

SIETAS model: Swarm-informed triggering, earthquake diffusion, and multiscaling applied to the current Campi Flegrei unrest

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Earthquake diffusion is commonly interpreted either as the migration of fluids or as the propagation of triggering cascades within branching processes. In the latter framework, anomalous diffusion emerges naturally from the long-memory kernels of epidemic-type aftershock sequence (ETAS) models. Here, we introduce a swarm-informed ETAS (SIETAS) model designed to describe volcanic seismic swarms. The model combines a temporally truncated power-law triggering kernel with a spatially localized logistic kernel, thereby introducing intrinsic temporal and spatial scales into the triggering process. Using an AI earthquake catalog from the ongoing Campi Flegrei unrest, we show that SIETAS provides a substantially better description of seismicity than the standard ETAS formulation. We further demonstrate that earthquake diffusion exhibits clear multiscaling behavior characterized by three effective scaling regimes. Analytical arguments based on branching-process theory and continuous-time random walks indicate that the SIETAS kernels generate a crossover from transient anomalous diffusion to an effectively normal-diffusive regime, followed by saturation due to finite subcriticality. We discuss how the observed multiscaling behavior may emerge from the interplay between short-time triggering, finite temporal memory, finite spatial localization, and secondary components of the triggering kernel. These results establish a connection between swarm-informed earthquake triggering and multiscale diffusion phenomena in volcanic environments.

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I. INTRODUCTION

Understanding how seismic activity propagates in space and time remains a central problem in statistical seismology and complex systems. Earthquake occurrence is commonly described through a set of phenomenological laws, including the Omori law for temporal clustering [1], the Gutenberg-Richter magnitude distribution [2], and the spatial clustering of aftershocks [3].

A very intriguing observation is that the aftershock area expands as in an anomalous diffusive process [4]. We note that a diffusion process can be described by the relation between the average distance from the expansion center and the time elapsed from the beginning of the diffusion:

$$\langle r^2 \rangle \propto t^\alpha. \quad (1)$$

When $\alpha = 1$, we have standard Brownian diffusion, whereas for $\alpha < 1$, we observe a subdiffusive random walk, and for $\alpha > 1$ a superdiffusive one. There are two different interpretations of earthquake diffusion:

(1) The first one attributes the expansion of the earthquake area to fluid migration. In fact, the pore-pressure increase can reduce the effective normal stress on active faults (e.g., Ref. [5]). A large amount of literature has been produced explaining empirical observations with stress and fluid diffusion in tectonic regime [6–10], for aseismic slip [11] or for volcanic swarms [12,13]. Triggered earthquakes can reach distances varying from few meters up to several kilometers away from the initial pore-pressure perturbation and at timescales that may vary from days up to months, or even years [14,15].

(2) The second interpretation views earthquake diffusion as a Levy flight [16] attributing this behavior to the cascading process governing seismic occurrence [17]. More precisely, in the standard epidemic-type aftershock sequence (ETAS) model [18–24], the triggering kernel is typically described by a power-law decay in time [1] and a spatial power-law distribution [3]. The authors of Ref. [17] showed that the space-time propagation of seismicity in this model can be mapped onto a continuous-time random walk (CTRW),

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leading to anomalous diffusion of aftershock sequences. Statistical analyses of global earthquake catalogs confirmed such interpretation [25], evidencing that earthquake diffusion is a multiscaling process [26].

While these mechanisms have been extensively investigated for tectonic earthquake sequences, their applicability to volcanic seismic swarms remains less clear. As one of the most compelling types of clustered seismicity, seismic swarms evolve without being preceded or dominated by a major mainshock, typically reflecting transient physical mechanisms like fluid transport, volcanic reactivation, or aseismic fault displacement. First characterized by Mogi [27] as spatiotemporal groupings of earthquakes within a confined region, swarms diverge from standard tectonic sequences because they comprise multiple events of similar magnitudes rather than a singular large event followed by decaying aftershocks [27,28]. Even though this definition dates back several decades, it remains highly accurate in identifying the core features of swarm behavior, even when analyzed through modern, high-resolution earthquake monitoring networks. Seismicity is rarely steady over time, often manifesting as localized bursts or phases of accelerated activity [28,29]. These seismic swarms are generally restricted to narrow geographical areas and frequently develop near active fault networks, geothermal systems, or volcanic complexes [30,31]. While conventional mainshock-aftershock sequences are primarily propelled by coseismic stress transfer, swarms are widely understood as a fault system's reaction to transient external forcings. The primary driving mechanisms include pore-pressure diffusion driven by subsurface fluids [5,32], the movement of magma or hydrothermal fluids in volcanic systems [33,34], and slow aseismic slip that continuously loads smaller locked asperities [35–38].

Volcanic swarms often exhibit finite-duration activation episodes, repeated bursts of seismicity, and strong spatial localization associated with hydrothermal and magmatic structures. These observations suggest the existence of intrinsic temporal and spatial scales that may not be adequately captured by the scale-free triggering kernels commonly adopted in standard ETAS formulations. In what follows, we show that the Campi Flegrei seismicity recorded during the current unrest exhibits a multiscaling behavior that can be interpreted in terms of a cascading process. In fact, for volcanic seismicity, empirical kernels often differ from the standard tectonic case. In particular, temporal triggering follows a power law with an exponential cutoff [39], while the spatial kernel is described a logisticlike expression [40]. We refer to the ETAS model equipped with these kernels as the swarm-informed ETAS (SIETAS) model. SIETAS has been already applied to explain the temporal occurrence of earthquakes at the Taupo volcanic zone [41].

