

Generalized parity measurements and efficient large multicomponent cat state preparation with quantum signal processing

Sina Zeytinoglu^{*}

*Physics and Informatics Laboratory, NTT Research, Inc., 940 Stewart Drive, Sunnyvale, California 94085, USA
and Institute of Theoretical Physics, TU Wien, Wiedner Hauptstraße 8-10/136, A-1040 Vienna, Austria*



(Received 28 July 2024; revised 24 July 2025; accepted 13 March 2026; published 26 May 2026)

Generalized parity measurements are instrumental for the preparation of nontrivial quantum states and the detection of errors in error-correction codes. However, realizing such measurements in current experimental platforms becomes prohibitively difficult with growing system size. Here, using algorithmic methods of quantum signal processing, we detail a simple and efficient control protocol for implementing generalized parity measurements. Crucially, our measurement protocol does not depend on the size of the system and implements the desired measurement in constant time. We benchmark the performance of the proposed measurement protocol in preparing large multicomponent cat states on superconducting circuit quantum electrodynamics platforms. We show that a 20-component cat state with 400 photons can be prepared with success probability $>2\%$ and fidelity $\approx 90\%$ limited by the cavity decay rate and higher-order qubit-cavity coupling. Our results demonstrate the feasibility of generalized parity measurements in current bosonic and spin systems, paving the way to improvements in state-preparation and error-correction capabilities.

DOI: [10.1103/xsgf-txnq](https://doi.org/10.1103/xsgf-txnq)

I. INTRODUCTION

Measurements are of great importance for quantum technologies. On the one hand, measurements can be used to generate [1–4] and verify [5,6] entanglement in quantum systems. On the other hand, measurements allow experimentalists to act on a quantum system conditionally on the measurement outcomes. This feedback loop is at the heart of efforts to control quantum mechanical systems [7,8], with applications ranging from system identification [9] to quantum error correction [10,11], and universal information processing paradigms such as measurement-based quantum computation [12–15].

A class of measurements that are particularly important for a broad range of technological applications are generalized parity measurements (GPMs), whose binary outcomes determine whether the number of excitations in the measured system is an integer multiple of a modulus $r \in \mathbb{N}$ up to a remainder $k \in \mathbb{N}$. GPMs broaden the scope of conventional parity measurements (i.e., $r = 2$ [3]), which play a crucial role in current efforts in resource state preparation and error detection for quantum error correction [16–24]. Initial work in Ref. [1] showed how GPMs can be used to prepare entangled states such as the multiqubit Greenberger–Horne–Zeilinger (GHZ) and W states, and other Dicke states, which are

of interest for quantum communication applications [1,25], metrology [26–29], and variational quantum optimization algorithms [30,31]. GPMs were brought back into focus by new and exciting applications in error-correcting codes encoded in bosonic [32–35] and qubit systems [36], as well as applications in photon-number-resolved measurements [37], a crucial ingredient in boson sampling [38] and quantum simulation [39,40].

Despite promising use cases and recent developments in the control theory of bosonic systems with so-called SNAP gates and displacement operators [41,42], the realization of GPMs on near-term platforms is hampered by the difficulty of their implementation, especially in larger systems that hold the most promise for technological applications. Current proposals for implementing GPMs based on circuit or SNAP gate constructions either require a frequency comb [1,32,43–46] or utilize a single continuous-time control field, whose time dependence should be optimized through a suitable cost function [37,47–51]. The former approach becomes technically challenging with growing system size because the number of frequency peaks over a fixed measurement bandwidth increases with the system size. The latter optimal control approach circumvents frequency crowding but, like most open-loop control algorithms [50–52], suffers from two important limitations that become more severe for large systems: (1) The classical computational cost of finding an optimal control protocol becomes prohibitive as the system size and the number of control parameters grow and (2) the optimized control protocol lacks a discernible structure that yields itself to generalization. For the particular example of GPMs, problem (1) entails that it becomes more difficult to implement the desired measurement on a system with a large number of excitations, and problem (2) entails that the optimization of the control protocol should be repeated for each r and k

^{*}Contact author: sinazeytinoglu@hotmail.com,
sina.zeytinoglu@tuwien.ac.at

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independently [37]. Resolving these two limitations would improve the scalability and feasibility of GPMs, dramatically increasing their impact on near-term quantum technologies.

