

Barkhausen noise from domain walls with internal degrees of freedom

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Domain walls in disordered ferromagnets—interfaces separating oppositely magnetized regions—are commonly modeled as elastic interfaces in random media. However, real domain walls possess internal degrees of freedom related to the local magnetization direction at the wall's midpoint, which are not captured by simple elastic interface models. Here, we investigate how these internal degrees of freedom influence Barkhausen noise arising from the jerky field-driven motion of linelike domain walls in disordered thin ferromagnetic films with strong perpendicular magnetic anisotropy. Using a recently developed reduced model that incorporates internal wall dynamics, we simulate systems two orders of magnitude larger than those accessible via full micromagnetic simulations. We focus on Bloch line dynamics and identify a disorder-driven mechanism where Bloch lines accumulate near the edges of moving wall segments during avalanches, thereby impeding further propagation. This mechanism leads to distinctive statistical features in Barkhausen avalanches, including pronounced bumps in the cutoffs of the avalanche distributions, skewness of the average avalanche shape evolving with the avalanche duration, and reduced domain wall roughness at large scales.

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I. INTRODUCTION

Barkhausen noise, originating from the jerky field-driven motion of domain walls (DWs) in disordered ferromagnets, is a paradigmatic example of crackling noise [1,2]. When subject to a slowly varying external magnetic field, the DWs in the magnet respond by exhibiting a sequence of jumps (or *avalanches*) characterized by broad power law distributions of the jump sizes and durations. This phenomenology is similar to that exhibited by a broad class of physical systems driven out of equilibrium by external forces, ranging from strain bursts in plastically deforming crystals (i.e., dislocation avalanches [3–7]) to intermittently propagating crack fronts in disordered solids [8,9]. In addition to fundamental interest motivated by the need to understand the theoretical foundations of the complex, collective DW dynamics that is typically understood to be linked to an underlying nonequilibrium phase transition [10,11], Barkhausen noise is of significant practical relevance as it is widely used in the industry for non-destructive testing of ferromagnetic components (e.g., steel) [12–14].

From the modeling perspective, one can identify several model types that have been used to study the Barkhausen effect and related phenomena. Describing DWs as elastic interfaces in random media has made it possible to

understand many key aspects of statistical properties of Barkhausen noise, e.g., the different universality classes one might observe for polycrystalline and amorphous ferromagnets in two- and three-dimensional samples [11,15,16]. Related classes of models are given by Ising spin models [17–19], Burridge-Knopoff-type spring-block models [20], and phase field models [21]. Full micromagnetic simulations [22] and atomistic spin dynamics simulations [23] are the most detailed and realistic numerical descriptions of DWs and their dynamics, and have the advantage of being able to describe also the internal precessional dynamics within the moving DWs taking place beyond the Walker breakdown [24,25]. In extended DWs, the latter happens via repeated nucleation, propagation, and annihilation of Bloch lines [26–29] within the DWs. These features are excluded by construction in the other models mentioned above as they do not describe magnetization explicitly as a vector field varying smoothly in space, hence failing to capture the internal structure of the DWs. However, micromagnetic simulations suffer from the limitation that they can in practice be performed only for length scales up to a few micrometers (atomistic spin dynamics are limited to even shorter length scales), thus severely limiting the range of scales one can reach to properly describe the scale-free domain wall jumps responsible for the Barkhausen effect.

Here, to overcome these limitations, we employ a recently developed reduced model of extended DWs in disordered ferromagnetic thin films with perpendicular magnetic anisotropy (PMA), combining features of the elastic line approach with those of full micromagnetic simulations [30], to study Barkhausen noise. The model allows to reach at least two orders of magnitude larger system sizes than what has been possible in full micromagnetic simulations [26], thus helping to reach a broad scaling range not affected by finite-size

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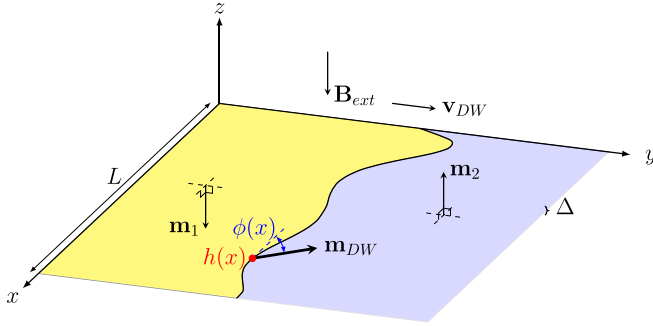


FIG. 1. A schematic illustration of the simulated system of a PMA thin film (thickness Δ) in the xy plane. A line-like DW, described by the displacement field $h(x)$ and the in-plane magnetization angle $\phi(x)$, separates two domains with opposite magnetizations $\mathbf{m}_1$ and $\mathbf{m}_2$. The external magnetic field B_{ext} is applied in the negative z direction, inducing DW motion in the positive y direction.

effects. In addition to a variable describing the local DW position, the model includes internal degrees of freedom in the form of an angular variable defined along the DW describing the local direction of the DW midpoint magnetization. This makes it possible to describe nucleation, propagation, and annihilation of Bloch lines within the DW moving in the precessional regime above the Walker breakdown. The model has recently been shown to exhibit also intriguing spatiotemporal chaos and roughening even in the absence of quenched disorder [31]. We study the role of these Bloch line dynamics in determining the properties of the Barkhausen noise, considering the avalanche distributions as well as the average temporal and spatial avalanche shapes. We identify a mechanism where for strong enough disorder Bloch lines tend to accumulate during avalanche propagation around the edges of the moving parts of the DW, hence limiting further propagation of the avalanche. Our detailed analysis of the statistical properties of the Barkhausen avalanches shows that this Bloch line accumulation mechanism gives rise to several key features, including pronounced bumps in the cutoffs of the avalanche size and duration distributions, skewness of the temporal average avalanche shape evolving with the avalanche duration, and reduced domain wall roughness on large scales. Overall, our findings emphasize the importance of the often overlooked role of internal degrees of freedom in the dynamics of extended DWs.

The paper is organized as follows: In Sec. II, the reduced model of thin film DWs employed in this study is presented. Section III presents the simulation results of Barkhausen noise for different disorder strengths, together with analysis of the role of Bloch line dynamics in the Barkhausen jump statistics. Section IV finishes the paper with discussion and conclusions.

II. REDUCED MODEL OF A THIN FILM DOMAIN WALL

We consider a line-like DW separating two domains with reduced magnetizations $\mathbf{m}_1 \equiv \mathbf{M}_1/M_s = -\mathbf{e}_z$ (with M_s the saturation magnetization) and $\mathbf{m}_2 = \mathbf{e}_z$, respectively, in a disordered ferromagnetic thin film with PMA (see Fig. 1). The DW is driven toward the positive y direction by an external magnetic field $\mathbf{B}_{\text{ext}} = -B_{\text{ext}}\mathbf{e}_z$. Unlike general micromagnetic

simulations solving the time evolution of the reduced magnetization $\mathbf{m}(\mathbf{r}, t)$ everywhere in the sample, the reduced model of DWs [30] considered here includes only degrees of freedom of the line-like DW. Hence, it does not describe magnetization dynamics within the surrounding domains (e.g., spin waves that are sometimes emitted from a moving DW [32]). However, the fact that in addition to the local DW position $h(x, t)$ the model includes an angular variable $\phi(x, t)$ describing the direction of the local in-plane magnetization $\mathbf{m}_{\text{DW}}$ within the DW implies the possibility of describing Bloch line dynamics within the DW, not possible in typical elastic line models neglecting the internal degrees of freedom of the DW. Another key advantage of the reduced model is its computational efficiency as compared to full micromagnetic simulations, making it possible to simulate systems of much larger sizes L than what is possible in full micromagnetic simulations.

