

Entanglement suppression, quantum statistics, and symmetries in spin-3/2 baryon scatterings

Tao-Ran Hu^{1,*}, Katsuyoshi Sone^{2,†}, Feng-Kun Guo^{1,3,‡}, Tetsuo Hyodo^{2,§,||} and Ian Low^{4,5,¶}

¹*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China*

²*Department of Physics, Tokyo Metropolitan University, Hachioji 192-0397, Japan*

³*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China*

⁴*High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

⁵*Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA*



(Received 16 June 2025; accepted 14 November 2025; published 16 December 2025)

We explore the interplay among entanglement suppression, quantum statistics, and enhanced symmetries in the nonrelativistic S -wave scattering involving the lowest-lying spin-3/2 baryons, which can be considered as four-dimensional qudits. These baryons form a ten-dimensional representation (decuplet) under the $SU(3)$ light-flavor symmetry and, in this limit, are considered indistinguishable under strong interactions. Treating the S matrix in the spin-3/2 baryon-baryon scattering as a quantum logic gate in the spin space, we study the consequence of entanglement suppression and compute the entanglement power of the S matrix. When the entanglement power vanishes, the S matrix is either an Identity or a SWAP gate and spin-flavor symmetries and/or nonrelativistic conformal invariance emerge, as previously observed in spin-1/2 baryons. In the case of scattering identical particles, the entanglement power never vanishes due to constraints from spin statistics, which we interpret as projection-valued measurements onto symmetric or antisymmetric Hilbert space and define the entanglement power accordingly. When the entanglement power is nonvanishing but sits at a global or local minimum, enhanced symmetries still emerge and the S matrix can be interpreted as an Identity or a SWAP gate acting on the restricted Hilbert space allowed by quantum statistics. In general, when scattering identical spin- s particles, we identify an enhanced $SU(2s + 1)_{\text{spin}}$ symmetry for the Identity gate.

DOI: [10.1103/x52w-4rbs](https://doi.org/10.1103/x52w-4rbs)

I. INTRODUCTION

One remaining fundamental challenge in the Standard Model of particle physics is the nonperturbative dynamics of the strong interaction. Since all quarks and gluons are confined inside color-singlet hadrons, in order to gain more insights, one needs to study hadron properties and hadron-hadron interactions to reveal patterns from either experimental data or lattice quantum chromodynamics (QCD) calculations. For instance, let us consider the nucleon-nucleon interaction at low energies. A pair of nucleons, being spin-1/2 and isospin-1/2 particles, in S waves can form either an isospin-singlet $^3S_1$ or an isospin-triplet $^1S_0$ combination. In principle, the interactions in these pairs can be very much different. However, from experimental data on nuclear force, the interaction strengths in these pairs are approximately equal [1]. This was traditionally

explained by considering the large- N_c , with N_c the number of colors in QCD, limit [3]; however, recently, an explanation using the so-called entanglement suppression was proposed [4]. It is thus interesting to explore whether the entanglement-suppression hypothesis that minimizing the entanglement in a quantum scattering process leads to an enlarged symmetry absent in the original action of the theory is universal in the infrared region of QCD.

Quantum entanglement is the most prominent feature of quantum mechanics. Yet its significance in the context of particle scatterings and quantum field theory was not greatly appreciated until recently [5–14]. One of the most fascinating results is the observation that entanglement suppression, or maximization, in 2-to-2 scattering processes could lead to enhanced symmetries in scattering dynamics [4,15–23]. Other intriguing developments include using entanglement to understand the flavor patterns in the Standard Model [24] or the dynamics of electroweak phase transition [25]. These studies open up a frontier to understand fundamental interactions of Nature from the information-theoretic perspective and offer insights into observables in high-energy colliders.

Demanding the S matrix suppresses spin entanglement leads to emergent symmetries that are not present in the fundamental Lagrangian of QCD. In particular, it was found that [15] when the S matrix is an Identity gate in the two-qubit space, the resulting symmetry is the spin-flavor symmetry [26–29], while a SWAP gate gives rise to the nonrelativistic conformal invariance [30,31]. More generally, when the u , d , and s quarks are taken to be degenerate in mass, the

*Contact author: hutaoran21@mails.ucas.ac.cn

†Contact author: sone-katsuyoshi@ed.tmu.ac.jp

‡Contact author: fkguo@itp.ac.cn

§Contact author: hyodo@rcnp.osaka-u.ac.jp

||Present address: Research Center for Nuclear Physics (RCNP), Ibaraki, Osaka 567-0047, Japan.

¶Contact author: ilow@northwestern.edu

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

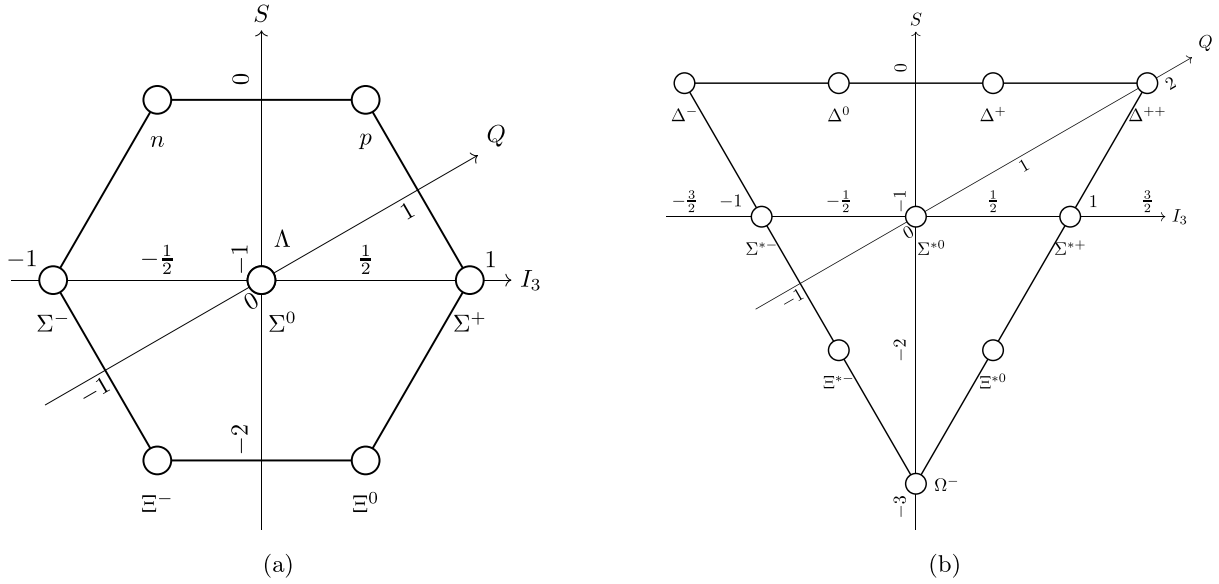


FIG. 1. Displayed in the plots are (a) the spin-1/2 ground-state baryon octet and (b) the spin-3/2 baryon decuplet in the SU(3) flavor symmetry framework. The axes Q , S , and I_3 represent charge, strangeness, and the third component of isospin, respectively.

nucleons are part of the eight-dimensional octet representation under the SU(3) flavor symmetry, which contains other spin-1/2 baryons, as shown in Fig. 1(a). Analyzing the 2-to-2 scatterings of the spin-1/2 octet baryons, which include 64 channels in total, revealed more instances of enhanced symmetries due to entanglement suppression in the spin space. This suggests that entanglement suppression may be regarded as an emergent phenomenological property of low-energy hadronic dynamics, realized either when the scattering exhibits larger symmetries than naively expected or when it approaches the nonrelativistic conformal limit. The correlation between entanglement and symmetry goes beyond low-energy QCD, as similar connections were also pointed out in the fully relativistic scattering of two qubits in the context of two-Higgs-doublet models [17,23], which are prototypical models for electroweak symmetry breaking. Moreover, Ref. [32] recently studied the appearance of symmetries in terms of the conservation of nonstabilizerness, also known as magic, during the scattering processes [33–40].

Building on the observations in the spin-1/2 octet baryons, in this work we will focus on the nonrelativistic, S -wave scattering of the lowest-lying spin-3/2 light baryons, which form a ten-dimensional representation (decuplet) under the SU(3) flavor symmetry, and study the relation between entanglement suppression and enhanced symmetries. In Fig. 1(b), we show the electric charge (Q) and the strangeness quantum number (S) of the decuplet baryons, which are conserved in QCD. Such a system is interesting for a variety of reasons. First, the spin-3/2 baryon can be viewed as a four-dimensional qudit in the spin space [41] and the two-body S matrix would be a two-qudit quantum gate. There have been attempts to study entanglement and symmetry in two-qutrit systems, such as the pion-pion scattering analyzed as a two-qutrit system in the isospin space [42] or the deuteron-deuteron scattering as a system of two spin-1 particles [43]. However, in the limit of exact isospin symmetry, pions are indistinguishable particles under strong interactions. They obey Bose-Einstein (BE) statistics

and the S matrix only acts on the sub-Hilbert space with totally symmetric wave functions, which needs to be dealt with care. Similar subtleties arise in identical deuteron-deuteron scattering as well [44]. In the limit of exact SU(3) flavor symmetry, both the spin-1/2 octet and the spin-3/2 decuplet baryons need to follow the Fermi-Dirac (FD) statistics. The case of spin-1/2 baryons (qubits) has been analyzed in detail in Ref. [16]. However, we will see for spin-3/2 baryons (qudits) a rich and intricate interplay among entanglement, quantum statistics, and symmetry appears.

Although our calculation is general and could be applied to any two-qudit systems, it is insightful to compare with experimental data, lattice QCD studies, and large- N_c analyses on the scattering of spin-3/2 baryons. For instance, based on the deuteron in $^3S_1$ NN scattering and the existence of a virtual state in $^1S_0$ NN scattering, four more zero-strangeness dibaryons were predicted using an SU(6) model long ago [45], including two $\Delta\Delta$ dibaryons with $(J, I) = (3, 0)$ and $(0, 3)$. There has been experimental evidence for the $(J, I) = (3, 0)$ dibaryon, namely, $d^*(2380)$ reported by the WASA-at-COSY Collaboration [46–49]. In addition, lattice calculation for the $\Omega\Omega$ system is available for the $^1S_0$ channel, leading to a large scattering length $a = 4.6$ fm [51]. Finally, large- N_c consequence on the decuplet baryons (mainly in the strangeness $S = 0$ sector) has recently been discussed in Ref. [52].

The paper is organized as follows. In Sec. II, we present the general construction of the S matrix for the nonrelativistic scattering of spin-3/2 decuplet baryons. In Sec. III, the definition and detailed calculation of the entanglement power in the spin space for distinguishable particles are discussed. In Sec. IV, we discuss the conditions for entanglement suppression in decuplet-baryon scatterings, followed by Sec. V for the identification of enhanced symmetries using group-theoretic reasoning. These symmetries are verified in the Lagrangian approach using nonrelativistic effective field theories in Sec. VI. Then, in Sec. VII we present a dedicated analysis for identical particle scatterings. Summary and

outlook are given in Sec. VIII. We also present two Appendixes: Appendix A on the classification of the spin and flavor quantum numbers for decuplet scatterings and Appendix B on the explicit calculation of entanglement power in neutron-neutron scattering.

II. S MATRIX FOR S-WAVE SCATTERINGS

In this paper, we primarily study the scattering of two light decuplet baryons in the low-energy regime [53], which is dominated by S -wave interactions. Therefore, the total angular momentum of the system is given by the total spin of the two baryons. Also, we consider the leading order (LO) of a nonrelativistic effective field theory (NREFT). At this order, all decuplet baryons are stable as their widths come from self-energy corrections at loop level, and no SU(3) breaking operators exist as such operators must contain powers of the quark mass matrix and are subleading, suppressed by $\mathcal{O}(m_s/\Lambda_{\text{QCD}})$, with m_s and Λ_{QCD} being the strange quark mass and the nonperturbative QCD scale, respectively. Thus, we proceed with the discussion assuming exact flavor SU(3) symmetry though the effects of symmetry breaking can be important in certain aspects of actual baryon scatterings. Consequently, all emergent symmetries predicted in this framework should also be regarded as approximate in reality, just as SU(3) flavor symmetry itself is.

Our focus lies in understanding the spin and flavor symmetries of the system, so we define our S matrix as

$$\hat{S} = \sum_{J,F} \mathcal{J}_J \otimes \mathcal{F}_F e^{2i\delta_{JF}}, \quad (1)$$

where $\mathcal{J}_J \otimes \mathcal{F}_F$'s denote the projection operators onto subspaces belonging to definite irreducible representations (irreps) of the spin group SU(2) and the flavor group SU(3). Specifically, J can be regarded as the total spin of this irrep. Therefore, the S matrix is constructed using these projectors and the corresponding phase shifts $\delta_{JF}(p)$'s as in Eq. (1), which in general depend on the center-of-mass momentum p of the scattering, although we often suppress the p dependence for brevity.

The decomposition of S matrix into irreps is determined from group theory, by the tensor product of the spin and flavor properties of the individual particles in the 2-to-2 scattering, similar to the addition of angular momenta in quantum mechanics. For two particles belonging to the irreps A and B , the tensor product $A \otimes B$ can be decomposed into a direct sum of irreps:

$$A \otimes B = \oplus_i C_i. \quad (2)$$

The operator $\mathcal{P}_{C_i}$ projects the tensor product $A \otimes B$ into a specific irrep C_i :

$$\mathcal{P}_{C_i}|C_j\rangle = \delta_{ij}|C_j\rangle, \quad \mathcal{P}_{C_i}|A\rangle \otimes |B\rangle = |C_i\rangle. \quad (3)$$

The projectors can be expressed as polynomials of Casimir operators $\mathbf{a} \cdot \mathbf{b} \equiv \sum_\alpha a^\alpha \otimes b^\alpha$ whose eigenvalues label the irreps. Here, $\mathbf{a} = \{a^\alpha\}$ and $\mathbf{b} = \{b^\alpha\}$ denote the generators of the spin SU(2) or flavor SU(3) in the corresponding representations A and B .

Focusing on the spin space for demonstration purposes, let us consider the scattering of two distinguishable spin-1/2

particles, such as the neutron-proton scattering in the S wave, where the total angular momentum is simply given by the total spin:

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1, \quad (4)$$

so there are two projectors for the two total spin values $J = 0$ and 1. The Casimir operator is $\mathbf{t}_{1/2} \cdot \mathbf{t}_{1/2}$, with $\mathbf{t}_s$ the SU(2) generators in the $(2s+1)$ -dimensional representation. Therefore, $\mathbf{t}_{1/2} = \boldsymbol{\sigma}/2$, with $\boldsymbol{\sigma}$ the Pauli matrices. Now we can write down the S matrix for a $1/2 \otimes 1/2$ system:

$$\hat{S} = \mathcal{J}_0 e^{2i\delta_0} + \mathcal{J}_1 e^{2i\delta_1}, \quad (5)$$

where

$$\begin{aligned} \mathcal{J}_0 &= \frac{1}{4}[1 - 4(\mathbf{t}_{1/2} \cdot \mathbf{t}_{1/2})], \\ \mathcal{J}_1 &= \frac{1}{4}[3 + 4(\mathbf{t}_{1/2} \cdot \mathbf{t}_{1/2})]. \end{aligned} \quad (6)$$

It is easy to check

$$\mathcal{J}_0|J=0\rangle = |J=0\rangle, \quad \mathcal{J}_0|J=1\rangle = 0, \quad (7)$$

$$\mathcal{J}_1|J=0\rangle = 0, \quad \mathcal{J}_1|J=1\rangle = |J=1\rangle, \quad (8)$$

where $|J=0\rangle$ and $|J=1\rangle$ denote the states with total spin $J=0$ and $J=1$, respectively.

