

Probing optomechanical decoherence from an acoustic environment with a superconducting qubit

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Decoherence in macroscopic quantum systems arises mainly from interactions with the environmental bath, offering key insights into the quantum-classical boundary. In this paper, we study the acoustic-environment-induced effects in a circuit quantum electrodynamics system, where an LC oscillator is optomechanically coupled to a long mechanical strip and capacitively coupled to a superconducting qubit. The mechanical strip functions as an acoustic environment, comprising a continuum of bath modes. The qubit population, serving as a probe of acoustic-environment-induced dephasing (AEID), exhibits collapse and revival dynamics in its Rabi oscillations. The AEID effect is extracted using a Ramsey-like interference scheme, manifesting as a dynamical decay in amplitude and a modification in the oscillation frequency. Our scheme opens avenues for studying the mechanisms of decoherence introduced by acoustic environments and to explore the optomechanically induced nonlinearity by collective acoustic bath modes.

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I. INTRODUCTION

Cavity optomechanics, which studies the interplay between mechanical motion and light, has found numerous applications in fundamental and applied physics [1–5], such as quantum mechanical phenomena of macroscopic objects [6–10], highly sensitive measurements of forces [11], displacement [12,13], and gravitational wave detection [14–17]. In cavity optomechanical devices, the intracavity radiation pressure force might couple several mechanical modes. Early studies have focused on the standard optomechanical system, which involves only a single optical mode coupled to a single mechanical degree of freedom. Recently, progress in fabrication techniques has made it possible to couple multiple mechanical modes to electromagnetic radiation by embedding nano- or microscale mechanical resonators in optical cavities [18,19] or superconducting quantum circuit systems [20–22]. This brings the multimode optomechanical systems to the fore as candidates for studying quantum many-body effects [9,10,23–30].

Mechanical systems are typically modeled as a discrete spectrum of acoustic modes, whereas only a few studies have addressed the dense spectrum or continuum of bath modes (CBMs)—for example, in interacting optical and bulk acoustic-wave resonators [31], or in environment-induced non-Markovian dynamics and the formation of nonclassical states [32]. For a continuum of mechanical modes, the

mechanical system can be considered as an acoustic environment. When an optical resonator is optomechanically coupled to the acoustic environment, initial photon number superposition states of the optical mode undergo dephasing (rather than dissipation), with a rate determined by the geometric size of the acoustic system [33–35]. The acoustic-environment-induced dephasing (AEID) can be effectively interpreted as analogs of gravitationally induced decoherence. Moreover, causal entanglement dynamics can be observed for two or more spatially separated optical modes (e.g., LC circuit oscillators) optomechanically coupled to the acoustic environment, giving rise to “quantum gravity” induced entanglement [30,34]. Thus, such systems can potentially serve as a platform to simulate the weak field, scalar matter-graviton interaction action [34,35]. For the systems involving only optical modes and acoustic bath modes, extracting the AEID rate and the CBM-induced effects can be a challenging task in itself. A potential approach is to couple the optical resonator to a weak probe field. Nevertheless, since the linear cavity-probe coupling Hamiltonian does not commute with the optomechanical interaction Hamiltonian, the photon number is no longer conserved, which in turn alters the inherent AEID rate.

It has been established that the collapse and revival dynamics of a qubit can serve as a probe for the non-Markovian character of a photonic bath, particularly when the qubit is directly coupled to a one-dimensional photonic waveguide through a Jaynes-Cummings-type linear interaction [36]. In this work, we consider an experimentally feasible circuit quantum electrodynamics (QED) architecture [37–41], where an LC oscillator is coupled via optomechanical interaction to a long mechanical strip, while simultaneously interacting dispersively with a two-level artificial atom—a superconducting qubit [40–45]. In this setup, the mechanical strip acts as an acoustic environment, giving rise to the AEID effect on the LC oscillator, instead of an energy dissipation [36]. The superconducting qubit is then employed as a probe to investigate the

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optomechanically induced decoherence originating from this acoustic bath.

By using a three-step procedure analogous to the Ramsey interferometry, the AEID effect can be extracted from the qubit population, i.e., the probabilities for measuring the qubit in the excited or ground state. We first show the typical dynamical behavior of the qubit population by coupling the circuit QED system to the acoustic environment. For the LC oscillator initially in the coherent state, the AEID effect manifests itself in the Rabi oscillations of the qubit population, which show periodic collapse and revival dynamics modulated by the optomechanically induced nonlinearity from the acoustic bath modes and a dynamical decay of oscillation amplitude. We thus provide an operational, quantitative measure of decoherence with a circuit QED setup. We also consider the dissipation for both the LC oscillator and the qubit, where the lifetime around a hundred to a few hundred microseconds has been reported for superconducting circuits, demonstrating the experimental feasibility for observing the AEID effect with the setup.

The paper is organized as follows. In Sec. II, we introduce the theoretical model for the circuit QED system. In Sec. III, we first propose the three-step measurement scheme, and calculate the time-evolution operator of the system and the qubit population by including the AEID effect. We show in Sec. III A the pure AEID effect for different system-environment coupling strengths and then include the qubit and cavity dissipations in Sec. III B. In Sec. IV, we investigate the scenario with the LC oscillator initially in a coherent state and show the collapse and revival dynamics under the AEID effect. Finally, we summarize our results in Sec. V.

II. MODEL AND HAMILTONIAN

We consider a superconducting transmon qubit (with excited and ground states $|e\rangle$ and $|g\rangle$, respectively) interacting with an LC circuit oscillator, which serves as the cavity mode a . The LC oscillator is optomechanically coupled to a long flexible mechanical strip, which contains $N \gg 1$ longitudinal modes b_j ($j = 1, 2, \dots, N$) and acts as an acoustic environment [33,34], as depicted in Fig. 1(a). The optomechanical interaction is facilitated by metallized segments (with lengths ΔL) centrally located on the mechanical strip. The strip of an overall length $L \gg \Delta L$ is clamped at both ends with a large tensile force F applied to them. The strip's transverse width W and thickness T_h are characterized by the relationship $T_h \ll W \ll L$, ensuring the strip's slender profile. The lower capacitor plate of the LC circuit is fixed in place and dimensioned to match the metallized segments in length ΔL and width W , and is separated from the metallized segments by a small equilibrium vacuum gap $d \ll W$. Then, the unperturbed capacitance of the LC circuit, i.e., in the absence of strip flexing, is given by $C_0 = \epsilon_0 W \Delta L / d$, with ϵ_0 the vacuum permittivity.

Without the effect of the oscillating mechanical strip, the qubit-oscillator interaction is typically governed by the Jaynes-Cummings Hamiltonian, i.e., $H_{ac} = \hbar \omega_{eg} |e\rangle\langle e| + \hbar \omega_c a^\dagger a + (\hbar \Omega a |e\rangle\langle g| + \text{H.c.})$, where ω_{eg} and ω_c are the frequencies of the qubit and LC oscillator, respectively. Ω is the single-photon qubit-field coupling strength and is assumed to

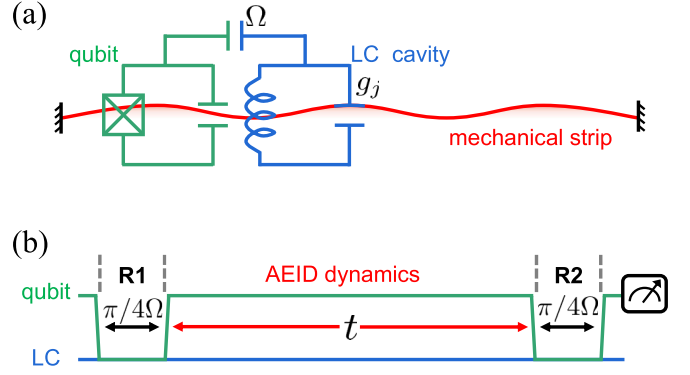


FIG. 1. (a) Schematic representation of the model. An LC circuit oscillator interacts with a transmon qubit and is optomechanically coupled to a long mechanical strip, which acts as the acoustic environment. (b) Pulse sequence. The transition frequency of the qubit is first tuned into resonance with the oscillator in the first time interval **R1**. In the second step, the qubit is biased to interact dispersively with the LC oscillator, and then the qubit-oscillator system undergoes dephasing as a result of interactions with the surrounding acoustic environment. After the evolution time t , the qubit is tuned into resonance with the oscillator again for another time interval **R2**. The qubit's state is measured finally.

