

Tailoring interaction ranges in atom arrays

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We introduce a method to synthetically engineer the range of dipolar interactions in tweezer atom arrays by effectively modifying the electromagnetic environment with far-detuned relay atoms. We derive equations of motion for the atoms of interest after adiabatic elimination of the relay atoms. We show the effectiveness of the scheme for realistic experimental parameter regimes with circular and low-angular-momentum Rydberg-atom states.

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Rydberg-atom arrays are a powerful platform for quantum simulation and computation [1–4]. Their ability to scale to large system sizes [5–9], maintain long coherence times [7,10–14], and achieve reconfigurable lattice geometries [14–18] makes them well suited for studying complex quantum many-body phenomena. Analog quantum simulations [2,16,19–21] have provided insights into quantum magnetism [22–25], symmetry-protected topological phases [26], coherent excitation transfer [27–30], emergent gauge fields [31–34], and many-body localization [35,36]. Circular Rydberg atoms (CRAs), i.e., states with maximal angular momentum, already used in cavity quantum electrodynamics [37] and quantum sensing [38,39], are promising for quantum simulation and computation [40–42] due to their long lifetimes [43].

Tailoring interactions is a challenge for analog quantum simulators. Ion-trap platforms have demonstrated variable-range interactions in the long-range regime, characterized by a power-law decay ($\sim 1/r^b$) with exponents between $0 \lesssim b \lesssim 3$ [44–50]. In Rydberg atoms, exponents are generally fixed by near-field dipole-dipole couplings, $1/r^3$ [1,51], or to an effective van der Waals scaling of $1/r^6$. Recent advances have shown potential for tunability, including modified pair interactions [52,53], controlled excitation transport [29], and enhanced superradiance in hybrid Rydberg-cavity systems [54]. The ability to engineer interaction ranges in the regimes described here opens unique opportunities: It allows to probe information propagation under long-range couplings, e.g., the emergence and shape of light cones (Lieb-Robinson bounds) [55–57] in regimes previously inaccessible [58–61]. Furthermore, it offers possibilities of observing nontrivial topological phases [62] and of studying effects based on the interplay

between long-range couplings and disorder, including rich quantum spin-glass physics and slow nonequilibrium dynamics [63–65].

In this Letter, we propose a method to engineer an effective variable-range spin exchange model, with an effective power-law behavior in the range $3 \lesssim b \lesssim 6$. We achieve this by modifying the electromagnetic environment with far-detuned relay atoms. Through adiabatic elimination of relay atoms, we demonstrate that the interaction can be finely tuned and depends solely on their position. We demonstrate the feasibility of the scheme with an array of CRAs [66] with far-detuned low-angular-momentum Rydberg states as relays.

Setup. We propose a synthetic implementation of the spin-1/2 model Hamiltonian

$$\hat{H}^{\text{eff}} = \sum_{i < j} J_{ij} (\hat{\sigma}_i^{\dagger} \hat{\sigma}_j + \text{H.c.}), \quad (1)$$

with a tunable coupling matrix J_{ij} between spins $i, j = 1, \dots, N$. Here, $\hat{\sigma}_i |\uparrow\rangle_i = |\downarrow\rangle_i$ is the lowering operator for spin i . We consider all spins to be represented by two internal atomic states of N “main atoms” and denote their transition frequency as ω . In our scheme, the tunability of the coupling matrix is achieved by placing N_R “relay atoms” in the vicinity of the main atoms, as sketched in Fig. 1. The total atom number is $M = N + N_R$. The relay atoms are modeled as two-level systems with states $|\alpha\rangle_{\mu}$ and $|\beta\rangle_{\mu}$ (as convention, we use the two indices μ, ν for the range $\mu, \nu = N + 1, \dots, M$) and transition frequency ω_R . In a specific implementation below, we will use two consecutive circular Rydberg states as main atoms, and two low-angular-momentum Rydberg levels as relay atom levels. The position of an atom a is given by the vector $\mathbf{r}_a$. The dipole-dipole coupling depends on atomic distances $r_{ab} = |\mathbf{r}_a - \mathbf{r}_b|$ and the relative orientation of the dipoles, here parametrized by the angles θ_{ab} between the connection vectors $\mathbf{r}_a - \mathbf{r}_b$ and the quantization (z) axis set by static fields (see Fig. 1).