In this work, we investigate whether finite-memory and finite-range triggering kernels can provide a more appropriate description of volcanic seismic swarms. Using a high-resolution AI-based catalog of the ongoing Campi Flegrei unrest [42], we first show that the SIETAS model provides a substantially improved description of seismicity compared with the standard ETAS formulation. We then demonstrate that Campi Flegrei seismicity exhibits clear multiscaling signatures in earthquake diffusion and explore

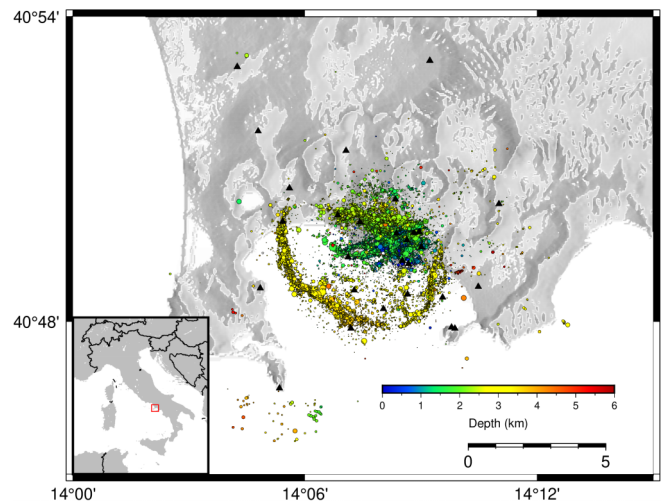


FIG. 1. The map of Campi Flegrei reporting the location of the earthquakes in the AI catalog. Circles represent the seismic events, the size is proportional to the magnitude, and the color is the focal depth. (Inset) The map of Italy highlighting the location of Campi Flegrei caldera in a red square.

how these observations may emerge from the finite temporal and spatial scales introduced by the SIETAS triggering kernels. Finally, we discuss the implications of these results for the interpretation of seismic diffusion in volcanic environments.

II. THE CATALOG

In the present study, we use the seismic catalog provided by the authors of Ref. [42], a high-resolution record of earthquakes occurring in the Campi Flegrei area in the time window 2022–2025 during the current unrest. It includes low- to moderate-magnitude events, detected through improved identification and relocation techniques that allow for a more detailed reconstruction of the spatial and temporal distribution of seismicity. The catalog reveals a pronounced clustering of events in specific areas of the caldera, highlighting the presence of active structures and ongoing processes related to the pressurization of the magmatic-hydrothermal system. Furthermore, the high level of completeness of the dataset enables the analysis of temporal variations in seismicity rates, offering valuable insights into the evolution of the unrest and its potential implications for hazard assessment.

The seismic catalog spans from 21 January 2022 to 20 March 2025 and includes more than 54 000 earthquakes identified using a machine-learning approach. The location of the earthquakes is shown in Fig. 1. The events cover a wide magnitude range, approximately from about -4.27 up to 4.38 , capturing both microseismicity and the largest shocks recorded during the current unrest phase. The magnitude used in the catalog is a relative magnitude constrained using the duration magnitude used in the manual official catalog used for training the machine-learning algorithm [42]. The magnitude as a function of time is shown in Fig. 2(b).

The completeness magnitude, equal to -0.16 , has been estimated following a method based on the variability

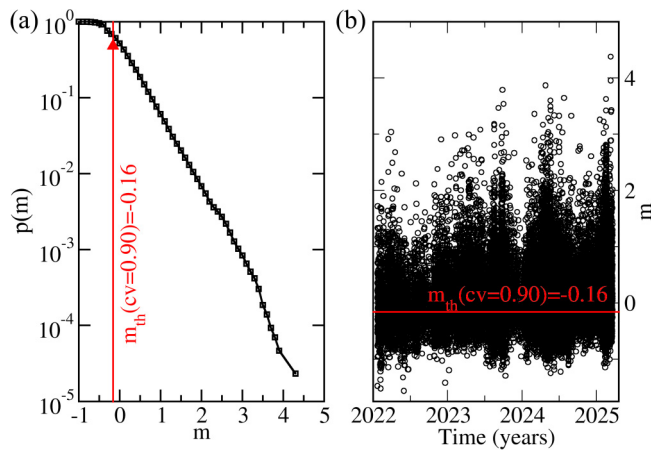


FIG. 2. (a) The Gutenberg-Richter distribution for the AI catalog for Campi Flegrei. (b) The earthquake magnitude as a function of time. The red line represents the minimum magnitude threshold considered in this study and corresponds to $CV = 0.9$, following Godano *et al.* [44].

coefficient [43,44]. c_V is defined as the ratio between the standard deviation and the average value of a given variable (in this case the magnitude). For an exponential distribution, it assumes the value 1. As a consequence, evaluating c_V as a function of a threshold magnitude m_{th} , if c_V overcomes a given threshold (here fixed at 0.90 [45]), then m_{th} equals the completeness magnitude. Figure 2(a) shows the magnitude distribution for comparison. Restricting at $m \geq m_{th}$, the completeness magnitude does not change with time substantially, as shown in Fig. 2(b).

III. A SWARM-INFORMED SPACE-TIME ETAS (SIETAS) MODEL

To account for the strongly clustered, finite memory, and spatially localized character of the Campi Flegrei seismic swarms, we extend the temporal swarm-informed ETAS formulation by introducing a spatial kernel inspired by the empirical swarm-distance distribution reported by Ref. [40]. The resulting model is a marked space-time Hawkes process of ETAS type, in which both the temporal and spatial triggering components are replaced by swarm-informed kernels. Let $(t_i, \mathbf{x}_i, m_i)$, $i = 1, \dots, N$, denote the occurrence time, hypocentral projection (or epicentral location) $\mathbf{x}_i \in W \subset \mathbb{R}^2$, and magnitude of the observed earthquakes in the space-time window $[T_1, T_2] \times W$. The conditional intensity is defined as

$$\lambda(t, \mathbf{x} | \mathcal{H}_t) = \mu(\mathbf{x}) + \sum_{t_i < t} K e^{a(m_i - M_0)} g(t - t_i) f(\|\mathbf{x} - \mathbf{x}_i\|), \quad (2)$$

where $\mathcal{H}_t$ is the history up to time t , $\mu(\mathbf{x})$ is the background rate, M_0 is the reference magnitude, K is the productivity constant, and α controls the magnitude dependence of triggering. The novelty of Eq. (2) lies in the choice of the normalized temporal kernel g and spatial kernel f , both designed to represent swarmlike triggering rather than standard mainshock-aftershock behavior.

Following the temporal formulation introduced for volcanic swarm activity [39,41], we define

$$g(u) = \frac{e^{-u/\tau}(\eta + u^{-p})}{Z_t(\tau, p, \eta)}, \quad u > 0, \quad (3)$$

where $\tau > 0$ is a characteristic memory timescale, $p < 1$ controls the short-time singularity, and $\eta \geq 0$ adds a smooth component that allows an initial burst followed by a more persistent activation. The temporal kernel is normalized to unity. The explicit expression of the normalization constant and the cumulative triggering function are reported in Appendix A.