be real for simplicity. In the dispersive interaction regime, where the qubit-oscillator detuning $\Delta = \omega_{eg} - \omega_c$ is much larger than Ω , the effective interaction Hamiltonian is alternatively given by $H_{ac} = \hbar \chi (a^\dagger a + 1) |e\rangle\langle e| - \hbar \chi a^\dagger a |g\rangle\langle g|$ with $\chi = \Omega^2 / \Delta$. By including the coupling to the mechanical strip, the general Hamiltonian of the system reads [4]

$$H = H_{ac} + H_{cb} + H_{dis}, \quad (1)$$

with

$$H_{cb} = \sum_j \hbar \omega_j \left(b_j^\dagger b_j + \frac{1}{2} \right) + \sum_j \hbar g_j \left(a^\dagger a + \frac{1}{2} \right) (b_j + b_j^\dagger) \quad (2)$$

and

$$H_{dis} = -i\hbar \frac{\gamma}{2} |e\rangle\langle e| - i\hbar \frac{\kappa}{2} a^\dagger a, \quad (3)$$

which is included for considering the inherent cavity and qubit dissipation. The parameters $\omega_j = \pi j \sqrt{\frac{F}{2mL}}$ are the mechanical normal-mode frequencies with $m = \rho_m W T_h L / 2$ the effective mass and ρ_m the mass density of the strip [33]; γ and κ denote the decay rates of the excited state $|e\rangle$ and a single cavity photon, respectively. $g_j = \omega_c \lambda_j$ is the single-photon optomechanical coupling strength between the LC oscillator and the j th mode of the elastic strip b_j [33,34], with $\lambda_j = -\frac{1}{2d} \left(\frac{\hbar}{2m\omega_j} \right)^{1/2} \sin\left(\frac{\pi j}{2}\right) \text{sinc}\left(\frac{\omega_j}{\omega_u}\right)$ and $\omega_u = \sqrt{2FL/m} / \Delta L$ the upper cutoff frequency, beyond which the coupling between the optical mode and the acoustic bath is significantly suppressed. The upper cutoff frequency is approximately $\omega_u / 2\pi \approx 10 \text{ MHz}$ for the set of parameters [33,34]: $d = 10^{-7} \text{ m}$, $F = 10^{-5} \text{ N}$, $\rho_m = 10^3 \text{ kg/m}^3$, $W = 10^{-6} \text{ m}$, $T_h = 10^{-7} \text{ m}$, $\Delta L = 10^{-5} \text{ m}$, and $L = 2 \times 10^{-2} \text{ m}$ and then the lower cutoff frequency is $\omega_1 / \omega_u = 10^{-3}$.

The Hamiltonian H_{cb} [Eq. (2)] can be diagonalized by the polaron transform given by unitary transformation: $U = e^{-iPa^\dagger a}$ with $P = i \sum_j \frac{g_j}{\omega_j} (b_j^\dagger - b_j)$ [46]. Then, one obtains $H_{cb} = \sum_j \hbar \omega_j (b_j^\dagger b_j + \frac{1}{2}) - \Delta_{\text{Kerr}} a^\dagger a a^\dagger a$, with $\Delta_{\text{Kerr}} = \sum_j \frac{g_j^2}{\omega_j}$. The photon nonlinearity is then determined by the spectral density of the acoustic bath. In what follows, we propose an effective scheme to extract the nonlinearity and the decoherence rate brought about by the acoustic environment.

III. THE RAMSEY INTERFEROMETRIC MEASUREMENT SCHEME

To manifest the effect of the acoustic environment on the qubit population, as shown in Fig. 1(b), we consider the Ramsey-like measurement scheme with three steps.

(1) Suppose that the qubit is initially in the excited state $|e\rangle$ and the cavity mode is initially in the vacuum. The qubit-field interaction is first tuned into resonance. The first time interval **R1** gives rise to the initial state of the qubit and field $|\psi(0)\rangle = (|e0\rangle - i|g1\rangle)/\sqrt{2}$ [the resonance interaction gives rise to $|e, 0\rangle \rightarrow \cos(\Omega t_c)|e, 0\rangle - i \sin(\Omega t_c)|g, 1\rangle$ and obtains $(|e0\rangle - i|g1\rangle)/\sqrt{2}$ after the time $t_c = \pi/4\Omega$]. The field-bath interaction is neglected for a strong resonant qubit-field coupling, i.e., Ω^{-1} is much shorter than the general decoherence time. This assumption is reasonable since Ω is typically a few tens of MHz, while the decay rates of the qubit and the LC oscillator can be $(\kappa, \gamma)/\Omega \sim 10^{-4}$ for a typical circuit QED setup [47–49].

(2) The qubit is then biased back to the frequency where the qubit-field interaction is far off-resonant and dispersive. At the same time, the field is coupled to the mechanical strip, whose initial density operator expressed in a coherent-state basis is given by

$$\rho_b(0) = \prod_j \frac{1}{\pi n_j} \int d\alpha_j^2 \exp(-|\alpha_j|^2/n_j) |\alpha_j\rangle \langle \alpha_j|, \quad (4)$$

where $n_j = (e^{\beta \hbar \omega_j} - 1)^{-1}$ and $\beta = (k_B T_m)^{-1}$, with k_B the Boltzmann's constant and T_m the bath temperature. The temporal evolution of the system is governed by the nonunitary evolution operator $U(t) = \exp(-iHt)$ (see Appendix A for more details), which is explicitly given by

$$\begin{aligned} U(t) = & e^{-i[\chi(a^\dagger a + 1)|e\rangle \langle e| - \chi a^\dagger a |g\rangle \langle g|]t} e^{-\frac{\kappa}{2} a^\dagger a t - \frac{\gamma}{2} t |e\rangle \langle e|} \\ & \times e^{\sum_j i[\omega_j t - \sin(\omega_j t)](g_j/\omega_j)^2 (a^\dagger a + \frac{1}{2})^2} \\ & \times e^{-\sum_j \frac{g_j}{\omega_j} (a^\dagger a + \frac{1}{2})(\zeta b^\dagger - \zeta^* b)} e^{-\sum_j i\omega_j t (b_j^\dagger b_j + \frac{1}{2})}, \end{aligned} \quad (5)$$

where $\zeta = 1 - e^{-i\omega_j t}$. Then, after an interaction time t , the reduced density operator for the qubit-field system is obtained by performing the partial trace over the acoustic environment's degrees of freedom (refer to Appendix B for a detailed derivation), leading to

$$\begin{aligned} \rho_{ac}(t) = & \text{Tr}_b[U(t)|\psi(0)\rangle \langle \psi(0)| \otimes \rho_b(0) U^\dagger(t)] \\ = & \frac{1}{2} e^{-\gamma t} |e0\rangle \langle e0| + \frac{1}{2} e^{-\kappa t} |g1\rangle \langle g1| \\ & + \frac{1}{2} i \exp[-i2\chi t - i2p(t) + d(t)] e^{-\frac{\kappa+\gamma}{2} t} |e0\rangle \langle g1| \\ & - \frac{1}{2} i \exp[i2\chi t + i2p(t) + d(t)] e^{-\frac{\kappa+\gamma}{2} t} |g1\rangle \langle e0|, \end{aligned} \quad (6)$$

where

$$p(t) = \sum_j \left[\frac{g_j^2}{\omega_j} t - \frac{g_j^2}{\omega_j^2} \sin(\omega_j t) \right] \quad (7)$$

and

$$d(t) = -2 \sum_j \frac{g_j^2}{\omega_j^2} \sin^2\left(\frac{\omega_j t}{2}\right) \coth\left(\frac{\beta \hbar \omega_j}{2}\right) \quad (8)$$

characterize the interaction of the system with the acoustic environment. In more details, $p(t)$ can be regarded as a dynamical modulation of the oscillatory frequency between the states $|e0\rangle$ and $|g1\rangle$. While $d(t)$ introduces the dephasing (or rephasing) effect due to random acoustic noises and only leads to the decay of the off-diagonal elements of $\rho_{ac}(t)$. Accompanied by qubit's spontaneous emission and cavity decay, the system will evolve into the classical state in the long-time regime.

