

Fundamental limits on polarization entanglement distribution in optical fiber

Stefano Pirandola Department of Computer Science, *University of York*, York YO10 5GH, United Kingdom

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Characterizing the ultimate rates of entanglement distribution is essential for both foundational research and the practical deployment of quantum technologies. To investigate these limits, we introduce an erasure-Pauli channel model describing the distribution of polarization entanglement in optical fiber. For this channel, we derive bounds on the rates of entanglement distribution and related quantum resources under optimal local operations and two-way classical communication (two-way assisted capacities). This framework allows us to determine the optimal repeaterless performance achievable over realistic optical fibers affected by polarization mode dispersion, thereby providing a rigorous benchmark for long-distance polarization-based quantum communication. Finally, we show that both our model and capacity bounds remain robust under the inclusion of detector dark counts.

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I. INTRODUCTION

The distribution of entanglement is a fundamental primitive for a wide range of quantum technologies, including quantum communication and security [1–5], quantum sensing [6,7], and distributed quantum computing [8–13]. Among the various physical implementations, polarization qubits transmitted through optical fiber represent one of the most practical and scalable platforms for long-distance entanglement distribution [14,15]. Here, an important open question is to determine the ultimate rates at which polarization-based entanglement can be distributed when the dominant decoherence mechanisms introduced by the optical fiber and the detection system are taken into account. This is the aim of this work.

We provide a general model for this type of distribution, that we call “erasure-Pauli” channel. This is defined as a flagged mixture of photon loss and a Pauli polarization channel. While erasure channels and Pauli channels have been extensively studied [10], their explicit combination in this form does not appear to have been previously analyzed in full generality [16]. For this channel, we bound the ultimate rates for distributing entanglement, transmitting quantum information, and generating secret keys. In the specific case where the model reduces to an erasure-dephasing channel, these bounds provide the exact formula of the two-way assisted capacities.

We then apply our formulas to establish the ultimate rates for distributing polarization in optical fiber, where Alice prepares a Bell pair in polarization, keeps one optical qubit, and sends the other to Bob through a standard optical fiber. In this setting, polarization mode dispersion (PMD) is the main cause of decoherence [22–24]. In this setting, we identify two

main regimes: one that is depolarizing dominated, strongly limited to short distances, and another one that is dephasing dominated, enabled by active polarization control and able to achieve much longer distances. For the latter, we study the performance of the two-way entanglement distribution capacity and show that this ultimate rate is very robust to the presence of dark counts in the receiving setup.

II. ERASURE-PAULI CHANNEL

Let us discuss a general type of quantum channel that models loss and noise in fiber-based distribution of entanglement with polarization qubits. Here, we need to account for two main effects: (1) the loss of the optical carrier (with a signal or loss event detectable by the receiver) and (2) the errors induced on the polarization degrees of freedom of the surviving carriers. This process can be modeled by an erasure-Pauli channel. In this general model, the state ρ of an optical polarization qubit undergoes the erasure-Pauli transformation:

$$\mathcal{E}_{\text{EP}}(\rho) = (1 - \eta)|e\rangle\langle e| + \eta\mathcal{P}(\rho), \quad (1)$$

where η is the transmissivity (probability that the photon is received), $|e\rangle$ is an orthogonal vacuum state (flagging a no detection event at the receiver), and $\mathcal{P}$ is a Pauli channel acting on the polarization of the transmitted qubits (see Fig. 1). Recall that the Kraus representation of a qubit Pauli channel $\mathcal{P}$ can be written as [10]

$$\mathcal{P}(\rho) = \sum_{k=0}^3 p_k P_k \rho P_k^\dagger, \quad (2)$$

where $\mathbf{p} := \{p_k\}$ is a probability distribution over the Pauli errors $P_k \in \{I, X, Y, Z\}$, with I the 2×2 identity.