The empirical analysis of swarm geometries suggests a two-regime spatial organization, characterized by an approximately flat core and an exponentially localized tail [40]. In order to embed this behavior into a proper two-dimensional point-process framework, we define the isotropic spatial kernel as

$$f(r) = \frac{C_s}{1 + \exp[\beta_s(r - r_c)]}, \quad r \geq 0, \quad (4)$$

where $r = \|\mathbf{x} - \mathbf{x}_i\|$, $\beta_s > 0$ controls the spatial localization, and $r_c > 0$ is the characteristic core radius. The normalization constant is obtained by imposing

$$\int_{\mathbb{R}^2} f(\|\mathbf{x}\|) d\mathbf{x} = 2\pi \int_0^\infty r f(r) dr = 1, \quad (5)$$

and its analytical derivation is reported in Appendix B. The normalized spatial kernel reads

$$f(r) = \frac{\beta_s^2}{2\pi[-\text{Li}_2(-e^{\beta_s r_c})]} \frac{1}{1 + \exp[\beta_s(r - r_c)]}. \quad (6)$$

Since both kernel are normalized, the productivity term $K e^{a(m_i - M_0)}$ retains its direct interpretation as the expected number of first-generation offspring triggered by event i in an unbounded domain. Assuming a truncated Gutenberg-Richter distribution [46], the corresponding branching ratio is

$$n = K \frac{\beta_m}{\beta_m - \alpha} \frac{1 - e^{-(\beta_m - \alpha)(M_{\max} - M_0)}}{1 - e^{-\beta_m(M_{\max} - M_0)}}, \quad (7)$$

while in the limit $M_{\max} \rightarrow \infty$ and $\beta_m > \alpha$ this reduces to

$$n = K \frac{\beta_m}{\beta_m - \alpha}. \quad (8)$$

The condition $n < 1$ corresponds to the subcritical regime. The derivation is given in Appendix C.

A. Statistical and physical motivation of the SIETAS kernels

The SIETAS kernels are not introduced solely as flexible alternatives to the conventional ETAS functions. Their forms are motivated by recurring statistical features observed in volcanic seismic swarms. For the temporal component, stacked swarm occurrence rates from several volcanic systems exhibit an initial power-law decay followed by an exponential depletion at longer times [39]. The factor u^{-p} in Eq. (3) preserves the strong short-time clustering that is also characteristic of ordinary aftershock sequences. In contrast to the modified Omori kernel, however, the factor $\exp(-u/\tau)$ introduces a finite-memory timescale. Thus,

$$g_S(u) \sim u^{-p}, \quad u \ll \tau, \quad (9)$$

whereas

$$g_S(u) \sim \exp(-u/\tau), \quad u \gg \tau. \quad (10)$$

This distinction is relevant for volcanic swarms, which are commonly associated with transient forcing by magma intrusion, hydrothermal-fluid migration, or aseismic deformation rather than with the instantaneous stress redistribution produced by a single dominant mainshock. The weak or negligible short-time offset is consistent with a rapid response of preexisting fractures to fluid-induced stress changes and fault lubrication. The long-time exponential taper may instead reflect the finite duration of the external forcing and the subsequent relaxation of the induced stress field. The smooth term η allows the kernel to represent a secondary, less singular component of swarm activation, such as the rate enhancement or bump observed in some Campi Flegrei swarm sequences. The spatial kernel is similarly based on empirical swarm geometry. A comparative analysis of volcanic systems worldwide identified an approximately uniform distribution at short radial distances followed by a rapidly decaying outer region [40]. This behavior differs from the scale-free spatial power-law tail conventionally adopted in ETAS. In the SIETAS formulation, r_c defines the characteristic extent of the inner triggering region, while β_s controls the localization outside this region. In the generalized logistic formulation, ν provides additional flexibility in the transition between the inner region and the tail.

Physically, r_c may be interpreted as the scale over which stress or pore-pressure perturbations remain efficiently correlated through the local fracture and hydrothermal network. A larger value can be associated with a more extensive or better-connected permeable structure, whereas a smaller value is compatible with a confined conduit, fault patch, or localized deformation source. The rapid outer decay represents the finite spatial extent of the structures activated during an individual swarm. The consequences of these modifications are illustrated directly in Fig. 3. In panel (a), the SIETAS temporal kernel (red curve) and the standard ETAS modified Omori kernel (green curve) share a decreasing short-time behavior, whereas the SIETAS kernel captures the overall depletion of the reconstructed triggering density more closely, although its exponential cutoff produces a sharper long-time decay than the relatively gradual empirical tail. In panel (b), the SIETAS

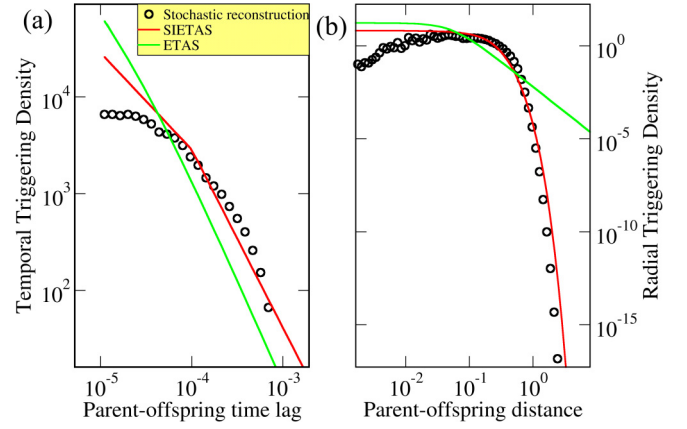


FIG. 3. Comparison between empirically reconstructed triggering distributions and the fitted SIETAS and standard ETAS kernels. The empirical distributions (open circles) were reconstructed using the stochastic declustering procedure of Zhuang *et al.* [50], in which each possible parent-offspring pair is weighted by its estimated triggering probability. (a) Temporal triggering distribution as a function of the parent-offspring time lag. (b) Radial spatial triggering distribution as a function of the parent-offspring distance. The SIETAS kernels (red curves) reproduce the reconstructed triggering distributions more closely than the corresponding standard ETAS kernels (green curves), with the improvement being particularly evident for the temporal triggering process and for the localized spatial decay.

spatial kernel reproduces the localized radial decay of the reconstructed distribution, whereas the standard ETAS power-law kernel retains a much broader long-distance tail.