Furthermore, to achieve a quantitative understanding of the AEID effect, one can further solve Eqs. (7) and (8) by assuming a sufficiently dense bath spectrum. When the number of bath modes is sufficiently large ($N \gg 1$), the discrete sum over the acoustic bath modes can be approximated by a continuous frequency integral [33]:

$$\pi \sum_j \lambda_j^2 f(\omega_j) \approx C \int_{\omega_1}^{\infty} d\omega \frac{f(\omega)}{\omega} \text{sinc}^2\left(\frac{\omega}{\omega_u}\right), \quad (9)$$

where $f(\omega_j)$ represents ω_j -dependent functions in Eqs. (7) and (8) and $C = \hbar(8d^2\sqrt{\rho_m W T_h F})^{-1}$ is a frequency-independent coupling constant. Using Eq. (9), $p(t)$ and $d(t)$ then take the forms

$$\begin{aligned} p(t) \approx & \frac{C\omega_c^2 t}{\pi} \int_{\omega_1}^{\infty} d\omega \omega^{-2} \text{sinc}^2\left(\frac{\omega}{\omega_u}\right) \\ & - \frac{C\omega_c^2}{\pi} \int_{\omega_1}^{\infty} d\omega \omega^{-3} \sin(\omega t) \text{sinc}^2\left(\frac{\omega}{\omega_u}\right), \quad (10) \\ d(t) \approx & -\frac{2C\omega_c^2}{\pi} \int_{\omega_1}^{\infty} d\omega \frac{\sin^2\left(\frac{\omega t}{2}\right)}{\omega^3} \coth\left(\frac{\beta \hbar \omega}{2}\right) \text{sinc}^2\left(\frac{\omega}{\omega_u}\right), \end{aligned} \quad (11)$$

which can be numerically evaluated by appropriately selecting C , ω_u , and ω_1 . Notably, for an idealized discrete and lossless spectrum [Eq. (8)], full rephasing occurs periodically at integer multiples of the fundamental period $2\pi/\omega_1$, where $d(t) = 0$. In a realistic, quasicontinuous spectrum [Eq. (11)], however, the slow-wave revival timescale is set by the lower cutoff ω_1 : A smaller ω_1 implies a longer revival. It is crucial to note that ω_1 is a physical parameter determined by the system's geometry and mass. The high-frequency cutoff ω_u , in contrast, stems from the finite capacitor length ΔL . For acoustic wavelengths much less than ΔL , the optomechanical coupling averages to zero, establishing ω_u . This cutoff does not govern the revival time.

(3) At the last step, the qubit is tuned again to be resonant with the LC oscillator, for a resonant interaction time $t_c = \pi/4\Omega$ [denoted as the process **R2** in Fig. 1(b)], giving rise to the transformations $|e0\rangle \rightarrow \frac{1}{\sqrt{2}}(|e0\rangle - i|g1\rangle)$ and $|g1\rangle \rightarrow \frac{1}{\sqrt{2}}(|g1\rangle - i|e0\rangle)$. It follows that the density operator for

the qubit-field system is given by

$$\begin{aligned} \rho_{ac}(t) = & \frac{1}{4}e^{-\gamma t}(|e0\rangle - i|g1\rangle)(\langle e0| + i\langle g1|) \\ & + \frac{1}{4}e^{-\kappa t}(|g1\rangle - i|e0\rangle)(\langle g1| + i\langle e0|) \\ & + \frac{1}{4}i\exp[-2ip(t) + d(t) - i2\chi t] \\ & \times e^{-\frac{\kappa+\gamma}{2}t}(|e0\rangle - i|g1\rangle)(\langle g1| + i\langle e0|) \\ & - \frac{1}{4}i\exp[2ip(t) + d(t) + i2\chi t] \\ & \times e^{-\frac{\kappa+\gamma}{2}t}(|g1\rangle - i|e0\rangle)(\langle e0| + i\langle g1|). \end{aligned} \quad (12)$$

After the three-step procedure, one can trace over the LC oscillator's degrees of freedom, and obtain the reduced density operator for the qubit $\rho_a(t) = P_e(t)|e\rangle\langle e| + P_g(t)|g\rangle\langle g|$, where the population inversion is characterized by $I(t) = P_e(t) - P_g(t)$ and is given by

$$I(t) = -\cos[2p(t) + 2\chi t]e^{d(t)}e^{-\frac{\kappa+\gamma}{2}t}. \quad (13)$$

Note that the Hamiltonian we consider remains effective only if no atomic spontaneous emission or photon loss events are observed. When either of the quantum jump events occurs, the system state reduces to $|g0\rangle$. However, since the qubit-oscillator interaction maintains the U(1) symmetry (i.e., $[a^\dagger a + |e\rangle\langle e|, H_c] = 0$), for experimental demonstration, it is convenient to measure the excited-state probability P_e instead, which is given by

$$P_e(t) = \frac{e^{-\gamma t} + e^{-\kappa t}}{4} + \frac{I(t)}{2}. \quad (14)$$

As a result, the AEID effect can be revealed by a postselection method, namely by excluding the data with respect to the qubit in the ground state $|g\rangle$.

A. Pure dephasing effect

To get an intuitive understanding of the physics, we now temporarily ignore the damping of the LC oscillator and the qubit (i.e., $\gamma/2\pi = \kappa/2\pi = 0$), and focus on the pure AEID effect. Under this condition, the population of the excited state $|e\rangle$ reduces to

$$P_e(t) = \frac{1}{2} - \frac{1}{2}\cos[2p(t) + 2\chi t]e^{d(t)}. \quad (15)$$

P_e shows coherent Rabi-like oscillations with the dominant frequency being cooperatively determined by the acoustic environment $[\sim p(t)]$ and the Lamb energy shift χ . The oscillation amplitude is modulated by the AEID according to $e^{d(t)}$. In the long-time limit, one readily obtains $e^{d(t)} \rightarrow 0$ and the steady state populations $P_e^{(s)} = P_g^{(s)} = 1/2$ [or $I(t) \rightarrow 0$], where the AEID effect eliminates the off-diagonal elements of the density matrix, namely, the quantum interference of the superposition state $(|e0\rangle - i|g1\rangle)/\sqrt{2}$, thus Eq. (6) eventually reduced to $(|e0\rangle\langle e0| + |g1\rangle\langle g1|)/2$.

In Fig. 2, we show the excited-state probability $P_e(t)$ versus the rescaled time $\omega_u t$ for two distinct qubit-field coupling strengths $\Omega/2\pi = 1$ MHz and $\Omega/2\pi = 10$ MHz, with the coupling constant $C = 4 \times 10^{-14}$ and upper cutoff frequency $\omega_u/2\pi = 10$ MHz. It shows that P_e oscillates rapidly around 0.5, and is modulated by the AEID revealed by the oscillating amplitude envelope $\sim [1 + e^{d(t)}]/2$, see the red dashed-dotted

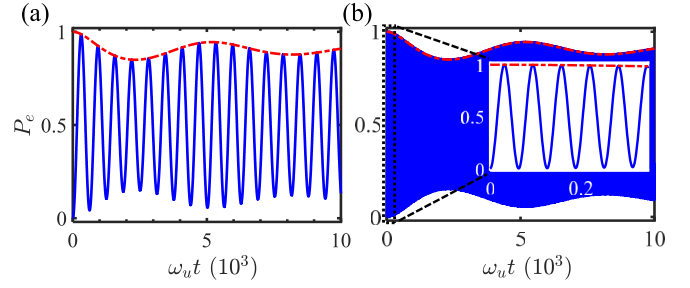


FIG. 2. The probability of the qubit in the excited state P_e as a function of the evolution time $\omega_u t$ for (a) $\Omega/2\pi = 1$ MHz and (b) $\Omega/2\pi = 10$ MHz with the decay rates $\gamma/2\pi = \kappa/2\pi = 0$. The red dashed-dotted curve shows the oscillating envelope following the form $\{1 + \exp[d(t)]\}/2$. Other parameters are $\omega_u/2\pi = 10$ MHz, $\omega_l/2\pi = 10$ kHz, $\beta\hbar\omega_u = 10$, $\omega_c/2\pi = 5$ GHz, $C = 4 \times 10^{-14}$, and $\Delta = 20\Omega$.

curve. The modulated envelope is independent of the qubit-field coupling strength $\Omega/2\pi$. An increase in Ω only varies the oscillation frequency $\sim 2\chi$. For $\Omega/2\pi = 1$ MHz, the dephasing effect can be discerned within just a few tens of oscillation cycles.

On the other hand, an increase in the coupling constant C enhances the AEID effect, where the coupling constant C is independent of the frequencies of the acoustic bath modes. In Fig. 3, we show the time evolution of P_e for different C . In all cases, we again find that P_e oscillates around the mean value of 1/2. For the coupling strength $C = 4 \times 10^{-14}$, the oscillation amplitude is weakly modulated by $e^{d(t)}$ and displays “slow-wave” dynamics, see the envelope denoted by the red dashed-dotted line in Fig. 3(a). As C increases (i.e., from left to right), the dephasing effect ($\sim e^{d(t)}$) enhances such that the amplitude of the Rabi-like oscillations in P_e decays quickly in the short time $\omega_u t < 2000$. It is remarkable that $P_e(t)$ undergoes a nontrivial collapse and revival dynamics around 0.5 for $C = 4 \times 10^{-13}$ and $C = 8 \times 10^{-13}$ [see Figs. 3(b) and 3(c)]. For a larger $C = 2 \times 10^{-12}$, as shown in Fig. 3(d), the AEID effect is stronger such that $e^{d(t)} \rightarrow 0$ in a shorter time, then the oscillation disappears at about $\omega_u t \approx 800$ without further revivals.