III. CAPACITY BOUNDS

Let us bound the ultimate rates for entanglement distribution, quantum information transmission, and secret-key

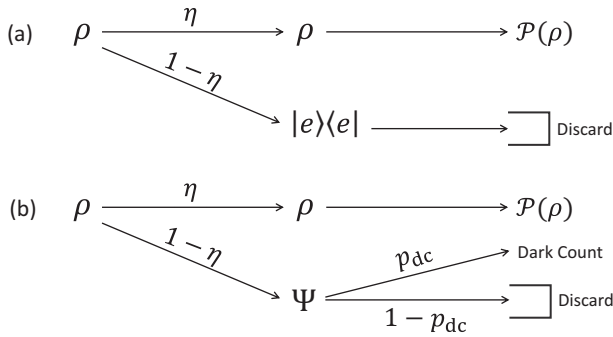


FIG. 1. Schematic of an erasure-Pauli channel in panel (a) and an erasure-Pauli channel with dark counts in panel (b).

agreement over an erasure-Pauli channel with generic transmissivity η and error distribution $\mathbf{p}$. These rates are provided by the various two-way assisted capacities, i.e., the two-way assisted entanglement distribution capacity D_2 , the two-way assisted quantum capacity Q_2 , and the two-way assisted secret-key capacity K_2 [25,26]. For simplicity, let us denote by $\mathcal{C}_2$ the generic two-way assisted capacity, which could represent D_2 , Q_2 , or K_2 . While our capacity bounds apply generally to all the considered communication tasks, in the following sections, we focus on their application to entanglement distribution over optical fiber.

For an erasure-Pauli channel $\mathcal{E}_{EP}$, we show the bounds

$$\eta[1 - H(\mathbf{p})] \leq \mathcal{C}_2(\mathcal{E}_{EP}) \leq \eta\Phi, \quad (3)$$

where $H(\mathbf{p})$ is the Shannon entropy over the Pauli error distribution $\mathbf{p} := \{p_k\}$ and

$$\Phi := \begin{cases} 1 - H_2(p_{\max}), & \text{if } p_{\max} \geq \frac{1}{2} \\ 0, & \text{otherwise,} \end{cases} \quad (4)$$

where H_2 is the binary Shannon entropy computed over the largest Pauli error $p_{\max} := \max_k \{p_k\}$.

Proof. For the direct part (lower bound), we note that the parties may implement the following entanglement distribution strategy based on backward classical communication. Assume that Alice sends a total of N systems to Bob. When Bob receives the flag state $|e\rangle\langle e|$, Bob discards the system; otherwise, he keeps it. This operation can be implemented by Bob applying a dichotomic positive operator-valued measure (POVM) $\{|e\rangle\langle e|, I - |e\rangle\langle e|\}$ on each received system and informing Alice that instances were correctly received. On the remaining ηN systems, each affected by the Pauli channel $\mathcal{P}$, Alice and Bob perform an optimal two-way assisted entanglement distillation protocol. For large N , they can distill $D_2(\mathcal{P})$ ebits per surviving system. Therefore, $D_2(\mathcal{E}_{EP}) \geq \eta D_2(\mathcal{P})$. From Ref. [25], we know that $D_2(\mathcal{P}) \geq 1 - H(\mathbf{p})$, where $H(\mathbf{p})$ is the Shannon entropy over the Pauli probability distribution. Therefore,

$$K_2(\mathcal{E}_{EP}) \geq Q_2(\mathcal{E}_{EP}) = D_2(\mathcal{E}_{EP}) \geq \eta[1 - H(\mathbf{p})]. \quad (5)$$

For the converse part (upper bound), we note that we can rewrite the erasure-Pauli channel as a channel ensemble $\{\pi_i, \mathcal{E}_i\}$, so that the state undergoes the Pauli channel $\mathcal{E}_0 = \mathcal{P}$ with probability $\pi_0 = \eta$, and a complete erasure (CE) channel $\mathcal{E}_1 = \mathcal{E}_{CE}$ with probability $\pi_1 = 1 - \eta$. This is

