaller coupling strengths to generate maximal multipartite entanglement at fixed drive strength Ω and dissipation rate γ . The time at which this entanglement is attained remains approximately constant across all system sizes. This indicates that the entanglement generation time depends on both the coupling strength and system size, and that the decrease in J^{Opt} is compensated by the enhanced collective interactions in larger systems to maintain this timescale. In the strong-coupling regime, it has been shown that entanglement exhibits oscillations with a period set by the coupling strength [64,74].

Moreover, we present the interplay between system-size scaling and resonant driving fields in Fig. 6(b), and how they affect entanglement dynamics under fixed coupling strength and dissipation rate. As the system-size increases, the time required to reach maximal entanglement decreases. However, this acceleration comes at the cost of a slightly stronger drive. The enhanced collective interaction in larger systems, under fixed coupling strength, mimics the behavior of small system sizes with increased coupling strength, and this effect alters the system's spectral structure and requires a higher drive strength to sustain maximal entanglement near higher-order EPs [32,33]. Therefore, multipartite entanglement generation in large non-Hermitian qubit systems requires weaker coupling strength due to enhanced collective interactions, and at fixed coupling, this collective effect demands a slightly stronger drive to accelerate the entanglement generation.

VI. EXPERIMENTAL PROPOSAL

We propose the realization of non-Hermitian superconducting qubits and their integration with Hermitian qubit subsystems using circuit QED architectures comprising n -transmon qubits embedded in a three-dimensional cavity and coupled through resonator-mediated interactions [75]. Each transmon is modeled as a three-level system with states $|g\rangle$, $|e\rangle$, and $|f\rangle$, where the ground state $|g\rangle$ functions as a probe and the qubit subspace is defined by the excited-state manifold $\{|e\rangle, |f\rangle\}$. A coherent drive is applied between $|e\rangle$ and $|f\rangle$ with a Rabi frequency Ω , and dissipation is introduced from $|e\rangle$ to $|g\rangle$ at rate γ via individual readout resonators to

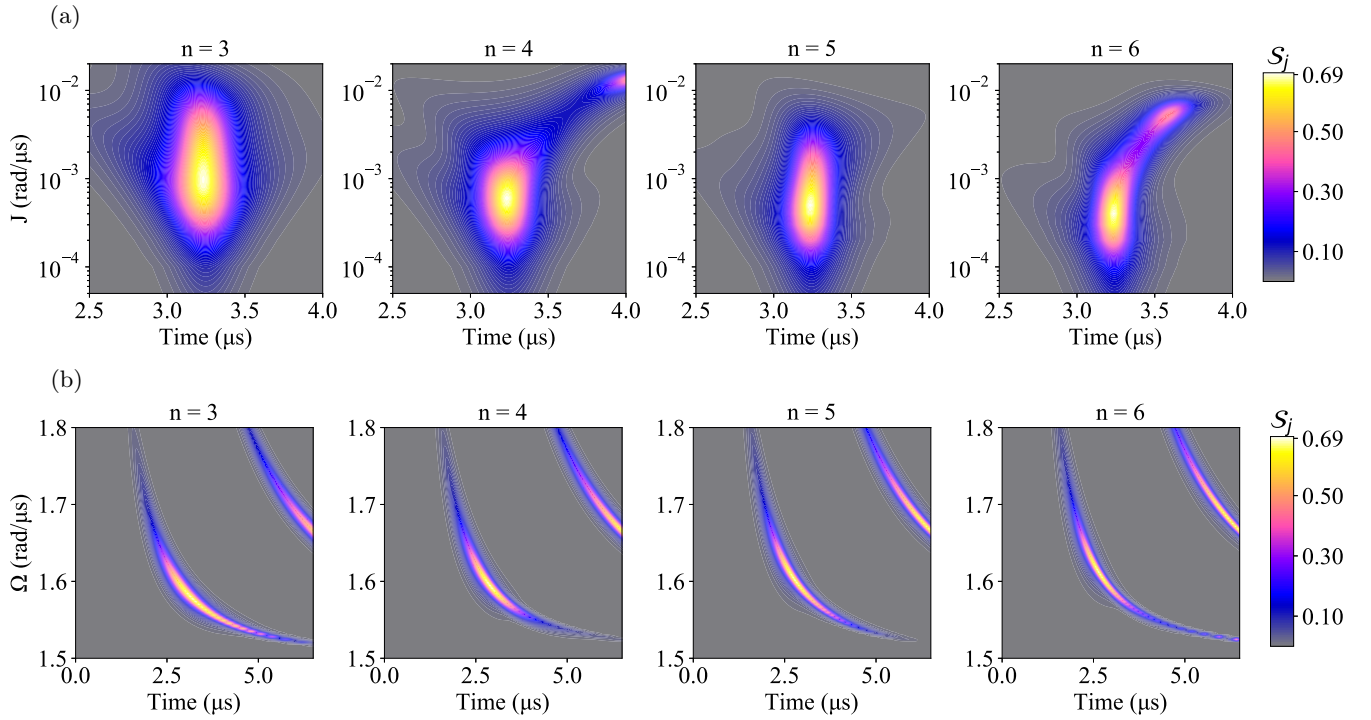


FIG. 6. Time evolution of single-site entanglement entropy in non-Hermitian qubit systems of sizes $n = 3, 4, 5$, and 6 . Panels (a) (top row) show the entanglement entropy as a function of coupling strength J and time, at fixed drive amplitude $\Omega = 1.576$ rad/ μ s. Panels (b) (bottom row) show the entanglement entropy as a function of drive amplitude Ω and time, at fixed coupling strength $J = 10^{-3}$ rad/ μ s. The dissipation rate is fixed at $\gamma = 6$ rad/ μ s in all cases. As the system size increases, the optimal coupling strength J that maximizes the entanglement entropy decreases, while the time at which maximal entropy occurs remains nearly constant. Larger systems exhibit slightly faster entanglement growth when driven with higher amplitudes Ω .

ensure selective monitoring of each qubit [4,7,69]. The interplay between coherent drive and dissipation allows to access the passive $\mathcal{PT}$ -symmetric regime, where higher-order EPs could emerge. By continuously monitoring the system's evolution and post-selecting trajectories without quantum jumps, the n qubits evolve under an effective non-Hermitian Hamiltonian Eq. (1).

