

Topology and criticality in non-Hermitian multimodal optical resonators through engineered losses

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Non-Hermitian topological matter provides a platform for engineering phenomena that go beyond the capabilities of Hermitian systems, enabling the use of losses to engineer topological phenomena. Non-Hermitian models often rely on artificial platforms made of engineered lattices because controlling losses in natural compounds is challenging. Although typical models for non-Hermitian photonic matter are often single mode, photonic systems are often multimodal, producing mixing between different normal modes in each site. Here, we explore a family of multimodal non-Hermitian lattices, featuring multiple resonant modes. We show that these multimodal models are capable of featuring topological modes and criticality, similar to the artificial single-mode models often considered. We analyze the robustness of these non-Hermitian topological modes to fluctuation of local losses, disorder, and artificial gauge field. We show that these effects can be captured via both a full microscopic model and effective multiorbital models. Specifically, we show that due to their multiorbital nature, the localization properties of non-Hermitian multiorbital models can be controlled by an external gauge field. Our results demonstrate that internal orbital degrees of freedom provide a promising strategy to engineer controllable non-Hermitian topology and criticality.

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I. INTRODUCTION

Topological phases enable robust wave phenomena in both natural materials and engineered systems, originating from the coexistence of symmetry-protected gapless edge states that coexist with gaped bulk spectra [1–14]. In photonics [15–17], non-Hermitian states provide a strategy for engineering topology, where gain, loss, and synthetic gauge fields [18,19] enable phenomena absent in Hermitian settings, such as exceptional points [20] and non-Hermitian skin effect [18,20]. Beyond their interest from a fundamental perspective, non-Hermitian phenomena enable applications including topological lasers [21–27], resilient optical localization [28], precision sensing [29,30], and topologically protected braiding of complex energy bands [31].

Most models for topological non-Hermitian systems in photonics are built upon single-mode resonators. In contrast, complex photonic resonating networks can be designed to have multimodal nature [32,33], supporting several internal spatial or polarization modes with distinct coupling behavior. This multimodal character introduces extended internal degrees of freedom [34], leading to multiple orbital modes in mesoscopic systems [35–38], exciton-polariton microcavity lattices [39,40], laser-written photonic waveguide

arrays supporting higher-order transverse modes [17,41], resonator-based photonic lattices hosting p -like cavity modes [42,43], and acoustic metamaterial lattices employing vortex or angular-momentum modes [44,45]. These higher-order local modes act as synthetic orbitals and enable orbital band formation, topological edge localization, and nonlinear dynamical effects across distinct wave platforms. Specifically, these internal modes act like orbitals and bring about new behavior that single-mode models [34,35,46] cannot achieve, including complex band structures [47], topology specific to multiorbital models [48], and mode-dependent localization transitions [49]. The loss and geometry of the systems can also be used to control which modes are active [50], giving a way to design and probe topological features, offering a potential framework to explore multiorbital non-Hermitian topological effects.

In this article, we demonstrate the emergence of topological modes and criticality in an engineered multimodal non-Hermitian photonic system. Specifically, we show that topologically protected modes associated with quasiperiodic losses emerge in multiorbital scenarios and that mobility edges emerge in a range of loss modulation strength. Furthermore, we address the impact of disorder in the multiorbital non-Hermitian scenario, showing that frequency disorder has a substantially bigger impact than loss disorder. We finally show that, due to the multiorbital nature of the model, an external gauge field enables controlling the localization properties of the states.

Our article is organized as follows. Section II introduces our model and characterizes its energy spectrum. In Sec. III, we present a low-energy effective theory to capture the multiorbital spectra, including its emergent topological modes.

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Section IV investigates the transition from extended to localized states as a function of loss strength, while Sec. V shows the influence of disorder on the spectra, and Sec. VI demonstrates the effect of the gauge field on the localization transition of the multimodal system. Finally, in Sec. VII we summarize our conclusions. Our findings demonstrate the interplay of dissipation, internal mode structure, and topology, offering strategies for realizing tunable photonic non-Hermitian photonic states.

II. TOPOLOGICAL MODES FROM ENGINEERED LOSSES IN A MULTIMODAL SYSTEM

A. Topological modes in a single-orbital non-Hermitian model

For the sake of concreteness, we start by briefly reviewing a minimal model featuring topological modes, solely originating from modulated losses [51–56], and recently realized in photonic resonators [28]. The Hamiltonian of this system takes the form

$$H = \gamma \sum_{n=0}^{N-2} (a_n^\dagger a_{n+1} + \text{H.c.}) + i \sum_{n=0}^{N-1} v_l \sin(2\pi n\alpha + \varphi) a_n^\dagger a_n, \quad (1)$$

where α is the frequency of modulation, φ is the phase of modulation, v_l is the amplitude of modulation, γ is the symmetric coupling between sites, and $a_n^\dagger, a_n$ are the creation and annihilation operators on site n . The term $iv_l \sin(2\pi n\alpha + \varphi)$ controls the strength of the local loss on site n . This system can be realized as the non-Hermitian generalization of the Aubry-Andre-Harper (AAH) model, a paradigmatic quasiperiodic model featuring topological modes [9,11,57]. The one-dimensional (1D) Hermitian AAH model can be mapped onto a two-dimensional quantum Hall system [4,58–60], where the quasiperiodic modulation plays the role of an additional synthetic dimension, giving rise to nontrivial topological band structures characterized by Chern numbers. The model is often extended to study generalized quasiperiodicity by varying the incommensurate modulation frequency α . When $\alpha = p/q$ is rational and the total number of sites N is a multiple of q , the system is periodic. In contrast, quasiperiodicity arises when α is irrational [28,51].

In the following, we take $\alpha = 1/4$ and phase offset $\varphi = 3\pi/4$, leading to a topological phase as illustrated in Figs. 1(a) and 1(b). This phase is protected by an emergent particle-hole symmetry and Chern number [54,61], and hosts robust zero-energy edge modes $\text{Re}(E) = 0$ clearly visible in Fig. 1(b) [54,55,62]. These zero modes persist even under increased loss, demonstrating their topological protection. It is worth noting that the system lacks parity-time symmetry, and thus all eigenvalues acquire nonzero imaginary components due to non-Hermiticity.

B. Topological modes in a multiorbital non-Hermitian model

We now move on to consider a multiorbital generalization of the non-Hermitian AAH model. This situation naturally arises in quasi-one-dimensional systems, where each site in the original one-dimensional chain [Fig. 1(a)] is replaced by a lattice of $W \times W$ resonators. Here, W controls the size of each resonator island, while N denotes the total number of

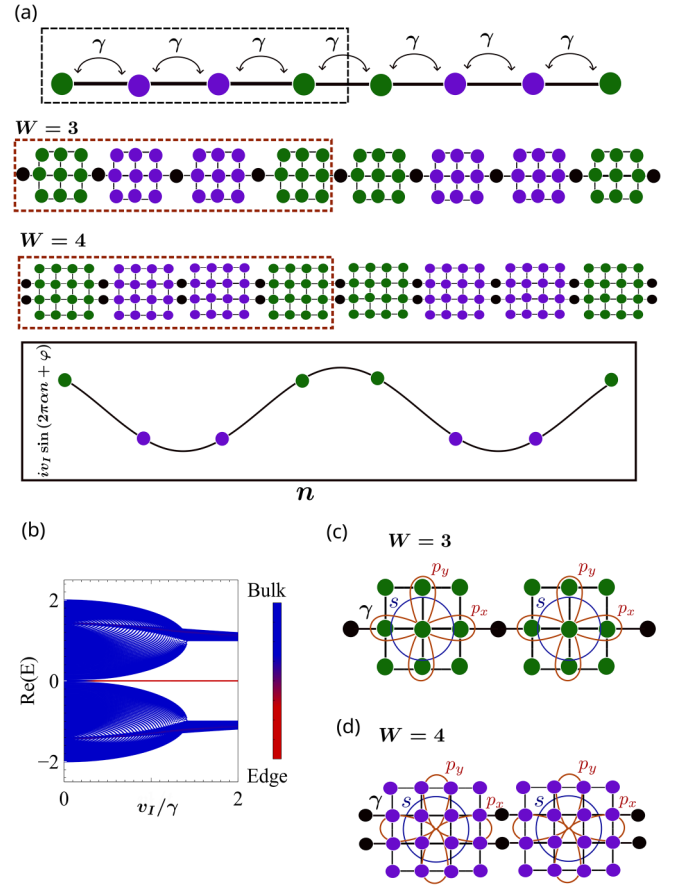


FIG. 1. (a) Schematics of the gain-loss lattice featuring a four-site unit cell (black dashed box), and the multimodal model with square islands (red dashed box), showing $W = 3, 4, 3 \times 3$, and 4×4 islands. The local loss is $iv_n = iv_l \sin(2\pi n\alpha + \varphi)$, with $\alpha = 1/4$ and $\varphi = 3\pi/4$ shown as a sine curve in the box. (b) Real spectrum vs v_l/γ for $N = 200$, which is symmetric about $\text{Re}(E) = 0$ with robust zero-energy edge modes across the full range of v_l . (c), (d) The s , p_x , and p_y orbital overlaps for two island (γ is the intersite coupling). For $W = 3$, adjacent p_y orbitals meet only at nodal points (negligible coupling); for $W = 4$, they overlap with finite coupling.