Here, we resolve the above-mentioned limitations by developing a useful formal connection between control protocols that implement GPMs and parametrized circuits developed for the design of efficient quantum algorithms, often referred to as quantum signal processing (QSP) [53–56]. We utilize QSP as a methodological bridge between pulse-level and circuit-level control approaches for designing binary-outcome quantum measurements, to obtain a simple and experimentally relevant control protocol whose efficiency is comparable to the best quantum algorithms designed for an all-to-all coupled universal quantum computer [30]. Most importantly, the use of algorithmic circuits allows us to address the difficulties associated with open-loop optimal control approaches. In particular, the parametrization of QSP circuits is independent of the size of the measured system. Leveraging this structure, we discover *analytical expressions* that describe control protocols to implement *any* GPM in *constant* time, independently of the system size, r , and k .

The proposed approach is applicable to bosonic or spin systems, where the number of excitations is conserved and a single ancillary qubit is coupled to the excitation-number operator [57–64]. We numerically demonstrate the effectiveness of our approach in preparing >90% fidelity multicomponent cat states [32,65] of up to 400 photons on the current superconducting circuit quantum electrodynamics (cQED) platforms and discuss the main obstacles in front of preparing states with even larger photon numbers. We thus demonstrate how QSP methods can drastically reduce the difficulty of designing quantum control protocols, paving the way to high-fidelity nonclassical states with hundreds of excitations.

II. GENERALIZED PARITY MEASUREMENTS (GPMs) FROM QSP PROTOCOLS

Formally, generalized parity measurements $\text{GP}_{r,k}$ are projective measurements whose binary outcome projects the system $\mathcal{S}$ onto or out of a subspace $\mathcal{S}_{r,k}$ spanned by states with m excitations, where $m \in \mathbb{N}$ obeys the modular congruence relation $m = k \pmod{r}$, with moduli r and remainder k . In the following, we denote the two projective measurement operators associated with $\text{GP}_{r,k}$ as $\Pi_{\mathcal{S}_{r,k}}$ and $\Pi_{\bar{\mathcal{S}}_{r,k}}$, where $\bar{\mathcal{S}}_{r,k}$ is the complement of $\mathcal{S}_{r,k}$.

Naimark's dilation theorem [66,67] implies that any binary-outcome quantum measurement on $\mathcal{S}$ can be implemented through a unitary that acts on the extended system $\mathcal{Q} + \mathcal{S}$, where $\mathcal{Q}$ is a qubit, followed by a projective measurement on $\mathcal{Q}$. For $\text{GP}_{r,k}$, the suitable unitary is

$$\hat{U}_{r,k} \equiv \mathbf{I}_{\mathcal{Q}} \otimes \Pi_{\mathcal{S}_{r,k}} + iX \otimes \Pi_{\bar{\mathcal{S}}_{r,k}}. \quad (1)$$

Here, $\mathbf{I}_{\mathcal{Q}}$ is the identity operator acting on $\mathcal{Q}$ and Pauli operators X and Z also act on $\mathcal{Q}$. Intuitively, $\hat{U}_{r,k}$ flips the state of $\mathcal{Q}$ only for the component of $\mathcal{S}$ that is not in $\mathcal{S}_{r,k}$. Thus, initiating $\mathcal{Q}$ in the state $|0\rangle$, applying $\hat{U}_{r,k}$, and measuring $\mathcal{Q}$ on the computational basis implements a quantum channel with the measurement operators [67–69]

$$M_{0(1)}(\hat{U}_{r,k}) \equiv (|0(1)\rangle \otimes \mathbf{I}_{\mathcal{S}}) \hat{U}_{r,k} (|0\rangle \otimes \mathbf{I}_{\mathcal{S}}) = \Pi_{\mathcal{S}_{r,k}(\bar{\mathcal{S}}_{r,k})}, \quad (2)$$

where the subscripts 0 and 1 denote the relevant measurement outcome of $\text{GP}_{r,k}$, as desired. The main result of this work is an analytical expression that describes a control protocol that implements a good approximation of $\hat{U}_{r,k}$ (1) independently of the size of $\mathcal{S}$, (2) for any r and k , and (3) in constant time.

