

Scalar spin chirality Nernst effect

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The scalar spin chirality, which characterizes the fundamental unit of noncoplanar spin structures, plays an important role in rich chiral physics of magnetic materials. In particular, the intensive research efforts over the past two decades have demonstrated that the scalar spin chirality is the source of various novel Hall transports in solid-state systems, offering a primary route to bring about chiral phenomena in condensed matter physics. However, in all of the previous studies, the scalar spin chirality has been given as a static quantity, serving only a passive role in the transport properties of materials. It remains an open question whether or not the scalar spin chirality itself can exhibit a Hall-type transport. In this work, we show that the answer is yes: The scalar spin chirality is Hall transported in Kagome ferromagnets and antiferromagnets under an external bias, engendering a phenomenon which we dub the scalar spin chirality Nernst effect. Notably, this effect is present even in the absence of any spin-orbit coupling. The analytical theory for the scalar spin chirality Nernst effect is corroborated by atomistic spin simulations. Our findings call for the need to lift the conventional assumption that the scalar spin chirality is a static quantity in order to discover the active roles of the scalar spin chirality in transport properties.

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Introduction. A triple product of spins $\chi = \hat{\mathbf{S}}_i \cdot (\hat{\mathbf{S}}_j \times \hat{\mathbf{S}}_k)$, known as the scalar spin chirality (SSC) [see Fig. 1(a)], is increasingly recognized as a key concept for understanding magnetic systems. It characterizes the most elementary unit of noncoplanar spin structures and plays a crucial role in understanding rich chiral physics of magnetic materials. In quantum magnetism, it has been introduced to describe the chiral spin state which breaks time-reversal symmetry in relation to superconductivity [1]. Also, the SSC has been found to contribute to the orbital magnetization through electron hopping among triplets of noncoplanar spins [2–7]. In the transport properties of solid-state systems, the SSC is known to induce an effective magnetic field through the Berry curvature and also serve as a scattering source on particles in solids such as conduction electrons [8–11], magnons [12], and phonons [13]. Consequently, it leads to the Hall-like transports of the aforementioned particles in the frustrated magnets [8,10,12,14–18] and in chiral magnets [11,19–24] through both intrinsic and skew scattering mechanisms. It is worth noting that, in previous studies, the SSC has been given as an immobile quantity that passively influences the dynamics of the particles in solids. Meanwhile, recent studies have extended the concept of Hall-like transport to include the transverse transport

of physical quantities carried by (quasi)particles, such as the orbital Hall effect [25–31], (electric) dipole Hall effect [32], and (magnetic) octupole Hall effect [33], motivating investigation into Hall-like transport of SSCs.

While the transport of macroscopic chiral spin textures, such as magnetic skyrmions, has been extensively studied, the transport properties of SSCs themselves have remained unexplored. In particular, it is an open question whether a dynamic SSC can exhibit a Hall-like transport, which we aim to address in this work. From the symmetry point of view, SSC, $\chi = \hat{\mathbf{S}}_i \cdot (\hat{\mathbf{S}}_j \times \hat{\mathbf{S}}_k)$ possesses the same symmetry properties as B_z or S_z with the z axis being normal to the triangular plane defined by the three sites, since it remains scalar under proper or improper spin rotations and transforms as the z component of a magnetic field under spatial transformation. Since S_z exhibit Hall transports via the spin-orbit coupling, e.g., the spin Hall effect and the spin Nernst effect, it is natural to ask

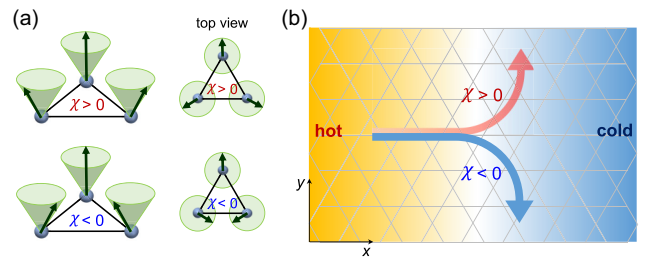


FIG. 1. Schematic illustration of (a), the spin configurations with positive SSC ($\chi > 0$) and negative SSC ($\chi < 0$), and (b), the SSC Nernst effect.