$\mathcal{E}_{CE}(\rho) = |e\rangle\langle e|$ for any input state ρ . Because these two channels are teleportation stretchable with resource Choi states σ_i , we can write the following upper bound for the two-way assisted capacity of the ensemble [18,25]:

$$\mathcal{C}_2(\{\pi_i, \mathcal{E}_i\}) \leq \sum_i \pi_i E_R(\sigma_i), \quad (6)$$

where E_R is the relative entropy of entanglement [27–29]. Because for the CE channel we have $E_R(\sigma_1) = 0$ (entanglement breaking) and for the Pauli channel we have $E_R(\sigma_0) = \Phi$ as in Eq. (4) (see also Ref. [25]), we find

$$\mathcal{C}_2(\mathcal{E}_{EP}) = \mathcal{C}_2(\{\pi_i, \mathcal{E}_i\}) \leq \eta\Phi. \quad (7)$$

The bounds in Eq. (3) can be simplified for specific distributions of the Pauli errors. In fact, for isotropic errors $\mathbf{p} = \{1 - 3p/4, p/4, p/4, p/4\}$, we have an erasure-depolarizing channel $\mathcal{E}_{EDP}$ with probability p . This channel takes the form as in Eq. (1) but where $\mathcal{P}$ simplifies to $\mathcal{P}(\rho) = (1 - p)\rho + pI/2$. Then, for Z errors only, i.e., for $\mathbf{p} = \{1 - p, 0, 0, p\}$, we have an erasure-dephasing channel $\mathcal{E}_{EDH}$ with probability p . This is given by Eq. (1), with $\mathcal{P}$ reducing to $\mathcal{P}(\rho) = (1 - p)\rho + pZ\rho Z$. For these specific channels, we easily derive [30]

$$\mathcal{C}_2(\mathcal{E}_{EDP}) \leq \begin{cases} \eta[1 - H_2(3p/4)], & \text{if } p \leq 2/3, \\ 0, & \text{otherwise,} \end{cases} \quad (8)$$

$$\mathcal{C}_2(\mathcal{E}_{EDH}) = \eta[1 - H_2(p)]. \quad (9)$$

It is important to identify the necessary and sufficient condition for which the two-way assisted capacity is zero. For an erasure-Pauli channel, we may write

$$\mathcal{C}_2(\mathcal{E}_{EP}) = 0 \iff \eta = 0 \quad \text{OR} \quad p_{\max} \leq 1/2. \quad (10)$$

For the case of an erasure-depolarizing channel, we therefore have $\mathcal{C}_2(\mathcal{E}_{EDP}) = 0$ if and only if $\eta = 0$ or $p \geq 2/3$. For the erasure dephasing, $\mathcal{C}_2(\mathcal{E}_{EDH}) = 0$ if and only if $\eta = 0$ (no transmission) or $p = 1/2$ (complete dephasing).

The proof of Eq. (10) is easy. In fact, the necessary condition ($\Leftarrow$) trivially comes from Eqs. (3) and (4). For the sufficient condition ($\Rightarrow$), note that the Choi state of the erasure-Pauli channel is

$$\sigma_{\mathcal{E}_{EP}} = (1 - \eta)\tau_e + \eta\sigma_{\mathcal{P}}, \quad (11)$$

where $\tau_e = (I/2) \otimes |e\rangle\langle e|$ and $\sigma_{\mathcal{P}}$ is the Choi state of the Pauli channel. The state $\sigma_{\mathcal{E}_{EP}}$ is Alice and Bob's joint output state when Alice transmits one qubit of a Bell pair. Assuming that Bob performs the POVM $\{|e\rangle\langle e|, I - |e\rangle\langle e|\}$ on his qubit, he will project on $\sigma_{\mathcal{P}}$ with probability η . Conditionally on successful projection, they will then share distillable entanglement $E_D(\sigma_{\mathcal{P}}) > 0$ for $p_{\max} > 1/2$. In fact, the Choi state of a Pauli channel has nonpositive partial transposition [31] for $p_{\max} > 1/2$ and this is equivalent to distillability in dimension 2×2 . Therefore, on average, the parties will share $\eta E_D(\sigma_{\mathcal{P}}) > 0$ ebits. So if $\eta > 0$ and $p_{\max} > 1/2$, we have $\mathcal{C}_2(\mathcal{E}_{EP}) \geq \eta E_D(\sigma_{\mathcal{P}}) > 0$. This statement is equivalent to showing the direct implication in Eq. (10).