Let us elaborate on our approach to synthesizing $\hat{U}_{r,k}$. Quantum signal processing has become a general framework for designing optimal implementations of quantum algorithms [53–56]. Despite its expressiveness, the core of QSP can be understood by considering a composite pulse sequence that acts on a single qubit [53,70]. To this end, consider the following single-qubit parametrized unitary of depth d [53,71,72]:

$$U_{\vec{\Phi}}(\theta) \equiv e^{i\Phi_0 X} e^{i\theta Z} e^{i\Phi_1 X} e^{i\theta Z} \dots e^{i\theta Z} e^{i\Phi_d X}, \quad (3)$$

where $\theta \in \mathbb{R}$ and $\Phi_j \equiv (\vec{\Phi})_j \in \mathbb{R}$ is a so-called processing phase associated with the processing unitary $e^{i\Phi_j X}$ [70]. Most importantly for this work, the fundamental theorem of QSP says that $U_{\vec{\Phi}}(\theta)$ has the representation

$$U_{\vec{\Phi}}(\theta) \doteq \begin{pmatrix} P_{\vec{\Phi}}(e^{i\theta}) & iQ_{\vec{\Phi}}(e^{i\theta}) \\ iQ_{\vec{\Phi}}(e^{-i\theta}) & P_{\vec{\Phi}}(e^{-i\theta}) \end{pmatrix}, \quad (4)$$

where P and Q are real Laurent polynomials whose coefficients can be directly related to $\vec{\Phi}$ [53,73,74].

It is straightforward to establish the relationship between Eq. (3) and the desired unitary $\hat{U}_{r,k}$ in Eq. (1). To this end, we define the parametrized unitary $\tilde{U}_{\vec{\Phi}}$ that acts on $\mathcal{Q} + \mathcal{S}$ and has the same form as the single-qubit unitary in Eq. (3) except the following replacements:

$$e^{i\theta Z} \leftarrow V(\hat{m} - k, r) \equiv \exp \left[-i \frac{\pi}{r} Z \otimes (\hat{m} - k) \right], \quad (5)$$

where $\hat{m}$ is the excitation-number operator for $\mathcal{S}$. The coefficient π/r ensures that the circuit is invariant (up to an overall sign) when the number of excitations is shifted by a multiple of r [i.e., $\tilde{U}_{\vec{\Phi}}(\hat{m} - k, r) = \tilde{U}_{\vec{\Phi}}(\hat{m} - k + r, r)$], a feature required by the modular equivalence relation defining $\mathcal{S}_{r,k}$. Figure 1 depicts the schematic circuit of the control protocol implementing $\tilde{U}_{\vec{\Phi}}$. Then, given an eigenstate $|\lambda_m\rangle$ of $\hat{m}$ with an eigenvalue m , the following relation is easily verified:

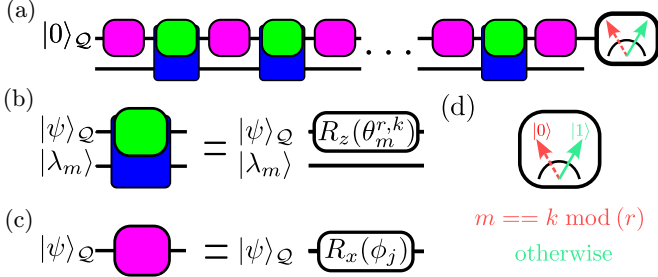
$$\tilde{U}_{\vec{\Phi}}(\hat{m} - k, r)(\mathbf{I} \otimes |\lambda_m\rangle) = U_{\vec{\Phi}}(\theta_m^{r,k}) \otimes |\lambda_m\rangle, \quad (6)$$

where $\theta_m^{r,k} \equiv \frac{\pi}{r}(m - k)$.

