

# Tripartite hybrid quantum systems: Skyrmion-mediated quantum interactions between single nitrogen-vacancy centers and superconducting qubits

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Nitrogen-vacancy (NV) centers in diamond and superconducting qubits are two promising solid-state quantum systems for quantum science and technology, but the realization of controlled interfaces between individual solid-state spins and superconducting qubits remains fundamentally challenging. Here, we propose and analyze a hybrid quantum system consisting of a magnetic skyrmion, an NV center, and a superconducting qubit, where the solid-state qubits are both positioned in proximity to the skyrmion structure in a thin magnetic disk. We show that it is experimentally feasible to achieve strong (coherent or dissipative) coupling between the NV center and the superconducting qubit by using the *quantized gyration mode of the skyrmion* as an intermediary. This allows coherent information transfer and nonreciprocal responses between the NV center and the superconducting qubit at the single quantum level with high controllability. The proposed platform provides a scalable pathway for implementing quantum protocols that synergistically exploit the complementary advantages of spin-based quantum memories, microwave-frequency superconducting circuits, and topologically protected magnetic excitations.

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## I. INTRODUCTION

Quantum information processing demands high-quality quantum systems with extended coherence times and external controllability. Typical solid-state quantum systems for quantum technology include electronic spins in diamond [1–8] and superconducting qubits [9–14]. Single nitrogen-vacancy (NV) centers serve as superior quantum memory platforms owing to their exceptional coherence lifetimes and immunity to inhomogeneous broadening inherent in spin ensembles [1,2,15–23]. In contrast, superconducting qubits stand out as high-performance quantum processors due to their unparalleled gate fidelity and microwave control precision [10,12,24–26]. Therefore, it is quite appealing to integrate these two kinds of solid-state quantum systems in a single hybrid quantum setup [9–12], which would allow the realization of long-lived quantum memories for superconducting qubits. However, the direct magnetic coupling between a single spin and a single superconducting qubit is typically only a few hertz, much smaller than the relevant decoherence rates [9,27]. This remains a fundamental barrier in constructing NV-superconducting qubit hybrid architectures.

Recent theoretical and experimental proposals demonstrate that magnonic modes in micromagnet spheres [28–61] can

achieve strong coupling at the single quantum level ( $\sim$  MHz scale) to both NV centers and superconducting qubits, offering a potential quantum interface between these solid-state qubits [33,58–61]. This coupling mechanism could bridge individual spin defects with macroscopic quantum circuits through magnon-mediated interactions. Nevertheless, a fundamental scaling dichotomy arises: NV centers necessitate sub-100-nm-scale magnetic confinement (radius  $<100$  nm) for optimal coupling enhancement [58,60], while superconducting qubits (SQ) require micron-scale magnetic structures (radius  $>1$   $\mu$ m) to maximize interaction strength [57]. This imposes mutually exclusive scaling constraints on the hybrid system design. In addition, a concurrent integration challenge emerges from the inherent incompatibility between three-dimensional micromagnet geometries and planar superconducting resonator architectures, where submicrometer alignment precision exceeds the practical limits of conventional flip-chip bonding technique.

To overcome the above-mentioned challenges, we here propose to exploit skyrmion structures in thin magnetic disks [62–103] for a quantum interface between NV centers and superconducting qubits. Recent studies have begun to investigate the construction of qubits using the quantized helicity degree of freedom in skyrmions and their interactions to other quantum systems [104–107]. In addition to the helicity degree of freedom, a magnetic skyrmion texture also exhibits a distinct excitation mode known as the skyrmion gyration mode (GM) [108–111]. Compared to magnon modes in micromagnetic spheres, the skyrmion gyration mode in a thin disk has its own distinct features. First, the stray field profiles of the gyration mode are concentrated near the skyrmion core, where the strong localized magnetic fields can significantly enhance the

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coupling to other quantum systems. Second, the confinement of the skyrmion gyration mode within a two-dimensional disk makes it an ideal candidate for integration with other quantum systems on a chip. It is therefore highly appealing to employ the quantized gyration modes of skyrmions as a quantum interface for solid-state qubits.

We consider a tripartite hybrid quantum system consisting of a magnetic skyrmion, an NV center, and a flux-tunable transmon qubit. Stabilized skyrmions have been extensively studied, with their observation and control demonstrated in materials like  $\text{MnSi}_{1-x}\text{Ge}_x$ ,  $\text{VOSe}_2\text{O}_5$ ,  $\text{Cu}_2\text{OSeO}_3$ , and  $\text{GaV}_4\text{S}_8$  [112–131]. The NV center, positioned above the skyrmion, is coupled to the stray field of the gyration mode through the magnetic dipole coupling. Below the magnetic disk, a flux-tuned transmon qubit is positioned. The magnetic field generated by the disk passes through the superconducting quantum interference device (SQUID), producing a magnetic flux that enables strong coupling between the skyrmion and the transmon qubit. These types of coupling differ fundamentally from the previously studied NV-phonon [19,132–136], NV-magnon [33,58,60], skyrmion-magnon [106], SQ-cold-atom [137–139], and SQ-magnon [44,57] couplings. We show that by making use of the virtual excitations of the gyration mode, strong (coherent or dissipative) coupling between the NV center and the transmon qubit can be achieved, with a coupling strength of megahertz. This allows quantum information transfer and nonreciprocal response between the two solid-state qubits. This work provides a promising quantum platform for on-chip quantum information processing with purely solid-state few-body hybrid quantum systems.

The structure of this paper is as follows. Section II presents a comprehensive analysis of the classical dynamical behavior of the skyrmion gyration modes, while Sec. III explores their quantization properties. Sections IV and V delve into the detailed coupling mechanisms between the skyrmion gyration mode and two key quantum components: NV centers and transmon qubits. Section VI introduces an indirect coupling scheme between NV centers and transmon qubits mediated by the skyrmion gyration mode. Section VII demonstrates the experimental feasibility of the proposed theoretical model through multidimensional analysis. Finally, Sec. VIII concludes the paper.

## II. CLASSICAL PROPERTIES OF THE SKYRMION GYRATION MODE

### A. The stationary skyrmion

Skyrmions are topological solitons in magnetic materials characterized by a centrosymmetric spiral structure. The stabilization of skyrmions arises from the competition among various interactions in magnetic materials, including exchange interactions, Dzyaloshinskii-Moriya (DM) interactions, easy-axis anisotropy, and Zeeman interactions. Materials with DM interactions, known as chiral materials, exhibit skyrmions that have been theoretically predicted and experimentally observed. The energy density functional of skyrmions can be expressed in terms of the normalized magnetization  $\mathbf{m} = \mathbf{M}/M_S$  [140,141]

$$\mathcal{F}_{\text{Sk}} = A_{\text{ex}}(\nabla \mathbf{m})^2 + \mathcal{F}_D - K(\mathbf{m} \cdot \mathbf{u})^2 - \mu_0 M_S \mathbf{m} \cdot \mathbf{H}_e, \quad (1)$$

where  $M_S$  represents the saturation magnetization of the magnetic material. The first term represents the energy density contributed by the isotropic exchange interaction, with  $A_{\text{ex}}$  being the exchange stiffness constant. The second term accounts for the energy density contributed by the DM interaction, which includes two types: bulk DM interaction and interface DM interaction, denoted as  $\mathcal{F}_D^b = D\mathbf{m} \cdot (\nabla \times \mathbf{m})$  and  $\mathcal{F}_D^i = D[m_z(\nabla \cdot \mathbf{m}) - (\mathbf{m} \cdot \nabla)m_z]$ , respectively, with  $D$  being the DM constant. The former stabilizes Bloch skyrmions, while the latter stabilizes Néel-type skyrmions. The third term corresponds to the uniaxial anisotropy perpendicular to the surface, with  $K$  being the anisotropy constant and  $\mathbf{u}$  denoting the anisotropy axis. The final term is the Zeeman energy density caused by the external magnetic field  $\mathbf{H}_e$ . The energy of the skyrmion can be expressed as the integral of its energy density  $\mathcal{H}_{\text{Sk}} = \int d\mathbf{r} \mathcal{F}_{\text{Sk}}$ . The dynamics of the skyrmion is described by the Landau-Lifshitz-Gilbert (LLG) equation [142]:

$$\frac{d\mathbf{m}}{dt} = -\gamma_e \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}, \quad (2)$$

with the effect field  $\mathbf{H}_{\text{eff}} = -[1/(\mu_0 M_S)]\delta \mathcal{H}_{\text{Sk}}/\delta \mathbf{m}$ , the Gilbert damping coefficient  $\alpha$ , and the gyromagnetic ratio  $\gamma_e$ .

Since the LLG equation is nonlinear, obtaining an analytical solution for the skyrmion's magnetization field  $\mathbf{m}$  is challenging. Without loss of generality, we consider the Belavin-Polyakov (BP) model here, which is an approximation of the skyrmion [143,144]. For convenience, we write the normalized magnetization vector as  $\mathbf{m} = (m_x, m_y, m_z)$ . In the BP model, the components of the magnetization vector  $\mathbf{m}$  are given by

$$m_x = c \frac{2x \cos \Phi_0 - 2y \sin \Phi_0}{c^2 + (x^2 + y^2)}, \quad (3a)$$

$$m_y = c \frac{2x \sin \Phi_0 + 2y \cos \Phi_0}{c^2 + (x^2 + y^2)}, \quad (3b)$$

$$m_z = \mathcal{P} \frac{c^2 - (x^2 + y^2)}{c^2 + (x^2 + y^2)}, \quad (3c)$$

where  $\mathcal{P} = \pm 1$  represents the core polarity of the skyrmion and  $\Phi_0$  denotes the skyrmion phase. The symbol  $c$  represents the reduced radius of the skyrmion, defined as  $c = R_{\text{Sk}}/R$ , where  $R_{\text{Sk}}$  is the actual radius of the skyrmion and  $R$  is the disk radius. For Bloch skyrmions,  $\Phi_0 = \pm\pi/2$  and the skyrmion chirality is defined as  $\mathcal{C} = \sin \Phi_0$ ; for Néel skyrmions,  $\Phi_0 = 0, \pi$  and the skyrmion chirality is defined as  $\mathcal{C} = \cos \Phi_0$ . Here, we focus on Bloch skyrmions, setting the chirality  $\mathcal{C} = 1$  and the polarity  $\mathcal{P} = -1$ .

### B. The Thiele equation

Here, we focus on the low-frequency dynamics of the skyrmion (gyrotropic eigenmodes), where it behaves as a rigid particle, preserving its internal structure during motion. To characterize this motion, we introduce the collective coordinate of the skyrmion, specifically its center of mass  $\mathcal{R}_c = (X, Y) = \int \mathbf{r} \rho(\mathbf{r}) d\mathbf{r} / \int \rho(\mathbf{r}) d\mathbf{r}$ , where  $\rho(\mathbf{r})$  represents the topological charge density of the skyrmion. Using the collective coordinate  $\mathcal{R}_c$ , the magnetization configuration of the skyrmion at any given moment can be expressed as  $\mathbf{m}(\mathbf{r}, t) = \mathbf{m}[\mathbf{r} - \mathcal{R}_c(t)]$ . Since the magnetization configuration  $\mathbf{m}$

TABLE I. The table lists the parameters used in Fig. 1. Apart from the parameters listed in the table, the other parameters are as follows: saturation magnetization  $M_S = 10^6$  A/m, length of side 30 nm, disk thickness  $h_G = 1$  nm, skyrmion chirality  $\mathcal{C} = 1$ , skyrmion core polarity  $\mathcal{P} = -1$ , and skyrmion topological charge  $Q = -1$ .

| No.    | $A_{\text{ex}}$ ( $10^{-11}$ J/m) | $D$ ( $10^{-3}$ J/m <sup>2</sup> ) | $K$ ( $10^6$ J/m <sup>3</sup> ) | $B_z$ (mT) | $\alpha$  |
|--------|-----------------------------------|------------------------------------|---------------------------------|------------|-----------|
| Fig. 1 | 1.5                               | 3                                  | 1                               | 50         | $10^{-4}$ |

remains unchanged during motion, we can substitute  $\mathbf{m}(\mathbf{r}, t) = \mathbf{m}[\mathbf{r} - \mathbf{R}_c(t)]$  into Eq. (2), and integrate over the skyrmion's magnetization profile to derive the equation of motion for the collective coordinate  $\mathbf{R}_c$ , which yields

$$\dot{\mathbf{R}}_c \times \mathbf{G} + \mathbf{F} = \bar{\mathbf{D}} \dot{\mathbf{R}}_c. \quad (4)$$

The gyrocoupling vector  $\mathbf{G}$  is defined as  $\mathbf{G} = (0, 0, G) = 4\pi h_G M_S Q / \gamma_e \mathbf{e}_z$  and related to the material's thickness  $h_G$ , saturation magnetization  $M_S$ , and topological charge  $Q = (1/4\pi) \int d\mathbf{r} (\partial_x \mathbf{m} \times \partial_y \mathbf{m}) \cdot \mathbf{m}$ . The symbol  $\bar{\mathbf{D}}$  is the dissipation tensor, with its matrix elements defined as  $\bar{D}_{i,j} = (\alpha h_G M_S / \gamma_e) \int d\mathbf{r} \partial_i \mathbf{m} \cdot \partial_j \mathbf{m}$ , where  $i, j = x, y$ . Notably, in contrast to magnetic quasiparticles like magnetic vortices, skyrmions exhibit a substantial inertial mass  $\mathcal{M}$  [145,146]. With the introduction of the skyrmion's inertial mass  $\mathcal{M}$ , the equation of motion for the center of mass  $\mathbf{R}_c$  can be phenomenologically formulated as [145,146]

$$\mathcal{M} \ddot{\mathbf{R}}_c + \mathbf{v} \times \mathbf{G} + k \mathbf{R}_c = 0, \quad (5)$$

with  $\mathbf{v} = \dot{\mathbf{R}}_c$  and  $\alpha = 0$ . The third term corresponds to the restoring force arising from the harmonic potential  $U = 1/2k(X^2 + Y^2)$ .

### C. The classical dynamics of the gyration mode

In this work, we focus on Bloch skyrmions characterized by a chirality  $\mathcal{C} = 1$ , core polarity  $\mathcal{P} = -1$ , and a resulting topological charge  $Q = -1$ . The clockwise (CW) gyration mode of the skyrmion is analyzed by taking the parameters in Table I and using micromagnetic simulations. The micromagnetic simulations are performed using the object-oriented micromagnetic framework (OOMMF) simulator [147] and the Ubermag Python package [148]. Without loss of generality, we first prepare a stable Bloch skyrmion in a square disk. The dynamical modes of the skyrmion can be excited by magnetic fields, strain fields, and so on. Here, the excitation is performed with an in-plane magnetic field defined as

$\mathbf{H} = (H_x(t), 0, 0)$ , where

$$H_x(t) = H_0 \text{sinc}[2\pi f_x(t - t_0)]. \quad (6)$$

The driving amplitude, the cutoff frequency, and the time shift are  $B_x = \mu_0 H_0 = 30$  mT,  $f_x = 16$  GHz, and  $t_0 = 1$  ns, respectively. The time-domain image and the frequency-domain image of the magnetic field pulse are shown in Figs. 1(a) and 1(b), respectively. For the micromagnetic simulation results, the evolution of the skyrmion center-of-mass coordinates  $X(t)$  or  $Y(t)$  over time is analyzed using the Fourier transform to obtain the spectrum of the skyrmion gyration modes. The results of the micromagnetic simulations are displayed in Fig. 1.

Figures 1(c) and 1(d) depict the time evolution of the skyrmion's center-of-mass coordinates and its motion trajectory within the disk, respectively. From the spectrum in Fig. 2(a), the resonance frequency of the gyration mode can be extracted as 0.95 GHz. The mode function corresponding to each eigenfrequency can be obtained by the site-to-site Fourier transform technique [108,149]. The normalized mode functions of the skyrmion clockwise gyration mode are shown in Figs. 2(b)–2(d). This is consistent with our theoretical derivation in Sec. III B.

### D. The resonant frequency of the skyrmion gyration mode

Next, micromagnetic simulations are employed to quantitatively examine the relationship between the gyration mode frequencies and the material parameters as well as the geometry. Here, exchange interactions, DM interactions,  $z$ -direction uniaxial anisotropy, Zeeman interactions, and static magnetic interactions of the magnetic material are considered; i.e., the total energy of the system is  $E_{\text{tot}} = E_{\text{ex}} + E_D + E_{\text{ani}} + E_{\text{ze}} + E_{\text{De}}$ . First, we prepare a Bloch skyrmion in a square disk and then apply an in-plane magnetic field pulse to excite the dynamical mode of the skyrmion. The in-plane magnetic field pulse is defined as  $\mathbf{H} = (H_x(t), 0, 0)$ , where  $H_x(t) = H_0 \text{sinc}[2\pi f_x(t - t_0)]$ . The driving amplitude, the cutoff frequency, and the time shift are  $B_x = \mu_0 H_0 = 5$  mT,  $f_x = 16$  GHz, and  $t_0 = 1$  ns, respectively. All parameters used in the micromagnetic simulation are listed in Table II, and all the results of the simulations are presented in Fig. 3.

Using micromagnetic simulations, we quantitatively demonstrate that the eigenfrequency of the skyrmion's clockwise gyration mode is inversely correlated with the exchange interaction strength, uniaxial anisotropy, and external magnetic field, while exhibiting a positive dependence on the DM interaction [Figs. 3(a)–3(d)]. The skyrmion radius decreases

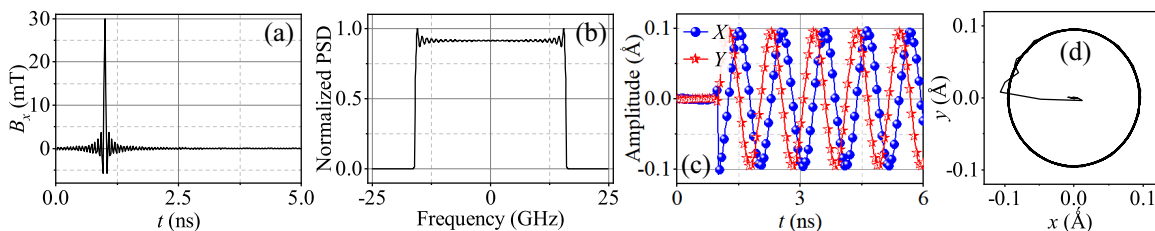


FIG. 1. The waveforms of the in-plane magnetic field pulse applied are shown in both the time domain and frequency domain in panels (a) and (b), respectively. Panel (c) shows the evolution of the collective coordinates  $X$  and  $Y$  over time. The trajectory of the clockwise gyration mode of the skyrmion is plotted in panel (d).

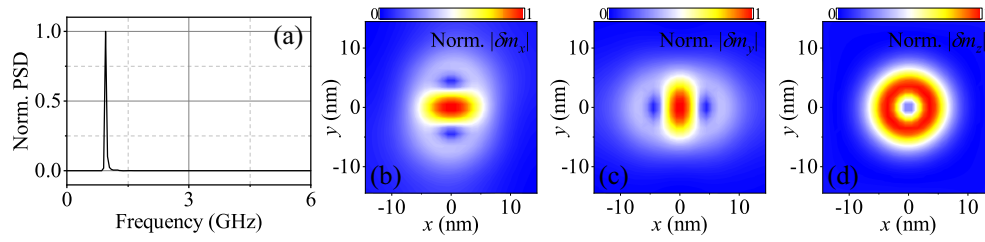


FIG. 2. Panel (a) is the spectrum of the clockwise gyration mode of the skyrmion, and the corresponding normalized mode functions of this mode are shown in panels (b)–(d).

with increasing exchange interaction, anisotropy, or magnetic field, but expands with enhanced DM interaction [Figs. 3(f)–3(i)]. Importantly, geometric effects introduce distinct scaling behaviors [Figs. 3(e) and 3(j)]: Under fixed material parameters, reduced boundary confinement in larger disks leads to skyrmion expansion and a corresponding decrease in gyration frequency, whereas under fixed geometric conditions, variations in material parameters result in a positive correlation between skyrmion radius and gyration frequency. These results illustrate the significant impact of the relative scale between the skyrmion size and the system geometry on its dynamics.