B. Log-likelihood

The conditional log-likelihood for the observed catalog is

$$\ell(\theta) = \sum_{j=1}^N \ln \lambda(t_j, \mathbf{x}_j | \mathcal{H}_{t_j}) - \int_{T_1}^{T_2} \int_W \lambda(t, \mathbf{x} | \mathcal{H}_t) d\mathbf{x} dt, \quad (11)$$

where

$$\theta = (K, a, \tau, p, \eta, \beta_s, r_c, \text{parameters of } \mu). \quad (12)$$

Using the SIETAS kernels, the conditional log-likelihood can be written as

$$\begin{aligned} \ell(\theta) = & \sum_{j=1}^N \ln \left[\mu(\mathbf{x}_j) + \sum_{i: t_i < t_j} K e^{\alpha(m_i - M_0)} g(t_j - t_i) f(\|\mathbf{x}_j - \mathbf{x}_i\|) \right] - (T_2 - T_1) \int_W \mu(\mathbf{x}) d\mathbf{x} \\ & - \sum_{i=1}^N K e^{\alpha(m_i - M_0)} Q(\mathbf{x}_i; W) G_t(T_2 - t_i). \end{aligned} \quad (13)$$

In the idealized case of an unbounded spatial domain, or when boundary effects are negligible, $Q(\mathbf{x}_i; W) \approx 1$, making easier the $\ell(\theta)$ calculation. The analytical derivation is reported in Appendix D.

The parameters were estimated by minimizing the negative conditional log-likelihood using a deterministic,

derivative-free coordinate-search algorithm implemented in Fortran. The optimization was carried out in a transformed parameter space. Parameters constrained to be positive were represented by exponential transformations, whereas parameters restricted to finite intervals were represented through bounded logistic transformations. For the SIETAS model, the

transformed parameter vector is

$$\mathbf{z} = (z_\mu, z_n, z_\alpha, z_\tau, z_p, z_\eta, z_{\beta_s}, z_{r_c}, z_v).$$

The units of τ and r_c are days and kilometers, respectively, while β_s is expressed in inverse kilometers. The branching ratio is optimized directly rather than the productivity coefficient K . At each likelihood evaluation, K is obtained from

$$K = \frac{n}{\frac{1}{N} \sum_{i=1}^N \exp[\alpha(m_i - M_0)]}, \quad (14)$$

which ensures that the average first-generation productivity over the observed magnitude distribution equals the fitted branching ratio. The upper bound $n < 0.99$ keeps the fitted process within the subcritical regime. Starting from an admissible initial vector, each transformed parameter is perturbed sequentially in the positive direction and, if this does not improve the objective function, in the negative direction. A trial point is retained whenever it produces a lower negative log-likelihood. If a complete cycle over all parameters produces no improvement, all step sizes are multiplied by 0.7. The optimization is terminated after at most 100 complete cycles or when the largest step size becomes smaller than 10^{-3} . The spatial normalization of the logistic kernel is evaluated by numerical quadrature. The spatial edge-correction factors

$$Q(\mathbf{x}_i; W) = \int_W f(\|\mathbf{x} - \mathbf{x}_i\|) d\mathbf{x} \quad (15)$$

are evaluated numerically for every earthquake using a regular 200×200 midpoint grid covering the observation window. The cumulative temporal kernel entering the compensator is also evaluated numerically. No analytical gradients, Markov chain Monte Carlo (MCMC) sampling, or Bayesian integration were used.

C. Reference standard ETAS model

For comparison, we also consider the standard space-time ETAS model. In this formulation, the conditional intensity is

$$\lambda_{\text{ETAS}}(t, \mathbf{x} | \mathcal{H}_t) = \mu(\mathbf{x}) + \sum_{t_i < t} K e^{\alpha(m_i - M_0)} g_E(t - t_i) f_E(\|\mathbf{x} - \mathbf{x}_i\|), \quad (16)$$

where the temporal triggering kernel follows the modified Omori law,

$$g_E(u) = \frac{(p-1)c^{p-1}}{(u+c)^p}, \quad u > 0, \quad p > 1, \quad c > 0, \quad (17)$$

and the spatial triggering kernel is given by the normalized isotropic power-law form

$$f_E(r) = \frac{q-1}{\pi d} \left(1 + \frac{r^2}{d}\right)^{-q}, \quad r \geq 0, \quad q > 1, \quad d > 0. \quad (18)$$

These kernels satisfy

$$\int_0^\infty g_E(u) du = 1, \quad \int_{\mathbb{R}^2} f_E(\|\mathbf{x}\|) d\mathbf{x} = 1. \quad (19)$$

Therefore, as for the SIETAS model, the factor $K e^{\alpha(m_i - M_0)}$ represents the expected number of direct offspring triggered by event i in an unbounded domain.

In the Campi Flegrei application, the same spatial observation window and the same uniform background assumption are used for both ETAS and SIETAS, so that the comparison is based only on the different temporal and spatial triggering kernels.

D. Hybrid models and ablation analysis

To disentangle the respective contributions of the temporal and spatial modifications introduced in SIETAS, we also consider two intermediate hybrid models. The first hybrid model combines the SIETAS temporal kernel [Eq. (3)] with the standard ETAS spatial kernel [Eq. (18)]. Its conditional intensity is

$$\begin{aligned} \lambda_{\text{S, E}_s}(t, \mathbf{x} | \mathcal{H}_t) \\ = \mu(\mathbf{x}) + \sum_{t_i < t} K e^{\alpha(m_i - M_0)} g_S(t - t_i) f_E(\|\mathbf{x} - \mathbf{x}_i\| | m_i). \end{aligned} \quad (20)$$

The second hybrid model combines the standard ETAS temporal kernel [Eq. (17)] with the generalized SIETAS spatial kernel [Eq. (4)]:

$$\begin{aligned} \lambda_{\text{E}_t, \text{S}_s}(t, \mathbf{x} | \mathcal{H}_t) \\ = \mu(\mathbf{x}) + \sum_{t_i < t} K e^{\alpha(m_i - M_0)} g_E(t - t_i) f_S(\|\mathbf{x} - \mathbf{x}_i\|). \end{aligned} \quad (21)$$

All four formulations are fitted to exactly the same catalog, magnitude threshold, temporal interval, spatial observation window, and coordinate dequantization. Consequently, their likelihood and information-criterion values can be directly compared. This ablation analysis allows us to quantify separately the contribution of the swarm-informed temporal kernel and that of the generalized spatial kernel.

E. Application of SIETAS model to the Campi Flegrei seismicity

Here, we compare the SIETAS model with the standard ETAS [18–24, 47–49] model that assumes the Omori law [1] as the time kernel and a power-law spatial kernel [3]. Also, we compare the two intermediate hybrid models to the Campi Flegrei catalog as ablation analysis.