Thus, in order for the measurement operator $M_0(\tilde{U}_{\vec{\Phi}})$ [defined as in Eq. (2)] to approximate $\Pi_{\mathcal{S}_{r,k}}$, it is sufficient to find QSP processing phases $\vec{\Phi}_r$ that satisfy

$$P_0^{r,k}(m) \equiv \langle 0 | U_{\vec{\Phi}_r}(\theta_m^{r,k}) | 0 \rangle \approx \begin{cases} 1, & m - k = 0 \pmod{r}, \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

Yet, the phase sequence $\vec{\Phi}_r$ does not immediately guarantee that the measurement operator $M_1(\tilde{U}_{\vec{\Phi}_r})$ approximates $\Pi_{\bar{\mathcal{S}}_{r,k}}$. Instead, unitarity of $\tilde{U}_{\vec{\Phi}_r}$ entails that in general $M_1(\tilde{U}_{\vec{\Phi}_r})|\lambda_m\rangle \approx e^{i\eta_m}|\lambda_m\rangle$ for $m - k \neq 0 \pmod{r}$. However, it is straightforward to cancel this excitation-number-dependent phase factor $e^{i\eta_m}$, using two consecutive measurements without requiring resetting or feedback [75,76] (see Supplemental Material [74]).



III. FINDING THE OPTIMAL QSP PROTOCOLS

Here, we use the mathematical results from the QSP literature to design constant-time control protocols that satisfy the condition in Eq. (7). The real-valuedness condition in Eq. (7) can be achieved by considering symmetric QSP phase sequences $\vec{\Phi}_{\text{sym}} = (\phi_0, \phi_1, \dots, \phi_1, \phi_0)$ [73]. Then, it is easily verified that $U_{\vec{\Phi}_{\text{sym}}}^T(\theta) = U_{\vec{\Phi}_{\text{sym}}}(\theta)$ and $Q^*(e^{i\theta}) = Q(e^{i\theta})$. Consequently, the diagonal elements of the following single-qubit unitary are real-valued:

$$U_{\vec{\Phi}_{\text{sym}}}^{\mathbf{R}}(\theta) \equiv iXU_{\vec{\Phi}_{\text{sym}}}(\theta) = U_{\frac{\pi}{4}\hat{e}_0 + \vec{\Phi}_{\text{sym}}}(\theta), \quad (8)$$

where $\hat{e}_0 \equiv (1, 0, \dots, 0)$. We denote the control protocol obtained through the replacement in Eq. (5) as $\tilde{U}_{\vec{\Phi}_{\text{sym}}}^{\mathbf{R}}$.

An additional advantage of restricting ourselves to symmetric QSP protocols is that the optimization problem associated with finding a symmetric $\vec{\Phi}_r$, see Supplemental Material Ref. [74] is well conditioned around the initial condition $\vec{\Phi}_0 = (0, \dots, 0)$ [73,77]. By fitting the optimal phase sequences $\vec{\Phi}_r$ to trigonometric functions, we find simple analytical expressions for $\vec{\Phi}_r$ that satisfy Eq. (7). Denoting $\vec{\Phi}_r = (\phi_r^{\text{edge}}, \vec{\Phi}_r^{\text{bulk}}, \phi_r^{\text{edge}})$, we find

$$(\vec{\Phi}_r^{\text{bulk}})_j = \phi_r^{\text{bulk}}(x_j) = \frac{4}{\pi r} \left[1 + \frac{1}{2} \cos\left(\frac{4}{r}x_j\right) \right],$$

$$\phi_r^{\text{edge}} = \frac{1}{2} \left[\frac{\pi}{2} - \sum_{j=1}^{\bar{r}} (\vec{\Phi}_r^{\text{bulk}})_j \right], \quad (9)$$

where $x_j = j - \lceil \frac{\bar{r}}{2} \rceil$ with $j \in \{1, \dots, \bar{r}\}$ and $\bar{r} \equiv 2 \times \lceil \frac{\bar{r}}{2} \rceil - 1$. The functions $\langle 0|U_{\vec{\Phi}_r}^{\mathbf{R}}(\theta)|0\rangle$ for $r = 8, 13$, and 41 are shown in Fig. 2(a). As indicated by the vertical dashed lines, $\langle 0|U_{\vec{\Phi}_r}^{\mathbf{R}}(\theta)|0\rangle$ approaches 0 for $\theta \approx \theta_m^{r,0}$ when m is not a multiple of r . The eigenvalues $P_0^{r,0}(m)$ are depicted in Fig. 2(b) for $r = 12, 13$ and as a function of the number of excitations $m <$