The model we consider here was originally developed in Ref. [30]. Below, we repeat the description of its derivation for completeness. The model is obtained by viewing the Landau-Lifshitz-Gilbert (LLG) equation governing the magnetization dynamics in terms of the polar angles θ and ϕ of the magnetization vector $\mathbf{m} = \cos\theta\mathbf{e}_z + \sin\theta(\cos\phi\mathbf{e}_x + \sin\phi\mathbf{e}_y)$ as a dissipative Euler-Lagrange equation. The Lagrangian $\mathcal{L} = T - E$ includes a “kinetic” part $T[\theta, \phi] = (M_s/\gamma) \int \dot{\phi} \dot{\theta} \sin\theta dx$, with γ the gyromagnetic ratio, and the energy functional $E[\theta, \phi] = \int A_{\text{ex}}[\nabla\theta^2 + \nabla\phi^2 \sin^2\theta - K_u \cos^2\theta B_a M_s] dx + E_d[\theta, \phi]$, where A_{ex} , K_u , and B_a are the exchange stiffness, uniaxial anisotropy constant, and the applied magnetic field, respectively, with the latter including contributions due to the external field B_{ext} as well as due to quenched random-field disorder (see below). $E_d[\theta, \phi]$ is the demagnetizing energy, which in a local approximation reads $E_d[\theta, \phi] = -\frac{1}{2}\mu_0 M_s^2 \int [N_n(\mathbf{m} \cdot \mathbf{n})^2 + m_z^2] dx$, where $\mathbf{n}$ is the unit vector perpendicular to the DW and N_n is given to lowest order in the film thickness Δ as $(\Delta/\pi D) \ln 2$ [33]. To describe effects due to the damping term in the LLG equation, the dissipation functional $F[\theta, \phi] = (\alpha_G M_s/2\gamma) \int (\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta) dx$, where α_G is the Gilbert damping constant, is considered. We then change variables to coordinates comoving with the DW, $\mathbf{x} = \mathbf{r}(s, t) + \rho\mathbf{n}(s, t)$, where $\mathbf{r}(s)$ is a parametrized curve describing the instantaneous DW profile and ρ is the projected signed distance from x to $\mathbf{r}(s)$. Assuming the typical tanh form for the z component of the magnetization profile across the DW, i.e., $\theta(\rho) = 2 \arctan[\exp(-\rho/D)]$ with D the DW width, we can write $\mathcal{L}$ and F in terms of $\mathbf{r}$, ϕ , and D by integrating over ρ , from which one can derive dynamical equations. By choosing the parametrization $\mathbf{r}(x, t) = x\mathbf{e}_x + h(x, t)\mathbf{e}_y$ and assuming small slopes $\partial h/\partial x$ and a constant $D = \sqrt{A_{\text{ex}}/(K_u - \frac{1}{2}\mu_0 M_s^2)}$, we obtain coupled dynamical equations for $h(x, t)$ and $\phi(x, t)$:

$$\dot{\phi} + \alpha_G \frac{\dot{h}}{D} = 2 \frac{\gamma A_{\text{ex}}}{M_s D} h'' - \gamma B_a, \quad (1)$$

$$\alpha_G \dot{\phi} - \frac{\dot{h}}{D} = 2 \frac{\gamma A_{\text{ex}}}{M_s} \phi'' - \frac{\gamma N_n}{2} \mu_0 M_s [2(\phi - \chi)], \quad (2)$$

where primes denote differentiation with respect to x , and $\chi = \arctan h'$ is the angle of the DW with respect to a flat, horizontal configuration. The applied field $B_a(\mathbf{r}) = B_{\text{ext}} + \eta(\mathbf{r})$

includes quenched disorder $\eta(\mathbf{r})$ modeled as a random out-of-plane magnetic field, where the disorder is drawn from a normal distribution with mean 0 and standard deviation σ , i.e., the disorder strength. The disorder field $\eta(\mathbf{r})$ is short-range correlated, $\langle \eta(\mathbf{r})\eta(\mathbf{r}') \rangle = \sigma^2 \exp(-|\mathbf{r} - \mathbf{r}'|^2/\xi^2)$, with ξ the correlation length, and is implemented by convolving an uncorrelated array of random numbers with a Gaussian kernel. Linear interpolation is employed in the h direction to compute the value of η at a given point $h(x)$. We notice that when setting the spatial derivatives to zero, Eqs. (1) and (2) reduce to those of the one-dimensional model [34]. On the other hand, when neglecting the internal degrees of freedom (i.e., by setting $\dot{\phi} = 0$ and $\phi = \chi$), the model becomes essentially the quenched Edwards-Wilkinson (qEW) equation, $\dot{h} = \Gamma_0 h'' + \eta + F_{\text{ext}}$, with Γ_0 the interface stiffness and F_{ext} the external driving force [35–37], i.e., a simple linear elastic interface driven through a disordered medium with an external force.

To solve Eqs. (1) and (2) numerically, we define the complex quantity $z = h - i\phi$. Measuring length in units of D , time in units of $1/\gamma\mu_0 M_s$ and magnetic field in units of $\mu_0 M_s$, Eqs. (1) and (2) can be written in the form

$$(\alpha_G + i)\dot{z} = \tilde{K}z'' - B_a + i(N_n/2)\sin[2(\phi - \chi)], \quad (3)$$

where $\tilde{K} = [(2K_u - \mu_0 M_s^2)/(\mu_0 M_s^2)]$. Considering periodic boundary conditions along x , time integration is implemented with the classical fourth-order Runge-Kutta method with a constant time step of $1/(2\gamma\mu_0 M_s)$. The solver is written in C++ and parallelized to run on several CPU cores. During the simulations, we monitor the spatially averaged DW velocity $v_{\text{DW}} = \frac{1}{L} \int_L \dot{h} dx$, and adjust the external driving field B_{ext} in a way mimicking the quasistatic constant velocity driving protocol, aimed at ensuring that subsequent avalanches are not overlapping in time. When v_{DW} is lower than a small threshold value $v_{\text{DW}}^{\text{th}}$, B_{ext} is slowly ramped up until v_{DW} exceeds $v_{\text{DW}}^{\text{th}}$. When $v_{\text{DW}} > v_{\text{DW}}^{\text{th}}$, the magnitude of B_{ext} is decreased according to $|\dot{B}_{\text{ext}}| = -kv_{\text{DW}}$. Physically, the parameter k can be understood to be proportional to the magnitude of the demagnetizing field [11] and has the effect of restricting the maximum size of the avalanches. Here, it is chosen so that the lateral extent of the largest observed avalanches stays well below L to avoid finite-scale effects.