IV. POLARIZATION NOISE MODELING

The dominant noise mechanism affecting polarization-based entanglement distribution in optical fiber is PMD. This originates from random birefringence caused by structural imperfections and environmental fluctuations along the fiber. It induces stochastic unitary polarization rotations and, for photons with finite spectral bandwidth, leads to effective phase decoherence due to differential group delay between polarization modes [24].

In general, this noise mechanism can be modeled as a Pauli channel acting on the polarization degrees of freedom, with a probability distribution that depends on the length and features of the fiber. Including the loss mechanism associated with the propagation of the carrier, the overall channel is therefore an erasure-Pauli channel.

The basic type of PMD is a depolarizing-dominated Pauli model, with an isotropic error distribution in X , Z , and Y . This regime corresponds to considering an overall erasure-depolarizing channel, where the transmissivity is given by the usual formula $\eta(d) = 10^{-\alpha d/10}$, where d is the fiber distance in km and α is the loss rate in dB/km. The depolarizing probability can be expressed as [32]

$$p(d) = p_\infty(1 - e^{-d/L}), \quad (12)$$

where p_∞ is the floor value (equal to 1 for complete depolarization) and L is the polarization decoherence length. Distribution is not possible for $p(d) \geq 2/3$, corresponding to a maximum fibre distance of $d_{\max} = L \ln 3$. In the depolarizing-dominated Pauli model, L has a characteristic scale of 10–100 m, so the range for entanglement distribution is very limited.

Luckily, the effect of PMD can be mitigated with active polarization control [33]. Active polarization control suppresses the random unitary rotations, leaving PMD-induced phase decoherence as the dominant mechanism. This is a dephasing-dominated Pauli model that is characterized by a much larger value of the decoherence length $L = L_{\text{DH}}$. We can write [34]

$$L_{\text{DH}} = \frac{2\tau^2}{D_{\text{PMD}}^2}, \quad (13)$$

where $\tau \simeq (2\pi\Delta\nu)^{-1}$ is the photon coherence time (with $\Delta\nu$ being the bandwidth) and D_{PMD} is the PMD coefficient in ps/ $\sqrt{\text{km}}$. In telecom fibers, $D_{\text{PMD}} \simeq 0.01$ – 10 ps/ $\sqrt{\text{km}}$ [24]. So, for a bandwidth $\Delta\nu = 100$ GHz ($\tau \simeq 1.6$ ps), we have a range $L_{\text{DH}} \simeq 0.05$ – 5×10^4 km. The dephasing probability can be written as in Eq. (12) but with the different floor value $p_\infty = 1/2$ (equivalent to complete dephasing). In principle, this model would allow for entanglement distribution at any distance, since the threshold $p(d) = 1/2$ corresponds to $L_{\text{DH}}^{\max} = \infty$.

Given a fiber with transmissivity $\eta(d)$ and PMD probability $p(d)$, we can express the previous bounds in terms of the distance d . Here, we are interested in the best performance, so we consider the dephasing-dominated regime. In Fig. 2, we plot the two-way assisted capacity of the erasure-dephasing channel. For typical parameters, we can see that the distribution of polarization entanglement is indeed feasible with high rates at long distances. For example, with a clock of 1 GHz and the parameters considered in Fig. 2, we see that Alice and