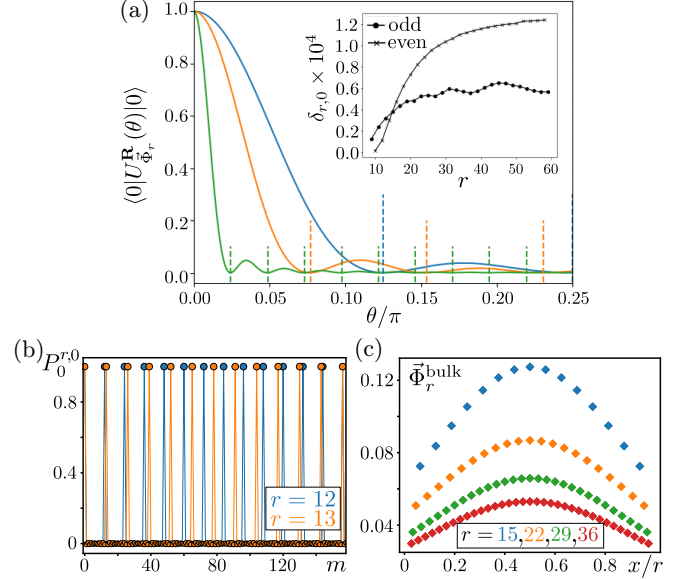


FIG. 2. (a) The polynomial transformation $\langle 0|U_{\vec{\Phi}_r}^{\mathbf{R}}(\theta)|0\rangle$ for $r = 8, 13$, and 41 (indicated by green, orange, and blue lines) using the approximate QSP angles in Eq. (9). The vertical dashed lines in matching colors indicate the values of θ corresponding to $\theta_m^{r,k}$. Inset: The approximation error $\delta_{r,0} \times 10^4$ for $6 < r < 60$ (see main text). Ideally, $P_0^{r,k}(m)$ should take the value 1 for $m = k \bmod r$ and 0 for $m \neq k \bmod r$. (b) The eigenvalues $P_0^{r,0}(m)$ [see Eq. (7)] for $m \in [0, 160]$ and $r = 12$ (in blue) and $r = 13$ (in red). The eigenvalues $P_0^{12,0}$ and $P_0^{13,0}$ closely approximate 1 when the number m of excitations is a multiple of 12 and 13, respectively. Both eigenvalues are approximately 1 when $m = 156 = 12 \times 13$. (c) The QSP phases $\vec{\Phi}_r^{\text{bulk}}$ that implement $GP_{r,k}$, given by the analytical function in Eq. (9).

160. That $P_0^{r,k}(m) = 1$ for $m = k \bmod r$ can be understood as a result of the sum condition $\Phi_r^{\text{tot}} \equiv \sum_{j=1}^{\bar{r}+2} (\vec{\Phi}_r)_j = \pi/2$ [see Eq. (9)]. Because $\bar{r}$ is odd, this sum rule implies $U_{\vec{\Phi}_r}^{\mathbf{R}}(\theta \in \{0, \pi\}) = \mathbf{I}$. That is, $\mathcal{Q}$ stays in its initial state for all $\theta_m^{r,k} = 0, \pi \bmod 2\pi$, a condition satisfied for $m = rn + k$ for all integers n . Overall, the optimized phases $\vec{\Phi}_r$ approximate the desired target function in Eq. (7) with very small trace infidelity $\delta_{r,k}$, see Supplemental Material Ref. [74], as shown in the inset of Fig. 2(a). Finally, we plot the phases $\vec{\Phi}_r^{\text{bulk}}$ in Fig. 2(b) to display their regular and symmetric structure.

Crucially, given access to an evolution generated by $\chi Z \otimes \hat{m}$, the time required to implement the generalized parity measurement $GP_{r,k}$ does not scale with the number of excitations or r . This result follows directly from the facts that (1) the sum Φ_r^{tot} is independent of r and (2) the time required to implement $\bar{r}$ repetitions of $V(\hat{m} - k, r)$ is also independent of r . Thus, naively, the time to implement $GP_{r,k}$ scales only as $O(k)$ as a result of the time required to implement the single-qubit rotations $e^{i\frac{\pi}{2}kZ}$ in Eq. (5). However, the desired single-qubit rotations along the z axis can be implemented virtually (i.e., without any time cost) by changing the phase of the qubit drive [78]. As a result, the proposed implementation of GPMs completes in constant time that is determined only by the interaction strength χ .