B. Including the cavity and qubit dissipation

By considering the cavity mode and qubit dissipation, the nonunitary evolution results in $P_e(t) + P_g(t) = (e^{-\gamma t} + e^{-\kappa t})/2$ being less than 1. Thus, without coupling to the acoustic environment (i.e., $C = 0$), the amplitude of the oscillatory P_e simply undergoes an exponential decay [as depicted in Figs. 4(a) and 4(b)] and has a simple form $P_e(t) = e^{-\kappa t}[1 - \cos(2\chi t)]/2$, with $\gamma = \kappa$. In comparison, by considering the AEID with $C = 4 \times 10^{-13}$ and setting the decay rates of $\kappa = \gamma = \{5, 10, 20, 50\}$ kHz, as shown in Fig. 4(c), the collapse and revival behavior remain observable in Rabi oscillations of $P_e(t)$ within practical time windows, even under typical transmon decay rates of 5–20 kHz. However, increasing the decay rates to $\gamma = \kappa = 50$ kHz will result in a full collapse of P_e without further revival. Furthermore, we show in Fig. 4(d) P_e versus κ and $\omega_u t$ with $\gamma = \kappa$. It shows that, as $\omega_u t$ increases and as κ (or γ) decreases, a transition from suppression to

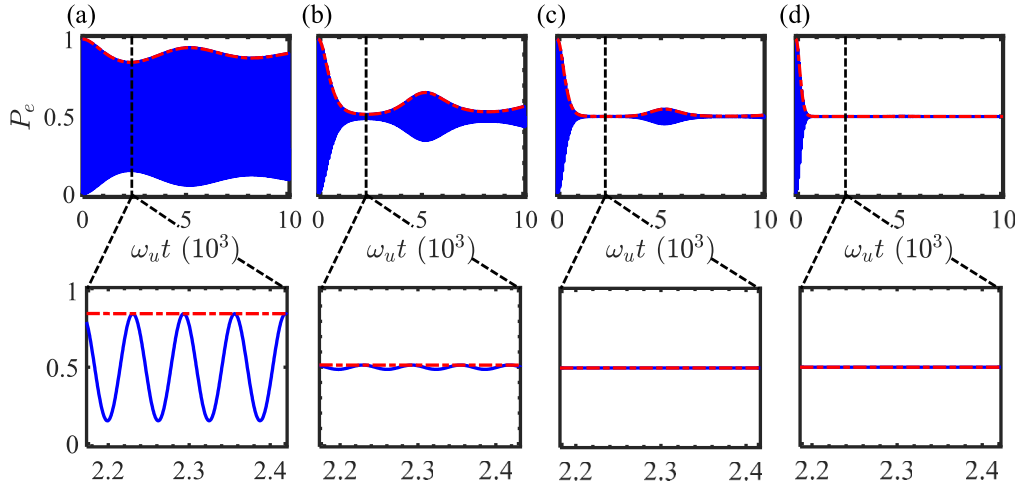


FIG. 3. P_e as a function of $\omega_u t$ for (a) $C = 4 \times 10^{-14}$, (b) $C = 4 \times 10^{-13}$, (c) $C = 8 \times 10^{-13}$, and (d) $C = 2 \times 10^{-12}$ with the decay rates $\gamma/2\pi = \kappa/2\pi = 0$ and the qubit-oscillator coupling strength $\Omega/2\pi = 10$ MHz. On the bottom row, we also show the oscillations near the first local minimum of the oscillation amplitude. The red dashed-dotted curves denote the simulations of the dynamical amplitude by $\{1 + \exp[d(t)]\}/2$. Other parameters are the same as in Fig. 2.

occurrence of the rephasing dynamics (i.e., revival of the Rabi oscillations) can be seen. The colors appear blended due to the presence of a highly oscillatory pattern in the 2D density plots.

IV. INITIAL PHOTON NUMBER SUPERPOSITION STATES

We now extend our analysis to the case where the cavity field is initially prepared in a superposition of Fock states $|n\rangle$, such as the coherent state $|\alpha\rangle = \sum_{n=0}^{\infty} C_n |n\rangle$ with $C_n = e^{-\bar{n}/2} (\alpha^n / \sqrt{n!})$ and average photon number $\bar{n} = |\alpha|^2$. We employ the three-step Ramsey interferometric

measurement scheme as before. The resonant interaction processes in **R1** and **R2** give rise to $|e, n\rangle \rightarrow \cos[\Omega' t_{1(2)}] |e, n\rangle - i \sin[\Omega' t_{1(2)}] |g, n+1\rangle$ and $|g, n+1\rangle \rightarrow \cos[\Omega' t_{1(2)}] |g, n+1\rangle - i \sin[\Omega' t_{1(2)}] |e, n\rangle$ with $\Omega' = \Omega\sqrt{n+1}$, where $t_{1(2)}$ are pulse durations in **R1(2)** and are assumed to be equal: $t_1 = t_2 = t_p$. In the second step, the LC oscillator once more engages with the acoustic environment and interacts with the qubit over a time interval t . After some calculations (as detailed in Appendix C), we obtain the population of the qubit's excited and ground states as

$$P_e(t) = e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} \left[1 - \frac{1}{2} \sin^2(2\Omega\sqrt{n+1}t_p) S(t) \right],$$

$$P_g(t) = 1 - P_e(t), \quad (16)$$

with

$$S(t) = 1 + \cos\{2(n+1)[p(t) + \chi t]\} e^{d(t)}, \quad (17)$$

where we have temporarily neglected the cavity and qubit dissipation. Then, the inversion of population reads

$$I(t) = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} \{ \cos^2(2\Omega t_p \sqrt{n+1}) - [S(t) - 1] \sin^2(2\Omega t_p \sqrt{n+1}) \}. \quad (18)$$

Figures 5(a)–5(d) show numerical results for the time evolution of $P_e(t)$ with $|\alpha|^2 = \{9, 16, 25\}$ by setting $t_p = \pi/4\Omega|\alpha|$ and $C = 4 \times 10^{-13}$, and show the case with $C = 0$ for comparison [see the gray curves in Figs. 5(a)–5(d)]. $P_e(t)$ exhibits a two-fold periodic collapse and revival dynamics, due to the cooperative effect by the interference between the weighted photon number states and the AEID dynamics, which is explained in detail as follows [50–54].

For the coherent state with $\bar{n} \gg 1$, the average photon number $\bar{n}$ makes dominant contribution to the phases $2\Omega\sqrt{n+1}t_p$ and $2(n+1)(p(t) + \chi t)$ in Eq. (16), as such the population of

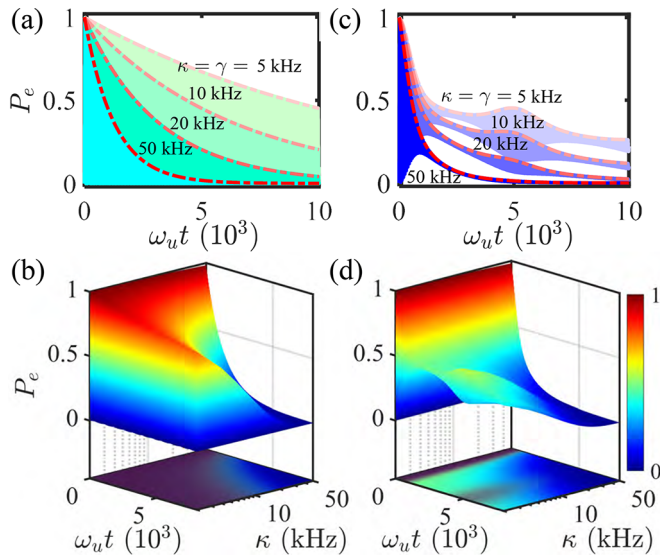


FIG. 4. P_e vs $\omega_u t$ for (a) $C = 0$ and (c) $C = 4 \times 10^{-13}$ with the decay rates $\kappa = \gamma = \{5, 10, 20, 50\}$ kHz. Panels (b) and (d) display P_e as a function of κ and $\omega_u t$ with $\gamma = \kappa$. The red dashed-dotted curves denote the simulations of the envelope with $\{(e^{-\gamma t} + e^{-\kappa t})/2 + \exp[d(t) - \frac{\kappa+\gamma}{2}t]\}/2$. Other parameters are the same as in Fig. 3.

