

Remote synchronization and critical-like breakdown in star networks of oscillator populations

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Remote synchronization, in which indirectly connected nodes synchronize while the mediating hub remains desynchronized, is a counterintuitive form of collective dynamics in complex networks. Most previous studies have treated each node as a single oscillator, whereas nodes in realistic systems often represent populations of interacting dynamical units. Here, we study remote synchronization in a star network whose nodes are Kuramoto oscillator populations. Each node contains many internally coupled oscillators with coupling strength K , and different nodes interact through an internode coupling strength L . We show that remote synchronization emerges only at an intermediate level of internal synchrony: The leaf nodes become strongly synchronized with each other, while the hub remains weakly synchronized with them, when the internal order parameter is neither close to zero nor close to one. Increasing K further causes an abrupt breakdown of the remote-synchronized state. Detrended fluctuation analysis reveals enhanced scaling behavior and critical-like long-range temporal correlations near this breakdown boundary. A two-parameter scan in the (L, K) plane confirms that the remote-synchronized regime occupies a finite region associated with intermediate internal order and relatively low detrended fluctuation analysis exponent. We further provide a phenomenological effective-phase interpretation, showing that internal coupling controls the coherence, susceptibility, and stability of each oscillator population. These results demonstrate that internal population dynamics can regulate both the emergence and critical-like breakdown of remote synchronization.

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I. INTRODUCTION

Synchronization is a fundamental collective phenomenon in complex systems and has been observed in a broad range of physical, biological, and technological systems, including coupled lasers, Josephson-junction arrays, chemical oscillators, power grids, neuronal circuits, and brain networks [1–5]. It provides a powerful framework for understanding how interacting dynamical units coordinate their motion and how network topology, coupling architecture, and intrinsic dynamics jointly shape coherent states [2,3,6–8]. Among the nontrivial synchronization patterns found in complex networks, remote synchronization is particularly counterintuitive. It refers to a state in which two or more nonadjacent nodes synchronize with one another, while the intermediate node that mediates their interaction remains desynchronized from them [9–11]. In a star network, this corresponds to strong synchronization among peripheral leaf nodes without strong synchronization between the leaves and the central hub. Previous studies have shown that remote synchronization

can be induced or modulated by network structure, frequency mismatch, coupling strength, time delay, amplitude dynamics, phase frustration, stochastic perturbations, and external forcing [9–16]. It has also been discussed as a possible mechanism for indirect coordination and long-range functional coupling in complex systems, including brain networks where distant regions may exhibit coherent activity despite weak or absent direct structural connections [10,17,18].

Most theoretical studies of remote synchronization have represented each node as a single oscillator. This representation is analytically and numerically convenient and captures the minimal ingredients required for remote synchronization in starlike topologies [9,11]. However, many realistic systems are coarse grained: A network node is often not an elementary dynamical unit, but a population of interacting units. For example, a brain region contains a large number of neurons, a functional module consists of many interacting components, and a mesoscopic node in a complex system may represent the collective activity of an underlying subsystem [19–22]. Brain networks provide a natural example of this population-level organization: Cortical regions and electrophysiological recording sites reflect collective activity rather than individual dynamical units. The internal coordination within such populations can vary across dynamical states and can influence large-scale functional interactions between brain regions [20,23,24]. Thus, the effective node-level phase or amplitude is not prescribed *a priori*, but emerges from the internal collective dynamics of the population. Related

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multilevel modeling approaches have also shown that internal dynamics within cortical areas can shape the emergence of large-scale functional organization in brain networks [25].

These considerations raise a natural question: How does the internal synchronization of oscillator populations regulate remote synchronization at the network level? When each node contains many interacting oscillators, internal coupling changes not only the coherence within each node, but also the effective collective phase through which different nodes interact. It is therefore unclear whether remote synchronization requires highly coherent node populations, whether it can survive when the internal dynamics is weakly ordered, or whether it is favored by an intermediate internal state. Clarifying this issue is important for extending remote synchronization from idealized networks of individual oscillators to more realistic multiscale and coarse grained systems.

To address this question, we investigate remote synchronization in a star network whose nodes are oscillator populations. Each node contains many internally coupled Kuramoto oscillators, while different nodes interact through the star-network topology. This framework allows us to distinguish internal synchrony within each node from node-level synchrony between different nodes. By tuning the internal and internode coupling strengths, we show that remote synchronization emerges only in an intermediate regime of internal synchrony. In this regime, the leaf nodes synchronize strongly with each other, whereas the hub remains weakly synchronized with them. When the internal coupling is further increased, the remote-synchronized state breaks down abruptly, accompanied by enhanced detrended fluctuation analysis (DFA) scaling and critical-like long-range temporal correlations near the breakdown boundary. A two-parameter scan confirms that the remote-synchronized regime occupies a finite region associated with intermediate internal order. Finally, a phenomenological effective-phase interpretation explains how internal coupling controls the coherence, susceptibility, and stability of each oscillator population. Together, these results extend remote synchronization from networks of individual oscillators to networks of oscillator populations and show that internal population dynamics can govern both the emergence and the critical-like breakdown of remote synchronization.

II. MODEL AND METHODS

A. Star network of oscillator populations

We consider a star network consisting of one hub node and four leaf nodes. The hub is denoted by $i = 0$, and the leaf nodes are denoted by $i = 1, \dots, 4$. The structural connectivity between nodes is described by the adjacency matrix A_{ij} , where $A_{0j} = A_{j0} = 1$ for $j = 1, \dots, 4$, and all other off-diagonal elements are zero. Different from conventional star-network models in which each node is represented by a single oscillator, here each node contains a population of M Kuramoto oscillators. Thus, the system has two dynamical levels: the microscopic oscillators inside each node and the macroscopic collective dynamics between different nodes.

The phase of oscillator a in node i is denoted by $\theta_{i,a}(t)$, where $a = 1, \dots, M$. The dynamics is governed by a

population-based Kuramoto model [26],

$$\dot{\theta}_{i,a} = \omega_{i,a} + Kr_i \sin(\Phi_i - \theta_{i,a}) + \frac{L}{N} \sum_{j=0}^{N-1} A_{ij} r_j \sin(\Phi_j - \theta_{i,a}), \quad (1)$$

where $N = 5$ is the number of network nodes. The first term $\omega_{i,a}$ is the natural frequency of oscillator a in node i . The second term describes the internal coupling among oscillators within the same node, with coupling strength K . The third term describes the interaction between different nodes through the star-network topology, with internode coupling strength L . The quantities $r_i(t)$ and $\Phi_i(t)$ are the internal order parameter and the collective phase of node i , respectively, defined below.