The model parameters are set to correspond to a 0.5 nm thick Co film within a Pt/Co/Pt multilayer [38]: $K_u = 8.4 \times 10^5$ J/m³, $A_{\text{ex}} = 1.4 \times 10^{-11}$ J/m, $M_s = 9.1 \times 10^5$ A/m, and $\alpha = 0.27$. The disorder correlation length is set to $\xi = 37$ nm. The DW is discretized using a resolution of 6 nm $\approx D$ along the x axis. The system size $L = 600$ μm is chosen to maximize L with the constraint that it should still be computationally feasible to collect good enough statistics in the simulations. Strikingly, the achieved L is approximately two orders of magnitude larger than what is typically possible in conventional micromagnetic simulations [26].

To test the possible disorder dependence of Barkhausen noise in our model, we perform simulations with three different disorder strengths, $\sigma = 3, 7$, and 10 mT. The ramp rates of B_{ext} in-between avalanches are chosen to be 1.9, 3.7, and 4.8 T/s for the three σ values, respectively. The corresponding values for k used in our simulations are $k = 0.054, 0.18$, and 0.31 T/mm. For all simulations, the threshold value of

$v_{\text{DW}}^{\text{th}} = 0.002$ m/s is considered. While we focus here on the quasistatic driving protocol discussed above, the Appendix shows example simulations where a sinusoidally oscillating external field is considered, resulting in hysteresis loops displaying steps corresponding to Barkhausen jumps.

After an initial transient, the time-dependent external field $B_{\text{ext}}(t)$ fluctuates around the critical field B_c of the depinning transition [30]. To speedup the approach toward the steady state, we initialize the external field to a value close to B_c , i.e., $B_{\text{ext}}(0) \approx B_c$. We then discard the first 200 μs of the simulation to remove the remaining initial transient. In order to obtain enough statistics, we collect data from several simulation runs, such that the total duration of the Barkhausen noise signal is 20, 60, and 60 ms for $\sigma = 3, 7$, and 10 mT, respectively.

In order to analyze the overall spin rotation activity in the system with contributions from both DW propagation and the internal dynamics within the DW, we define the *total activity* A_{tot} :

$$A_{\text{tot}} = \frac{C}{L} \int_0^L \int_{-\infty}^{+\infty} |\dot{\mathbf{m}}| d\rho ds \approx \frac{1}{L} \int_0^L \sqrt{\dot{h}^2 + D^2 \dot{\phi}^2} dx, \quad (4)$$

where C is an arbitrary scaling constant. Full micromagnetic simulations suggest that in the precessional regime the internal DW dynamics can give a significant contribution to A_{tot} , sometimes even exceeding that related to domain wall propagation [26]. On the other hand, below the Walker breakdown (i.e., for $\dot{\phi} = 0$), A_{tot} as defined in Eq. (4) coincides with the spatially averaged DW velocity v_{DW} . The $A_{\text{tot}}(t)$ signal is then used to define avalanches, such that their sizes are taken to be

$$S_{A_{\text{tot}}} = \int_0^{T_{A_{\text{tot}}}} [A_{\text{tot}}(t) - A_{\text{tot}}^{\text{th}}] dt, \quad (5)$$

where $A_{\text{tot}}^{\text{th}}$ is an activity threshold and the integral is over the avalanche duration $T_{A_{\text{tot}}}$, i.e., the time period during which $S_{A_{\text{tot}}}$ continuously exceeds $A_{\text{tot}}^{\text{th}}$. For avalanches extracted from the spatially averaged DW velocity signal $v_{\text{DW}}(t)$ and those defined from the in-plane internal activity signal, $A_{\text{xy}}(t) = 1/L \int_0^L |\dot{\phi}| dx$ (see the Supplemental Material in Ref. [39]).

III. RESULTS

Figure 2 shows an example of an individual avalanche in the intermediate-disorder case of $\sigma = 7$ mT extracted using $A_{\text{tot}}^{\text{th}} = 0.002$ m/s. The top panel shows both the magnetization configuration of the DW after the avalanche and the DW position before the avalanche of duration $T_{A_{\text{tot}}} = 3.7$ μs , with the color in the former indicating the internal in-plane magnetization within the DW. Notice the presence of several Bloch lines within the DW, visible as relatively sharp transition regions between red and cyan, corresponding to $\phi = 0$ and $\phi = \pi$, respectively. The DW position before the avalanche is shown as the dashed black line, with the difference between the two DW configurations illustrating the area swept over by the DW in the avalanche. The bottom panel of Fig. 2 shows the $A_{\text{tot}}(t)$ signal of the same avalanche, displaying the typical irregular shape of critical avalanches. An animation of a similar avalanche for $\sigma = 7$ mT illustrating Bloch line dynamics during the avalanche is included as Supplemental Material [39].

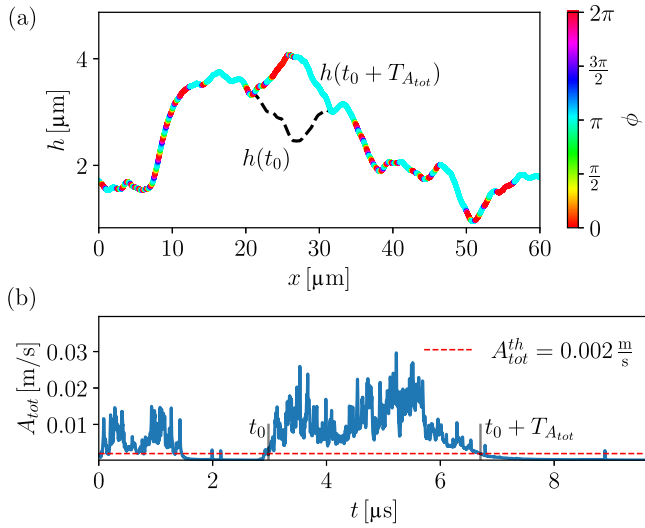


FIG. 2. (a) Part of a DW before (dashed black line) and after (solid line with color indicating the local in-plane magnetization within the DW) an avalanche of duration $T_{A_{tot}} = 3.7 \mu\text{s}$ and size $S_{A_{tot}} = 3.7 \times 10^{-8} \text{ m}$, extracted from the $A_{tot}(t)$ signal shown in panel (b) with a threshold value $A_{tot}^{th} = 0.002 \text{ m/s}$. The disorder strength considered is $\sigma = 7 \text{ mT}$.

The Bloch line dynamics within the intermittently moving DW is further illustrated in Fig. 3, showing space-time plots of the ϕ variable for $\sigma = 7 \text{ mT}$ in panel (a) and 10 mT in panel (b), considering a uniform initial state with $\phi = 0$ at $t = 0$. For both $\sigma = 7$ and 10 mT , the system quickly nucleates several pairs of Bloch lines, again visible as transition regions between $\phi = 0$ and $\phi = \pi$, with the average Bloch line density a little larger in the $\sigma = 10 \text{ mT}$ case [Fig. 3(b)]. The resulting spatiotemporal pattern is quite rich, with pairs of Bloch lines repeatedly nucleating, propagating, and annihilating within the moving DW. In contrast, for $\sigma = 3 \text{ mT}$ (not shown), the corresponding plot would in a typical case remain uniformly red, as the disorder is not strong enough to nucleate Bloch lines. Instead, the low depinning field due to the weak disorder of $\sigma = 3 \text{ mT}$ is below the Walker field, and no precessional DW dynamics is observed as the DW is driven by B_{ext} fluctuating around the small B_c . Hence, in that case $A_{tot}(t) \approx v_{\text{DW}}$, i.e., the DW propagates without internal degrees of freedom getting excited, and the model described by Eqs. (1) and (2) reduces to the qEW equation.