The discussion can be straightforwardly extended to two distinguishable spin-3/2 particle scatterings in the S wave. The total spin of two such particles ranges from 0 to 3, which indicates that the projectors should be cubic polynomials of the Casimir operator. They read as

$$\begin{aligned} \mathcal{J}_0 &= \frac{33}{128} + \frac{31}{96}(\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2}) - \frac{5}{72}(\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2})^2 \\ &\quad - \frac{1}{18}(\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2})^3, \end{aligned} \quad (9)$$

$$\begin{aligned} \mathcal{J}_1 &= -\frac{81}{128} - \frac{117}{160}(\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2}) + \frac{9}{40}(\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2})^2 \\ &\quad + \frac{1}{10}(\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2})^3, \end{aligned} \quad (10)$$

$$\begin{aligned} \mathcal{J}_2 &= \frac{165}{128} + \frac{23}{96}(\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2}) - \frac{17}{72}(\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2})^2 \\ &\quad - \frac{1}{18}(\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2})^3, \end{aligned} \quad (11)$$

$$\begin{aligned} \mathcal{J}_3 &= \frac{11}{128} + \frac{27}{160}(\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2}) + \frac{29}{360}(\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2})^2 \\ &\quad + \frac{1}{90}(\mathbf{t}_{3/2} \cdot \mathbf{t}_{3/2})^3, \end{aligned} \quad (12)$$

where $\mathbf{t}_{3/2}$ represent the SU(2) generators in the four-dimensional representation. The S matrix in the spin space is then

$$\hat{S} = \mathcal{J}_0 e^{2i\delta_0} + \mathcal{J}_1 e^{2i\delta_1} + \mathcal{J}_2 e^{2i\delta_2} + \mathcal{J}_3 e^{2i\delta_3}. \quad (13)$$

Of course, the full decuplet-decuplet S matrix involves both SU(2) (spin) and SU(3) (flavor) projectors. However, as we will demonstrate in Sec. IV A, when a system is constrained by quantum statistics, as is the case for octet and decuplet baryons in the limit of exact SU(3) flavor symmetry, it is often sufficient to consider only the spin space (or, equivalently, only the flavor space), which greatly simplifies the analysis.

III. ENTANGLEMENT POWER FOR DISTINGUISHABLE PARTICLES

Entanglement measures have been developed to quantify the degree of entanglement present in a given quantum system. In this section, we focus on systems with distinguishable particles, and identical particles will be discussed in Sec. IV A.

For a bipartite state $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$, a widely used entanglement measure is the linear entropy, defined as (see, e.g., Refs. [16,42])

$$\mathcal{E}(|\psi\rangle) = 1 - \text{Tr}_A [\rho_A^2], \quad (14)$$

where $\rho = |\psi\rangle\langle\psi|$ is the density matrix and $\rho_A = \text{Tr}_B[\rho]$ is the reduced density matrix obtained after tracing over subsystem B . $\mathcal{E}(|\psi\rangle)$ is semipositive definite and vanishes only when $|\psi\rangle$ is a tensor product, i.e., $|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$, as shown in the Appendix of Ref. [19].

While entanglement measure describes how entangled a given state is, entanglement power characterizes the capacity of an operator $\hat{U}$ to generate entanglement. It is defined by averaging the entanglement of the output states produced by acting $\hat{U}$ on all possible product states [54]:

$$E(\hat{U}) = \overline{\mathcal{E}(\hat{U}|\psi_A\rangle \otimes |\psi_B\rangle)} \equiv \int d\omega_A d\omega_B \mathcal{E}(\hat{U}|\psi_A\rangle \otimes |\psi_B\rangle), \quad (15)$$

where the integral is taken over all elements belonging to irreps A and B , with $d\omega_{A,B}$ denoting the corresponding normalized integration measures:

$$\int d\omega_A = \int d\omega_B = 1. \quad (16)$$

In general, a quantum state in an $(m+1)$ -dimensional Hilbert space resides on the manifold $\mathbb{CP}^m$, which can be parametrized as

$$|\psi_{m+1}\rangle = (n_0, n_1 e^{iv_1}, \dots, n_m e^{iv_m}), \quad v_i \in [0, 2\pi), \quad (17)$$

where

$$\begin{cases} n_0 = \cos \theta_1 \sin \theta_2 \sin \theta_3 \cdots \sin \theta_m, \\ n_1 = \sin \theta_1 \sin \theta_2 \sin \theta_3 \cdots \sin \theta_m, \\ n_2 = \cos \theta_2 \sin \theta_3 \cdots \sin \theta_m, \\ \dots \dots \dots \\ n_m = \cos \theta_m, \end{cases} \quad \theta_i \in \left[0, \frac{\pi}{2}\right). \quad (18)$$

The commonly used Fubini-Study measure is [55,56]

$$\begin{aligned} d\omega_{m+1} &= \frac{m!}{\pi^m} \prod_{i=1}^m d\theta_i dv_i \cos \theta_i \sin^{2i-1} \theta_i, \\ \int d\omega_{m+1} &= 1, \end{aligned} \quad (19)$$

which is properly normalized. Note that for a spin- s particle, $m = 2s$.

Entanglement power serves as a state-independent measure of entanglement generation, which is semipositive definite and vanishes if and only if $\hat{U}|\psi\rangle$ remains a tensor-product state for all $|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$. The entanglement power $E(\hat{S})$ of an S matrix can then be defined as

$$E(\hat{S}) = 1 - \int d\omega_A d\omega_B \text{Tr}_A [\rho_A^2], \quad (20)$$

where

$$\rho = |\psi_{\text{out}}\rangle\langle\psi_{\text{out}}|, \quad |\psi_{\text{out}}\rangle = \hat{S}|\psi_{\text{in}}\rangle, \quad |\psi_{\text{in}}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle. \quad (21)$$

For distinguishable particles, the S matrix acts on the full Hilbert space spanned by the initial state $|\psi_{\text{in}}\rangle$. The unitarity of the S matrix ensures that the density matrix is properly normalized: $\text{Tr}[\rho] = 1$. In the spin space, this averaging integration in Eq. (20) can be understood as originating from the fact that the directions of the spins in the initial states can be arbitrary.

We first take the $1/2 \otimes 1/2$ system for example. $|\psi_A\rangle$ and $|\psi_B\rangle$ are two distinguishable spin-1/2 particles. Setting $m+1 = 2s+1 = 2$ in Eq. (17),

$$|\psi_2\rangle = (\cos \theta_1, \sin \theta_1 e^{iv_1})^T, \quad (22)$$

with $\theta_1 \in [0, \pi/2)$ and $v_1 \in [0, 2\pi)$. The integral measure $d\omega_2$ then reads as

$$d\omega_2 = \frac{1}{\pi} d\theta_1 \cos \theta_1 \sin \theta_1 dv_1. \quad (23)$$

This parametrization is identical to the usual Bloch sphere parametrization for a qubit with (θ, ϕ) as $\theta_1 = \theta/2$ and $v_1 = \phi$.

Now we can calculate the entanglement power for various cases. For distinguishable particles, the result for the $1/2 \otimes 1/2$ S matrix in Eq. (5) has been known [4,16]:

$$E(\hat{S}) = \frac{1}{6} \sin^2 [2(\delta_0 - \delta_1)], \quad (24)$$

and $E(\hat{S}) = 0$ yields two possible solutions (modulo multiple times π):

$$|\delta_0 - \delta_1| = 0 \text{ or } \frac{\pi}{2}. \quad (25)$$

Note that we restrict the phase shifts to the range $\delta \in [0, \pi)$ because they only appear in the form of $e^{2i\delta}$. Moreover, the solutions for $|\delta_0 - \delta_1| = \pi/2$ are $\delta_0 = 0$ and $\delta_1 = \pi/2$ (or vice versa), as will be proven in Sec. VI. We highlight here these two particularly important phase shift values associated with the noninteracting limit and unitary limit [57], respectively; see more discussions in Sec. VI.

For the scattering of distinguishable spin-3/2 particles, the S matrix is given in Eq. (13) and we obtain

$$\begin{aligned} E(\hat{S}) &= \frac{1}{200000} \{ 77482 - 2100 \cos [2(\delta_0 + \delta_1 - \delta_2 - \delta_3)] - 2100 \cos [2(\delta_0 - \delta_1 + \delta_2 - \delta_3)] \\ &\quad - 2100 \cos [2(\delta_0 - \delta_1 - \delta_2 + \delta_3)] - 1200 \cos [2(\delta_0 - 2\delta_1 + \delta_2)] - 4200 \cos [2(\delta_0 + \delta_2 - 2\delta_3)] \\ &\quad - 8400 \cos [2(\delta_1 - 2\delta_2 + \delta_3)] - 375 \cos [4(\delta_0 - \delta_1)] - 10800 \cos [2(\delta_0 - \delta_2)] - 625 \cos [4(\delta_0 - \delta_2)] \\ &\quad - 875 \cos [4(\delta_0 - \delta_3)] - 2175 \cos [4(\delta_1 - \delta_2)] - 26376 \cos [2(\delta_1 - \delta_3)] \\ &\quad - 5481 \cos [4(\delta_1 - \delta_3)] - 10675 \cos [2(\delta_2 - \delta_3)] \}, \end{aligned} \quad (26)$$

which vanishes when all the cosines are valued at +1 and leads to the following nonentangling solutions:

$$\begin{aligned} \delta_0 = \delta_2 = \delta_{\text{even}}, \quad \delta_1 = \delta_3 = \delta_{\text{odd}}, \\ |\delta_{\text{even}} - \delta_{\text{odd}}| = 0 \text{ or } \frac{\pi}{2}. \end{aligned} \quad (27)$$

We observe an intriguing feature of the solutions: the phase shifts for all channels with even total spin are equal, and similarly, those for odd total spin are also equal, with the two sets differing by either 0 or $\pi/2$. This turns out to be a general pattern that we prove in the next section.

IV. ENTANGLEMENT SUPPRESSION IN DECUPLER SCATTERING

A. A general result for two qudits

In this section, we begin with the observation made at the end of the previous section and formulate a general result concerning entanglement suppression for two-qudit systems. Although we focus on spin-3/2 particles as four-dimensional qudits, the generalization to arbitrary qudits is straightforward. Additionally, we consider the connection between quantum statistics and quantum entanglement. The discussion here provides an alternative perspective to the general reasoning in Ref. [22].

A natural starting point is to consider the form of the operators implied by the phase-shift relations in Eq. (27). If $|\delta_{\text{even}} - \delta_{\text{odd}}| = 0$, meaning all four phase shifts are equal, the resulting S matrix is simply an Identity operator with an overall phase, which clearly has zero entanglement power (it leaves any tensor-product state invariant). The other solution $|\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2$ is particularly interesting, with the nonentangling S matrix proportional to

$$-(\mathcal{J}_0 - \mathcal{J}_1 + \mathcal{J}_2 - \mathcal{J}_3) = \sum_{\text{odd } J} \mathcal{J}_J - \sum_{\text{even } J} \mathcal{J}_J. \quad (28)$$

Note that channels with an odd total spin J correspond to the symmetric part of the $3/2 \otimes 3/2$ tensor-product space, while those with an even J correspond to the antisymmetric part, so the above operator is precisely a SWAP gate [15], which is a quantum logic gate exchanging the two substates, thus yielding 1 when acting on symmetric states and -1 on antisymmetric states.

In general, the SWAP operator can be defined as follows for a bipartite system with equal dimensions: $\mathcal{H}_R \otimes \mathcal{H}_R$, where R is a certain irrep of a group G . The tensor product decomposes as

$$R \otimes R = (\oplus_i S_i) \oplus (\oplus_j A_j), \quad (29)$$

where S_i and A_j denote symmetric and antisymmetric irreps, respectively. The SWAP is then given by the sum of projectors onto symmetric irreps (S_i) minus the sum of projectors onto

antisymmetric ones (A_j):

$$\begin{aligned} \text{SWAP} &= \sum_i S_i - \sum_j A_j \equiv \mathcal{P}_S - \mathcal{P}_A, \\ \sum_i S_i &\equiv \mathcal{P}_S, \quad \sum_j A_j \equiv \mathcal{P}_A, \end{aligned} \quad (30)$$

where $\mathcal{P}_S$ and $\mathcal{P}_A$ are the projection operators into the symmetric and antisymmetric parts of the bipartite Hilbert space, respectively,

$$\begin{aligned} \mathcal{P}_S |S_i\rangle &= |S_i\rangle, \quad \mathcal{P}_S |A_i\rangle = 0, \\ \mathcal{P}_A |A_i\rangle &= |A_i\rangle, \quad \mathcal{P}_A |S_i\rangle = 0, \end{aligned} \quad (31)$$

which implies that $|S_i\rangle$ and $|A_j\rangle$ are eigenstates of SWAP with +1 and -1 eigenvalues:

$$\text{SWAP} |S_i\rangle = |S_i\rangle, \quad \text{SWAP} |A_j\rangle = -|A_j\rangle. \quad (32)$$

Note that $\mathcal{P}_S$ and $\mathcal{P}_A$ are complete in the sense that $\mathcal{P}_S + \mathcal{P}_A = 1$ is the Identity operator over the entire Hilbert space. In fact, if we choose a basis consisting of symmetric and antisymmetric base vectors, $\mathcal{P}_S$ is an Identity operator over the symmetric part of the Hilbert space and vanishes over the antisymmetric part. Conversely for $\mathcal{P}_A$.

Under the definition in Eq. (30), one can show that SWAP interchanges the two qudits. Let us consider an arbitrary initial product state written as

$$|\psi\rangle = |a\rangle \otimes |b\rangle, \quad |a\rangle = \sum_i a_i |\phi_i\rangle, \quad |b\rangle = \sum_j b_j |\phi_j\rangle, \quad (33)$$

where $\{|\phi_i\rangle\}$ is a set of basis vectors for $\mathcal{H}_R$. The SWAP gate acts on it as follows:

$$\begin{aligned} \text{SWAP} |\psi\rangle &= \text{SWAP} \left(\sum_i a_i |\phi_i\rangle \otimes \sum_j b_j |\phi_j\rangle \right) \\ &= \text{SWAP} \left(\sum_{i,j} \frac{a_i b_j + b_i a_j}{2} |\phi_i\rangle \otimes |\phi_j\rangle \right. \\ &\quad \left. + \sum_{i,j} \frac{a_i b_j - b_i a_j}{2} |\phi_i\rangle \otimes |\phi_j\rangle \right) \\ &= \sum_{i,j} \frac{a_i b_j + b_i a_j}{2} |\phi_i\rangle \otimes |\phi_j\rangle \\ &\quad - \sum_{i,j} \frac{a_i b_j - b_i a_j}{2} |\phi_i\rangle \otimes |\phi_j\rangle \\ &= |b\rangle \otimes |a\rangle, \end{aligned} \quad (34)$$

which is simply the product state with the two qudits interchanged.