For each node, the macroscopic population state is characterized by the complex Kuramoto order parameter [27,28]

$$Z_i(t) = r_i(t) e^{i\Phi_i(t)} = \frac{1}{M} \sum_{a=1}^M e^{i\theta_{i,a}(t)}. \quad (2)$$

Here, $r_i(t) = |Z_i(t)| \in [0, 1]$ measures the internal synchrony of the oscillator population in node i , and $\Phi_i(t) = \arg[Z_i(t)]$ denotes the collective phase of this population. When $r_i(t) \approx 0$, the oscillators inside node i are nearly incoherent; when $r_i(t) \approx 1$, they are nearly fully synchronized. Intermediate values of $r_i(t)$ correspond to partially synchronized internal dynamics.

In the simulations, each node contains $M = 500$ oscillators. The hub has a higher characteristic frequency than the leaf nodes. Specifically, the node frequencies are set as

$$\Omega_{\text{hub}} = 20, \quad (\Omega_1, \Omega_2, \Omega_3, \Omega_4) = (9.61, 9.66, 9.58, 9.56). \quad (3)$$

The natural frequencies $\omega_{i,a}$ are assigned around the corresponding node frequency Ω_i , with the frequency dispersion controlled by a scale parameter. Unless otherwise stated, the frequency scale is set to 0.2.

B. Node-level phase synchronization

Remote synchronization is evaluated at the level of the node populations. For each node i , the collective phase $\Phi_i(t)$ extracted from Eq. (2) is used to quantify phase locking between different nodes. The phase-locking value (PLV) between nodes i and j is defined as [29]

$$\text{PLV}_{ij} = \left| \frac{1}{T} \sum_t \exp[i(\Phi_i(t) - \Phi_j(t))] \right|, \quad (4)$$

where the summation is performed over the time points after discarding the transient. $\text{PLV}_{ij} = 1$ indicates perfect phase locking, whereas $\text{PLV}_{ij} = 0$ indicates the absence of phase locking.

For the star network, we distinguish synchronization among leaf nodes from synchronization between the hub and the leaves, following the standard characterization of remote synchronization in starlike networks [9–11]. The average

leaf-leaf synchronization is defined as

$$R_{ll} = \frac{1}{6} \sum_{1 \leq i < j \leq 4} \text{PLV}_{ij}, \quad (5)$$

where the factor 6 is the number of leaf-leaf pairs. The average hub-leaf synchronization is defined as

$$R_{hl} = \frac{1}{4} \sum_{j=1}^4 \text{PLV}_{0j}. \quad (6)$$

A remote-synchronized state is identified by strong synchronization among the leaf nodes together with much weaker synchronization between the hub and the leaves. Thus, remote synchronization corresponds to a regime in which

$$R_{ll} \simeq 1, \quad R_{hl} \ll R_{ll}. \quad (7)$$

For quantitative identification in the parameter scans, we use the criterion

$$R_{ll} > 0.8, \quad R_{hl} < 0.5. \quad (8)$$

The first condition ensures strong synchronization among the leaf nodes, whereas the second condition excludes states in which the hub is also strongly synchronized with the leaves.

To quantify the overall internal coherence of the oscillator populations, we also compute the time-averaged internal order parameter [27,28]

$$\langle r_i \rangle = \frac{1}{N} \sum_{i=0}^{N-1} \frac{1}{T} \sum_t r_i(t), \quad (9)$$

where the time average is again performed after removing the transient. This quantity characterizes whether the node populations are internally incoherent, partially synchronized, or nearly fully synchronized.

C. Detrended fluctuation analysis

To characterize temporal correlations in the internal population dynamics, we apply DFA to the time series of the internal order parameter $r_i(t)$ [26,30–32]. For a given time series $x(t) = r_i(t)$, we first subtract its temporal mean and construct the integrated profile

$$Y(k) = \sum_{t=1}^k [x(t) - \bar{x}]. \quad (10)$$

The profile $Y(k)$ is then divided into nonoverlapping windows of size s . In each window, a local linear trend is fitted and subtracted, and the root-mean-square fluctuation $F(s)$ is calculated. If the time series exhibits scale-invariant temporal correlations, the fluctuation function follows

$$F(s) \sim s^\alpha, \quad (11)$$

where α is the DFA exponent. A larger DFA exponent indicates stronger long-range temporal correlations [31,32]. In the parameter scans, the reported DFA exponent is averaged over the leaf nodes unless otherwise stated.

D. Numerical simulations

The phase equations are integrated numerically with sampling frequency 200 Hz. For each simulation, the total simulation time is 360 s, and the initial transient is discarded before computing synchronization and DFA measures. To examine how internal synchrony controls remote synchronization, we first fix the internode coupling strength L and scan the internal coupling strength K . We then perform a two-parameter scan in the (L, K) plane to determine the region in which remote synchronization emerges. For each parameter set, we compute the PLV matrix, R_{ll} , R_{hl} , the mean internal order parameter $\langle r_i \rangle$, and the DFA exponent.

III. RESULTS

A. Remote synchronization in a star network of oscillator populations

We first examine whether remote synchronization can emerge in a star network whose nodes are oscillator populations. Figure 1(a) shows the schematic illustration of the model. The network consists of one hub and four leaf nodes. Each node contains a population of Kuramoto oscillators, whose internal coherence is controlled by the coupling strength K . The interactions between different nodes follow the star topology and are controlled by the internode coupling strength L . This model therefore allows us to investigate how synchronization within each node affects synchronization between different nodes.

To reveal the role of internal coupling, we fix the internode coupling strength at $L = 70$ and vary the internal coupling strength K . The corresponding results are shown in Fig. 1(b). For small K , the internal coupling is too weak to generate coherent collective dynamics inside each node. In this regime, both the leaf-leaf synchronization R_{ll} and the hub-leaf synchronization R_{hl} remain low, and no remote synchronization is observed. The mean internal order parameter $\langle r_i \rangle$ is also close to zero, indicating that the oscillator populations inside the nodes are nearly incoherent.

As K increases, the system enters a remote-synchronized regime, highlighted by the gray shaded region in Fig. 1(b). In this regime, the leaf nodes become strongly phase locked with each other, with R_{ll} approaching unity. In contrast, the hub remains much more weakly synchronized with the leaves, as indicated by a substantially smaller R_{hl} . Therefore, the system exhibits the characteristic signature of remote synchronization: strong synchronization among indirectly connected leaf nodes without comparable synchronization between the leaves and the mediating hub. Importantly, this remote-synchronized regime appears when the internal order parameter $\langle r_i \rangle$ takes intermediate values. Thus, remote synchronization is not associated with either nearly incoherent or nearly fully synchronized node populations, but instead emerges when each node is internally partially synchronized.