A. Avalanche distributions

The avalanche size distributions $P(S_{A_{tot}})$ for the three σ values are shown in the left column of Fig. 4, considering in each case three different threshold values A_{tot}^{th} , i.e., $A_{tot}^{th} = 0.001$, 0.002 , and 0.004 m/s . The solid lines are fits of the form

$$P(S_{A_{tot}}) = A S_{A_{tot}}^{-\tau} f(S_{A_{tot}}/S_{A_{tot}}^*), \quad (6)$$

where A is a normalization constant, τ is the avalanche size exponent, $f(x)$ is a scaling function that defines the shape of the large-avalanche cutoff, and $S_{A_{tot}}^*$ is the cutoff avalanche size. To capture the slight “bump” present in the cutoffs of the distributions [40], we consider the scaling function

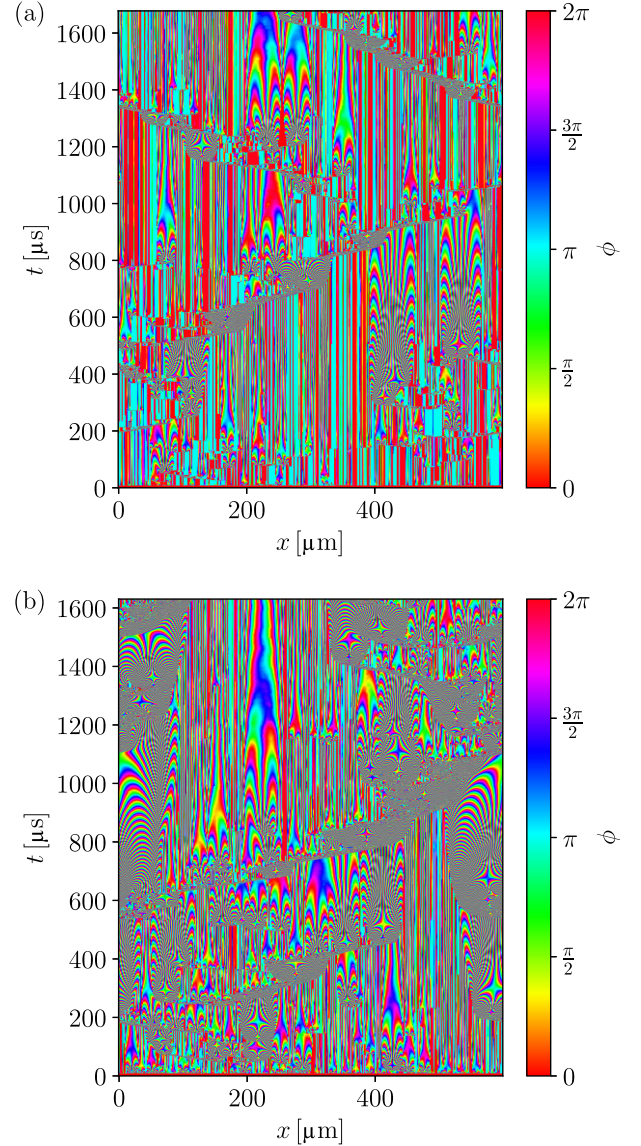


FIG. 3. Space-time maps of $\phi(x, t)$ for (a) $\sigma = 7 \text{ mT}$ and (b) $\sigma = 10 \text{ mT}$, both shown over the time period of one individual simulation run.

$f(x) = \exp(B\sqrt{x} - Cx^\delta)$ in Eq. (6) [41], with B , C , and δ treated as fitting parameters.

For all size distributions, a scaling regime spanning at least two orders of magnitude in $S_{A_{tot}}$ can be found. This scaling regime is limited on the small- $S_{A_{tot}}$ side by the disorder correlation length ξ as avalanches with their spatial extent below ξ do not exhibit scaling, while the large- $S_{A_{tot}}$ cutoff $S_{A_{tot}}^*$ is controlled by k . For the weak disorder of $\sigma = 3 \text{ mT}$, considering $A_{tot}^{th} = 0.002 \text{ m/s}$, which coincides with $v_{th} = 0.002 \text{ m/s}$ used to control the external magnetic field in the driving protocol employed, we find $\tau = 1.10 \pm 0.10$ [indicated by the black dashed line in Fig. 4(a)], which is close to the literature value for the qEW equation, $\tau_{\text{qEW}} \approx 1.11$ [41]. This is in agreement with the observation of the absence of Bloch line dynamics within the DW for $\sigma = 3 \text{ mT}$, implying that the DW behaves as a simple elastic line without internal degrees of freedom.

