

Quantum sensing in the vicinity of dissipative phase transition

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(Received 5 May 2026; accepted 11 August 2026; published 1 September 2026)

Recently, there has been considerable interest in studying quantum sensing near the critical points of a system evolving unitarily and undergoing a phase transition. However, there are many quantum systems in which dissipation is very important. The quantum phase transition in such systems results as a result of competition between nonlinearity, drive, and dissipation. These quantum phase transitions belong to the class of nonequilibrium phase transitions. In this work, we present a detailed study of quantum sensing of the system parameters near the critical points of a dissipative phase transition. We illustrate the key results by considering a coherently driven nonlinear Kerr medium interacting with environment. The first-order quantum phase transition is characterized by the closing of the gap in eigenvalues of the underlying Liouvillian. The steady-state sensing away from the critical point has standard quantum limit scaling with respect to the effective system size, whereas Heisenberg scaling is found at the critical point. The latter is subject to a critical slowing down. We have also examined sensing of system parameters using the metastable states and find better sensing performance at intermediate timescales, i.e., when the system has not yet settled in steady state. Detailed results on the scaling behavior in different time domains are given.

DOI: [10.1103/x7gn-pqzb](https://doi.org/10.1103/x7gn-pqzb)

I. INTRODUCTION

Quantum sensing leverages quantum resources, like entanglement, to achieve a better sensitivity than classical sensing [1,2], and has significant impact on developments across various disciplines [3–5]. Theories concerning the precision of and the resources required for different sensing protocols have been well developed [6,7]. Quantum sensing can be implemented in a variety of platforms such as superconducting circuits [8,9], solid-state spin systems [10,11], magnonics [12], photonics [13–15], cavity optomechanics [16,17], and cavity QED systems [18,19].

Recently, there is a growing interest in studying quantum sensing near the critical points of phase transitions in unitarily evolving systems. The system has multiple phases, and the quantum fluctuations diverge at the critical points, introducing a diverging susceptibility around the transition and hence an enhanced sensing performance [20,21]. Previously, it was argued that such an enhancement is often diminished by the critical slowing down, which reduces the sensing capability to at best the Heisenberg scaling [22–24]. The scaling is defined as the quantum Fisher information (QFI) scaling quadratically

with the resources including the number of atoms and the sensing time. More recently, it has been shown theoretically that the critical slowing down issue can be overcome for certain systems, enabling a super-Heisenberg scaling of the QFI [25,26]. The effect of weak dissipation on the sensing performance of the unitarily evolving systems has been considered [27–29].

A dissipative phase transition is a nonequilibrium transition in an open quantum system, where the competition between coherent driving and dissipation leads to distinct steady states [30–32]. Such transitions are characterized by the closing of the Liouvillian gap; i.e., the gap between the zero eigenvalue, corresponding to the steady state, and the leading nonzero eigenvalue of the Liouvillian superoperator that governs the dynamics of the system. The behavior near the critical point can lead to a diverging susceptibility and an enhanced sensing performance, similar to the case of unitary phase transitions, but with the added complexity of dissipation and nonequilibrium dynamics [33–36]. A Heisenberg scaling of the QFI can be achieved at the critical point depending on the operating conditions [37,38].

In this paper, we present a comprehensive study of the sensing performance of a versatile system, from quantum optics and condensed matter physics, that exhibits a first-order dissipative phase transition. We characterize the sensing performance in different regimes of operation using both steady states and metastable states. The organization of the paper is as follows. In Sec. II, we introduce the driven-dissipative quantum Duffing oscillator model. We consider a coherently driven cavity containing a Kerr nonlinear medium. The leakage from the cavity is the source of dissipation.

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At the mean-field level, such a system exhibits well-studied bistability [39–42], whereas quantum characteristics are quite different. Recently, an extensive experimental work examined the nature of the quantum dissipative phase transition in this model [43]. We analyze the Liouvillian gap structure and discuss its first-order dissipative phase transition and effective two-level behavior in the metastable parameter region, which contrasts bistability behavior obtained in mean-field approximation. We also introduce the basics of QFI for quantifying the sensing performance of the system [2,44]. In Sec. III, we present the sensing performance of the system for both the frequency of the cavity mode or detuning parameter and the nonlinearity. We use the steady state of the system. We find standard quantum limit (SQL) scaling with respect to the effective system size when the operating point is away from the critical point and a Heisenberg scaling while working near the critical point. In Sec. IV, we find that the metastable state can provide an enhanced sensing performance compared to the steady state if the sensing time is properly chosen compared with the characteristic time of the system. The latter is determined by the first nonvanishing eigenvalue of the Liouvillian. In Sec. V, we provide a comparison of the sensing performance at the critical point while using steady states and metastable states. The metastable states provide better sensing performance with a shorter sensing time. Finally, in Sec. VI, we summarize our results.

II. MODEL AND THE NONEQUILIBRIUM STATES

We introduce the driven-dissipative quantum Duffing oscillator model for the realization of a first-order dissipative phase transition and use the QFI as the main tool for quantifying the sensing performance of the key parameters of the system. The quantum Duffing oscillator is a single-mode bosonic system with Kerr nonlinearity usually implemented in a cavity. It is described by the Hamiltonian

$$H = \Delta a^\dagger + U a^\dagger a a^\dagger a + \Omega(a^\dagger + a), \quad (1)$$

and when dissipation is included, the dynamics of the system is described by the master equation

$$\frac{d\rho}{dt} = -i[H, \rho] + \gamma \mathcal{D}[a]\rho \equiv \mathcal{L}\rho. \quad (2)$$

Here, Δ is the detuning between the external drive and the cavity frequency, a is the photon annihilation operator, U is the Kerr nonlinearity, and Ω is the Rabi frequency of the external drive. ρ is the density matrix of the system, γ is the rate of energy dissipation due to cavity leakage, $\mathcal{D}[a]\rho = a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}$, and $\mathcal{L}$ is the Liouvillian super-operator whose eigenvalue spectrum determines the nature of the dissipative phase transition. The system can be realized in a variety of platforms such as superconducting circuits [38,43,45], magnetic systems [41,46], and optical cavities [47–49]. The experimental feasibility of the system and the rich physics of the first-order dissipative phase transition attracted great interest for quantum sensing and metrology [50,51].