When K is increased further, the remote-synchronized state loses stability. Near the upper boundary of the shaded region, R_{ll} drops abruptly from a value close to 1 to a much lower value, accompanied by a simultaneous reduction of R_{hl} . For example, R_{ll} decreases from approximately 0.99 at $K = 2.84$ to approximately 0.32 at $K = 2.85$, indicating a

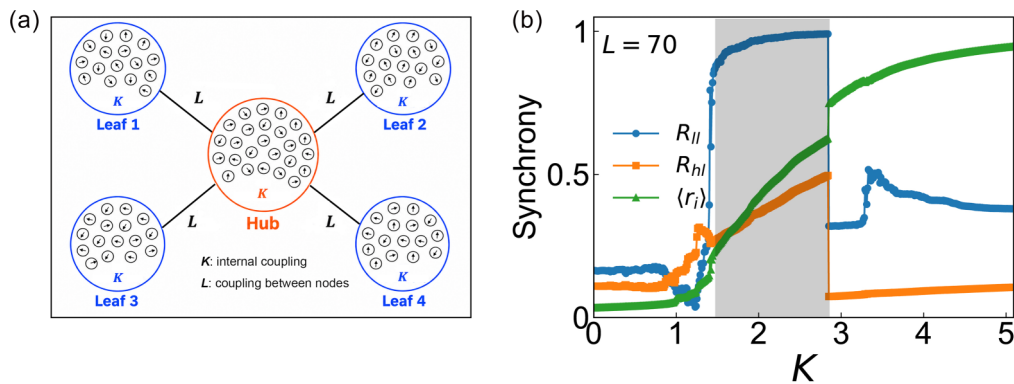


FIG. 1. Emergence of remote synchronization controlled by internal synchrony. (a) Schematic of the star network of oscillator populations. Each node contains many Kuramoto oscillators coupled internally with strength K , while the hub and leaves are coupled through the star topology with internode coupling strength L . (b) Leaf-leaf synchronization R_{ll} , hub-leaf synchronization R_{hl} , and mean internal order parameter $\langle r_i \rangle$ as functions of K for fixed $L = 70$. The gray shaded region marks the remote-synchronized regime identified by the criterion $R_{ll} > 0.8$ and $R_{hl} < 0.5$, where R_{ll} is close to unity while R_{hl} remains much smaller. This regime appears at intermediate internal order, indicating that remote synchronization is facilitated by partial internal synchrony rather than by fully incoherent or fully synchronized node populations. At larger K , R_{ll} drops abruptly, signaling the breakdown of remote synchronization.

sudden breakdown of remote synchronization. At the same time, $\langle r_i \rangle$ continues to increase, showing that stronger internal ordering does not necessarily promote remote synchronization. These results demonstrate that internal synchrony acts as a key control parameter for remote synchronization in oscillator-population networks: An intermediate level of internal synchrony facilitates remote synchronization, whereas excessive internal coupling destroys it. We further verified that this mechanism is not restricted to the particular hub frequency used here. Additional simulations with different hub frequencies show that remote synchronization persists over a finite range of hub-leaf frequency detunings and, whenever it occurs, remains associated with intermediate internal synchrony (see Appendix A).

B. Dynamical states across the remote-synchronization transition

To further understand how the remote-synchronized state emerges and breaks down, we examine three representative values of the internal coupling strength: $K = 1.0$, $K = 2.84$, and $K = 2.85$. These values correspond, respectively, to a low- K nonsynchronized state, a remote-synchronized state, and a post-transition state after the abrupt breakdown of remote synchronization. For each case, we analyze the node-level PLV matrix, the macroscopic phases $\Phi_i(t)$, and the internal order parameters $r_i(t)$, as shown in Fig. 2.

For $K = 1.0$, the internal coupling is weak and the oscillator populations inside the nodes remain largely incoherent. As shown in Fig. 2(a), both the leaf-leaf and hub-leaf PLVs are low, indicating that no clear node-level synchronization is formed. The corresponding macroscopic phases $\Phi_i(t)$ in Fig. 2(d) do not show a stable phase relationship among the leaf nodes. Consistently, the internal order parameters in Fig. 2(g) remain close to zero, confirming that each node is internally weakly synchronized. Thus, insufficient internal coherence prevents the formation of a stable collective node-level phase and suppresses remote synchronization.

At $K = 2.84$, the system enters the remote-synchronized regime. The PLV matrix in Fig. 2(b) shows strong synchronization among all leaf nodes, whereas the PLVs between the hub and the leaves remain much lower. This pattern is the defining feature of remote synchronization in the star network. The macroscopic phase traces in Fig. 2(e) further confirm this behavior: The phases of the leaf nodes evolve almost together, while the hub phase remains separated from them. At the same time, the internal order parameters in Fig. 2(h) take intermediate values rather than approaching either zero or one. This indicates that the remote-synchronized state is supported by partially synchronized oscillator populations.

When K is increased only slightly from 2.84 to 2.85, the remote-synchronized state collapses. As shown in Fig. 2(c), the strong leaf-leaf synchronization observed at $K = 2.84$ disappears, and the PLV values among the leaf nodes are substantially reduced. The macroscopic phases in Fig. 2(f) become desynchronized again, showing no stable collective locking among the leaves. However, in contrast to the low- K case, the internal order parameters in Fig. 2(i) are relatively high, indicating that the oscillator populations are now more internally ordered. Therefore, the loss of remote synchronization at large K is not caused by insufficient internal coherence, but rather by excessive internal ordering that changes the effective node-level dynamics.

These representative dynamical states reveal that remote synchronization in oscillator-population networks is confined to an intermediate internal-synchrony regime. When the internal synchrony is too weak, the nodes cannot generate coherent collective phases. When the internal synchrony becomes too strong, the remote-synchronized state is destroyed. Only between these two limits do the leaf nodes achieve strong mutual synchronization while remaining weakly synchronized with the hub.

C. Critical-like breakdown of remote synchronization

The abrupt collapse of remote synchronization near the upper boundary of the remote-synchronized regime suggests that