For all models, the background intensity was assumed spatially uniform,

$$\mu(\mathbf{x}) = \frac{\mu_0}{|W|}, \quad (22)$$

where $|W|$ denotes the area of the spatial observation window. Spatial edge-correction factors were evaluated numerically for each event and for the corresponding fitted spatial kernel. Table I reports the maximum-likelihood parameter estimates, branching ratios, log-likelihoods, and Akaike information criterion values for the four formulations. The two hybrid models isolate the improvements associated with the temporal and spatial components, whereas the complete SIETAS formulation evaluates their combined contribution. In particular, replacing the standard modified Omori kernel with the exponentially truncated SIETAS temporal kernel changes the log-likelihood from 101 233 to 103 124, while introducing the generalized logistic spatial kernel changes it from 101 233

TABLE I. Maximum-likelihood parameter estimates and model-comparison statistics for the standard ETAS model, the two hybrid formulations, and the complete SIETAS model. The notation S_tE_s denotes the SIETAS temporal kernel combined with the standard ETAS spatial kernel, while E_tS_s denotes the standard ETAS temporal kernel combined with the generalized SIETAS spatial kernel. Parameters not included in a given formulation are indicated by “—”.

Model	K	μ_0	α	p	c	τ	q	d	γ	β_s	r_c	η	$\ln L$	AIC
ETAS	0.80	0.70	0.32	1.15	1.75×10^{-5}	—	1.35	2.83×10^{-3}	0.027	—	—	—	101 233	−186 466
S_tE_s	0.84	2.47	0.29	0.82	—	0.57	1.42	0.005	0.012	—	—	6.73×10^{-4}	103 124	−206 230
E_tS_s	0.81	1.11	0.35	1.10	1.60×10^{-5}	—	—	—	—	13.59	0.10	—	103 677	−207 338
SIETAS	0.84	3.69	0.29	0.84	—	0.89	—	—	—	13.15	0.10	2.7×10^{-3}	107 007	−213 996

to 103 677. The complete SIETAS formulation achieves a log-likelihood of 107 007 and an AIC of −213 996. These results indicate that full SIETAS model is the best model for the description of the considered data. Also, the ablation analysis therefore provides a direct quantitative assessment of the relative contributions of the temporal and spatial triggering kernels.

F. Comparison with empirically reconstructed triggering distributions

To assess whether the proposed triggering kernels accurately describe the observed triggering process, we reconstructed the empirical temporal and spatial offspring distributions using the stochastic declustering approach of Zhuang *et al.* [50]. For each ordered earthquake pair $i < j$, the probability that earthquake i directly triggered earthquake j is

$$P_{ij} = \frac{Ke^{\alpha(m_i - M_0)} g(t_j - t_i) f(r_{ij})}{\lambda(t_j, \mathbf{x}_j | \mathcal{H}_{t_j})}, \quad (23)$$

while the probability that event j is a background event is

$$P_{0j} = \frac{\mu(\mathbf{x}_j)}{\lambda(t_j, \mathbf{x}_j | \mathcal{H}_{t_j})}. \quad (24)$$

These probabilities were used as statistical weights to reconstruct the empirical distributions of parent-offspring time lags and spatial separations.

Figure 3 compares these reconstructed distributions with the fitted SIETAS and standard ETAS kernels. The reconstructed temporal distribution exhibits a clear finite temporal scale and a marked depletion of triggering probability at large time lags. The exponentially truncated SIETAS kernel captures this departure from the scale-free decay predicted by standard ETAS and provides a closer overall description of the reconstructed distribution. However, its single exponential cutoff produces a sharper long-time decay than the relatively gradual decrease observed in the reconstructed distribution. This discrepancy indicates that a kernel characterized by a single cutoff timescale remains somewhat restrictive, even though it improves the overall likelihood. A more flexible temporal formulation, such as a stretched-exponential cutoff or a mixture of characteristic timescales, could provide a better description of the long-time behavior and will be investigated in future work. The reconstructed spatial distribution similarly reveals a localized triggering region followed by a relatively sharp decrease in triggering probability, which is reproduced more closely by the logistic SIETAS kernel

than by the standard ETAS power-law kernel. Overall, these comparisons show that the likelihood improvement reflects a more realistic description of the triggering process, while also highlighting the remaining limitation of the SIETAS temporal kernel.

IV. MULTISCALING IN EARTHQUAKE DIFFUSION

Multiscaling properties of earthquake diffusion can be investigated by evaluating the quantity [26,51]

$$\langle r_{ij}^q \rangle(\theta) = \frac{\sum_i \sum_j r_{ij}^q \delta(\theta - \theta_{ij})}{\sum_i \sum_j \delta(\theta - \theta_{ij})}, \quad (25)$$

where r_{ij} is the spatial distance between the i th and the j th earthquakes, θ_{ij} is their temporal separation, and $\delta(\cdot)$ denotes the Dirac delta.

$\langle r_{ij}^q \rangle$ is shown in Fig. 4 as a function of θ for the different q values. As expected, $\langle r_{ij}^q \rangle$ exhibits a regime scaling as a power law followed by a flat regime because of the saturation.

The values of the fitted exponents are shown in Fig. 5 as a function of q .

Adopting the multifractal formalism the exponent ϵ has to vary linearly with q and we have [52]

$$\epsilon = \min_h [hq + D(h) - 3], \quad (26)$$

where $D(h)$ denotes the singularity spectrum associated with the scaling exponent h . In Fig. 5, we observe three approximately linear regimes, suggesting the presence of different effective scaling behaviors in earthquake diffusion. The interpretation of the fitted intercepts in terms of fractal dimensions should, however, be treated with caution. In particular, values formally exceeding the embedding dimension do not necessarily imply a literal fractal dimension larger than 3, but rather indicate a tendency toward a space-filling regime or saturation of the multifractal spectrum. Within this interpretation, the blue and green branches are compatible with an effectively space-filling organization of seismicity, characterized by different singularity strengths. In one case, the singularity strength is very small, $h = 9.4 \times 10^{-2}$, whereas in the second case it is larger, $h = 0.38$, indicating different scaling behaviors of the underlying measure. The third branch (red fit in Fig. 5) corresponds instead to an effective fractal dimension $D(h) = 2.82$ with singularity strength $h = 5.4 \times 10^{-2}$. The physical interpretation of these different scaling regimes will be discussed in Sec. VI.