We focus on a regime with atomic distances much smaller than transition wavelengths, i.e., near-field dipole-dipole interactions dominate. We consider dipole-allowed transitions

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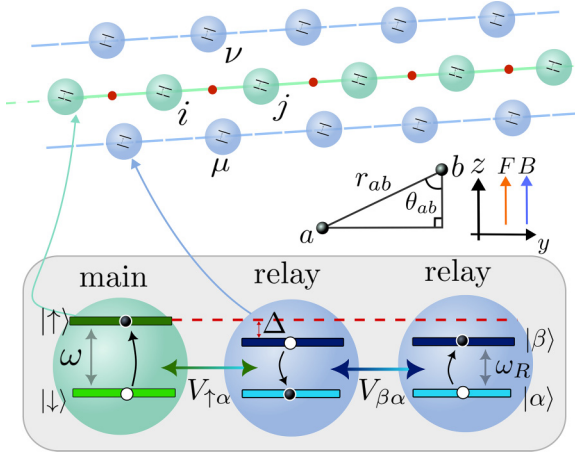


FIG. 1. Setup: N main atoms (green, $i, j = 1, \dots, N$) are placed on a line; relay atoms (blue, $\mu, \nu = N + 1, \dots, M$) are located symmetrically above and below (see text for detailed parametrization). We define the pairwise atom spacing r_{ab} and angles θ_{ab} as sketched. A static electric (magnetic) field F (B) points along the z axis. Gray box: For main (relay) atoms, we consider two-level systems with states $|\downarrow\rangle, |\uparrow\rangle$ ($|\alpha\rangle, |\beta\rangle$) and spacing ω (ω_R). We include dipole-dipole couplings between all atoms: main-relay and relay-relay; however, main-main couplings vanish in a “magic angle” configuration with $\cos \theta_{ij} = \sqrt{1/3}$.

between the states $|\downarrow\rangle_i \leftrightarrow |\uparrow\rangle_i$ and $|\alpha\rangle_\mu \leftrightarrow |\beta\rangle_\mu$, which lead to coherent energy transfer between all atoms by virtual photon exchange. This is governed by the pairwise Hamiltonian $\hat{H}_{mn'}^{\text{DD}} = \sum_{a,b} V_{(mn)(m'n')}(r_{ab}, \theta_{ab})(\hat{\sigma}_{mn}^a \hat{\sigma}_{m'n'}^b + \text{H.c.})$ with $\hat{\sigma}_{mn}^a = |m\rangle\langle n|_a$ [67]. Here, we define $\hat{\sigma}_{mn}^a$ for the different internal state combinations $(mn)(m'n') \in \{(\uparrow\downarrow)(\uparrow\downarrow), (\uparrow\downarrow)(\beta\alpha), (\beta\alpha)(\beta\alpha)\}$ with corresponding index ranges $a, b = 1, \dots, N$ for main and $a, b = N + 1, \dots, M$ for relay atoms, respectively. For conciseness, we shorten the notation $(mn)(m'n')$ to mn' . In our convention, $V_{(mn')}(r_{ab}, \theta_{ab}) = C_3^{(mn')}(3 \cos^2(\theta_{ab}) - 1)/\xi_{ab}^3$, with $\xi_{ab} = 2\pi r_{ab}/\lambda_{mn}$ for a transition wavelength λ_{mn} and with constants $C_3^{(mn')}$ depending on the transition dipole moments.