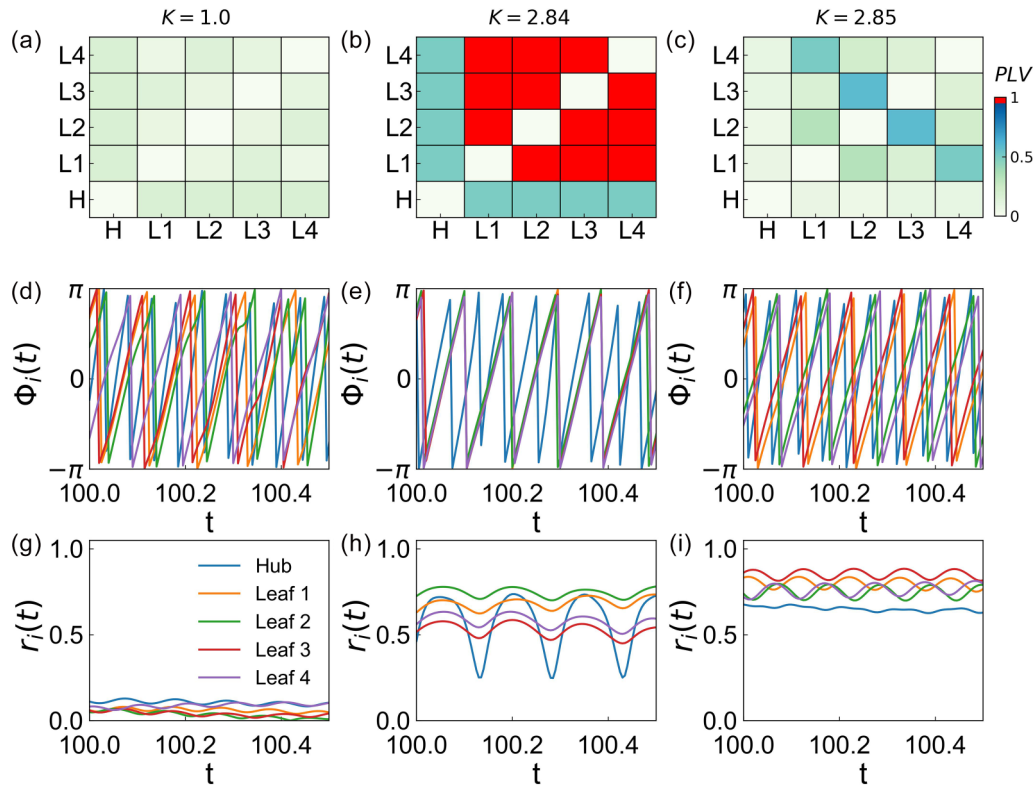


FIG. 2. Dynamical states before, during, and after remote synchronization. The columns show three representative internal coupling strengths: $K = 1.0$, $K = 2.84$, and $K = 2.85$, with fixed $L = 70$. (a)–(c) PLV matrices between node-level collective phases. At $K = 2.84$, the leaf nodes are strongly synchronized with each other while remaining weakly synchronized with the hub, indicating remote synchronization. This pattern is absent at $K = 1.0$ and disappears abruptly at $K = 2.85$. (d)–(f) Collective phases $\Phi_i(t)$ of the five nodes. The leaf phases evolve coherently only in the remote-synchronized state. (g)–(i) Internal order parameters $r_i(t)$ of the node populations. Remote synchronization appears at an intermediate level of internal synchrony, whereas too weak or too strong internal ordering suppresses the remote-synchronized state.

the system undergoes a sharp dynamical transition. To characterize this transition, we analyze the temporal correlations of the internal population dynamics using DFA. Specifically, we apply DFA to the time series of the internal order parameter $r_i(t)$, which reflects the temporal fluctuations of the collective state inside each node.

Figure 3(a) shows the dependence of the DFA exponent on the internal coupling strength K for fixed $L = 70$. Within the remote-synchronized regime, the DFA exponent remains relatively low, indicating weak long-range temporal correlations in the internal order dynamics. However, when K crosses the upper boundary of the remote-synchronized regime, the

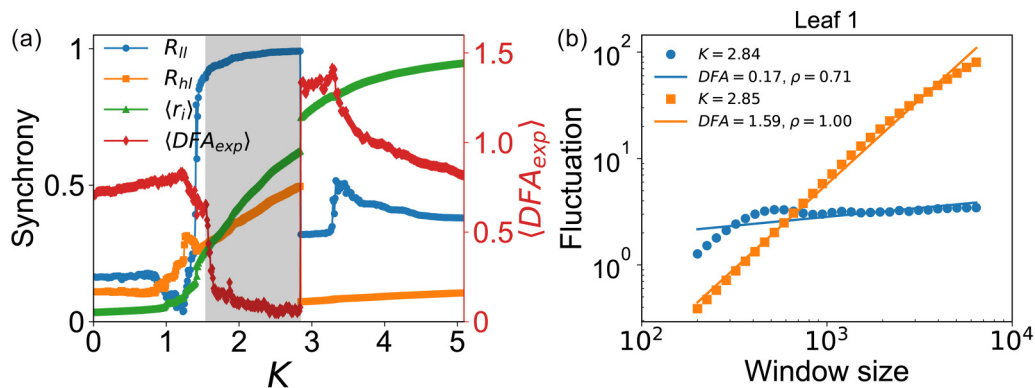


FIG. 3. Critical-like temporal correlations at the breakdown of remote synchronization. (a) DFA exponent as a function of the internal coupling strength K for fixed $L = 70$. The DFA exponent remains low inside the remote-synchronized regime but increases sharply when the leaf-leaf synchronization R_{ll} abruptly collapses, indicating a critical-like transition at the boundary of remote synchronization. (b) DFA fluctuation functions at $K = 2.84$ and $K = 2.85$. The former corresponds to the remote-synchronized state and shows weak scaling, whereas the latter corresponds to the post-transition state and displays clear power-law-like scaling in log-log coordinates. These results show that critical-like long-range temporal correlations emerge when remote synchronization breaks down.

DFA exponent increases sharply. This increase occurs simultaneously with the abrupt decrease of R_{ll} , showing that the loss of leaf-leaf synchronization is accompanied by strongly enhanced temporal correlations. Thus, the breakdown of remote synchronization is not a gradual loss of coherence, but is associated with a critical-like dynamical transition.

To further examine this transition, we compare the DFA fluctuation functions at two nearby values of K : $K = 2.84$ and $K = 2.85$. As shown in Fig. 3(b), $K = 2.84$ lies inside the remote-synchronized regime. In this case, the fluctuation function shows weak scaling behavior, and the corresponding DFA exponent is small. In contrast, $K = 2.85$ lies just beyond the remote-synchronized regime, after the abrupt collapse of leaf-leaf synchronization. At this value, the fluctuation function exhibits an approximately linear relation in log-log coordinates, indicating a power-law-like scaling behavior. The corresponding DFA exponent is much larger, revealing enhanced long-range temporal correlations in the internal order dynamics.

These results indicate that the remote-synchronized state itself is not characterized by strong critical-like fluctuations. Instead, critical-like temporal correlations emerge near the boundary where remote synchronization breaks down. In other words, as the internal coupling strength increases, the system first enters a remote-synchronized regime supported by partial internal synchrony; when K is further increased, the system crosses a transition boundary at which remote synchronization collapses and the internal dynamics becomes strongly correlated over long timescales. This suggests that the critical-like behavior is linked to the breakdown of remote synchronization rather than to the stable remote-synchronized state itself.