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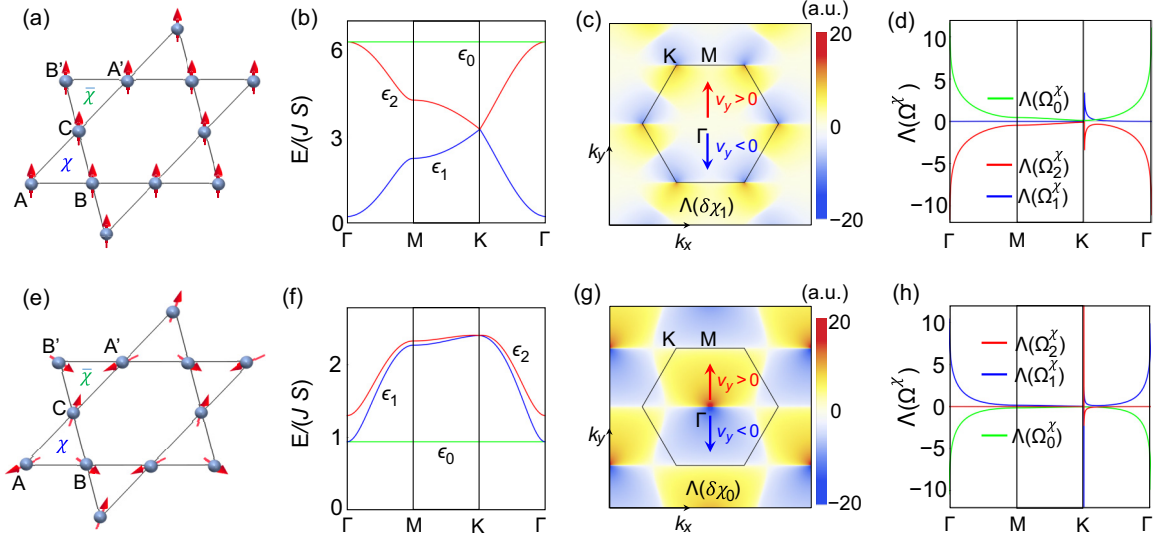


FIG. 2. (a) Ground state spin configuration, (b) magnon dispersion, (c) nonequilibrium SSC of the lowest magnonic state $\delta\chi_1$ in a log-scale [$\Lambda(\delta\chi_1) \equiv \text{sgn}(\delta\chi_1)\log(1 + |\delta\chi_1|)$] in the presence of temperature gradient along the x direction, and (d) log-scale SSC Berry curvature [$\Lambda(\Omega^x) \equiv \text{sgn}(\Omega^x)\log(1 + |\Omega^x|)$] of the collinear Kagome FM. (e) Ground state spin configuration, (f) magnon dispersion, (g) nonequilibrium SSC of the lowest magnon state $\delta\chi_0$ in a log scale in the presence of temperature gradient along the x direction, and (h) log-scale SSC Berry curvature of the coplanar Kagome AFM.

whether the SSC can support Hall transport, and furthermore, whether this can occur even without spin-orbit coupling.

The SSC is defined on a triangular cluster consisting of three spins at three distinct sites. In edge-sharing lattices like triangular and square lattices, SSC contributions from neighboring triangles cancel each other when spin canting angles are small [12,34]. In contrast, the Kagome lattice, as a simple corner-sharing structure, avoids such cancellation. Thus, in this work, we focus on Hall-like transport of the SSC in Kagome lattice magnets in the main text. We also discuss its manifestation in honeycomb magnets, where SSC is defined on a triangle formed by next-nearest-neighbor bonds (see the Supplemental Material).