It has also been shown that, for bipartite systems $\mathcal{H}_R \otimes \mathcal{H}_R$, the Identity and the SWAP are the only two gates that would always map a product state to another product state [15,58–60]; these are the only two minimally entangling quantum gates. If we consider the group $G = \text{SU}(2)_{\text{spin}}$, this

directly provides the minimal-entanglement condition for the S -wave scattering between two spin- s particles. The total spin can range from 0 to $2s$, corresponding to $(2s + 1)$ phase shifts. Entanglement power of its S matrix vanishes, $E(\hat{S}) = 0$, if and only if the phase shifts for even total spins are all equal (δ_{even}), the phase shifts for odd total spins are all equal (δ_{odd}), and

$$|\delta_{\text{even}} - \delta_{\text{odd}}| = 0 \text{ or } \frac{\pi}{2}. \quad (35)$$

This is the entanglement-suppression condition for distinguishable particles in the spin space.

In the cases of octet-octet and decuplet-decuplet scatterings, the S matrix involves the projection operators in both the spin and flavor spaces, as shown in Eq. (1). In this case, the group $G = \text{SU}(2)_{\text{spin}} \otimes \text{SU}(3)_{\text{flavor}}$ and an irrep R of G can be written as the tensor product of irreps $R = R_{\text{spin}} \otimes R_{\text{flavor}}$. As we focus on minimal entanglement in the spin space, it is convenient to define the Identity and SWAP operators acting on the spin and the flavor spaces separately. For instance, consider the action of $\text{SWAP}_{\text{spin}} \otimes \text{SWAP}_{\text{flavor}}$ on the qudits:

$$\text{SWAP}_{\text{spin}} \otimes \text{SWAP}_{\text{flavor}} |s_1, f_1\rangle \otimes |s_2, f_2\rangle = |s_2, f_2\rangle \otimes |s_1, f_1\rangle, \quad (36)$$

where s_i and f_i refer to the spin and flavor quantum numbers, respectively.

If the qudits further obey quantum statistics—BE for bosons or FD for fermions—we obtain

$$|s_2, f_2\rangle \otimes |s_1, f_1\rangle = \begin{cases} +|s_1, f_1\rangle \otimes |s_2, f_2\rangle, & \text{BE,} \\ -|s_1, f_1\rangle \otimes |s_2, f_2\rangle, & \text{FD.} \end{cases} \quad (37)$$

Therefore, we immediately conclude [16]

$$\text{SWAP}_{\text{spin}} \otimes \text{SWAP}_{\text{flavor}} = \begin{cases} +1_{\text{spin}} \otimes 1_{\text{flavor}}, & \text{BE,} \\ -1_{\text{spin}} \otimes 1_{\text{flavor}}, & \text{FD.} \end{cases} \quad (38)$$

Moreover, one can similarly derive

$$\text{SWAP}_{\text{spin}} \otimes 1_{\text{flavor}} = \begin{cases} +1_{\text{spin}} \otimes \text{SWAP}_{\text{flavor}}, & \text{BE,} \\ -1_{\text{spin}} \otimes \text{SWAP}_{\text{flavor}}, & \text{FD.} \end{cases} \quad (39)$$

From the above discussion, we arrive at the following conclusion: in the presence of quantum statistics, the ability of an operator to generate entanglement in the spin space is equivalent to its ability to entangle in the flavor space [16], or more generally in the rest of all other spaces. This will greatly simplify the analysis of decuplet-baryon scattering in the next section.

B. Spin-3/2 baryon scatterings

Light decuplet baryons have spin 3/2 and belong to the **10** irrep of the $\text{SU}(3)$ flavor group. The spin and flavor structures of the decuplet-decuplet system can be decomposed, respectively, as

$$\frac{3}{2} \otimes \frac{3}{2} = \underbrace{0 \oplus 2}_{\text{antisymmetric}} \oplus \underbrace{1 \oplus 3}_{\text{symmetric}}, \quad (40)$$

$$\mathbf{10} \otimes \mathbf{10} = \underbrace{27 \oplus 28}_{\text{symmetric}} \oplus \underbrace{\bar{\mathbf{10}} \oplus \mathbf{35}}_{\text{antisymmetric}}. \quad (41)$$

Due to FD statistics, spin-symmetric components project into flavor-antisymmetric irreps, while spin-antisymmetric

components project into flavor-symmetric irreps. Hence, the corresponding decuplet-decuplet S matrix takes the form

$$\begin{aligned} \hat{S} = & \mathcal{J}_0 \otimes (\mathcal{F}_{27} e^{2i\delta_{0,27}} + \mathcal{F}_{28} e^{2i\delta_{0,28}}) \\ & + \mathcal{J}_1 \otimes (\mathcal{F}_{\bar{\mathbf{10}}} e^{2i\delta_{1,\bar{\mathbf{10}}}} + \mathcal{F}_{35} e^{2i\delta_{1,35}}) \\ & + \mathcal{J}_2 \otimes (\mathcal{F}_{27} e^{2i\delta_{2,27}} + \mathcal{F}_{28} e^{2i\delta_{2,28}}) \\ & + \mathcal{J}_3 \otimes (\mathcal{F}_{\bar{\mathbf{10}}} e^{2i\delta_{3,\bar{\mathbf{10}}}} + \mathcal{F}_{35} e^{2i\delta_{3,35}}), \end{aligned} \quad (42)$$

where $\mathcal{F}_F$ is the projector into a certain flavor irrep F . Recall that $\sum_J \mathcal{J}_J = 1_{\text{spin}}$ and $\sum_F \mathcal{F}_F = 1_{\text{flavor}}$.

As discussed in the previous section, the nonentangling S matrix in the spin space must be proportional to the Identity or SWAP gate in the spin space. The Identity gate can be achieved when all eight phase shifts are equal:

$$\delta_{0,27} = \delta_{2,27} = \delta_{0,28} = \delta_{2,28} = \delta_{1,\bar{\mathbf{10}}} = \delta_{3,\bar{\mathbf{10}}} = \delta_{1,35} = \delta_{3,35}, \quad (43)$$

in which case

$$\begin{aligned} \hat{S} & \propto (\mathcal{J}_0 + \mathcal{J}_2) \otimes (\mathcal{F}_{27} + \mathcal{F}_{28}) + (\mathcal{J}_1 + \mathcal{J}_3) \otimes (\mathcal{F}_{\bar{\mathbf{10}}} + \mathcal{F}_{35}) \\ & = \frac{1_{\text{spin}} - \text{SWAP}_{\text{spin}}}{2} \otimes \frac{1_{\text{flavor}} + \text{SWAP}_{\text{flavor}}}{2} \\ & \quad + \frac{1_{\text{spin}} + \text{SWAP}_{\text{spin}}}{2} \otimes \frac{1_{\text{flavor}} - \text{SWAP}_{\text{flavor}}}{2} \\ & = \frac{1}{2} [1_{\text{spin}} \otimes 1_{\text{flavor}} - \text{SWAP}_{\text{spin}} \otimes \text{SWAP}_{\text{flavor}}] \\ & = 1_{\text{spin}} \otimes 1_{\text{flavor}}, \end{aligned} \quad (44)$$

where in the second line above we have used the definition of a SWAP gate in Eq. (30) to write $\text{SWAP}_{\text{spin}}$ and $\text{SWAP}_{\text{flavor}}$ as the difference between the projectors in the symmetric and the antisymmetric irreps,

$$\text{SWAP}_{\text{spin}} = -(\mathcal{J}_0 - \mathcal{J}_1 + \mathcal{J}_2 - \mathcal{J}_3),$$

$$\text{SWAP}_{\text{flavor}} = \mathcal{F}_{27} + \mathcal{F}_{28} - \mathcal{F}_{\bar{\mathbf{10}}} - \mathcal{F}_{35}, \quad (45)$$

while in the last line we have imposed FD statistics on the decuplet baryons and applied Eq. (38). On the other hand, the $\text{SWAP}_{\text{spin}}$ gate can be achieved only when

$$\begin{aligned} \delta_{0,27} = \delta_{2,27} = \delta_{0,28} = \delta_{2,28}, \\ \delta_{1,\bar{\mathbf{10}}} = \delta_{3,\bar{\mathbf{10}}} = \delta_{1,35} = \delta_{3,35}, \\ |\delta_{0,27} - \delta_{1,\bar{\mathbf{10}}}| = \frac{\pi}{2}, \end{aligned} \quad (46)$$

so that

$$\begin{aligned} \hat{S} & \propto -(\mathcal{J}_0 + \mathcal{J}_2) \otimes (\mathcal{F}_{27} + \mathcal{F}_{28}) + (\mathcal{J}_1 + \mathcal{J}_3) \otimes (\mathcal{F}_{\bar{\mathbf{10}}} + \mathcal{F}_{35}) \\ & = -\frac{1_{\text{spin}} - \text{SWAP}_{\text{spin}}}{2} \otimes \frac{1_{\text{flavor}} + \text{SWAP}_{\text{flavor}}}{2} \\ & \quad + \frac{1_{\text{spin}} + \text{SWAP}_{\text{spin}}}{2} \otimes \frac{1_{\text{flavor}} - \text{SWAP}_{\text{flavor}}}{2} \\ & = \frac{1}{2} [\text{SWAP}_{\text{spin}} \otimes 1_{\text{flavor}} - 1_{\text{spin}} \otimes \text{SWAP}_{\text{flavor}}] \\ & = \text{SWAP}_{\text{spin}} \otimes 1_{\text{flavor}}. \end{aligned} \quad (47)$$

Although we include in Eq. (42) all possible flavor irreps, not all scattering channels involve every irrep. This is because scattering processes in QCD conserve both electric

charge Q and strangeness S . As a result, an initial baryon pair in a given (Q, S) sector can only scatter into final states within the same sector. Consequently, imposing vanishing entanglement power on these channels yields less stringent constraints than in Eqs. (43) and (46), a fact that has been observed in the scattering of spin-1/2 octet baryons [16]. For instance, in the $(Q, S) = (0, 0)$ sector, which includes two pairs $(\Delta^-\Delta^+, \Delta^0\Delta^0)$, only **27**, **28**, and **35** irreps are involved. Therefore, in this sector, the condition of entanglement suppression leads to

$$\delta_{0,27} = \delta_{2,27} = \delta_{0,28} = \delta_{2,28}, \quad \delta_{1,35} = \delta_{3,35},$$

$$|\delta_{0,27} - \delta_{1,35}| = 0 \text{ or } \frac{\pi}{2}. \quad (48)$$

The classification of (Q, S) sectors for decuplet-decuplet scattering and their corresponding flavor irreps involved can be found in Appendix A.

Finally, some scattering sectors contain only one flavor irrep, corresponding to single-channel identical particle scatterings. For such scatterings, entanglement suppression exhibits a completely unique property, which we will discuss in Sec. VII.

V. GROUP-THEORETIC IDENTIFICATION OF SYMMETRIES

Entanglement suppression has revealed relations between phase shifts, which implicitly hint at unique symmetries. As demonstrated in Ref. [16] for light octet baryons, these constraints manifest as emergent symmetries across different scattering sectors when implemented at the Lagrangian level. These symmetries can also be determined via group-theoretic counting by combining irreps of $SU(2)_{\text{spin}}$ and $SU(3)_{\text{flavor}}$ into irreps of the larger emergent symmetry, which provide further insights into the interplay between entanglement suppression and quantum statistics. We start by illustrating the counting in the spin-1/2 baryons and then apply it to the spin-3/2 cases.

A. Spin-1/2 octet baryons

Recall that octet baryons belong to the **8** irrep of the $SU(3)$ flavor group. The spin and flavor structures of two octet baryons can be decomposed as

$$SU(2)_{\text{spin}}: \quad \frac{1}{2} \otimes \frac{1}{2} = \underbrace{0}_{\text{antisymmetric}} \oplus \underbrace{1}_{\text{symmetric}}, \quad (49)$$

$$SU(3)_{\text{flavor}}: \quad \mathbf{8} \otimes \mathbf{8} = \underbrace{\mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{27}}_{\text{symmetric}} \oplus \underbrace{\mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}}}_{\text{antisymmetric}}. \quad (50)$$

There is a single six-dimensional sector with $(Q, S) = (0, -2)$ in which scatterings involve all six flavor irreps. Minimizing the entanglement power in this sector leads to two enhanced symmetries [16]: (1) an $SU(16)$ spin-flavor symmetry associated with an Identity gate S matrix and (2) an $SU(8)$ flavor symmetry corresponding to a SWAP gate S matrix. Let us consider them in turn.

When the S matrix is an Identity gate, one observes the universality of interaction strengths across all channels [16],

$$\delta_1 = \delta_{8_S} = \delta_{27} = \delta_{8_A} = \delta_{10} = \delta_{\overline{10}}, \quad (51)$$

and a large $SU(16)$ symmetry emerges, representing a full spin-flavor symmetry under which the eight baryons and the two spin states combine to form a **16**, the fundamental representation. Hence, the scattering of two spin-1/2 baryons in this case can be decomposed as

$$SU(16)_{\text{spin+flavor}}: \quad \mathbf{16} \otimes \mathbf{16} = \underbrace{\mathbf{136}}_{\text{symmetric}} \oplus \underbrace{\mathbf{120}}_{\text{antisymmetric}}. \quad (52)$$

FD statistics dictates that the scattering can only occur in the antisymmetric channel, **120**, which can be obtained by combining the symmetric $\mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{27}$ with the $^1S_0$ spin singlet and the antisymmetric $\mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}}$ with the $^3S_1$ spin triplet:

$$(1 + 8 + 27) \times 1 + (8 + 10 + 10) \times 3 = 120. \quad (53)$$

We emphasize that this is a consequence of the scattering dynamics being universal in all spin and flavor channels.

The SWAP gate is obtained when [16]

$$\delta_1 = \delta_{8_S} = \delta_{27} = \delta_S, \quad \delta_{8_A} = \delta_{10} = \delta_{\overline{10}} = \delta_A,$$

$$|\delta_S - \delta_A| = \frac{\pi}{2}, \quad (54)$$

in which case all scattering phases within the symmetric or antisymmetric irreps are of equal strength but there is a difference between the symmetric and antisymmetric irreps. In this case, Ref. [16] identified an $SU(8)$ flavor symmetry where all flavor-symmetric states combine into one single irrep and similarly for all flavor-antisymmetric states:

$$SU(8)_{\text{flavor}}: \quad \mathbf{8} \otimes \mathbf{8} = \underbrace{\mathbf{36}}_{\text{symmetric}} \oplus \underbrace{\mathbf{28}}_{\text{antisymmetric}}. \quad (55)$$

The counting works out as expected:

$$1 + 8 + 27 = 36 \quad \text{and} \quad 8 + 10 + 10 = 28. \quad (56)$$

The $SU(8)$ flavor symmetry dictates that there are only two independent scattering channels: the flavor-symmetric and the antisymmetric ones.