The two boundaries of the remote-synchronized regime have different dynamical characters. At the lower- K boundary, the onset of remote synchronization is continuouslike: As K increases, the internal order parameter gradually grows, the collective phases of the node populations become better defined, and the leaf-leaf synchronization R_{ll} increases smoothly. This transition reflects the gradual formation of coherent node-level dynamics from weakly synchronized oscillator populations. In contrast, the upper- K boundary corresponds to an abrupt, discontinuouslike breakdown of the remote-synchronized state. Across this boundary, R_{ll} drops sharply, while the DFA exponent increases rapidly, indicating that the selective leaf-leaf synchronized state loses stability and gives way to a different dynamical regime. Thus, the emergence and the loss of remote synchronization are governed by different mechanisms: The former is associated with the progressive buildup of internal coherence, whereas the latter is associated with the instability of the remote-synchronized state.

To further test whether the enhanced DFA scaling reflects genuine temporal correlations, we performed a shuffled-surrogate analysis. The shuffled surrogates preserve the distribution of $r_i(t)$, but destroy its temporal ordering. The DFA exponents of the shuffled leaf-node time series are strongly reduced, confirming that the large DFA exponents near the breakdown boundary arise from temporal correlations in the internal population dynamics (see Appendix B). These results support a critical-like interpretation of the

breakdown boundary: The collapse of the remote-synchronized state is accompanied by enhanced long-range temporal correlations in the internal population dynamics. Here, the term “critical-like” refers to this dynamical enhancement of temporal correlations near the loss of stability of the remote-synchronized state.

D. Phase diagram in the internal-external coupling plane

The above results were obtained by fixing the internode coupling strength L and varying the internal coupling strength K . To examine whether the emergence of remote synchronization persists over a broader parameter range, we next perform a two-parameter scan in the internal-external coupling plane. For each pair of (L, K) , we compute the leaf-leaf synchronization R_{ll} , the hub-leaf synchronization R_{hl} , the mean internal order parameter $\langle r_i \rangle$, and the DFA exponent. The results are summarized in Fig. 4. The black-white outlined region marks the remote-synchronized regime, where the leaf nodes are strongly synchronized with each other while the hub remains relatively weakly synchronized with the leaves.

Figure 4(a) shows the distribution of the leaf-leaf synchronization R_{ll} in the (L, K) plane. Large R_{ll} values appear in more than one parameter region. In particular, R_{ll} is also high when both K and L are large, indicating strong synchronization among the leaf nodes. However, high R_{ll} alone is not sufficient to identify remote synchronization, because the hub may also be synchronized with the leaves in this regime. Therefore, the remote-synchronized state must be identified by comparing R_{ll} with the hub-leaf synchronization R_{hl} .

Figure 4(b) shows the corresponding hub-leaf synchronization R_{hl} . In the upper-right region of the phase diagram, both R_{ll} and R_{hl} are large, indicating a more globally synchronized state rather than remote synchronization. In contrast, inside the black-white outlined region, R_{ll} is close to unity whereas R_{hl} remains much smaller. This contrast identifies this outlined region as the remote-synchronized regime: The leaf nodes are strongly synchronized with each other, but the hub remains relatively weakly synchronized with the leaves. Thus, remote synchronization emerges only in a restricted region of intermediate K and L , rather than throughout the entire high- R_{ll} region.

Figure 4(c) shows the mean internal order parameter $\langle r_i \rangle$. Inside the black-white outlined remote-synchronized region, $\langle r_i \rangle$ takes intermediate values. This confirms that remote synchronization is not simply generated by increasing the internal coherence of each node. Instead, it requires an appropriate internal collective state: The node populations must be coherent enough to support stable node-level phases, but not so strongly ordered that the system is driven into global synchronization or loses the remote-synchronized pattern.

Finally, Fig. 4(d) displays the DFA exponent in the same parameter plane. The black-white outlined remote-synchronized region is associated with relatively low DFA values, indicating that the stable remote-synchronized state does not exhibit strong long-range temporal correlations in the internal order dynamics. Enhanced DFA exponents appear near the boundary of this outlined region, suggesting that critical-like temporal correlations are mainly associated with

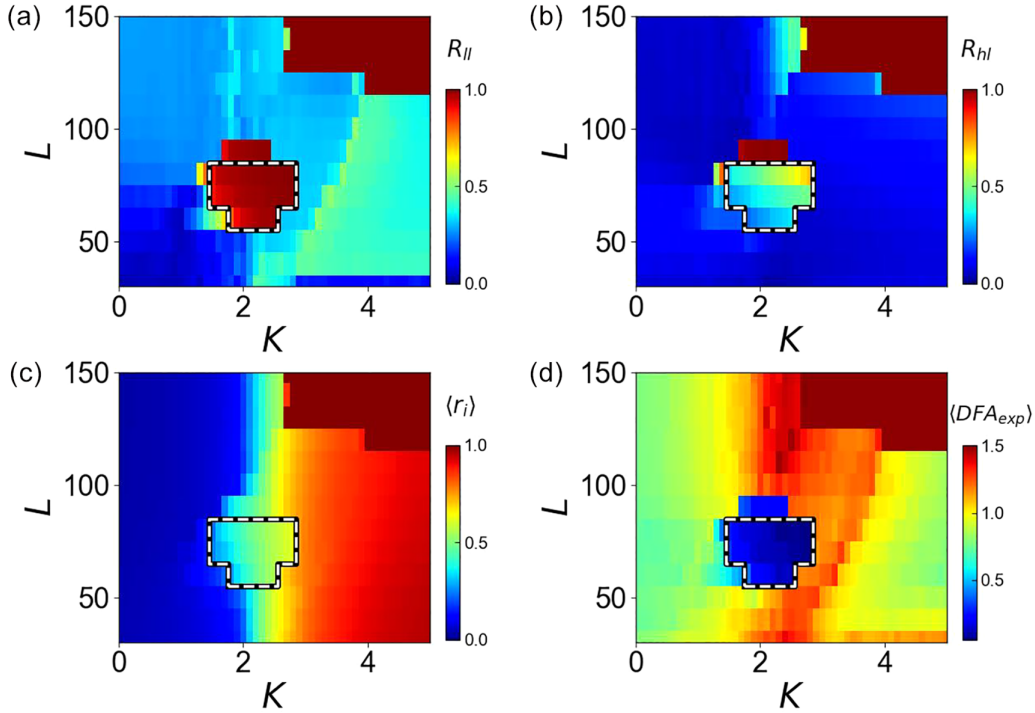


FIG. 4. Phase diagram in the internal-external coupling plane. (a) Leaf-leaf synchronization R_{ll} , (b) hub-leaf synchronization R_{hl} , (c) mean internal order parameter $\langle r_i \rangle$, and (d) DFA exponent $\langle \text{DFA}_{\text{exp}} \rangle$ as functions of K and L . The black-white outlined region marks the remote-synchronized regime, where the leaf nodes are strongly synchronized with each other while the hub remains much more weakly synchronized with the leaves. The outlined region is identified using the quantitative criterion $R_{ll} > 0.8$ and $R_{hl} < 0.5$. Although high R_{ll} also appears in the upper-right region, this region is not remote synchronization because R_{hl} is also high, indicating a more globally synchronized state. The remote-synchronized regime is associated with intermediate internal order and relatively low DFA exponent. Enhanced DFA values appear near the boundary of this region, suggesting critical-like temporal correlations at the breakdown of remote synchronization.

the breakdown of remote synchronization rather than with the remote-synchronized state itself.