such islands in the chain. These islands are coupled with each other through one or more interconnecting sites. Figure 1(a) illustrates schematic representations for multimodal systems with $W = 3$ and $W = 4$. The black sites indicate the connecting points between islands and are assumed to have zero local loss, in contrast to the lossy islands. The loss on the islands is given by

$$H_L = \sum_{n,l} iv_l \sin(2\pi n\alpha + \varphi) a_{n,l}^\dagger a_{n,l}, \quad (2)$$

where n labels the island and l labels the site in each island. To compare with the minimal model, we first take $\alpha = 1/4$, $\varphi = 3\pi/4$, and n is the index of the island. The intrainland hopping structure is depicted in Figs. 1(c) and 1(d). These quasi-one-dimensional configurations are referred to as superlattices. The non-Hermitian multimodal model is directly realizable in programmable photonic lattices composed of coupled single-mode resonators with independently tunable on-site losses and coupling strengths, as demonstrated

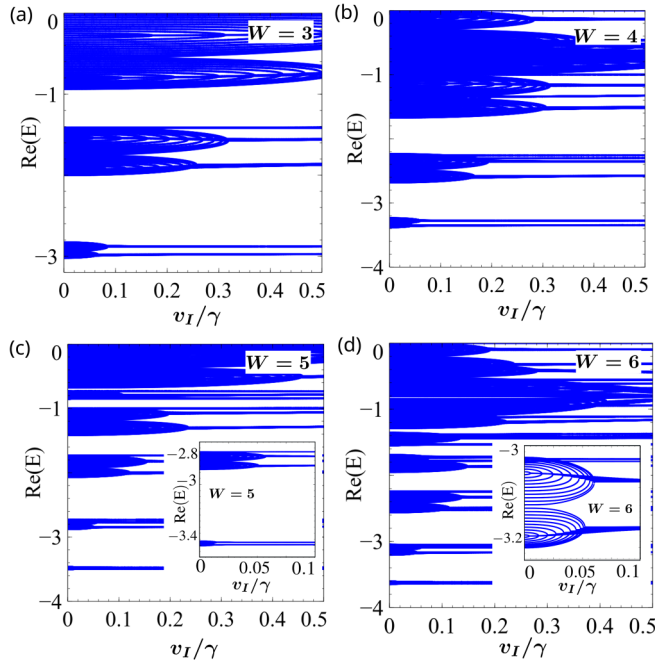


FIG. 2. (a)–(d) Real part of the energy spectrum as a function of modulation strength v_I for different sizes of the island $W = 3, 4, 5$, and 6 . The systems have bulk modes with topological band gaps in between them. Panels (a) and (c) show similar band spectrum for $W = 3, 5$, as seen in the inset of $W = 5$ and Panels (b) and (d) show similar band spectrum for $W = 4, 6$, as seen in the inset of $W = 6$. The system has particle-hole symmetry, leading to an energy spectrum symmetric with respect to $\text{Re}(E) = 0$. We took $N = 80$, $\alpha = 1/4$, and $\varphi = 3\pi/4$.

experimentally [28,63]. This strategy to engineer a multi-orbital system is different from a generic optical platform featuring intrinsically many modes [38]. In contrast, our system starts from coupled individual single-mode resonators, leading to an effective mesoscopic multimodal system.

Coupled-resonator superlattices support normal modes that emerge from the interactions between individual resonant units. As shown in Figs. 1(c) and 1(d), each square island acts as a localized resonator, and their coupling gives rise to extended modes between islands that span the structure, visible in the energy spectrum of Fig. 2. These modes are shaped by the geometry and symmetry of the lattice, providing a natural framework for understanding the spectral properties of the system. Each island supports distinct resonant patterns that resemble orbital-like modes such as s , p_x , and p_y , depending on its size and boundary conditions [64,65]. As illustrated in Figs. 1(c) and 1(d), both islands of odd and even sizes support these modes, but their coupling behaviors differ. In particular, the p_y -like modes in islands of odd sizes disappear because of the node at $y = 0$. In contrast, in the case of islands with W even, the p_y orbitals can be featured by finite hybridization.

Now, we use the lattices shown in Fig. 1; all the islands with odd sizes have superlattice structure similar to that of $W = 3$ and the islands with even sizes look similar to $W = 4$. Focusing on the models shown in Fig. 1, we now study the energy spectrum as in Fig. 2 for periodic boundary conditions, where we show the real part of the bulk modes for

systems with $W = 3, 4, 5, 6$ versus the modulation amplitude v_I . We see that different band gaps appear at different $\text{Re}(E)$, and the energy spectra are symmetric with respect to $\text{Re}(E) = 0$. In addition, the energy spectrum of $W = 3$ is similar to that of $W = 5$, unlike the energy spectrum of $W = 4$, which is analogous at low energies to the spectrum of $W = 6$. The insets shown in Figs. 2(c) and 2(d) highlight the fact that the lowest-energy mode and first excited energy modes for the odd-sized islands ($W = 3, 5$) and even-sized islands ($W = 4, 6$) are alike. Note that all these systems shown in Figs. 2(a)–2(d) have an imaginary component of the energy that is not shown. The losses are implemented microscopically at each site according to an island-dependent non-Hermitian Aubry-André-Harper profile.

Similar to the single-orbital non-Hermitian model shown in Fig. 1(a), the multi-orbital model also exhibits emergent protected edge modes, stemming from a topological protection [28,51,54,61]. This is illustrated in Figs. 3(a) and 3(b). For $W = 3$, the energy spectrum of the s orbitals shows a bulk band gap with a highlighted edge mode in the gap. This edge mode is topological and characterized by a second-order Chern number [54,61]. A similar bulk gap and topological edge mode are observed for the p_x orbitals, again associated with a second-order Chern number [54,61]. In contrast, the p_y orbitals are nondispersive in this case, with no gap in the bulk modes. The same qualitative behavior holds for $W = 4$ for the s and p_x orbitals. However, in this case the p_y orbitals also develop a bulk band gap and support a topological edge mode characterized by a second-order Chern number [54,61]. The topological and trivial band gaps are indicated in Fig. 3. Note that, when transitioning from the topological gap to the trivial gap, the edge modes merge with the bulk modes in $\text{Re}(E)$ while remaining localized at the edges, a behavior characteristic of the imaginary AAH model [56].

III. NON-HERMITIAN MULTIMODAL SYSTEM IN THE LOW-ENERGY REGIME

We now focus on systems with $W = 3$ and $W = 4$ under lower-strength on-site loss conditions. In Fig. 3, we analyze the topological edge modes and observe that the energy spectra are analogous for modes originating from the lowest-energy s -like mode in both cases. However, differences arise in the modes that route from the p_x , p_y -manifold. These differences stem from the fact that, for $W = 3$, the effective hopping in the p_y manifold is almost zero, as those orbitals have a node at $y = 0$, the coordinates where the only coupling between islands takes place. In contrast, for $W = 4$, the coupling between islands happens at $y \neq 0$, where the p_y orbitals take a finite value, which gives rise to a finite effective hopping, and thus a nonzero bandwidth, between the p_y orbitals.