Next, there are several three-dimensional sectors (e.g., $p\Lambda - p\Sigma^0 - n\Sigma^+$) that scatter into $\mathbf{8}_S \oplus \mathbf{27} \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}}$ sectors. Entanglement suppression in the three-dimensional sectors arises from the conditions [16]:

$$\delta_{8_S} = \delta_{27} \equiv \delta_S, \quad \delta_{8_A} = \delta_{10} = \delta_{\overline{10}} \equiv \delta_A,$$

$$|\delta_S - \delta_A| = 0 \text{ or } \frac{\pi}{2}. \quad (57)$$

Since the **1** irrep is absent in this case, the numbers of symmetric and antisymmetric flavor states are

$$\text{symmetric: } 27 + 8 = 35,$$

$$\text{antisymmetric: } 8 + 10 + 10 = 28. \quad (58)$$

It turns out that the $SO(8)$ group has a rank-2 traceless symmetric irrep with dimension $35 = 8(8+1)/2 - 1$ and a rank-2 antisymmetric irrep with dimension $28 = 8(8-1)/2$, which match the counting above. Indeed, the $SO(8)$ flavor symmetry was discovered in Ref. [16] via the Lagrangian approach in the case of the Identity gate. One can verify that, in the case of the S -matrix being a SWAP gate, the Lagrangian also maintains the same $SO(8)$ symmetry.

B. Spin-3/2 decuplet baryons

Now we identify the possible emergent symmetries in the spin-3/2 baryon scatterings. In Sec. IV B, we demonstrated that the S matrix in the spin space is an Identity gate if all scattering phases are equal, as Eq. (43) shows. Again the universal strength of interactions in all channels suggests a large symmetry under which every scattering channel belongs to the same irrep. The total number of spin and flavor states is

$$(27 + 28) \times (1 + 5) + (10 + 35) \times (3 + 7) = 780, \quad (59)$$

where the number of spin states is given by $(2J + 1)$, $J = 0, 1, 2, 3$. This number matches the dimension of the rank-2 antisymmetric representation of $SU(40)$, arising from the product of two fundamental representations (40):

$$SU(40)_{\text{spin+flavor}}: \quad 40 \otimes 40 = \underbrace{820}_{\text{symmetric}} \oplus \underbrace{780}_{\text{antisymmetric}}. \quad (60)$$

This indicates that the ten spin-3/2 baryons, each with four spin states, fully occupy 40 of $SU(40)$. Moreover, 780 is precisely what the FD statistics demands, as the two-baryon system must be antisymmetric under state exchange!

On the other hand, the SWAP solution for the S matrix in Eq. (46) requires the scattering dynamics to be universal only within the symmetric or antisymmetric flavor channels, which is similar to the spin-1/2 baryon case in Eq. (54). This suggests a symmetry group where all $55 = 27 + 28$ flavor-symmetric states combine into one single irrep and similarly for all $45 = 10 + 35$ flavor-antisymmetric states. Thus, there is an enhanced $SU(10)$ flavor symmetry:

$$SU(10)_{\text{flavor}}: \quad 10 \otimes 10 = \underbrace{55}_{\text{symmetric}} \oplus \underbrace{45}_{\text{antisymmetric}}. \quad (61)$$

As discussed in Sec. IV A, the ability of the S matrix to entangle spin states is correlated with its ability to entangle flavor states, due to quantum statistics. Thus, FD statistics dictates that an enhanced symmetry in flavor space implies an enhanced symmetry in spin space as well, which is manifest in the observation that $J = 0$ and $J = 2$ states now share the same scattering dynamics and similarly for $J = 1$ and $J = 3$ states. This results in an emergent $SU(4)_{\text{spin}}$ symmetry, under which $J = 0, 2$ states form a six-dimensional rank-2 antisymmetric irrep and $J = 1, 3$ states form a ten-dimensional symmetric irrep:

$$SU(4)_{\text{spin}}: \quad 4 \otimes 4 = \underbrace{10}_{\text{symmetric}} \oplus \underbrace{6}_{\text{antisymmetric}}. \quad (62)$$

In the end, the enhanced symmetry is $SU(4)_{\text{spin}} \times SU(10)_{\text{flavor}}$.

It is interesting to compare the SWAP solutions between spin-1/2 octet baryons and spin-3/2 decuplet baryons. In the former case, the enhanced symmetry is $SU(8)_{\text{flavor}}$, while in the latter case it is $SU(4)_{\text{spin}} \times SU(10)_{\text{flavor}}$. Again, this is due to the fact that there is more than one single spin channel contributing to the symmetric or antisymmetric flavor states. As a result, the symmetry in the spin space is enhanced from $SU(2)$ to $SU(4)$. This emergent spin symmetry is not affected by whether exact flavor $SU(3)$ symmetry is assumed, and it remains exact since its starting point, the spin $SU(2)$ symmetry, is itself exact as long as no spin symmetry breaking

effect, e.g., spin-orbital coupling, is considered. Considering a single scattering channel, e.g., $\Delta^0 \Delta^+$, it will still exhibit an $SU(4)$ spin symmetry after suppressing entanglement. The role of assuming exact flavor $SU(3)$ symmetry is to relate the scattering in this channel to that in other channels, thereby allowing the emergence of a larger $SU(40)$ spin-flavor symmetry. If flavor symmetry is not taken into account at all, then the emergent symmetry from entanglement suppression is simply the $SU(4)$ spin symmetry (in this channel solely). Moreover, when scattering spin- s baryons with $s > 1/2$, we generally expect the spin symmetry to be enhanced to $SU(2s + 1)_{\text{spin}}$. We will see in Sec. VII that the existence of two spin channels within the symmetric or antisymmetric flavor states also gives rise to a richer phenomenology when scattering two identical particles.

VI. FIELD-THEORETIC IDENTIFICATION OF SYMMETRIES

As shown in Ref. [16], an explicit identification of emergent symmetries can be achieved in the Lagrangian approach. This requires establishing a connection between the phase shifts and the parameters in the Lagrangian. Since we are interested in the very low-energy phenomenon, our description does not need to contain explicit pseudoscalar meson degrees of freedom. In this context, one often considers NREFT with short-range interactions, which can be matched to the effective range expansion (ERE) [61–63].

A. NREFT for decuplet baryons

Operators in an NREFT Lagrangian are ordered in powers of momenta or derivatives, and the LO Lagrangian contains only constant contact terms. The LO Lagrangian for decuplet-decuplet interaction in NREFT reads as [64]

$$\begin{aligned} \mathcal{L} = & c_1 (T_{abc}^\dagger T_{abc}) (T_{\text{def}}^\dagger T_{\text{def}}) \\ & + c_2 (T_{abc}^\dagger \Sigma^\alpha T_{abc}) (T_{\text{def}}^\dagger \Sigma^\alpha T_{\text{def}}) \\ & + c_3 (T_{abc}^\dagger \Sigma^{\alpha\beta} T_{abc}) (T_{\text{def}}^\dagger \Sigma^{\alpha\beta} T_{\text{def}}) \\ & + c_4 (T_{abc}^\dagger \Sigma^{\alpha\beta\gamma} T_{abc}) (T_{\text{def}}^\dagger \Sigma^{\alpha\beta\gamma} T_{\text{def}}) \\ & + c_5 (T_{abc}^\dagger T_{abd}) (T_{\text{def}}^\dagger T_{\text{cef}}) \\ & + c_6 (T_{abc}^\dagger \Sigma^\alpha T_{abd}) (T_{\text{def}}^\dagger \Sigma^\alpha T_{\text{cef}}) \\ & + c_7 (T_{abc}^\dagger \Sigma^{\alpha\beta} T_{abd}) (T_{\text{def}}^\dagger \Sigma^{\alpha\beta} T_{\text{cef}}) \\ & + c_8 (T_{abc}^\dagger \Sigma^{\alpha\beta\gamma} T_{abd}) (T_{\text{def}}^\dagger \Sigma^{\alpha\beta\gamma} T_{\text{cef}}), \end{aligned} \quad (63)$$

where c_i 's are eight Wilson coefficients, also known as low-energy constants (LECs) in the context of low-energy effective field theories. Here, the decuplet baryons are represented by the totally symmetric three-index tensor T :

$$\begin{aligned} T_{111} &= \Delta^{++}, \quad T_{112} = \frac{1}{\sqrt{3}} \Delta^+, \quad T_{122} = \frac{1}{\sqrt{3}} \Delta^0, \quad T_{222} = \Delta^-, \\ T_{113} &= \frac{1}{\sqrt{3}} \Sigma^{*+}, \quad T_{123} = \frac{1}{\sqrt{6}} \Sigma^{*0}, \quad T_{223} = \frac{1}{\sqrt{3}} \Sigma^{*-}, \end{aligned}$$

$$T_{133} = \frac{1}{\sqrt{3}} \Xi^{*0}, \quad T_{233} = \frac{1}{\sqrt{3}} \Xi^{*-},$$

$$T_{333} = \Omega^{-}. \quad (64)$$

We also introduced unity, Σ^α , $\Sigma^{\alpha\beta}$, and $\Sigma^{\alpha\beta\gamma}$, corresponding to scalar, vector, rank-2, and rank-3 tensor operators, respectively. The operators $\Sigma^\alpha/2$ are simply the generators of SU(2) in the four-dimensional representation: $\Sigma^\alpha/2 = t_{s=3/2}^\alpha$, and

$$\Sigma^{\alpha\beta} = \frac{1}{8}(\Sigma^\alpha \Sigma^\beta + \Sigma^\beta \Sigma^\alpha - 10\delta^{\alpha\beta}),$$

$$\Sigma^{\alpha\beta\gamma} = \frac{1}{36\sqrt{3}}[5(\Sigma^\alpha \Sigma^\beta \Sigma^\gamma + \Sigma^\gamma \Sigma^\alpha \Sigma^\beta + \Sigma^\beta \Sigma^\gamma \Sigma^\alpha + \Sigma^\alpha \Sigma^\gamma \Sigma^\beta + \Sigma^\beta \Sigma^\alpha \Sigma^\gamma + \Sigma^\gamma \Sigma^\beta \Sigma^\alpha) - 82(\Sigma^\alpha \delta^{\beta\gamma} + \Sigma^\gamma \delta^{\alpha\beta} + \Sigma^\beta \delta^{\gamma\alpha})]. \quad (65)$$

It will be convenient to project the contact operators into irreps of SU(3) flavor symmetry, and the corresponding SU(3)-symmetric Wilson coefficients are [64]

$$C_{0,27} = \frac{1}{27}(-54c_1 + 810c_2 - 405c_3 + 6300c_4 + 6c_5 - 90c_6 + 45c_7 - 700c_8), \quad (66)$$

$$C_{2,27} = -2c_1 + 6c_2 + 9c_3 + \frac{140}{3}c_4 + \frac{2}{9}c_5 - \frac{2}{3}c_6 - c_7 - \frac{140}{27}c_8, \quad (67)$$

$$C_{0,28} = \frac{1}{3}(-6c_1 + 90c_2 - 45c_3 + 700c_4 - 6c_5 + 90c_6 - 45c_7 + 700c_8), \quad (68)$$

$$C_{2,28} = -2c_1 + 6c_2 + 9c_3 + \frac{140}{3}c_4 - 2c_5 + 6c_6 + 9c_7 + \frac{140}{3}c_8, \quad (69)$$

$$C_{1,\overline{10}} = \frac{1}{3}(-6c_1 + 66c_2 - 9c_3 - 420c_4 + 2c_5 - 22c_6 + 3c_7 + 140c_8), \quad (70)$$

$$C_{3,\overline{10}} = -2c_1 - 18c_2 - 3c_3 - \frac{20}{3}c_4 + \frac{2}{3}c_5 + 6c_6 + c_7 + \frac{20}{9}c_8, \quad (71)$$

$$C_{1,35} = \frac{1}{3}(-6c_1 + 66c_2 - 9c_3 - 420c_4 - 2c_5 + 22c_6 - 3c_7 - 140c_8), \quad (72)$$

$$C_{3,35} = -2c_1 - 18c_2 - 3c_3 - \frac{20}{3}c_4 - \frac{2}{3}c_5 - 6c_6 - c_7 - \frac{20}{9}c_8. \quad (73)$$

The ERE at LO [61–63] establishes the relationship between the phase shift δ and the Wilson coefficient C [16,19]:

$$\lim_{p \rightarrow 0} p \cot \delta = -\frac{1}{a} = -\frac{2\pi}{\mu C} - \Lambda, \quad (74)$$

where p is the magnitude of the center-of-mass momentum, a is the S -wave scattering length, $\mu = m/2$ is the reduced mass with m being the decuplet baryon mass, and Λ is the ultraviolet cutoff.

Utilizing Eq. (74), we can deduce the Wilson coefficient C for certain phase shift values. In particular, the noninteracting and unitarity limit cases are of great importance at LO:

$$\delta = 0: \quad a = 0 \quad \text{or} \quad C = 0, \quad (75)$$

$$\delta = \frac{\pi}{2}: \quad a = \infty \quad \text{or} \quad C = -\frac{2\pi}{\mu\Lambda}. \quad (76)$$

Both the free theory ($\delta = 0$) and the unitarity limit ($\delta = \pi/2$) exhibit an enhanced symmetry: they are invariant under the Schrödinger group [15,31], the nonrelativistic conformal group that leaves the Schrödinger equation invariant. Additionally, these two cases correspond to the fixed points of the renormalization group flow for nonrelativistic two-body scattering governed by short-range interactions [30].

Furthermore, if we know the relation between the phase shifts of different channels, we can also use Eq. (74) to derive relations between the corresponding Wilson coefficients. In particular, entanglement suppression provides two relations between the phase shifts (denoting the two relevant channels as α and β):

$$\delta_\alpha = \delta_\beta: \quad a_\alpha = a_\beta \quad (77)$$

or

$$|\delta_\alpha - \delta_\beta| = \frac{\pi}{2}: \quad \frac{1}{a_\alpha a_\beta} = \lim_{p \rightarrow 0} p^2 \cot \delta_\alpha \cot \delta_\beta = \lim_{p \rightarrow 0} (-p^2). \quad (78)$$

The former implies that the two channels have identical scattering lengths, meaning that their corresponding Wilson coefficients are equal at a given cutoff. The latter, in the limit of $p \rightarrow 0$, indicates that one of the scattering lengths diverges (i.e., its phase shift reaches $\pi/2$), which in turn requires the phase shift of the other channel to vanish, and then their Wilson coefficients are given by Eqs. (75) and (76).