#### E. The dissipation of skyrmion gyroscopic modes: Micromagnetic simulations

The Thiele equation (5) governs the skyrmion gyration mode, with the dissipation described by the dissipation tensor  $\bar{D}$ . The dissipation tensor  $\bar{D}$  is associated with the phenomenological Gilbert damping parameter  $\alpha$ , which can be measured experimentally. The Gilbert damping parameter describes the dissipation mechanism arising from the coupling of the skyrmion to other degrees of freedom, such as electrons, phonons, and magnons [150]. To analyze the dissipation of the skyrmion gyration mode, we use the parameters listed in Table III and perform micromagnetism simulations, the results of which are displayed in Fig. 4.

As shown in Fig. 4(a), when the parameters and geometry of the magnetic material are fixed, the frequency of the clockwise gyration mode of the skyrmion does not change with variations in the Gilbert damping parameter. However, when the Gilbert damping parameter reaches 0.1, we observe a slight decrease in the eigenfrequency of the gyration mode. This can be attributed to the increased spectral broadening caused by the larger Gilbert damping parameter, which leads to a shift in the eigenfrequency. In addition, Figure 4(b) shows

that the size of the skyrmion remains unaffected by changes in the Gilbert damping parameter.

The dissipation of the gyration mode is analyzed via the spectrum, where the full width at half maximum (FWHM) of the resonance peak reflects the system's energy dissipation rate. Specifically, a larger FWHM corresponds to faster energy loss in the system, indicating stronger dissipation. Figure 4(c) presents the spectrum of the clockwise gyration mode as a function of the Gilbert damping parameter. Notably, the bandwidth of the gyration mode increases significantly only when the Gilbert damping parameter exceeds  $10^{-2}$ . As the Gilbert damping parameter increases, the FWHM also increases, with a significant rise occurring only when the parameter  $\alpha$  exceeds  $10^{-2}$  [Fig. 4(d)].

The Gilbert damping parameter of  $\text{Cu}_2\text{OSeO}_3$  can reach  $10^{-4}$  at a temperature of 5 K [151]. Thus, the system's dissipation can be estimated to be approximately 50 MHz. As the temperature increases, the Gilbert damping parameter also increases, exceeding  $10^{-2}$  at approximately 57 K [151]. The dissipation of the skyrmion gyration mode in the magnetic material  $\text{Cu}_2\text{OSeO}_3$  has been measured to be approximately 100 MHz (57 K) [152,153], in good agreement with the results from our simulations [Fig. 4(d)]. Additionally, selecting materials without DM interactions is expected to further lower the Gilbert damping parameter, thereby reducing the system's dissipation. Moreover, with the quantum device operating in the mK range, the dissipation of the gyration mode is expected to decrease further in this ultralow temperature environment.

### III. QUANTUM PROPERTIES OF THE SKYRMION GYRATION MODE

#### A. Quantized gyration modes: Harmonic oscillatorlike objects

In Sec. II, we examined classical gyration modes. We now turn our attention to the analysis of quantizing the

TABLE II. The table lists the parameters used in Fig. 3. Apart from the varying parameters listed in the table, the other parameters are as follows: saturation magnetization  $M_S = 10^6$  A/m, disk thickness  $h_G = 1$  nm, skyrmion chirality  $C = 1$ , skyrmion core polarity  $\mathcal{P} = -1$ , and skyrmion topological charge  $Q = -1$ . For chiral magnetic materials, the Gilbert damping parameter is taken here as  $\alpha = 10^{-4}$ .

| No.                 | $A_{\text{ex}}$ ( $10^{-11}$ J/m) | $D$ ( $10^{-3}$ J/m <sup>2</sup> ) | $K$ ( $10^6$ J/m <sup>3</sup> ) | $B_z$ (mT) | Length of side (nm) |
|---------------------|-----------------------------------|------------------------------------|---------------------------------|------------|---------------------|
| Figs. 3(a) and 3(f) | 0.9–1.7                           | 3                                  | 1                               | 50         | 70                  |
| Figs. 3(b) and 3(g) | 1.5                               | 2–2.8                              | 1                               | 50         | 70                  |
| Figs. 3(c) and 3(h) | 1.5                               | 2                                  | 0.8–0.98                        | 50         | 70                  |
| Figs. 3(d) and 3(i) | 1.5                               | 2.5                                | 1                               | 0–90       | 70                  |
| Figs. 3(e) and 3(j) | 1.5                               | 3                                  | 1                               | 30         | 50–95               |

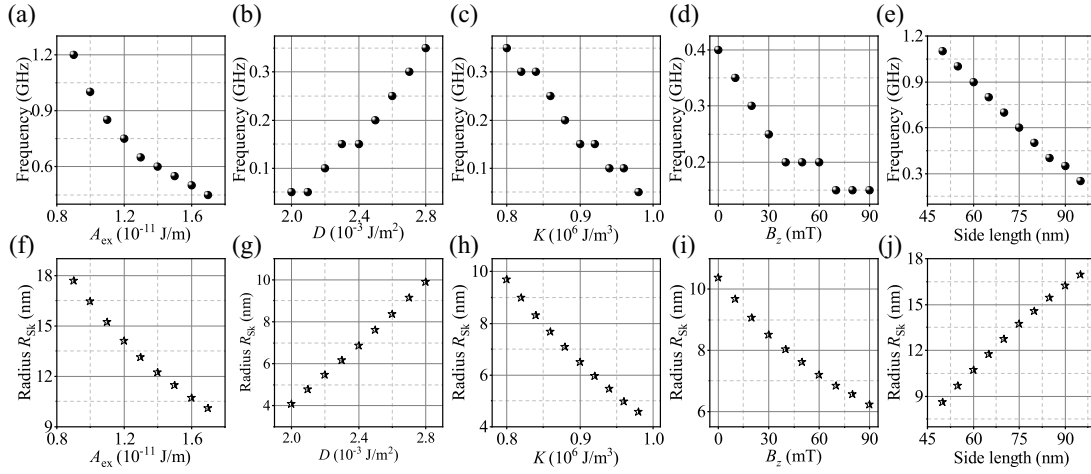


FIG. 3. The dependence of the clockwise gyration mode frequency on the exchange interaction  $A_{\text{ex}}$ , DM interaction  $D$ , uniaxial anisotropy  $K$ , applied magnetic field  $B_z$ , and disk geometry is presented. The parameters used are presented in Table II.

gyration modes, which exhibit characteristics similar to those of quantum harmonic oscillators. As discussed earlier, within the framework of the rigid body model, the coordinates  $\mathcal{R}_c = (X, Y)$  of the skyrmion's center satisfy the Thiele equation (5) [145,154]. Introducing the gauge potential  $\mathbf{A} = G(A_X, A_Y, 0)$ , the Hamiltonian corresponding to the Thiele equation can be expressed as

$$H_{\text{GM}} = \frac{(p_X + GA_X)^2}{2\mathcal{M}} + \frac{(p_Y + GA_Y)^2}{2\mathcal{M}} + U, \quad (7)$$

where the gauge potential  $\mathbf{A}$  satisfies the condition  $\nabla \times \mathbf{A} = \mathbf{G}$ , specifically fulfilling the equation  $\partial_X A_Y - \partial_Y A_X = 1$ .

To clarify the physical interpretation of the gyration mode, we select the symmetric gauge potential  $\mathbf{A} = G(-Y, X, 0)/2$  for a more detailed analysis. In the symmetric gauge potential, the Hamiltonian of the system can be expressed as

$$H_{\text{GM}} = \frac{(p_X - G'Y)^2}{2\mathcal{M}} + \frac{(p_Y + G'X)^2}{2\mathcal{M}} + \frac{1}{2}k(X^2 + Y^2), \quad (8)$$

with  $G' = G/2$ . Expanding the Hamiltonian (8) and utilizing the definition of orbital momentum  $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ , we get

$$H_{\text{GM}} = \frac{p_X^2}{2\mathcal{M}} + \frac{p_Y^2}{2\mathcal{M}} + \frac{1}{2}\mathcal{M}\omega^2(X^2 + Y^2) + \omega'_0 L_z, \quad (9)$$

where  $\omega^2 = \omega_0'^2 + \omega_h'^2$ ,  $\omega_0' = G'/\mathcal{M}$ , and  $\omega_h = \sqrt{k/\mathcal{M}}$ .

Next, we discuss the quantization of the Hamiltonian (9). For convenience, in the subsequent analysis we decompose the Hamiltonian into two parts: a two-dimensional harmonic oscillator Hamiltonian and an orbital momentum Hamiltonian

$$\hat{H}_{\text{SHO}} = \frac{\hat{p}_X^2}{2\mathcal{M}} + \frac{\hat{p}_Y^2}{2\mathcal{M}} + \frac{1}{2}\mathcal{M}\omega^2(\hat{X}^2 + \hat{Y}^2), \quad (10a)$$

TABLE III. The table lists the parameters used in Fig. 4. Apart from the varying parameters listed in the table, the constant shared parameters are as follows: saturation magnetization  $M_s = 10^6$  A/m, length of side 70 nm, disk thickness  $h_G = 1$  nm, skyrmion chirality  $\mathcal{C} = 1$ , skyrmion core polarity  $\mathcal{P} = -1$ , and skyrmion topological charge  $Q = -1$ .

| No.    | $A_{\text{ex}} (10^{-11} \text{ J/m})$ | $D (10^{-3} \text{ J/m}^2)$ | $K (10^6 \text{ J/m}^3)$ | $B_z \text{ (mT)}$ | $\alpha$               |
|--------|----------------------------------------|-----------------------------|--------------------------|--------------------|------------------------|
| Fig. 4 | 1.5                                    | 3                           | 1                        | 30                 | $10^{-4}$ to $10^{-1}$ |

$$\hat{H}_L = \omega'_0(\hat{X}\hat{p}_Y - \hat{Y}\hat{p}_X). \quad (10b)$$

It is worth noting that the Hamiltonian  $\hat{H}_{\text{SHO}}$  and  $\hat{H}_L$  commute,  $[\hat{H}_{\text{SHO}}, \hat{H}_L] = 0$ , implying that they share a common eigenstate. Introducing the annihilation operators in the  $x$  and  $y$  directions,  $\hat{a}_X = 1/\sqrt{2}[\beta\hat{X} + i\hat{p}_X/(\hbar\beta)]$  and  $\hat{a}_Y = 1/\sqrt{2}[\beta\hat{Y} + i\hat{p}_Y/(\hbar\beta)]$ , we can obtain

$$\hat{H}_{\text{SHO}} = \hbar\omega(\hat{a}_X^\dagger\hat{a}_X + \hat{a}_Y^\dagger\hat{a}_Y + 1), \quad (11a)$$

$$\hat{H}_L = i\hbar\omega'_0(\hat{a}_X\hat{a}_Y^\dagger - \hat{a}_X^\dagger\hat{a}_Y), \quad (11b)$$

where  $\beta \equiv \sqrt{m\omega/\hbar}$ . We define  $\hat{a}_{\text{CW}} = 1/\sqrt{2}(\hat{a}_X - i\hat{a}_Y)$  and  $\hat{a}_{\text{CCW}} = 1/\sqrt{2}(\hat{a}_X + i\hat{a}_Y)$ , corresponding to the annihilation operators for the CW and counterclockwise (CCW) gyration modes, respectively. The operators  $\hat{a}_{\text{CW}}$  and  $\hat{a}_{\text{CCW}}$  satisfy the commutation relations  $[\hat{a}_{\text{CW}}, \hat{a}_{\text{CW}}^\dagger] = 1 = [\hat{a}_{\text{CCW}}, \hat{a}_{\text{CCW}}^\dagger]$ . Thus, the Hamiltonian  $\hat{H}_{\text{GM}} = \hat{H}_{\text{SHO}} + \hat{H}_L$  can be simplified to

$$\hat{H}_{\text{GM}} = \hbar\omega_{\text{CW}}\hat{a}_{\text{CW}}^\dagger\hat{a}_{\text{CW}} + \hbar\omega_{\text{CCW}}\hat{a}_{\text{CCW}}^\dagger\hat{a}_{\text{CCW}}, \quad (12)$$

where  $\omega_{\text{CW}} = \omega + \omega'_0$  and  $\omega_{\text{CCW}} = \omega - \omega'_0$ . Based on the analysis in Sec. II C, we can conclude that  $G \propto Q < 0 \rightarrow \omega'_0 < 0$ ; i.e., the eigenfrequency of the CW gyration mode is smaller than that of the CCW gyration mode for Bloch skyrmions with core polarity  $\mathcal{P} = -1$ . Here, we consider only the CW gyration mode  $\hat{a}_{\text{CW}}$  and, for convenience, drop the subscripts from the operators [Figs. 1(c) and 1(d)]. The free Hamiltonian can then be written as ( $\hbar = 1$  is used hereafter)

$$\hat{H}_{\text{GM}} = \omega_{\text{GM}}\hat{a}^\dagger\hat{a}, \quad (13)$$

where  $\omega_{\text{GM}} \equiv \omega_{\text{CW}}$  denotes the gyration frequency.

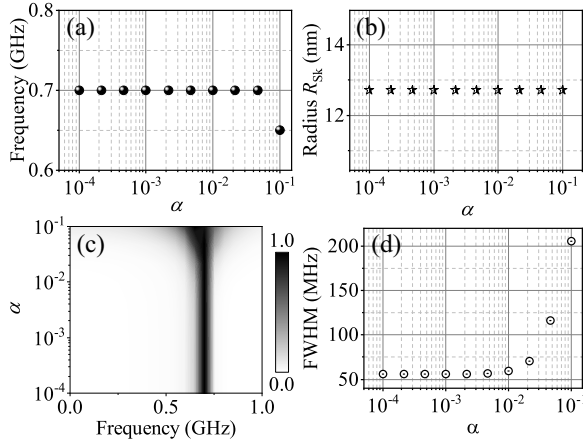


FIG. 4. Panels (a) and (b) illustrate the dependence of the resonance frequency of the clockwise gyration mode and the skyrmion radius on the Gilbert damping parameter  $\alpha$ , respectively. The dependence of the spectrum and FWHM on the Gilbert damping parameter  $\alpha$  is shown in panels (c) and (d). The parameters used are presented in Table III.

### B. Quantum magnetization fluctuations due to skyrmion gyration modes

Next, we focus on the quantum magnetization fluctuation induced by the gyration mode. Based on the analysis in Sec. II, we know that the magnetization at any moment can be expressed as  $\mathbf{m}(\mathbf{r}, t)$ . When the gyration mode is excited, the coordinates of the skyrmion center can be expressed as  $\mathbf{R}_c(t) = r_c[\cos(\omega_{\text{GM}}t)\mathbf{e}_x + \mathcal{P}\sin(\omega_{\text{GM}}t)\mathbf{e}_y]$ . Then the magnetization at time  $t$  can be written as  $\mathbf{m}(\mathbf{r}, t) = \mathbf{m}[\mathbf{r} - \mathbf{R}_c(t)] \approx \mathbf{m}(\mathbf{r}) - [\mathbf{R}_c(t) \cdot \nabla]\mathbf{m}(\mathbf{r})$ . The variation of the magnetization due to the excitation of the gyration mode can be expressed as  $\delta\mathbf{m}_l(\mathbf{r}, t) = \delta\mathbf{m}_l(\mathbf{r})\exp(i\omega_l t) + \text{c.c.}$ , where  $\delta\mathbf{m}_l(\mathbf{r})$  represents the mode function of the gyration mode [108,149]. This equation holds under the condition  $|\delta\mathbf{m}_l(\mathbf{r})| \ll 1$ . Therefore, the magnetization field can be regarded as a superposition of a series of eigenmodes, represented as  $\delta\mathbf{m}(\mathbf{r}, t) = \sum_{l=\text{CW,CCW}} \delta\mathbf{m}_l(\mathbf{r})\exp(i\omega_l t) + \text{c.c.}$ . Here, we focus on the CW mode  $\hat{a}$ , with the subscripts omitted for simplicity

$$\delta\mathbf{m}(\mathbf{r}, t) = \delta\mathbf{m}(\mathbf{r})e^{i\omega_{\text{GM}}t} + \text{c.c.}, \quad (14)$$

where  $\delta\mathbf{m}(\mathbf{r}) = (-r_c/2)[\partial_x\mathbf{m}(\mathbf{r}) - i\mathcal{P}\partial_y\mathbf{m}(\mathbf{r})]$ . Quantizing  $\delta\mathbf{m}(\mathbf{r}, t)$  yields a quantized magnetization intensity

$$\hat{\mathbf{m}} \equiv \delta\mathbf{m}(\mathbf{r})\hat{a} + \delta\mathbf{m}^*(\mathbf{r})\hat{a}^\dagger. \quad (15)$$

The amplitude of the gyration mode, denoted as  $r_c$ , will be determined in the subsequent analysis. Utilizing the BP model of the skyrmion Eq. (3), a specific expression for the mode function  $\delta\mathbf{m} = (\delta m_x, \delta m_y, \delta m_z)$  can be computed, which is given as follows:

$$\delta m_x = -\frac{r_c}{2} \left\{ -\frac{2iR_{\text{Sk}}[R_{\text{Sk}}^2 + (x+iy)^2]}{[R_{\text{Sk}}^2 + (x^2+y^2)]^2} \right\}, \quad (16a)$$

$$\delta m_y = -\frac{r_c}{2} \left\{ \frac{2R_{\text{Sk}}[R_{\text{Sk}}^2 - (x+iy)^2]}{[R_{\text{Sk}}^2 + (x^2+y^2)]^2} \right\}, \quad (16b)$$

$$\delta m_z = -\frac{r_c}{2} \left\{ \frac{4R_{\text{Sk}}^2(x+iy)}{[R_{\text{Sk}}^2 + (x^2+y^2)]^2} \right\}. \quad (16c)$$

Figure 5 illustrates the distribution of the mode function described by Eq. (16), which is consistent with the micromagnetic simulations (Fig. 2).

Next, we identify the gyration radius  $r_c$  corresponding to a single quantum excitation of the gyration mode. The local magnetization can be written as [154–156]

$$\mathbf{m}_{\text{ex}}(\mathbf{r}) = \mathbf{e}_3 \sqrt{1 - \frac{2g\mu_B}{M_S}|\psi(\mathbf{r})|^2} + \sqrt{\frac{g\mu_B}{M_S}}[\psi(\mathbf{r})\mathbf{e}_+ + \psi^*(\mathbf{r})\mathbf{e}_-], \quad (17)$$

where  $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$  represents the local orthogonal basis, and  $\mathbf{e}_3 = \mathbf{m}(\mathbf{r})$  corresponds to the local equilibrium direction. The vectors  $\mathbf{e}_\pm$  are defined as  $\mathbf{e}_\pm = (\mathbf{e}_1 \pm i\mathbf{e}_2)/\sqrt{2}$ . The complex amplitude of the spin wave is represented by  $\psi(\mathbf{r})$ , and the probability density of magnon excitations is given by  $|\psi(\mathbf{r})|^2$ . To determine the local orthogonal basis  $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ , we first simplify the expression  $\mathbf{e}_3 = \mathbf{m}(\mathbf{r}) = \cos\Phi\mathbf{e}_\phi + \sin\Phi\mathbf{e}_z$ , where  $\mathbf{e}_3$  is defined in the plane spanned by  $\mathbf{e}_\phi$  and  $\mathbf{e}_z$ . Then, we have  $\mathbf{e}_2 = \sin\Phi\mathbf{e}_\phi - \cos\Phi\mathbf{e}_z$ , and the third local basis vector is given by  $\mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_\rho$ . From Eq. (17), we obtain  $\delta\mathbf{m}(\mathbf{r}) = \sqrt{g\mu_B/M_S}[\psi(\mathbf{r})\mathbf{e}_+ + \psi^*(\mathbf{r})\mathbf{e}_-]$ , from which we determine the wave function  $\psi(\mathbf{r})$  in terms of the mode function

$$\begin{aligned} \psi(\mathbf{r}) &= \sqrt{\frac{M_S}{g\mu_B}} \frac{\delta\mathbf{m}(\mathbf{r})}{2} \cdot \mathbf{e}_+ \\ &= -\frac{r_c R_{\text{Sk}}}{\sqrt{2}} \sqrt{\frac{M_S}{g\mu_B}} \frac{-i\cos\phi + \sin\phi}{R_{\text{Sk}}^2 + x^2 + y^2}. \end{aligned} \quad (18)$$

The wave function satisfies the normalization condition over the entire disk, namely,  $\int d\mathbf{r}|\psi(\mathbf{r})|^2 = 1$ . Thus, the gyration radius corresponding to a single excitation of the gyration mode is given by

$$r_c = \sqrt{\frac{2g\mu_B(R^2 + R_{\text{Sk}}^2)}{\pi h_G R^2 M_S}}. \quad (19)$$

Using the parameters  $R_{\text{Sk}}/R = 0.1$ ,  $h_G = 5$  nm, and  $M_S = 10^6$  A/m, we get  $r_c \approx 0.5$  Å.