We now address the emergence of topological modes in the p_y manifold. Due to the presence of mirror symmetry, the p_x and p_y manifolds are decoupled. This implies that the effective model in the p_x manifold is analogous to the effective model in the s manifold and therefore we expect the emergence of topological modes. Figs. 3(c) and 3(d) show that topological edge modes emerge for specific values of the phase parameter φ . Thus, by tuning φ , one can effectively pump topological modes through the system. This is an analogous

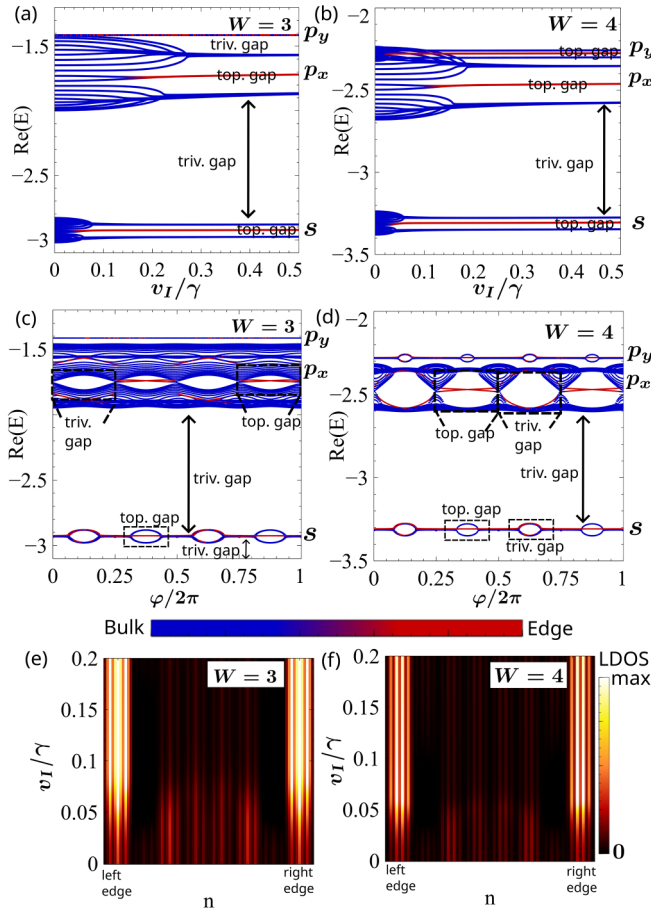


FIG. 3. Real part of the energy spectrum vs modulation strength v_I when $\alpha = 1/4$, $\varphi = 3\pi/4$ as a function of the loss amplitude v_I [panels (a) and (b)] and the phase φ [panels (c) and (d)], with $W = 3$ for panels (a) and (c) and $W = 4$ for panels (b) and (d). We have highlighted the states of s orbitals and p orbitals for the systems under consideration. (a), (c) For $W = 3$, the p -orbital state has a band where many bulk and edge modes coalesce even for small losses compared to $W = 4$ [panels (b) and (d)]. (c), (d) The edge modes are seen in the band gap for a certain value of φ ; we choose $v_I = 0.2\gamma$. The spectral density of the lowest edge mode vs n , where n is the index of the sites in the chain, for the $W = 3$ [panel (e)] system and the $W = 4$ [panel (f)] system. It is observed that with the increase in the amplitude of loss v_I , the edge modes get localized at the edge islands exponentially and the localization length is a function of v_I . We took $N = 8$ for panels (e) and (f).

phenomenology to what is observed in the s manifold, with the key difference that the bandwidths and on-site energies of the p manifold are different for the decoupled p_x and p_y sectors.

We further investigate the evolution of edge modes as a function of modulation strength v_I . We analyze the spectral density defined as $D(\omega, n) = \sum_{\alpha} \delta(\omega - \text{Re}(E_{\alpha})) \langle n | \Psi_{\alpha,R}(n) \rangle \langle \Psi_{\alpha,L}(n) |$, where $|\Psi_{\alpha,L}(n)\rangle$ and $|\Psi_{\alpha,R}(n)\rangle$ are the biorthogonal left and right eigenvectors of the non-Hermitian Hamiltonian. The previous spectral density corresponds to the conventional local density of states in the case of a Hermitian Hamiltonian and enables to image the localization in real space of the different modes. Our results

show that, as the loss strength increases, the edge modes become exponentially localized at the centers of the boundary islands and the localization length is a function of the loss strength, as evident from Figs. 3(e) and 3(f).

To gain deeper insights into the lowest and first excited modes of our system, we now analyze the low-energy effective models that represent these modes. The confinement of each different island generates local normal modes that are coupled through the arms of the islands. Specifically, for an isolated island, the Hamiltonian of the system can be written in the diagonal form as

$$H = \gamma \sum_{(ij)} a_i^{\dagger} a_j = \sum_{\alpha} \epsilon_{\alpha} \psi_{\alpha}^{\dagger} \psi_{\alpha}, \quad (3)$$

where ψ_{α} are the eigenmodes of a single island. Specifically, for the square island we consider the lowest-energy modes that correspond to an s -like orbital or the form $\psi_s \sim \cos(\kappa x) \cos(\kappa y) a_{\uparrow}^{\dagger}$, a p_x -like orbital of the form $\psi_{p_x} \sim \cos(\kappa y) \sin(2\kappa x) a_{\uparrow}^{\dagger}$ and a p_y -like orbital of the form $\psi_{p_y} \sim \cos(\kappa x) \sin(2\kappa y) a_{\uparrow}^{\dagger}$, where $\kappa = \pi/L$.

The normal modes of the isolated islands described above allow for the definition of an effective Hamiltonian for the coupled islands as

$$H = H_s + H_{p_x} + H_{p_y}, \quad (4)$$

where

$$H_s = t_{s,1} \sum_n (\psi_{s,n}^{\dagger} \psi_{s,n+1} + \text{H.c.}) + t_{s,2} \sum_n (\psi_{s,n}^{\dagger} \psi_{s,n+2} + \text{H.c.}) + \sum_n \epsilon_{s,n} \psi_{s,n}^{\dagger} \psi_{s,n}, \quad (5)$$

$$H_{p_x} = t_{p_x,1} \sum_n (\psi_{p_x,n}^{\dagger} \psi_{p_x,n+1} + \text{H.c.}) + t_{p_x,2} \sum_n (\psi_{p_x,n}^{\dagger} \psi_{p_x,n+2} + \text{H.c.}) + \sum_n \epsilon_{p_x,n} \psi_{p_x,n}^{\dagger} \psi_{p_x,n}, \quad (6)$$

$$H_{p_y} = t_{p_y,1} \sum_n (\psi_{p_y,n}^{\dagger} \psi_{p_y,n+1} + \text{H.c.}) + t_{p_y,2} \sum_n (\psi_{p_y,n}^{\dagger} \psi_{p_y,n+2} + \text{H.c.}) + \sum_n \epsilon_{p_y,n} \psi_{p_y,n}^{\dagger} \psi_{p_y,n}, \quad (7)$$

where $\epsilon_{s,n}$, $\epsilon_{p_x,n}$, $\epsilon_{p_y,n}$ correspond to the complex on-site energies projected on the s , p_x , p_y orbitals, $t_{s,1}$, $t_{p_x,1}$, $t_{p_y,1}$ correspond to the nearest-island hopping of the effective normal-mode Hamiltonian, and $t_{s,2}$, $t_{p_x,2}$, $t_{p_y,2}$ correspond to the next-to-nearest-island hopping of the effective normal-mode Hamiltonian. It is worth noting that while the hoppings in the microscopic site models are uniform, when projected to the normal modes, an orbital-dependent hopping will appear. In Fig. 4, we present schematic representations of these effective models for modes arising from the s and p orbitals, along with the real part of their corresponding energy spectrum.

We first focus on the effective model of the s -like orbitals, which emerge at the lowest energies. Figure 4(a) shows the lowest-energy mode in the quasi-one-dimensional system, namely, the s -like orbital, and in the limit of small losses. We take a loss modulation that leads to a unit cell