The symmetry of the Lagrangian can be made more transparent in one operator basis than in another. This will be helpful for analyzing emergent symmetries in the next subsections. Therefore, we rewrite the Lagrangian using an alternative set of operators:

$$\begin{aligned} \mathcal{L} = & d_1(T_{abc}^\dagger T_{\text{def}}^\dagger)(T_{abc} T_{\text{def}}) \\ & + d_2(T_{abc}^\dagger \Xi^{\alpha*} T_{\text{def}}^\dagger)(T_{abc} \Xi^\alpha T_{\text{def}}) \\ & + d_3(T_{abc}^\dagger \Xi^{\alpha\beta*} T_{\text{def}}^\dagger)(T_{abc} \Xi^{\alpha\beta} T_{\text{def}}) \\ & + d_4(T_{abc}^\dagger \Xi^{\alpha\beta\gamma*} T_{\text{def}}^\dagger)(T_{abc} \Xi^{\alpha\beta\gamma} T_{\text{def}}) \\ & + d_5(T_{abc}^\dagger T_{\text{def}}^\dagger)(T_{abd} T_{\text{cef}}) \\ & + d_6(T_{abc}^\dagger \Xi^{\alpha*} T_{\text{def}}^\dagger)(T_{abd} \Xi^\alpha T_{\text{cef}}) \\ & + d_7(T_{abc}^\dagger \Xi^{\alpha\beta*} T_{\text{def}}^\dagger)(T_{abd} \Xi^{\alpha\beta} T_{\text{cef}}) \\ & + d_8(T_{abc}^\dagger \Xi^{\alpha\beta\gamma*} T_{\text{def}}^\dagger)(T_{abd} \Xi^{\alpha\beta\gamma} T_{\text{cef}}), \end{aligned} \quad (79)$$

where d_i 's are the LECs in the new operator basis and Ξ 's, along with their complex conjugates Ξ^* 's, are defined as

$$\Xi^\alpha = \exp\left(-i\pi \frac{\Sigma^y}{2}\right) \Sigma^\alpha. \quad (80)$$

In this new basis, the corresponding Wilson coefficients read as

$$C_{0,27} = -8d_1 + \frac{8}{9}d_5, \quad C_{2,27} = \frac{4}{3}(-9d_3 + d_7), \quad (81)$$

$$C_{0,28} = -8(d_1 + d_5), \quad C_{2,28} = -12(d_3 + d_7), \quad (82)$$

$$C_{1,\overline{10}} = \frac{40}{3}(-3d_2 + d_6), \quad C_{3,\overline{10}} = -\frac{400}{9}(3d_4 - d_8), \quad (83)$$

$$C_{1,35} = -\frac{40}{3}(3d_2 + d_6), \quad C_{3,35} = -\frac{400}{9}(3d_4 + d_8). \quad (84)$$

In the following, we will switch between Eqs. (63) and (79) as needed.

B. $27 \oplus 28 \oplus \overline{10} \oplus 35$ sectors

Having set up the theoretical framework, we can now analyze the emergent symmetries that entanglement suppression predicts in various sectors, starting from the global minimum in sectors containing all four flavor irreps $27 \oplus 28 \oplus \overline{10} \oplus 35$.

Let us first examine the sectors that involve all eight spin-flavor irreps. These include

$$\begin{aligned} &(\Delta^-\Delta^{++}, \Delta^0\Delta^+), \quad (\Delta^-\Sigma^{*+}, \Delta^0\Sigma^{*0}, \Delta^+\Sigma^{*-}), \\ &(\Delta^-\Xi^{*0}, \Delta^0\Xi^{*-}, \Sigma^{*-}\Sigma^{*0}), \\ &(\Delta^+\Xi^{*-}, \Delta^0\Xi^{*0}, \Sigma^{*0}\Sigma^{*0}, \Sigma^{*+}\Sigma^{*-}), \\ &(\Sigma^{*-}\Xi^{*-}, \Omega^-\Delta^-), \quad (\Sigma^{*-}\Xi^{*0}, \Sigma^{*0}\Xi^{*-}, \Omega^-\Delta^0), \end{aligned} \quad (85)$$

and their I_3 -conjugate partners (see Appendix A), when they exist.

The first nonentangling condition requires all eight phase shifts, and consequently all eight Wilson coefficients, to be equal, which sets $c_{2,\dots,8} = 0$. As a result, Eq. (63) reduces to

$$\mathcal{L} = c_1(T_{abc}^\dagger T_{abc})(T_{\text{def}}^\dagger T_{\text{def}}) \quad (86)$$

or equivalently, in the particle basis,

$$\mathcal{L} = c_1(\mathbf{T}^{\dagger i} \cdot \mathbf{T}^i)^2, \quad (87)$$

where

$$\mathbf{T} = (\Delta^{++}, \Delta^+, \Delta^0, \Delta^-, \Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}, \Xi^{*0}, \Xi^{*-}, \Omega^-), \quad (88)$$

and $i = 1, 2, 3, 4$ denotes the spin index. This Lagrangian exhibits a maximal $SU(40)$ symmetry, which was identified using group-theoretic reasoning in Sec. VB.

Another two minimal-entanglement solutions from Eq. (46) are

$$\begin{aligned} C_{0,27} = C_{2,27} = C_{0,28} = C_{2,28} &= -\frac{2\pi}{\mu\Lambda}, \\ C_{1,\overline{10}} = C_{3,\overline{10}} = C_{1,35} = C_{3,35} &= 0 \end{aligned} \quad (89)$$

and

$$\begin{aligned} C_{0,27} = C_{2,27} = C_{0,28} = C_{2,28} &= 0, \\ C_{1,\overline{10}} = C_{3,\overline{10}} = C_{1,35} = C_{3,35} &= -\frac{2\pi}{\mu\Lambda}. \end{aligned} \quad (90)$$

These two solutions correspond to the Lagrangians

$$\begin{aligned} \mathcal{L} &= \frac{2\pi}{\mu\Lambda} \left[\frac{1}{4}(T_{abc}^\dagger T_{abc})(T_{\text{def}}^\dagger T_{\text{def}}) - \frac{1}{4}(T_{abc}^\dagger T_{\text{def}})(T_{\text{def}}^\dagger T_{abc}) \right] \\ &= \frac{2\pi}{\mu\Lambda} \left[\frac{1}{4}(\mathbf{T}^{\dagger i} \cdot \mathbf{T}^i)^2 - \frac{1}{4}(\mathbf{T}^{\dagger i} \cdot \mathbf{T}^j)(\mathbf{T}^{\dagger j} \cdot \mathbf{T}^i) \right] \end{aligned} \quad (91)$$

and

$$\begin{aligned} \mathcal{L} &= \frac{2\pi}{\mu\Lambda} \left[\frac{1}{4}(T_{abc}^\dagger T_{abc})(T_{\text{def}}^\dagger T_{\text{def}}) + \frac{1}{4}(T_{abc}^\dagger T_{\text{def}})(T_{\text{def}}^\dagger T_{abc}) \right] \\ &= \frac{2\pi}{\mu\Lambda} \left[\frac{1}{4}(\mathbf{T}^{\dagger i} \cdot \mathbf{T}^i)^2 + \frac{1}{4}(\mathbf{T}^{\dagger i} \cdot \mathbf{T}^j)(\mathbf{T}^{\dagger j} \cdot \mathbf{T}^i) \right], \end{aligned} \quad (92)$$

respectively. A Fierz transformation has been employed to simplify these operators:

$$\begin{aligned} \frac{1}{4}\delta_{ij}\delta_{kl} + \frac{1}{20}(\Sigma^\alpha)_{ij}(\Sigma^\alpha)_{kl} + \frac{1}{6}(\Sigma^{\alpha\beta})_{ij}(\Sigma^{\alpha\beta})_{kl} \\ + \frac{3}{200}(\Sigma^{\alpha\beta\gamma})_{ij}(\Sigma^{\alpha\beta\gamma})_{kl} = \delta_{il}\delta_{kj}. \end{aligned} \quad (93)$$

Equations (91) and (92) exhibit an $SU(4)_{\text{spin}} \times SU(10)_{\text{flavor}}$ symmetry, in addition to the Schrödinger symmetry contained in Eqs. (89) and (90). Moreover, observe that the $(\mathbf{T}^{\dagger i} \cdot \mathbf{T}^i)^2$ interaction preserves the spins of the incoming baryons after scattering, while $(\mathbf{T}^{\dagger i} \cdot \mathbf{T}^j)(\mathbf{T}^{\dagger j} \cdot \mathbf{T}^i)$ interchanges the spins. In other words, Eqs. (91) and (92) are proportional to

$$\frac{1_{\text{spin}} - \text{SWAP}_{\text{spin}}}{2} \quad \text{and} \quad \frac{1_{\text{spin}} + \text{SWAP}_{\text{spin}}}{2}, \quad (94)$$

which are precisely the antisymmetric and symmetric projectors in spin space, respectively, again consistent with Eqs. (89) and (90).

C. Other sectors

In this subsection, we employ another set of LECs d_i , which will facilitate the identification of the emergent symmetry. The following sectors involve the 27 , 28 , and 35 irreps: $(\Delta^-\Delta^+, \Delta^0\Delta^0)$, $(\Delta^0\Sigma^{*-}, \Delta^-\Sigma^{*0})$, $(\Delta^-\Xi^{*-}, \Sigma^{*-}\Sigma^{*-})$, $(\Xi^{*-}\Xi^{*-}, \Omega^-\Sigma^{*-})$, $(\Xi^{*-}\Xi^{*0}, \Omega^-\Sigma^{*0})$, and their I_3 -conjugate partners, if they exist. The phase-shift relations are given by

$$\delta_{0,27} = \delta_{2,27} = \delta_{0,28} = \delta_{2,28}, \quad \delta_{1,35} = \delta_{3,35}, \quad (95)$$

$$\delta_{0,27} - \delta_{1,35} = 0, \quad \frac{\pi}{2}, \quad \text{or} \quad -\frac{\pi}{2}. \quad (96)$$

The Lagrangian corresponding to the case where $\delta_{0,27} = \delta_{1,35}$ is

$$\begin{aligned} \mathcal{L} &= d_1(T_{abc}^\dagger T_{\text{def}}^\dagger)(T_{abc} T_{\text{def}}) \\ &+ d_2(T_{abc}^\dagger \Xi^{\alpha*} T_{\text{def}}^\dagger)(T_{abc} \Xi^\alpha T_{\text{def}}) \\ &+ \frac{2}{3}d_1(T_{abc}^\dagger \Xi^{\alpha\beta*} T_{\text{def}}^\dagger)(T_{abc} \Xi^{\alpha\beta} T_{\text{def}}) \\ &+ d_4(T_{abc}^\dagger \Xi^{\alpha\beta\gamma*} T_{\text{def}}^\dagger)(T_{abc} \Xi^{\alpha\beta\gamma} T_{\text{def}}) \\ &+ \left(\frac{3}{5}d_1 - 3d_2\right)(T_{abc}^\dagger \Xi^{\alpha*} T_{\text{def}}^\dagger)(T_{abd} \Xi^\alpha T_{\text{cef}}) \\ &+ \left(\frac{9}{50}d_1 - 3d_4\right)(T_{abc}^\dagger \Xi^{\alpha\beta\gamma*} T_{\text{def}}^\dagger)(T_{abd} \Xi^{\alpha\beta\gamma} T_{\text{cef}}). \end{aligned} \quad (97)$$

For $\delta_{0,27} = \pi/2$ and $\delta_{1,35} = 0$, the Lagrangian becomes

$$\begin{aligned}\mathcal{L} = & \frac{\pi}{4\mu\Lambda} (T_{abc}^\dagger T_{\text{def}}^\dagger) (T_{abc} T_{\text{def}}) \\ & + d_2 (T_{abc}^\dagger \Xi^{\alpha*} T_{\text{def}}^\dagger) (T_{abc} \Xi^\alpha T_{\text{def}}) \\ & + \frac{\pi}{6\mu\Lambda} (T_{abc}^\dagger \Xi^{\alpha\beta*} T_{\text{def}}^\dagger) (T_{abc} \Xi^{\alpha\beta} T_{\text{def}}) \\ & + d_4 (T_{abc}^\dagger \Xi^{\alpha\beta\gamma*} T_{\text{def}}^\dagger) (T_{abc} \Xi^{\alpha\beta\gamma} T_{\text{def}}) \\ & - 3d_2 (T_{abc}^\dagger \Xi^{\alpha*} T_{\text{def}}^\dagger) (T_{abd} \Xi^\alpha T_{\text{cef}}) \\ & - 3d_4 (T_{abc}^\dagger \Xi^{\alpha\beta\gamma*} T_{\text{def}}^\dagger) (T_{abd} \Xi^{\alpha\beta\gamma} T_{\text{cef}}). \quad (98)\end{aligned}$$

Similarly, when $\delta_{0,27} = 0$ and $\delta_{1,35} = \pi/2$, the Lagrangian takes the form

$$\begin{aligned}\mathcal{L} = & d_2 (T_{abc}^\dagger \Xi^{\alpha*} T_{\text{def}}^\dagger) (T_{abc} \Xi^\alpha T_{\text{def}}) \\ & + d_4 (T_{abc}^\dagger \Xi^{\alpha\beta\gamma*} T_{\text{def}}^\dagger) (T_{abc} \Xi^{\alpha\beta\gamma} T_{\text{def}}) \\ & + \left(\frac{3\pi}{20\mu\Lambda} - 3d_2 \right) (T_{abc}^\dagger \Xi^{\alpha*} T_{\text{def}}^\dagger) (T_{abd} \Xi^\alpha T_{\text{cef}}) \\ & + \left(\frac{9\pi}{200\mu\Lambda} - 3d_4 \right) (T_{abc}^\dagger \Xi^{\alpha\beta\gamma*} T_{\text{def}}^\dagger) (T_{abd} \Xi^{\alpha\beta\gamma} T_{\text{cef}}). \quad (99)\end{aligned}$$

In all three scenarios displayed above, it can be seen that at least two terms in the Lagrangian can be eliminated, indicating the presence of an enhanced symmetry. Only d_1 , d_2 , and d_4 remain as free parameters, and d_1 is further fixed if the interaction strength is specified in the latter two cases. An explicit identification of the symmetry in the Lagrangian remains to be further investigated. However, there are indications from the group-theoretic arguments presented in Sec. V. Notably, the flavor-symmetric (spin-antisymmetric) sector exhibits precisely the same symmetry structure as in the preceding subsection. This provides conclusive evidence for the existence of $\text{SU}(4)_{\text{spin}} \times \text{SU}(10)_{\text{flavor}}$ symmetry in the $27 \oplus 28$ channels. As for the 35 channels, an $\text{SU}(4)$ spin symmetry can be verified, since $3 + 7 = 10$ spin states in this flavor irrep share the same coupling strength, thus forming a rank-2 symmetric representation of $\text{SU}(4)$.

Finally, the following one-dimensional sectors involve the 28 and 35 irreps: $\Delta^-\Delta^0$, $\Delta^-\Sigma^{*-}$, $\Omega^-\Xi^{*-}$, and their I_3 -conjugate partners. An $\text{SU}(4)$ spin symmetry can always be concluded in these flavor irreps by counting numbers. Additional enlarged symmetries are suggested by relations between phase shifts or Wilson coefficients in the LO effective Lagrangian. However, identifying the underlying group-theoretic structure remains challenging.