Our scheme depends on the detuning between the main and the relay atoms (as further explained in the paragraph on *adiabatic elimination* below), which in our implementation is controlled by the strength of a static electric field F aligned along the z axis, which modifies the bare Hamiltonian of each atom via a second-order Stark shift to $\hat{H}_0 = \sum_i (\omega + p_{\uparrow}^{(2)} F^2) |\uparrow\rangle\langle\uparrow|_i + \sum_\mu (\omega_R + p_{\beta}^{(2)} F^2) |\beta\rangle\langle\beta|_\mu$, where $p_{\uparrow, \beta}^{(2)}$ are the differential second-order Stark shifts for the respective states. Note that we use a convention where the state $\otimes_i |\downarrow\rangle_i \otimes_\mu |\alpha\rangle_\mu$ corresponds to zero energy. The electric field F provides control over the detuning from a Förster resonance condition [68–71] between the main and the relay states, $\Delta = (\omega_R + p_{\beta}^{(2)} F^2) - (\omega + p_{\uparrow}^{(2)} F^2)$. Summarizing all Hamiltonian contributions, the dynamics of the M atoms is governed by $\hat{H}_M = \hat{H}_0 + \hat{H}_{\uparrow\downarrow}^{\text{DD}} + \hat{H}_{\uparrow\alpha}^{\text{DD}} + \hat{H}_{\alpha\beta}^{\text{DD}}$.

In every realistic model, dissipation should be taken into account. We consider full dynamics of the density operator ρ

governed by the Lindblad master equation ($\hbar \equiv 1$)

$$\dot{\rho} = -i[\hat{H}_M, \rho] + \sum_\eta \mathcal{L}_\eta \rho, \quad (2)$$

with the superoperator for a noise channel η defined as $\mathcal{L}_\eta \rho \equiv [\hat{L}_\eta \rho, \hat{L}_\eta^\dagger] + [\hat{L}_\eta, \rho \hat{L}_\eta^\dagger]$ with Lindblad jump operators $\hat{L}_\eta$. For the experimental realization described below, we include dissipation due to thermally induced repopulation between the high-lying Rydberg states (blackbody radiation, BBR) and spontaneous emission. The corresponding Lindblad operators and respective rates are defined as $\hat{L}_{mn} \equiv \sqrt{\gamma_{mn}} |m\rangle\langle n|$. Note that in our implementation, the energy spacing between all states for which BBR is relevant is very similar, and we consider only a single BBR rate $\gamma \approx \gamma_{mn}$ for all mn combinations.

The geometric configuration of our proposed setup is sketched in Fig. 1: We consider all main atoms placed on a line, such that $\theta_{ij} = \arccos(\sqrt{1/3})$ for $i, j = 1, \dots, N$. In such a “magic angle” configuration, direct dipole-dipole coupling terms (and higher-order van der Waals interactions induced between the atomic levels) vanish, and $\hat{H}_{\uparrow\downarrow}^{\text{DD}} = 0$. Still, effective interactions of the form of Hamiltonian (1) are induced via virtual excitations of the relay atoms. The goal of this article is to show how a controlled placement of the relay atoms leads to a tunable coupling matrix J_{ij} .

Adiabatic elimination. We derive an effective equation of motion for the subspace density matrix of the N main atoms ρ_N , using adiabatic elimination [72–75]. Following the procedure in Ref. [75], the evolution on the Hilbert subspace of the relay atoms is governed by the equation $\dot{\rho} = -i\hat{\mathbf{t}}^T \mathbf{M} \hat{\mathbf{t}} \rho + i\rho \hat{\mathbf{t}}^T \mathbf{M}^* \hat{\mathbf{t}} + 2\gamma \hat{\mathbf{t}}^T \rho \hat{\mathbf{t}}$. Here, we defined the vector of relay lowering operators $\hat{\mathbf{t}}$ with elements $(\hat{\mathbf{t}})_\mu = |\alpha\rangle\langle\beta|_\mu$. The complex $N_R \times N_R$ matrix $\mathbf{M}$ is given by the elements $(\mathbf{M})_{\mu\nu} = (\Delta - i\gamma)\delta_{\mu\nu} + (1 - \delta_{\mu\nu})V_{\alpha\beta}(r_{\mu\nu}, \theta_{\mu\nu})$. Validity of the adiabatic elimination requires the N_R eigenvalues $\mathbf{M}$ to be the largest energy scale in the system [75]. This condition ensures that the relay atom dynamics takes place on a much faster timescale than the main atom dynamics (see the Supplemental Material [76]). Then, the evolution of the density operator projected on the Hilbert space of the main atoms is given by