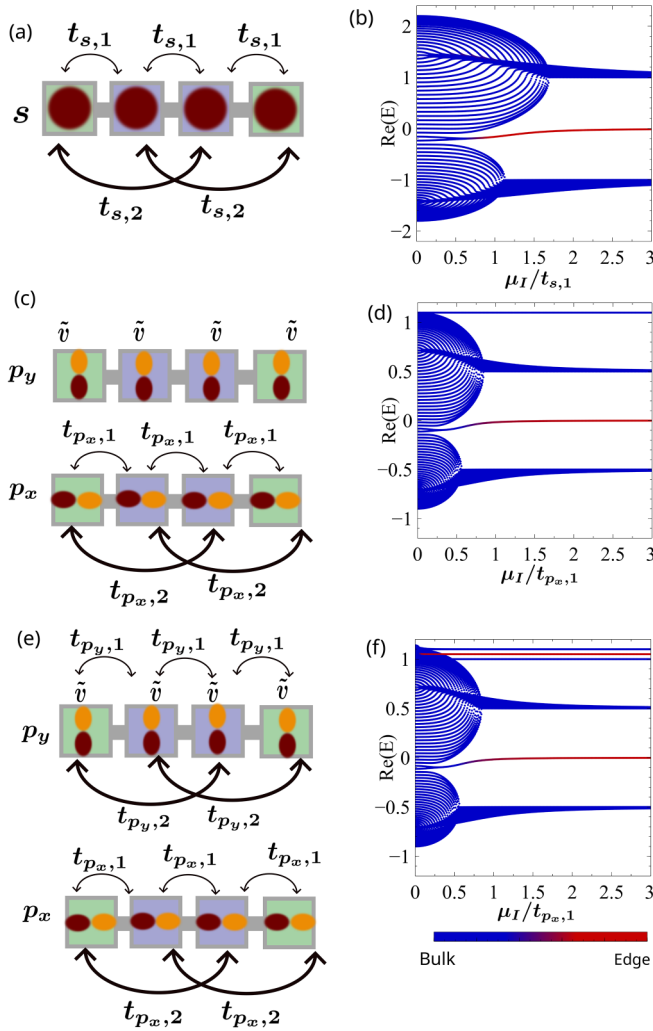


FIG. 4. Schematic of the low-energy effective model in the s manifold [panel (a)] for $W = 3, 4$, p_x , p_y manifold for $W = 3$ [panel (c)], and p_x , p_y manifold for $W = 4$ [panel (e)], with $i\mu_l \sin(2\pi\alpha n + \varphi)$ as modulated on-site loss, for $\alpha = 1/4$ and $\varphi = 3\pi/4$. Real part of the energy spectrum for the effective model in the s manifold [panel (b)], p_x , p_y manifold for $W = 3$ [panel (d)], p_x , p_y manifold for $W = 4$ [panel (f)]. Note that the main difference between $W = 3$ and $W = 4$ in the main manifold is the absence of sizable coupling between p_y orbitals in $W = 3$.

composed of four islands with on-site loss modulated as $i\mu_n = i\mu_l \sin(2\pi\alpha n + \varphi)$, where $\alpha = 1/4$ and $\varphi = 3\pi/4$. The energy spectrum of the effective s -like model can be compared with the full calculation involving the superlattice. It is clearly seen that the effective model accurately captures the characteristics of the lowest-energy mode observed in the full multimodal system, as seen in Figs. 3(a), 3(b), and 4(b).

Although the effective model for the s mode remains the same for islands of even and odd sizes, the effective models for the p_y manifold differ quantitatively. This difference arises because of the distinct coupling mechanisms that govern these modes in the respective systems. The p_x orbitals are coupled in both systems, resulting in the formation of a dispersive band. In contrast, for odd-sized islands, the p_y orbitals remain mostly uncoupled due to the node at $y = 0$, which leads to a vanishing hopping through the bridge. It is also worth noting

that while in the case of the isolated island the on-site energy of the p_x and p_y orbitals is identical due to C_4 symmetry, in the chain of islands the presence of the bridge breaks C_4 symmetry and the spectra of the p_x and p_y orbitals become unequal. In general stemming from mirror symmetry $\mathcal{M} : y \rightarrow -y$, the effective models for the p_x , p_y manifold shown in Figs. 4(c) and 4(e) consist of two decoupled chains, one corresponding to the p_x orbitals and the other to the p_y orbitals. Figure 4(c) depicts the effective model for the system with odd-sized islands, and its validity is supported by the corresponding energy spectrum shown in Fig. 4(d). Similarly, Fig. 4(e) presents the effective model for the system with even-sized islands, which is validated by its energy spectrum in Fig. 4(f).

IV. CRITICALITY AND LOCALIZATION-DELOCALIZATION TRANSITIONS

The quasiperiodic Hermitian and non-Hermitian AAH models, described by Eq. (1), exhibit a localization transition of all bulk modes at a finite critical value of the modulation amplitude of the real or imaginary part of the on-site energy. In the Hermitian case, this localization transition arises due to self-duality [2,4,6,66,67]. However, introducing even a slight modification in the hopping terms, and specifically, by including next-nearest-neighbor couplings, self-duality is broken. Consequently, the system no longer exhibits a sharp localization transition but instead features a mobility edge that segregates localized and extended states [51,68–73]. As the effective model of our systems features longer-range couplings, even though its microscopic model only had first neighbors, we expect the emergence of a mobility edge as we show below.

We now investigate how the localization-delocalization transition manifests in the multimodal version of the model. The localization transition can be directly inferred by calculating the inverse participation ratio (IPR) of the eigenstates [70]. For the non-Hermitian system, the definition of IPR for a biorthonormal state takes a modified form as shown below [10,28,74,75]:

$$\text{IPR}(|\psi\rangle) = \sum_l |\langle\psi_L|l\rangle\langle l|\psi_R\rangle|^2, \quad (8)$$

where $|l\rangle$ is a state localized on site l . For $N \rightarrow \infty$, we have $\text{IPR} = 0$ for an extended state and $\text{IPR} \sim 1/N$, with N being the number of sites where the state is localized, for a localized state. For quantifying localization, we use a reference value for the maximum IPR as $6/N$, following Ref. [69].

We now analyze the localization transition in the multimodal system as a function of the modulation amplitude v_l . For an irrational value of α , Figs. 5(a)–5(c) show that all bulk modes become localized, but at different values of v_l . This phenomenology for the s manifold can be rationalized from the low-energy effective model of the system introduced in Figs. 4(a) and 4(b), where the modulation frequency characterizes a quasicrystalline structure. Figure 5(d) presents the real part of the energy spectrum of this effective model, which demonstrates similarity between the microscopic system and the effective model. The existence of a mobility edge thus stems from the finite second neighbor hopping in the effective model, which naturally accounts for the results of the

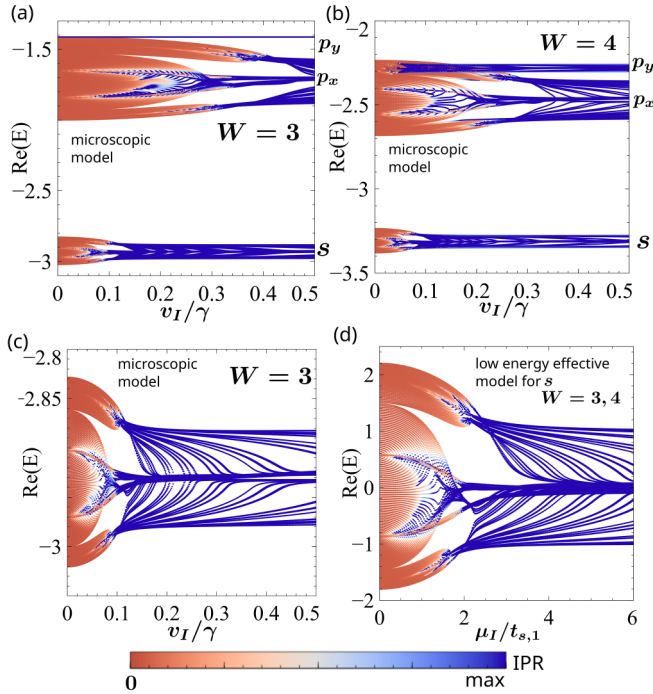


FIG. 5. Real part of the energy spectra for our quasi-one-dimensional model and the effective model as a function of on-site losses, with IPR as the color. Panels (a) and (b) show that the localization transition for $W = 3$ and $W = 4$ has a mobility edge with a nonzero slope, whereas panel (c) shows the enlarged view of the localization transition for the s orbital for $W = 3$, which has a mobility edge. (d) Localization transition for the low-energy effective model of the bands resulting from the combination of the s orbitals. We took $N = 80$, $\alpha = (\sqrt{5} - 1)/2$, and $\varphi = 0.4\pi$ in panels (a)–(d). Note that the edge modes are omitted in this work because they are localized already at small v ; only bulk modes undergo the localization transition.

superlattice model. A similar study of the localization transition can be conducted for the p manifold. Although the non-Hermitian AAH model with a single mode has a known critical point at $v_I = \pm 2.0\gamma$, such a universal critical value does not emerge in the multimodal case. This is due to the presence of mobility edges, as the localization transitions for the real and imaginary parts of the on-site potentials in the multimodal AAH model occur at different values and lack an explicit analytical continuation between them [51].

V. IMPACT OF DISORDER

To determine whether the edge modes observed in our multimodal system are topologically trivial or nontrivial, we test their robustness against disorder [51,53,56]. To analyze the effect of disorder on the energy spectrum, we define the spatially resolved spectral density as

$$D(\omega, n) = \sum_{\alpha} \delta(\omega - \text{Re}(E_{\alpha})) \langle n | \Psi_{\alpha,R} \rangle \langle \Psi_{\alpha,L} | n \rangle, \quad (9)$$

where ω is the resonant frequency and $|\Psi_{\alpha,R}\rangle$ and $|\Psi_{\alpha,L}\rangle$ are the right and left eigenvectors of the Hamiltonian, respectively. Although this expression projects onto the real part

of the eigenenergies, a similar formulation can be defined for the imaginary part. The spatially resolved spectral density $D(\omega, n)$ gives direct access to the number of eigenstates with a given real energy value and is analogous to the local density of states in Hermitian systems. In experimental contexts, particularly in photonic platforms, the modulus of this quantity corresponds to the light intensity observed in optical cavities.