### C. Quantized magnetic fields generated by the excitation of the gyration mode

In Sec. III B, we derived that the quantum magnetization fluctuation resulting from gyration mode excitation is  $\hat{\mathbf{m}} \equiv \delta\mathbf{m}(\mathbf{r})\hat{a} + \delta\mathbf{m}^*(\mathbf{r})\hat{a}^\dagger$ . The quantized magnetic field generated by the magnetization  $\hat{\mathbf{m}}$  is

$$\mathcal{B}(\mathbf{r}) = \mu_0 M_S \int d\mathbf{r}' \Gamma(\mathbf{r}, \mathbf{r}') \hat{\mathbf{m}}(\mathbf{r}'), \quad (20)$$

with the tensorial magnetostatic Green function  $\Gamma(\mathbf{r}, \mathbf{r}') = -\nabla\nabla'G(\mathbf{r}, \mathbf{r}')$ . The operators  $\nabla$  and  $\nabla'$  are defined as  $\nabla = (\partial_x, \partial_y, \partial_z)$  and  $\nabla' = (\partial_{x'}, \partial_{y'}, \partial_{z'})$ , respectively, and  $G(\mathbf{r}, \mathbf{r}') = 1/(4\pi|\mathbf{r} - \mathbf{r}'|)$  represents the Coulombic Green's function. Defining the mode function of the quantized magnetic field

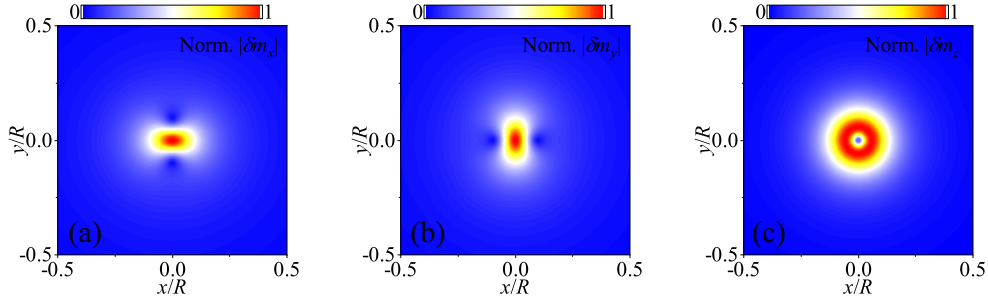


FIG. 5. Panels (a), (b), and (c) show the components of the normalized mode function  $|\delta m_x|$ ,  $|\delta m_y|$ , and  $|\delta m_z|$  of the skyrmion described by Eq. (16), respectively.

as

$$\tilde{\mathbf{B}} = \mu_0 M_S \int d\mathbf{r}' \Gamma(\mathbf{r}, \mathbf{r}') \delta \mathbf{m}(\mathbf{r}'), \quad (21)$$

Eq. (20) can then be simplified to

$$\hat{\mathbf{B}} = \tilde{\mathbf{B}} \hat{a} + \tilde{\mathbf{B}}^* \hat{a}^\dagger. \quad (22)$$

For the general case of  $\delta \mathbf{m}$ , the mode function can be simplified to

$$\tilde{\mathbf{B}} = \frac{\mu_0 M_S}{4\pi} \int d\mathbf{r}' \left\{ \frac{3(\mathbf{r} - \mathbf{r}') [\delta \mathbf{m} \cdot (\mathbf{r} - \mathbf{r}')] }{|\mathbf{r} - \mathbf{r}'|^5} - \frac{\delta \mathbf{m}}{|\mathbf{r} - \mathbf{r}'|^3} \right\}. \quad (23)$$

The distance between the field point  $\mathbf{r} = (x, y, z)$  and the source point  $\mathbf{r}' = (x', y', z')$  is given by  $r = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$ . The physical interpretation of this integral is as follows: By subdividing a magnetic system into infinitesimal volume elements, treating each as a magnetic dipole, and integrating the dipole field from each element, we can derive the total magnetic field produced by the system.

#### IV. COUPLING BETWEEN SKYRMIONS AND NV CENTERS

##### A. The device

As depicted in Fig. 6(a), we examine a thin disk with a radius  $R$  significantly greater than its thickness  $h_G$ , in which the skyrmion is stabilized within the disk. As discussed previously, gyration modes of the skyrmion exist, where the skyrmion oscillates in a circular motion around the equilibrium position, analogous to a rigid particle. Additionally, the skyrmion gyration modes can be quantized as bosons. A diamond particle is positioned atop the disk, with an NV center embedded within the particle. The distance between the NV center and the disk surface is denoted by  $d_G$ . The energy-level structure of the NV center features a triplet state with  $S = 1$  [Fig. 6(b)]. In the presence of a magnetic field  $B_z$  along the  $z$  direction, the Hamiltonian of the NV center is given by  $\hat{H}_{\text{NV}} = D\hat{S}_z^2 + \delta\hat{S}_z$ , where  $D$  represents the zero-field splitting between the subenergy levels  $|0\rangle$  and  $|\pm 1\rangle$ , and  $\delta = \gamma_e B_z$  denotes the Zeeman splitting due to the applied magnetic field  $B_z$ . Selecting the energy levels  $|-1\rangle$  and  $|0\rangle$  as a qubit, the Hamiltonian of the NV center simplifies to  $\hat{H}_{\text{NV}} = \omega_{\text{NV}} \hat{\sigma}_z / 2$ ,

with the resonant frequency  $\omega_{\text{NV}} = D - \delta$ . The Pauli operator of the NV center is defined as  $\hat{\sigma}_z = |-1\rangle\langle -1| - |0\rangle\langle 0|$ .

##### B. The magnetic field experienced by the NV center

The coupling between the NV centers and the skyrmion gyration modes is achieved through the stray field around the microdisks, as shown in Fig. 6(c). Figure 6(d) illustrates the spatial distribution of the magnetic field mode function  $\text{Re}[\tilde{\mathbf{B}}]$  above the disk. The arrows indicate the direction of the in-plane normalized magnetic field. The contour represents the  $z$  component of the field, with positive and negative signs corresponding to the positive and negative directions along the  $z$  axis. Near the periphery of the disk, the magnetic field strength gradually decreases, and a centrosymmetric pattern in the  $z$  component of the field is observed.

Considering an NV center placed directly above the disk, at the field point  $\mathbf{r} = (0, 0, H_{\text{SN}})$  with  $H_{\text{SN}} = d_G + h_G/2$ , the magnetic field at this location can be simplified to

$$\tilde{\mathbf{B}}_x = \frac{\mu_0 M_S}{4\pi} \int d\mathbf{r} \left\{ \frac{3\mathcal{G}_{\text{SN}x}}{r^5} - \frac{\delta m_x}{r^3} \right\}, \quad (24a)$$

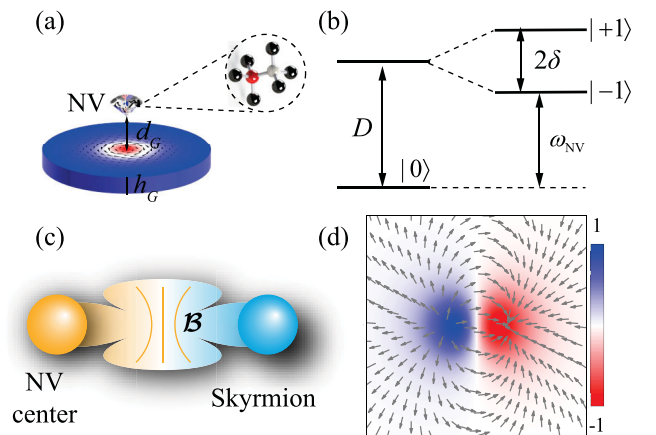


FIG. 6. (a) The coupling model between the skyrmion and the NV center. (b) The energy levels of the NV center. (c) The coupling mechanism in this hybrid quantum system. Panel (d) shows the distribution of the field  $\text{Re}[\tilde{\mathbf{B}}]$  at a position 5 nm above a disk with a radius of  $R = 50$  nm and a thickness of  $h_G = 5$  nm. Here, the reduced skyrmion radius is defined as  $c = R_{\text{sk}}/R = 0.1$ , where  $R_{\text{sk}}$  denotes the skyrmion radius.

$$\tilde{B}_y = \frac{\mu_0 M_S}{4\pi} \int d\mathbf{r} \left\{ \frac{3\mathcal{G}_{\text{SN}y}}{r^5} - \frac{\delta m_y}{r^3} \right\}, \quad (24b)$$

$$\tilde{B}_z = \frac{\mu_0 M_S}{4\pi} \int d\mathbf{r} \left\{ \frac{3\mathcal{G}_{\text{SN}}(z - H_{\text{SN}})}{r^5} - \frac{\delta m_z}{r^3} \right\}, \quad (24c)$$

where  $\mathcal{G}_{\text{SN}} \equiv x\delta m_x + y\delta m_y + (z - H_{\text{SN}})\delta m_z$ . To simplify the expressions, the prime notation for the position coordinates has been excluded.

### C. The interaction Hamiltonian

The interaction between the gyration mode and the NV center is mediated by magnetic dipole coupling, with the corresponding interaction Hamiltonian given by

$$\hat{H}_{\text{SN}} = -\gamma_e \hat{\mathbf{B}} \cdot \hat{\mathbf{S}} = \sum_{j=x,y,z} (\lambda_j \hat{a} + \text{H.c.}) \hat{S}_j, \quad (25)$$

where  $\lambda_j = -\gamma_e \tilde{B}_j$ ,  $\gamma_e$ , and  $\hat{\mathbf{S}} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$  are the coupling strength, the gyromagnetic ratio, and the spin operator of the NV center, respectively.

Next, we derive the expressions for the coupling strengths. First, we evaluate the coupling in the  $x$  direction. By substituting Eq. (24a) into the coupling strength  $\lambda_x \equiv -\gamma_e \tilde{B}_x$  and simplifying, we obtain

$$\hat{H}_{\text{SN}}^x = (\lambda_x \hat{a} + \lambda_x^* \hat{a}^\dagger) \hat{S}_x, \quad (26)$$

with the coupling strength

$$\lambda_x = -\gamma_e \frac{\mu_0 M_S}{4\pi R} \pi r_c \int dz \int \rho d\rho \left[ -\frac{3}{r^5} \frac{2c^2 \rho^2}{(c^2 + \rho^2)^2} (z - H_{\text{SN}}) + i \frac{3}{r^5} \frac{c \rho^2}{c^2 + \rho^2} - i \frac{2}{r^3} \frac{c^3}{(c^2 + \rho^2)^2} \right], \quad (27)$$

where the integrals in azimuthal coordinates  $\phi$  have been evaluated, terms with zero contribution have been omitted, and the integral has been nondimensionalized using the disk radius  $R$ .

Applying the same calculation for the  $y$  direction, we obtain the interaction Hamiltonian

$$\hat{H}_{\text{SN}}^y = (\lambda_y \hat{a} + \lambda_y^* \hat{a}^\dagger) \hat{S}_y, \quad (28)$$

with coupling strength

$$\lambda_y = -\gamma_e \frac{\mu_0 M_S}{4\pi R} \pi r_c \int dz \int \rho d\rho \left[ -i \frac{3}{r^5} \frac{2c^2 \rho^2}{(c^2 + \rho^2)^2} (z - H_{\text{SN}}) - \frac{3}{r^5} \frac{c \rho^2}{c^2 + \rho^2} + \frac{2}{r^3} \frac{c^3}{(c^2 + \rho^2)^2} \right]. \quad (29)$$

For the  $z$  direction coupling  $\lambda_z \equiv -\gamma_e \tilde{B}_z$ , the calculation yields  $\lambda_z = 0$ , indicating that the coupling strength in the  $z$  direction is zero. Comparing Eq. (27) with Eq. (29), we notice that  $\text{Re}[\lambda_x] = \text{Im}[\lambda_y]$  and  $\text{Im}[\lambda_x] = -\text{Re}[\lambda_y]$ . We then define the coupling strength as  $\Lambda_{\text{SN}} = |\lambda_x| = |\lambda_x^*| = |\lambda_y| = |\lambda_y^*|$ . For convenience, defining the dimensionless integral

$$\mathcal{F}_{\text{SN}} = \left| \int dz \int \rho d\rho \left[ -\frac{3}{r^5} \frac{2c^2 \rho^2}{(c^2 + \rho^2)^2} (z - H_{\text{SN}}) + i \frac{3}{r^5} \frac{c \rho^2}{c^2 + \rho^2} - i \frac{2}{r^3} \frac{c^3}{(c^2 + \rho^2)^2} \right] \right|, \quad (30)$$

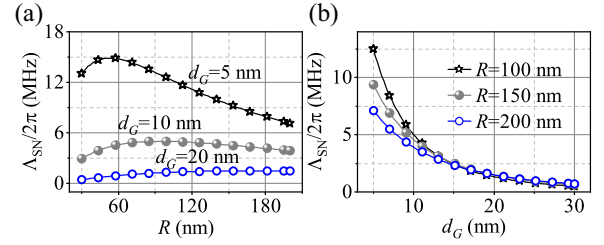


FIG. 7. Panel (a) presents the variation of the coupling strength with the disk radius  $R$  while keeping  $d_G$  constant, and panel (b) examines the variation with the distance  $d_G$  for fixed radius  $R$ .

the coupling strength can be reduced to

$$\Lambda_{\text{SN}} = \frac{\gamma_e \mu_0 M_S r_c}{4R} \mathcal{F}_{\text{SN}}. \quad (31)$$

Based on the above analysis, we obtain  $\lambda_x = \Lambda_{\text{SN}} e^{i\phi_x}$  and  $\lambda_y = \Lambda_{\text{SN}} e^{i\phi_y}$ , with phases  $\phi_x = \arg(\lambda_x)$  and  $\phi_y = \arg(\lambda_y) = \phi_x + \pi/2$ . The interaction Hamiltonian can then be written as  $\hat{H}_{\text{SN}} = \Lambda_{\text{SN}} (e^{i\phi_x} \hat{a} + e^{-i\phi_x} \hat{a}^\dagger) \hat{S}_x + i \Lambda_{\text{SN}} (e^{i\phi_x} \hat{a} - e^{-i\phi_x} \hat{a}^\dagger) \hat{S}_y$ . Using  $\hat{S}_x = (\hat{S}_+ + \hat{S}_-)/2$  and  $\hat{S}_y = (\hat{S}_+ - \hat{S}_-)/(2i)$ , and applying the rotating wave approximation, the interaction Hamiltonian can be simplified to  $\hat{H}_{\text{SN}} = \Lambda_{\text{SN}} (e^{i\phi_x} \hat{a} \hat{S}_+ + e^{-i\phi_x} \hat{a}^\dagger \hat{S}_-)$ . Here, the constant phase  $\phi_x$  can be neglected, as it does not affect the system dynamics. By selecting  $|-1\rangle$  and  $|0\rangle$  as a qubit, the Hamiltonian describing the coupling between the skyrmion and the NV center is given by

$$\hat{H}_{\text{SN}} = \Lambda_{\text{SN}} (\hat{a} \hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_-), \quad (32)$$

where the Pauli operators are defined as  $\hat{\sigma}_+ = |-1\rangle\langle 0|$  and  $\hat{\sigma}_- = |0\rangle\langle -1|$ . Note that if we treat the skyrmion gyration mode as a classical field, the effect of the skyrmion gyration mode on the NV center is equivalent to a classical drive  $\hat{H}_{\text{dr}} = \Omega_d (\hat{\sigma}_+ + \hat{\sigma}_-)$ , with the drive amplitude  $\Omega_d$ , rather than an interaction at the single-quantum level described by the Jaynes-Cummings (JC) model [157].

Figures 7(a) and 7(b) display the coupling strength  $\Lambda_{\text{SN}}$  as a function of the disk radius  $R$ , and the distance from the NV center to the disk surface  $d_G$ . The coupling strength between the skyrmion and the NV center decreases with increasing distance  $d_G$ . Here, we assume a typical saturation magnetization of  $M_S = 10^6$  A/m [108,149]. The coupling strength between the gyration mode and NV center at the single-quantum level can reach several megahertz, potentially exceeding a dozen megahertz.

## V. COUPLING BETWEEN SKYRMIONS AND TRANSMON QUBITS

### A. Calculation of the magnetic flux

As shown in Fig. 8(a), we consider the configuration in which the transmon qubit is parallel to the disk. The transmon qubit contains two Josephson junctions,  $E_J^1$  and  $E_J^2$ , which constitute an SQUID shunted by a capacitor  $C$ . By harnessing the magnetic flux generated by the field around the microdisk through the SQUID, the coupling between the skyrmion gyration mode and the transmon qubit is established, as depicted in Fig. 8(b). We first calculate the magnetic flux through the

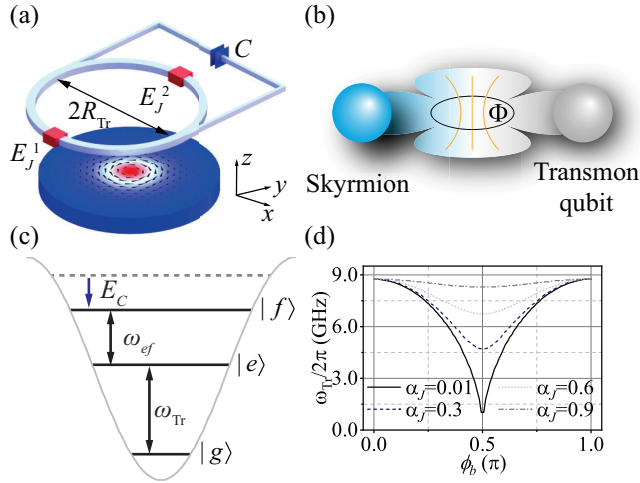


FIG. 8. Panel (a) illustrates the setup consisting of a skyrmion and a transmon qubit. Panel (b) illustrates the coupling mechanism between the skyrmions and the transmon qubits. Panel (c) depicts the energy levels of transmon qubits. Panel (d) plots the resonance frequencies of the transmon qubit as a function of  $\alpha_J$  and  $\phi_b$ . The typical transmon parameters are a Josephson energy  $E_J^{\text{max}}/h = 50$  GHz and a charging energy of  $E_C/h = 200$  MHz.