D. Large- N_c limit and dibaryons

In the large- N_c limit, where the number of colors $N_c \rightarrow \infty$, the intrinsic $\text{SU}(2)_{\text{spin}} \times \text{SU}(3)_{\text{flavor}}$ symmetry is enlarged to the $\text{SU}(6)$ spin-flavor symmetry. In this scenario, the spin-1/2 octet and spin-3/2 decuplet baryons combine to form a 56-dimensional representation described by a completely symmetric three-index field $\Psi_{\alpha\beta\gamma}$. The LO effective La-

grangian contains only two operators (3):

$$\mathcal{L} = -(\tilde{a} \Psi_{\alpha\beta\gamma}^\dagger \Psi_{\alpha\beta\gamma} \Psi_{\mu\nu\rho}^\dagger \Psi_{\mu\nu\rho} + \tilde{b} \Psi_{\alpha\beta\gamma}^\dagger \Psi_{\alpha\beta\rho} \Psi_{\mu\nu\rho}^\dagger \Psi_{\mu\nu\gamma}). \quad (100)$$

In this limit of $\text{SU}(6)$ spin-flavor symmetry, the eight Wilson coefficients are related to $\tilde{a}$ and $\tilde{b}$ in the above [52]:

$$\begin{aligned}C_{0,27} = C_{1,\overline{10}} &= 2\left(\tilde{a} + \frac{\tilde{b}}{27}\right), \\ C_{2,27} = C_{1,35} &= 2\left(\tilde{a} - \frac{\tilde{b}}{27}\right), \\ C_{0,28} = C_{3,\overline{10}} &= 2\left(\tilde{a} - \frac{\tilde{b}}{3}\right), \\ C_{2,28} = C_{3,35} &= 2\left(\tilde{a} + \frac{\tilde{b}}{3}\right). \quad (101)\end{aligned}$$

We observe that each line in Eq. (101) equates one spin-symmetric component with one spin-antisymmetric component, in contrast to the global entanglement suppression prediction, for which

$$\begin{aligned}C_{0,27} = C_{2,27} = C_{0,28} = C_{2,28}, \\ C_{1,\overline{10}} = C_{3,\overline{10}} = C_{1,35} = C_{3,35}. \quad (102)\end{aligned}$$

The only solution that simultaneously satisfies both $\text{SU}(6)$ symmetry and entanglement suppression is to choose $C_{0,27} = C_{1,\overline{10}}$, which immediately implies that all Wilson coefficients are equal. As can be directly seen from Eq. (101), this corresponds to setting $\tilde{b} = 0$. Once again, an emergent $\text{SU}(40)$ symmetry appears in the decuplet-decuplet sector. In fact, the Lagrangian in Eq. (100) with $\tilde{b} = 0$ also exhibits an $\text{SU}(16)$ symmetry in the octet-octet sector and an $\text{SU}(56)$ symmetry when considering the full octet + decuplet system—necessary for consistency in the large- N_c limit. While an approximately vanishing $\tilde{b}$ has been observed in lattice calculations in the octet-octet sector [29,65], such a result has not yet been seen in the sectors involving decuplet baryons.

Let us also discuss the possible $\Delta\Delta$ dibaryons in the framework of entanglement suppression. The $\Delta\Delta$ systems with $(J, I) = (3, 0)$ and $(0, 3)$ are in the irreps $(J, F) = (3, \overline{10})$ and $(0, 28)$, respectively, in the three-flavor case. The potential existence of the $(3, \overline{10})$ resonance $d^*(2380)$ suggests that this channel approaches the unitarity limit, which might serve as an additional input for the sector $(\Delta^-\Delta^{++}, \Delta^0\Delta^+)$ that involves the flavor $\overline{10}$ irrep. Demanding $\delta_{3,\overline{10}} = \pi/2$ in this sector leads to two possibilities: (1) all eight irreps reach the unitarity limit or (2) spin-antisymmetric irreps are noninteracting and spin-symmetric irreps are at the unitarity limit, as shown in Eq. (90). The former solution means that $d^*(2380)$ is in the 780 multiplet under $\text{SU}(40)$; in particular, this would predict $d^*(2380)$ to have a partner of $(J, I) = (0, 3)$.

VII. IDENTICAL PARTICLES AND SUBUNITARY S MATRICES

A special scenario that deserves a dedicated analysis is the scattering of two identical particles into a single flavor channel. For spin-1/2 octet baryons, such a case includes

$\{nn, pp, \Sigma^+\Sigma^+, \Sigma^-\Sigma^-, \Xi^-\Xi^-, \Xi^0\Xi^0\}$, all of which scatter into the symmetric **27** irrep of $SU(3)_{\text{flavor}}$. In this case, FD statistics constrains the total angular momentum in the S -wave scattering to be in the antisymmetric spin singlet $^1S_0$ channel, which is already maximally entangled [16]. The S matrix contains a single channel:

$$\hat{S}_A = \mathcal{J}_0 \otimes \mathcal{F}_{27} e^{2i\delta_{0,27}}, \quad (103)$$

which is neither Identity nor SWAP. Notice that FD statistics restricts the physical states to the antisymmetric part of the total Hilbert space; in the symmetric part of the Hilbert space, there can be neither free propagation nor scattering dynamics. Therefore, $\hat{S}_A$ is not a unitary operator over the entire Hilbert space; instead, it vanishes over the symmetric Hilbert (sub)space and is unitary only over the antisymmetric Hilbert (sub)space. In this case, we call $\hat{S}_A$ “subunitary” over the antisymmetric Hilbert space. Moreover, the entanglement power of $\hat{S}_A$ in Eq. (103) is constant, independent of the scattering phase $\delta_{0,27}$.

For spin-3/2 decuplet baryons, a more interesting scenario appears because both $\mathcal{J} = 0$ and $\mathcal{J} = 2$ are antisymmetric, so there can be two different angular momentum channels, thereby allowing for interferences between the two when computing the entanglement power. For instance, the $(Q, S) = (-2, -6)$ sector, i.e., $\Omega\Omega$, scatters into a single flavor-symmetric irrep **28** with two phase shifts $\delta_{0,28}$ and $\delta_{2,28}$:

$$\hat{S}_A = \mathcal{J}_0 e^{2i\delta_{0,28}} + \mathcal{J}_2 e^{2i\delta_{2,28}}. \quad (104)$$

In this case, the entanglement power is not a constant anymore and depends on the phase difference $\delta_{0,28} - \delta_{2,28}$. There are two other such channels for spin-2/3 baryons: $\Delta^-\Delta^-$ and $\Delta^{++}\Delta^{++}$, which are the I_3 conjugate of each other. A similar scenario arises in the spin-1 deuteron-deuteron (dd) scattering, which was studied in Ref. [43]. In what follows, we will base our discussion on FD statistics; the generalization to BE statistics can be carried out trivially.

A. Density matrix and entanglement power for identical particles

Formally speaking, the subunitary S matrix can be viewed as a unitary S matrix projected into the antisymmetric Hilbert space:

$$\hat{S}_A \equiv \mathcal{P}_A \hat{S} \mathcal{P}_A = \hat{S} \mathcal{P}_A = \mathcal{P}_A \hat{S}, \quad [\hat{S}, \mathcal{P}_A] = 0, \quad (105)$$

where $\mathcal{P}_A$ is the projection operator in the antisymmetric Hilbert space defined in Eq. (30). Notice that $\hat{S}_A^\dagger \hat{S}_A = \mathcal{P}_A$, implying unitarity over the antisymmetric Hilbert space, which is manifest when we go to a basis where $\mathcal{P}_A$ is an Identity operator over the antisymmetric Hilbert space [66]. It is easy to see that $\hat{S}_A$ in Eq. (104) indeed satisfies $\hat{S}_A^\dagger \hat{S}_A = \mathcal{J}_0 + \mathcal{J}_2 = \mathcal{P}_A$. The resulting density matrix constructed out of $|\psi_{\text{out}}\rangle_A \equiv \hat{S}_A |\psi_{\text{in}}\rangle$ is not normalized to 1:

$$\begin{aligned} \rho &= |\psi_{\text{out}}\rangle_A \langle\psi_{\text{out}}| = \hat{S}_A |\psi_{\text{in}}\rangle \langle\psi_{\text{in}}| \hat{S}_A^\dagger, \\ \text{Tr}[\rho] &= \langle\psi_{\text{out}}|\psi_{\text{out}}\rangle_A = \langle\psi_{\text{in}}|\mathcal{P}_A|\psi_{\text{in}}\rangle < 1. \end{aligned} \quad (106)$$

This situation is similar to the projection-valued measurement in quantum mechanics, in which case the properly normalized

density matrix is [67]

$$\tilde{\rho} = \frac{\mathcal{P}_A \rho \mathcal{P}_A}{\text{Tr}[\rho \mathcal{P}_A]} = \frac{|\psi_{\text{out}}\rangle_A \langle\psi_{\text{out}}|}{\langle\psi_{\text{in}}|\mathcal{P}_A|\psi_{\text{in}}\rangle} = \frac{\rho}{\langle\psi_{\text{in}}|\mathcal{P}_A|\psi_{\text{in}}\rangle}. \quad (107)$$

Then, we have $\text{Tr}[\tilde{\rho}] = 1$.

The calculation of entanglement power for subunitary S -matrices also needs to be modified. The definition of entanglement power in Eq. (15) employs an averaging procedure over the entire Hilbert space, which is clearly not applicable now given the restriction of the subunitary S matrix to the antisymmetric Hilbert space. Instead, we propose that the entanglement power only averages over the physically accessible sub-Hilbert space, where the subunitary S matrix projects into, and define the k th-order-weighted entanglement power for linear entropy:

$$\begin{aligned} E_k(\hat{S}) &= \frac{1}{\int d\omega_A d\omega_B \langle\psi_{\text{in}}|\mathcal{P}_A|\psi_{\text{in}}\rangle^k} \\ &\times \int d\omega_A d\omega_B \langle\psi_{\text{in}}|\mathcal{P}_A|\psi_{\text{in}}\rangle^k (1 - \text{Tr}_A[\tilde{\rho}_A^2]) \\ &= 1 - \frac{\int d\omega_A d\omega_B \langle\psi_{\text{in}}|\mathcal{P}_A|\psi_{\text{in}}\rangle^k \text{Tr}_A[\tilde{\rho}_A^2]}{\int d\omega_A d\omega_B \langle\psi_{\text{in}}|\mathcal{P}_A|\psi_{\text{in}}\rangle^k}. \end{aligned} \quad (108)$$

It can be verified that the above expression satisfies all essential conditions required for an “entanglement power.” For example, when the two subsystems have the same dimension n , the original entanglement power ranges from 0 to $1 - 1/n$ [21], and the values of E_k also fall within this range. Additionally, for neutron-neutron (nn) scattering, $E_k(\hat{S}) = 1/2$ for any k (see Appendix B for a proof), which is precisely what we expect given that the S -wave nn system is always in a maximally entangled $^1S_0$ spin singlet state.

It is computationally straightforward to calculate E_k with $k = 2$, as the $\langle\psi_{\text{in}}|\mathcal{P}_A|\psi_{\text{in}}\rangle^2$ factors cancel out the denominator in Eq. (107):

$$E_2(\hat{S}) = 1 - \frac{\int d\omega_A d\omega_B \text{Tr}_A[\rho_A^2]}{\int d\omega_A d\omega_B \langle\psi_{\text{in}}|\mathcal{P}_A|\psi_{\text{in}}\rangle^2}, \quad (109)$$

using which we can compute the entanglement power for the $\Omega\Omega$ S matrix in Eq. (104):

$$E_2(\hat{S}) = \frac{1}{48} [25 - \cos[4(\delta_{0,28} - \delta_{2,28})]]. \quad (110)$$

We find that $E_2(\hat{S})$ never vanishes, since the $\Omega\Omega$ state itself is highly entangled already. Nevertheless, there are two minima, which take place when

$$|\delta_{0,28} - \delta_{2,28}| = 0 \text{ or } \frac{\pi}{2}. \quad (111)$$

Since the lattice result for the scattering length in $^1S_0$ channel of $\Omega\Omega$ system is rather large, about 4.6 fm at a nearly physical pion mass $m_\pi \simeq 146$ MeV [51], it may suggest $\delta_{0,28} = \pi/2$, so $\delta_{2,28} = 0$ or $\pi/2$.

B. Identity and SWAP gates for identical particles

Let us interpret the two minima found in Eq. (111). Substituting $\delta_{0,28} = \delta_{2,28}$ into Eq. (104) yields

$$\hat{S}_A \propto \mathcal{J}_2 + \mathcal{J}_0 = \mathcal{P}_A, \quad (112)$$

which is the projection operator onto antisymmetric states $\sum_i \mathcal{A}_i$ and, as commented previously, can be interpreted as the Identity operator 1_A in the sub-Hilbert space spanned by antisymmetric states. On the other hand, substituting $\delta_{0,28} = \pi/2$ and $\delta_{2,28} = 0$, we obtain

$$\hat{S}_A = \mathcal{J}_2 - \mathcal{J}_0 \equiv \text{SWAP}_A, \quad (113)$$

which is the analog of the SWAP gate acting on the antisymmetric Hilbert space. Observe that $(\text{SWAP}_A)^2 = \mathcal{P}_A = 1_A$, just like $(\text{SWAP})^2 = 1$ for the unrestricted Hilbert space.

These considerations lead to

$$\mathcal{J}_0 = \frac{\mathcal{P}_A - \text{SWAP}_A}{2}, \quad \mathcal{J}_2 = \frac{\mathcal{P}_A + \text{SWAP}_A}{2}. \quad (114)$$

Moreover, $|J=0\rangle$ and $|J=2\rangle$ states are eigenstates of SWAP_A with ∓ 1 eigenvalues:

$$\text{SWAP}_A |J=2\rangle = |J=2\rangle, \quad \text{SWAP}_A |J=0\rangle = -|J=0\rangle, \quad (115)$$

which is parallel to the definition of SWAP in Eq. (32) for the unrestricted Hilbert space. The analogy can be developed further by considering the $J_z = 0$ states in both $J=0$ and $J=2$:

$$|\alpha\rangle = \frac{1}{\sqrt{2}}(|J=2; J_z=0\rangle + |J=0; J_z=0\rangle), \quad (116)$$

$$|\beta\rangle = \frac{1}{\sqrt{2}}(|J=2; J_z=0\rangle - |J=0; J_z=0\rangle), \quad (117)$$

and then the action of SWAP_A becomes

$$\text{SWAP}_A |\alpha\rangle = |\beta\rangle, \quad \text{SWAP}_A |\beta\rangle = |\alpha\rangle, \quad (118)$$

which indicates that it exchanges the states $|\alpha\rangle$ and $|\beta\rangle$.

Notice that it is the existence of two scattering channels in $J=0$ and $J=2$ that makes it possible to define an operator like SWAP_A in two-qudit scattering. For a two-qubit system such as the nn scattering, only one channel exists and one cannot introduce the SWAP_A operator.

C. Emergent symmetries for identical particles

It is important to point out that the solution in Eq. (111) indeed yields enhanced symmetries. When the S matrix is $\mathcal{P}_A$, which is the Identity gate over the antisymmetric Hilbert space, there is a “spin symmetry” since $\delta_{0,28} = \delta_{2,28}$ and the scattering dynamics is independent of the total spin of the system.

The symmetry group can be identified as follows. As the spin of the Ω baryon is $3/2$, there are four spin states with $J_z = \pm 1/2, \pm 3/2$, and the spin symmetry group is $\text{SU}(4)$ under which the four states form the fundamental representation **4**. Then, the relation $\delta_{0,28} = \delta_{2,28}$ is enforced by the $\text{SU}(4)$ spin symmetry as the $J=0$ and $J=2$ states now furnish the rank-2 antisymmetric irrep **6**. Out of the two possibilities shown in Eq. (62), **6** is realized because of FD statistics. On the other hand, Schrödinger symmetry emerges when the S matrix is SWAP_A , $|\delta_{0,28} - \delta_{2,28}| = \pi/2$, and $\delta_{0,28} = \pi/2$, as suggested by the lattice result in Ref. [51]. In other words, although the entanglement power in Eq. (110) never vanishes for identical particle scattering, there are still enhanced symmetries when the entanglement power is minimized.