In the mean-field approximation, the quantum fluctuations are neglected, and the steady-state behavior of the system is determined in terms of the appropriately scaled input driving

field intensity I and the mean number of photons α in the cavity [42]:

$$I = \frac{\alpha}{2} \left[1 + \left(\tilde{\Delta} + \frac{\alpha}{2} \right)^2 \right], \quad (3)$$

where scaled quantities $I = \frac{2U\Omega^2}{(\gamma/2)^3}$, $\alpha = 4\frac{U|\langle a \rangle|^2}{\gamma/2}$, $\tilde{\Delta} = \frac{(\Delta+U)}{\gamma/2}$. Here, α marks the energy shift per photon due to the Kerr nonlinearity rescaled by the dissipation. To pin down the shift effect with a decreasing nonlinearity U , the photon number expectation value $|\langle a \rangle|^2$ needs to be increased inversely, requiring a stronger drive Ω . Roughly, the drive strength Ω is proportional to the amplitude of the oscillator $|\langle a \rangle|$. I justifies this intuition that similar Kerr effects are achieved, with $\Omega^2 \propto U^{-1}$ in an analytical way. It serves as a measure of the rescaled drive strength and characterizes the drive requirement to achieve a similar Kerr effect (up to a difference in $\tilde{\Delta}$) with a decreasing nonlinearity U . Note that in the limit $U \rightarrow 0$, $I = (1 + 4\Delta^2/\gamma^2)\alpha/2$. When the driving field increases, as illustrated in Fig. 1(a), a bistable behavior emerges: More than one value of α exists and there are two stable steady states. The two states are referred to as a near-vacuum state and a bright state according to the photon number expectation value. The region between the two turning points of the mean-field solutions is the mean-field bistable region. The two turning points mark the hysteresis boundaries. Around the turning points of the bistable region $I_\pm$, the system has a diverging susceptibility scaling as $\frac{d\alpha}{d\tilde{\Delta}} \sim |I - I_\pm|^{-1/2}$, referring to a sensitive photon number response to the detuning near the critical points. Around the critical magnitude I^c where the bistability first emerges, $I_+ = I_-$ and the susceptibility scales as $\frac{d\alpha}{d\tilde{\Delta}} \sim |I - I^c|^{-2/3}$ [41,42].

However, in the quantum regime, the fluctuations are important. The system is predicted and observed to have a unique steady state defined by $\mathcal{L}\rho_{ss} = 0$ [53,54]. Meanwhile, the other bistable candidate becomes a low-lying long-lived excited state resulting in a slow relaxation in the observables to the steady-state value [30,43,55,56]. The steady state and the excited state can be understood as a near-vacuum state and a bright state. They exchange their roles at a critical point when the drive strength is tuned, characterizing a dissipative phase transition. The critical point can be strictly defined in the thermodynamic limit

$$U = U_0\varepsilon, \quad \Omega = \Omega_0\varepsilon^{-1/2}, \quad \varepsilon \rightarrow 0. \quad (4)$$

U_0, Ω_0 are fixed rescaled parameters. $\varepsilon \rightarrow 0$ scales the system to the thermodynamic limit and $1/\varepsilon$ can be interpreted as the effective system size when mapped onto a many-resonator driven-dissipative Bose-Hubbard model [30,43,55].

Different steady and metastable states are shown in Fig. 1(a). The unique quantum steady state ρ_{ss} switches from a low-photon-number state to a high-photon-number state at the critical point $\Omega_{0,c} \approx 1.031\gamma$. The rescaled critical frequency is determined by finding the fixed point of the steady-state scaled photon number expectation value $\varepsilon\langle n \rangle_{ss}$ [43]. The photon number expectation value has a steep increase at the critical point. With a finite-size scaling, the jump becomes sharper with a decreasing ε , as directly demonstrated in previous literature [55] and in the thermodynamic limit it would be a true jump. It indicates a first-order phase transition at