A. Mitigating coherent control errors through sequential GPMs

The effect of coherent control errors on the implemented measurement operators can be analyzed using the subadditivity of coherent errors [79]. In compliance with common experiments, we consider multiplicative errors on the processing phases of the form $\phi_j \leftarrow \phi_j(1 + \epsilon_{\phi,j})$ and additive timing errors of the form $\Delta t_j \leftarrow \pi/(r\chi) + \epsilon_{t,j}/\chi$ due to the jitter of the control clock [80]. Denoting the erroneous unitary $\tilde{U}_{\Phi_r}^{\mathbf{R}}$ and introducing $m_{\max}$ as the maximum number of excitations relevant for a given application, see Supplemental Material Ref. [74], it is straightforward to show that

$$|\tilde{U}_{\Phi_r}^{\mathbf{R}} - \tilde{U}_{\Phi_r}^{\mathbf{R}}| \leq m_{\max}(\bar{r} + 1)|\epsilon_{t,\max}| + \pi/2|\epsilon_{\phi,\max}|, \quad (10)$$

where $|\cdot|$ denotes the matrix norm [79], and $\epsilon_{\phi,\max}$ and $\epsilon_{t,\max}$ are the maximum phase and timing errors, respectively. Thus, our protocol is especially robust against errors in the processing phases, while the timing errors become increasingly important as we increase r and $m_{\max}$. However, the worsening effect of $\epsilon_{t,j}$ with increasing r can be mitigated for certain large r by using an adaptive sequence of s GPMs $\{\text{GP}_{r_i,k_i}\}$, where r_i and k_i are informed by the Chinese remainder theorem [81,82]. We emphasize that high-fidelity implementations of sequential $\text{GP}_{r,k}$ enable excitation-number measurements, see Supplemental Material Ref. [74] and optimal Fock state preparation [83] (see Supplementary Material [74]).

IV. MULTI-COMPONENT CAT-STATE PREPARATION

We evaluate the performance of the proposed QSP-based implementation of $\text{GP}_{l,k}$ to prepare $\bar{n}$ -excitation l -component cat states $|\phi_{\text{cat}}^{l,k,\bar{n}}\rangle$, defined as [65,84,85]

$$|\phi_{\text{cat}}^{l,k,\bar{n}}\rangle \equiv \frac{\Pi_{S_{l,k}}|\alpha = \sqrt{\bar{n}}\rangle}{\sqrt{p_{\text{succ}}^0}}, \quad (11)$$

where $|\alpha = \sqrt{\bar{n}}\rangle$ is a coherent state of a spin or a bosonic system with an average of $\bar{n}$ excitations. Note that the normalization constant is related to the probability p_{succ} of successfully preparing a cat state given that we can implement $\Pi_{S_{r,k}}$ as a measurement operator, see Supplemental Material Ref. [74].

We study a concrete implementation of the cat state preparation protocol on a superconducting circuit QED device. In order to implement the unitary $\tilde{U}_{\Phi_l}^{\mathbf{R}}$ in Eq. (8), we can use the native qubit-cavity coupling Hamiltonian $H_{\chi} \equiv \chi Z \otimes n_C$ in the highly dispersive regime [46,86] with n_C denoting the cavity-photon-number operator. For each state-preparation trial, we reduce the unwanted effects of errors on the ancilla qubit Q by accepting the result only if three consecutive GPMs succeed.