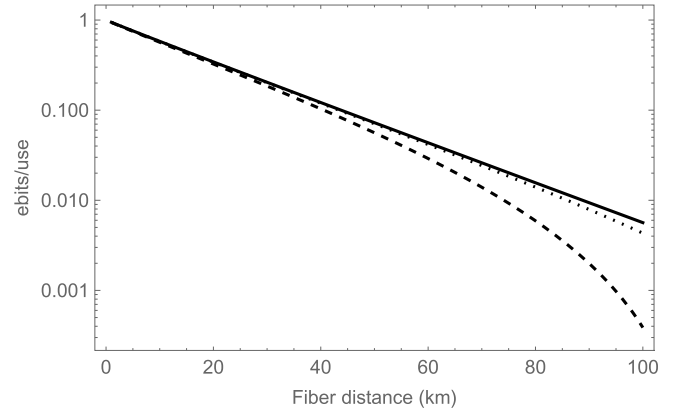


FIG. 2. We plot the capacity $C_2(\mathcal{E}_{\text{EDH}})$ in ebits/use of the erasure-dephasing channel vs fiber distance in km (solid line). $\mathcal{E}_{\text{EDH}}$ describes entanglement distribution in fiber with active polarization control (dephasing-dominated regime). We also plot an upper bound for $\mathcal{E}_{\text{EDH}}^{\text{dc}}$, which accounts for dark counts, with $p_{\text{dc}} = 10^{-2}$ (dashed line) and $p_{\text{dc}} = 10^{-3}$ (dotted line). Fiber parameters are $\alpha = 0.2$ dB/km, $\Delta\nu = 100$ GHz, and $D_{\text{PMD}} = 0.1$ ps/ $\sqrt{\text{km}}$.

Bob can share about 5×10^6 ebits per second at 100 km of fiber.

V. ERASURE PAULI WITH DARK COUNTS

We can extend the previous model to incorporate receiver-side imperfections, most notably detector dark counts. In this more realistic setting, a no-photon (erasure) event may trigger a spurious detection click, which is then assigned a random polarization outcome. Operationally, this converts part of the erasure component into an effective depolarizing contribution, thereby increasing the overall Pauli error probability.

In the presence of dark counts with probability $p_{\text{dc}} > 0$, the erasure-Pauli channel becomes (see also Fig. 1)

$$\mathcal{E}_{\text{EP}}^{\text{dc}}(\rho) = (1 - \eta)\Psi + \eta\mathcal{P}(\rho), \quad (14)$$

$$\Psi := (1 - p_{\text{dc}})|e\rangle\langle e| + p_{\text{dc}}I/2. \quad (15)$$

This channel induces a postselected (click-conditioned) effective qubit channel

$$\mathcal{E}_{\text{click}}(\rho) = \frac{\eta\mathcal{P}(\rho) + (1 - \eta)p_{\text{dc}}I/2}{\eta + (1 - \eta)p_{\text{dc}}}, \quad (16)$$

which closely resembles the formula of the quantum bit error rate in quantum key distribution [5].

It is important to note that we may rewrite Eq. (15) as an equivalent erasure-Pauli channel as follows:

$$\mathcal{E}_{\text{EP}}^{\text{dc}}(\rho) = (1 - \eta')|e\rangle\langle e| + \eta'\mathcal{P}'(\rho), \quad (17)$$

where we have defined the effective transmissivity

$$\eta' := \eta + (1 - \eta)p_{\text{dc}} = 1 - (1 - \eta)(1 - p_{\text{dc}}) \quad (18)$$

and the effective Pauli channel $\mathcal{P}'$ with distribution

$$p'_k = rp_k + (1 - r)/4, \quad r := \frac{\eta}{\eta'}. \quad (19)$$

It is clear that Eqs. (3), (4), and (10) are extended to dark counts by replacing $\eta \rightarrow \eta'$ and $\mathbf{p} = \{p_k\} \rightarrow \mathbf{p}' = \{p'_k\}$.

In the depolarizing regime, we derive Eq. (17), where $\mathcal{P}'$ is a depolarizing channel with probability $p' = 1 - r(1 - p)$, so we extend Eq. (8), and the associated threshold, up to replacing $\eta \rightarrow \eta'$ and $\mathbf{p} \rightarrow \mathbf{p}'$. However, in the dephasing regime, the original dephasing channel is transformed into a Pauli channel $\mathcal{P}'$ with probability distribution as in the form of Eq. (19). As a result, instead of Eq. (9), we apply the more general Eqs. (3), (4), and (10). These equations will govern the performance of the erasure-dephasing channel with dark counts $\mathcal{E}_{\text{EDH}}^{\text{dc}}$.