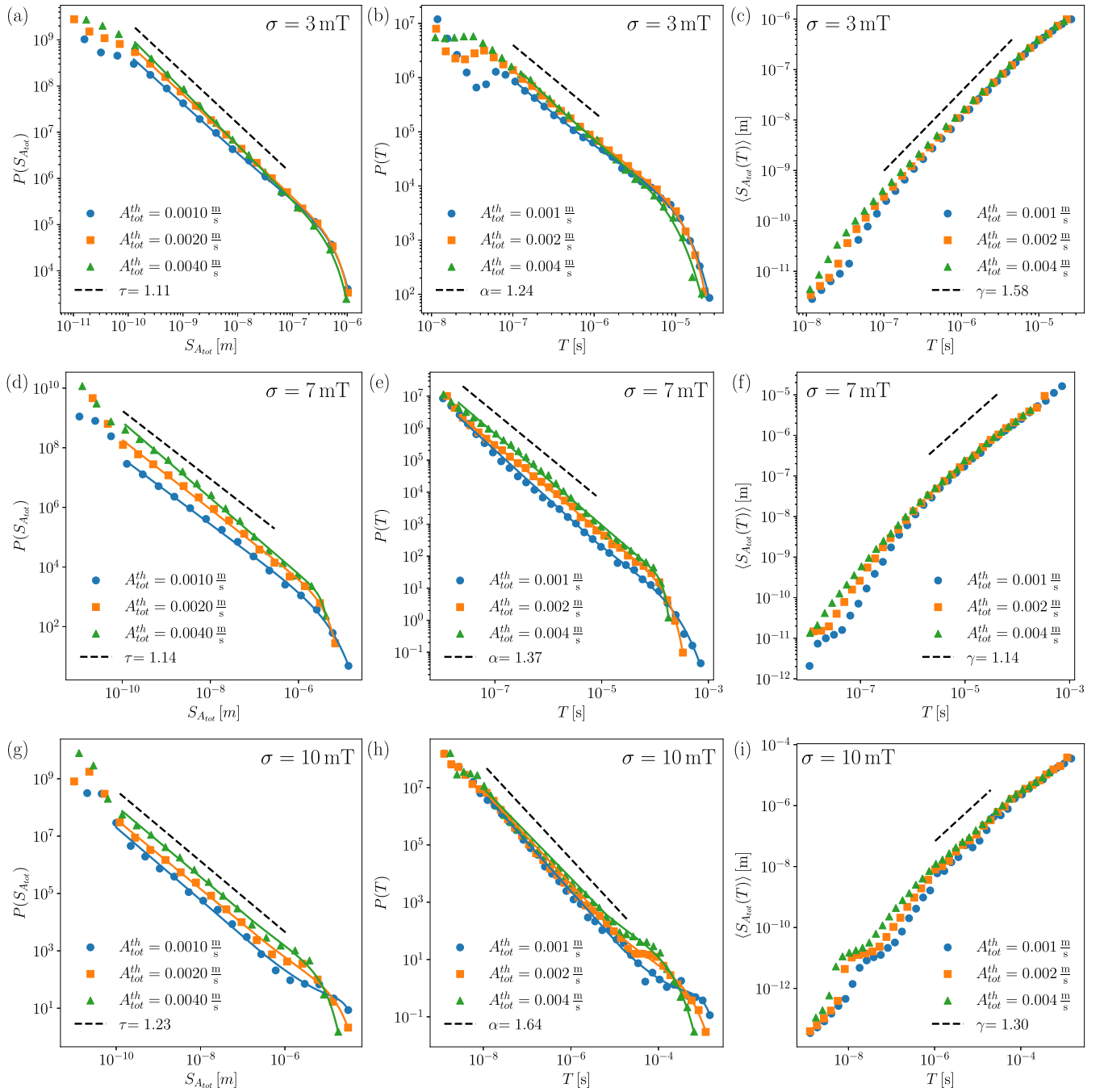


FIG. 4. Distributions of avalanche sizes $P(S_{A_{tot}})$, avalanche durations $P(T)$, and scaling of the average avalanche size with the duration, $\langle S_{A_{tot}}(T) \rangle$, for the three different σ values. Panels (a), (b), and (c) show $P(S_{A_{tot}})$, $P(T)$, and $\langle S_{A_{tot}}(T) \rangle$ for $\sigma = 3$ mT, respectively, while panels (d), (e), and (f) show the same data for $\sigma = 7$ mT, and panels (g), (h), and (i) for $\sigma = 10$ mT. In each case, data for three different threshold values A_{tot}^{th} are shown (as indicated by the legends), with solid lines corresponding to fits of Eq. (6) for the $P(S_{A_{tot}})$'s and fits of Eq. (6) with the substitutions $S_{A_{tot}} \rightarrow T$, $\tau \rightarrow \alpha$, and $f(x) \rightarrow g(x)$ for the $P(T)$'s. Slopes of the black dashed lines indicate the power law exponents obtained from the fits for $A_{tot}^{th} = 0.002$ m/s.

Considering $A_{tot}^{th} = 0.001$ m/s leads to a slightly smaller τ , likely due to merging of subsequent avalanches due to the low A_{tot}^{th} value. For systems with larger σ values where the internal degrees of freedom are excited and Bloch line dynamics takes place within the DW, we observe slightly larger values of τ increasing with σ , with the estimated values for $A_{tot}^{th} = 0.002$ m/s being $\tau = 1.14 \pm 0.10$ and $\tau = 1.23 \pm 0.10$ for $\sigma = 7$ and 10 mT, respectively [dashed black lines

in Figs. 4(d) and 4(g)]. We then proceed to analyze the avalanche duration distributions, using from now on the notation $T \equiv T_{A_{tot}}$ for the avalanche duration for simplicity. The distributions $P(T) \propto T^{-\alpha} g(T/T^*)$ are shown in the middle column of Fig. 4. In the fits (solid lines), we again consider the scaling function $g(x)$ of the form of $g(x) = \exp(B'\sqrt{x} - C'x^{\delta'})$, with B' , C' , and δ' the fitting parameters. Compared to the size distributions, the $P(T)$'s

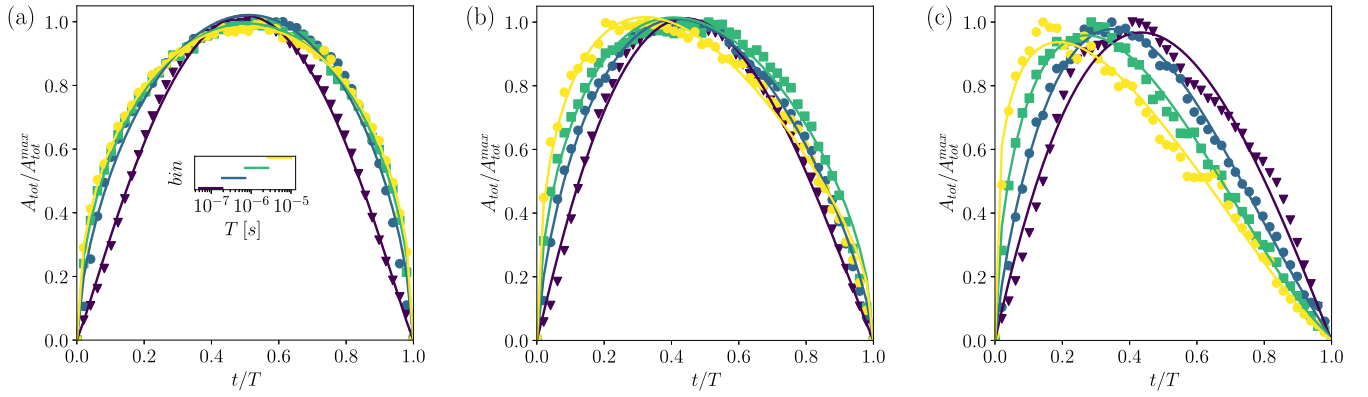


FIG. 5. Average temporal avalanche shapes $\langle A_{\text{tot}}(t/T) \rangle$ for different values of σ , with $\sigma = 3, 7$, and 10 mT in panels (a), (b), and (c), respectively. The solid lines show fits of the form of Eq. (7), with the fitting parameters reported in Fig. 6. The average shapes are calculated for separate duration bins indicated in the inset of panel (a).

exhibit a somewhat narrower scaling range (as is typical for critical avalanches), and a more clear-cut separation in the values of the scaling exponent α for the different σ values: We find $\alpha = 1.24 \pm 0.10$ (in good agreement with the qEW value of $\alpha \approx 1.19$ [41]), $\alpha = 1.37 \pm 0.10$, and $\alpha = 1.64 \pm 0.10$ for $\sigma = 3, 7$, and 10 mT, respectively [dashed black lines illustrating the α values obtained for $A_{\text{tot}}^{\text{th}} = 0.002$ m/s in Figs. 4(b), 4(e), and 4(h)]. We note that both $P(S_{A_{\text{tot}}})$ and $P(T)$ develop pronounced bumps in their cutoffs in the limit of large σ , suggesting the presence of an additional mechanism limiting the avalanche growth and inducing a characteristic scale in the strong-disorder regime, which is different from the simple cutoff mechanism due to the k parameter described above.

We then proceed to analyze the scaling of the average avalanche size $\langle S_{A_{\text{tot}}}(T) \rangle$ with the avalanche duration T , shown in the right column of Fig. 4. One expects scaling of the form $\langle S_{A_{\text{tot}}}(T) \rangle \propto T^\gamma$, where γ is usually found to obey the scaling relation $\gamma = (\alpha - 1)/(\tau - 1)$. Figure 4(c) shows that for $\sigma = 3$ mT, one finds a scaling regime with $\gamma = 1.58 \pm 0.10$, which satisfies the scaling relation reasonably well considering the τ and α values found above for $\sigma = 3$ mT, and is in good agreement with the known qEW value $\gamma_{\text{qEW}} \approx 1.56$ [8]. However, considering the larger σ values, the situation becomes less clear: The obtained values of γ , extracted from rather narrow scaling ranges, are smaller than that for $\sigma = 3$ mT, i.e., $\gamma \approx 1.14$ [Fig. 4(f)] and $\gamma \approx 1.3$ [Fig. 4(i)] for $\sigma = 7$ and 10 mT, respectively. Moreover, those values do not satisfy the scaling relation $\gamma = (\alpha - 1)/(\tau - 1)$. This suggest a breakdown of “simple” avalanche scaling in the presence of strong disorder, something we will explore in more detail below.