$$\dot{\rho}_N = -i[\hat{H}^{\text{eff}} + \hat{H}_0^{\text{eff}}, \rho_N] + \mathcal{L}^{\text{eff}} \rho_N, \quad (3)$$

where both the coherent and dissipative dynamics are now governed by effective Hamiltonians and Lindblad superoperators $\hat{H}^{\text{eff}}$, $\hat{H}_0^{\text{eff}}$, and $\mathcal{L}^{\text{eff}}$, respectively.

The effective Hamiltonian takes the form of Eq. (1) with modified couplings:

$$J_{ij} = -\text{Re}[\mathbf{V}_i^T \mathbf{M}^{-1} \mathbf{V}_j]. \quad (4)$$

Here, we defined vectors with N_R elements corresponding to the couplings between the main atom i and relay atoms μ , $(\mathbf{V}_i)_\mu = V_{\uparrow\alpha}(r_{i\mu}, \theta_{i\mu})$. Expression (4) is a consequence of second-order perturbation theory, where dynamics for the main atoms are modified by virtual dynamics in the ensemble of relay atoms. This is also seen more directly when neglecting the interactions between relay atoms. Then, Eq. (4) simplifies to

$$J_{ij} = \frac{1}{\Delta} \sum_\mu V_{\uparrow\alpha}(r_{i\mu}, \theta_{i\mu}) V_{\uparrow\alpha}(r_{j\mu}, \theta_{j\mu}). \quad (5)$$