SQUID, induced by the magnetic field generated by the disk. For an SQUID placed horizontally, only the magnetic field in the  $z$  direction generates an effective magnetic flux. We define the coordinates of any point on the disk as  $(x, y, z)$  and the coordinates of any point on the SQUID as  $(x_f, y_f, z_f)$ . According to Eq. (22), the distribution of the magnetic field in the  $z$  direction on the surface of the SQUID can be simplified as

$$\hat{B}_z = \tilde{B}_z \hat{a} + \tilde{B}_z^* \hat{a}^\dagger = \frac{\mu_0 M_S}{4\pi} (\tilde{B}_z \hat{a} + \tilde{B}_z^* \hat{a}^\dagger), \quad (33)$$

where  $\tilde{B}_z \equiv \mu_0 M_S / (4\pi) \tilde{B}_z$ , and

$$\tilde{B}_z = \int d\mathbf{r} \left\{ \frac{3(z - z_f) \mathcal{G}_{\text{ST}}}{r^5} - \frac{\delta m_z}{r^3} \right\}, \quad (34)$$

with  $\mathcal{G}_{\text{ST}} = [(x - x_f)\delta m_x + (y - y_f)\delta m_y + (z - z_f)\delta m_z]$ . The distance between the field point and the source point is denoted as  $r = \sqrt{(x - x_f)^2 + (y - y_f)^2 + (z - z_f)^2}$ . Here, we have used  $\tilde{\mathbf{B}} = [\mu_0 M_S / (4\pi)] \tilde{\mathbf{B}}$ . The magnetic flux through the SQUID is given by  $\hat{\Phi} = \int \tilde{B}_z \mathbf{e}_z \cdot d\mathbf{S}$ . Defining  $\tilde{\mathcal{F}}_\Phi = \int dx_f \int dy_f \tilde{B}_z(x_f, y_f) = \mathcal{F}_\Phi e^{i\phi_{\text{GM}}}$ , we can obtain

$$\hat{\Phi} = \frac{\mu_0 M_S R^2}{4\pi} \mathcal{F}_\Phi (e^{i\phi_{\text{GM}}} \hat{a} + e^{-i\phi_{\text{GM}}} \hat{a}^\dagger), \quad (35)$$

where the modulus  $\mathcal{F}_\Phi = |\tilde{\mathcal{F}}_\Phi|$  and the phase  $\phi_{\text{GM}} = \arg(\tilde{\mathcal{F}}_\Phi)$ . The constant phase  $\phi_{\text{GM}}$  typically does not influence the dynamics of the system and can thus usually be ignored.

### B. The free Hamiltonian of the transmon qubit

The Hamiltonian of a transmon qubit can be written as

$$\hat{H}_{\text{TTQ}} = 4E_C \hat{N}^2 + \mathcal{E}_{\text{JJ}}, \quad (36)$$

where the inductive energy  $\mathcal{E}_{\text{JJ}}$  is defined as

$$\mathcal{E}_{\text{JJ}} = -E_J^{\text{max}} \mathcal{S}(\phi_{\text{ext}}) \cos[\hat{\delta} - \arctan(\alpha_J \tan \phi_{\text{ext}})], \quad (37)$$

which is nonlinearly related to the superconducting phase difference  $\hat{\delta}$  and the magnetic flux  $\phi_{\text{ext}} = \pi \Phi_{\text{ext}} / \Phi_0$ . The parameters involved in Eqs. (36) and (37) are the imbalance between the Josephson energies or SQUID asymmetry  $\alpha_J = |E_J^1 - E_J^2| / E_J^{\text{max}}$  with  $E_J^{\text{max}} = E_J^1 + E_J^2$ ; the flux quantum  $\Phi_0 = h/2e$  with Planck constant  $h$  and elementary charge  $e$ ; charging energy  $E_C$ ;  $\mathcal{S}(\phi_{\text{ext}}) = \sqrt{\cos^2 \phi_{\text{ext}} + \alpha_J^2 \sin^2 \phi_{\text{ext}}}$ ; and the charge number operator  $\hat{N}$  conjugate to  $\hat{\delta}$ . Utilizing the trigonometric relationship

$$\cos(A - B) = \cos(A) \cos(B) + \sin(A) \sin(B), \quad (38a)$$

$$\cos(\arctan A) = \frac{1}{\sqrt{1 + A^2}}, \quad (38b)$$

$$\sin(\arctan A) = \frac{A}{\sqrt{1 + A^2}}, \quad (38c)$$

the Hamiltonian (36) can be simplified to

$$\hat{H}_{\text{TTQ}} = 4E_C \hat{N}^2 - E_J^{\text{max}} |\cos \phi_{\text{ext}}| (\cos \hat{\delta} + \alpha_J \tan \phi_{\text{ext}} \sin \hat{\delta}), \quad (39)$$

where the external magnetic flux  $\phi_{\text{ext}}$  is defined as  $\phi_{\text{ext}} = \phi_b + \phi$ . The first term,  $\phi_b$ , denotes the extra flux to modulate the transmon qubits, and the second term,  $\phi = \pi \Phi / \Phi_0$ , denotes the quantized flux resulting from the excitation of the gyration mode. With  $\phi \ll \phi_b$ , the Hamiltonian (39) can be reduced to the free and interaction terms

$$\hat{H}_{\text{Tr}} = 4E_C \hat{N}^2 - s E_J^{\text{max}} (\cos \phi_b \cos \hat{\delta} + \alpha_J \sin \phi_b \sin \hat{\delta}), \quad (40a)$$

$$\hat{H}_{\text{ST}} = s E_J^{\text{max}} \phi (\sin \phi_b \cos \hat{\delta} - \alpha_J \cos \phi_b \sin \hat{\delta}), \quad (40b)$$

where  $s = \text{sgn}[\cos \phi_{\text{ext}}] \approx \text{sgn}[\cos \phi_b]$ .

Next, we quantize the free Hamiltonian  $\hat{H}_{\text{Tr}}$ , while the quantization of the interaction Hamiltonian  $\hat{H}_{\text{ST}}$  is discussed in Sec. V C. Defining  $\tan x = \alpha_J \sin \phi_b / \cos \phi_b = \alpha_J \tan \phi_b$ , the Hamiltonian  $\hat{H}_{\text{Tr}}$  reduces to

$$\hat{H}_{\text{Tr}} = 4E_C \hat{N}^2 - E_J^{\text{max}} \mathcal{S}(\phi_b) \cos \tilde{\delta}, \quad (41)$$

with  $\mathcal{S}(\phi_b) = \sqrt{\cos^2 \phi_b + \alpha_J^2 \sin^2 \phi_b}$  and  $\tilde{\delta} = \hat{\delta} - \arctan(\alpha_J \tan \phi_b)$ . When the qubit is in the transmon regime, i.e.,  $E_J^{\text{max}} \mathcal{S}(\phi_b) \gg E_C$ , and the zero-point fluctuation of the phase satisfies  $\tilde{\delta}_{\text{zpf}} = [2E_C / (E_J^{\text{max}} \mathcal{S}(\phi_b))]^{1/4} \ll 1$ , then the Hamiltonian of the qubit can be simplified as

$$\hat{H}_{\text{Tr}} = 4E_C \hat{N}^2 - E_J^{\text{max}} \mathcal{S}(\phi_b) \left( -\frac{1}{2} \tilde{\delta}^2 + \frac{1}{24} \tilde{\delta}^4 \right), \quad (42)$$

where the constant term has been neglected. Introducing the bosonic field operator of the qubit excitation

$$\hat{N} = i \left[ \frac{E_J^{\text{max}} \mathcal{S}(\phi_b)}{32E_C} \right]^{1/4} (\hat{b}^\dagger - \hat{b}), \quad (43a)$$

$$\tilde{\delta} = \left[ \frac{2E_C}{E_J^{\text{max}} \mathcal{S}(\phi_b)} \right]^{1/4} (\hat{b}^\dagger + \hat{b}), \quad (43b)$$

the Hamiltonian  $\hat{H}_{\text{Tr}}$  can be written as

$$\hat{H}_{\text{Tr}} = \omega_{\text{Tr}} \hat{b}^\dagger \hat{b} - \frac{1}{2} E_C \hat{b}^\dagger \hat{b}^\dagger \hat{b} \hat{b}. \quad (44)$$

The energy levels of the Hamiltonian (44) are shown in Fig. 8(c). Selecting the ground state  $|g\rangle$  and the first-excited

TABLE IV. The table lists the parameters used in Fig. 9.

| No.                 | $x_c$ (nm)  | $y_c$ (nm) | $z_c/h_G$ | $c$ ( $R_{Sk}/R$ ) | $E_J^{\max}/h$ (GHz) | $E_C/h$ (MHz) | $R_{Tr}$ (nm)    | $\alpha_J$ | $\phi_b$ |
|---------------------|-------------|------------|-----------|--------------------|----------------------|---------------|------------------|------------|----------|
| Figs. 9(a) and 9(b) | 50          | 0          | 2         | 0.1                | 50                   | 200           | 50               | 0.1–0.9    | $0-\pi$  |
| Fig. 9(c)           | –100 to 100 | 0          | 3         | 0.1, 0.2, and 0.3  | 50                   | 200           | 50               | 0.06       | $\pi/2$  |
| Fig. 9(d)           | –100 to 100 | 0          | 3         | 0.1                | 50                   | 200           | 50, 100, and 200 | 0.06       | $\pi/2$  |

state  $|e\rangle$  as a qubit, the Hamiltonian (44) can be further simplified as

$$\hat{H}_{Tr} = \frac{\omega_{Tr}}{2} \hat{\sigma}_z^T, \quad (45)$$

with  $\omega_{Tr} = \sqrt{8E_J^{\max} \mathcal{S}(\phi_b) E_C} - E_C$  and the Pauli operator  $\hat{\sigma}_z^T = |e\rangle\langle e| - |g\rangle\langle g|$ . As shown in Fig. 8(d), the resonance frequency  $\omega_{Tr}$  can be controlled by the parameters  $\alpha_J$  and  $\phi_b$ , allowing modulation in the GHz range.

### C. The interaction Hamiltonian

In this section, we will discuss the quantization of the interaction Hamiltonian  $\hat{H}_{ST}$ . Using Eq. (38) and  $\tilde{\delta} = \hat{\delta} - \arctan(\alpha_J \tan \phi_b)$ , the interaction Hamiltonian can be simplified to

$$\hat{H}_{ST} = \frac{E_J^{\max} \phi}{\mathcal{S}(\phi_b)} \left[ \frac{\sin(2\phi_b)}{2} (1 - \alpha_J^2) \cos \tilde{\delta} - \alpha_J \sin \tilde{\delta} \right]. \quad (46)$$

With  $E_J^{\max} \mathcal{S}(\phi_b) \gg E_C$ , and the zero-point fluctuation of the phase  $\tilde{\delta}_{zpf} = [2E_C/(E_J^{\max} \mathcal{S}(\phi_b))]^{1/4} \ll 1$ , then the Hamiltonian  $\hat{H}_{ST}$  can be simplified as

$$\hat{H}_{ST} = \frac{E_J^{\max} \phi}{\mathcal{S}(\phi_b)} \left[ \frac{\sin(2\phi_b)}{2} (1 - \alpha_J^2) \left( -\frac{1}{2} \tilde{\delta}^2 \right) - \alpha_J \tilde{\delta} \right], \quad (47)$$

where the constant terms and higher-order terms ( $\tilde{\delta}^4$  and  $\tilde{\delta}^3$ ) have been ignored due to their negligible effects (Appendix A). The transmon qubit has an anharmonic energy-level structure  $\{|g\rangle, |e\rangle, |f\rangle, \dots\}$ , and we typically select the ground state  $|g\rangle$  and the first-excited state  $|e\rangle$  to construct a qubit (see Sec. VB). In the Hilbert space spanned by  $|g\rangle$  and  $|e\rangle$ , the interaction Hamiltonian is quantized as

$$\hat{H}_{ST} = -\Lambda_{ST}^L (\hat{a} + \hat{a}^\dagger) \hat{\sigma}_+^T \hat{\sigma}_-^T - \Lambda_{ST}^T (\hat{a} \hat{\sigma}_+^T + \hat{a}^\dagger \hat{\sigma}_-^T), \quad (48)$$

where the coupling strengths are defined as

$$\Lambda_{ST}^L = \frac{\mu_0 M_S R^2 (1 - \alpha_J^2) \sin(2\phi_b)}{8\hbar \Phi_0} \sqrt{\frac{2E_C E_J^{\max}}{S^3(\phi_b)}} \mathcal{F}_\Phi, \quad (49a)$$

$$\Lambda_{ST}^T = \frac{\mu_0 M_S R^2 \alpha_J}{4\hbar \Phi_0} \left[ \frac{2E_C E_J^{\max}}{S^5(\phi_b)} \right]^{1/4} \mathcal{F}_\Phi. \quad (49b)$$

The Pauli operators are given by  $\hat{\sigma}_+^T = (\hat{\sigma}_-^T)^\dagger = |e\rangle\langle g|$ . Here, the constant phase  $\phi_{GM}$  is neglected. Figures 9(a) and 9(b) illustrate that the coupling strength can be modulated by adjusting  $\alpha_J$  and the additional flux  $\phi_b$ .

Next, the discussions focus on the transverse coupling strength  $\Lambda_{ST}^T$ . Since the transmon qubit and the gyration mode are coupled through the magnetic flux  $\hat{\Phi}$ , the coupling strength can be tuned by adjusting the spatial position of the SQUID, altering the radius of the SQUID, or increasing the

skyrmion radius. These modifications can enhance the magnetic field passing through the SQUID, thereby strengthening the coupling. Here, we use the central coordinates of the SQUID,  $(x_c, y_c, z_c)$ , to represent its spatial position. Assuming  $y_c = 0$  and  $z_c = 3h_G$ , it can be observed from Figs. 9(c) and 9(d) that when  $x_c = 0$ , the coupling strength is minimized. This is attributed to the fact that the  $\hat{B}_z$  fields in the  $+z$  and  $-z$  directions simultaneously pass through the SQUID, resulting in an effective magnetic flux close to zero [Fig. 6(d)]. When the SQUID is offset from the disk center, the  $\hat{B}_z$  fields in the  $+z$  and  $-z$  directions passing through it are no longer equal. At an optimal position, the magnetic field passing through the SQUID will predominantly be in either the  $+z$  or  $-z$  direction, maximizing the magnetic flux and thus achieving the maximum coupling strength.

In Fig. 9(c), as the radius of the skyrmion increases, the magnetic field passing through the SQUID intensifies, leading to an increase in magnetic flux. Consequently, the coupling strength increases with the enlargement of the skyrmion radius. In Fig. 9(d), as the radius of the SQUID  $R_{Tr}$  increases, the optimal position for coupling strength shifts outward. This adjustment aims to ensure that the magnetic field passing through the SQUID is predominantly in a single direction. In other words, a larger SQUID can be employed, requiring its loop to cover only half of the disk. This is feasible because

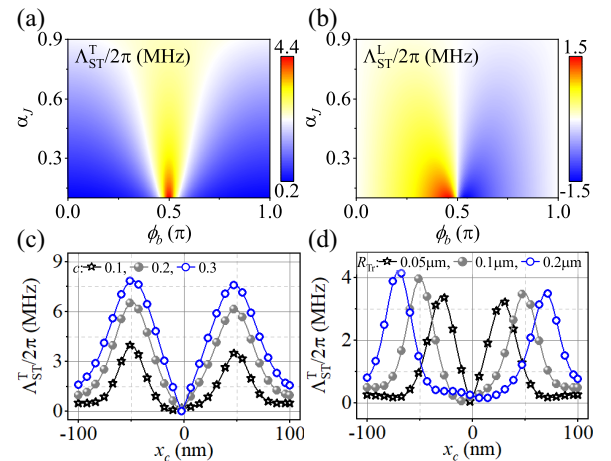


FIG. 9. Panels (a) and (b) show the variations of transverse  $\Lambda_{ST}^T$  and longitudinal  $\Lambda_{ST}^L$  couplings with the parameters of the superconducting qubit, respectively. Panels (c) and (d) illustrate the transverse coupling strength  $\Lambda_{ST}^T$  as a function of the SQUID's spatial position, the reduced skyrmion radius  $c$ , and the SQUID size  $R_{Tr}$ . The parameters of the disk are fixed as follows: radius  $R = 100$  nm, thickness  $h_G = 5$  nm, and saturation magnetization  $M_S = 10^6$  A/m. The other parameters used here are presented in detail in Table IV.

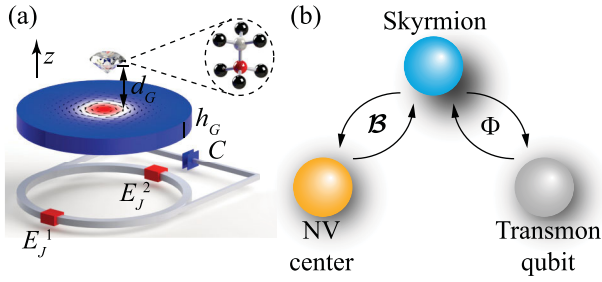


FIG. 10. (a) A three-body hybrid quantum system comprising a skyrmion, an NV center, and a transmon qubit. (b) The coupling mechanism in this hybrid quantum system.

the  $\hat{B}_z$  field is primarily concentrated at the disk center, with nearly zero field at the edges.

## VI. INDIRECT COUPLING BETWEEN THE NV CENTER AND THE SUPERCONDUCTING QUBIT

In this section, we analyze a tripartite hybrid quantum system comprising a skyrmion, an NV center, and a transmon qubit. As illustrated in Fig. 10(a), the diamond particle and the transmon qubit are positioned above and beneath the micromagnetic disk, respectively. As discussed in Secs. IV and V, the skyrmion gyration mode is coupled to the NV center through magnetic dipole interaction, and to the transmon qubit via flux-mediated coupling. The skyrmion gyration mode can serve as a mediator enabling both coherent and dissipative coupling between NV centers and transmon qubits. Coherent coupling enables quantum state conversion between NV centers and transmon qubits, whereas dissipative coupling facilitates nonreciprocal quantum state conversion between them. In the following, we provide a detailed discussion of the indirect coherent and dissipative couplings between NV centers and transmon qubits. The longitudinal coupling term  $\Lambda_{ST}^L(\hat{a} + \hat{a}^\dagger)\hat{\sigma}_+^T\hat{\sigma}_-^T$  has been neglected due to the large detuning condition ( $\omega_{GM}, \omega_{NV}, \omega_{Tr} \gg \Lambda_{SN}, \Lambda_{ST}^T, \Lambda_{ST}^L$ ) or by modulating the qubit parameters  $\{\phi_b, \alpha_J\}$ .