Here, we theoretically demonstrate that the SSC exhibits intrinsic Hall transport in Kagome ferromagnets and antiferromagnets, within the theoretical framework where the SSC is described in terms of the magnon operators. We show that the external biases, such as a temperature gradient or a magnon-current injection, induce a magnonic SSC flow, perpendicular to the direction of the external perturbation. We refer to this phenomenon as the magnon SSC Nernst effect. Specifically, nonequilibrium magnonic states with positive and negative SSCs exhibit opposite Nernst transports [Fig. 1(b)]. Interestingly, unlike magnon Hall-like phenomena explored in the previous studies, such as the magnon Hall effect [12,35–40], the magnon spin Nernst effect [41–44], and the magnon orbital Nernst effect [45], the magnon SSC Nernst effect does not rely on the magnon Berry curvature and therefore does not require spin-orbit coupling; even if the magnon Berry curvature is zero everywhere, the magnon SSC Nernst effect still exists. Thus, the magnon SSC Nernst effect is an inherent property of the Kagome magnets without requiring any specific symmetry breaking or spin-orbit-coupling related effects. We further verify this effect through atomistic spin dynamics simulations. We envision that our findings trigger

further investigations of novel transport phenomena where the SSC plays active roles, enriching the chiral physics of solid-state systems.

Model construction. Here, we consider the following spin Hamiltonian in the Kagome lattice:

$$H = -J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_i (\mathbf{S}_i \cdot \mathbf{e}_i)^2, \quad (1)$$

where J is the exchange coupling and K is the easy-axis magnetic anisotropy along $\mathbf{e}_i$. In this study, we explore two distinct magnetic systems: the collinear Kagome ferromagnet (FM) ($J > 0$, $K > 0$, and a uniform easy-axis, e.g., $\mathbf{e}_i = \hat{\mathbf{z}}$) and the coplanar Kagome antiferromagnet (AFM) with 120° spin order ($J < 0$ and $K > 0$). In the coplanar Kagome AFM model, the easy-axis directions are sublattice dependent and collinear to the spin directions shown in Fig. 2(e). In the main text, we focus on the Hamiltonian of Eq. (1) without considering the external magnetic field and the Dzyaloshinskii-Moriya interaction (DMI), resulting in vanishing magnon Berry curvatures across the entire momentum space. The effects of the magnetic field and the DMI are discussed in the Supplemental Material.

The ground states of both FM and AFM systems possess no SSC. Our interest is in the thermally excited SSC on top of these ground states and their Nernst transports. Since elementary excitations of the considered magnetic systems are magnons, it is natural to describe the SSC in terms of the magnon operators. By employing the Holstein-Primakoff approach, we obtain the magnon dispersions of the FM and the AFM models, which are shown in Figs. 2(b) and 2(f), respectively. For the collinear FM, we use the model parameter of $\text{V}_3\text{Br}_8\text{Li}$: $J = 1.13$ meV, $K = 0.152$ meV, and $S = 5/2$ [46]. For the coplanar AFM, we simply flip the sign of the exchange constant ($J = -1.13$ meV), while keeping the same values of

K and S as in the collinear FM. In both models, the magnon spectra consist of two dispersive bands (ϵ_1 and ϵ_2) and a flat band ϵ_0 , with no band gap between the different bands. For the Kagome FM, the flat band is located at the top of the band dispersion [$\epsilon_0^{\text{FM}} = \epsilon_2^{\text{FM}}(\Gamma) = 6JS + 2KS$], while for the coplanar AFM it is located at the bottom of the band dispersion [$\epsilon_0^{\text{AFM}} = \epsilon_1^{\text{AFM}}(\Gamma) = S\sqrt{2K(3J + 2K)}$]. In both models, the ground state and the magnon Hamiltonians are invariant under the spatial inversion and the combined operation of time reversal and a spin rotation by 180° around the axis perpendicular to the ground state spin direction. As a result, both models exhibit vanishing magnon Berry curvature in the entire Brillouin zone, leading to the absence of the magnon Hall effect.