We investigate the spatially resolved spectral density of the multimodal system with $W = 3$ as a function of the disorder strength. We consider two types of disorder, loss disorder and detuning disorder, incorporated into the Hamiltonian via the additional term:

$$H_D = w_R \sum_n \chi_{n,R} a_n^{\dagger} a_n + i w_I \sum_n \chi_{n,I} a_n^{\dagger} a_n, \quad (10)$$

where w_R and w_I parametrize the strengths of the detuning and loss disorder, respectively. The disorder variables $\chi_{n,R}$ and $\chi_{n,I}$ are random values sampled independently from Gaussian distributions with zero mean and unit variance. To understand the different roles of the two types of disorders, we study the effects of loss and detuning disorder separately.

To evaluate the robustness of the edge modes, we analyze the average spectral density, as shown in Fig. 6. In Figs. 6(a) and 6(b), we present the average spectral density for the system in the presence of bulk loss disorder [Fig. 6(a)] and edge loss disorder [Fig. 6(b)], as a function of w_I . As the disorder strength increases, the band gaps associated with the bulk p_x orbitals remain open, and the edge modes remain robust, indicating topological protection.

In contrast, Figs. 6(c) and 6(d) show the effects due to detuning disorder w_R , again averaged over disorder realizations for the bulk [Fig. 6(c)] and edge [Fig. 6(d)] regions. It is observed that the gap is modified by the presence of disorder, and the edge modes distort with increasing disorder strength. Visibly, Fig. 6(d) reveals that the spatial spread of the edge modes increases proportionally to w_R , indicating their sensitivity to detuning disorder. These results show that edge modes in the multimodal system are topologically protected against loss disorder but are vulnerable to detuning disorder.

VI. IMPACT OF A GAUGE FIELD

Synthetic gauge fields in photonic systems can be engineered using diverse mechanisms that imprint complex phases onto hopping amplitudes [76–79], effectively mimicking the influence of magnetic fields [80,81] on neutral particles. These approaches include laser-assisted tunneling [82], dynamic modulation of resonator couplings [83,84], Floquet driving [85–87], and magnetoelectric Stark shifts [88]. Additional methods such as strain-induced gauge potentials [89], nonplanar optical cavities [90,91], geometric phase engineering [8,92], and magnetooptical materials [15,93,94] have also been successfully employed to break reciprocity and realize artificial magnetic flux. These techniques have enabled the realization of topological lattice models, such as the Harper-Hofstadter [80,95–97], Haldane [98], and the Su-Schrieffer-Heeger Hamiltonian [99,100] across various platforms, including ultracold atoms in optical lattices, waveguide arrays, ring-resonator lattices, and superconducting circuits. In particular, gauge fields have been demonstrated

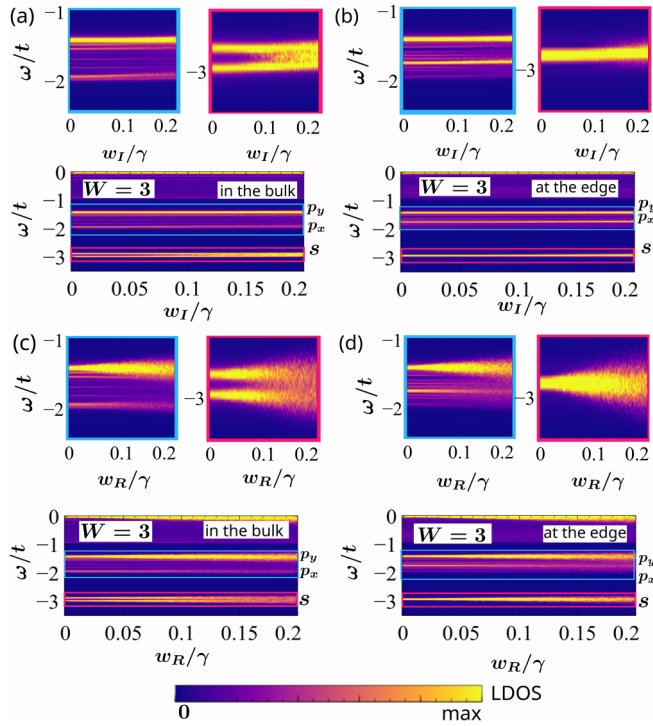


FIG. 6. Spectral density as a function of the disorder strength in the local loss w_I [panels (a) and (b)] and disorder in the resonant frequency w_R [panels (c) and (d)] for $W = 3$. Panels (a) and (c) show the effect of disorder on the bulk and panels (b) and (d) show the effect at the edge. It is observed that the existence of a finite disorder decreases the bulk gap [panels (a) and (c)], but without destroying it. The edge modes for the s -like mode and the p -like mode are robust to the existence of disorder in loss as shown in panel (b). In contrast, they develop a finite splitting in the presence of detuning disorder [panel (d)]. The insets provide an enlarged view of the edge states for s and p orbitals. We took $N = 80$ and $v_I = 0.4t$, $\alpha = 1/4$, $\varphi = 3\pi/4$, and the results are averaged over 100 realizations.

in programmable platforms like the one we study by using discretized Floquet evolution [101].

In our scenario, an important role of the gauge field at the level of the effective model is to mix the p_x and p_y manifolds, thus affecting the full dispersion of the multiorbital system. In the microscopic basis, the presence of this gauge field, the hopping terms in the Hamiltonian acquire Peierls phase factors and are modified as follows:

$$\mathcal{H} = \sum_{\alpha\beta} \gamma_{\alpha\beta} a_{\alpha}^{\dagger} a_{\beta} = \sum_{\alpha\beta} \gamma e^{i\phi_{\alpha\beta}} a_{\alpha}^{\dagger} a_{\beta}, \quad (11)$$

where $\phi_{\alpha\beta} = \int_{\mathbf{r}_{\alpha}}^{\mathbf{r}_{\beta}} \mathbf{A} \cdot d\mathbf{r}$ is the Peierls phase acquired during hopping from site β to α , and $\mathbf{\Omega} = \nabla \times \mathbf{A}$ represents the effective magnetic field corresponding to the applied gauge potential. For the sake of concreteness, we take the Landau gauge $\mathbf{A} = (-B_{\text{micro}}, 0, 0)$, which yields the Peierls phase $\phi_{\alpha\beta} = 2\pi B_{\text{micro}}(x_{\alpha} - x_{\beta})(\frac{y_{\alpha} + y_{\beta}}{2})$, where B_{micro} characterizes the strength of the applied synthetic magnetic field in the microscopic system. Here, (x_{α}, y_{α}) and (x_{β}, y_{β}) denote the spatial coordinates of the lattice sites α and β , respectively.

Introducing a gauge field into the gain-loss and quasiperiodic models induces substantial changes in the energy

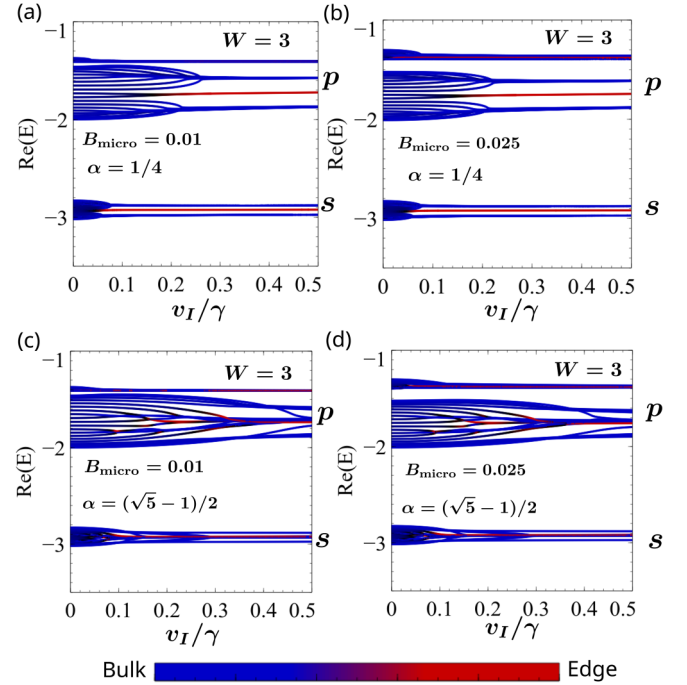


FIG. 7. (a)–(d) Real part of the energy spectrum vs the amplitude of modulation v_I for different values of B_{micro} and frequency of modulation α . Energy spectrum of a unit cell with four islands (with $\alpha = 1/4$) [panels (a) and (b)] and a quasiperiodic system (with $\alpha = (\sqrt{5} - 1)/2$) [panels (c) and (d)], of size $W = 3$ and gauge field of strength $B_{\text{micro}} = 0.01$ and $B_{\text{micro}} = 0.025$, respectively. We can see that adding the gauge field affects the spectrum for the bulk modes with p orbitals, while the s orbitals remain unaffected.

spectrum. As shown in Fig. 7, the bulk modes originating from the s orbitals remain largely unaffected, as expected from the vanishing angular momenta of the s manifold. However, in the presence of a gauge field, the eigenstates in each island become eigenstates of the angular momenta $L_z = \pm 1$, leading to the formation of chiral combinations $p_x + ip_y$ and $p_x - ip_y$, directly stemming from the interorbital coupling that was previously absent for $B_{\text{micro}} = 0$ in the model.