### A. Skyrmion-mediated coherent coupling between an NV center and a transmon qubit

The Hamiltonian of the hybrid quantum system, comprising a skyrmion, an NV center, and a transmon qubit, is given by

$$\hat{H}_{NST} = \omega_{GM}\hat{a}^\dagger\hat{a} + \frac{\omega_{NV}}{2}\hat{\sigma}_z + \frac{\omega_{Tr}}{2}\hat{\sigma}_z^T + \hat{H}_{SN} + \hat{H}_{ST}. \quad (50)$$

Here,  $\omega_{GM}$ ,  $\omega_{NV}$ , and  $\omega_{Tr}$  denote the resonance frequencies of the gyration mode, NV center, and transmon qubit, respectively. When the frequencies of the gyration modes are significantly detuned from those of the NV center and transmon qubit, their indirect coupling can be realized through the exchange of virtual excitations of the gyration modes, described by the effective Hamiltonian (see Appendix B)

$$\hat{H}_{NT}^{\text{coh}} = \frac{\omega_{NV}^{\text{eff}}}{2}\hat{\sigma}_z + \frac{\omega_{Tr}^{\text{eff}}}{2}\hat{\sigma}_z^T + \Lambda_{NT}(\hat{\sigma}_+\hat{\sigma}_-^T + \hat{\sigma}_+^T\hat{\sigma}_-), \quad (51)$$

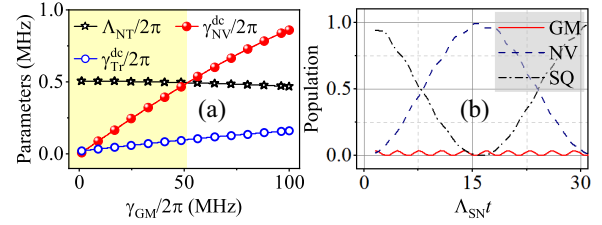


FIG. 11. (a) Indirect coupling between an NV center and a transmon qubit with  $\Lambda_{SN}/2\pi = 12.5$  MHz and  $\Lambda_{ST}/2\pi = 5.05$  MHz. (b) State conversion between NV centers and transmon qubits via the exchange of virtual excitations of the gyration mode with  $\Lambda_{SN} = \Lambda_{ST}$ . The parameters used are  $\gamma_{NV}^{\text{dp}}/2\pi = 10$  kHz,  $\gamma_{NV}^{\text{dc}}/2\pi \approx 0$ , and  $\Delta_{GM} = 10\Lambda_{SN}$ . Here, we take the typical relaxation and dephasing times for the transmon as  $T_1 = T_2 = 50$   $\mu$ s [10].

with the effective frequency  $\{\omega_{NV}^{\text{eff}} = \omega_{NV} - \alpha^2\Delta_{GM}, \omega_{Tr}^{\text{eff}} = \omega_{Tr} - \beta^2\Delta_{GM}\}$ , the detuning  $\Delta_{GM} = \omega_{GM} - \omega_{Tr}$ , and the indirect coupling strength  $\Lambda_{NT} = \Lambda_{ST}\alpha$ . Here, the parameters  $\alpha \approx \Lambda_{SN}/|\Delta_{GM}|$  and  $\beta \approx \Lambda_{ST}/|\Delta_{GM}|$  satisfy  $\alpha, \beta \ll 1$ . The dissipation of gyration modes increases the decay rates of NV centers and transmon qubits, with effective decay rates of  $\Gamma_{NV}^{\text{dc}} = \gamma_{NV}^{\text{dc}} + \alpha^2\gamma_{GM}$  and  $\Gamma_{Tr}^{\text{dc}} = \gamma_{Tr}^{\text{dc}} + \beta^2\gamma_{GM}$ , respectively. The symbols  $\gamma_{NV}^{\text{dc}}$  ( $\gamma_{Tr}^{\text{dc}}$ ) and  $\gamma_{NV}^{\text{dp}}$  ( $\gamma_{Tr}^{\text{dp}}$ ) represent the intrinsic decay rate and dephasing rate of the NV center (transmon qubit), respectively. Figure 11(a) illustrates the dependence of the indirect coupling strength on the gyration mode dissipation. The coupling strength satisfies  $\Lambda_{NT} > \Gamma_{NV}^{\text{dc}}, \gamma_{NV,Tr}^{\text{dp}}$  in the shaded region, indicating that skyrmion-mediated indirect coupling between the NV center and the transmon qubit can reach the strong coupling regime. State conversion between an NV center and a transmon qubit can be achieved through the exchange of virtual excitations of the gyration mode [Fig. 11(b)]. In other words, quantum information can be transferred from the quantum processor (transmon qubits) to the quantum memory (NV centers).

### B. Skyrmion-mediated dissipative coupling between an NV center and a transmon qubit

We next discuss the realization of nonreciprocal interactions between NV centers and transmon qubits via skyrmion-mediated dissipative coupling. By applying a microwave drive  $\hat{H}_{qd} = -\Omega_1(e^{i\omega_1 t}\hat{\sigma}_- + e^{-i\omega_1 t}\hat{\sigma}_+) - \Omega_2(e^{i\omega_2 t}\hat{\sigma}_- + e^{-i\omega_2 t}\hat{\sigma}_+)$  to the NV center, the Hamiltonian of the hybrid quantum system becomes

$$\hat{H}_{NSTD} = \hat{H}_{NST} + \hat{H}_{qd}, \quad (52)$$

where  $\Omega_{1/2}$  and  $\omega_{1/2}$  are the driving strengths and frequencies, respectively. Transforming to the rotating frame of drive  $\Omega_1$  yields  $\hat{H}_{NSTD} = \Delta_{NV}/2\hat{\sigma}_z + \Delta_{Tr}/2\hat{\sigma}_z^T + \Delta_{GM}\hat{a}^\dagger\hat{a} + \Lambda_{SN}(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-) - \Lambda_{ST}^T(\hat{a}\hat{\sigma}_+^T + \hat{a}^\dagger\hat{\sigma}_-^T) - \Omega_1(\hat{\sigma}_- + \hat{\sigma}_+) - \Omega_2[e^{i(\omega_2 - \omega_1)t}\hat{\sigma}_- + e^{-i(\omega_2 - \omega_1)t}\hat{\sigma}_+]$ , where  $\Delta_{NV} = \omega_{NV} - \omega_1$ ,  $\Delta_{Tr} = \omega_{Tr} - \omega_1$ , and  $\Delta_{GM} = \omega_{GM} - \omega_1$ . The first drive is converted into a time-independent term, assumed to be the most significant, defined as  $\hat{H}_0 = -\Omega_1(\hat{\sigma}_- + \hat{\sigma}_+)$ . By employing the transformation  $\hat{H}_{JR} = \exp(i\hat{H}_0 t)(\hat{H}_{NSTD} - \hat{H}_0)\exp(-i\hat{H}_0 t)$ , tuning the drive frequency to satisfy  $\omega_1 - \omega_2 = 2\Omega_1$ , and assuming that drive  $\Omega_1$  is sufficiently strong,

the Hamiltonian  $\hat{H}_{\text{NSTD}}$  can be reduced to [158]

$$\begin{aligned} \hat{H}_{\text{JR}} = & \frac{\Omega_2}{2} \hat{\sigma}_z + \frac{\Delta_{\text{Tr}}}{2} \hat{\sigma}_z^T + \Delta_{\text{GM}} \hat{a}^\dagger \hat{a} \\ & + \bar{\Lambda}_{\text{SN}} (\hat{a} + \hat{a}^\dagger) \hat{\sigma}_x - \Lambda_{\text{ST}}^T (\hat{a} \hat{\sigma}_+^T + \hat{a}^\dagger \hat{\sigma}_-^T), \end{aligned} \quad (53)$$

with  $\bar{\Lambda}_{\text{SN}} = \Lambda_{\text{SN}}/2$ . In other words, this system incorporates both JC coupling and effective Rabi coupling.

Next, this system is utilized to achieve a nonreciprocal response between an NV center and a transmon qubit. Considering the dissipation of the gyration mode, the system's master equation is given by

$$\dot{\hat{\rho}} = -i[\hat{H}_{\text{JR}}, \hat{\rho}] + \gamma_{\text{GM}} D[\hat{a}] \hat{\rho}, \quad (54)$$

where the Lindblad operator is defined as  $D[\hat{O}] = \hat{O} \hat{\rho} \hat{O}^\dagger - \{\hat{O}^\dagger \hat{O}, \hat{\rho}\}/2$ . Based on Eq. (53), we derive the quantum Langevin equation for the skyrmion gyration mode  $\hat{a}$  as

$$\begin{aligned} \partial_t \hat{a} = & -i(\Delta_{\text{GM}} \hat{a} + \bar{\Lambda}_{\text{SN}} \hat{\sigma}_+ + \bar{\Lambda}_{\text{SN}} \hat{\sigma}_- - \Lambda_{\text{ST}}^T \hat{\sigma}_-^T) \\ & - \frac{\gamma_{\text{GM}}}{2} \hat{a} + \sqrt{\gamma_{\text{GM}}} \hat{a}_{\text{in}}, \end{aligned} \quad (55)$$

where  $\hat{a}_{\text{in}}$  represents the noise operator. When the dissipation of the gyration mode is large,  $\gamma_{\text{GM}} \gg \max\{\bar{\Lambda}_{\text{SN}}, \Lambda_{\text{ST}}^T\}$ , the gyration mode can be adiabatically eliminated by setting  $\partial_t \hat{a} = 0$ . Then, we can get [159]

$$\hat{a} = \frac{-i(\bar{\Lambda}_{\text{SN}} \hat{\sigma}_+ + \bar{\Lambda}_{\text{SN}} \hat{\sigma}_- - \Lambda_{\text{ST}}^T \hat{\sigma}_-^T)}{i\Delta_{\text{GM}} + \gamma_{\text{GM}}/2}. \quad (56)$$

Substituting Eq. (56) into the master equation [Eq. (54)], we obtain the master equation for the dynamics involving only NV centers and transmon qubits, given by

$$\dot{\hat{\rho}} = -i[\hat{H}_{\text{NT}}, \hat{\rho}] + \Gamma_{\text{NT}} D[\hat{\Xi}] \hat{\rho}, \quad (57)$$

with  $\hat{H}_{\text{NT}} = \bar{\Omega}_{\text{NV}}/2 \hat{\sigma}_z + \bar{\Omega}_{\text{Tr}}/2 \hat{\sigma}_z^T + \bar{\Lambda}_{\text{NT}} \hat{\sigma}_x \hat{\sigma}_x^T$  and  $\hat{\Xi} = \hat{\sigma}_+ + \hat{\sigma}_- - \eta_{\text{NT}} \hat{\sigma}_-^T$ . The effective parameters of the system are defined as  $\eta_{\text{NT}} = \Lambda_{\text{ST}}^T / \bar{\Lambda}_{\text{SN}}$ ,  $\bar{\Omega}_{\text{NV}} = \Omega_2$ ,  $\bar{\Omega}_{\text{Tr}} = \Delta_{\text{Tr}} - \eta_{\text{NT}}^2 \Delta_{\text{GM}} \bar{\Lambda}_{\text{SN}}^2 / (\Delta_{\text{GM}}^2 + \gamma_{\text{GM}}^2/4)$ ,  $\bar{\Lambda}_{\text{NT}} = \eta_{\text{NT}} \Delta_{\text{GM}} \bar{\Lambda}_{\text{SN}}^2 / (\Delta_{\text{GM}}^2 + \gamma_{\text{GM}}^2/4)$ , and  $\Gamma_{\text{NT}} = \gamma_{\text{GM}} \bar{\Lambda}_{\text{SN}}^2 / (\Delta_{\text{GM}}^2 + \gamma_{\text{GM}}^2/4)$ . Starting from the quantum master equation [Eq. (57)], the equation of motion for the system's expectation value is obtained as

$$\begin{aligned} \partial_t \langle \hat{\sigma}_- \rangle = & (-i\bar{\Omega}_{\text{NV}} - \Gamma_{\text{NT}}) \langle \hat{\sigma}_- \rangle + \Gamma_{\text{NT}} \langle \hat{\sigma}_+ \rangle \\ & + \left( i\bar{\Lambda}_{\text{NT}} + \eta_{\text{NT}} \frac{\Gamma_{\text{NT}}}{2} \right) \langle \hat{\sigma}_+^T \hat{\sigma}_z \rangle \\ & + \left( i\bar{\Lambda}_{\text{NT}} - \eta_{\text{NT}} \frac{\Gamma_{\text{NT}}}{2} \right) \langle \hat{\sigma}_-^T \hat{\sigma}_z \rangle, \end{aligned} \quad (58a)$$

$$\begin{aligned} \partial_t \langle \hat{\sigma}_-^T \rangle = & \left( -i\bar{\Omega}_{\text{Tr}} - \eta_{\text{NT}}^2 \frac{\Gamma_{\text{NT}}}{2} \right) \langle \hat{\sigma}_-^T \rangle \\ & + \left( i\bar{\Lambda}_{\text{NT}} - \eta_{\text{NT}} \frac{\Gamma_{\text{NT}}}{2} \right) \langle \hat{\sigma}_+ \hat{\sigma}_z^T \rangle \\ & + \left( i\bar{\Lambda}_{\text{NT}} - \eta_{\text{NT}} \frac{\Gamma_{\text{NT}}}{2} \right) \langle \hat{\sigma}_- \hat{\sigma}_z^T \rangle. \end{aligned} \quad (58b)$$

Equation (58) illustrates that the dynamics between the NV center and the transmon qubit are mutually influential but asymmetric, enabling the realization of nonreciprocal responses between the NV center and the transmon qubit. Figures 12(a) and 12(b) illustrate state conversion between

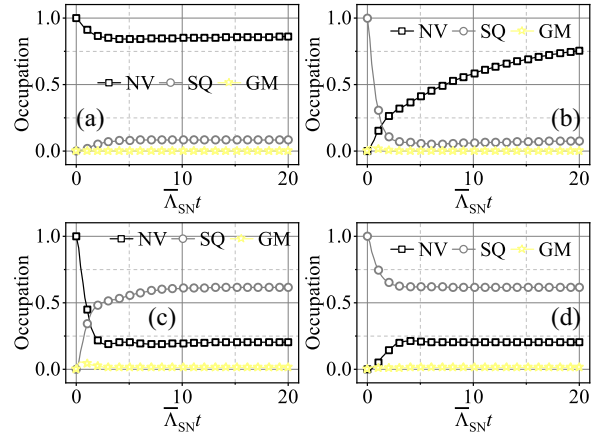


FIG. 12. Panels (a) and (b) show the nonreciprocal state conversion between the NV centers and the transmon qubits when the NV centers are driven, with parameters  $\eta_{\text{NT}} = 3$ ,  $\Delta_{\text{GM}} = \Omega_2 = \Delta_{\text{Tr}} = 0.5\bar{\Lambda}_{\text{SN}}$ , and  $\gamma_{\text{GM}} = 10 \max(\bar{\Lambda}_{\text{SN}}, \Lambda_{\text{ST}})$ . Panels (c) and (d) illustrate the nonreciprocal state conversion between the NV center and the transmon qubit under the driven-transmon condition. The parameters are set to  $\Delta_{\text{GM}} = \Omega_2 = \Delta_{\text{NV}} = 0.5\bar{\Lambda}_{\text{SN}}$ , and  $\gamma_{\text{GM}} = 10 \max(\Lambda_{\text{SN}}, \bar{\Lambda}_{\text{ST}})$ .

the two, corresponding to the excitation of the NV center and the transmon qubit, respectively. State conversion between the NV center and the transmon qubit is demonstrably nonreciprocal, as the NV center has difficulty transferring quantum states to the transmon qubit, whereas the transmon qubit can efficiently transfer quantum states to the NV center.

Similarly, modifying the drive from the NV center to the transmon qubit also yields a nonreciprocal state conversion between these systems. By performing a similar derivation, the system's effective Hamiltonian when the transmon qubit is driven is given by

$$\begin{aligned} \hat{H}_{\text{DTr}} = & \frac{\Delta_{\text{NV}}}{2} \hat{\sigma}_z + \frac{\Omega_2}{2} \hat{\sigma}_z^T + \Delta_{\text{GM}} \hat{a}^\dagger \hat{a} \\ & + \Lambda_{\text{SN}} (\hat{a} \hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_-) - \bar{\Lambda}_{\text{ST}}^T (\hat{a} + \hat{a}^\dagger) \hat{\sigma}_x^T, \end{aligned} \quad (59)$$

with  $\bar{\Lambda}_{\text{ST}}^T = \Lambda_{\text{ST}}^T/2$ . Numerical simulations based on Eq. (59), presented in Figs. 12(c) and 12(d), demonstrate the nonreciprocal state conversion when transmon qubit is driven. Comparison with Figs. 12(a) and 12(b) shows that the physical mechanism of nonreciprocity depends on the system's asymmetric dissipative interactions, rather than on which subsystem the drive is applied to.

It is worth noting that Eq. (58) reveals a competition between coherent and dissipative channels. Coherent coupling favors reciprocal state conversion between the NV center and the transmon qubit, whereas dissipative coupling favors nonreciprocal conversion. The competition between coherent and dissipative coupling is numerically simulated using the master equation Eq. (54), and the simulation results are shown in Fig. 13. The system is tuned by varying the detuning  $\Delta_{\text{GM}}$  or the dissipation  $\gamma_{\text{GM}}$ . For convenience, we define  $\eta_{\text{CD}} = \bar{\Lambda}_{\text{NT}}/\Gamma_{\text{NT}}$  as the ratio of coherent to dissipative coupling strengths. Figures 13(a) and 13(b) show the case  $\eta_{\text{CD}} = 300$ , where coherent coupling dominates and the state conversion between the NV center and the transmon qubit is

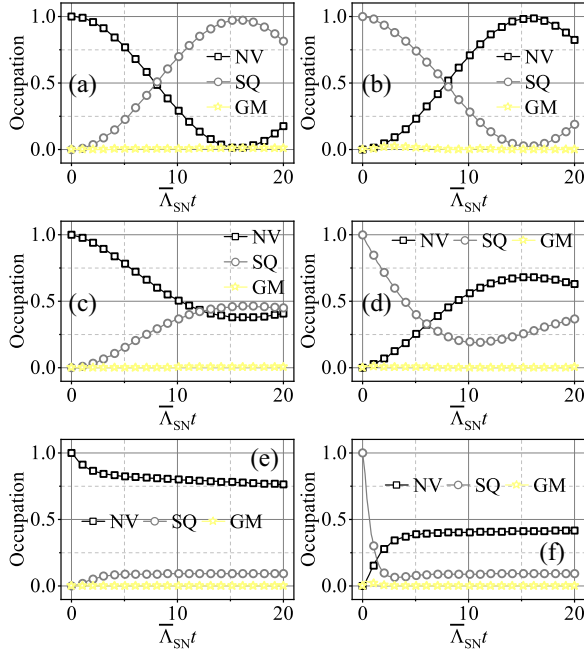


FIG. 13. Panels (a) and (b) ( $\Delta_{GM} = 30\bar{\Lambda}_{SN}$ ,  $\gamma_{GM} = 0.1\Lambda_{ST}^T$ ), (c) and (d) ( $\Delta_{GM} = 30\bar{\Lambda}_{SN}$ ,  $\gamma_{GM} = 5\Lambda_{ST}^T$ ), and (e) and (f) ( $\Delta_{GM} = 0$ ,  $\gamma_{GM} = 10\Lambda_{ST}^T$ ) illustrate the state conversion between the NV center and transmon qubit. The corresponding ratios  $\eta_{CD}$  of coherent to dissipative coupling are  $\eta_{CD} = 300$ ,  $\eta_{CD} = 6$ , and  $\eta_{CD} = 0$ , respectively. The other parameters are set to  $\eta_{NT} = 3$ ,  $\Omega_2 = 0$ , and  $\Delta_{Tr} = \eta_{NT}^2 \Delta_{GM} \bar{\Lambda}_{SN}^2 / (\Delta_{GM}^2 + \gamma_{GM}^2/4)$ .

reciprocal. As shown in Figs. 13(c) and 13(d), for  $\eta_{CD} = 6$ , the coherent and dissipative couplings are comparable, and asymmetry emerges in the state conversion. Figures 13(e) and 13(f) present the case of purely dissipative coupling ( $\eta_{CD} = 0$ ), which yields pronounced nonreciprocity in the state conversion. Therefore, adjusting the detuning  $\Delta_{GM}$  or the dissipation  $\gamma_{GM}$  enables the controlled transition of state conversion between the NV center and the transmon qubit from reciprocal to nonreciprocal.

## VII. EXPERIMENTAL FEASIBILITY

In this setup, the disk has a radius  $R = 100$  nm and a thickness  $h_G = 5$  nm, respectively, satisfying the condition  $R \gg h_G$ , which corresponds to a thin disk [146,160–164]. The distance between the NV center and the disk surface is  $d_G = 5$  nm. Consequently, the coupling strength between them is  $\Lambda_{SN}/2\pi \approx 12.5$  MHz. In contrast to the NV center, the transmon qubit is situated beneath the disk with the center coordinates of the SQUID being  $x_c = R/2$ ,  $y_c = 0$ , and  $z_c = 2h_G$ . The diameter of the SQUID is taken as 100 nm. Typical experimentally feasible parameters for transmon qubits include an externally applied flux  $\phi_b = \pi/2$  and a Josephson energy imbalance of  $\alpha_J = 0.06$  [10,24]. The transverse coupling strength is calculated to be  $\Lambda_{ST}^T/2\pi \approx 5.05$  MHz.