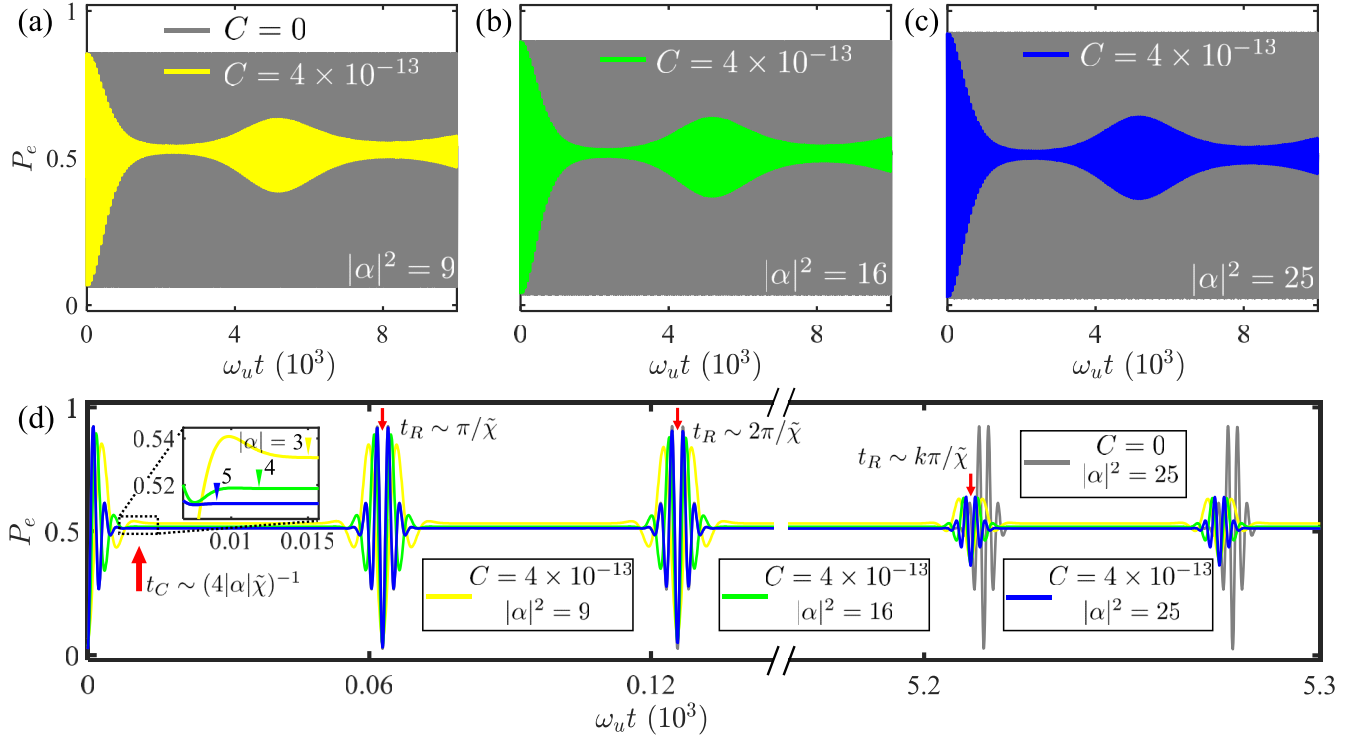


FIG. 5. P_e as a function of $\omega_u t$ for (a) $|\alpha|^2 = 9$, (b) $|\alpha|^2 = 16$, and (c) $|\alpha|^2 = 25$ with $C = 0$ (gray curves) and $C = 4 \times 10^{-13}$ (yellow, green, and blue curves). (d) Magnified image of panels (a)–(c), which shows the periods of oscillation collapse and revival. The acoustic-induced shift $\sum_j (g_j^2/\omega_j)$ relative to the intrinsic dispersive shift χ is about 6×10^{-4} for the device parameters that yield $C = 4 \times 10^{-13}$. Other parameters are the same as in Fig. 3.

the excited state can be approximately given by

$$P_e(t) = e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} \left[1 - \frac{1}{2} \sin^2(2\Omega\sqrt{\bar{n}}t_p) S(t) \right], \quad (19)$$

with

$$S(t) = 1 + \cos\{2\bar{n}[p(t) + \chi t]\} e^{d(t)}. \quad (20)$$

For $\sin(2\Omega|\alpha|t_p) = 1$, i.e., $t_p = \pi/4\Omega|\alpha|$, Eq. (19) reduces to

$$P_e(t) = \frac{1}{2} - e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} \frac{\cos\{2\bar{n}[p(t) + \chi t]\}}{2} e^{d(t)}, \quad (21)$$

and the inversion is $I(t) = e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} \cos\{2\bar{n}[p(t) + \chi t]\} e^{d(t)}$, where $e^{d(t)}$ leads to the envelope modulation and is independent of the specific photon number n . In addition, we note that, after a short transient period, the function $p(t)$ can be approximated as $p(t) \approx \sum_j (g_j^2 t/\omega_j)$. Consequently, in the regime $\omega_j t \gg 1$, the inversion is well approximated by

$$I(t) = e^{d(t)} e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} \cos[\omega(\bar{n})t], \quad (22)$$

where $\omega(\bar{n}) = 2\bar{n}\tilde{\chi}$ is the dominant oscillation frequency for $I(t)$ with

$$\tilde{\chi} \approx \chi + \sum_j \frac{g_j^2}{\omega_j}. \quad (23)$$

Note that the second part in Eq. (23) is exactly the photon nonlinearity introduced by the acoustic environment and

strongly depends on the spectral density of the acoustic bath. However, due to photon number fluctuation for the coherent state $\delta n \sim |\alpha|$, there will be a range of oscillation frequencies around the dominant oscillation frequency $\omega(\bar{n})$ for the spread of the probabilities $|C_n|^2 = e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!}$ about $\bar{n}$. As a result, the oscillatory terms with frequencies in the range $\omega(\bar{n} - \delta n)$ to $\omega(\bar{n} + \delta n)$ become important. Without the $e^{d(t)}$ term, the destructive interference between these weighted oscillatory terms leads to the dephasing of the Rabi oscillations for $\bar{n} \gg 1$, where the collapse time can be estimated from the time-frequency uncertainty relation and is given by

$$t_C \sim \frac{1}{|\omega(\bar{n} + \delta n) - \omega(\bar{n} - \delta n)|} = \frac{1}{4\sqrt{\bar{n}}\tilde{\chi}}, \quad (24)$$

with $\omega(\bar{n} \pm \delta n) = 2(\bar{n} \pm \delta n)\tilde{\chi}$. On the other hand, one can expect a revival of the oscillations whenever neighboring phases differ by some multiple of 2π , i.e., $[\omega(\bar{n} + 1) - \omega(\bar{n})]t_R = 2k\pi$ ($k = 0, 1, 2, \dots$), corresponding to a constructive interference of the oscillating terms around $\bar{n}$. The revival time t_R is thus given by

$$t_R \sim \frac{2k\pi}{|\omega(\bar{n} + 1) - \omega(\bar{n})|} = k \frac{\pi}{\tilde{\chi}}, \quad (25)$$

which is independent of $\bar{n} \gg 1$. As shown in Figs. 5(a)–5(d), as $\bar{n}$ increases, the collapse time of the oscillations in P_e (scaling as $\bar{n}^{1/2}$) becomes shorter, see the inset in Fig. 5(d). The peaks of the revival oscillations remain centered at $\omega_u t_R \sim k\pi\omega_u/\tilde{\chi}$ (independent of the specific value of $\bar{n}$), and are equally spaced by $\delta(\omega_u t) \sim \pi\omega_u/\tilde{\chi}$. For the (experimentally feasible) parameters under consideration here [33]: the upper

cutoff frequency $\omega_u/2\pi \approx 10$ MHz, $\Omega/2\pi = 10$ MHz, $\Delta = 20\Omega$, and the lower cutoff frequency $\omega_1 = 10^{-3}\omega_u$, the revival time is spaced by $\delta(\omega_u t) \approx 20\pi$. For $\omega_u t \ll 10^3$ [i.e., $\omega_j t \ll 1$ and then $\sin(\omega_j t) \sim \omega_j t$], the effect of the acoustic noise $\sim p(t)$ on the dominant oscillation frequency can be neglected, i.e., $\omega(\bar{n}) \approx 2\bar{n}\chi$ and the peak centers for $C \neq 0$ and $C = 0$ are almost coincident [see the gray curve in Fig. 5(d)]. However, for $\omega_u t_R = 83\pi\omega_u/\tilde{\chi}$, where the oscillation envelope reaches the local maximum, one finds that the peaks of the revival oscillations is deviated from $\omega_u t \sim 83\pi\omega_u/\chi$ due to the influence of the acoustic noise [i.e., $\tilde{\chi} = \chi + \sum_j (g_j^2/\omega_j)$], where the acoustic-induced shift $\sum_j (g_j^2/\omega_j)$ relative to the intrinsic dispersive shift χ is about 6×10^{-4} for $C \sim 4 \times 10^{-13}$. Thus, by comparing with the noise-free oscillations (the gray curves) and examining the phase advance, one can resolve the collective phonon nonlinearities $\sum_j (g_j^2/\omega_j)$ introduced by the acoustic bath modes.