Taken together, the phase diagrams demonstrate that remote synchronization in a star network of oscillator populations is not determined by leaf-leaf synchronization alone. Instead, it requires a selective synchronization pattern in which the leaves are strongly synchronized with each other while the hub remains relatively desynchronized from them. The black-white outlined region in Fig. 4 therefore represents the remote-synchronized regime in the (L, K) plane. This regime is associated with intermediate internal order and relatively low DFA exponent, while crossing its boundary leads to the collapse of remote synchronization and the emergence of critical-like temporal correlations.

IV. MECHANISTIC INTERPRETATION

The above numerical results suggest that remote synchronization in oscillator-population networks is controlled by the internal collective state of each node. Unlike a conventional star network of single oscillators, here the effective node-level dynamics is not fixed in advance. Instead, it emerges from the population order parameter $Z_i(t) = r_i(t)e^{i\Phi_i(t)}$. Therefore, changing the internal coupling strength K affects not only the degree of synchrony within each node, but also the effective interaction between different nodes.

A useful way to interpret the dynamics is to view each oscillator population as a collective phase oscillator with a

K -dependent coherence and susceptibility. At the node level, the collective phase dynamics can be phenomenologically written as

$$\dot{\Phi}_i \simeq \Omega_i + L\chi_i(K) \sum_{j=0}^{N-1} A_{ij}r_j \sin(\Phi_j - \Phi_i) + \eta_i(t; K), \quad (12)$$

where r_j measures the coherence of the signal emitted by node j , $\chi_i(K)$ represents the effective response of node i to internode input, and $\eta_i(t; K)$ denotes residual fluctuations originating from the finite-size and partially coherent internal dynamics of the oscillator population. Equation (12) is not intended as an exact reduction of the full system, but it captures the main physical ingredients revealed by the simulations: The internode coupling is mediated by population-level coherent signals, and both the strength and stability of these signals depend on the internal synchronization level.

This interpretation explains why remote synchronization does not occur when K is too small. In this regime, the internal order parameters $r_i(t)$ are close to zero, indicating that the oscillators inside each node are nearly incoherent. As a result, the population does not generate a stable collective phase, and the coherent signal transmitted from one node to another is weak. Although the star topology still couples the hub and the leaves, the effective node-level interaction is insufficient to establish stable phase locking among the leaf nodes. Consequently, both R_{ll} and R_{hl} remain low.

When K is increased to an intermediate range, each node develops a partially coherent collective state. The internal order parameter becomes large enough to define a stable node-level phase, but the population is not yet fully locked into a rigid oscillator. In this regime, the leaf nodes, which have similar intrinsic frequencies, receive a common hub-mediated input through the star topology. This common input can align the collective phases of the leaves with each other. At the same time, the hub has a substantially different characteristic frequency and therefore does not become strongly phase locked with the leaves. The combined effect is a selective synchronization pattern: The leaves synchronize with each other, while the hub remains relatively desynchronized. This is the remote-synchronized regime observed in Figs. 1 and 2.

For larger K , the internal populations become more strongly ordered and behave increasingly like rigid collective oscillators. In this limit, increasing internal coherence no longer enhances remote synchronization. Instead, it changes the effective node-level dynamics and can destabilize the selective leaf-leaf synchronized pattern. The system then leaves the remote-synchronized regime and moves toward a different dynamical state, in which the strong leaf-leaf synchronization is lost or replaced by a more globally coherent state depending on the value of L . This explains why high internal synchrony is not sufficient for remote synchronization and why the remote-synchronized state occupies only a finite region in the (L, K) plane.

The sharp increase of the DFA exponent near the boundary of the remote-synchronized regime can be understood as a signature of critical-like slowing down near the loss of stability of the remote-synchronized state. Let $\mathbf{x}(t)$ denote small perturbations around the remote-synchronized collective state. Near this state, the coarse grained dynamics can be written schematically as

$$\dot{\mathbf{x}} = J(K, L)\mathbf{x} + \boldsymbol{\eta}(t; K), \quad (13)$$

where $J(K, L)$ is the effective Jacobian of the node-level dynamics. Inside the remote-synchronized regime, the leading eigenvalue of J has a negative real part, so perturbations decay and the internal order dynamics shows relatively weak long-range temporal correlations. As the system approaches the boundary of the remote-synchronized regime, the leading eigenvalue approaches zero,

$$\text{Re } \lambda_{\max}(J) \rightarrow 0^-, \quad (14)$$

and the relaxation time increases as

$$\tau_{\text{corr}} \sim \frac{1}{|\text{Re } \lambda_{\max}|}. \quad (15)$$

This increase in the relaxation time enhances slow fluctuations in $r_i(t)$, leading to stronger scaling behavior in the DFA fluctuation function. After the boundary is crossed, the remote-synchronized state loses stability, R_{II} drops abruptly, and the system enters a post-transition dynamical regime with enhanced temporal correlations. This interpretation also explains why the onset and breakdown of remote synchronization are asymmetric: The onset is controlled by the gradual emergence of coherent population-level phases, whereas the breakdown is controlled by the loss of stability of an already formed selective synchronization pattern.

This mechanism clarifies the relationship between remote synchronization and critical-like dynamics in the present system. Remote synchronization itself is not the critical-like state. Rather, it is a stable selective synchronization pattern supported by partial internal synchrony. Critical-like temporal correlations appear near the boundary where this selective pattern breaks down. Thus, the internal population dynamics plays a dual role: It enables remote synchronization at intermediate coherence and controls the critical-like breakdown of this state when the internal coupling becomes too strong.

V. DISCUSSION AND CONCLUSION

In this work, we have investigated remote synchronization in a star network whose nodes are oscillator populations rather than individual oscillators. This setting introduces an additional dynamical layer: Each node has its own internal collective state, and the effective node-level phase emerges from the interactions among many microscopic oscillators [26–28]. Our results show that this internal population dynamics plays a decisive role in determining whether remote synchronization can emerge and how it breaks down. This is particularly relevant for coarse grained systems, such as brain networks, in which macroscopic nodes represent collective population activity rather than elementary dynamical units.

The central finding is that remote synchronization is not favored by either nearly incoherent or nearly fully synchronized node populations. When the internal coupling K is too small, the oscillators inside each node remain weakly coherent, and the node populations cannot generate stable collective phases that support node-level synchronization. When K is too large, the internal populations become strongly ordered, but the selective leaf-leaf synchronization pattern is destroyed. Between these two limits, the system exhibits a remote-synchronized regime in which the leaf nodes are strongly synchronized with each other while the hub remains relatively weakly synchronized with the leaves. Thus, partial internal synchrony provides a favorable dynamical condition for remote synchronization.