Next, we analyze the strengths of coherent and dissipative coupling between the NV center and the transmon qubit, considering the intrinsic dissipation parameters of NV centers and transmon qubits as  $\gamma_{NV}^{dc}/2\pi \approx 1$  Hz,

$\gamma_{NV}^{dp}/2\pi = 10$  kHz,  $\gamma_{Tr}^{dc}/2\pi = 20$  kHz, and  $\gamma_{Tr}^{dp}/2\pi = 10$  kHz [1,2,10,134,165]. For gyration-mode-mediated coherent coupling, with gyration-mode dissipation  $\gamma_{GM}/2\pi = 50$  MHz (see Sec. II E), the gyration-mode-mediated indirect coupling  $\Lambda_{NT}/2\pi = 0.5$  MHz between the NV center and the transmon qubit is obtained, with effective decay rates for the NV center and the transmon qubit given by  $\Gamma_{NV}^{dc}/2\pi = 0.48$  MHz and  $\Gamma_{Tr}^{dc}/2\pi = 0.1$  MHz, respectively. In other words, the indirect coupling between NV centers and transmon qubits can enter the strong coupling regime. For gyration-mode-mediated dissipative coupling, with  $\Omega_2 = \Delta_{Tr} = \Delta_{GM} = 0$ , the dissipative coupling strength simplifies to  $\Gamma_{NT} = 4\bar{\Lambda}_{SN}^2/\gamma_{GM}$ . In the case of large dissipation ( $\gamma_{GM} \gg \bar{\Lambda}_{SN}$ ), we set  $\gamma_{GM}/2\pi = 300$  MHz, yielding a dissipative coupling strength of  $\Gamma_{NT}/2\pi = 0.52$  MHz, which satisfies  $\Gamma_{NT} > \gamma_{NV,Tr}^{dc,dp}$ , indicating that the system can enter the strong coupling regime.

## VIII. CONCLUSION

We present a three-body hybrid quantum system integrating skyrmions, NV centers, and transmon qubits. By utilizing the gyration mode as a quantum bus, we establish both coherent and dissipative couplings between the NV center and transmon qubit that can reach the strong coupling regime. This enables bidirectional quantum state transfer with reciprocity control—reciprocal conversion can be realized through coherent coupling, while nonreciprocal state transfer is achievable via dissipative coupling mechanisms. The proposed hybrid platform provides a potential for developing integrated quantum computing architectures and advancing quantum information processing technologies at the chip scale.

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## DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

## APPENDIX A: THE EFFECTS OF HIGHER-ORDER TERMS

In this Appendix, we will analyze the effect of higher-order terms ( $\delta^4$  and  $\delta^3$ ) that were ignored in Sec. V C. The interaction Hamiltonian containing only higher-order terms is

$$\hat{H}'_{ST} = \frac{E_J^{\max} \phi}{S(\phi_b)} \left[ \frac{\sin(2\phi_b)}{2} \frac{1 - \alpha_J^2 \delta^4}{24} + \frac{\alpha_J \delta^3}{6} \right]. \quad (A1)$$

Next, the first and second terms in Eq. (A1) are calculated separately. Substituting Eq. (35) and Eq. (43) into

$$\hat{H}'_1 = \frac{E_J^{\max} \phi}{S(\phi_b)} \frac{\sin(2\phi_b)}{2} (1 - \alpha_J^2) \left( \frac{1}{24} \delta^4 \right) \quad (A2)$$

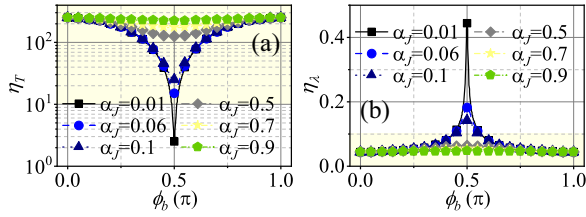


FIG. 14. Panels (a) and (b) display the variation of  $\eta_T$  and  $\eta_\lambda$  with the parameters  $\alpha_J$  and  $\phi_b$ , respectively. The other parameters are  $E_J^{\max}/h = 50$  GHz and  $E_C/h = 200$  MHz.

and simplifying it, we obtain

$$\hat{H}'_1 = \Lambda_{ST}^{L'} (\hat{a} + \hat{a}^\dagger) (\hat{b}^\dagger \hat{b} + \frac{1}{2} \hat{b}^\dagger \hat{b}^\dagger \hat{b} \hat{b}), \quad (\text{A3})$$

where  $\Lambda_{ST}^{L'} \equiv \eta_\lambda \Lambda_{ST}^L$  and  $\eta_\lambda = \sqrt{E_C/[2S(\phi_b)E_J^{\max}]}$ . Applying the same calculation, the second term in Eq. (A1) can be reduced to

$$\hat{H}'_2 = \Lambda_{ST}^{T'} (\hat{b}^\dagger \hat{b} \hat{b}^\dagger \hat{a} + \hat{b} \hat{b}^\dagger \hat{b} \hat{a}^\dagger), \quad (\text{A4})$$

with  $\Lambda_{ST}^{T'} \equiv \eta_\lambda \Lambda_{ST}^T$ . Here, the ground state  $|g\rangle$  and the first-excited state  $|e\rangle$  are treated as qubits, then we can get  $\hat{b} \rightarrow \hat{\sigma}_-^T$  and  $\hat{b}^\dagger \rightarrow \hat{\sigma}_+^T$ . Utilizing  $\hat{\sigma}_+^T \hat{\sigma}_+^T \hat{\sigma}_-^T \hat{\sigma}_-^T = 0$ ,  $\hat{\sigma}_+^T \hat{\sigma}_-^T \hat{\sigma}_+^T = \hat{\sigma}_+^T$ , and  $\hat{\sigma}_-^T \hat{\sigma}_+^T \hat{\sigma}_-^T = \hat{\sigma}_-^T$ , the correction term of the interaction Hamiltonian due to the higher-order terms can be simplified to

$$\hat{H}_{ST} = \eta_\lambda \Lambda_{ST}^L (\hat{a} + \hat{a}^\dagger) \hat{\sigma}_+^T \hat{\sigma}_-^T + \eta_\lambda \Lambda_{ST}^T (\hat{a} \hat{\sigma}_+^T + \hat{a}^\dagger \hat{\sigma}_-^T). \quad (\text{A5})$$

The interaction Hamiltonian, including the correction term, is

$$\begin{aligned} \hat{H}_{ST}^{\text{corr}} = & -\Lambda_{ST}^L (1 - \eta_\lambda) (\hat{a} + \hat{a}^\dagger) \hat{\sigma}_+^T \hat{\sigma}_-^T \\ & - \Lambda_{ST}^T (1 - \eta_\lambda) (\hat{a} \hat{\sigma}_+^T + \hat{a}^\dagger \hat{\sigma}_-^T). \end{aligned} \quad (\text{A6})$$

This shows that the corrections introduced by the higher-order terms result in a reduction of the coupling strength. We define the parameter  $\eta_T = E_J^{\max} S(\phi_b)/E_C$  to describe the operational region of the transmon qubit, where  $\eta_T \gg 1$  means the transmon qubit operates in the transmon regime. Figure 14(a)

illustrates the variation of  $\eta_T$  with  $\phi_b$  and  $\alpha_J$ , where the shaded area represents the transmon regime. It can be seen that superconducting qubits may operate beyond the transmon regime when  $\alpha_J \rightarrow 0$  and  $\phi_b \rightarrow \pi/2$ . In this case, the effect of higher-order terms will become significant [Fig. 14(b)]. The shaded areas in Fig. 14(b) represent regions where the correction terms can be safely ignored. Here, we take  $\alpha_J = 0.06$  and  $\phi_b = \pi/2$ . With these parameter values, the qubits operate in the transmon regime [Fig. 14(a)]. However, in this case, the correction term contributes an effect greater than 0.1, so the coupling strength after correction is  $\Lambda_{ST}^{\text{Tcorr}}/2\pi = \Lambda_{ST}^T (1 - \eta_\lambda) = 4.14$  MHz.

## APPENDIX B: SKYRMION-MEDIATED COHERENT COUPLING BETWEEN AN NV CENTER AND A TRANSMON QUBIT

Based on the previous analysis, the Hamiltonian of a hybrid quantum system comprising a skyrmion, an NV center, and a transmon qubit is given by Eq. (50). By transforming to a rotating frame with frequency  $\omega_{\text{Tr}}$ , the Hamiltonian takes the form  $\hat{H}_{\text{NST}} = \Delta_{\text{GM}} \hat{a}^\dagger \hat{a} + \Delta_{\text{NV}}/2 \hat{\sigma}_z + \Lambda_{\text{SN}} (\hat{a} \hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_-) - \Lambda_{\text{ST}} (\hat{a} \hat{\sigma}_+^T + \hat{a}^\dagger \hat{\sigma}_-^T)$ , where the detunings are given by  $\Delta_{\text{GM}} = \omega_{\text{GM}} - \omega_{\text{Tr}}$  and  $\Delta_{\text{NV}} = \omega_{\text{NV}} - \omega_{\text{Tr}}$ . Next, we consider the adiabatic elimination of gyration modes under large detuning conditions. From the Hamiltonian  $\hat{H}_{\text{NST}}$ , the system's quantum Langevin equation can be derived as

$$\dot{\hat{a}} = -\left(i\Delta_{\text{GM}} + \frac{\gamma_{\text{GM}}}{2}\right) \hat{a} - i\Lambda_{\text{SN}} \hat{\sigma}_- + i\Lambda_{\text{ST}} \hat{\sigma}_-^T + \sqrt{\gamma_{\text{GM}}} \hat{a}_{\text{in}}, \quad (\text{B1a})$$

$$\dot{\hat{\sigma}}_- = -\left(i\Delta_{\text{NV}} + \frac{\gamma_{\text{NV}}^{\text{dc}}}{2}\right) \hat{\sigma}_- + i\Lambda_{\text{SN}} \hat{a} \hat{\sigma}_z + \sqrt{\gamma_{\text{NV}}^{\text{dc}}} \hat{\sigma}_-^{\text{in}}, \quad (\text{B1b})$$

$$\dot{\hat{\sigma}}_-^T = -\frac{\gamma_{\text{Tr}}^{\text{dc}}}{2} \hat{\sigma}_-^T - i\Lambda_{\text{ST}} \hat{a} \hat{\sigma}_z^T + \sqrt{\gamma_{\text{Tr}}^{\text{dc}}} \hat{\sigma}_-^{T,\text{in}}, \quad (\text{B1c})$$

where  $\hat{a}_{\text{in}}$ ,  $\hat{\sigma}_-^{\text{in}}$ , and  $\hat{\sigma}_-^{T,\text{in}}$  denote the corresponding noise operators. Here, we examine the large-detuning regime with  $\Delta_{\text{GM}} \gg \Lambda_{\text{SN}}, \Lambda_{\text{ST}}, \gamma_{\text{GM}}$ . Consequently, the formal integral of the equation of motion (B1) is given by

$$\hat{a}(t) = \hat{a}(0) \exp(-\mathcal{X}_{\text{GM}} t) + \exp(-\mathcal{X}_{\text{GM}} t) \times \int_0^t d\tau [-i\Lambda_{\text{SN}} \hat{\sigma}_-(\tau) + i\Lambda_{\text{ST}} \hat{\sigma}_-^T(\tau) + \sqrt{\gamma_{\text{GM}}} \hat{a}_{\text{in}}(\tau)] \exp(\mathcal{X}_{\text{GM}} \tau), \quad (\text{B2a})$$

$$\hat{\sigma}_-(t) = \hat{\sigma}_-(0) \exp(-\mathcal{X}_{\text{NV}} t) + \exp(-\mathcal{X}_{\text{NV}} t) \times \int_0^t d\tau [i\Lambda_{\text{SN}} \hat{a}(\tau) \hat{\sigma}_z(\tau) + \sqrt{\gamma_{\text{NV}}^{\text{dc}}} \hat{\sigma}_-^{\text{in}}(\tau)] \exp(\mathcal{X}_{\text{NV}} \tau), \quad (\text{B2b})$$

$$\hat{\sigma}_-^T(t) = \hat{\sigma}_-^T(0) \exp\left(-\frac{\gamma_{\text{Tr}}^{\text{dc}}}{2} t\right) + \exp\left(-\frac{\gamma_{\text{Tr}}^{\text{dc}}}{2} t\right) \times \int_0^t d\tau (-i\Lambda_{\text{ST}} \hat{a}(\tau) \hat{\sigma}_z^T(\tau) + \sqrt{\gamma_{\text{Tr}}^{\text{dc}}} \hat{\sigma}_-^{T,\text{in}}(\tau)) \exp\left(\frac{\gamma_{\text{Tr}}^{\text{dc}}}{2} \tau\right), \quad (\text{B2c})$$

where  $\mathcal{X}_{\text{GM}} = i\Delta_{\text{GM}} + \gamma_{\text{GM}}/2$  and  $\mathcal{X}_{\text{NV}} = i\Delta_{\text{NV}} + \gamma_{\text{NV}}^{\text{dc}}/2$ . We examine the regime in which the bosonic mode is significantly detuned. Under this condition, the dynamics of the NV center and the transmon qubit are only weakly affected by the gyration mode. Consequently, we can get

$$\hat{\sigma}_-(t) = \hat{\sigma}_-(0) \exp\left[-\left(i\Delta_{\text{NV}} + \frac{\gamma_{\text{NV}}^{\text{dc}}}{2}\right) t\right] + \Sigma_-^{\text{in}}(t), \quad (\text{B3a})$$

$$\hat{\sigma}_-^T(t) = \hat{\sigma}_-^T(0) \exp\left(-\frac{\gamma_{\text{Tr}}^{\text{dc}}}{2} t\right) + \Sigma_-^{T,\text{in}}(t), \quad (\text{B3b})$$

where  $\Sigma_-^{\text{in}}(t)$  and  $\Sigma_-^{T,\text{in}}(t)$  denote the associated noise terms. Substituting Eq. (B3) into the integral equation of the gyration mode (B2a) and utilizing the condition  $\gamma_{\text{GM}} \gg \gamma_{\text{NV}}^{\text{dc}}, \gamma_{\text{Tr}}^{\text{dc}}$ , one

can get

$$\hat{a}(t) = \frac{-i\Lambda_{SN}\hat{\sigma}_-(t) + i\Lambda_{ST}\hat{\sigma}_-^T(t)}{i\Delta_{GM} + \gamma_{GM}/2} + \tilde{A}_{in}(t), \quad (B4)$$

where  $\tilde{A}_{in}(t)$  denotes the modified noise term. Substituting Eq. (B4) into Eqs. (B1b) and (B1c), along with the condition  $\hat{\sigma}_z\hat{\sigma}_- = -\hat{\sigma}_-$ , yields the expression

$$\dot{\hat{\sigma}}_- = -\left(i\Delta_{NV}^{\text{eff}} + \frac{\Gamma_{NV}^{\text{dc}}}{2}\right)\hat{\sigma}_- + i\Lambda_{ST}\alpha\hat{\sigma}_z\hat{\sigma}_-^T + \tilde{\Sigma}_{-}^{\text{in}}, \quad (B5a)$$

$$\dot{\hat{\sigma}}_-^T = -\left(i\Delta_{Tr}^{\text{eff}} + \frac{\Gamma_{Tr}^{\text{dc}}}{2}\right)\hat{\sigma}_-^T + i\Lambda_{ST}\alpha\hat{\sigma}_z^T\hat{\sigma}_- + \tilde{\Sigma}_{-}^{T,\text{in}}, \quad (B5b)$$

where  $\tilde{\Sigma}_{-}^{\text{in}}$  and  $\tilde{\Sigma}_{-}^{T,\text{in}}$  denote the modified noise terms of the NV center and the transmon qubit, respectively. Thus, the

effective Hamiltonian describing the interaction between the NV center and the transmon qubit is given by

$$\hat{H}_{NT}^{\text{coh}} = \frac{\Delta_{NV}^{\text{eff}}}{2}\hat{\sigma}_z + \frac{\Delta_{Tr}^{\text{eff}}}{2}\hat{\sigma}_z^T + \Lambda_{NT}(\hat{\sigma}_+\hat{\sigma}_-^T + \hat{\sigma}_+^T\hat{\sigma}_-). \quad (B6)$$

Here, we have defined the effective parameters as  $\Delta_{NV}^{\text{eff}} = \Delta_{NV} - \alpha^2\Delta_{GM}$ ,  $\Delta_{Tr}^{\text{eff}} = -\beta^2\Delta_{GM}$ ,  $\Gamma_{NV}^{\text{dc}} = \gamma_{NV}^{\text{dc}} + \alpha^2\gamma_{GM}$ ,  $\Gamma_{Tr}^{\text{dc}} = \gamma_{Tr}^{\text{dc}} + \beta^2\gamma_{GM}$ ,  $\Lambda_{SN} = \Lambda_{ST}\alpha$ ,  $\alpha \approx \Lambda_{SN}/|\Delta_{GM}|$ , and  $\beta \approx \Lambda_{ST}/|\Delta_{GM}|$ . Since the large detuning condition is considered, we can get  $\alpha, \beta \ll 1$ . Returning to the original representation, we obtain the effective Hamiltonian

$$\hat{H}_{NT}^{\text{coh}} = \frac{\omega_{NV}^{\text{eff}}}{2}\hat{\sigma}_z + \frac{\omega_{Tr}^{\text{eff}}}{2}\hat{\sigma}_z^T + \Lambda_{NT}(\hat{\sigma}_+\hat{\sigma}_-^T + \hat{\sigma}_+^T\hat{\sigma}_-), \quad (B7)$$

where  $\omega_{NV}^{\text{eff}} = \omega_{NV} - \alpha^2\Delta_{GM}$  and  $\omega_{Tr}^{\text{eff}} = \omega_{Tr} - \beta^2\Delta_{GM}$ .