Moreover, the envelope of the oscillations is modulated by the AEID effect [see Figs. 5(a)–5(c)], exhibiting the second fold “slow-wave” (collapse and revival) dynamics. The characteristic time for the slow-wave dynamics of the envelope can be derived from extreme values of the exponential factor $e^{d(t)}$. Under the conditions of $k_B T \gg \hbar\omega_1$ and $\omega_u \gg \omega_1$, Eq. (11) can be approximately given by

$$d(t) \approx -\frac{2C\omega_c^2}{\pi\beta\hbar} \left[\frac{1}{3\omega_1^3} - \frac{\cos(\eta\omega_u t)}{3\omega_1^3} + \frac{t \sin(\eta\omega_u t)}{6\omega_1^2} + \frac{t^2 \cos(\eta\omega_u t)}{6\omega_1} - \frac{t^3}{6} \left(\frac{\pi}{2} - \text{Si}(\eta\omega_u t) \right) \right], \quad (26)$$

where $\eta = \omega_1/\omega_u$ and $\text{Si}(\eta\omega_u t) = \int_0^{\eta\omega_u t} \frac{\sin(x)}{x} dx$ is the sine integral function of the variable x . As such, the peaks and valleys of the envelope can be determined by finding the zeros of the time derivative of $d(t)$, i.e., by solving $d'(t) = 0$, leading to $\sin(\eta\omega_u t) + \eta\omega_u t \cos(\eta\omega_u t) = (\eta\omega_u t)^2 [\frac{\pi}{2} - \text{Si}(\eta\omega_u t)]$. For $C = 4 \times 10^{-13}$ and $\eta = 10^{-3}$, the peaks and valleys of the envelope can be found at $\omega_u t/10^3 \approx 5.18, 11.24, \dots$ and $\omega_u t/10^3 \approx 2.32, 8.18, \dots$, respectively. As C increases, the valleys may become increasingly flat, corresponding to a complete collapse of oscillatory behavior. Notably, the collapse and revival behavior can then be observed in both the dominant oscillation and the overall envelope.

Previously, we have set $t_p = \pi/4\Omega|\alpha|$ such that $P_e \approx 0$ at $t = 0$. Furthermore, we show P_e versus $\omega_u t$ and $2\Omega|\alpha|t_p/\pi$ in Figs. 6(a) and 6(b) with $|\alpha|^2 = 2$ and $|\alpha|^2 = 25$, respectively. Now, the initial population of excited state $P_e(t = 0)$ depends on $\Omega|\alpha|t_p$, and is given by [i.e., Eq. (16) with $t = 0$]

$$P_e(0) = e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} \cos^2(2\theta_n), \quad (27)$$

where $\theta_n = \Omega\sqrt{n+1}t_p$. For $|\alpha|^2 = 2$, $P_e(0)$ is sensitive to the underlying photon number distribution. However, one finds that the temporal evolution ($\omega_u t$ dependence) of P_e can still exhibit similar oscillatory behavior as in Fig. 5(a) for $t_p = \pi/(4\Omega|\alpha|)$. While for the coherent state with $|\alpha|^2 \gg 1$, Eq. (27) can be further approximated as $P_e(0) \approx \cos^2(2\Omega|\alpha|t_p)$, which is simply a cosine function and $P_e(0) \approx 0$ for $2\Omega|\alpha|t_p = \pi/2, 3\pi/2$. As an example, for $|\alpha|^2 = 25$,

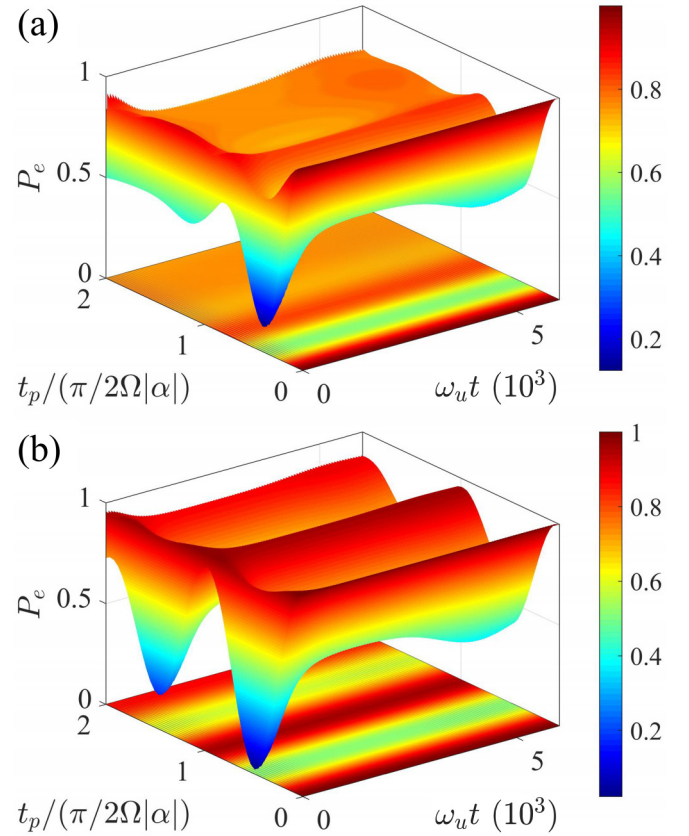


FIG. 6. P_e as a function of $\omega_u t$ and $t_p/(\pi/2\Omega|\alpha|)$ for (a) $|\alpha|^2 = 2$ and (b) $|\alpha|^2 = 25$, with the decay rate $\kappa/2\pi = \gamma/2\pi = 0$, $\Omega/2\pi = 10$ MHz, and $C = 4 \times 10^{-13}$. Other parameters are the same as in Fig. 5.

the amplitude of the dominant oscillations along $\omega_u t$ become maximum for $2\Omega|\alpha|t_p = \pi/2$ [see also Fig. 5(c)], and slightly decreases alternatively for $2\Omega|\alpha|t_p = 3\pi/2$. For $2\Omega|\alpha|t_p = \pi$, where $P_e(0) \approx 1$, the periodic collapses and revivals dynamics in P_e and the AEID effect cannot be observed.

Finally, we examine the effect of the cavity and qubit dissipation on the collapse and revival dynamics, where the probabilities of finding the qubit in the states $|e\rangle$ and $|g\rangle$ are revised as

$$P_e(t) = e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{(\bar{n}e^{-\kappa t})^n}{n!} \left\{ (e^{-\gamma t} \cos^4 \theta_n + e^{-\kappa t} \sin^4 \theta_n) - \frac{1}{2} e^{d(t) - \frac{\kappa+\gamma}{2}t} \cos[2(n+1)\tilde{\chi}t] \sin^2(2\theta_n) \right\}, \quad (28)$$

$$P_g(t) = e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{(\bar{n}e^{-\kappa t})^n}{n!} \left\{ \cos^2 \theta_n \sin^2 \theta_n (e^{-\gamma t} + e^{-\kappa t}) + \frac{1}{2} e^{d(t) - \frac{\kappa+\gamma}{2}t} \cos[2(n+1)\tilde{\chi}t] \sin^2(2\theta_n) \right\}. \quad (29)$$

In Fig. 7, we show P_e as a function of $\omega_u t$ by considering the set of the decay rates $\kappa/2\pi = \gamma/2\pi = \{0, 10, 10^2, 10^3\}$ Hz, the average photon number $|\alpha|^2 = 25$ and the qubit-field coupling strength $\Omega/2\pi = 10$ MHz. For $|\alpha|^2 \gg 1$, the dominant oscillations decay exponentially as $P_e \propto e^{-\gamma t} e^{\bar{n}(e^{-\gamma t} - 1)}$.

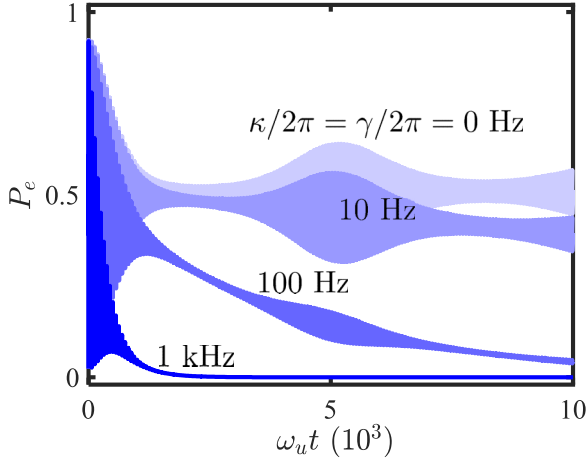


FIG. 7. P_e as a function of $\omega_u t$ for the decay rates $\kappa/2\pi = \gamma/2\pi = \{0, 10, 10^2, 10^3\}$ Hz at $|\alpha|^2 = 25$. Here, the coupling strengths $C = 4 \times 10^{-13}$ and $\Omega/2\pi = 10$ MHz. Other parameters are the same as in Fig. 5.

Moreover, as the decay rates κ and γ increase, the AEID-induced slow-wave dynamics in the envelope is progressively suppressed, see the envelope peak around $\omega_u t \sim 5 \times 10^3$. As a result, the slow-wave dynamics in the envelope survives for the decay rates $\kappa/2\pi = \gamma/2\pi \sim 0.1$ kHz [47–49], and almost disappears for $\kappa/2\pi = \gamma/2\pi = 1$ kHz.