Scalar spin chirality Nernst effect. Despite the trivial magnon topology with vanishing magnon Berry curvature everywhere, here we demonstrate that our models exhibit an intrinsic Nernst effect of the magnon SSC, namely the magnon SSC Nernst effect. The SSC operator is defined as the counterclockwise triple products around the upper and lower triangular unit cells: $\hat{\chi} = \hat{\mathbf{S}}^A \cdot (\hat{\mathbf{S}}^B \times \hat{\mathbf{S}}^C)$ and $\hat{\bar{\chi}} = \hat{\mathbf{S}}^{A'} \cdot (\hat{\mathbf{S}}^{B'} \times \hat{\mathbf{S}}^{C'})$ [see Figs. 2(a) and 2(e)]. Because the Hamiltonian is time-reversal symmetric and the SSC operator is invariant under cyclic permutations, the equilibrium SSC profiles $\chi_{n,\mathbf{k}} = \langle n | \hat{\chi}_{\mathbf{k}} | n \rangle$ and $\bar{\chi}_{n,\mathbf{k}} = \langle n | \hat{\bar{\chi}}_{\mathbf{k}} | n \rangle$, where $|n\rangle$ is n th eigenstate of magnons, are odd functions in the momentum space with C_3 rotational symmetry (see the Supplemental Material). Consequently, their integrals over the Brillouin zone, i.e., the SSCs of upper and lower triangles, vanish in equilibrium. Additionally, it is important to note that $\bar{\chi}_{n,\mathbf{k}} = \chi_{n,-\mathbf{k}}$ due to the fact that the upper and lower triangles are connected by spatial inversion. Combining this with $\chi_{n,-\mathbf{k}} = -\chi_{n,\mathbf{k}}$, we deduce that the two spin chiralities are opposite, yielding $\bar{\chi}_{n,\mathbf{k}} = -\chi_{n,\mathbf{k}}$. Therefore, when summing over the upper and lower triangles, the momentum profile of the net SSC, $\chi_{n,\mathbf{k}}^{\text{net}} \equiv \chi_{n,\mathbf{k}} + \bar{\chi}_{n,\mathbf{k}}$, cancels out and vanishes in equilibrium.

Although the net SSC profile vanishes in equilibrium, a nonequilibrium bias might generate intrinsic SSC Hall transport through the interband superpositions. To check this possibility, we perform a heuristic calculation of the momentum-specific nonequilibrium magnon SSC density induced by the temperature gradient along the x direction using the Kubo response formalism as described in Ref. [47]. Because $\chi_{n,\mathbf{k}}^{\text{net}} = 0$, the intraband contribution, representing the distribution shift effect of $\chi_{n,\mathbf{k}}^{\text{net}}$ vanishes. However, the interband contribution of the nonequilibrium magnon SSC from the two triangles is identical and does not cancel out. In Figs. 2(c) and 2(g), we present the momentum-resolved nonequilibrium SSC ($\delta\chi_{n,\mathbf{k}}$) for the lowest magnonic state (see the Supplemental Material for calculation details and the nonequilibrium magnon SSC profiles of all three magnonic states). The opposite values of $\delta\chi_{n,\mathbf{k}}$ between positive and negative k'_y s indicate that the nonequilibrium states with positive and negative SSC propagate along the transverse direction with opposite velocities, hinting at the potential existence of the magnon SSC Hall transport.

To investigate the magnon SSC Nernst response rigorously by going beyond the aforementioned heuristic approach, we invoke the generalized Berry curvature method [48] for the SSC, $\Omega_{n,\mathbf{k}}^{\chi(\bar{\chi})}$, which is directly related to the SSC Hall-type

conductivities (see the Supplemental Material). We note that the SSC Berry curvature is not associated with a topologically protected integer; rather, it merely serves as a measure of the Nernst response of the magnon SSC within the Kubo response framework. Although the magnon Berry curvature is zero everywhere, the off-diagonal elements $\langle n | \hat{\chi} | m \rangle$ ($m \neq n$) allow the finite magnon SSC Berry curvatures via interband superpositions. The calculated magnon SSC Berry curvatures exhibit a broad peak around the Γ point, where a dispersive band (ϵ_1 or ϵ_2) intersects with the flat band (ϵ_0), and relatively a narrower peak around the K point, where the two dispersive bands converge [see Figs. 2(d) and 2(h)]. At the high symmetry Γ and K points, the magnon SSC Berry curvature diverges due to the band touching. To suppress the numerical divergence, we introduce a small level broadening term that does not affect the resulting value of magnon SSC Nernst conductivities. Notably, since $\bar{\chi}$ and χ are related by spatial inversion, magnon SSC Berry curvatures of upper and lower triangles exhibit identical behavior, $\Omega_{n,\mathbf{k}}^{\chi} = \Omega_{n,\mathbf{k}}^{\bar{\chi}}$, when time-reversal symmetry is present ($D = 0$) (see the Supplemental Material). This indicates that the net magnon SSC Nernst transport does not vanish when averaging over the upper and lower triangles.