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- [1] N. Bar-Gill, L. M. Pham, A. Jarmola, D. Budker, and R. L. Walsworth, Solid-state electronic spin coherence time approaching one second, *Nat. Commun.* **4**, 1743 (2013).
- [2] M. W. Doherty, N. B. Manson, P. Delaney, F. Jelezko, J. Wrachtrup, and L. C. L. Hollenberg, The nitrogen-vacancy colour centre in diamond, *Phys. Rep.* **528**, 1 (2013).
- [3] A. Sipahigil, R. E. Evans, D. D. Sukachev, M. J. Burek, J. Borregaard, M. K. Bhaskar, C. T. Nguyen, J. L. Pacheco, H. A. Atikian, C. Meuwly, R. M. Camacho, F. Jelezko, E. Bielejec, H. Park, M. Lončar, and M. D. Lukin, An integrated diamond nanophotonics platform for quantum-optical networks, *Science* **354**, 847 (2016).
- [4] Y.-I. Sohn, S. Meesala, B. Pingault, H. A. Atikian, J. Holzgrafe, M. Gündoğan, C. Stavrakas, M. J. Stanley, A. Sipahigil, J. Choi, M. Zhang, J. L. Pacheco, J. Abraham, E. Bielejec, M. D. Lukin, M. Atatüre, and M. Lončar, Controlling the coherence of a diamond spin qubit through its strain environment, *Nat. Commun.* **9**, 2012 (2018).
- [5] C. Bradac, W. Gao, J. Forneris, M. E. Trusheim, and I. Aharonovich, Quantum nanophotonics with group IV defects in diamond, *Nat. Commun.* **10**, 5625 (2019).
- [6] T. Neuman, M. Eichenfield, M. E. Trusheim, L. Hackett, P. Narang, and D. Englund, A phononic interface between a superconducting quantum processor and quantum networked spin memories, *npj Quantum Inf.* **7**, 121 (2021).
- [7] I. B. W. Harris and D. Englund, Coherence of group-IV color centers, *Phys. Rev. B* **109**, 085414 (2024).
- [8] G. D. Fuchs, G. Burkard, P. V. Klimov, and D. D. Awschalom, A quantum memory intrinsic to single nitrogen-vacancy centres in diamond, *Nat. Phys.* **7**, 789 (2011).
- [9] Z.-L. Xiang, S. Ashhab, J. Q. You, and F. Nori, Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems, *Rev. Mod. Phys.* **85**, 623 (2013).
- [10] M. Kjaergaard, M. E. Schwartz, J. Braumüller, P. Krantz, J. I.-J. Wang, S. Gustavsson, and W. D. Oliver, Superconducting qubits: Current state of play, *Annu. Rev. Condens. Matter Phys.* **11**, 369 (2020).
- [11] A. A. Clerk, K. W. Lehnert, P. Bertet, J. R. Petta, and Y. Nakamura, Hybrid quantum systems with circuit quantum electrodynamics, *Nat. Phys.* **16**, 257 (2020).
- [12] G. Burkard, M. J. Gullans, X. Mi, and J. R. Petta, Superconductor-semiconductor hybrid-circuit quantum electrodynamics, *Nat. Rev. Phys.* **2**, 129 (2020).
- [13] F. Arute, K. Arya, R. Babbush, D. Bacon, J. C. Bardin, R. Barends, R. Biswas, S. Boixo, F. G. S. L. Brandao, and D. A. Buell, Quantum supremacy using a programmable superconducting processor, *Nature (London)* **574**, 505 (2019).
- [14] M. Pita-Vidal, A. Bargerbos, R. Žitko, L. J. Splitthoff, L. Grünhaupt, J. J. Wesdorp, Y. Liu, L. P. Kouwenhoven, R. Aguado, B. van Heck, A. Kou, and C. K. Andersen, Direct manipulation of a superconducting spin qubit strongly coupled to a transmon qubit, *Nat. Phys.* **19**, 1110 (2023).
- [15] I. Aharonovich, A. D. Greentree, and S. Praver, Diamond photonics, *Nat. Photon.* **5**, 397 (2011).
- [16] E. R. MacQuarrie, T. A. Gosavi, N. R. Jungwirth, S. A. Bhave, and G. D. Fuchs, Mechanical spin control of nitrogen-vacancy centers in diamond, *Phys. Rev. Lett.* **111**, 227602 (2013).
- [17] P. Ovarthaiyapong, K. W. Lee, B. A. Myers, and A. C. B. Jayich, Dynamic strain-mediated coupling of a single diamond spin to a mechanical resonator, *Nat. Commun.* **5**, 4429 (2014).
- [18] J. Teissier, A. Barfuss, P. Appel, E. Neu, and P. Maletinsky, Strain coupling of a nitrogen-vacancy center spin to a diamond mechanical oscillator, *Phys. Rev. Lett.* **113**, 020503 (2014).
- [19] D. A. Golter, T. Oo, M. Amezcua, I. Lekavicius, K. A. Stewart, and H. Wang, Coupling a surface acoustic wave to an electron spin in diamond via a dark state, *Phys. Rev. X* **6**, 041060 (2016).
- [20] D. A. Golter, T. Oo, M. Amezcua, K. A. Stewart, and H. Wang, Optomechanical quantum control of a nitrogen-vacancy center in diamond, *Phys. Rev. Lett.* **116**, 143602 (2016).
- [21] B. Chen, X. Hou, F. Ge, X. Zhang, Y. Ji, H. Li, P. Qian, Y. Wang, N. Xu, and J. Du, Calibration-free vector magnetometry using nitrogen-vacancy center in diamond integrated with optical vortex beam, *Nano Lett.* **20**, 8267 (2020).
- [22] J. F. Barry, J. M. Schloss, E. Bauch, M. J. Turner, C. A. Hart, L. M. Pham, and R. L. Walsworth, Sensitivity optimization for NV-diamond magnetometry, *Rev. Mod. Phys.* **92**, 015004 (2020).
- [23] L. Orphal-Kobin, K. Unterguggenberger, T. Pregnolato, N. Kemf, M. Matalla, R.-S. Unger, I. Ostermay, G. Pieplow, and T. Schröder, Optically coherent nitrogen-vacancy defect

- centers in diamond nanostructures, *Phys. Rev. X* **13**, 011042 (2023).
- [24] A. Blais, A. L. Grimsmo, S. M. Girvin, and A. Wallraff, Circuit quantum electrodynamics, *Rev. Mod. Phys.* **93**, 025005 (2021).
- [25] J. Clarke and F. K. Wilhelm, Superconducting quantum bits, *Nature (London)* **453**, 1031 (2008).
- [26] G. Wendin, Quantum information processing with superconducting circuits: A review, *Rep. Prog. Phys.* **80**, 106001 (2017).
- [27] D. Marcos, M. Wubs, J. M. Taylor, R. Aguado, M. D. Lukin, and A. S. Sørensen, Coupling nitrogen-vacancy centers in diamond to superconducting flux qubits, *Phys. Rev. Lett.* **105**, 210501 (2010).
- [28] Q. Bin, H. Jing, Y. Wu, F. Nori, and X.-Y. Lü, Nonreciprocal bundle emissions of quantum entangled pairs, *Phys. Rev. Lett.* **133**, 043601 (2024).
- [29] M. F. Colombano, G. Arregui, F. Bonell, N. E. Capuj, E. Chavez-Angel, A. Pitanti, S. O. Valenzuela, C. M. Sotomayor-Torres, D. Navarro-Urrios, and M. V. Costache, Ferromagnetic resonance assisted optomechanical magnetometer, *Phys. Rev. Lett.* **125**, 147201 (2020).
- [30] C. Gonzalez-Ballester, J. Gieseler, and O. Romero-Isart, Quantum acoustomechanics with a micromagnet, *Phys. Rev. Lett.* **124**, 093602 (2020).
- [31] C. Gonzalez-Ballester, D. Hümmer, J. Gieseler, and O. Romero-Isart, Theory of quantum acoustomagnonics and acoustomechanics with a micromagnet, *Phys. Rev. B* **101**, 125404 (2020).
- [32] M. Harder, Y. Yang, B. M. Yao, C. H. Yu, J. W. Rao, Y. S. Gui, R. L. Stamps, and C.-M. Hu, Level attraction due to dissipative magnon-photon coupling, *Phys. Rev. Lett.* **121**, 137203 (2018).
- [33] X.-L. Hei, P.-B. Li, X.-F. Pan, and F. Nori, Enhanced tripartite interactions in spin-magnon-mechanical hybrid systems, *Phys. Rev. Lett.* **130**, 073602 (2023).
- [34] A. Kani, B. Sarma, and J. Twamley, Intensive cavity-magnomechanical cooling of a levitated macromagnet, *Phys. Rev. Lett.* **128**, 013602 (2022).
- [35] A. Kani, F. Quijandría, and J. Twamley, Magnonic Einstein-de Haas effect: Ultrafast rotation of magnonic microspheres, *Phys. Rev. Lett.* **129**, 257201 (2022).
- [36] D. Lachance-Quirion, Y. Tabuchi, A. Gloppe, K. Usami, and Y. Nakamura, Hybrid quantum systems based on magnonics, *Appl. Phys. Express* **12**, 070101 (2019).
- [37] J. Li, S.-Y. Zhu, and G. S. Agarwal, Magnon-photon-phonon entanglement in cavity magnomechanics, *Phys. Rev. Lett.* **121**, 203601 (2018).
- [38] J. Li, Y.-P. Wang, W.-J. Wu, S.-Y. Zhu, and J. You, Quantum network with magnonic and mechanical nodes, *PRX Quantum* **2**, 040344 (2021).
- [39] F. Pirmoradian, B. Zare Rameshti, M. Miri, and S. Saeidian, Topological magnon modes in a chain of magnetic spheres, *Phys. Rev. B* **98**, 224409 (2018).
- [40] J. Qian, C. H. Meng, J. W. Rao, Z. J. Rao, Z. An, Y. Gui, and C. M. Hu, Non-Hermitian control between absorption and transparency in perfect zero-reflection magnonics, *Nat. Commun.* **14**, 3437 (2023).
- [41] Z. Shen, G.-T. Xu, M. Zhang, Y.-L. Zhang, Y. Wang, C.-Z. Chai, C.-L. Zou, G.-C. Guo, and C.-H. Dong, Coherent coupling between phonons, magnons, and photons, *Phys. Rev. Lett.* **129**, 243601 (2022).
- [42] R.-C. Shen, J. Li, Z.-Y. Fan, Y.-P. Wang, and J. Q. You, Mechanical bistability in Kerr-modified cavity magnomechanics, *Phys. Rev. Lett.* **129**, 123601 (2022).
- [43] Y. Tabuchi, S. Ishino, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Hybridizing ferromagnetic magnons and microwave photons in the quantum limit, *Phys. Rev. Lett.* **113**, 083603 (2014).
- [44] Y. Tabuchi, S. Ishino, A. Noguchi, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Coherent coupling between a ferromagnetic magnon and a superconducting qubit, *Science* **349**, 405 (2015).
- [45] Y.-P. Wang, G.-Q. Zhang, D. Zhang, T.-F. Li, C.-M. Hu, and J. Q. You, Bistability of cavity magnon polaritons, *Phys. Rev. Lett.* **120**, 057202 (2018).
- [46] Z.-Q. Wang, Y.-P. Wang, J. Yao, R.-C. Shen, W.-J. Wu, J. Qian, J. Li, S.-Y. Zhu, and J. Q. You, Giant spin ensembles in waveguide magnonics, *Nat. Commun.* **13**, 7580 (2022).
- [47] S. P. Wolski, D. Lachance-Quirion, Y. Tabuchi, S. Kono, A. Noguchi, K. Usami, and Y. Nakamura, Dissipation-based quantum sensing of magnons with a superconducting qubit, *Phys. Rev. Lett.* **125**, 117701 (2020).
- [48] Y. Yang, Y.-P. Wang, J. W. Rao, Y. S. Gui, B. M. Yao, W. Lu, and C.-M. Hu, Unconventional singularity in anti-parity-time symmetric cavity magnonics, *Phys. Rev. Lett.* **125**, 147202 (2020).
- [49] Y. Yang, J. Yao, Y. Xiao, P.-T. Fong, H.-K. Lau, and C.-M. Hu, Anomalous long-distance coherence in critically driven cavity magnonics, *Phys. Rev. Lett.* **132**, 206902 (2024).
- [50] H. Y. Yuan, P. Yan, S. Zheng, Q. Y. He, K. Xia, and M.-H. Yung, Steady Bell state generation via magnon-photon coupling, *Phys. Rev. Lett.* **124**, 053602 (2020).
- [51] T. Yu, Y.-X. Zhang, S. Sharma, X. Zhang, Y. M. Blanter, and G. E. W. Bauer, Magnon accumulation in chirally coupled magnets, *Phys. Rev. Lett.* **124**, 107202 (2020).
- [52] X. Zhang, C.-L. Zou, L. Jiang, and H. X. Tang, Strongly coupled magnons and cavity microwave photons, *Phys. Rev. Lett.* **113**, 156401 (2014).
- [53] D. Zhang, X.-M. Wang, T.-F. Li, X.-Q. Luo, W. Wu, F. Nori, and J. Q. You, Cavity quantum electrodynamics with ferromagnetic magnons in a small yttrium-iron-garnet sphere, *npj Quantum Inf.* **1**, 15014 (2015).
- [54] X. Zhang, C.-L. Zou, L. Jiang, and H. X. Tang, Cavity magnomechanics, *Sci. Adv.* **2**, e1501286 (2016).
- [55] V. Bittencourt, I. Liberal, and S. Viola Kusminskiy, Optomagnonics in dispersive media: Magnon-photon coupling enhancement at the epsilon-near-zero frequency, *Phys. Rev. Lett.* **128**, 183603 (2022).
- [56] Ö. O. Soykal and M. E. Flatté, Strong field interactions between a nanomagnet and a photonic cavity, *Phys. Rev. Lett.* **104**, 077202 (2010).
- [57] M. Kounalakis, G. E. W. Bauer, and Y. M. Blanter, Analog quantum control of magnonic cat states on a chip by a superconducting qubit, *Phys. Rev. Lett.* **129**, 037205 (2022).
- [58] X.-L. Hei, X.-L. Dong, J.-Q. Chen, C.-P. Shen, Y.-F. Qiao, and P.-B. Li, Enhancing spin-photon coupling with a micromagnet, *Phys. Rev. A* **103**, 043706 (2021).
- [59] F.-Z. Ji and J.-H. An, Kerr nonlinearity induced strong spin-magnon coupling, *Phys. Rev. B* **108**, L180409 (2023).

- [60] T. Neuman, D. S. Wang, and P. Narang, Nanomagnonic cavities for strong spin-magnon coupling and magnon-mediated spin-spin interactions, *Phys. Rev. Lett.* **125**, 247702 (2020).
- [61] W. Xiong, M. Tian, G.-Q. Zhang, and J. Q. You, Strong long-range spin-spin coupling via a Kerr magnon interface, *Phys. Rev. B* **105**, 245310 (2022).
- [62] N. Nagaosa and Y. Tokura, Topological properties and dynamics of magnetic skyrmions, *Nat. Nanotechnol.* **8**, 899 (2013).
- [63] A. Fert, N. Reyren, and V. Cros, Magnetic skyrmions: Advances in physics and potential applications, *Nat. Rev. Mater.* **2**, 17031 (2017).
- [64] H. Ochoa and Y. Tserkovnyak, Quantum skyrmionics, *Int. J. Mod. Phys. B* **33**, 1930005 (2019).
- [65] C. Reichhardt, C. J. O. Reichhardt, and M. V. Milošević, Statics and dynamics of skyrmions interacting with disorder and nanostructures, *Rev. Mod. Phys.* **94**, 035005 (2022).
- [66] M. Hirschberger, T. Nakajima, S. Gao, L. Peng, A. Kikkawa, T. Kurumaji, M. Kriener, Y. Yamasaki, H. Sagayama, H. Nakao, K. Ohishi, K. Kakurai, Y. Taguchi, X. Yu, T.-H. Arima, and Y. Tokura, Skyrmion phase and competing magnetic orders on a breathing kagomé lattice, *Nat. Commun.* **10**, 5831 (2019).
- [67] N. D. Khanh, T. Nakajima, X. Yu, S. Gao, K. Shibata, M. Hirschberger, Y. Yamasaki, H. Sagayama, H. Nakao, L. Peng, K. Nakajima, R. Takagi, T.-h. Arima, Y. Tokura, and S. Seki, Nanometric square skyrmion lattice in a centrosymmetric tetragonal magnet, *Nat. Nanotechnol.* **15**, 444 (2020).
- [68] X. Yu, F. Kagawa, S. Seki, M. Kubota, J. Masell, F. S. Yasin, K. Nakajima, M. Nakamura, M. Kawasaki, N. Nagaosa, and Y. Tokura, Real-space observations of 60-nm skyrmion dynamics in an insulating magnet under low heat flow, *Nat. Commun.* **12**, 5079 (2021).
- [69] S. Seki, M. Suzuki, M. Ishibashi, R. Takagi, N. D. Khanh, Y. Shiota, K. Shibata, W. Koshibae, Y. Tokura, and T. Ono, Direct visualization of the three-dimensional shape of skyrmion strings in a noncentrosymmetric magnet, *Nat. Mater.* **21**, 181 (2022).
- [70] M. Hirschberger, B. G. Szigeti, M. Hemmida, M. M. Hirschmann, S. Esser, H. Ohsumi, Y. Tanaka, L. Spitz, S. Gao, K. K. Kolincio, H. Sagayama, H. Nakao, Y. Yamasaki, L. Forró, H.-A. Krug von Nidda, I. Kezsmarki, T.-H. Arima, and Y. Tokura, Lattice-commensurate skyrmion texture in a centrosymmetric breathing kagome magnet, *npj Quantum Mater.* **9**, 45 (2024).
- [71] N. Romming, A. Kubetzka, C. Hanneken, K. von Bergmann, and R. Wiesendanger, Field-dependent size and shape of single magnetic skyrmions, *Phys. Rev. Lett.* **114**, 177203 (2015).
- [72] S. Meyer, M. Perini, S. von Malottki, A. Kubetzka, R. Wiesendanger, K. von Bergmann, and S. Heinze, Isolated zero field sub-10 nm skyrmions in ultrathin Co films, *Nat. Commun.* **10**, 3823 (2019).
- [73] D. Cortés-Ortuño, N. Romming, M. Beg, K. von Bergmann, A. Kubetzka, O. Hovorka, H. Fangohr, and R. Wiesendanger, Nanoscale magnetic skyrmions and target states in confined geometries, *Phys. Rev. B* **99**, 214408 (2019).
- [74] R. Brüning, A. Kubetzka, K. von Bergmann, E. Y. Vedmedenko, and R. Wiesendanger, Nanoscale skyrmions on a square atomic lattice, *Phys. Rev. B* **105**, L241401 (2022).
- [75] Y. Zhou, E. Iacocca, A. A. Awad, R. K. Dumas, F. C. Zhang, H. B. Braun, and J. Åkerman, Dynamically stabilized magnetic skyrmions, *Nat. Commun.* **6**, 8193 (2015).
- [76] X. Zhang, Y. Zhou, and M. Ezawa, Magnetic bilayer-skyrmions without skyrmion Hall effect, *Nat. Commun.* **7**, 10293 (2016).
- [77] B. Göbel, I. Mertig, and O. A. Tretiakov, Beyond skyrmions: Review and perspectives of alternative magnetic quasiparticles, *Phys. Rep.* **895**, 1 (2021).
- [78] C. Schütte and M. Garst, Magnon-skyrmion scattering in chiral magnets, *Phys. Rev. B* **90**, 094423 (2014).
- [79] Z. Wang, H. Y. Yuan, Y. Cao, Z.-X. Li, R. A. Duine, and P. Yan, Magnonic frequency comb through nonlinear magnon-skyrmion scattering, *Phys. Rev. Lett.* **127**, 037202 (2021).
- [80] Z. Jin, X. Yao, Z. Wang, H. Y. Yuan, Z. Zeng, W. Wang, Y. Cao, and P. Yan, Nonlinear topological magnon spin Hall effect, *Phys. Rev. Lett.* **131**, 166704 (2023).
- [81] J. Iwasaki, M. Mochizuki, and N. Nagaosa, Universal current-velocity relation of skyrmion motion in chiral magnets, *Nat. Commun.* **4**, 1463 (2013).
- [82] B. Zhang, W. Wang, M. Beg, H. Fangohr, and W. Kuch, Microwave-induced dynamic switching of magnetic skyrmion cores in nanodots, *Appl. Phys. Lett.* **106**, 102401 (2015).
- [83] Y. Liu, N. Lei, C. Wang, X. Zhang, W. Kang, D. Zhu, Y. Zhou, X. Liu, Y. Zhang, and W. Zhao, Voltage-driven high-speed skyrmion motion in a skyrmion-shift device, *Phys. Rev. Appl.* **11**, 014004 (2019).
- [84] X. Yao, J. Chen, and S. Dong, Controlling the helicity of magnetic skyrmions by electrical field in frustrated magnets, *New J. Phys.* **22**, 083032 (2020).
- [85] X.-G. Wang, G.-H. Guo, V. K. Dugaev, J. Barnaś, J. Berakdar, S. S. P. Parkin, A. Ernst, and L. Chotorlishvili, Steering skyrmions with microwave and terahertz electric pulses, *Phys. Rev. B* **107**, 094404 (2023).
- [86] S. Okumura, V. P. Kravchuk, and M. Garst, Instability of magnetic skyrmion strings induced by longitudinal spin currents, *Phys. Rev. Lett.* **131**, 066702 (2023).
- [87] H. Ochoa, S. K. Kim, O. Tchernyshyov, and Y. Tserkovnyak, Gyrotropic elastic response of skyrmion crystals to current-induced tensions, *Phys. Rev. B* **96**, 020410 (2017).
- [88] J. Hagemeister, N. Romming, K. von Bergmann, E. Y. Vedmedenko, and R. Wiesendanger, Stability of single skyrmionic bits, *Nat. Commun.* **6**, 8455 (2015).
- [89] Y. D. Kato, Y. Okamura, M. Hirschberger, Y. Tokura, and Y. Takahashi, Topological magneto-optical effect from skyrmion lattice, *Nat. Commun.* **14**, 5416 (2023).
- [90] X. Zhang, Y. Zhou, M. Ezawa, G. P. Zhao, and W. Zhao, Magnetic skyrmion transistor: Skyrmion motion in a voltage-gated nanotrack, *Sci. Rep.* **5**, 11369 (2015).
- [91] S. Luo, M. Song, X. Li, Y. Zhang, J. Hong, X. Yang, X. Zou, N. Xu, and L. You, Reconfigurable skyrmion logic gates, *Nano Lett.* **18**, 1180 (2018).
- [92] U. K. Röbler, A. N. Bogdanov, and C. Pfleiderer, Spontaneous skyrmion ground states in magnetic metals, *Nature (London)* **442**, 797 (2006).
- [93] A. B. Butenko, A. A. Leonov, U. K. Röbler, and A. N. Bogdanov, Stabilization of skyrmion textures by uniaxial distortions in noncentrosymmetric cubic helimagnets, *Phys. Rev. B* **82**, 052403 (2010).