Although realizing implementation of GPMs seems straightforward at first sight, when the number of photons is large, realizing the desired dynamics becomes difficult due to multiple sources of unwanted coherent processes. A crucial obstacle to the implementation of the unitary $\tilde{U}_{\Phi_l}$ is the requirement to cancel the always-on qubit-cavity interactions during the implementation of the QSP processing unitaries. While H_{χ} can be switched off using additional drives [87],

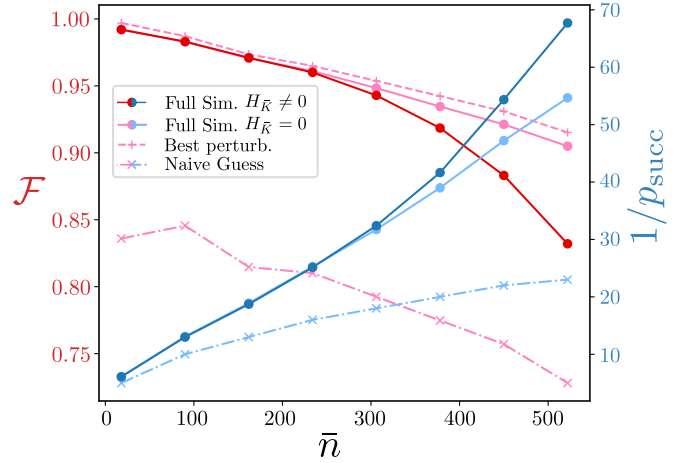


FIG. 3. The fidelity $\mathcal{F}$ and the inverse success probability $1/p_{\text{succ}}$, for a successful three-times repeated $\bar{n}$ photon $[\sqrt{\bar{n}}]$ -component cat state-preparation protocol. The full lines correspond to full master equation simulations, with red and pink (blue and light blue) full disks representing the data resulting from simulations with $H_{\bar{K}} \neq 0$ and $H_{\bar{K}} = 0$, respectively. By repeating the state-preparation protocol three times, we obtain a fidelity that closely approximates the perturbative result that takes into account only cavity decay (pink dashed line). For $\bar{n} \leq 300$, substantially improving the naive guess for the fidelity of a one-time repeated state-preparation protocol (pink dotted-dotted line). The maximal inverse success probability $1/p_{\text{succ}}^0$ for preparing a $[\sqrt{\bar{n}}]$ -component $\bar{n}$ -photon cat state (blue dotted-dashed line) scales with the square root of the number of legs (see main text). Repeating the state preparation three times reduces p_{succ}^0 for $\bar{n} = 378$ by half.

our numerical results indicate that the adverse effects of always-on H_{χ} can be drastically reduced through the following replacement of the single-qubit processing unitary:

$$e^{i\phi_j X} \leftarrow e^{i(\phi_j X - \bar{n}\chi t_{\phi_j} Z)}, \quad (12)$$

where t_{ϕ_j} is the time required to implement $e^{i\phi_j X}$. In our simulations, we observe that this replacement does not result in a significant error for nonclassical photonic states with at least up to 500 photons (see the pink full line in Fig. 3). The replacement in Eq. (12) is especially effective because the photon-number variance of the initial coherent state is $\propto \bar{n}$.

Another source of coherent errors is the presence of the always-on qubit-state-dependent Kerr nonlinearity $H_{\bar{K}} \propto \bar{K}\sigma_z n_C^2$, where $\bar{K} \ll \chi$ [58,74,88]. The effects of this nonlinear coupling can also be reduced by modifying the single-qubit rotation in the same way as in Eq. (12). However, $H_{\bar{K}}$ also introduces coherent errors in the implementation of V in Eq. (5). These errors are the main limitation of our QSP-based cat-state-preparation protocol. We find that, for the parameters we chose, the effect of the always-on $H_{\bar{K}}$ becomes relevant for $\bar{n} > 350$ (see Fig. 3). Finally, we note that the effective Kerr nonlinearity $H_K \propto n_C^2$ commutes with all components of the QSP-based GPM. Therefore, the evolution due to Kerr nonlinearity can be canceled before or after implementing $\text{GP}_{r,k}$ [88].

In Fig. 3, we present the inverse success probability p_{succ}^{-1} and the fidelity $\mathcal{F} \equiv \sqrt{\langle \phi_{\text{des}} | \sigma_{\text{final}} | \phi_{\text{des}} \rangle}$ of the heralded state σ_{final} , in the presence of all relevant decoherence channels

in the superconducting cQED setting. For simplicity, we consider only the preparation of $\lceil\sqrt{\bar{n}}\rceil$ -component cat states with average photon number $\bar{n}$. The most important system parameters for understanding our results are the cavity lifetime T_c and the qubit-cavity coupling parameter χ . For the results presented here, we chose $T_c = 25$ ms [89] and $\chi = 2\pi \times 41$ kHz [46] (see Supplemental Material Ref. [74] for all system parameters). Because the clock jitter of arbitrary waveform generators can be reduced down to 50 ps [80], we neglect the timing errors in our simulations up to 500 photons.