V. CONCLUSION

In summary, we have considered a circuit QED system, where an LC oscillator optomechanically couples to the continuum of bath modes and dispersively interacts with a superconducting qubit. The acoustic bath modes lead to dephasing of photon number superposition states, which can be extracted by measuring the qubit population with a three-step Ramsey interferometric measurement scheme. The time evolution of the qubit population shows dynamical oscillations with periodic collapses and revivals for the LC oscillator is initially in the coherent state. The AEID effect influences both the frequency and amplitude of the Rabi oscillations, which strongly depend on the spectral density of the acoustic bath and the optomechanically induced nonlinearity.

We have also shown that the cavity and qubit dissipation may suppress the revivals caused by rephasing, accelerating the decay of the qubit population oscillations. For the LC oscillator is initialized in a coherent state, observing long-time envelope revivals requires low dissipation rates corresponding to millisecond-scale coherence times, which are achievable in state-of-the-art superconducting circuit platforms [47–49]. By contrast, in the vacuum state case, the revival signatures remain observable for typical transmon decay rates on the order of 10–20 kHz, significantly relaxing the experimental constraints.

Looking forward, the mechanism demonstrated in Ref. [36]—utilizing qubit dynamics to probe a structured photonic bath—presents a promising and complementary perspective. A future synthesis of these approaches, potentially embedding an artificial atom directly within an optomechanical circuit, could enable the simultaneous

probing of both dissipative and dephasing interactions with a non-Markovian environment. This would offer a more complete toolkit for quantum acoustics and the characterization of complex decoherence pathways.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: THE EVOLUTION OPERATOR

In this Appendix, we derive the dynamical evolution operator for the second step, where the LC oscillator interacts with a superconducting qubit and optomechanically couples to a long flexible mechanical strip. The Hamiltonian of the system including the cavity and qubit dissipation is

$$H = \hbar\chi(a^\dagger a + 1)|e\rangle\langle e| - \hbar\chi a^\dagger a|g\rangle\langle g| + \sum_j \hbar\omega_j \left(b_j^\dagger b_j + \frac{1}{2}\right) + \sum_j \hbar g_j \left(a^\dagger a + \frac{1}{2}\right)(b_j + b_j^\dagger) - i\hbar\frac{\gamma}{2}|e\rangle\langle e| - i\hbar\frac{\kappa}{2}a^\dagger a. \quad (\text{A1})$$

The parameters χ and g_j are as those given in the main text. The time-evolution operator for the system is simply $U(t) = e^{-iHt/\hbar}$. By applying a unitary transformation with the operator [30,33,55]

$$T = \exp \left[\sum_j k \left(a^\dagger a + \frac{1}{2} \right) (b_j^\dagger - b_j) \right] \quad (\text{A2})$$

(where $k = g_j/\omega_j$) to the Hamiltonian, we find that the photon number operator is invariant, while the mechanical operators are transformed into

$$T b_j T^\dagger = b_j - k \left(a^\dagger a + \frac{1}{2} \right), \quad T b_j^\dagger T^\dagger = b_j^\dagger - k \left(a^\dagger a + \frac{1}{2} \right). \quad (\text{A3})$$

It follows that the time-evolution operator $U(t)$ is transformed into the form as

$$U'(t) = T U(t) T^\dagger = e^{-i[\chi(a^\dagger a + 1)|e\rangle\langle e| - \chi a^\dagger a|g\rangle\langle g|]t - \frac{\gamma}{2}t|e\rangle\langle e| - \frac{\kappa}{2}t a^\dagger a} \times e^{-\sum_j i\omega_j t (b_j^\dagger b_j + \frac{1}{2}) + \sum_j i\omega_j t k^2 (a^\dagger a + \frac{1}{2})^2}. \quad (\text{A4})$$

Then, the original time-evolution operator $U(t)$ is obtained by applying an inverse transformation to $U'(t)$,

$$U(t) = T^\dagger U'(t) T = T^\dagger e^{-i[\chi(a^\dagger a + 1)|e\rangle\langle e| - \chi a^\dagger a|g\rangle\langle g|]t - \frac{\gamma}{2}t|e\rangle\langle e| - \frac{\kappa}{2}t a^\dagger a} \times e^{\sum_j i\omega_j t k^2 (a^\dagger a + \frac{1}{2})^2} \left(e^{-\sum_j i\omega_j t (b_j^\dagger b_j + \frac{1}{2})} T \right). \quad (\text{A5})$$

By applying the rotating transformation

$$\begin{aligned} & \exp \left(- \sum_j i\omega_j t \left(b_j^\dagger b_j + \frac{1}{2} \right) \right) T \exp \left(\sum_j i\omega_j t \left(b_j^\dagger b_j + \frac{1}{2} \right) \right) \\ &= \exp \left[\sum_j k \left(a^\dagger a + \frac{1}{2} \right) (b_j^\dagger e^{-i\omega_j t} - b_j e^{i\omega_j t}) \right] \end{aligned} \quad (\text{A6})$$

to the last line in Eq. (A5), the time-evolution operator $U(t)$ can be written in a useful form after some simple algebra:

$$\begin{aligned} U(t) &= e^{-i[\chi(a^\dagger a + 1)|e\rangle\langle e| - \chi a^\dagger a |g\rangle\langle g|]t} e^{-\frac{\kappa}{2} t a^\dagger a - \frac{\gamma}{2} t |e\rangle\langle e|} \\ &\quad \times e^{\sum_j i[\omega_j t - \sin(\omega_j t)](g_j/\omega_j)^2 (a^\dagger a + \frac{1}{2})^2} \\ &\quad \times e^{-\sum_j \frac{g_j}{\omega_j} (a^\dagger a + \frac{1}{2})(\zeta b_j^\dagger - \zeta^* b)} \\ &\quad \times e^{-\sum_j i\omega_j t (b_j^\dagger b_j + \frac{1}{2})}, \end{aligned} \quad (\text{A7})$$

where $\zeta = 1 - e^{-i\omega_j t}$.

APPENDIX B: TIME EVOLUTION FOR AN INITIAL VACUUM STATE

Based on Eq. (A7), we are able to solve the time evolution of a more general initial system-environment state. We first consider the system is initially in the state

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|e0\rangle - i|g1\rangle) \otimes |\{\alpha_j\}\rangle, \quad (\text{B1})$$

where $|\{\alpha_j\}\rangle = |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes \cdots \otimes |\alpha_\infty\rangle$ and $|\alpha_j\rangle$ denotes the j th acoustic bath mode in a coherent state. Note that an arbitrary initial bath state can be expressed in terms of the coherent-state basis. The time evolution for such a state is given by

$$\begin{aligned} |\psi(t)\rangle &= U(t)|\psi(0)\rangle \\ &= \frac{1}{\sqrt{2}} e^{-i\chi t - \frac{\gamma}{2} t} \exp \left\{ \sum_j i[\omega_j t - \sin(\omega_j t)](g_j/\omega_j)^2 \left(\frac{1}{2} \right)^2 - i\omega_j t/2 \right\} \\ &\quad \times \exp \left\{ \sum_j \frac{1}{4} \frac{g_j}{\omega_j} [\alpha_j^* (1 - e^{i\omega_j t}) - \alpha_j (1 - e^{-i\omega_j t})] \right\} |e0\rangle \left| \left\{ \alpha_j e^{-i\omega_j t} - \frac{1}{2} \frac{g_j}{\omega_j} (1 - e^{-i\omega_j t}) \right\} \right\rangle \\ &\quad - i \frac{1}{\sqrt{2}} e^{i\chi t - \frac{\gamma}{2} t} \exp \left\{ \sum_j i[\omega_j t - \sin(\omega_j t)](g_j/\omega_j)^2 \left(\frac{3}{2} \right)^2 - i\omega_j t/2 \right\} \\ &\quad \times \exp \left\{ \sum_j \frac{3}{4} \frac{g_j}{\omega_j} [\alpha_j^* (1 - e^{i\omega_j t}) - \alpha_j (1 - e^{-i\omega_j t})] \right\} |g1\rangle \left| \left\{ \alpha_j e^{-i\omega_j t} - \frac{3}{2} \frac{g_j}{\omega_j} (1 - e^{-i\omega_j t}) \right\} \right\rangle. \end{aligned} \quad (\text{B2})$$