It is interesting to study the effect that dark counts have on the long-distance entanglement distribution. In Fig. 2, we see that the upper bound for $\mathcal{E}_{\text{EDH}}^{\text{dc}}$ is not far from the capacity in the absence of dark counts. In fact, the deviation becomes evident for longer distances and high values for the dark count probability, such as 10^{-2} , as in the figure. For smaller values of dark counts, on the order of 10^{-3} or less, the deviation becomes negligible.

VI. CONCLUSIONS

We have introduced the model of erasure-Pauli channel, including some specific forms it may take when the noise has symmetry (depolarizing or dephasing). For this general channel, we have established fundamental bounds for the tasks of entanglement distribution, quantum information transmission, and secret-key distribution. We have then applied the model to the distribution of polarization entanglement in optical fiber, discussing two main regimes of operation, depolarizing dominated and dephasing dominated.

A key novelty of our work is the formulation of a unified information-theoretical model that combines photon loss, polarization decoherence, and detector dark

counts within a single quantum-channel description. This enables a capacity-based treatment of realistic polarization entanglement distribution in optical fiber, going beyond previous analyses that considered these impairments separately or only at the level of detection statistics.

In the presence of active polarization control, where the PMD regime is dephasing dominated, we found that high rates can be achieved over long fiber lengths. We also verify that this result is robust to the presence of dark counts in the receiving station by suitably modifying the erasure-Pauli model to account for these extra imperfections. Overall, our results provide an information-theoretical framework to quantify ultimate distance and rate limits for polarization entanglement distribution in optical fiber, clarifying when repeaterless architectures remain viable or need to be assisted by repeaters.

As future work, it would be interesting to explore the limits of entanglement distribution in free-space paths, where the performance is much more limited due to geometric diffraction, pointing errors, and turbulence.

ACKNOWLEDGMENT

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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- [26] The two-way assisted capacities represent the optimal rates that are achievable for tasks such as qubit transmission (Q_2), entanglement distribution (D_2), and key generation (K_2). More precisely, these are the maximum rates at which qubits, ebits, or key bits can be distributed over a quantum channel, assuming that Alice and Bob do not have preshared entanglement but can perform the most general local operations assisted by two-way classical communication. For example, in a QKD scenario, this means that Bob can send classical information to Alice on how to prepare the quantum systems at the input of the channel, and Alice can send classical information to Bob on how to measure them at the output. These exchanges of information are potentially unlimited. The one described represents a repeaterless scenario, i.e., a communication configuration where Alice and Bob are directly connected by the intermediate quantum channel without any third party in the middle. In contrast, in a repeater-based scenario, the communication between Alice and Bob would be mediated by a middle relay or repeater, say Charlie, connected to Alice via channel $\mathcal{E}_1$ and to Bob via channel $\mathcal{E}_2$. In this case, the end-to-end rate between Alice and Bob could exceed the value of a repeaterless rate achieved over a direct quantum channel $\mathcal{E} = \mathcal{E}_2 \circ \mathcal{E}_1$, where Charlie does not intervene.
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- $$\rho = \begin{pmatrix} \rho_{HH} & \rho_{HV} \\ \rho_{VH} & \rho_{VV} \end{pmatrix}, \quad (20)$$
- where $\rho_{HV} = \langle H|\rho|V\rangle$ is the off-diagonal polarization coherence. Under first-order PMD, polarization coherence decays as $\rho_{HV} \rightarrow e^{-d/L} \rho_{HV}$. A depolarizing channel acts on the off-diagonal term as $\rho_{HV} \rightarrow (1-p)\rho_{HV}$, while a dephasing channel acts as $\rho_{HV} \rightarrow (1-2p)\rho_{HV}$. Equating these coherence-reduction factors to the PMD case above, we find
- $$1 - p/p_\infty = e^{-d/L}, \quad (21)$$
- where $p_\infty = 1(1/2)$ for depolarizing (dephasing).
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