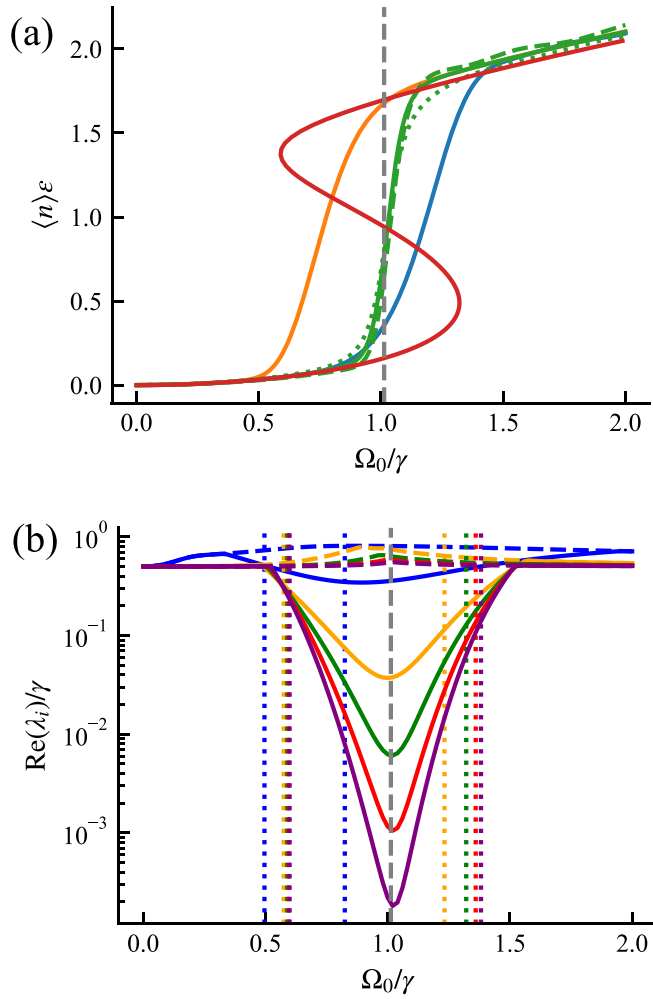


FIG. 1. (a) The scaled photon number $\langle n \rangle/N$. The dotted lines are at $\varepsilon = 1/3$, the solid lines are at $\varepsilon = 1/5$, and the dashed lines are at $\varepsilon = 1/25$. The red line is the classical result and the green lines are the quantum steady states. The blue line represents the states initially in a steady state at $\Omega_{0,\text{init}} = 0$ evolved for $\gamma t = 10$ after a quench to the indicated Ω_0 . The orange line represents the states initially in a steady state at $\Omega_{0,\text{init}} = 3\gamma$ evolved for $\gamma t = 10$ after a quench to the indicated Ω_0 . (b) The Liouvillian gaps $\lambda_{1,2}$ are represented by the solid lines and the dashed lines, respectively. From top to bottom, ε is 1, $1/3$, $1/5$, $1/7$, $1/9$. The gray vertical line indicates the critical frequency $\Omega_{0,c} \approx 1.031\gamma$ and the colored vertical dotted lines indicate the classical hysteresis boundaries for the corresponding ε . Throughout this study, we fix $\Delta = 3\gamma$, $U_0 = -\gamma$. The states are analyzed with aid of the QUTIP package [52].

the critical point. We refer to Fig. 1(a), where we show the gradually sharper jump for three values of ε (see the green solid, dotted, and dashed lines). For the states quenched from the vacuum and bright states, the relaxation dynamics shows two qualitatively different behaviors depending on the quench point. The states quenched outside the mean-field bistable region relax to the steady-state observable value with a timescale of order $1/\gamma$. With the same evolution time, the states quenched inside the mean-field bistable region do not yet relax to the steady-state value. This indicates the existence

of a slow timescale and the metastable states in the mean-field bistable region.

The steady states and the metastable states can be analyzed with a spectral decomposition of the Liouvillian superoperator $\mathcal{L}$. Eigenstates of the system can be obtained with a numerical diagonalization of $\mathcal{L}$ over the vectorized density matrix space [57], yielding the eigenvalues λ_i and the corresponding eigenmodes ρ_i . The steady state corresponds to $\lambda_0 = 0$ and all other eigenvalues have negative real parts, which indicates the decay rate and characteristic timescale $\tau_i = -1/\text{Re}(\lambda_i)$ of the corresponding eigenmodes. In Fig. 1(b), we plot the real part of the first two nonvanishing eigenvalues $\text{Re}(\lambda_{1,2})$ as a function of Ω_0 for different ε . Despite that λ_2 remains around 0.5γ , there are two regions with different behaviors of the excited states: (1) a region around the critical point, where $\text{Re}(\lambda_1) \ll \text{Re}(\lambda_2)$. Here, the first excited state is a long-lived mode and the system has a slow relaxation to the steady state, and λ_1 is real [30]. The timescale of the system is set by $\tau_1 \gg \tau_2$, which is much longer than those associated with higher modes. (2) This region is away from the critical point, where $\text{Re}(\lambda_1) = \text{Re}(\lambda_2)$. Here, λ_1 and λ_2 are complex conjugates. In region (1), this set the stage for the isolation of an effective two-level system and the existence of metastable states at times longer than τ_2 but shorter than or comparable to τ_1 . As the system approaches the thermodynamic limit, the gap of the system given by $\text{Re}(\lambda_1)$ closes at the critical point $\Omega_{0,c} \approx 1.031\gamma$ exponentially [55]. This is consistent with the first-order criticality in this driven-dissipative model. It is worth noting that the scaling given by Eq. (4) does not change I by ε in Eq. (3), and therefore the mean-field features are expected to share similarities with the thermodynamic limit of the quantum system. With a decreasing ε , the mean-field bistability region would match the region (1) previously defined with Liouvillian spectrum [55]. The comparison between them is shown in Fig. 1(b).