B. Average temporal avalanche shapes

In addition to the avalanche distributions studied above, the average temporal avalanche shape $\langle A_{\text{tot}}(t/T) \rangle$ is another key quantity often used to characterize avalanches. These shapes, obtained by calculating the mean A_{tot} as a function of normalized time t/T for four different duration bins to examine the evolution of the shape, are shown in Fig. 5. The avalanches are extracted from the $A_{\text{tot}}(t)$ signal considering $A_{\text{tot}}^{\text{th}} = 0.002$ m/s,

and normalized with their maximum values. To characterize the average avalanche shapes, we fit them with the function

$$\left\langle A_{\text{tot}}\left(\frac{t}{T}\right)\right\rangle \propto \left[\frac{t}{T}\left(1 - \frac{t}{T}\right)\right]^{\gamma-1} \left[1 - a\left(\frac{t}{T} - \frac{1}{2}\right)\right], \quad (7)$$

where γ is the scaling exponent characterizing also the scaling of $\langle S_{A_{\text{tot}}}(T) \rangle$ with T and a quantifies the magnitude of the temporal asymmetry of the shape [8].

For $\sigma = 3$ mT [Fig. 5(a)], $\langle A_{\text{tot}}(t/T) \rangle$ is almost symmetric (i.e., $a \approx 0$), and the shapes of the different duration bins collapse on top of each other with the exception of the shortest duration bin where the avalanches are affected by the disorder correlation length ξ . Also, the values of γ obtained from the fits are close to that found above from the size versus duration scaling, again with the exception of the shortest duration bin (see Fig. 6). Within error bars, these features all agree with the known results for the qEW equation [8].

Interestingly, for the two cases with larger σ , the avalanche shapes are clearly asymmetric in time, with $a > 0$ [Figs. 5(b) and 5(c)]. After the avalanche is triggered, it quickly reaches its peak activity followed by a slower decay of spin rotation activity. This is accompanied by an effective, T -dependent value of γ [see again Fig. 6(a)]. Moreover, the skewness of the shapes [and hence the asymmetry parameter a , see Fig. 6(b)] increases with T for both $\sigma = 7$ mT and especially $\sigma = 10$ mT. One should note that Eq. (7) assumes a small

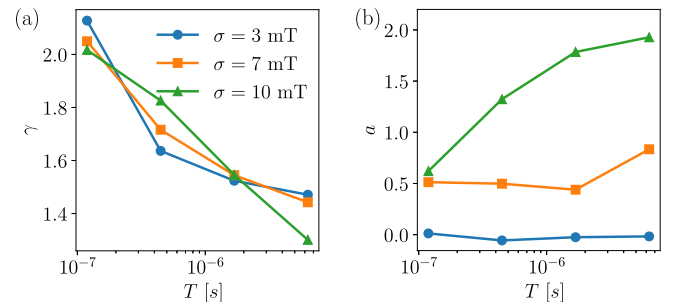


FIG. 6. Fitting parameters (a) γ and (b) a of the function of Eq. (7) obtained from the fits shown as solid lines in Fig. 5.

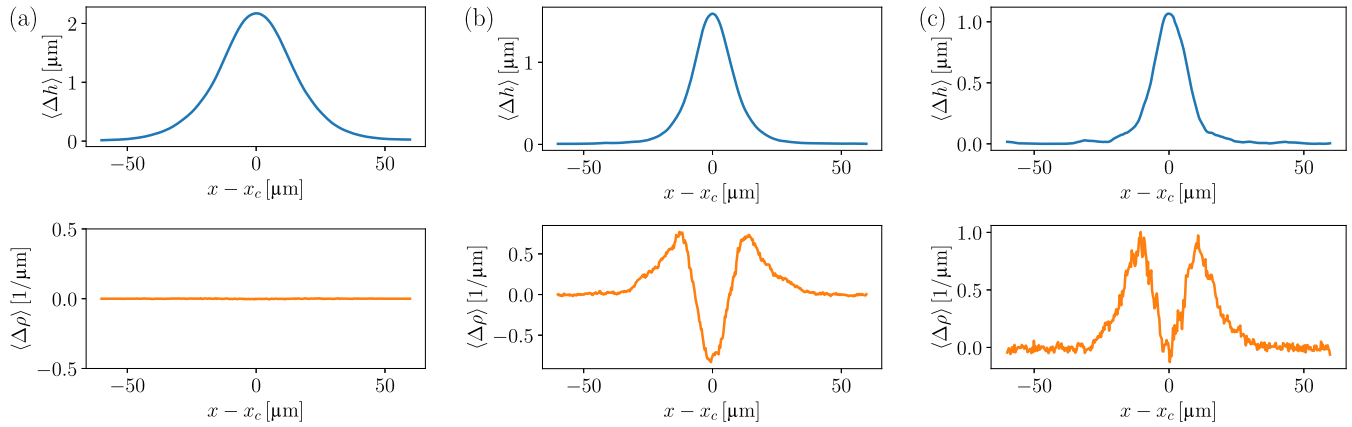


FIG. 7. Top panels: The average spatial avalanche propagation $\langle \Delta h \rangle$ as a function of the distance from the avalanche center, $x - x_c$, for (a) $\sigma = 3$ mT, (b) $\sigma = 7$ mT, and (c) $\sigma = 10$ mT. Bottom panels: The corresponding average change of the Bloch line density, $\langle \Delta \rho \rangle$, as a function of $x - x_c$. The plots show averages over avalanches that have durations $T \in [2.7 \times 10^{-6} \text{ s}, 1 \times 10^{-5} \text{ s}]$.

skewness, and hence the rather large skewness observed for long avalanches and strong disorder results in somewhat distorted estimates of the fitting parameters together with fits that do not perfectly fit the data. Nevertheless, these results give a strong indication that for strong disorder, there seems to be a disorder-dependent mechanism that slows down the avalanche propagation once the avalanche has grown large enough, and that this effect is stronger for avalanches with a larger T .

C. Average spatial avalanche shapes and Bloch line accumulation

In an attempt to understand the features of the avalanche dynamics discussed above, we examine the effect of Bloch lines on Barkhausen avalanches more closely and analyze the spatial shapes of the avalanches. Here, the spatial shape of an avalanche is defined as follows: The width w of an avalanche is taken to be the width of the region where the change of $h(x)$ during the avalanche, denoted by Δh , is greater than a threshold value $\Delta h_{\text{th}} = 10$ nm. The center of an avalanche, x_c , is defined as the point halfway between its edges. The average spatial shape of the avalanche is determined as a mean of Δh as a function of the distance from the center of the avalanche, $\langle \Delta h(x - x_c) \rangle$. The resulting spatial shapes are shown in the top panels of Fig. 7, averaged over avalanches with duration $T \in [2.7 \times 10^{-6} \text{ s}, 1 \times 10^{-5} \text{ s}]$, corresponding to the longest duration bin of the four bins used in the temporal shape plots in Fig. 5. All shapes resemble a Gaussian, which, however, gets more narrow upon increasing σ , suggesting the presence of a disorder-induced mechanism limiting the lateral growth of avalanches.