For the bath modes initially in the thermal states $\rho_b = \prod_j \frac{1}{\pi n_j} \int d\alpha_j^2 \exp(-|\alpha_j|^2/n_j) |\alpha_j\rangle\langle\alpha_j|$ and $\rho_{ac} = \frac{1}{2}(|e0\rangle - i|g1\rangle)(\langle e0| + i\langle g1|)$, the initial density operator is $\rho(0) = \rho_{ac} \otimes \rho_b$. The system state at time t can be derived as

$$\begin{aligned} \rho(t) &= U(t)|\rho(0)\rangle U^\dagger(t) \\ &= \frac{1}{2} \prod_j \frac{e^{\beta\hbar\omega_j} - 1}{\pi} \int d\alpha_j^2 \exp[-|\alpha_j|^2(e^{\beta\hbar\omega_j} - 1)] [\rho^{(1)}(t) + \rho^{(2)}(t) + \rho^{(3)}(t) + \rho^{(4)}(t)], \end{aligned} \quad (\text{B3})$$

with

$$\begin{aligned} \rho^{(1)}(t) &= e^{-\gamma t} |e0\rangle\langle e0| \otimes \left| \alpha_j e^{-i\omega_j t} - \frac{1}{2} \frac{g_j}{\omega_j} (1 - e^{-i\omega_j t}) \right\rangle \left\langle \alpha_j e^{-i\omega_j t} - \frac{1}{2} \frac{g_j}{\omega_j} (1 - e^{-i\omega_j t}) \right|, \\ \rho^{(2)}(t) &= e^{-\kappa t} |g1\rangle\langle g1| \otimes \left| \alpha_j e^{-i\omega_j t} - \frac{3}{2} \frac{g_j}{\omega_j} (1 - e^{-i\omega_j t}) \right\rangle \left\langle \alpha_j e^{-i\omega_j t} - \frac{3}{2} \frac{g_j}{\omega_j} (1 - e^{-i\omega_j t}) \right|, \\ \rho^{(3)}(t) &= i e^{-i2\chi t - \frac{\gamma}{2} t - \frac{\kappa}{2} t} \exp \left\{ - \sum_j 2i[\omega_j t - \sin(\omega_j t)] \left(\frac{g_j}{\omega_j} \right)^2 \right\} \exp \left\{ - \sum_j \frac{1}{2} \frac{g_j}{\omega_j} [\alpha_j^* (1 - e^{i\omega_j t}) - \alpha_j (1 - e^{-i\omega_j t})] \right\} \\ &\quad \otimes |e0\rangle\langle g1| \otimes \left| \alpha_j e^{-i\omega_j t} - \frac{1}{2} \frac{g_j}{\omega_j} (1 - e^{-i\omega_j t}) \right\rangle \left\langle \alpha_j e^{-i\omega_j t} - \frac{3}{2} \frac{g_j}{\omega_j} (1 - e^{-i\omega_j t}) \right|, \end{aligned}$$

$$\rho^{(4)}(t) = -ie^{i2\chi t - \frac{\gamma}{2}t - \frac{\kappa}{2}t} \exp \left\{ \sum_j 2i[\omega_j t - \sin(\omega_j t)] \left(\frac{g_j}{\omega_j} \right)^2 \right\} \exp \left\{ \sum_j \frac{1}{2} \frac{g_j}{\omega_j} [\alpha_j^* (1 - e^{i\omega_j t}) - \alpha_j (1 - e^{-i\omega_j t})] \right\} \\ \otimes |g1\rangle\langle e0| \otimes \left| \alpha_j e^{-i\omega_j t} - \frac{3}{2} \frac{g_j}{\omega_j} (1 - e^{-i\omega_j t}) \right\rangle \left\langle \alpha_j e^{-i\omega_j t} - \frac{1}{2} \frac{g_j}{\omega_j} (1 - e^{-i\omega_j t}) \right|.$$

Using the identity $\int d\alpha^2 \exp(x\alpha + y\alpha^* - z|\alpha|^2) = \frac{\pi}{z} \exp(xy/z)$ and tracing out the phononic degrees of freedom, we obtain the reduced density matrix of the superconducting qubit and the LC oscillator:

$$\rho_{ac}(t) = \frac{1}{2} e^{-\gamma t} |e0\rangle\langle e0| + \frac{1}{2} e^{-\kappa t} |g1\rangle\langle g1| + \frac{1}{2} i \exp[-2ip(t) - i2\chi t + d(t)] e^{-\frac{\kappa+\gamma}{2}t} |e0\rangle\langle g1| \\ - \frac{1}{2} i \exp[2ip(t) + i2\chi t + d(t)] e^{-\frac{\kappa+\gamma}{2}t} |g1\rangle\langle e0|, \quad (\text{B4})$$

with

$$p(t) = \sum_j \left[\frac{g_j^2}{\omega_j} t - \frac{g_j^2}{\omega_j^2} \sin(\omega_j t) \right], \quad (\text{B5})$$

$$d(t) = -2 \sum_j \frac{g_j^2}{\omega_j^2} \sin^2 \left(\frac{\omega_j t}{2} \right) \coth \left(\frac{\beta \hbar \omega_j}{2} \right). \quad (\text{B6})$$

APPENDIX C: TIME EVOLUTION FOR AN INITIAL COHERENT STATE

When the LC oscillator is initially in the coherent state, the system state after the first resonant pulse reads

$$|\psi(0)\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} [\cos(\Omega t_1 \sqrt{n+1}) |e, n\rangle - i \sin(\Omega t_1 \sqrt{n+1}) |g, n+1\rangle] \otimes |\{\alpha_j\}\rangle. \quad (\text{C1})$$

Then, during the second step, the system-bath coupling leads to the evolution given by $|\psi(t)\rangle = U(t)|\psi(0)\rangle$. Upon performing the similar calculations as before, we obtain the reduced density operator

$$\rho_{ac}(t) = \text{Tr}_b[|\psi(t)\rangle\langle\psi(t)|] \\ = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \frac{\alpha^{*m}}{\sqrt{m!}} \left\{ \exp[i(n+m+1)(n-m)p(t) + (n-m)^2 d(t)] \exp \left[-i\chi(n-m)t - \frac{\kappa}{2}(n+m)t - \gamma t \right] \right. \\ \times [\cos(\Omega t_1 \sqrt{n+1}) \cos(\Omega t_1 \sqrt{m+1})] |e, n\rangle\langle e, m| \\ + \exp[i(n+m+3)(n-m)p(t) + (n-m)^2 d(t)] \exp \left[i\chi(n-m)t - \frac{\kappa}{2}(n+m+2)t \right] \\ \times [\sin(\Omega t_1 \sqrt{n+1}) \sin(\Omega t_1 \sqrt{m+1})] |g, n+1\rangle\langle g, m+1| \\ + i \exp[i(n+m+2)(n-m-1)p(t) + (n-m-1)^2 d(t)] \exp \left[-i\chi(n+m+2)t - \frac{\kappa}{2}(n+m+1)t - \frac{\gamma}{2}t \right] \\ \times [\cos(\Omega t_1 \sqrt{n+1}) \sin(\Omega t_1 \sqrt{m+1})] |e, n\rangle\langle g, m+1| \\ - i \exp[i(n+m+2)(n-m+1)p(t) + (n-m+1)^2 d(t)] \exp \left[i\chi(n+m+2)t - \frac{\kappa}{2}(n+m+1)t - \frac{\gamma}{2}t \right] \\ \times [\sin(\Omega t_1 \sqrt{n+1}) \cos(\Omega t_1 \sqrt{m+1})] |g, n+1\rangle\langle e, m| \}. \quad (\text{C2})$$

We subsequently perform the pulse sequence **R2**, giving rise to

$$|e, n\rangle \rightarrow \cos(\Omega t_2 \sqrt{n+1}) |e, n\rangle - i \sin(\Omega t_2 \sqrt{n+1}) |g, n+1\rangle,$$

$$|g, n+1\rangle \rightarrow \cos(\Omega t_2 \sqrt{n+1}) |g, n+1\rangle - i \sin(\Omega t_2 \sqrt{n+1}) |e, n\rangle, \quad (\text{C3})$$

and then trace over the LC oscillator's degrees of freedom. The final reduced density operator for the qubit reads

$$\rho_a(t) = P_e(t)|e\rangle\langle e| + P_g(t)|g\rangle\langle g|, \quad (C4)$$

where, for $t_1 = t_2 = t_p$, the probabilities for the qubit in the excited state and ground state are

$$P_e = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} \left\{ e^{-\kappa n t - \gamma t} \cos^4(\Omega t_p \sqrt{n+1}) + e^{-\kappa(n+1)t} \sin^4(\Omega t_p \sqrt{n+1}) - \sin^2(2\Omega t_p \sqrt{n+1}) \right. \\ \left. \times \frac{1}{2} \cos\{2(n+1)[p(t) + \chi t]\} e^{d(t) - \frac{\kappa}{2}(2n+1)t - \frac{\gamma}{2}t} \right\}, \quad (C5)$$

$$P_g = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} \frac{1}{4} \sin^2(2\Omega t_p \sqrt{n+1}) \{ e^{-\kappa n t - \gamma t} + e^{-\kappa(n+1)t} + 2 \cos\{2(n+1)[p(t) + \chi t]\} e^{d(t) - \frac{\kappa}{2}(2n+1)t - \frac{\gamma}{2}t} \}. \quad (C6)$$

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