Having discussed the main features of the phase transition, we now turn to the sensing aspects using the critical region of the dissipative phase transition. We will use the quantum metrology framework. The QFI provides a quantitative measure of the sensitivity of a quantum state to parameter variations. If θ is the parameter whose sensitivity is desired, its QFI is defined as

$$F_\theta(\rho) \equiv \text{Tr}(\rho L_\theta^2), \quad (5)$$

where L_θ is the symmetric logarithmic derivative satisfying $\frac{d\rho}{d\theta} = \frac{1}{2}(\rho L_\theta + L_\theta \rho)$. It gives the maximum precision for estimating a parameter θ when using a quantum probe [2,7]. We will especially concentrate on the parameters detuning Δ and nonlinearity U of the system described in Eq. (1). In particular, we will examine the following questions: (1) how well the steady state ρ_{ss} can perform as a sensor, (2) whether the metastable state, i.e., the state at intermediate times $\tau_2 \ll t \sim \tau_1$, given approximately by $\rho(t) \approx \rho_{ss} + c_1 e^{-\lambda_1 t} \rho_1$ can provide an enhanced sensing performance compared to the one using the steady state, and (3) how the sensing performances change as the system approaches the thermodynamic limit $\varepsilon \rightarrow 0$. Most of our discussion would be based on the operating points on the bright-state side of the transition $\Omega_0 \geq \Omega_{0,c}$.

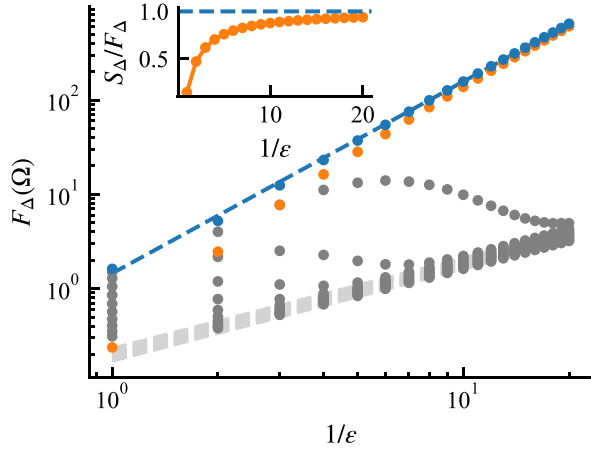


FIG. 2. The QFI $F_\theta(\rho_{ss})$ and the squared inverse of the error sensitivity over photon number $S_\theta(\rho_{ss})$ for $\theta = \Delta$. Main plot: as a function of $1/\varepsilon$. The blue dots are for the QFI and the orange dots are for the squared inverse of the error sensitivity at $\Omega_0 = \Omega_{0,c}$. The blue dashed line is a power-law fitting for the QFI with an exponent of 2.03. The gray dots are series for the QFI for $\Omega_0 = 1.1\gamma, 1.2\gamma, \dots, 2.0\gamma$ from top to bottom, and the gray dashed lines are power-law fittings for the QFI for $\Omega_0 \geq 1.3\gamma$ on $\varepsilon < 1/5$ and they have an exponent of approximately 0.97. Inset: the ratio between the squared inverse of the error sensitivity and the QFI at $\Omega_0 = \Omega_{0,c}$.

III. QUANTUM SENSING AT THE STEADY STATE

We will study the QFI at the steady state scaled with respect to the effective system size $1/\varepsilon$. The steady state provides SQL scaling away from the critical point and Heisenberg scaling at the critical point, the latter being subject to critical slowing down at the thermodynamic limit.

A. Thermodynamic scalings of F_Δ

The QFI of the steady state $F_\theta(\rho_{ss})$ for $\theta = \Delta$ is shown in Fig. 2 as a function of the effective system size $1/\varepsilon$. By applying a power-law fitting to the QFI $F_\theta(\rho) = a(1/\varepsilon)^b$, the QFI of the steady state $F_\theta(\rho_{ss})$ for $\theta = \Delta$ scales linearly in $1/\varepsilon$ away from the critical point. This matches the definition of SQL: QFI scales linearly with the effective system size. At the critical point, however, the QFI scales quadratically with $1/\varepsilon$, marking the emergence of Heisenberg scaling. For the steady states obtained close to the critical point, at smaller $1/\varepsilon$, the QFI follows the quadratic trend. At larger $1/\varepsilon$, the distance from the critical point to Ω_0 becomes relevant and the scaling crosses over to the linear trend. The closer the operating point is to the critical point, the larger the range of $1/\varepsilon$ over which the quadratic scaling is observed. The Heisenberg scaling at the critical point is subject to critical slowing down due to the exponentially closing Liouvillian gap with respect to $1/\varepsilon$ (see Fig. 1).

The photon number operator $n = a^\dagger a$ can be used to construct a practical estimator for Δ (and U below). Although measuring n is not optimal for estimating these parameters, its squared inverse of the error sensitivity

$$S_\theta(\rho_{ss}) \equiv \frac{|\partial_\theta \langle n \rangle|^2}{\langle n^2 \rangle - \langle n \rangle^2}$$

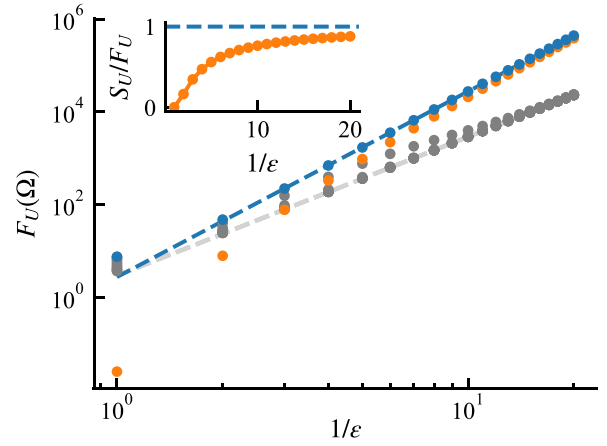


FIG. 3. The QFI $F_\theta(\rho_{ss})$ and the squared inverse of the error sensitivity over photon number $S_\theta(\rho_{ss})$ for $\theta = U$. Main plot: as a function of $1/\varepsilon$. The blue dots show the QFI and the orange dots show the squared inverse of the error sensitivity at $\Omega_0 = \Omega_{0,c}$. The blue dashed line is a power-law fitting for the QFI with an exponent of 4.00. The gray dots show the QFI for $\Omega_0 = 1.1\gamma, 1.2\gamma, \dots, 2.0\gamma$ from top to bottom, and the gray dashed lines are power-law fittings for the QFI for $\Omega_0 \geq 1.3\gamma$ on $\varepsilon < 1/5$ and they have an exponent of approximately 3.00. Inset: the ratio between the squared inverse of the error sensitivity and the QFI at $\Omega_0 = \Omega_{0,c}$.