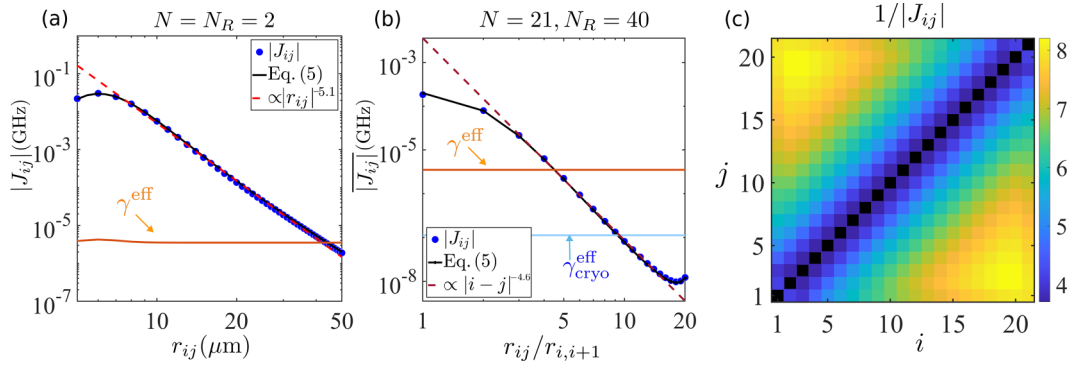


FIG. 2. (a), (b) Two examples of effective distance dependence of J_{ij} for a realization with CRAs. Blue points: effective model [Eq. (4)]; solid lines: analytical results [Eq. (5)]; and dashed lines: power-law fits. Nearly horizontal lines: effective decay rates at room ($T = 300$ K) and cryogenic ($T = 4$ K) temperatures, γ^{eff} and γ^{cryo} , respectively. (a) $|J_{ij}|$ between two main atoms as a function of distance r_{ij} ($r_{i\mu} = 6 \mu\text{m}$, $\theta_{i\mu} = 0$). (b) Averaged coupling $|J_{ij}|$ in a chain of 21 main atoms as a function of $r_{ij}/r_{i,i+1} = |i-j|$ ($r_{i,i+1} = 10 \mu\text{m}$, $r_{i\mu} = 12 \mu\text{m}$, $\theta_{i\mu} = 0$). (c) Full coupling matrix $1/|J_{ij}|$ (in log scale) as a function of site indices i and j (the diagonal elements are null and shown as black squares). Parameters: $C_3^{(\uparrow\alpha)} = \pi \times 2.25 \text{ GHz } \mu\text{m}^3$, $C_3^{(\beta\alpha)} = \pi \times 1.31 \text{ GHz } \mu\text{m}^3$, $p_{\uparrow}^{(2)} = -0.39 \text{ MHz}/(\text{V}/\text{cm})^2$, $p_{\beta\alpha}^{(2)} = -57 \text{ MHz}/(\text{V}/\text{cm})^2$, and $F = 3.5 \text{ V}/\text{cm}$.

We numerically confirm that neglecting relay atom interactions is an excellent approximation for our configuration in Figs. 2(a), 2(b), and 3(b). From expression (5) we observe that the effective pairwise interaction can be tuned in terms of distance, using the relay atom separations $r_{i\mu}$, $r_{j\nu}$, and in terms of sign, using the polar relay atom position angles $\theta_{i\mu}$, $\theta_{j\nu}$. Note that to compute the effective interaction strengths and ranges below, we numerically evaluate the full expression (4).

The effective Hamiltonian (3) also includes modified single-atom terms $\hat{H}_0^{\text{eff}} = \sum_i \delta^{\text{eff}} |\uparrow\rangle \langle \uparrow|_i$ with detunings $\delta^{\text{eff}} = (\omega + p_{\uparrow}^{(2)} F^2) - \text{Re}[\mathbf{V}_i^T \mathbf{M}^{-1} \mathbf{V}_i]$, which can be compensated by choosing an appropriate geometry [76]. The local dissipative rates in Lindblad operators for the main atoms modify to $\gamma^{\text{eff}} = \gamma + \text{Im}[\mathbf{V}_i^T \mathbf{M}^{-1} \mathbf{V}_i]$. We note that in the near-field regime, coherent interactions dominate, thus leaving the dissipative rates nearly unchanged.

Experimental implementation. Our proposal builds on an experimental platform that enables the preparation of arrays of interacting laser-trapped CRAs [66,78]. This platform uses circular Rydberg levels of rubidium, but other platforms have

also prepared [79] and trapped [80] circular Rydberg levels of strontium. With $n \sim 50$, the lifetimes of these levels reach tens of milliseconds in a cryogenic environment [43]. The lifetime of the circular Rydberg level with principal quantum number n , denoted by $|nC\rangle$, is solely limited by the microwave (MW) transition to the lower-lying $|(n-1)C\rangle$ state. At room temperature, MW BBR reduces the lifetime of CRAs to a few hundred microseconds, with the strongest contributions stemming from stimulated emission and absorption on the $|nC\rangle \rightarrow |(n \pm 1)C\rangle$ transitions. Note that relaxation rates can be significantly reduced by operating in a spontaneous-emission-inhibition structure [80,81], with the promise to reach in a cryogenic environment circular-Rydberg-level lifetimes in the minute range [40].

Quantum nondemolition measurements and local manipulations of circular Rydberg states have recently been performed through the interaction of the CRAs with relay atoms transiently excited to a low- l Rydberg level [82]. This experiment uses a Stark-tuned Förster resonance between the $|53C\rangle \rightarrow |54C\rangle$ and $|45S_{1/2}, m_J = 1/2\rangle \rightarrow |45P_{3/2}, m_J = 3/2\rangle$ transitions. We propose to implement our