This result extends the conventional picture of remote synchronization [9–11]. In most previous studies, each network node is represented as a single oscillator, so the node-level dynamics is directly specified by the oscillator phase or amplitude. Here, in contrast, the node-level dynamics is an emergent property of an underlying oscillator population. The formation of remote synchronization therefore depends not only on the star-network topology and the frequency mismatch between the hub and the leaves, but also on the internal coherence of each node. This finding suggests that remote synchronization in coarse grained or multilayer systems cannot be fully understood without considering the internal degrees of freedom hidden inside each network node [19,20,26].

Another important result is the critical-like breakdown of remote synchronization. As the internal coupling is increased beyond the remote-synchronized regime, the leaf-leaf synchronization drops abruptly. Detrended fluctuation analysis shows that this breakdown is accompanied by a sharp increase in the DFA exponent and by power-law-like scaling of the fluctuation function [30,31]. These results indicate enhanced

long-range temporal correlations near the boundary where remote synchronization collapses [32]. Importantly, the stable remote-synchronized state itself is associated with relatively low DFA values, whereas the critical-like temporal correlations appear near the breakdown boundary. Therefore, remote synchronization and critical-like dynamics play different roles in the present system: Remote synchronization is a stable selective synchronization pattern, while critical-like behavior emerges when this pattern loses stability.

The two-parameter phase diagram further confirms this interpretation. Remote synchronization occupies a finite region in the (L, K) plane, where the internal and internode coupling strengths are balanced. The black-white outlined region in Fig. 4 corresponds to a selective synchronization pattern with R_{ll} close to unity and R_{hl} much smaller. High leaf-leaf synchronization alone is not sufficient to define remote synchronization, because for large K and large L , both R_{ll} and R_{hl} can become large, indicating a more globally synchronized state. Thus, remote synchronization requires not only strong synchronization among the leaves, but also the relative desynchronization of the hub from the leaves. This distinction is essential for identifying remote synchronization in systems with population-level nodes.

The mechanistic interpretation proposed above provides a physical picture for these observations. The internal coupling K controls the coherence, susceptibility, and residual fluctuations of each oscillator population. For weak internal coupling, the node-level signal is too incoherent to mediate stable synchronization. For intermediate internal coupling, each node forms a partially coherent collective phase, allowing the similar leaf nodes to synchronize through the hub-mediated interaction while the frequency-mismatched hub remains separated. For strong internal coupling, the populations become more rigid and the selective remote-synchronized pattern loses stability. Near this loss of stability, the relaxation time of perturbations increases, giving rise to enhanced temporal correlations and critical-like DFA scaling [31,32].

Although the present study does not aim to model a specific neural system, the oscillator-population framework provides a useful theoretical perspective for understanding synchronization in multiscale brain networks and other coarse grained complex systems. In brain network models, a node often represents a cortical region, an electrode, or a functional module, each of which contains many interacting neural elements [19–21]. Such macroscopic nodes reflect population-level activity rather than elementary oscillators, and their internal coordination can vary across dynamical states, ranging from weakly coherent activity to partially synchronized and highly coherent regimes [20,23,24]. Our results suggest that the internal synchrony of such local populations may regulate the emergence of long-range functional coordination between distant regions [19,20,25]. In particular, remote coordination may be most effective when local populations are neither fully incoherent nor completely locked, but instead maintain an intermediate level of internal order. This provides a possible mechanism by which local population dynamics may shape large-scale synchronization patterns without requiring each brain region to behave as a single rigid oscillator. This perspective is consistent with the broader idea that complex

systems can benefit from a balance between local coherence and dynamical flexibility.

In conclusion, we have shown that remote synchronization can emerge in a star network of oscillator populations, but only under an appropriate level of internal synchrony. Partial internal synchrony enables strong synchronization among indirectly connected leaf nodes while keeping the hub relatively desynchronized. When the internal coupling is increased beyond this regime, the remote-synchronized state breaks down and critical-like long-range temporal correlations emerge. These results demonstrate that internal population dynamics acts as a control layer for both the emergence and the breakdown of remote synchronization. By extending remote synchronization from networks of individual oscillators to networks of oscillator populations, this work reveals a new route through which hidden internal degrees of freedom can shape collective synchronization in complex systems.

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DATA AVAILABILITY

Codes for generating the plots can be provided upon reasonable request.

APPENDIX A: ROBUSTNESS AGAINST HUB-LEAF FREQUENCY DETUNING

In the main text, the hub frequency is fixed at $\Omega_{\text{hub}} = 20$, while the leaf-node frequencies are set as

$$(\Omega_1, \Omega_2, \Omega_3, \Omega_4) = (9.61, 9.66, 9.58, 9.56).$$

To examine whether the observed remote synchronization depends on this specific choice of the hub frequency, we performed an additional robustness analysis by varying Ω_{hub} while keeping the leaf frequencies unchanged. The internode coupling strength was fixed at $L = 70$, and the internal coupling strength K was varied as in the main text.

Figure 5 shows the dependence of the leaf-leaf synchronization R_{ll} , the hub-leaf synchronization R_{hl} , and the mean internal order parameter $\langle r_i \rangle$ on K for different hub frequencies. The results show that remote synchronization is not restricted to a single frequency choice. Instead, it exists over a finite range of hub frequencies. For relatively small hub frequencies, the hub becomes less separated from the leaf populations and tends to synchronize more strongly with them, which weakens or destroys the selective remote-synchronized pattern. For excessively large hub frequencies, the hub becomes too strongly detuned from the leaves, and the remote-synchronized state also disappears. Thus, an appropriate level of hub-leaf frequency detuning is required for the formation of remote synchronization.