approaches the ultimate limit set by the QFI in the thermodynamic limit. This is shown in the inset of Fig. 2, where the ratio $S_\theta(\rho_{ss})/F_\theta(\rho_{ss})$ approaches 1. Generally, the error sensitivity of θ found using measurement by the observable A is lower bounded by the quantum Cramér-Rao bound as

$$\delta\theta \equiv \frac{\sqrt{\langle A^2 \rangle - \langle A \rangle^2}}{\partial_\theta \langle A \rangle} \geq 1/\sqrt{F_\theta(\rho)}.$$

Therefore, $S_\theta(\rho_{ss}) \leq F_\theta(\rho_{ss})$. S_θ then quantifies the experimental sensitivity with an upper bound given by the QFI.

B. Thermodynamic scalings of F_U

As shown in Fig. 3, by applying the same power-law fitting to F_U , the QFI scales cubically away from the critical point and quartically at the critical point with the effective system size $1/\varepsilon$. This is because the parameter U is correlated with the scaling constant ε , as shown in Eq. (4). To avoid this correlation, we consider sensing on a fixed rescaled parameter $U_0 = U/\varepsilon$ instead of U . Using a classical Fisher information analogy on the probability distribution $f(x; U_0)$ of the measurement outcome x ,

$$\begin{aligned} I_{U_0}(1/\varepsilon) &= \mathbb{E}[(\partial_{U_0} \ln f)^2] \\ &= \mathbb{E}\left[\left(\frac{\partial U}{\partial U_0} \partial_U \ln f\right)^2\right] \\ &= \varepsilon^2 I_U(1/\varepsilon). \end{aligned}$$

The QFI of the rescaled parameter F_{U_0} is expected to show a similar relation with F_U due to the correlation. Therefore, $F_{U_0} \sim \varepsilon^2 F_U$ and is expected to scale quadratically with $1/\varepsilon$ at the critical point and linearly away from the critical point. The

thermodynamic scaling of F_{U_0} is then similar to the one for F_{Δ} . Both U_0 and Δ are fixed parameters that do not change with ε .

IV. QUANTUM SENSING IN METASTABLE STATES

In this section, the time evolution of the QFI $F_{\theta}(\rho(t))$ of a metastable state is analyzed. The metastable state is obtained by initializing the system in the vacuum state and evolving it for time t after a quench to a target Ω_0 slightly above the critical Rabi frequency. An enhanced QFI compared to the steady state is observed at a finite-time window. We will further extract the scaling of $F_{\theta}(\rho(t))$ with respect to $1/\varepsilon$ in the enhancement time window. When comparing different ε , the sensing time must be scaled with τ_1 . The QFI of the time-evolved state away from the critical point is also studied. There the system has no metastability and the sensing is more of a transient nature. The QFI does not show clear scaling law with respect to $1/\varepsilon$.

We start by demonstrating that the quantum fluctuations in the metastable states are crucial for the sensing in the system. The QFI of the time-evolved state after a quench from the vacuum state to different Ω_0 and Δ is studied. In Fig. 4, the QFI F_{Δ} of the state quenched from the vacuum state onto different (Ω_0, Δ) for increasing times is plotted. In Figs. 4(a) and 4(b), with a small sensing time, the system is more sensitive around the mean-field hysteresis boundary with a finite QFI. This shows the quantum fluctuations smooth out the mean-field susceptibility divergence. In Figs. 4(c) and 4(d), as time increases, the ridge of maximal QFI shifts toward the dissipative phase transition line and its peak value grows. In Fig. 4(e), the QFI slice at $\Delta = 3\gamma$ is plotted for different times. It clearly shows that the QFI peak shifts from the mean-field hysteresis boundary to the phase transition line as time increases and the peak value increases. These observations are in contrast to the mean-field result where the sensitivity is divergent at the hysteresis boundary. This behavior arises because transient modes decay on a timescale τ_1 , and the system relaxes into a metastable state with strong quantum fluctuations. These fluctuations increase as the system approaches the phase transition line, thereby enhancing the sensing performance. At the same time, τ_1 becomes longer near criticality. Consequently, for longer sensing times the system can relax even when Ω_0 is closer to the critical point, allowing it to reach the metastable state with higher quantum fluctuations. As a result, the QFI attains larger values closer to the critical point if the time is sufficiently large.