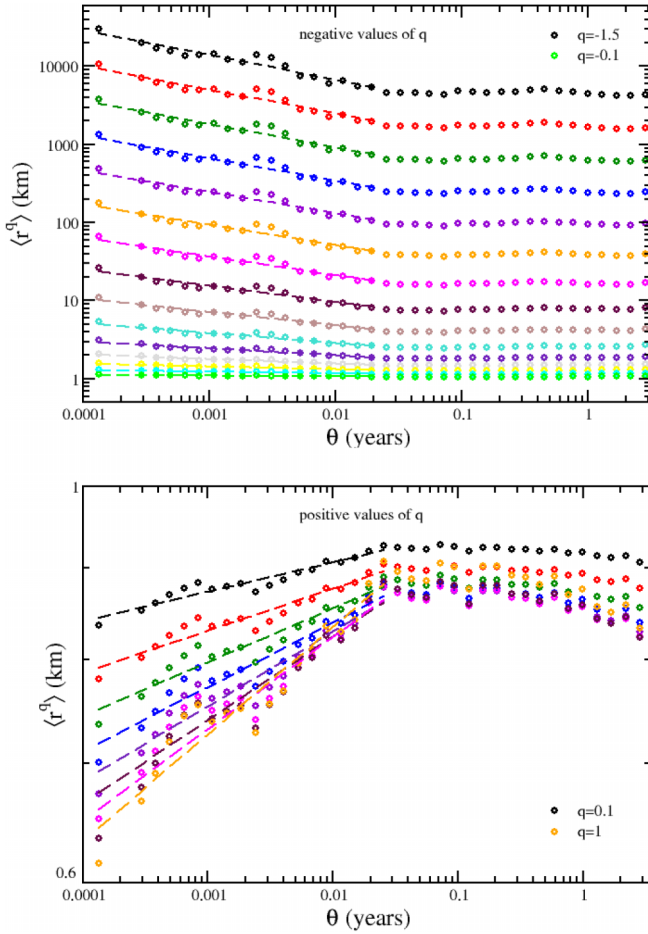


FIG. 4. $\langle r_{ij}^q \rangle$ vs θ for the different values of q . Dashed lines represent the power-law fits within the scaling regime.

V. TRIGGERING KERNELS AND EARTHQUAKES DIFFUSION

In order to provide a physical interpretation of our observations, let us to consider a separable triggering kernel of the

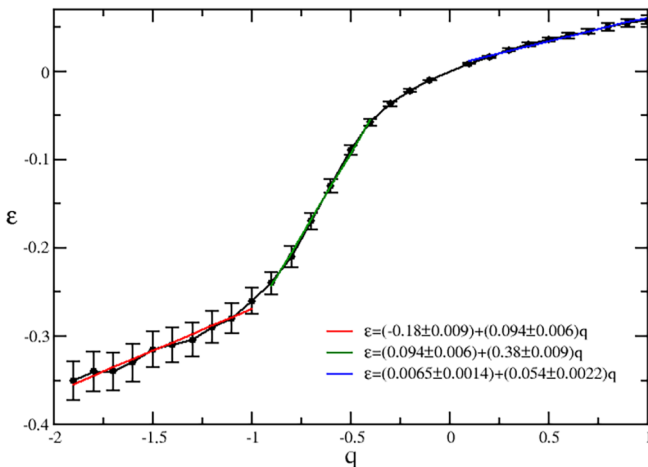


FIG. 5. The fitted exponents ϵ vs q . The dashed lines are the three linear fits representing the three different fractal regimes described (see text) by the relationships in the figure labels.

form

$$\phi(\mathbf{r}, t) = K \psi(t) \lambda(r), \quad (27)$$

where $\psi(t)$ is the temporal triggering kernel and $\lambda(r)$ is the spatial distribution of triggered events. Namely, $\psi(t)$ and $\lambda(r)$ are the equivalent of relationships (3) and (4) for time and space kernels, respectively.

We would like to emphasize that both kernels are characterized by two parameters, τ and r_c , representing the values of t and r beyond which the occurrence rates decay exponentially. The exponential truncation ensures the existence of finite moments for both the temporal and spatial occurrence rates.

On the contrary, for $t < \tau$ the temporal kernel decays as t^{-p} , whereas for $r < r_c$ the spatial kernel assumes a constant value.

In what follows, we neglect the normalization constants in order to simplify the notation without any loss of generality.

A. Propagation of triggered seismicity

Let $N(\mathbf{r}, t)$ denote the average density of events generated by a single initial earthquake. In the branching formulation, N satisfies

$$N(\mathbf{r}, t) = \phi(\mathbf{r}, t) + \int_0^t d\tau \int d\mathbf{r}' \phi(\mathbf{r} - \mathbf{r}', t - \tau) N(\mathbf{r}', \tau). \quad (28)$$

This equation accounts for direct aftershocks as well as higher generations.

Taking the Fourier transform in space and the Laplace transform in time yields

$$\tilde{N}(\mathbf{k}, s) = \frac{\tilde{\phi}(\mathbf{k}, s)}{1 - \tilde{\phi}(\mathbf{k}, s)}. \quad (29)$$

If the branching ratio is n , we can write

$$\tilde{\phi}(\mathbf{k}, s) = n \tilde{\psi}(s) \tilde{\lambda}(\mathbf{k}). \quad (30)$$

Because the temporal kernel has a finite mean waiting time due to the exponential cutoff, its strict small- s expansion is

$$\tilde{\psi}(s) \simeq 1 - \langle t \rangle s. \quad (31)$$

Similarly, since the spatial kernel has finite variance,

$$\tilde{\lambda}(k) \simeq 1 - Dk^2. \quad (32)$$

Substituting these expansions into the denominator of the propagator gives

$$1 - n \tilde{\psi}(s) \tilde{\lambda}(k) \simeq (1 - n) + n \langle t \rangle s + n Dk^2. \quad (33)$$

For a strictly subcritical process, $n < 1$, the term $1 - n$ introduces a finite branching cutoff. Therefore, the diffusive approximation is valid only over scales for which

$$n \langle t \rangle s + n Dk^2 \gg 1 - n. \quad (34)$$

In this regime, the denominator reduces effectively to

$$n \langle t \rangle s + n Dk^2, \quad (35)$$

which corresponds to normal diffusion,

$$\langle r^2(t) \rangle \sim t. \quad (36)$$

At longer times, however, the finite subcriticality limits the cascade and the spreading saturates.