The generalized magnon SSC Berry curvature induces a transverse current of the magnon SSC in the presence of a temperature gradient $J_y^x = -\alpha_{xy}^x \partial_x T$ [48,49]. Here, $\alpha_{xy}^x = \frac{k_B}{\hbar V} \sum_{n,\mathbf{k}} c_1(\rho_n) \Omega_{n,\mathbf{k}}^{\chi}$ is the Nernst conductivity of the magnon SSC, where k_B is the Boltzmann constant and $c_1(\rho) = (1 + \rho) \ln(1 + \rho) - \rho \ln \rho$. In Fig. 3, we illustrate the temperature dependence of the magnon SSC Nernst conductivity. This is our main result: the Kagome FM and AFM exhibit the magnon SSC Nernst transport even though the magnon Berry curvature is zero everywhere. In both models, the magnon SSC Nernst conductivity decreases as K decreases [Figs. 3(a) and 3(c)], as a reduction in K lowers the energy of the magnon bands and increases magnon occupation at a given temperature. Notably, in contrast to the collinear FM model, the flat band energy in the coplanar AFM model lies at the bottom of the band structure [Figs. 2(b) and 2(f)], leading to a sharp increase in the SSC Nernst conductivity as K decreases. Additionally, we emphasize that the magnon SSC Nernst conductivity remains finite in both models even in the $K \rightarrow 0$ limit. This indicates that the magnon SSC Nernst effect is a distinctive feature of the Kagome magnets, stemming exclusively from the exchange interaction and the lattice geometry.

In the Supplemental Material, we also calculate the DMI effect on the magnon SSC Nernst conductivity in both models. For the Kagome FM, the influence of the DMI on the magnon SSC Nernst conductivity can be summarized as follows. At low temperatures, where the contribution is primarily from the lowest band, the magnon SSC Nernst conductivity is minimally influenced by the DMI coefficient. However, as the temperature increases and the contribution from the higher bands becomes dominant, the conductivity undergoes a sign reversal. Notably, the sign of the DMI coefficient does not affect the magnon SSC Nernst conductivity from the perspective of symmetry. However, for the Kagome AFM, the magnon SSC Nernst conductivity either increases or decreases depending on the sign of the DMI coefficient, due to the

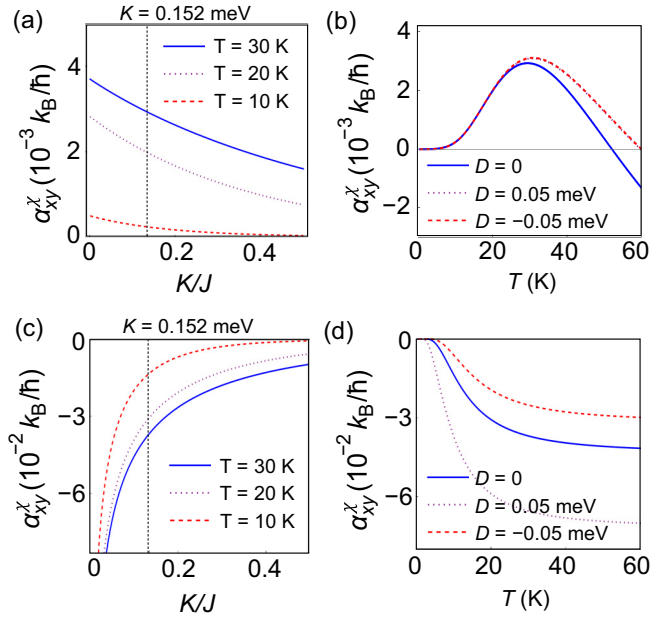


FIG. 3. SSC Nernst conductivity. (a) Easy-axis anisotropy, and (b) temperature, dependence of the SSC Nernst conductivity in the Kagome FM model. (c) Easy-axis anisotropy, and (d) temperature dependence of the SSC Nernst conductivity in the Kagome AFM model. The symbol D in (b) and (d) is the out-of-plane DMI parameter (see the Supplemental Material). The dashed lines in (a) and (c) represent $K = 0.152$ meV.