To distinguish the states that emerge from the introduction of the gauge field in the microscopic model, we evaluate the expectation value of the angular-momentum operator, defined as $l = \langle \frac{\partial H}{\partial B_{\text{micro}}} \rangle$. This quantity is positive for the $p_x + ip_y$ states and negative for the $p_x - ip_y$ states, which allows clear identification of their chiral nature. The application of a gauge field to a square island lifts the degeneracy of eigenstates formed by combinations of the p_x , p_y orbitals (i.e., $p_x + ip_y$ and $p_x - ip_y$ states), resulting in eigenvectors with distinct positive and negative angular momenta, as evident in Figs. 8(a) and 8(b). It is noteworthy that when the geometry of the island is modified to include connecting sites, a splitting of eigenvalues is observed for zero gauge field. This splitting stems from the inequality between the p_x and p_y orbitals, arising from the breaking of the C_4 symmetry. The gauge field thus competes with such a splitting, creating the $p_x \pm ip_y$ eigenstates for large gauge field, as shown in Figs. 8(c) and 8(d). The competition of the C_4 symmetry breaking and the gauge field can be further rationalized in a unit cell consisting of four islands,

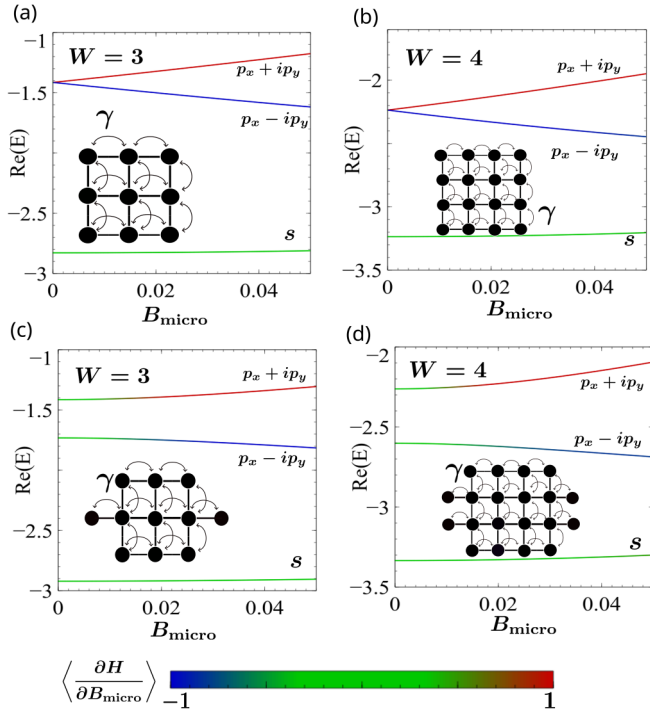


FIG. 8. (a)–(d) Real part of the energy spectrum vs the strength of the gauge field B_{micro} for an island. Panels (a)–(d) show the spectrum when we take a single island of sizes 3×3 and 4×4 , respectively; as shown in the insets, we can see that the modes due to p orbitals split to give two nondegenerate eigenvalues $p_x + ip_y$ and $p_x - ip_y$ with slopes $\langle \frac{\partial H}{\partial B_{\text{micro}}} \rangle$ as positive and negative [panels (a) and (b)], and similarly for panels (c) and (d). The color indicates the expectation value of the angular-momentum operator $l = \langle \frac{\partial H}{\partial B_{\text{micro}}} \rangle$. No on-site loss was introduced to the systems here.

each subjected to an imaginary on-site potential of the form $iv_l \sin(2\pi\alpha n + \varphi)$, where n is the island index. Incorporating the gauge field as described in Eq. (11), we observe that the bulk modes formed by the coupling of p_x and p_y orbitals evolve into new states with positive and negative expectation values of the angular-momentum operator $l = \langle \frac{\partial H}{\partial B_{\text{micro}}} \rangle$, as shown in Figs. 9(a) and 9(b).

We now turn to how such a gauge field can be included in the effective model. From the perspective of the normal-mode effective model, the presence of a gauge field B_{eff} is included in the Hamiltonian equation (4) as

$$H_{\text{gauge}} = iB_{\text{eff}} \sum_n \psi_{p_x,n}^\dagger \psi_{p_y,n} + \text{H.c.} \quad (12)$$

Applying the gauge field to the low-energy effective model induces a coupling between the originally decoupled p_x and p_y orbitals, resulting in modes with positive and negative values of the angular momenta $l = \langle \frac{\partial H}{\partial B_{\text{eff}}} \rangle$, as illustrated in Figs. 9(c) and 9(d).

The faithfulness of the effective model can be observed by studying the localization transition as a function of the gauge field and the loss modulation, as shown in Fig. 10. Since the synthetic gauge field does not affect the s manifold [Figs. 9(a) and 9(b)], we focus our study on its effect on the localization transition in the p manifold. Specifically, we study

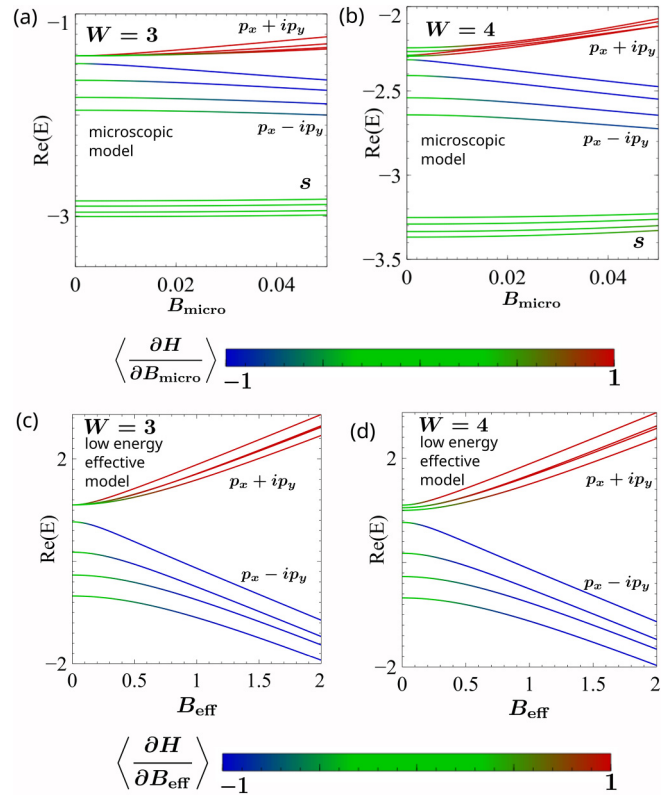


FIG. 9. Real part of the energy spectrum vs the strength of the gauge field B_{micro} in panels (a) and (b) and B_{eff} in panels (c) and (d), with local on-site losses. (a), (b) Energy spectrum of the microscopic model of a unit cell with four islands of sizes 3×3 and 4×4 , respectively, with a local loss modulated as $iv_l \sin(2\pi n/4 + 3\pi/4)$ with strength $v_l = 0.2\gamma$ (n is the index of the island). Energy spectrum of the low-energy effective models for the models in panels (a) and (b) (with local on-site losses $i\mu_l \sin(2\pi n/4 + 3\pi/4)$ and $\mu_l = 0.5t_{p_x,1}$) as described by Figs. 5(c) and 5(d). We can see that the p -orbital spectrum splits into nondegenerate states of $p_x + ip_y$ and $p_x - ip_y$ as expected.

the mobility edge phase diagrams for the multimodal system with the microscopic model [Figs. 10(a) and 10(c)] and its corresponding low-energy model for p orbitals [Figs. 10(b) and 10(d)], considering two different island sizes $W = 3$ [Figs. 10(a) and 10(c)] and $W = 4$ [Figs. 10(b) and 10(d)]. For each different width W separately, we observe that both the microscopic system and the effective model exhibit nearly identical localization behavior when the potential strength v_l , μ_l and the gauge field strength B_{micro} , B_{eff} are varied. The plotted quantity represents the percentage of localized states among those arising from the coupling of the p orbitals. This agreement indicates that the low-energy model accurately captures the essential physics of the localization transition for $W = 3$ and $W = 4$. Interestingly, the evolution of the mobility edges for $W = 3$ and $W = 4$ is qualitatively different, a phenomenon perfectly captured by the low-energy model. Specifically, for $W = 3$, the multimodal system shows a nontrivial trend; as B_{micro} , B_{eff} increase, the bulk modes become localized only at higher values of v_l , μ_l , indicating a field-dependent shift in the localization threshold.