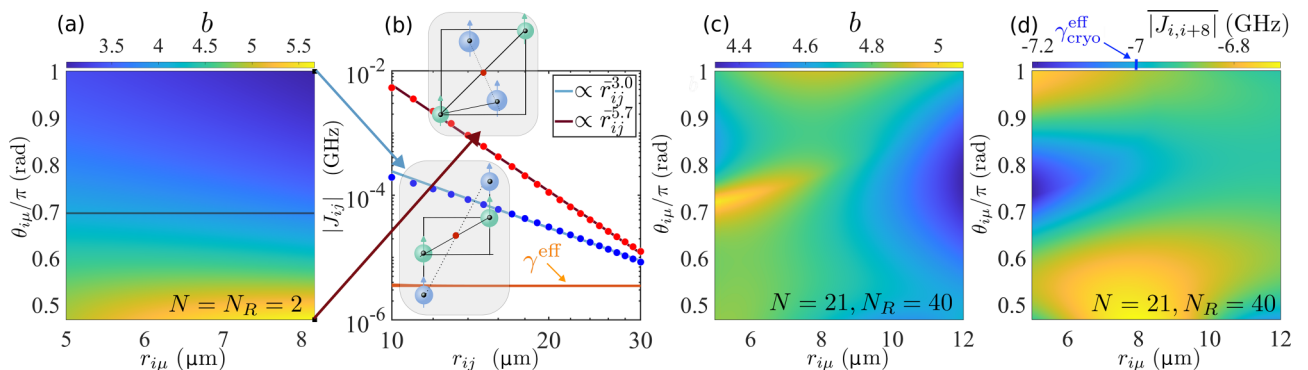


FIG. 3. (a), (c) Contour plots of the power-law exponent b as a function of $r_{i\mu}$ and $\theta_{i\mu}$. It is obtained by fitting $|J_{ij}| = a/r_{ij}^b$ and $|J_{ij}| = a/|i-j|^b$ for panels (a) and (c), respectively. (a) Pair of atoms, $N = N_R = 2$. (c) Chain configuration $N = 21, N_R = 40$. (b) Examples of the distance dependence of $|J_{ij}|$ at two specific configurations marked in panel (a). Insets: Corresponding geometrical configurations. (d) The averaged coupling strength over a distance of eight atoms, $|J_{i,i+8}|$ (in log scale). Parameters identical to Fig. 2.

method by mapping the states of the main atoms and relay atoms onto the same four levels: $\{\downarrow, \uparrow\} = \{53C, 54C\}$ and $\{\alpha, \beta\} = \{45S, 45P\}$. The Förster resonance ($\Delta = 0$) is reached at an electric field of $F = 1.6$ V/cm. With $F = 3.5$ V/cm, we have $\Delta = 550$ MHz (this choice is discussed in more detail in Ref. [76]).

To describe the relaxation of the CRAs, we add two neighboring circular Rydberg levels $|\uparrow\rangle = |55C\rangle$ and $|\downarrow\rangle = |52C\rangle$ to our model. The main atoms, now being four-level systems, relax via Lindblad jump operators of the form $\hat{L}_{mn} = \sqrt{\gamma_{mn}}|m\rangle\langle n|$, with $mn \in \{\uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow, \downarrow\downarrow\}$. At room temperature, all γ_{mn} rates are similar and we consider a single BBR rate $\gamma \approx 3.5$ kHz. At $T = 0$ K, only spontaneous emission must be considered, so $\gamma_{\downarrow\downarrow} = \gamma_{\uparrow\downarrow} = \gamma_{\uparrow\uparrow} = 0$, and for simplicity, we choose the natural decay rate of $|54C\rangle$ for the other rates to be $\gamma_{\downarrow\downarrow} = \gamma_{\uparrow\downarrow} = \gamma_{\uparrow\uparrow} \equiv \gamma^{\text{cryo}} = 24$ Hz.

The low- ℓ Rydberg levels, $|45S\rangle$ and $|45P\rangle$, decay toward the ground state or other Rydberg levels by the emission of optical or MW photons, respectively. At room temperature, the decay rates associated with these two processes are almost equal, with a common value $\gamma_\ell \approx 10$ kHz. For simplicity, we model the decay on the MW transitions with the Lindblad operators $\hat{L}_{mn} = \sqrt{\gamma_\ell}|m\rangle\langle n|$, with $mn \in \{\alpha\beta, \beta\alpha\}$. The decay on MW transitions is, however, strongly reduced at cryogenic temperatures. Only the decay toward the ground state is relevant in this case. As analyzed in Ref. [76], including additional losses of relay atoms shortens the timescale for observing modified interactions. Utilizing a repumping into the desired low- ℓ state is, in principle, feasible without adversely affecting the interaction measurement. Importantly, losses still limit analog quantum simulation timescales significantly.