A similar conclusion can be reached in the case of spin-1 dd scattering, which was first considered in Ref. [43]. Due to BE statistics, the S -wave dd scattering has total angular momentum $J=0$ and $J=2$, which is similar to the two spin channels in identical spin-3/2 $\Omega\Omega$ scattering (but here $J=0$ and $J=2$ are symmetric components). Using Eq. (109), the entanglement power for dd scattering can be written in terms of the phase difference in the $J=0$ and $J=2$ channel [68]:

$$E_2(\hat{S}) = \frac{171 - 70 \cos [2(\delta_0 - \delta_2)] - 20 \cos [4(\delta_0 - \delta_2)]}{594}. \quad (119)$$

Notice the dependence on both $2(\delta_0 - \delta_2)$ and $4(\delta_0 - \delta_2)$. Consequently, the entanglement power has a global minimum at $\delta_0 = \delta_2$ and a local minimum at $|\delta_0 - \delta_2| = \pi/2$. This is in contrast with the entanglement power for spin-3/2 $\Omega\Omega$ scattering in Eq. (110), which only depends on $4(\delta_{0,28} - \delta_{2,28})$ and both solutions, $|\delta_{0,28} - \delta_{2,28}| = 0$ or $\pi/2$, are global minima. It is intriguing that SWAP_S corresponds to the local minimum in dd scattering while $\mathcal{P}_S$ gives the global minimum. How general this observation is remains to be investigated further.

Following the discussion of enhanced symmetries in $\Omega\Omega$ scattering, the enhanced spin symmetry in dd scattering for $\delta_0 = \delta_2$ is $\text{SU}(3)_{\text{spin}}$, under which the $J_z = 0, \pm 1$ states form a **3** of $\text{SU}(3)$ and the group theory now dictates

$$\text{SU}(3)_{\text{spin}}: \quad \mathbf{3} \otimes \mathbf{3} = \underbrace{\mathbf{6}}_{\text{symmetric}} \oplus \underbrace{\mathbf{3}}_{\text{antisymmetric}}. \quad (120)$$

The symmetric **6** is realized in dd scattering because of BE statistics. The other scenario $|\delta_0 - \delta_2| = \pi/2$ can be realized if one of the channels flows to the unitarity limit and exhibits Schrödinger symmetry, as discussed previously.

More generally, when scattering two identical spin- s particles in the S wave, quantum statistics determines that the total angular momentum J can only take even values for both fermions and bosons [69]. According to the general result in Sec. IV A, the S matrix is an Identity gate when all scattering phases are equal. In this case, group-theoretic counting indicates that there is an enhanced $\text{SU}(2s+1)_{\text{spin}}$ symmetry, under which the $(2s+1)$ spin states furnish a fundamental representation. In this case,

$$\begin{aligned} \text{SU}(2s+1)_{\text{spin}}: \quad & (2s+1) \otimes (2s+1) \\ & = \underbrace{(s+1)(2s+1)}_{\text{symmetric}} \oplus \underbrace{s(2s+1)}_{\text{antisymmetric}}. \end{aligned} \quad (121)$$

The dimensions of rank-2 symmetric and antisymmetric irreps agree with the numbers of symmetric and antisymmetric J states, respectively,

$$\sum_{\text{even } J, J \leq 2s} (2J+1) = \begin{cases} (s+1)(2s+1), & \text{for integral } s, \\ s(2s+1), & \text{for half-integral } s. \end{cases} \quad (122)$$

VIII. SUMMARY AND OUTLOOK

In this paper, we explored the consequence of the conjectured entanglement suppression in the 2-to-2 S -wave scattering of spin-3/2 baryons, which are four-dimensional qudits in the spin space. This hypothesis posits that under

certain conditions, low-energy scatterings minimize entanglement in the spin degrees of freedom, which can lead to the emergence of enhanced symmetries. Spin-3/2 baryons transform as a ten-dimensional irrep under the light-flavor $SU(3)$ symmetry and are considered indistinguishable under strong interactions. Our analysis places particular emphasis on the interplay between spin and flavor symmetries due to the constraints of quantum statistics.

Using the general result that, for a bipartite system of equal dimensions, entanglement suppression takes place only for the Identity and SWAP gates, we presented the conditions on the scattering phases leading to entanglement suppression in spin-3/2 decuplet scattering. Using group-theoretic arguments, we identify a large $SU(40)_{\text{spin+flavor}}$ when the S matrix is an Identity gate, as well as an $SU(4)_{\text{spin}} \times SU(10)_{\text{flavor}}$ symmetry combined with the Schrödinger symmetry in the case of the SWAP gate. These observations are subsequently verified using nonrelativistic effective field theories for spin-3/2 baryons. These findings generalize the results from the scattering of spin-1/2 octet baryons.

We also carefully examine the scattering of identical particles and the related computation of entanglement power. While this case has been considered previously in the literature, there are subtleties arising from the fact that quantum statistics restricts that only the symmetric or antisymmetric portion of the two-particle Hilbert space is physically accessible, which needs to be dealt with deliberately. In particular, we propose that the entanglement power only averages over the symmetric or antisymmetric Hilbert space, depending on whether the interaction satisfies the BE or FD statistics, and proceed to demonstrate that one can define the corresponding Identity and SWAP gates for identical particle scattering. Furthermore, even though the entanglement power is generally nonvanishing due to quantum statistics, enhanced symmetries still appear at the global or local minimum. For the scattering of identical spin- s particles, we identify the enhanced symmetry groups to be $SU(2s+1)_{\text{spin}}$ and/or the nonrelativistic conformal group.

It is worth emphasizing that the scattering dynamics of systems beyond two identical qubits contains a rich phenomenology and offers important insights into the interplay between entanglement suppression, enhanced symmetries, and quantum statistics. This is because in two-qubit systems the S matrix in the symmetric or antisymmetric Hilbert space only contains one single scattering channel in the spin space, which renders the entanglement power constant. It is the existence of two or more scattering channels that results in the possibility of global or local minimum in the nonvanishing entanglement power. In the end, the entanglement power does not have to vanish to lead to enhanced symmetries. A global or local minimum is sufficient.

As a natural continuation of the current framework, octet-decuplet scattering involves a bipartite system with differing dimensions for the subsystems. This scenario will be studied in the future.

ACKNOWLEDGMENTS

We would like to thank Hao-Jie Jing for discussions regarding the generators of $SU(3)$ irreps and the construction

of Lagrangian. This work was initiated during the long-term workshop, HHIQCD2024, at the Yukawa Institute for Theoretical Physics (YITP-T-24-02) in Kyoto University, and we thank the workshop organizers and the participants for a stimulating meeting. T.R.H. was supported in part by the China National Training Program of Innovation and Entrepreneurship for Undergraduates under Grant No. 202414430004; K.S. and T.H. were supported by the Grants-in-Aid for Scientific Research from JSPS (Grants No. JP23H05439 and No. JP22K03637) and by JST, the establishment of university fellowships toward the creation of science technology innovation, Grant No. JPMJFS2123; T.R.H. and F.K.G. were supported in part by the National Natural Science Foundation of China under Grants No. 12125507, No. 12361141819, and No. 12447101, and by the Chinese Academy of Sciences under Grant No. YSBR-101; and I.L. was supported in part by the U.S. Department of Energy under Contracts No. DE-AC02-06CH11357, No. DE-SC0023522, No. DE-SC0010143, and No. 89243024CSC000002 (QuantISED Program).

DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: CLASSIFICATION OF DECUPLET-DECUPLET CHANNELS

The channels of the decuplet-decuplet scattering and their charge Q , total isospin I , total spin J , and corresponding flavor irreps F in the S -wave scattering are summarized in Tables I–VII. Each table is classified in terms of the strangeness (hypercharge) S (Y) of the system, and therefore the (Q, S) sectors can be found from the tables. The spin-antisymmetric (spin-symmetric) component $J_A = 0, 2$ ($J_S = 1, 3$) and their corresponding flavor-symmetric (flavor-antisymmetric) irreps $F_S = 27, 28$ ($F_A = \overline{10}, 35$) are shown in the $[J_A, F_S]$ ($[J_S, F_A]$) column in each table. The isospin, spin, and flavor irreps in the lower half of each table are omitted, as they are identical to those of the I_3 -conjugate channels presented in the upper half.

The flavor irrep $\overline{10}$ does not contain the $S = -4, -5$, and -6 sectors, and therefore the channels in Tables V–VII are within the 27, 28, and 35 irreps. Moreover, in the case of $S = -6$, the channel with $\Omega\Omega$ belongs to only 28 and their spin components are restricted to antisymmetric ones $J = 0, 2$, as discussed in Sec. VII. On the other hand, the channels including all possible flavor irreps can be found in the $S = 0, -1, -2$, and -3 sectors.

APPENDIX B: ENTANGLEMENT POWER IN nn SCATTERING

Here, we show that the k th-order-weighted entanglement power in Eq. (108) gives the desired result $E_k(\hat{S}) = 1/2$ for the nn scattering, independent of k . For the nn scattering, the S matrix is given by a single component

$$\hat{S} = \mathcal{J}_0 e^{2i\delta_0}, \quad (\text{B1})$$

TABLE I. The decuplet-decuplet scattering channels and their charge Q , total isospin I , total spin J , and corresponding flavor irreps F for the strangeness (hypercharge) $S = 0$ ($Y = 2$) sector. Channels in the bottom half are the I_3 conjugate of those in the upper half and have the same spin and flavor irreps.

Channels for $S = 0$	Q	I	$[J_A, F_S]$	$[J_S, F_A]$
$\Delta^- \Delta^-$	-2	3	[0, 28], [2, 28]	
$\Delta^- \Delta^0$	-1	2,3	[0, 28], [2, 28]	[1, 35], [3, 35]
$\Delta^- \Delta^+, \Delta^0 \Delta^0$	0	1,2,3	[0, 27], [0, 27], [2, 28], [2, 28]	[1, 35], [3, 35]
$\Delta^- \Delta^{++}, \Delta^0 \Delta^+$	1	0,1,2,3	[0, 27], [0, 27], [2, 28], [2, 28]	[1, 10], [1, 10], [1, 35], [3, 35]
$\Delta^0 \Delta^{++}, \Delta^+ \Delta^+$	2			
$\Delta^+ \Delta^{++}$	3			
$\Delta^{++} \Delta^{++}$	4			

TABLE II. Same as Table I but for the $S = -1$ ($Y = 1$) sector.

Channels for $S = -1$	Q	I	$[J_A, F_S]$	$[J_S, F_A]$
$\Delta^- \Sigma^{*-}$	-2	5/2	[0, 28], [2, 28]	[1, 35], [3, 35]
$\Delta^0 \Sigma^{*-}, \Delta^- \Sigma^{*0}$	-1	3/2, 5/2	[0, 27], [2, 27], [0, 28], [2, 28]	[1, 35], [3, 35]
$\Delta^- \Sigma^{*+}, \Delta^0 \Sigma^{*0}, \Delta^+ \Sigma^{*-}$	0	1/2, 3/2, 5/2	[0, 27], [2, 27], [0, 28], [2, 28]	[1, 10], [3, 10], [1, 35], [3, 35]
$\Delta^0 \Sigma^{*+}, \Delta^+ \Sigma^{*0}, \Delta^{++} \Sigma^{*-}$	1			
$\Delta^+ \Sigma^{*+}, \Delta^{++} \Sigma^{*0}$	2			
$\Delta^{++} \Sigma^{*+}$	3			

TABLE III. Same as Table I but for the $S = -2$ ($Y = 0$) sector.

Channels for $S = -2$	Q	I	$[J_A, F_S]$	$[J_S, F_A]$
$\Delta^- \Xi^{*-}, \Sigma^{*-} \Sigma^{*-}$	-2	2	[0, 27], [2, 27], [0, 28], [2, 28]	[1, 35], [3, 35]
$\Delta^- \Xi^{*0}, \Delta^0 \Xi^{*-}, \Sigma^{*-} \Sigma^{*0}$	-1	1,2	[0, 27], [2, 27], [0, 28], [2, 28]	[1, 10], [3, 10], [1, 35], [3, 35]
$\Delta^+ \Xi^{*-}, \Delta^0 \Xi^{*0}, \Sigma^{*0} \Sigma^{*0}, \Sigma^{*+} \Sigma^{*-}$	0	0,1,2	[0, 27], [2, 27], [0, 28], [2, 28]	[1, 10], [3, 10], [1, 35], [3, 35]
$\Delta^+ \Xi^{*0}, \Delta^{++} \Xi^{*-}, \Sigma^{*+} \Sigma^{*0}$	1			
$\Delta^{++} \Xi^{*0}, \Sigma^{*+} \Sigma^{*+}$	2			

TABLE IV. Same as Table I but for the $S = -3$ ($Y = -1$) sector.

Channels for $S = -3$	Q	I	$[J_A, F_S]$	$[J_S, F_A]$
$\Sigma^{*-} \Xi^{*-}, \Omega^- \Delta^-$	-2	3/2	[0, 27], [2, 27], [0, 28], [2, 28]	[1, 10], [3, 10], [1, 35], [3, 35]
$\Sigma^{*-} \Xi^{*0}, \Sigma^{*0} \Xi^{*-}, \Omega^- \Delta^0$	-1	1/2, 3/2	[0, 27], [2, 27], [0, 28], [2, 28]	[1, 10], [3, 10], [1, 35], [3, 35]
$\Sigma^{*0} \Xi^{*0}, \Sigma^{*+} \Xi^{*-}, \Omega^- \Delta^+$	0			
$\Sigma^{*+} \Xi^{*0}, \Omega^- \Delta^{++}$	1			

TABLE V. Same as Table I but for the $S = -4$ ($Y = -2$) sector.

Channels for $S = -4$	Q	I	$[J_A, F_S]$	$[J_S, F_A]$
$\Xi^{*-} \Xi^{*-}, \Omega^- \Sigma^{*-}$	-2	1	[0, 27], [2, 27], [0, 28], [2, 28]	[1, 35], [3, 35]
$\Xi^{*-} \Xi^{*0}, \Omega^- \Sigma^{*0}$	-1	0,1	[0, 27], [2, 27], [0, 28], [2, 28]	[1, 35], [3, 35]
$\Xi^{*0} \Xi^{*0}, \Omega^- \Sigma^{*+}$	0			

TABLE VI. Same as Table I but for the $S = -5$ ($Y = -3$) sector.

Channels for $S = -5$	Q	I	$[J_A, F_S]$	$[J_S, F_A]$
$\Omega^- \Xi^{*-}$	-2	1/2	$[0, \mathbf{28}], [2, \mathbf{28}]$	$[1, \mathbf{35}], [3, \mathbf{35}]$
$\Omega^- \Xi^{*0}$	-1			

with $\mathcal{J}_0 = \mathcal{P}_A$. The projection operator can be written as

$$\mathcal{J}_0 = |J=0\rangle \langle J=0|, \quad |J=0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle). \quad (\text{B2})$$

Let us write the initial state in the computational basis [16]:

$$|\psi_{\text{in}}\rangle = \alpha |\uparrow\uparrow\rangle + \beta |\uparrow\downarrow\rangle + \gamma |\downarrow\uparrow\rangle + \delta |\downarrow\downarrow\rangle, \quad (\text{B3})$$

where α, β, γ , and δ are complex coefficients satisfying $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Multiplying the projection operator $\mathcal{J}_0$ to the initial state $|\psi_{\text{in}}\rangle$, we obtain

$$\mathcal{J}_0 |\psi_{\text{in}}\rangle = \frac{\beta - \gamma}{\sqrt{2}} |J=0\rangle. \quad (\text{B4})$$

This means that the normalization factor is

$$\langle \psi_{\text{in}} | \mathcal{P}_A | \psi_{\text{in}} \rangle = \langle \psi_{\text{in}} | \mathcal{J}_0 | \psi_{\text{in}} \rangle = \frac{|\beta - \gamma|^2}{2}, \quad (\text{B5})$$

and the normalized density matrix is nothing but the projection operator

$$\tilde{\rho} = \frac{\hat{S}_A |\psi_{\text{in}}\rangle \langle \psi_{\text{in}}| \hat{S}_A^\dagger}{\langle \psi_{\text{in}} | \mathcal{P}_A | \psi_{\text{in}} \rangle} = \frac{\frac{\beta - \gamma}{\sqrt{2}} |J=0\rangle \langle J=0| \frac{(\beta - \gamma)^*}{\sqrt{2}}}{\frac{|\beta - \gamma|^2}{2}} = \mathcal{J}_0. \quad (\text{B6})$$

TABLE VII. Same as Table I but for the $S = -6$ ($Y = -4$) sector.