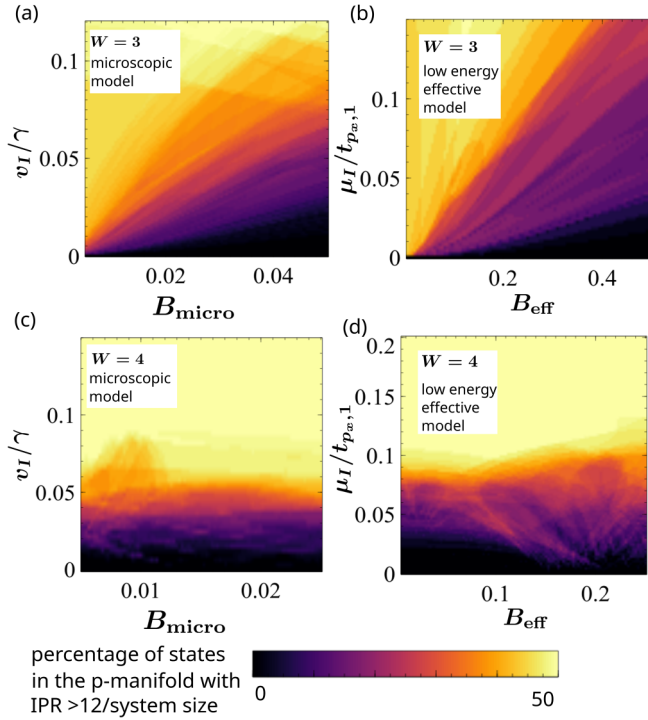


FIG. 10. Mobility edge as a function of the gauge field B_{micro} , B_{eff} and the loss modulation v_I , μ_I in the p manifold, for the full model [panels (a) and (c)] and the effective model [panels (b) and (d)]. Panels (a) and (b) correspond to $W = 3$ and panels (c) and (d) correspond to $W = 4$. The color represents the percentage of states that are localized with a limit of $\text{IPR} > 12/N$, where N is the size of the system. It is observed that the microscopic and effective models show a similar localization evolution, becoming different for the two different sets of islands.

It is important to emphasize that the observed edge localization transition originates from the quasiperiodic modulation and is not related to the non-Hermitian skin effect. The skin effect typically arises in systems with asymmetric hopping [102,103], whereas the present model retains reciprocal couplings. One strategy to generate nonreciprocal couplings is through an imaginary gauge field, which would generate an asymmetric hopping [22]. In the presence of such an imaginary gauge field, a non-Hermitian skin effect can appear. However, this is not the case of our setup, where the gauge field is real.

In summary, the application of a synthetic gauge field to the system introduces significant changes to the energy spectrum, as a result of the hybridization of the p_x and p_y manifolds. Such hybridization has a substantial impact on the localization of the eigenmodes upon inclusion of quasiperiodic loss, and particularly enables control on the localization properties of the modes through an external gauge field.

VII. CONCLUSION

Here, we showed the emergence of multimodal non-Hermitian topological states that can be realized on an optical superlattice platform. We analyzed the emergence of topological edge modes, localization-delocalization transitions, and

the impact on its spectral properties under the influence of loss, disorder, and an external gauge field. We demonstrated that topological modes and localization transitions emerge in multiorbital scenarios, showing that these phenomena go beyond single-mode models. We showed that the full bulk and edge modes can be captured using low-energy multiorbital effective models that accurately describe the full microscopic models. We showed that in the quasiperiodic limit, the topological properties depend sensitively on the type of disorder. Although the system remains robust against the disorder in the local loss, the detuning frequency disorder significantly affects the spectral structure, highlighting its role in controlling topological stability. Additionally, we showed that the inclusion of a synthetic gauge field in the multimodal system enables control of the hybridization of different orbitals of these multimodal platforms and particularly enables tuning a localization-delocalization transition through an external gauge field. Our results establish the emergence of topology and criticality in multiorbital photonic lossy systems, putting forward internal orbital degrees of freedom as a flexible knob to control non-Hermitian topology and criticality.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [104].

APPENDIX A: IMPACT OF A ZIGZAG ISLAND COUPLING

In the main text, we studied a multimodal lattice composed of square islands of size $W = 3$ (Fig. 1), whose bulk spectrum consists of modes derived from both s and p orbitals. In the absence of interorbital coupling, the p_x and p_y orbitals remain decoupled, resulting in a dispersive band associated with the p_x orbitals and a nondispersive (flat) band originating from the p_y orbitals, as shown in Figs. 3(a) and 3(c).

Here, we modify this system by creating an alternative zigzag coupling geometry between islands, which effectively introduces mixing between the p_x and p_y orbitals. We take a geometry where the coupling between islands is no longer linear, but are arranged at 90° as shown in Fig. 11(a). This reshaping of the lattice connectivity breaks the original mirror symmetry leading to interorbital hybridization. The on-site potential retains the form $iv_I \sin(2\alpha n + \varphi)$, where n labels the island index, with $\alpha = 1/4$ and $\varphi = 3\pi/4$.

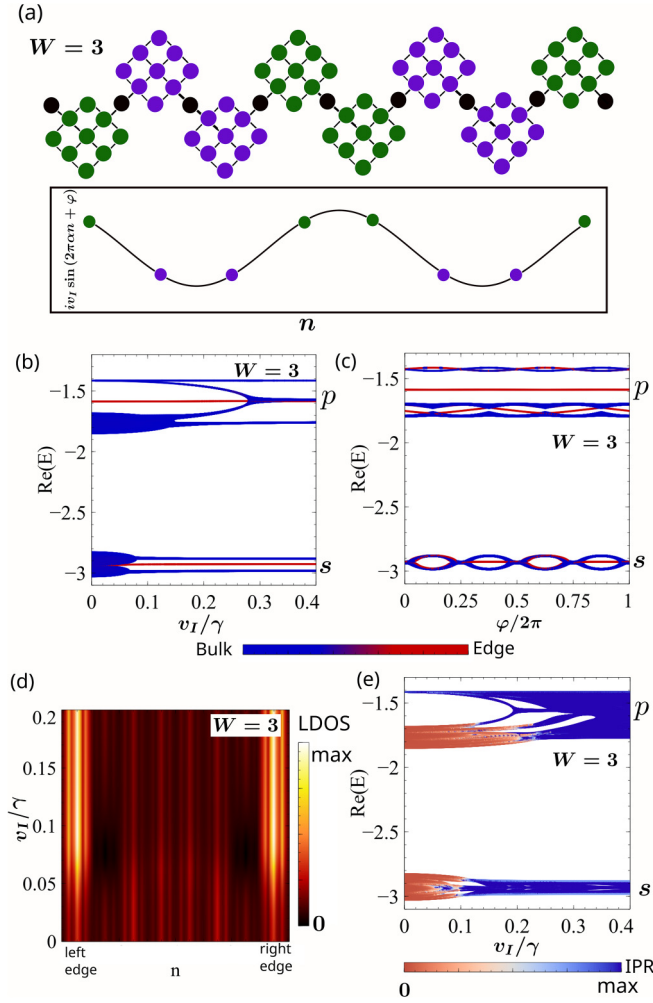


FIG. 11. (a) Schematic of the gain-loss lattice for the multimodal model with square islands ($W = 3$), composed of three islands under physical strain. Real part of the energy spectrum for $\alpha = 1/4$ and $\varphi = 3\pi/4$, shown as a function of the loss modulation strength v_I [panel (b)] and the phason φ [panel (c)] for $v_I = 0.2\gamma$. States originating from s and p orbitals are highlighted. For $W = 3$, the p -orbital sector forms a dispersive band due to hybridization between p_x and p_y orbitals, while the s -orbital mode remains unaffected by the applied physical strain. (d) Spectral density of the lowest bulk mode as a function of the site index n for $W = 3$ and $N = 8$. Increasing the loss amplitude v_I induces exponential localization on the edge islands, with a localization length that depends on v_I . (e) Real part of the energy spectrum for the quasi-one-dimensional model as a function of on-site loss strength, with the IPR encoded by color. For $W = 3$, the localization transition exhibits a mobility edge with a finite slope. We use $N = 80$, $\alpha = (\sqrt{5} - 1)/2$, and $\varphi = 0.4\pi$. Edge modes are omitted; as they are already localized at small v_I , only bulk modes undergo the localization transition.