Note that one could also encode the relay atom levels $|\alpha\rangle$ and $|\beta\rangle$ onto long-lived circular Rydberg levels instead of low- ℓ Rydberg levels at the expense of a slight increase of Δ . For instance, $\Delta = 1.5$ GHz with $\{\downarrow, \uparrow, \alpha, \beta\} = \{70C, 71C, 72C, 73C\}$. This would have the advantage of effectively suppressing decay to the ground state—which occurs only for low- ℓ states—leaving dephasing as a main decoherence effect [76].

A pair of atoms. Starting with the simple case of $N = 2$, we show an example on how relay atom placement modifies the effective coupling in Fig. 2(a). Here, we choose a geometry with $N_R = 2$ as sketched in Fig. 3(b) (insets): We parametrize the position of the relay atom μ using only its distance $r_{i\mu}$ from the main atom i , and the polar angle $\theta_{i\mu}$. The second relay atom ν is located in a mirrored position with respect to the main atom j . This geometry ensures resonance between the levels of the main atoms, as the relay atoms induce inverse shifts [76]. It also ensures that the sign of J_{ij} only depends on the detuning Δ . In the example in Fig. 2(a), we use $r_{i\mu} = 6$ μm , $\theta_{i\mu} = 0$ and find that the long-distance behavior of J_{ij} decays as a power-law with exponent $b \sim 5.1$. Note that the price for tunability of the interaction range via virtual relay excitation is a reduction in effective coupling strength. However, long-range couplings remain significantly above the effective state lifetimes at room temperature, indicated by the nearly horizontal line in Fig. 2(a).

To obtain an analytical understanding, using the parametrization for the relay atom positions and Eq. (5), we can express the coupling-strength dependence as a

function of $r_{i\mu}$ and $\theta_{i\mu}$ only. Using a Taylor expansion in r_{ij}^{-1} one finds [76]

$$J_{ij} = \frac{2}{\Delta} V_{\uparrow\alpha}(r_{i\mu}, \theta_{i\mu}) \sum_{\alpha \geq 4} \frac{B_\alpha(\theta_{ij}, r_{i\mu})}{r_{ij}^\alpha}. \quad (6)$$

The two leading-order coefficients are given by $B_4 = r_{i\mu} \cos(\theta_{ij} - \theta_{i\mu}) - 3r_{i\mu} \cos(\theta_{i\mu} + \theta_{ij})$ and $B_5 = \frac{33}{2}r_{i\mu}^2 \cos(2\theta_{i\mu}) + 15r_{i\mu}^2 \cos(2\theta_{ij}) + 5r_{i\mu}^2 \cos^2(\theta_{ij} - \theta_{i\mu})$. Further analytical expressions for J_{ij} are provided in Ref. [76]. For example, in the limit $r_{ij} \gg r_{i\mu}$, we see that in leading order J_{ij} exhibits a power-law decay $\sim r_{ij}^{-4}$.