A. Metastability sensitivity enhancement for F_{Δ}

Now, we focus on the sensing performance of the metastable states at longer times. As shown in Fig. 5, the QFI $F_{\Delta}(\rho(t))$ of the metastable state exhibits a peak at time $t_{\text{peak}} \approx 1.6\tau_1$ and decays monotonically to the steady-state value at long times. This suggests that the metastable state can provide an enhanced sensing performance compared to the steady state if the sensing time is properly tuned with respect to τ_1 . This peak region is enlarged in Figs. 6(a) and 6(c) and the temporal behavior of QFI for multiple ε is plotted. Around the peak, a power-law fitting is applied to the QFI

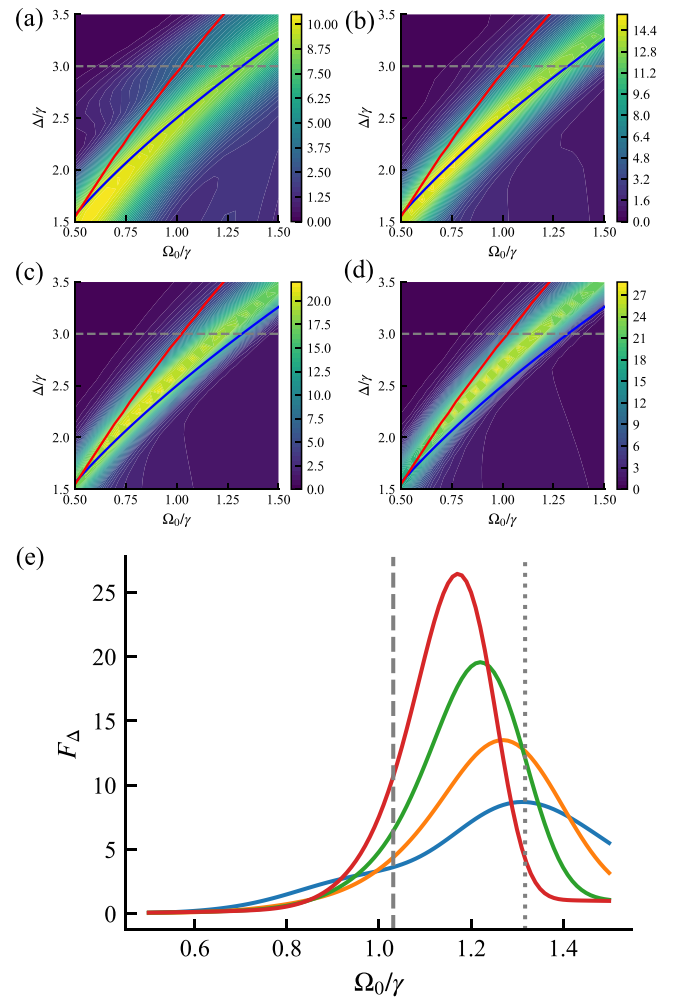


FIG. 4. The QFI F_{Δ} (color) as a function of (Δ, Ω_0) at $\varepsilon = 1/5$ for different time evolution. (a) $\gamma t = 5$, (b) $\gamma t = 10$, (c) $\gamma t = 20$, and (d) $\gamma t = 40$. The blue line is the mean-field hysteresis boundary from the dark state to the bright state. The red line is the phase transition line. The dashed line is a cut at $\Delta = 3\gamma$, at which panel (e) is plotted. (e) The QFI slice for previous times. The vertical dashed line indicates the phase transition line and the vertical dotted line indicates the mean-field hysteresis boundary. Here, the phase transition line is obtained by the finding the largest derivative of the scaled photon number $\frac{d\varepsilon(n)_{ss}}{d\Omega_0}$ over Ω_0 at $\varepsilon = 1/5$. In this paper, we denote $F_{\theta}(\rho)$ in unit of $1/\gamma^2$.

with different fixed times as shown in Figs. 6(b) and 6(d). The exponent b remains approximately unchanged during the enhancement time window lasting for a timescale $\sim \tau_1$, being approximately 1.70 for $\Omega_0 = 1.3\gamma$ and 2.22 for $\Omega_0 = 1.1\gamma$. This suggests that a shorter sensing time can be beneficial for sensing when the system is quenched around the critical point. The scaling of the QFI with respect to $1/\varepsilon$ can also be enhanced compared to the steady state during the enhancement time window. In addition, we note that despite the exponent being larger than 2 for $\Omega_0 = 1.1\gamma$, it is not a sign of super-Heisenberg scaling because $\tau_1 \sim \exp(1/\varepsilon)$. Achieving such a scaling from the vacuum state thus requires an exponentially long evolution time.

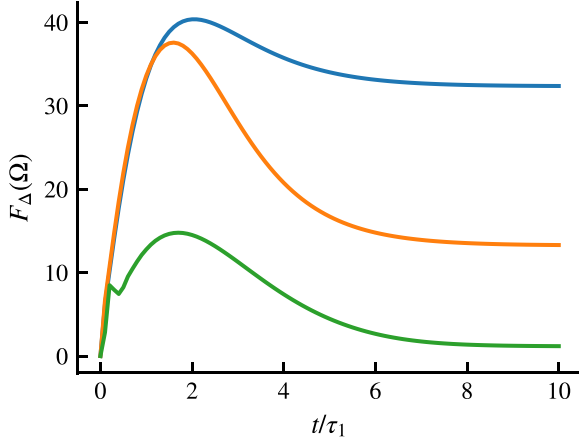


FIG. 5. A long-time evolution of the QFI F_Δ for $\varepsilon = 1/5$ showing its monotonic decay to the steady-state QFI after the peak. $\Omega_0 = 1.3\gamma, 1.1\gamma, 1.05\gamma$, bottom to top.