Over intermediate times, before the exponential cutoff becomes dominant, the exponential taper is not yet dominant and the singular part of the temporal kernel behaves as

$$\psi(t) \sim t^{-p}, \quad 0 < p < 1. \quad (37)$$

It is important to stress that this is not a true long-time tail. Since the full SIETAS kernel is exponentially truncated, all temporal moments are finite and the strict small- s expansion remains analytic. Therefore, the asymptotic long-time behavior is controlled by the normal-diffusive regime described above, with saturation due to subcriticality.

Nevertheless, over an intermediate range of scales, before the exponential cutoff is resolved, the Laplace transform is controlled by the singular part of the kernel. In this intermediate regime, one may write, in an effective sense,

$$1 - \tilde{\psi}(s) \simeq As^{1-p}, \quad (38)$$

where $A > 0$ depends on the normalization and on the cutoff scale. Hence,

$$\tilde{\psi}(s) \simeq 1 - As^{1-p}. \quad (39)$$

Combining this expression with

$$\tilde{\lambda}(k) \simeq 1 - Dk^2, \quad (40)$$

we obtain

$$\begin{aligned} 1 - n\tilde{\psi}(s)\tilde{\lambda}(k) &\simeq 1 - n(1 - As^{1-p})(1 - Dk^2) \\ &\simeq (1 - n) + nAs^{1-p} + nDk^2. \end{aligned} \quad (41)$$

Close to criticality, or over timescales for which the branching cascade dominates over the finite subcritical cutoff, this denominator has the effective fractional-diffusion form

$$As^{1-p} + Dk^2. \quad (42)$$

Consequently, the mean-square displacement exhibits a transient anomalous scaling,

$$\langle r^2(t) \rangle \sim t^{1-p}, \quad t \lesssim \tau. \quad (43)$$

For $p = 0.81$, this corresponds to an effective subdiffusive exponent $1 - p \simeq 0.19$.

Thus, the SIETAS temporal kernel does not predict genuine asymptotic subdiffusion. Rather, it predicts an intermediate-time anomalous regime controlled by the short-time singularity of the triggering rate, followed by a crossover to normal diffusion once the exponential cutoff becomes dominant, and finally by saturation due to the finite subcriticality of the branching process.

VI. PHYSICAL INTERPRETATION

This behavior suggests a two-regime dynamics for volcanic swarms occurrence:

(1) At intermediate times ($\theta < \tau$), triggering exhibits memory effects leading to subdiffusive spreading of seismicity. This could represent the earthquake diffusion within swarms in a cascading process, as observed for tectonic events. Namely, each event can trigger its own aftershocks of the next generation. This may provide a possible interpretation of the fractal behavior observed at intermediate

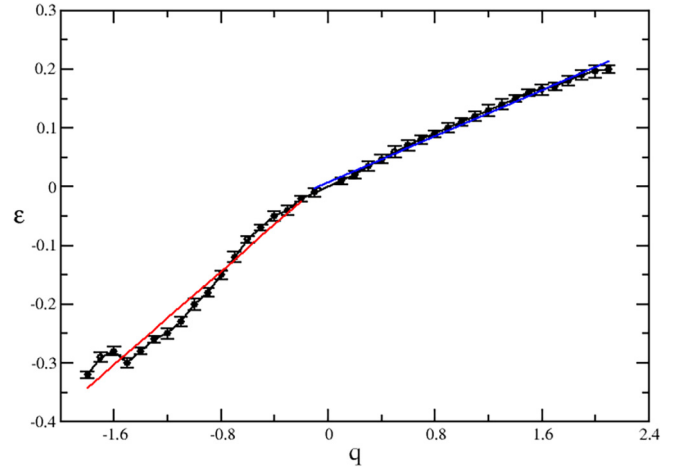


FIG. 6. Same as Fig. 5, but for the Etna catalog. In this case, only two scaling regimes are observed, consistently with the absence of the smooth component associated with η .

θ in Fig. 5 (green fit), suggesting a similarity with standard aftershock sequences. An alternative interpretation is that the migration may be driven by a transient slow slip acting as a primary driving engine. As an aseismic slip front migrates slowly along a fault plane, often modulated by high-pressure fluid diffusion, it continuously alters the local stress field, systematically triggering clusters of small-magnitude events as it encounters small, brittle asperities [38,53–55].

(2) At longer times ($\theta > \tau$), the exponential cutoff removes the long-time memory and the system crosses over to an effectively normal-diffusive regime, before eventual saturation due to the finite subcritical branching ratio.

The third fractal regime at small θ in Fig. 5 (red fit) may reflect the presence of an additional characteristic process not fully captured by the dominant power-law and exponential-cutoff scales. In principle, both r_c and η introduce additional crossover scales into the SIETAS triggering kernel.

However, r_c primarily represents the characteristic spatial extent of volcanic swarms [40], whereas the intermediate- q regime (green fit in Fig. 5) already appears compatible with swarmlike cascading diffusion. This suggests that the third scaling regime may instead be associated with the smooth component controlled by η .

Although the fitted value of η is small compared with the dominant singular contribution over most of the temporal range preceding the exponential cutoff, multifractal observables may still be sensitive to weak secondary components of the triggering kernel. Therefore, the possible association between η and the third scaling regime cannot be excluded, although it should be regarded as a qualitative interpretation rather than a direct demonstration.

As additional support for this interpretation, we performed the same analysis for the Etna catalog, for which the authors of Ref. [39] reported $\eta = 0$. In that case, only two scaling regimes are observed, approximately corresponding to the regimes $\theta < \tau$ and $\theta > \tau$. In fact, in Fig. 6 it is possible to observe a $\epsilon(q)$ very similar to the one observed

for Campi Flegrei for $q > -1$. However, in the case of Etna, the third regime, with a smaller slope, is not observed. This represents an indication that the third scaling regime observed for the Campi Flegrei catalog can be associated with η .

VII. CONCLUSION

When ETAS-type triggering involves truncated temporal kernels and spatial distributions with finite characteristic scales, the resulting space and time propagation of seismicity is expected to display transient anomalous diffusion followed by an effectively normal-diffusive regime and eventual saturation due to finite subcriticality. Such behavior may be particularly relevant for volcanic earthquake sequences where physical processes introduce natural temporal and spatial cutoffs.