We find that when the proposed state-preparation scheme is repeated three times, it allows us to prepare 20-component cat states of $\bar{n} = 378$ photons with $\mathcal{F} \approx 92\%$ and $p_{\text{succ}} = 2.4\%$. Our numerical results also show that the fidelity of the state preparation does not change substantially when the number of cat state components is increased, even in the presence of coherent and incoherent errors. In particular, we show that the photon-number state with $\bar{n} = 376$ photons can be prepared with $\approx 94\%$ fidelity and $\approx 1.1\%$ success probability by applying our protocol for $\text{GP}_{r,k}$ with $r = 94$ and $k = 0$ to an initial coherent state with $\bar{n} = 378$ photons, see Supplemental Material Ref. [74]. We emphasize that the success probability of the Fock state-preparation protocol scales only as $1/\sqrt{\bar{n}}$, see Supplemental Material Ref. [74], and the time required to implement the GPM is constant with respect to $\bar{n}$. As a result, assuming that the time required to initialize the cavity field to a coherent state with the desired $\bar{n}$ scales as $\sqrt{\bar{n}}$, the total time to prepare the Fock state scales as $O(\bar{n})$, matching the optimum scaling conjectured for an all-to-all coupled quantum computer in Ref. [90], and is consistent with the constant excitation ground-state-preparation protocol in Ref. [91].

Interestingly, the state-preparation infidelities obtained by applying the proposed GPM three times are about half the naive guess using the cavity decay rate. To explain this observation, we develop a perturbative expansion of the stochastic wavefunction trajectories [74,92]. Our scheme tracks the modification of the dissipation channels as a result of the time-dependent QSP protocol used to implement the GPM in a similar way to the input-state-dependent error probability introduced in Ref. [93]. As we further detail in the Supplementary Material, the perturbative analysis shows that the state-preparation protocol is more likely to fail when a single error occurs. Thus, the increase in fidelity comes at the cost of a small decrease in the success probability. Our analysis shows

that the observed error robustness of the state-preparation protocol can be understood as a trade-off between fidelity and success probability.

V. CONCLUSIONS AND OUTLOOK

In this article, we presented a way of leveraging the efficient resource usage of the algorithmic framework of QSP to (1) prescribe concrete control protocols to efficiently implement generalized parity measurements and (2) nondeterministically prepare large multicomponent cat states with high fidelity. Further developments in reducing the cavity decay rate [94] and canceling unwanted coherent terms can greatly improve the state-preparation results in the superconducting cQED setting. Our results demonstrate the potential of using algorithmic frameworks like QSP to design concrete control protocols for near-term experimental setups that allow access to constant-time implementations of $e^{-itZ \otimes H_S}$, such as circuit acoustodynamics [57], and other cavity QED settings [59,95], spin systems consisting of neutral atoms [60–62,96], trapped ions [63], and nitrogen vacancy centers [64]. Implementation of the proposed methods in these platforms will require the mitigation of a wide range of system-specific error mechanisms (e.g., the inhomogeneous broadening in the spin-based systems), and thus will benefit greatly from insights into filter design [71,97] and closed-loop optimal control [98]. Extensions of our approach to systems where $\mathcal{Q}$ and $\mathcal{S}$ are coupled through Jaynes-Cummings [56,99] and Rabi interactions can shorten the measurement time and allow better control of GKP error-correcting codes [100,101], respectively.

ACKNOWLEDGMENTS

The author is deeply grateful for enlightening discussions with Marek Pechal on the experimental implementation of generalized parity measurements in circuit QED and with Isaac Chuang, Arkopal Dutt, and Sho Sugiura on quantum signal processing during the initial stages of this project. The author acknowledges the financial and technical support of NTT PHI Labs.

DATA AVAILABILITY

A minimal code for realizing the circuit QED simulation can be found at [102]

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