DMI's influence on the flat band position (see the Supplemental Material).

Atomistic simulation results. To confirm our theoretical prediction of the magnon SSC Nernst effect, we perform atomistic spin dynamics simulations by solving the stochastic Landau-Lifshitz-Gilbert equation (see the Supplemental Material). Our simulation setup involves a rectangular stripe of a Kagome lattice with a finite width in the y direction and periodic boundary conditions along the longer x direction. The simulations are conducted with a nonuniform temperature profile shown in Fig. 4(a). The ensemble average of the net SSC accumulation for Kagome FMs and AFMs are shown in Figs. 4(b) and 4(c), respectively, where the red and blue dots represent positive and negative values of the net SSC.

We observe that the opposite SSC accumulation occurs at both edges of the sample in regions with a finite temperature gradient. Note that Kagome FM and AFM models show the opposite signs of the SSC edge accumulations. This sign difference of the two models agrees with our theoretical results that the two models possess the opposite signs of the SSC Nernst conductivities as shown in Fig. 3. Our numerical simulation confirms that a temperature gradient along the x direction induces the SSC accumulation due to the transverse SSC current flowing along the y direction. In the Supplemental Material, we show that the individual-site canting angles, which correspond to local S_z , offer no direct insight into SSC—they merely reflect the temperature profile and boundary conditions. Instead, they demonstrate that both the magnon Hall and spin Nernst effects vanish in our model because there is no asymmetric spin accumulation along the y direction.

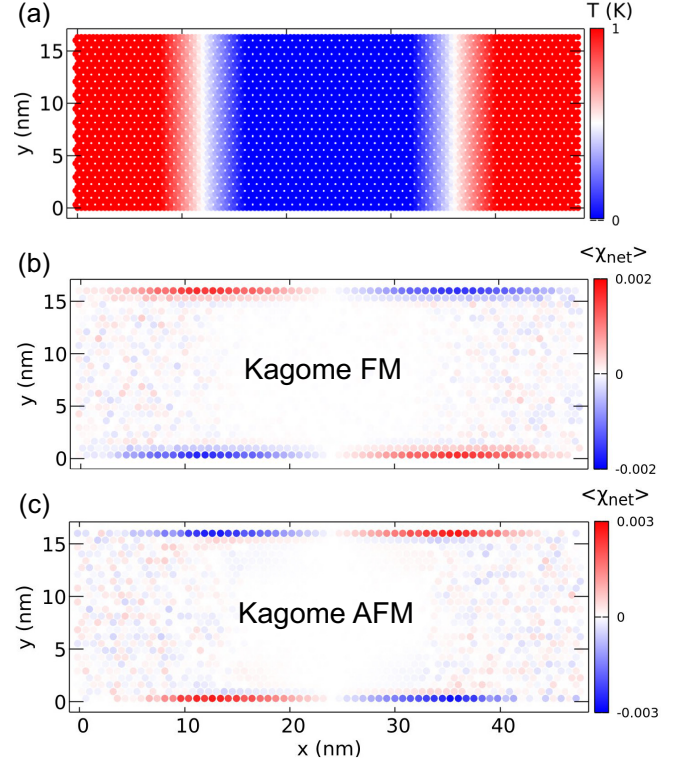


FIG. 4. Atomistic simulation results. (a) Temperature profile. (b), (c) net SSC accumulation profiles for Kagome FM and AFM, respectively. The red and blue dots in (b) and (c) represent positive and negative net SSC, respectively.

We note that the observed noise in Figs. 4(b) and 4(c) originates from thermal effects, as the Langevin dynamics of spins introduce random fluctuations. However, the absolute magnitude of the noise is only about ten percent of the maximum SSC accumulation, making it possible to distinguish the noise from the SSC signal in experimental studies.