In this work, we have extended this framework by explicitly introducing the modified kernels discussed within the ETAS formalism, leading to what we define as the SIETAS model. This extension allows us to account more realistically for the presence of intrinsic temporal and spatial limits in the triggering process, which are especially relevant in volcanic environments where the dynamics are controlled by complex physical processes.

Moreover, focusing on the Campi Flegrei area, by using an advanced AI-based catalog, we have shown that the seismicity exhibits a clear multiscaling behavior characterized by three distinct scaling regimes. The first two regimes can be qualitatively interpreted within the truncated ETAS/SIETAS framework, reflecting the interplay between short-term triggering and intermediate-scale diffusion processes. The third regime may instead be associated with the weak smooth component of the temporal kernel, although this interpretation remains qualitative and should be tested with synthetic catalogs generated from the fitted SIETAS modelcheck. This result highlights that, even during highly clustered phases, a non-negligible background component persists and contributes to the overall scaling properties of the system.

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DATA AVAILABILITY

The SIETAS implementation and the analysis material used in this study are publicly available in Ref. [56]. The repository contains the source code required to fit the SIETAS model.

APPENDIX A: NORMALIZATION OF THE TEMPORAL KERNEL

The normalization constant for the temporal kernel is

$$Z_t(\tau, p, \eta) = \int_0^\infty e^{-u/\tau} (\eta + u^{-p}) du = \eta\tau + \tau^{1-p}\Gamma(1-p), \quad (\text{A1})$$

so that

$$\int_0^\infty g(u) du = 1. \quad (\text{A2})$$

The exponential taper ensures finite temporal memory, while the power-law factor preserves the sharp short-time clustering typical of swarms. The cumulative temporal triggering function entering the compensator is

$$G_t(u) = \int_0^u g(v) dv = \frac{\eta\tau(1 - e^{-u/\tau}) + \tau^{1-p}\gamma(1-p, u/\tau)}{\eta\tau + \tau^{1-p}\Gamma(1-p)}, \quad (\text{A3})$$

where $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function.

APPENDIX B: NORMALIZATION OF THE SPATIAL KERNEL

The constant C_s is chosen so that f is a normalized density on $\mathbb{R}^2$:

$$\int_{\mathbb{R}^2} f(\|\mathbf{x}\|) d\mathbf{x} = 2\pi \int_0^\infty r f(r) dr = 1. \quad (\text{B1})$$

Substituting Eq. (4) into Eq. (B1) gives

$$1 = 2\pi C_s \int_0^\infty \frac{r}{1 + \exp[\beta_s(r - r_c)]} dr. \quad (\text{B2})$$

Introducing the change of variable $u = \beta_s r$, we obtain

$$\int_0^\infty \frac{r}{1 + \exp[\beta_s(r - r_c)]} dr = \frac{1}{\beta_s^2} \int_0^\infty \frac{u}{1 + \exp(u - \beta_s r_c)} du. \quad (\text{B3})$$

Using the standard Fermi-Dirac integral identity

$$\int_0^\infty \frac{u}{1 + \exp(u - \xi)} du = -\text{Li}_2(-e^\xi), \quad (\text{B4})$$

where Li_2 is the dilogarithm, Eq. (B2) becomes

$$1 = 2\pi C_s \frac{-\text{Li}_2(-e^{\beta_s r_c})}{\beta_s^2}. \quad (\text{B5})$$

Therefore,

$$C_s = \frac{\beta_s^2}{2\pi [-\text{Li}_2(-e^{\beta_s r_c})]}. \quad (\text{B6})$$

APPENDIX C: BRANCHING RATIO

If magnitudes follow a truncated Gutenberg-Richter distribution [46] on $[M_0, M_{\max}]$ with density

$$p_M(m) = \frac{\beta_m e^{-\beta_m(m-M_0)}}{1 - e^{-\beta_m(M_{\max}-M_0)}}, \quad M_0 \leq m \leq M_{\max}, \quad (\text{C1})$$

then the mean branching ratio is

$$n = K \mathbb{E}[e^{\alpha(M-M_0)}] = K \frac{\beta_m}{1 - e^{-\beta_m(M_{\max}-M_0)}} \int_0^{M_{\max}-M_0} e^{-(\beta_m-\alpha)z} dz. \quad (C2)$$

Hence, for $\beta_m \neq \alpha$,

$$n = K \frac{\beta_m}{\beta_m - \alpha} \frac{1 - e^{-(\beta_m-\alpha)(M_{\max}-M_0)}}{1 - e^{-\beta_m(M_{\max}-M_0)}}. \quad (C3)$$

APPENDIX D: LIKELIHOOD DERIVATION

Substituting Eq. (2) into Eq. (11) yields

$$\begin{aligned} \ell(\theta) = & \sum_{j=1}^N \ln \left[\mu(\mathbf{x}_j) + \sum_{i: t_i < t_j} K e^{a(m_i-M_0)} g(t_j - t_i) f(\|\mathbf{x}_j - \mathbf{x}_i\|) \right] - \int_{T_1}^{T_2} \int_W \mu(\mathbf{x}) d\mathbf{x} dt \\ & - \sum_{i=1}^N K e^{a(m_i-M_0)} \int_{t_i}^{T_2} g(t - t_i) \left[\int_W f(\|\mathbf{x} - \mathbf{x}_i\|) d\mathbf{x} \right] dt. \end{aligned} \quad (D1)$$

Defining the spatial edge-correction factor

$$Q(\mathbf{x}_i; W) = \int_W f(\|\mathbf{x} - \mathbf{x}_i\|) d\mathbf{x}, \quad (D2)$$

and using Eq. (A3), the compensator becomes

$$\int_{t_i}^{T_2} g(t - t_i) \left[\int_W f(\|\mathbf{x} - \mathbf{x}_i\|) d\mathbf{x} \right] dt = Q(\mathbf{x}_i; W) G_t(T_2 - t_i). \quad (D3)$$

Therefore,

$$\begin{aligned} \ell(\theta) = & \sum_{j=1}^N \ln \left[\mu(\mathbf{x}_j) + \sum_{i: t_i < t_j} K e^{a(m_i-M_0)} g(t_j - t_i) f(\|\mathbf{x}_j - \mathbf{x}_i\|) \right] - (T_2 - T_1) \int_W \mu(\mathbf{x}) d\mathbf{x} \\ & - \sum_{i=1}^N K e^{a(m_i-M_0)} Q(\mathbf{x}_i; W) G_t(T_2 - t_i). \end{aligned} \quad (D4)$$

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