The bottom panels of Fig. 7 show the corresponding average spatial change of the Bloch line density $\langle \Delta \rho(x - x_c) \rangle$. The density ρ of Bloch lines before and after an avalanche is determined from the spatial derivative of ϕ via $\rho \approx |\phi'|/\pi$. The system with $\sigma = 3$ mT serves again as a reference without Bloch lines and thus without changes in ρ during avalanches [bottom panel of Fig. 7(a)]. For larger values of σ [$\sigma = 7$ and 10 mT, shown in the bottom panels of Figs. 7(b) and 7(c), respectively], we observe significant space-dependent changes

in ρ during the avalanches. The shapes of the $\langle \Delta \rho(x - x_c) \rangle$ curves differ distinctly from the spatial shape curves in the top panels of Fig. 7, with two peaks of $\langle \Delta \rho(x - x_c) \rangle$ on the two sides of the avalanche. This occurs even though the nucleation of Bloch lines and the movement of the DW are closely linked, with Bloch lines nucleating in areas where the DW advances most rapidly. However, after nucleation, Bloch lines seem to have the tendency to propagate toward the two edges of the avalanche, forming two pileups of Bloch lines, visible as the two peaks in $\langle \Delta \rho(x - x_c) \rangle$. For $\sigma = 7$ mT, the effect of Bloch lines propagating to the edges of the avalanche dominates so that in the middle of the avalanche a region is created where ρ on the average even decreases during avalanche propagation. For $\sigma = 10$ mT, on the other hand, the increased nucleation rate of Bloch lines during avalanche propagation due to stronger disorder results in disappearance of the above-mentioned effect, such that ρ stays on the average roughly unchanged in the middle of the avalanche around $x = x_c$. The two peaks in $\langle \Delta \rho(x - x_c) \rangle$ at the edges of the avalanche are even more prominent for $\sigma = 10$ mT than for $\sigma = 7$ mT.

Finally, we analyze the scaling of the aspect ratio of the spatial avalanche shapes. Figure 8 shows the scaling of the average avalanche height $\langle \Delta h \rangle$ as a function of the avalanche width w (considering now an average over x within a window of width w as a function of w irrespective of T) on the log-log scale. The dashed lines in Fig. 8 indicate scaling of the form of

$$\langle \Delta h \rangle \sim w^\zeta, \quad (8)$$

where ζ is the roughness exponent indicated in the legend. Small avalanches appear to scale with a roughly similar ζ . In other words, the aspect ratio $\langle \Delta h \rangle/w$ of avalanches in systems with different σ 's differs only by a constant factor for small avalanches. The values of ζ in this small avalanche regime are close to the value $\zeta_{\text{qEW}} = 1.25$ expected for the qEW equation [37,42].

However, on larger scales the systems with larger values of σ exhibit a different behavior. Figure 8 suggests that there is a crossover scale $w_c \approx 10 \mu\text{m}$. Above that scale, avalanches in the intermediate-disorder system with $\sigma = 7$ mT seem

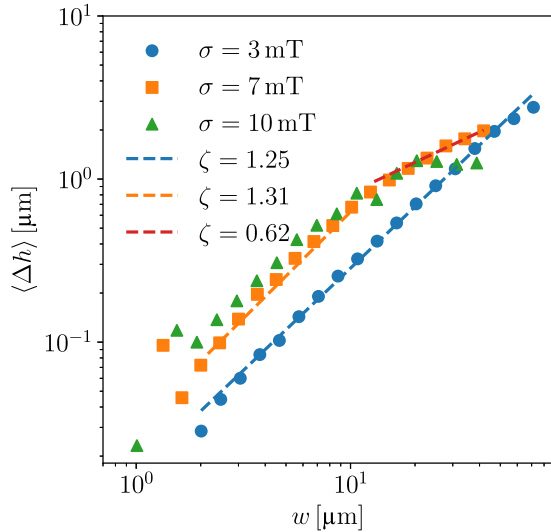


FIG. 8. Scaling of the average height $\langle \Delta h \rangle$ of an avalanche as a function of its width w . Dashed lines indicate fits of the scaling relation described in Eq. (8). The corresponding estimates for the roughness exponent ζ are shown in the legend.

to scale with a significantly smaller ζ , in agreement with results in Ref. [30]. Similar conclusions apply in the strongest-disorder system ($\sigma = 10\text{ mT}$), even if there is not enough statistics for an accurate estimate of ζ on the largest scales.

Overall, our results suggest that while weakly disordered systems exhibit the usual qEW scaling behavior of DWs essentially free of Bloch lines, strong disorder facilitates nucleation of pairs of Bloch lines during avalanche propagation. Subsequently, for large enough avalanches, these Bloch lines tend to pile up around the edges of the avalanche, hence limiting further lateral propagation of the avalanche above a scale w_c , which for our system is roughly $w_c \approx 10\text{ }\mu\text{m}$. Indeed, it is well known that the presence of Bloch lines within the DW affects its movement by slowing the DW down via the Walker breakdown mechanism [27]. This has a number of consequences for the observed avalanche statistics in the strong-disorder regime: (1) The avalanche size and duration distributions $P(S_{\text{A}_{\text{tot}}})$ and $P(T)$ for $\sigma = 10\text{ mT}$ both develop pronounced bumps around the cutoffs, likely due to the Bloch line accumulation effect limiting the growth of avalanches and inducing a characteristic scale to the avalanche dynamics. (2) The temporal average avalanche shapes especially for long avalanches exhibit clear skewness, which we argue is linked to the same effect via the Bloch line accumulation acting predominantly at the later stages of avalanche propagation due to its cumulative nature. The effect intensifies with increasing T as more Bloch lines accumulate to the pileups, resulting in an asymmetry increasing with T . (3) The crossover in DW roughness where the small-scale qEW scaling crosses over to a smaller ζ for scales exceeding w_c is probably due to the same effect, given that the Bloch line pileup mechanism damping the roughness growth operates only above w_c , implying that the qEW scaling remains valid for small length scales, i.e., for $w < w_c$.

IV. DISCUSSION AND CONCLUSIONS

To summarize, we have successfully demonstrated a simulation approach of Barkhausen noise using a reduced micromagnetic model of DW motion in thin films with PMA. Contrary to commonly used elastic interface models, the model used here includes internal degrees of freedom via an angular variable $\phi(x)$, which makes it possible to describe Bloch line dynamics along the intermittently moving DW. By employing the reduced model, we were able to simulate $600\text{ }\mu\text{m}$ long DWs, i.e., system sizes that are two orders of magnitude larger than what has been achieved in conventional micromagnetic simulations [26].

Considering different disorder strengths σ in the model, it is possible to probe different regimes of jerky DW propagation. For weak disorder, the small depinning field is below the Walker field, and hence the internal degrees of freedom are not excited. As a result, the linelike DW behaves similarly to a simple elastic interface in a random medium, i.e., the qEW equation. Stronger disorder facilitates nucleation of pairs of Bloch lines within the moving DW, which then exhibit rich dynamics consisting of repeated nucleation, propagation, and annihilation of Bloch lines within the DW moving in the precessional regime. The excitation of Bloch lines was found to influence the statistical properties of Barkhausen noise, with the related scaling exponents and avalanche shapes diverging from those observed in systems with weaker disorder and no Bloch lines. These findings imply that the often-used approximation of DW as a simple elastic interface is not universally valid. These disorder-dependent properties suggest that further related investigations may be worthwhile, including exploring whether the model exhibits a disorder-driven phase transition [43,44].