For a rough quantitative estimation of the SSC accumulation at finite temperatures in bulk Kagome magnets, we use the solution of the drift-diffusion equation for the SSC, $\chi(y) = \alpha_{xy}^x \tau \sinh(\frac{W-2y}{2\lambda}) / (\lambda \cosh(\frac{W}{2\lambda})) \partial_x T$, where W is the width of the sample, λ and τ are the SSC diffusion length and relaxation time, respectively. For chosen parameters, $\alpha_{xy}^x = 10^{-2} k_B/\hbar$, $W = 20$ nm, $\partial_x T = 10$ K/ μm , $\lambda = 1$ nm, and $\tau = 0.1$ ns. The resulting SSC accumulation at the edge is $\chi \sim 1 \times 10^{-3}/\text{cell}$ (see the Supplemental Material for SSC profiles by using other values of λ and τ).

Discussion. In this work, we have predicted the magnon SSC Nernst effect in Kagome FM and AFM models. Remarkably, this effect emerges even in the complete absence of the Berry curvature in Kagome magnets, in sharp contrast to the previously studied magnonic Hall-like phenomena that rely on a finite Berry curvature. The validity of our prediction is further supported by numerical simulations of finite-temperature spin dynamics. While we have employed a temperature gradient to drive the SSC transport, we note that an alternative mechanism, such as spin Hall effect-induced spin current injection, may also be utilized to pump the SSC [50,51]. We have further verified that the SSC Nernst effect is present in honeycomb lattice magnets with

vanishing magnon Berry curvature (see the Supplemental Material), but is absent in triangular lattice magnets, highlighting the need for a classification scheme based on lattice geometry.

For the potential experimental schemes for detecting the magnon SSC, we examine the electron orbital magnetization induced by the magnon SSC. It is known that the magnon SSC induces the so-called topological orbital moment [2–7], which is given phenomenologically by $L = \kappa^{\text{TO}} \chi$ with κ^{TO} the topological orbital susceptibility. To compute this, we utilize a simple tight-binding model as described in Ref. [7], and apply the modern theory of electron orbital magnetization [52–54] in the presence of a noncollinear spin texture due to the magnon SSC. For $\kappa^{\text{TO}} = 0.1 \mu_B$ with parameters used in previous section, the maximum value of the topological orbital magnetization is estimated to be on the order of $10^{-4} \mu_B/\text{cell}$ (see the Supplemental Material), which is comparable to the orbital accumulation in a light metal Ti that has been observed successfully by using magneto-optical Kerr effect [30]. Additionally, we remark that it is plausible to consider that the accumulation of the nonequilibrium SSC at the edge can induce an electromotive force in a proximate normal metal [51], making it detectable through an electrical measurement scheme. Furthermore, the spin chiralities in bulk materials can be measured by neutron scattering [55–58], the resonant inelastic x-ray scattering [59,60], and the topological magneto-optical effects [61].

Here, we note that the SSC Nernst effect bears qualitative similarities to the skyrmion Hall effect [62–64], as both

phenomena involve the transverse transport of noncollinear localized spin textures, their physical origins are distinct. The skyrmion Hall effect describes the transverse motion of stable solitons within a continuum theory, independent of sublattice degrees of freedom. In contrast, the magnon SSC Nernst effect arises from thermally excited magnons, mediated by interband superposition (magnon SSC Berry curvature), and is intrinsically tied to the sublattice structure (see the Supplemental Materials [65] for the details). Consequently, the SSC Nernst effect does not occur in triangular or square lattices. Nevertheless, the possibility of a deeper connection between these two phenomena within a unified transport framework remains an open question.

By lifting the common assumption that the SSC is a static quantity in this work, we have uncovered one transport phenomenon, the magnon SSC Nernst effect. We anticipate that our findings will stimulate further exploration of SSC-driven transport, advancing the understanding of chiral phenomena where both active and passive roles of the SSC are treated on equal footing.

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Correction: A contact author footnote was missing at publication and has been added.