Furthermore, the hybrid setup described by Eq. (21) is realized by decoupling some of the transmon qubits from the engineered lossy channels, while the remaining qubits are subject to continuous monitoring to induce non-Hermitian effects. The higher-order EPs give rise to sharp spectral transitions in the non-Hermitian qubits and enable the rapid generation of GHZ- and W-type multipartite entangled states under weak interqubit couplings, which can be mediated using microwave resonators and quantum routers [76–78].

Although real superconducting non-Hermitian qubits inevitably experience losses due to decoherence and fabrication-induced imperfections, our results show that the fast entangled states generated near higher-order EPs emerge on timescales much shorter than the intrinsic decoherence times and remain robust against coupling asymmetries and off-resonant driving errors. Therefore, the proposed entanglement mechanism is experimentally feasible with current superconducting circuit technologies [4,7].

Our results can also be simulated on IBM superconducting quantum processors, which are commonly used to implement a wide range of quantum phenomena, including quantum

communication [79], quantum algorithms [80,81], and foundational studies [82–84]. The main idea is embedding the system into a larger Hilbert space by introducing additional ancillary qubits [79,85,86]. This approach is based on the Stinespring dilation theorem [87], which guarantees that any physical transformation can be realized by applying a global unitary operation on the combined system-ancilla space, followed by measurement and conditional post-processing. The measurement outcomes on the ancilla correspond to continuous monitoring of the system and can induce the desired non-Hermitian evolution. In fact, the same logic of simulating non-Hermitian systems on IBM superconducting quantum processors is also used in Refs. [79,85].

VII. CONCLUSIONS

In conclusion, we have discussed the robustness of EP-induced multipartite entanglement in passive $\mathcal{PT}$ -symmetric non-Hermitian qubits and hybrid systems made up of Hermitian and non-Hermitian qubits. We considered all-to-all and nearest-neighbor interactions with uniform and nonuniform coupling strengths and examined strong driving effects where non-Hermitian qubits undergo spectral transitions protected by $\mathcal{PT}$ -symmetry.

We demonstrate that all-to-all coupled non-Hermitian qubits rapidly generate GHZ states which are characterized by narrow dips in pairwise concurrences, upper bounds of both residual three-tangle and entanglement entropies. In

contrast, nearest-neighbor coupled qubits produce W states on the same timescale. Furthermore, these entangled states are robust against off-resonant driving effects and can persist across a broad range of nonuniform coupling strengths, with optimal conditions being achieved under uniform coupling. In hybrid setups consisting Hermitian and non-Hermitian qubits, the introduced non-Hermiticity causes incompatible evolution times and prevents the creation of genuine tripartite entangled states, but biseparable entangled states can still be induced in the hybrid systems. This indicates that non-Hermiticity and its associated EPs are not only crucial for generating, speeding up, and maintaining multipartite entangled states, but also enable dissipation-driven switching between multipartite and biseparable entanglement.

We have also shown that driving the qubits with a strong Rabi frequency can sustain these entangled states in the $\mathcal{PT}$ -symmetric phase by counteracting losses, while strong interqubit coupling enhances the entanglement in the low dissipation regime. This indicates that the effects of non-Hermiticity, including EPs, arise from the interplay between driven and dissipative processes in our system. Moreover, we find that multipartite entanglement remains robust against decoherence induced by quantum jumps when analyzed at the trajectory level, where individual realizations preserve correlations that are otherwise suppressed in ensemble-averaged dynamics. In larger systems of non-Hermitian qubits, this multipartite entanglement generation occurs more rapidly and requires weaker coupling strengths due to enhanced collective interactions.

While our study focuses on passive $\mathcal{PT}$ -symmetric qubits and their Hamiltonian EPs, future research could consider Liouvillian EPs and effects of quantum jumps [16,17] or incorporate an additional gain to our system [1,11,34], particularly to enhance success rates of entanglement generation. Another promising direction would be the realization of the quantum Hatano-Nelson model in hybrid setups of Hermitian and non-Hermitian qubits [71,72]. Our model could also be implemented in superconducting circuits [4,7,88] and microwave quantum state router [76–78]. Furthermore, the result presented here can be extended to multipartite entangled states and their phase transition induced by skin effect [6].

ACKNOWLEDGMENTS

H.H.J. acknowledges support from the National Science and Technology Council (NSTC), Taiwan, under Grants No. NSTC-112-2119-M-001-007 and No. NSTC-112-2112-M-001-079-MY3, from Academia Sinica under Grant No. AS-CDA-113-M04, Taiwan, and from TG 1.2 of NCTS at NTU, Taiwan. H.-Y.K. is supported by the Ministry of Science and Technology, Taiwan, (with Grant No. MOST112-2112-M-003-020-MY3), and Higher Education Sprout Project of National Taiwan Normal University (NTNU).

DATA AVAILABILITY

The data supporting this study's findings are available upon request.

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