A more detailed analysis of Bloch line dynamics in the strong-disorder regime reveals that the Bloch lines nucleated within the moving parts of the DW during an avalanche tend to migrate toward and pile up close to the edges of the avalanche, hence limiting its further propagation. Thus, Bloch lines tend to damp the growth of large avalanches, something that is here visible in both dynamical and static properties of the DWs. The former include the avalanche size and duration distributions with disorder-dependent exponents and pronounced bumps in the cutoffs of the distributions and the clearly asymmetric average temporal avalanche shapes for large σ . For the latter, the roughness of the DW is characterized by a smaller roughness exponent in the large-scale strong-disorder regime dominated by Bloch lines, due to the damping of large avalanches responsible for roughness growth on large scales. From the point of view of the asymmetric temporal avalanche shapes, the damping mechanism due to Bloch lines discussed here exhibits some qualitative similarities with damping due to eddy currents in conducting ferromagnets, another effect that has been shown to give rise to temporally asymmetric average avalanche shapes [45]. We also point out that the fact that no clear disorder dependence of the exponents was found in recent micromagnetic simulations [26] suggests that the damping mechanism due to Bloch line accumulation discussed above requires larger avalanches and hence larger system sizes than those accessible in micromagnetic simulations. Indeed, the roughness exponents of Fig. 8 for

all σ values considered are consistent with that of the qEW equation for scales below $w_c \approx 10 \mu\text{m}$. This can be compared with the maximum system size of $L \approx 4 \mu\text{m}$ considered in Ref. [26]. On the other hand, the large-scale roughness exponent $\zeta \approx 0.62$ observed here has a value close to $\zeta_{\text{qKPZ}} \approx 0.63$ expected for the quenched Kardar-Parisi-Zhang (qKPZ) equation [i.e., the qEW equation amended with a nonlinear term $\lambda(h')^2$] [35,46,47] and also agrees with typical experimental observations [48]. While this may suggest an interpretation in terms of an effective nonlinearity in the elasticity of the DW emerging from the Bloch line accumulation mechanism, one should note that the avalanche exponents appear to be somewhat different from the qKPZ ones (see also the Supplemental Material [39]). Nevertheless, our observation could explain why the super-rough qEW behavior is often not observed in experiments, with the exception of the low-temperature experiments of Ref. [49] where a $\zeta > 1$ was found via scaling relations.

We finish by presenting three final remarks: First, our results emphasize the importance of going beyond the simple elastic interface models of extended DWs by including a description of their internal degrees of freedom, which have here proven to play a crucial role in determining the statistical properties of the Barkhausen avalanches. Second, our findings call for experiments where Barkhausen noise is studied in settings allowing for precise control of the disorder strength [50] and hence Bloch line nucleation, to probe both the “elastic interface regime” for weak disorder and the precessional Bloch line dominated regime for strong disorder. A more challenging direction would be to try to image the moving DWs with nanometer resolution, something that would make it possibly to directly observe the Bloch line dynamics within the DW, including the accumulation mechanism discussed above. This is likely to be easier in materials such as garnets where the DW widths are significantly larger [51,52]. Finally, the model considered here and originally developed in Ref. [30] could be generalized in a number of different ways, including different types of DWs in 2D systems (e.g., linelike DWs separating in-plane magnetized domains in thin films), considering membranelike DWs in 3D ferromagnets with internal degrees of freedom [28], including more realistic disorder fields interacting with the DW [53], and incorporating additional features like the Dzyaloshinskii-Moriya interaction [54,55].

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DATA AVAILABILITY

The data that support the findings of this article are openly available [56].

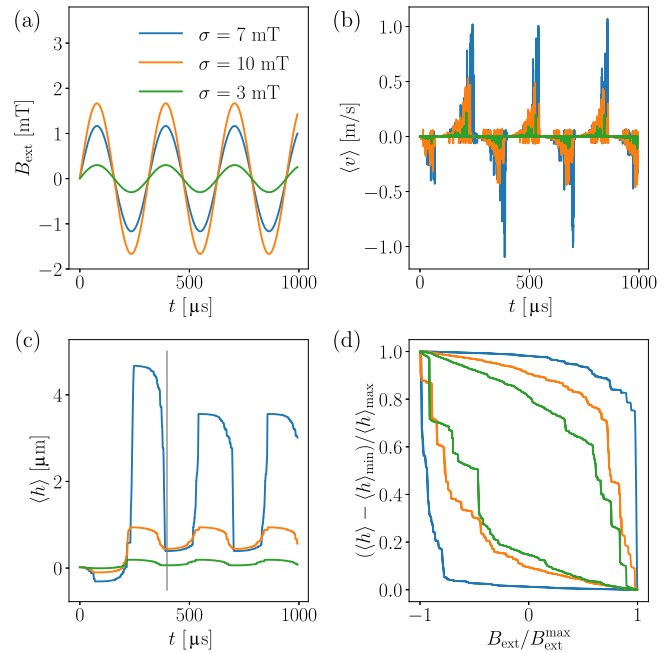


FIG. 9. Domain wall ($L = 60 \mu\text{m}$) under oscillating external field $B_{\text{ext}}(t)$ [panel (a)], mean velocity $\langle v \rangle(t)$ [panel (b)], mean displacement $\langle h \rangle(t)$ [panel (c)], and normalized hysteresis curves $\langle h \rangle(B_{\text{ext}})$ [panel (d)]. The hysteresis curves are constructed from the data after an initialization period of $410 \mu\text{s}$ indicated by the vertical line in panel (c). The simulations are made for the three different disorder strengths labeled in panel (a).

APPENDIX: HYSTERESIS DUE TO OSCILLATING FIELD

In addition to the approach of generating Barkhausen noise by the quasistatic external field protocol as described above, the DW model exhibits avalanche dynamics also when driven by an oscillating magnetic field. This is illustrated in Fig. 9, where we consider DWs with $L = 60 \mu\text{m}$ subject to a sinusoidal external field $B_{\text{ext}}(t) = B_{\text{ext}}^{\text{max}} \sin(2\pi t/T)$ with a period of $T = 315 \mu\text{s}$, chosen to be long enough to make the individual Barkhausen jumps of the DW visible as a staircaselike structure of the hysteresis loop. We consider here the same three disorder strength values as above, i.e., $\sigma = 3, 7$, and 10 mT , with the corresponding external field amplitudes chosen to be $B_{\text{ext}}^{\text{max}} = 0.3, 1.17$, and 1.67 mT , respectively [Fig. 9(a)]. The field amplitudes are chosen so that $B_{\text{ext}}(t)$ stays well below the depinning field for all three σ values. The resulting DW dynamics displays the typical experimental signatures of Barkhausen noise, i.e., bursty DW velocity signals [Fig. 9(b)] and staircaselike steps in the hysteresis loops corresponding to the Barkhausen avalanches [Fig. 9(d)]. Interestingly, in these zero-temperature simulations, after an initial transient [indicated by the vertical line in Fig. 9(c)], the DW dynamics appear in each case to lock onto a single irregular hysteresis loop shown in Fig. 9(d).

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