As shown in Figs. 11(b) and 11(c), the spectral features associated with the s -orbital modes remain unaffected by the geometric modification. In contrast, the p -orbital sector undergoes qualitative changes; the previously nondispersive band acquires dispersion, and increases with increasing loss amplitude v_I , reflecting hybridization between the p_x and p_y orbitals. The edge mode originating from the s -orbital sector

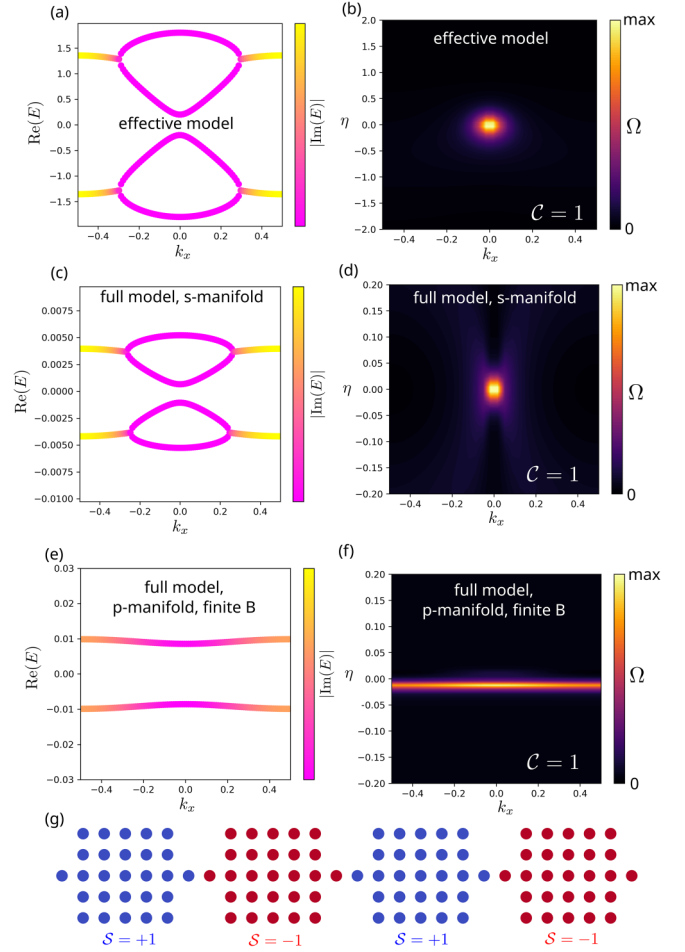


FIG. 12. Spectra [panels (a), (c), and (e)] and topological invariants for the minimal effective model [panels (a) and (b)], and the full lattice model in the s manifold [panels (c) and (d)] and the p -wave manifold [panels (e) and (f)]. Panel (g) shows the chiral operator used for the full lattice model. The results in panels (a) and (b) correspond to the minimal topological non-Hermitian model of Ref. [54]. It is observed that the full lattice model features an invariant in the s manifold [panels (c) and (d)] very similar to the minimal model [panels (a) and (b)]. The full lattice model in the p -wave manifold and in the presence of gauge field shows a different dispersion, tunable by the gauge field. We observe that for all the cases Chern number $C = 1$. Energy reference is set to zero in the gap for all cases.

remains topological in the presence of loss and becomes exponentially localized on the edge islands as v_I increases, as shown in Fig. 11(d).

Finally, we investigate the localization transition in the quasiperiodic regime of the geometrically modified multimodal system, taking $\alpha = (\sqrt{5} - 1)/2$ and $\varphi = 0.4\pi$. As shown in Fig. 11(e), the localization transition for the s -orbital sector is unchanged compared to the unstrained case. In contrast, the p -orbital sector exhibits interorbital coupling accompanied by a mobility edge, with a subset of modes remaining localized even at $v_I = 0$, as was the case in the presence of gauge field. These findings show that a zigzag geometric island arrangement leads to intrinsic interorbital

coupling in multimodal lattices, resulting in hybridized p -orbital bands with a mobility edge, while preserving the robustness of the s -orbital sector.

APPENDIX B: TOPOLOGICAL INVARIANT OF THE FULL LATTICE MODEL

We discuss here explicitly the calculation of the topological Chern invariant for the non-Hermitian model, comparing the results for a minimal model and the full lattice model. The spectra and topological invariant for the minimal and full lattice models are shown in Fig. 12, where Figs. 12(a), 12(c), and 12(e) are the spectral dispersion in a four-site/island unit cell, Figs. 12(b), 12(d), and 12(f) are the topological invariants, and Fig. 12(g) is the chiral operator for the full lattice model. It is worth first analyzing the minimal single-orbital model, analogous to the low-energy s -wave model discussed in our article, which corresponds exactly to the topological non-Hermitian model whose topological invariant is computed in Ref. [54]. Calculation of the Chern number requires identifying the chiral operator $\mathcal{S}$ such that $\mathcal{S}H_k\mathcal{S}^\dagger = -H_k$, and subsequently computing the Chern number of the effective Hamiltonian $H^{\text{eff}}(\eta, k) = \mathcal{S}(\eta - iH_k)$, where η parametrizes the imaginary part of a generalized family of boundary modes, so that the Chern invariant is then computed in the space (η, k) [54]. As a result, identifying the chiral operator $\mathcal{S}$ is a crucial step to compute the Chern number. Quantization of the Chern number is then guaranteed by a compactification of the η axis [54]. For the minimal model, such a chiral operator can be constructed explicitly as $\mathcal{S} = I \otimes \sigma_z$ [54]. We show in Fig. 12(a) the band structure of the effective model in the s manifold, and in Fig. 12(b) its Berry curvature computed with the procedure noted above. It is observed that the Berry curvature is concentrated around $(\eta, k_x) = 0$.

We now move on to compute the Berry curvature and Chern number for the full lattice model. It is worth noting

that this is a nontrivial procedure from a numerical point of view given the hidden nature of the Chern number in a 1D non-Hermitian system. In particular, the crucial step is to identify the chiral operator for those manifolds such that $\mathcal{S}H_k\mathcal{S}^\dagger = -H_k$. While this can be easily done in the minimal model, identifying such operator in the full lattice model is more nontrivial as it depends on the exact shape of the Wannier orbitals of the low-energy manifold. For those low-energy states, an approximate representation of the operator can be obtained by promoting the operator $\mathcal{S}$ to the whole chain of islands as $\mathcal{S} = I \otimes \sigma_z \otimes \mathcal{I}$, where $\mathcal{I}$ is the identity operator for all the island sites and half the links, as shown in Fig. 12(g). It is worth noting that we used a geometry with slightly elongated links so that the chiral operator defined above fulfills $\mathcal{S}^2 = I$.

We now discuss the s and p manifolds of the full lattice model. In the s manifold of the full lattice model, we obtain a low-energy dispersion [Fig. 12(c)] analogous to the minimal model [Fig. 12(a)]. The Berry curvature computed for the full lattice model [Fig. 12(d)] in the s manifold also shows a behavior analogous to the minimal model [Fig. 12(b)]. As the s -wave manifold is effectively not impacted by a gauge field, the previous picture is robust to inclusion of a gauge field. The situation in the p -wave manifold of the full lattice model is different. We show in Fig. 12(e) the spectral dispersion of the p -wave manifold in the presence of a gauge field, leading to a mixing of the p_x and p_y manifolds, which leads to a manifold of $p_x + ip_y$ states. We take the zero-energy reference for the topological invariant at one-quarter filling of the p manifold. The dispersion of the p manifold becomes highly tunable with the gauge field, which as a result also gives rise to a different distribution of the Berry curvature in the (k_x, η) plane, and, in particular, gauge-field-dependent. Interestingly, we find that the Chern number $\mathcal{C} = 1$, which is consistent with the existence of a single edge mode.

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