Channels for $S = -6$	Q	I	$[J_A, F_S]$	$[J_S, F_A]$
$\Omega^- \Omega^-$	-2	0	$[0, \mathbf{28}], [2, \mathbf{28}]$	

Because this normalized density matrix is independent of the angles ω_A and ω_B of the initial state, the entanglement power is independent of k :

$$E_k(\hat{S}) = 1 - \frac{\int d\omega_A d\omega_B \langle \psi_{\text{in}} | \mathcal{P} | \psi_{\text{in}} \rangle^k \text{Tr}_A [\tilde{\rho}_A^2]}{\int d\omega_A d\omega_B \langle \psi_{\text{in}} | \mathcal{P} | \psi_{\text{in}} \rangle^k} \\ = 1 - \text{Tr}_A [\tilde{\rho}_A^2]. \quad (\text{B7})$$

Moreover, this is the entanglement entropy of the maximally entangled Bell state, and therefore we find

$$E_k(\hat{S}) = \frac{1}{2} \quad (nn \text{ scattering}). \quad (\text{B8})$$

This result is to be expected because the initial state is constrained by FD statistics to be a maximally entangled Bell state and $\hat{S}$ is proportional to $\mathcal{J}_0$, which is an Identity gate over the antisymmetric Hilbert space. So, the S matrix preserves the entanglement of the initial state to be 1/2. When averaging over the antisymmetric Hilbert space, the entanglement power remains at 1/2. Without the proper volume normalization in Eq. (B7), the entanglement power would not have come out correctly.

Note that the simple result in Eq. (B6) is obtained because of the single-component S matrix as in Eq. (B1). For multi-component S matrix such as Eq. (104) for $\Omega\Omega$ scattering, the normalized density matrix depends on the phase shifts and the angles ω_A and ω_B , and the cancellation in Eq. (B7) does not hold.

- [1] The deuteron is a $^3S_1$ bound state with a binding energy of 2.2 MeV, while the $^1S_0$ nn pair has a virtual state about 66 keV below threshold (see, e.g., Ref. [2]). Were the attraction in the latter pair a little bit stronger, a bound state would also be formed. As a consequence of the existence of the shallow bound or virtual state, both channels have unnaturally large scattering lengths.
- [2] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, An accurate nucleon-nucleon potential with charge independence breaking, *Phys. Rev. C* **51**, 38 (1995).
- [3] D. B. Kaplan and M. J. Savage, The spin flavor dependence of nuclear forces from large- n QCD, *Phys. Lett. B* **365**, 244 (1996).
- [4] S. R. Beane, D. B. Kaplan, N. Klco, and M. J. Savage, Entanglement suppression and emergent symmetries of strong interactions, *Phys. Rev. Lett.* **122**, 102001 (2019).
- [5] A. Cervera-Lierta, J. I. Latorre, J. Rojo, and L. Rottoli, Maximal entanglement in high energy physics, *SciPost Phys.* **3**, 036 (2017).
- [6] Y. Afik and J. R. M. de Nova, Entanglement and quantum tomography with top quarks at the LHC, *Eur. Phys. J. Plus* **136**, 907 (2021).
- [7] R. Aoude, M.-Z. Chung, Y.-T. Huang, C. S. Machado, and M.-K. Tam, Silence of binary Kerr black holes, *Phys. Rev. Lett.* **125**, 181602 (2020).
- [8] R. Aoude, E. Madge, F. Maltoni, and L. Mantani, Probing new physics through entanglement in diboson production, *J. High Energy Phys.* **12** (2023) 017.
- [9] K. Sakurai and M. Spannowsky, Three-body entanglement in particle decays, *Phys. Rev. Lett.* **132**, 151602 (2024).
- [10] A. J. Barr, M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, Quantum entanglement and Bell inequality violation at colliders, *Prog. Part. Nucl. Phys.* **139**, 104134 (2024).
- [11] R. Aoude, G. Elor, G. N. Remmen, and O. Sumensari, Positivity in amplitudes from quantum entanglement, [arXiv:2402.16956](https://arxiv.org/abs/2402.16956).
- [12] I. Low and Z. Yin, An area law for entanglement entropy in particle scattering, [arXiv:2405.08056](https://arxiv.org/abs/2405.08056).
- [13] I. Low and Z. Yin, Elastic cross section is entanglement entropy, *Phys. Rev. D* **111**, 065027 (2025).
- [14] M. Blasone, S. De Siena, G. Lambiase, C. Matrella, and B. Micciola, Entanglement saturation in quantum electrodynamics scattering processes, [arXiv:2505.06878](https://arxiv.org/abs/2505.06878).
- [15] I. Low and T. Mehen, Symmetry from entanglement suppression, *Phys. Rev. D* **104**, 074014 (2021).

- [16] Q. Liu, I. Low, and T. Mehen, Minimal entanglement and emergent symmetries in low-energy QCD, *Phys. Rev. C* **107**, 025204 (2023).
- [17] M. Carena, I. Low, C. E. M. Wagner, and M.-L. Xiao, Entanglement suppression, enhanced symmetry, and a standard-model-like Higgs boson, *Phys. Rev. D* **109**, L051901 (2024).
- [18] Q. Liu and I. Low, Hints of entanglement suppression in hyperon-nucleon scattering, *Phys. Lett. B* **856**, 138899 (2024).
- [19] T.-R. Hu, S. Chen, and F.-K. Guo, Entanglement suppression and low-energy scattering of heavy mesons, *Phys. Rev. D* **110**, 014001 (2024).
- [20] K. Kowalska and E. M. Sessolo, Entanglement in flavored scalar scattering, *J. High Energy Phys.* **07** (2024) 156.
- [21] S. Chang and G. Jacobo, Consequences of minimal entanglement in bosonic field theories, *Phys. Rev. D* **110**, 096020 (2024).
- [22] N. McGinnis, Symmetry, entanglement, and the S -matrix, [arXiv:2504.21079](#).
- [23] M. Carena, G. Coloretti, W. Liu, M. Littmann, I. Low, and C. E. M. Wagner, Entanglement maximization and mirror symmetry in two-Higgs-doublet models, *J. High Energy Phys.* **08** (2025) 016.
- [24] J. Thaler and S. Trifinopoulos, Flavor patterns of fundamental particles from quantum entanglement? *Phys. Rev. D* **111**, 056021 (2025).
- [25] J. Liu, M. Tanaka, X.-P. Wang, J.-J. Zhang, and Z. Zheng, Scattering entanglement entropy and its implications for electroweak phase transitions, *Phys. Rev. D* **112**, 015028 (2025).
- [26] E. Wigner, On the consequences of the symmetry of the nuclear Hamiltonian on the spectroscopy of nuclei, *Phys. Rev.* **51**, 106 (1937).
- [27] E. Wigner, On the structure of nuclei beyond oxygen, *Phys. Rev.* **51**, 947 (1937).
- [28] E. P. Wigner, On coupling conditions in light nuclei and the lifetimes of beta-radioactivities, *Phys. Rev.* **56**, 519 (1939).
- [29] M. L. Wagman, F. Winter, E. Chang, Z. Davoudi, W. Detmold, K. Orginos, M. J. Savage, and P. E. Shanahan, Baryon-Baryon interactions and spin-flavor symmetry from lattice quantum chromodynamics, *Phys. Rev. D* **96**, 114510 (2017).
- [30] M. C. Birse, J. A. McGovern, and K. G. Richardson, A renormalization group treatment of two-body scattering, *Phys. Lett. B* **464**, 169 (1999).
- [31] T. Mehen, I. W. Stewart, and M. B. Wise, Conformal invariance for nonrelativistic field theory, *Phys. Lett. B* **474**, 145 (2000).
- [32] G. Busoni, J. Gargalionis, E. N. V. Wallace, and M. J. White, Emergent symmetry in a two-Higgs-doublet model from quantum information and nonstabilizerness, *Phys. Rev. D* **112**, 035022 (2025).
- [33] S. Bravyi and A. Kitaev, Universal quantum computation with ideal Clifford gates and noisy ancillas, *Phys. Rev. A* **71**, 022316 (2005).
- [34] C. E. P. Robin and M. J. Savage, Quantum complexity fluctuations from nuclear and hypernuclear forces, *Phys. Rev. C* **112**, 044004 (2025).
- [35] C. D. White and M. J. White, Magic states of top quarks, *Phys. Rev. D* **110**, 116016 (2024).
- [36] I. Chernyshev, C. E. P. Robin, and M. J. Savage, Quantum magic and computational complexity in the neutrino sector, *Phys. Rev. Res.* **7**, 023228 (2025).
- [37] Q. Liu, I. Low, and Z. Yin, Maximal magic for two-qubit states, [arXiv:2502.17550](#).
- [38] Q. Liu, I. Low, and Z. Yin, Quantum magic in quantum electrodynamics, [arXiv:2503.03098](#).
- [39] J. Gargalionis, N. Moynihan, S. Trifinopoulos, E. N. V. Wallace, C. D. White, and M. J. White, Spin versus magic: Lessons from gluon and graviton scattering, [arXiv:2508.14967](#).
- [40] Q. Liu, I. Low, and Z. Yin, A quantum computational determination of the weak mixing angle in the standard model, [arXiv:2509.18251](#).
- [41] For brevity, we would just refer to spin-3/2 baryons as “qudits” henceforth.
- [42] S. R. Beane, R. C. Farrell, and M. Varma, Entanglement minimization in hadronic scattering with pions, *Int. J. Mod. Phys. A* **36**, 2150205 (2021).
- [43] T. Kirchner, W. Elkamhawy, and H.-W. Hammer, Entanglement in few-nucleon scattering events, *Few-Body Syst.* **65**, 29 (2024).
- [44] The same considerations apply to the scattering of identical bosons in the two-Higgs-doublet model [21].
- [45] F. Dyson and N. H. Xuong, $Y = 2$ States in $su(6)$ theory, *Phys. Rev. Lett.* **13**, 815 (1964).
- [46] M. Bashkanov *et al.*, Double-Pionic fusion of nuclear systems and the ABC effect: Approaching a puzzle by exclusive and kinematically complete measurements, *Phys. Rev. Lett.* **102**, 052301 (2009).
- [47] P. Adlarson *et al.* (WASA-at-COSY), ABC effect in basic double-pionic fusion—observation of a new resonance? *Phys. Rev. Lett.* **106**, 242302 (2011).
- [48] P. Adlarson *et al.* (WASA-at-COSY), Evidence for a new resonance from polarized neutron-proton scattering, *Phys. Rev. Lett.* **112**, 202301 (2014).
- [49] There is also a nonresonant explanation for the experimental signal [50].
- [50] R. Molina, N. Ikeno, and E. Oset, Sequential single pion production explaining the dibaryon “ $d^*(2380)$ ” peak, *Chin. Phys. C* **47**, 041001 (2023).
- [51] S. Gongyo *et al.*, Most strange dibaryon from lattice QCD, *Phys. Rev. Lett.* **120**, 212001 (2018).
- [52] T. R. Richardson, M. R. Schindler, and R. P. Springer, The role of intermediate $\Delta\Delta$ states in nucleon–nucleon scattering in the large- N_c and unitary limits, and $\Delta\Delta$ and $\Omega\Omega$ scattering, *J. Phys. G: Nucl. Part. Phys.* **52**, 085101 (2025).
- [53] In principle, the octet-octet and octet-decuplet channels with lower thresholds can also couple to the decuplet-decuplet systems with the same quantum numbers. In the near-threshold region of the decuplet pair, such channels do not produce any sharp energy dependence, and thus effectively give an optical potential to the decuplet-decuplet interaction, manifesting as an imaginary part of the contact terms. For simplicity, we disregard the complexities associated with this imaginary component.
- [54] P. Zanardi, Entanglement of quantum evolutions, *Phys. Rev. A* **63**, 040304 (2001).
- [55] I. Bengtsson, J. Braennlund, and K. Życzkowski, CP^n , or, entanglement illustrated, *Int. J. Mod. Phys. A* **17**, 4675 (2002).
- [56] I. Bengtsson and K. Życzkowski, *Geometry of Quantum States: An Introduction to Quantum Entanglement* (Cambridge University Press, Cambridge, 2017).

- [57] At the unitarity limit, $p \cot \delta = 0$, where p is the magnitude of the center-of-mass momentum and δ is the phase shift, so that the inverse of the scattering amplitude is given only the unitarity term, and correspondingly $\delta = \pi/2$.
- [58] E. Alfsen and F. Shultz, Unique decompositions, faces, and automorphisms of separable states, *J. Math. Phys.* **51**, 052201 (2010).
- [59] N. Johnston, Characterizing operations preserving separability measures via linear preserver problems, *Lin. Multilin. Alg.* **59**, 1171 (2011).
- [60] S. Friedland, C.-K. Li, Y.-T. Poon, and N.-S. Sze, The automorphism group of separable states in quantum information theory, *J. Math. Phys.* **52**, 042203 (2011).
- [61] H. A. Bethe, Theory of the effective range in nuclear scattering, *Phys. Rev.* **76**, 38 (1949).
- [62] D. B. Kaplan, M. J. Savage, and M. B. Wise, Nucleon-nucleon scattering from effective field theory, *Nucl. Phys. B* **478**, 629 (1996).
- [63] H.-W. Hammer, S. König, and U. van Kolck, Nuclear effective field theory: Status and perspectives, *Rev. Mod. Phys.* **92**, 025004 (2020).
- [64] J. Haidenbauer, S. Petschauer, N. Kaiser, U.-G. Meißner, and W. Weise, Scattering of decuplet baryons in chiral effective field theory, *Eur. Phys. J. C* **77**, 760 (2017).
- [65] M. Illa *et al.* (NPLQCD), Low-energy scattering and effective interactions of two baryons at $m_\pi \sim 450$ meV from lattice quantum chromodynamics, *Phys. Rev. D* **103**, 054508 (2021).
- [66] As mentioned in Sec. IV A, such a basis consists of symmetric and antisymmetric base vectors.
- [67] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2012).
- [68] The denominator in Eq. (109) is different from that used in Ref. [43] to compute the entanglement power in the spin-1 dd scattering, where the corresponding denominator is $(\int d\omega_A d\omega_B \langle \psi_{\text{in}} | \mathcal{P}_S | \psi_{\text{in}} \rangle)^2$. We thank Hans-Werner Hammer for clarifications on this point.
- [69] Recall that when adding two identical half-integral angular momenta, the even total angular momentum is antisymmetric under the exchange of two particles. On the other hand, when adding two identical integral angular momenta, the symmetric states also have even total angular momentum!