- [94] S. Banerjee, J. Rowland, O. Erten, and M. Randeria, Enhanced stability of skyrmions in two-dimensional chiral magnets with Rashba spin-orbit coupling, *Phys. Rev. X* **4**, 031045 (2014).
- [95] O. Janson, I. Rousochatzakis, A. A. Tsirlin, M. Belesi, A. A. Leonov, U. K. Rößler, J. van den Brink, and H. Rosner, The quantum nature of skyrmions and half-skyrmions in  $\text{Cu}_2\text{OSeO}_3$ , *Nat. Commun.* **5**, 5376 (2014).
- [96] M. N. Wilson, A. B. Butenko, A. N. Bogdanov, and T. L. Monchesky, Chiral skyrmions in cubic helimagnet films: The role of uniaxial anisotropy, *Phys. Rev. B* **89**, 094411 (2014).
- [97] A. O. Leonov and M. Mostovoy, Multiply periodic states and isolated skyrmions in an anisotropic frustrated magnet, *Nat. Commun.* **6**, 8275 (2015).
- [98] A. O. Leonov, T. L. Monchesky, N. Romming, A. Kubetzka, A. N. Bogdanov, and R. Wiesendanger, The properties of isolated chiral skyrmions in thin magnetic films, *New J. Phys.* **18**, 065003 (2016).
- [99] S.-Z. Lin and S. Hayami, Ginzburg-Landau theory for skyrmions in inversion-symmetric magnets with competing interactions, *Phys. Rev. B* **93**, 064430 (2016).
- [100] R. Takashima, H. Ishizuka, and L. Balents, Quantum skyrmions in two-dimensional chiral magnets, *Phys. Rev. B* **94**, 134415 (2016).
- [101] X. S. Wang, H. Y. Yuan, and X. R. Wang, A theory on skyrmion size, *Commun. Phys.* **1**, 31 (2018).
- [102] V. Lohani, C. Hickey, J. Masell, and A. Rosch, Quantum skyrmions in frustrated ferromagnets, *Phys. Rev. X* **9**, 041063 (2019).
- [103] A. P. Petrović, C. Psaroudaki, P. Fischer, M. Garst, and C. Panagopoulos, *Colloquium*: Quantum properties and functionalities of magnetic skyrmions, *Rev. Mod. Phys.* **97**, 031001 (2025).
- [104] C. Psaroudaki and C. Panagopoulos, Skyrmion qubits: A new class of quantum logic elements based on nanoscale magnetization, *Phys. Rev. Lett.* **127**, 067201 (2021).
- [105] J. Xia, X. Zhang, X. Liu, Y. Zhou, and M. Ezawa, Universal quantum computation based on nanoscale skyrmion helicity qubits in frustrated magnets, *Phys. Rev. Lett.* **130**, 106701 (2023).
- [106] X.-F. Pan, P.-B. Li, X.-L. Hei, X. Zhang, M. Mochizuki, F.-L. Li, and F. Nori, Magnon-skyrmion hybrid quantum systems: Tailoring interactions via magnons, *Phys. Rev. Lett.* **132**, 193601 (2024).
- [107] X.-F. Pan, X.-L. Hei, X.-Y. Yao, J.-Q. Chen, Y.-M. Ren, X.-L. Dong, Y.-F. Qiao, and P.-B. Li, Skyrmion-mechanical hybrid quantum systems: Manipulation of skyrmion qubits via phonons, *Phys. Rev. Res.* **6**, 023067 (2024).
- [108] M. Mruczkiewicz, P. Gruszecki, M. Zelent, and M. Krawczyk, Collective dynamical skyrmion excitations in a magnonic crystal, *Phys. Rev. B* **93**, 174429 (2016).
- [109] Z. V. Gareeva and K. Y. Guslienko, Collective magnetic skyrmion gyrotropic modes in a dot chain, *J. Phys. Commun.* **2**, 035009 (2018).
- [110] X.-J. Liu, H. Wang, and Z.-K. Tang, Correlated gyrotropic modes of multi-skyrmion in elliptical ferromagnetic nanodisks, *J. Magn. Magn. Mater.* **510**, 166965 (2020).
- [111] X. Liu, Z. Li, Q. Wang, R. Ye, and P. Yan, Correlated gyrotropic motion of skyrmion clusters in ultrathin ferromagnetic nanodisks, *J. Magn. Magn. Mater.* **572**, 170649 (2023).
- [112] S. Mühlbauer, B. Binz, F. Jonietz, C. Pfleiderer, A. Rosch, A. Neubauer, R. Georgii, and P. Böni, Skyrmion lattice in a chiral magnet, *Science* **323**, 915 (2009).
- [113] W. Münzer, A. Neubauer, T. Adams, S. Mühlbauer, C. Franz, F. Jonietz, R. Georgii, P. Böni, B. Pedersen, M. Schmidt, A. Rosch, and C. Pfleiderer, Skyrmion lattice in the doped semiconductor  $\text{Fe}_{1-x}\text{Co}_x\text{Si}$ , *Phys. Rev. B* **81**, 041203 (2010).
- [114] X. Z. Yu, Y. Onose, N. Kanazawa, J. H. Park, J. H. Han, Y. Matsui, N. Nagaosa, and Y. Tokura, Real-space observation of a two-dimensional skyrmion crystal, *Nature (London)* **465**, 901 (2010).
- [115] S. Heinze, K. von Bergmann, M. Menzel, J. Brede, A. Kubetzka, R. Wiesendanger, G. Bihlmayer, and S. Blügel, Spontaneous atomic-scale magnetic skyrmion lattice in two dimensions, *Nat. Phys.* **7**, 713 (2011).
- [116] I. Kézsmárki, S. Bordács, P. Milde, E. Neuber, L. Eng, J. S. White, H. M. Rønnow, C. D. Dewhurst, M. Mochizuki, K. Yanai, H. Nakamura, D. Ehlers, V. Tsurkan, and A. Loidl, Neel-type skyrmion lattice with confined orientation in the polar magnetic semiconductor  $\text{GaV}_4\text{S}_8$ , *Nat. Mater.* **14**, 1116 (2015).
- [117] A. K. Nayak, V. Kumar, T. Ma, P. Werner, E. Pippel, R. Sahoo, F. Damay, U. K. Rößler, C. Felser, and S. S. P. Parkin, Magnetic antiskyrmions above room temperature in tetragonal Heusler materials, *Nature (London)* **548**, 561 (2017).
- [118] S. Woo, K. M. Song, X. Zhang, M. Ezawa, Y. Zhou, X. Liu, M. Weigand, S. Finizio, J. Raabe, M.-C. Park, K.-Y. Lee, J. W. Choi, B.-C. Min, H. C. Koo, and J. Chang, Deterministic creation and deletion of a single magnetic skyrmion observed by direct time-resolved X-ray microscopy, *Nat. Electron.* **1**, 288 (2018).
- [119] T. Kurumaji, T. Nakajima, M. Hirschberger, A. Kikkawa, Y. Yamasaki, H. Sagayama, H. Nakao, Y. Taguchi, T.-H. Arima, and Y. Tokura, Skyrmion lattice with a giant topological Hall effect in a frustrated triangular-lattice magnet, *Science* **365**, 914 (2019).
- [120] Y. Tokura and N. Kanazawa, Magnetic skyrmion materials, *Chem. Rev.* **121**, 2857 (2021).
- [121] V. T. Pham, N. Sisodia, I. D. Manici, J. Urrestarazu-Larrañaga, K. Bairagi, J. Pelloux-Prayer, R. Guedes, L. D. Buda-Prejbeanu, S. Auffret, A. Locatelli, T. O. Menteş, S. Pizzini, P. Kumar, A. Finco, V. Jacques, G. Gaudin, and O. Boulle, Fast current-induced skyrmion motion in synthetic antiferromagnets, *Science* **384**, 307 (2024).
- [122] A. Hrabec, J. Sampaio, M. Belmeguenai, I. Gross, R. Weil, S. M. Chérif, A. Stashkevich, V. Jacques, A. Thiaville, and S. Rohart, Current-induced skyrmion generation and dynamics in symmetric bilayers, *Nat. Commun.* **8**, 15765 (2017).
- [123] A. Casiraghi, H. Corte-León, M. Vafaei, F. Garcia-Sanchez, G. Durin, M. Pasquale, G. Jakob, M. Kläui, and O. Kazakova, Individual skyrmion manipulation by local magnetic field gradients, *Commun. Phys.* **2**, 145 (2019).
- [124] B. Dai, D. Wu, S. A. Razavi, S. Xu, H. He, Q. Shu, M. Jackson, F. Mahfouzi, H. Huang, Q. Pan, Y. Cheng, T. Qu, T. Wang, L. Tai, K. Wong, N. Kioussis, and K. L. Wang, Electric field manipulation of spin chirality and skyrmion dynamic, *Sci. Adv.* **9**, eade6836 (2023).
- [125] S. L. Zhang, W. W. Wang, D. M. Burn, H. Peng, H. Berger, A. Bauer, C. Pfleiderer, G. van der Laan, and T. Hesjedal,

- Manipulation of skyrmion motion by magnetic field gradients, *Nat. Commun.* **9**, 2115 (2018).
- [126] S. Woo, K. M. Song, H.-S. Han, M.-S. Jung, M.-Y. Im, K.-S. Lee, K. S. Song, P. Fischer, J.-I. Hong, J. W. Choi, B.-C. Min, H. C. Koo, and J. Chang, Spin-orbit torque-driven skyrmion dynamics revealed by time-resolved X-ray microscopy, *Nat. Commun.* **8**, 15573 (2017).
- [127] L. Peng, R. Takagi, W. Koshibae, K. Shibata, K. Nakajima, T.-h. Arima, N. Nagaosa, S. Seki, X. Yu, and Y. Tokura, Controlled transformation of skyrmions and antiskyrmions in a non-centrosymmetric magnet, *Nat. Nanotechnol.* **15**, 181 (2020).
- [128] N. Romming, C. Hanneken, M. Menzel, J. E. Bickel, B. Wolter, K. von Bergmann, A. Kubetzka, and R. Wiesendanger, Writing and deleting single magnetic skyrmions, *Science* **341**, 636 (2013).
- [129] C. Hanneken, F. Otte, A. Kubetzka, B. Dupé, N. Romming, K. von Bergmann, R. Wiesendanger, and S. Heinze, Electrical detection of magnetic skyrmions by tunnelling non-collinear magnetoresistance, *Nat. Nanotechnol.* **10**, 1039 (2015).
- [130] P.-J. Hsu, A. Kubetzka, A. Finco, N. Romming, K. von Bergmann, and R. Wiesendanger, Electric-field-driven switching of individual magnetic skyrmions, *Nat. Nanotechnol.* **12**, 123 (2017).
- [131] M. Perini, S. Meyer, A. Kubetzka, R. Wiesendanger, S. Heinze, and K. von Bergmann, Electrical detection of domain walls and skyrmions in Co films using noncollinear magnetoresistance, *Phys. Rev. Lett.* **123**, 237205 (2019).
- [132] P. Rabl, P. Cappellaro, M. V. G. Dutt, L. Jiang, J. R. Maze, and M. D. Lukin, Strong magnetic coupling between an electronic spin qubit and a mechanical resonator, *Phys. Rev. B* **79**, 041302 (2009).
- [133] S. D. Bennett, N. Y. Yao, J. Otterbach, P. Zoller, P. Rabl, and M. D. Lukin, Phonon-induced spin-spin interactions in diamond nanostructures: Application to spin squeezing, *Phys. Rev. Lett.* **110**, 156402 (2013).
- [134] P.-B. Li, Y. Zhou, W.-B. Gao, and F. Nori, Enhancing spin-phonon and spin-spin interactions using linear resources in a hybrid quantum system, *Phys. Rev. Lett.* **125**, 153602 (2020).
- [135] X.-F. Pan, X.-L. Hei, X.-L. Dong, J.-Q. Chen, C.-P. Shen, H. Ali, and P.-B. Li, Enhanced spin-mechanical interaction with levitated micromagnets, *Phys. Rev. A* **107**, 023722 (2023).
- [136] F. Fung, E. Rosenfeld, J. D. Schaefer, A. Kabcenell, J. Gieseler, T. X. Zhou, T. Madhavan, N. Aslam, A. Yacoby, and M. D. Lukin, Toward programmable quantum processors based on spin qubits with mechanically mediated interactions and transport, *Phys. Rev. Lett.* **132**, 263602 (2024).
- [137] K. R. Patton and U. R. Fischer, Ultrafast quantum random access memory utilizing single Rydberg atoms in a Bose-Einstein condensate, *Phys. Rev. Lett.* **111**, 240504 (2013).
- [138] K. R. Patton and U. R. Fischer, Hybrid of superconducting quantum interference device and atomic Bose-Einstein condensate: An architecture for quantum information processing, *Phys. Rev. A* **87**, 052303 (2013).
- [139] T. J. Volkoff and U. R. Fischer, Amplification of the quantum superposition macroscopicity of a flux qubit by a magnetized Bose gas, *Phys. Rev. A* **94**, 042320 (2016).
- [140] C. Psaroudaki and D. Loss, Skyrmions driven by intrinsic magnons, *Phys. Rev. Lett.* **120**, 237203 (2018).
- [141] C. Psaroudaki and D. Loss, Quantum depinning of a magnetic skyrmion, *Phys. Rev. Lett.* **124**, 097202 (2020).
- [142] T. Gilbert, A phenomenological theory of damping in ferromagnetic materials, *IEEE Trans. Magn.* **40**, 3443 (2004).
- [143] K. Guslienko and Z. Gareeva, Magnetic skyrmion low frequency dynamics in thin circular dots, *J. Magn. Magn. Mater.* **442**, 176 (2017).
- [144] A. A. Belavin and A. M. Polyakov, Metastable states of two-dimensional isotropic ferromagnets, *JETP Lett.* **22**, 245 (1975).
- [145] I. Makhfudz, B. Krüger, and O. Tchernyshyov, Inertia and chiral edge modes of a skyrmion magnetic bubble, *Phys. Rev. Lett.* **109**, 217201 (2012).
- [146] F. Büttner, C. Moutafis, M. Schneider, B. Krüger, C. M. Günther, J. Geilhufe, C. v. K. Schmising, J. Mohanty, B. Pfau, and S. Schaffert, Dynamics and inertia of skyrmionic spinstructures, *Nat. Phys.* **11**, 225 (2015).
- [147] M. J. Donahue and D. G. Porter, OOMMF User's Guide (ver. 1.0) [Online], Interagency Report No. NISTIR 6376, National Institute of Standards and Technology, Gaithersburg, MD, 1999, <https://doi.org/10.6028/NIST.IR.6376>.
- [148] M. Beg, M. Lang, and H. Fangohr, Ubermag: Towards more effective micromagnetic workflows, *IEEE Trans. Magn.* **58**, 1 (2022).
- [149] X. J. Liu, L. Guo, H. Wang, and Z. K. Tang, The internal dynamic modes of an antiskyrmion in ultrathin ferromagnetic nanodisks, *AIP Adv.* **10**, 075222 (2020).
- [150] C. Psaroudaki, S. Hoffman, J. Klinovaja, and D. Loss, Quantum dynamics of skyrmions in chiral magnets, *Phys. Rev. X* **7**, 041045 (2017).
- [151] I. Stasinopoulos, S. Weichselbaumer, A. Bauer, J. Waizner, H. Berger, S. Maendl, M. Garst, C. Pfeiderer, and D. Grundler, Low spin wave damping in the insulating chiral magnet  $\text{Cu}_2\text{OSeO}_3$ , *Appl. Phys. Lett.* **111**, 032408 (2017).
- [152] S. Khan, O. Lee, T. Dion, C. W. Zollitsch, S. Seki, Y. Tokura, J. D. Breeze, and H. Kurebayashi, Coupling microwave photons to topological spin textures in  $\text{Cu}_2\text{OSeO}_3$ , *Phys. Rev. B* **104**, L100402 (2021).
- [153] L. Liansberger, F. X. Haslbeck, A. Bauer, H. Berger, R. Gross, H. Huebl, C. Pfeiderer, and M. Weiler, Tunable cooperativity in coupled spin-cavity systems, *Phys. Rev. B* **104**, L100415 (2021).
- [154] J. H. Han, *Skyrmions in Condensed Matter* (Springer International Publishing, Cham, Switzerland, 2017).
- [155] J. Graf, H. Pfeifer, F. Marquardt, and S. Viola Kusminskiy, Cavity optomagnonics with magnetic textures: Coupling a magnetic vortex to light, *Phys. Rev. B* **98**, 241406(R) (2018).
- [156] T. Weber, D. M. Fobes, J. Waizner, P. Steffens, G. S. Tucker, M. Böhm, L. Beddrich, C. Franz, H. Gabold, R. Bewley, D. Voneshen, M. Skoulatos, R. Georgii, G. Ehlers, A. Bauer, C. Pfeiderer, P. Böni, M. Janoschek, and M. Garst, Topological magnon band structure of emergent Landau levels in a skyrmion lattice, *Science* **375**, 1025 (2022).
- [157] B. W. Shore and P. L. Knight, The Jaynes-Cummings model, *J. Mod. Opt.* **40**, 1195 (1993).
- [158] D. Ballester, G. Romero, J. J. García-Ripoll, F. Deppe, and E. Solano, Quantum simulation of the ultrastrong-coupling

- dynamics in circuit quantum electrodynamics, *Phys. Rev. X* **2**, 021007 (2012).
- [159] Y. Zhang, W. Nie, and Y.-X. Liu, Edge-state oscillations in a one-dimensional topological chain with dissipative couplings, *Phys. Rev. Appl.* **18**, 024038 (2022).
- [160] B. Paikaray, A. Joseph, C. Murapaka, and A. Haldar, Tunable microwave properties of a skyrmion in an isolated nanodisk, *J. Magn. Magn. Mater.* **529**, 167900 (2021).
- [161] B. Satywali, V. P. Kravchuk, L. Pan, M. Raju, S. He, F. Ma, A. P. Petrović, M. Garst, and C. Panagopoulos, Microwave resonances of magnetic skyrmions in thin film multilayers, *Nat. Commun.* **12**, 1909 (2021).
- [162] H. Du, R. Che, L. Kong, X. Zhao, C. Jin, C. Wang, J. Yang, W. Ning, R. Li, C. Jin, X. Chen, J. Zang, Y. Zhang, and M. Tian, Edge-mediated skyrmion chain and its collective dynamics in a confined geometry, *Nat. Commun.* **6**, 8504 (2015).
- [163] L. Sun, R. X. Cao, B. F. Miao, Z. Feng, B. You, D. Wu, W. Zhang, A. Hu, and H. F. Ding, Creating an artificial two-dimensional skyrmion crystal by nanopatterning, *Phys. Rev. Lett.* **110**, 167201 (2013).
- [164] D. A. Gilbert, B. B. Maranville, A. L. Balk, B. J. Kirby, P. Fischer, D. T. Pierce, J. Unguris, J. A. Borchers, and K. Liu, Realization of ground-state artificial skyrmion lattices at room temperature, *Nat. Commun.* **6**, 8462 (2015).
- [165] T. Roskopf, A. Dussaux, K. Ohashi, M. Loretz, R. Schirhagl, H. Watanabe, S. Shikata, K. M. Itoh, and C. L. Degen, Investigation of surface magnetic noise by shallow spins in diamond, *Phys. Rev. Lett.* **112**, 147602 (2014).