We analyze the tunability of the distance dependence in Figs. 3(a) and 3(b) as a function of $r_{i\mu}$ and $\theta_{i\mu}$. Depending on the placement of the relay atoms, we find a wide tunability of the effective exponent in the range $3 \lesssim b \lesssim 6$, which we extract from fits of the form $|J_{ij}| \equiv a/r_{ij}^b$. Here, we focus on relay atom distances $r_{i\mu}$ of at least 5 μm , for which we have confirmed that the adiabatic elimination remains valid [76]. The horizontal line indicates the location where $\theta_{i\mu} = \pi - \arccos(\sqrt{1/3})$, a point at which the direct dipole-dipole coupling between the relay atom and its neighboring main atom vanishes. We show that around $\theta_{i\mu} = \pi - \arccos(\sqrt{1/3}) \pm 0.05$, the effective interaction is too weak compared to the decay rate γ^{eff} in Ref. [76]. This exclusion region follows from the condition $J_{ij} < \gamma^{\text{eff}}$ at $r_{ij} = 25$ μm . We generally observe that the tunable power-law behavior should be resolvable experimentally over a wide parameter range, even at room temperature, as effective couplings remain large compared to effective decay rates, $|J_{ij}| \gg \gamma^{\text{eff}}$.

Chain of atoms. For analyzing effective couplings in the chain configuration, we choose equally spaced main atoms with distances $r_{i,i+1} = 10$ μm . We consider a succession of blocks of two main and two relay atoms in the same mirrored position as for $N = N_R = 2$ (see sketch in Fig. 1). Therefore, the relay atom distance $r_{i\mu}$ and relative relay atom angle $\theta_{i\mu}$ remain the only free parameters and are identical for all $i = 1, \dots, N$.

Figure 2(b) shows the averaged magnitude $|J_{ij}|$ as a function of the distance $|i - j|$ obtained from Eq. (4) (blue points). We choose $r_{i\mu} = 12$ μm and $\theta = \pi/2$. The black solid line corresponds to the expression neglecting relay-relay interactions, Eq. (5), and is in good agreement with Eq. (4). The dashed line represents a power-law fit of the form $|J_{ij}| = a/|i - j|^b$. We find that the power-law model remains a good approximation at long distances throughout our parameter regime [76]. At a distance of ~ 4 atoms, the effective coupling strength becomes smaller than the effective decay rate at room temperature, γ^{eff} . Therefore, observing long-range power-law interactions will require cryogenic temperatures, leading to the effective decay rate $\gamma_{\text{cryo}}^{\text{eff}}$ (blue horizontal line).

In Fig. 3(c), we analyze the tunability of b in the chain, as a function of $r_{i\mu}$ and $\theta_{i\mu}$. We demonstrate a smooth tunability in the range $b \sim 4$ –5. The shortest-range coupling is achieved for relay atoms placed close to the main atoms (small $r_{i\mu}$), where we find a relatively large variability of b as a function of $\theta_{i\mu}$. However, for $0.65\pi \lesssim \theta_{i\mu} \lesssim 0.85\pi$ nearest-neighbor main-relay couplings vanish, leading to very small $|J_{ij}|$, as analyzed in Fig. 3(d). There, we plot the averaged coupling over a distance of eight atoms. To assess whether the

effective power-law coupling is observable, we compare it to the effective main atom decay rate at cryogenic temperatures. For configurations with $\theta_{i\mu} \sim \pi/2$, we find strong effective long-range power-law couplings with variable exponents as a function of $r_{i\mu}$. For large relay atom distances $r_{i\mu} \gg r_{ij}$, we recover the $b \sim 4$ power-law behavior, as perturbatively expected from Eq. (6).

Conclusion. We have introduced a scheme to engineer tunable dipolar interactions between Rydberg atoms in tweezer arrays, by modifying the electromagnetic environment with a proper positioning of far-detuned relay atoms. Our approach enables control over the interaction range in the near-field regime, achieving effective power-law behavior at long distances with exponents tunable between 3 and 6. This tunability opens promising avenues for analog quantum simulation, especially when implemented with CRAs in cryogenic environments. Remarkably, our proposal could be readily transposed in dual-species Rydberg experiments [83] or molecule-Rydberg platforms [84], which rely on Förster resonances, too. Combining our scheme with tunable transverse fields would enable the realization of more complex long-range XY models, further broadening the scope of our setup. Future work will implement the scheme experimentally and explore more complex geometries, e.g., with

sign-alternating couplings, and potential applications in quantum gate operations using relay-mediated dynamics.

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Data availability. The data that support the findings of this article are openly available [85].

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