B. Metastability sensitivity enhancement for F_U

For sensing $\theta = U$, we can also find a stable enhancement of the QFI $F_U(\rho(t))$ in the metastable region, as illustrated in Figs. 7(a) and 7(c). The QFI peaks at $t_{\text{peak}} \approx 1.7\tau_1$ for quenching frequency $\Omega_0 = 1.3\gamma$ and $t_{\text{peak}} \approx 1.6\tau_1$ for $\Omega_0 = 1.1\gamma$ before it decays monotonically to the steady-state value. When we apply a power-law fitting to the QFI, the exponent b remains approximately constant during the enhancement time window: $b \approx 3.5$ for $\Omega_0 = 1.3\gamma$ and $b \approx 4.1$ for $\Omega_0 = 1.1\gamma$. This is shown in Figs. 7(b) and 7(d). These scalings are results of both the correlation of U and ε as stated in Sec. III B and

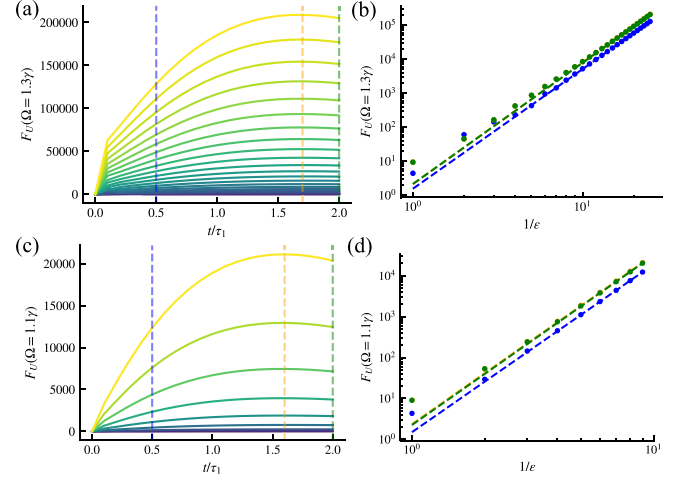


FIG. 7. The QFI of the metastable state $F_U(\rho(t))$ for $\theta = U$ at (a), (b) $\Omega_0 = 1.3\gamma$ and (c), (d) $\Omega_0 = 1.1\gamma$. The meanings of the plots are the same as Fig. 6 except that the orange line in panel (b) refers to $t_{\text{peak}} \approx 1.7\tau_1$. The exponents for the QFIs are approximately (b) 3.52, 3.57, 3.56 for $t/\tau_1 = 0.5, 1.7, 2.0$ and (d) 4.10, 4.14, 4.15 for $t/\tau_1 = 0.5, 1.7, 2.0$.

the enhancement of sensing scalings over a fixed parameter (as shown in the F_Δ study) due to the metastability.

C. Time-evolved state sensing away from the metastable region

In the region of Ω_0 where $\text{Re}(\lambda_1)$ is not separated from the rest of the spectrum, the system can no longer be treated as an effective two-level system and the metastability is not observed. Figure 8 shows the QFI of $\theta = \Delta$ for the time-evolved state initially at vacuum state. The QFI oscillates and there is no clear scaling law with respect to $1/\varepsilon$. Therefore, the sensing response in this regime is transient in nature.

V. COMPARISON BETWEEN STEADY-STATE AND METASTABLE-STATE SENSING

In this section, the QFI and the sensing time are compared between the steady state at the critical point and the metastable states. For the steady state at the critical point,

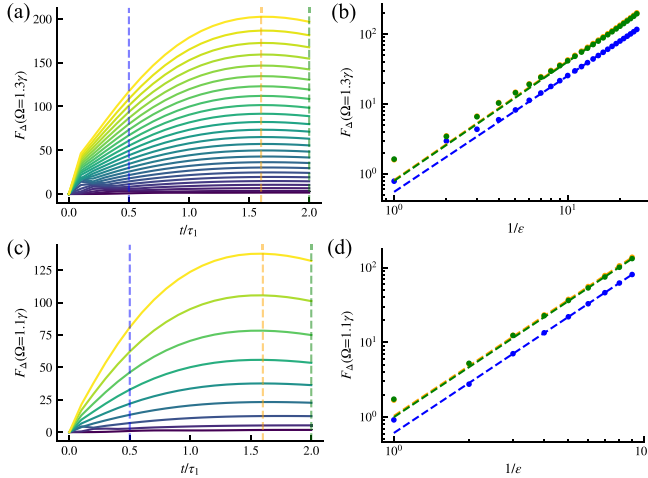


FIG. 6. The QFI of the metastable state $F_\theta(\rho(t))$ for $\theta = \Delta$ at (a), (b) $\Omega_0 = 1.3\gamma$ and (c), (d) $\Omega_0 = 1.1\gamma$. (a) As a function of t scaled with $\tau_1(\varepsilon)$ at $\varepsilon = 1/1, 1/2, \dots, 1/25$, bottom to top. Vertical dashed lines are $t/\tau_1 = 0.5, 1.6, 2$ sampled around the peak for plotting [panel (b)]. (b) As a function of $1/\varepsilon$ at $t/\tau_1 = 0.5, 1.6, 2$ for blue, orange, and green dots, respectively. The dashed lines are power-law fittings with exponents 1.66, 1.70, 1.70. (c) As a function of t scaled with $\tau_1(\varepsilon)$ at $\varepsilon = 1/1, 1/2, \dots, 1/9$, bottom to top. Vertical dashed lines are $t/\tau_1 = 0.5, 1.6, 2$ sampled around the peak for plotting [panel (d)]. (d) With the same meaning as panel (b). The exponents for the power-law fittings are all 2.22.