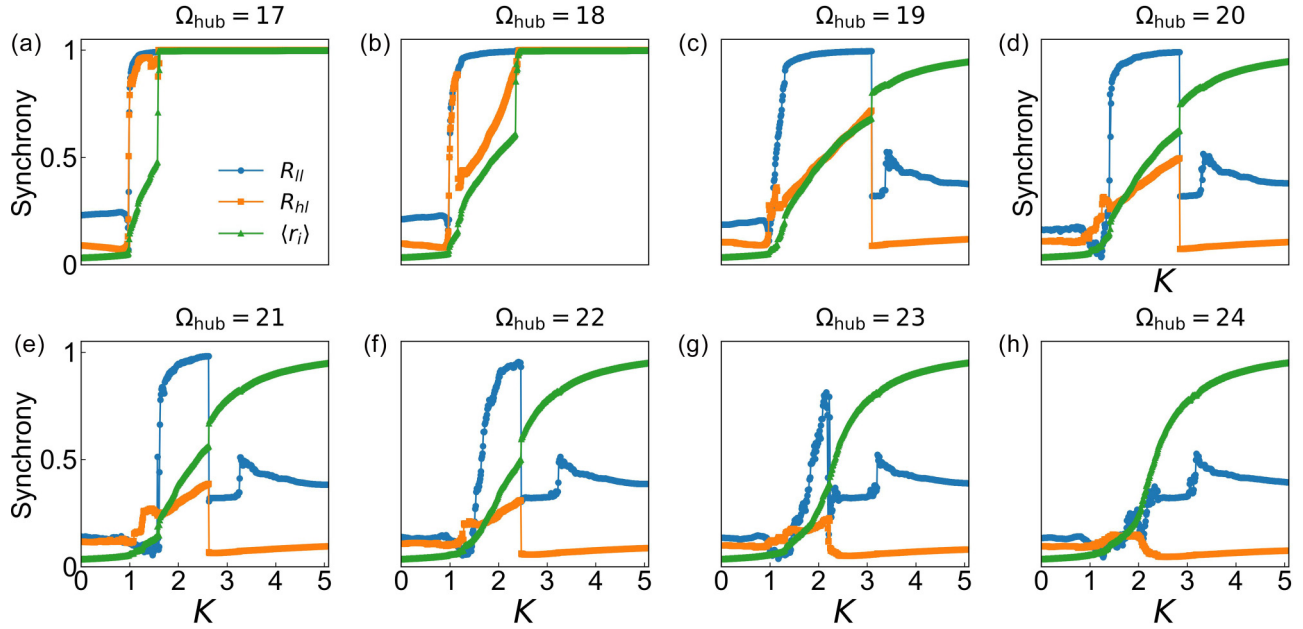


FIG. 5. Robustness of remote synchronization against changes in the hub frequency. The leaf-node frequencies are fixed at $(\Omega_1, \Omega_2, \Omega_3, \Omega_4) = (9.61, 9.66, 9.58, 9.56)$, while the hub frequency is varied from $\Omega_{\text{hub}} = 17$ to $\Omega_{\text{hub}} = 24$. For each value of Ω_{hub} , the leaf-leaf synchronization R_{ll} , the hub-leaf synchronization R_{hl} , and the mean internal order parameter $\langle r_i \rangle$ are plotted as functions of the internal coupling strength K , with fixed internode coupling strength $L = 70$. Remote synchronization appears only within an appropriate range of hub-leaf frequency detuning. When the hub frequency is too small or too large, the remote-synchronized state disappears. Whenever remote synchronization occurs, it is associated with an intermediate level of internal synchrony, confirming the robustness of the mechanism reported in the main text.

Importantly, whenever a remote-synchronized regime is observed, it appears when the internal order parameter $\langle r_i \rangle$ takes intermediate values. This confirms that the main conclusion of the paper is robust with respect to the choice of hub frequency: Remote synchronization is favored by partial internal synchrony rather than by either nearly incoherent or nearly fully synchronized node populations. The precise location and width of the remote-synchronized regime depend

on the frequency detuning, but the underlying mechanism remains unchanged.

APPENDIX B: SURROGATE TEST FOR DFA SCALING

To test whether the enhanced DFA scaling near the breakdown boundary reflects genuine temporal correlations rather than the marginal distribution of the time series, we performed

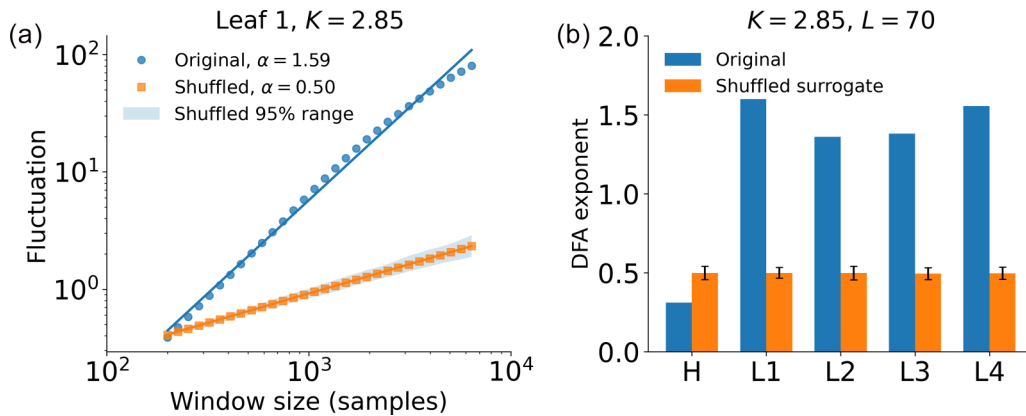


FIG. 6. Surrogate test for DFA scaling near the breakdown boundary of remote synchronization. The parameters are $K = 2.85$ and $L = 70$. (a) DFA fluctuation functions for the original internal order-parameter time series of Leaf 1 and for shuffled-surrogate time series. The shuffled surrogates preserve the amplitude distribution of $r_i(t)$, but destroy the temporal ordering. The original time series shows a large DFA exponent $\alpha \approx 1.59$, whereas the shuffled surrogate gives $\alpha \approx 0.50$. (b) Comparison of DFA exponents for the original and shuffled-surrogate time series across all nodes. The leaf nodes show much larger DFA exponents in the original data than in the shuffled surrogates, while the shuffled surrogates give exponents close to 0.5. This confirms that the enhanced DFA scaling of the leaf-node dynamics reflects temporal correlations rather than the marginal distribution of the time series.

a shuffled-surrogate analysis. For each node, the internal order-parameter time series $r_i(t)$ was randomly permuted in time. This procedure preserves the amplitude distribution of $r_i(t)$, but destroys the temporal ordering and therefore removes temporal correlations.

Figure 6 compares the DFA results obtained from the original time series and from shuffled-surrogate time series at $K = 2.85$ and $L = 70$, near the breakdown boundary of remote synchronization. For the original leaf-node time series, the DFA fluctuation function shows a strong scaling behavior. For example, for Leaf 1, the original time series gives a DFA exponent $\alpha \approx 1.59$. After shuffling the same time series, the

DFA exponent is reduced to $\alpha \approx 0.50$, close to the value expected for an uncorrelated sequence. The same reduction is observed for the other leaf nodes: The original leaf-node time series have large DFA exponents, whereas their shuffled surrogates all give DFA exponents close to 0.5.

These results show that the enhanced DFA exponents near the breakdown boundary do not arise from the marginal distribution of $r_i(t)$. Instead, they reflect genuine temporal correlations in the internal population dynamics. Thus, the surrogate analysis supports our interpretation that the breakdown of remote synchronization is accompanied by critical-like long-range temporal correlations.

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