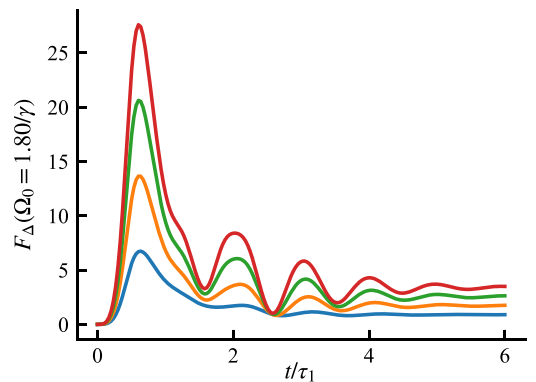


FIG. 8. The QFI of the state evolved from vacuum $F_\theta(\rho(t))$ for $\theta = \Delta$ as a function of t scaled with an identical τ_1 at $\varepsilon = 1/5, 1/10, 1/15, 1/20$.

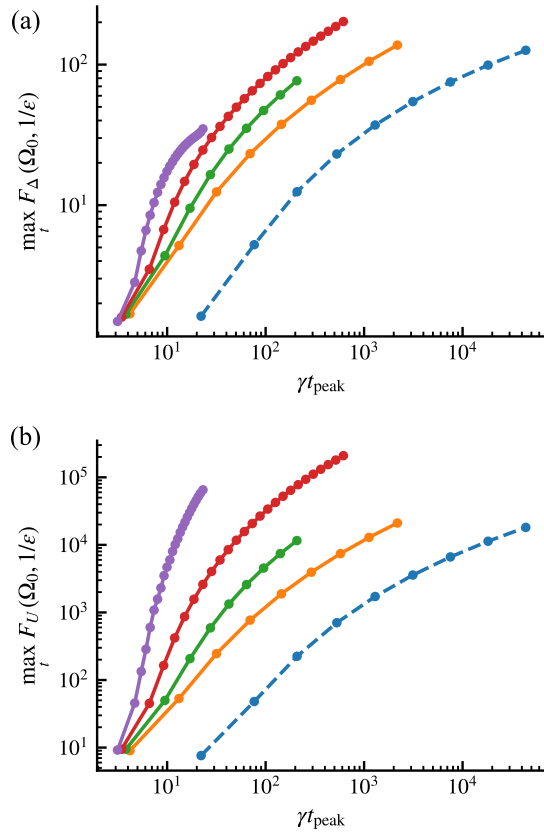


FIG. 9. The QFI F_θ for (a) $\theta = \Delta$ and (b) $\theta = U$. The lines represent the peak QFIs and the corresponding sensing times of the metastable states (solid) and the steady state (dashed). From left to right, the metastable states are quenched from vacuum to $\Omega_0 = 1.4\gamma$, 1.3γ , 1.2γ , 1.1γ . The dots in the lines correspond to $\varepsilon = 1/1, 1/2, \dots, 1/25$ for $\Omega_0 = 1.4\gamma, 1.3\gamma$ and $\varepsilon = 1/1, 1/2, \dots, 1/9$ for $\Omega_0 = 1.2\gamma, 1.1\gamma$, bottom to top.

we take $8\tau_1(\Omega_{0,c})$ as the sensing time and for the metastable state, we take t_{peak} as the sensing time. As shown in Fig. 9, within a fixed sensing time, metastable states can provide a larger QFI than the steady state at the critical point with a properly tuned Ω_0 and ε . Further, it is preferred to quench the Rabi frequency Ω_0 to an intermediate value between the critical point and the metastability boundary. This is because the states close to the critical point suffer from the critical slowing down and require much longer times to reach the states with the maximum QFI. On the other hand, when Ω_0 is set too close to the metastability boundary, the eigenvalues of the Liouvillian vary weakly when tuning ε . Therefore, the effective two-level dynamics and the QFI varies weakly with ε . This can be seen from the dense dots at the end of the line for $\Omega_0 = 1.4\gamma$. Hence, increasing the QFI at such a value of Ω_0 requires a larger decrease in ε and poses difficulties in

actual experiments since it requires very small nonlinearity and very large drive strength. We note that recent experiments can push up to $\gamma t \approx 287$ [43].

VI. CONCLUSION

In this work, we studied the sensing performance of a quantum Duffing oscillator with a coherent drive and cavity leakage. The system shows a first-order dissipative phase transition and an effective two-level behavior in a metastable region around the critical point. These are the consequences of the closing of the Liouvillian gap. The steady-state sensing of the detuning parameter and the rescaled Kerr nonlinearity exhibit Heisenberg scaling with respect to the effective system size at the critical point and SQL scaling away from the critical point. The Heisenberg scaling is subject to critical slowing down due to the Liouvillian-gap closing. When the system dynamics is taken into account, the QFI of the metastable state shows an enhancement compared to the steady-state value. The enhancement occurs at the characteristic timescale determined by the Liouvillian gap and persists for a duration at the same timescale. The maximum QFI during the enhancement increases as power law when the parameters are scaled with the effective system size within this enhancement window. Such scaling is not found outside the metastable region. With a given sensing time, a specific type of metastable states is preferred for achieving better sensing performance. These states are prepared at an intermediate value of Ω_0 between the critical point and the metastability boundary, and under a large effective system size. These states outperform sensing with the steady state at the critical point and with metastable states close to the critical point, due to the increasingly severe critical slowing down near criticality. They are also better realized experimentally than states close to the metastability boundary, which require very small nonlinearity and very large drive strength to scale up the QFI. Overall, these results characterize the sensing capability of the steady state of a system with a first-order dissipative phase transition. They also enable the use of metastable states to achieve improved sensing performance at shorter evolution times than steady-state protocols.

ACKNOWLEDGMENTS

Authors thank the partial support of NSF Grant No. 2426699 and the WELCH foundation Grant No. A-1943-20240404 for this work. Z.Z. acknowledges discussions with Q. Miao, T. Holdsworth, and M. Durjoy.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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