

Optical interference by amplitude measurement: Spectral analysis

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Optical interference usually requires two interfering paths to be balanced within the coherence length. For the case of unbalanced paths, interference effects can be recovered in the frequency-domain analysis of the superposed resultant optical field because optical frequency filtering in the spectral analysis of the optical field lengthens the coherence length to overcome the path imbalance. However, since optical detection usually is intensity measurement that loses the phase information of the detected optical field, the output photocurrent of intensity measurement cannot recover the lost interference effect in the postprocessing of the data such as electronic spectral analysis. On the other hand, if amplitude measurement can be used to measure the resultant optical field from the path-unbalanced superposition of two fields, the output photocurrent carries complete information of two fields and can restore interference in postdetection processing, even though no interference shows up in direct intensity measurement. In this paper, we use homodyne detection method for amplitude measurement to analyze the output of a path-unbalanced interferometer in the spectral domain. Since complete information about the superposed optical field is contained in the photocurrent, spectral analysis shows interference fringes in frequency that reveal information about the path imbalance. It is found that shot noise is the limiting factor in the visibility of interference but can be reduced with a squeezed state input in the unused port of the unbalanced interferometer. This approach is a different paradigm from the traditional one for studying the coherence between optical fields and should find wide application in remote sensing and precision measurement.

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I. INTRODUCTION

Interferometers are very sensitive and accurate scientific instruments that are widely used in scientific and engineering applications. There are many different structures and types of optical interferometers, such as the Michelson interferometer and Mach-Zehnder interferometer. Their basic working principle is to superpose coherent light waves in a common location where interference of the optical waves can occur and be observed in intensity. Since intensity measurement only records the energy of the field, phase information is lost, so interference must occur before detection. This is the reason why optical paths in an optical interferometer must be balanced within the coherence length for the observation of interference effect.

However, when the optical paths are not balanced, although no interference is observed in direct intensity measurement with a photodetector, spectral analysis by optical spectrometers of the superposed optical field can be performed to recover the interference effect in the frequency domain [1–3]. This even works for matter waves on neutron interferometers [4]. A detailed look shows that the act of optical spectrometers

is equivalent to optical filtering of the field to be analyzed, which effectively lengthens the coherence length of the detected fields and therefore satisfies the path balance condition for the observation of interference.

Optical filtering requires optical elements such as gratings and Fabry-Perot cavities and precise optical alignment, so the system can be complicated and sometimes limited by spectral resolution because higher spectral resolution is required to increase the coherence length for longer delays. However, electronic signal processing techniques are quite mature and widely used in electrical engineering. So, is it possible to recover the interference effect in a path-unbalanced interferometer by spectral analysis of the photocurrent from intensity measurement? The answer is no because intensity measurement records the energy distribution of the optical field, which shows no variation if there is no direct optical interference. On the other hand, if we can make an amplitude measurement whose output photocurrent contains complete information about the optical field, including the phase information, postprocessing of the photocurrent should be able to recover the interference effect. This was recently demonstrated in both a quantum SU(1, 1) interferometer [5] and a traditional Mach-Zehnder interferometer [6] by using the homodyne detection technique for the amplitude measurement. However, the analysis in these prior works is done in the time domain that depends on the response of the detectors, and the path imbalance information can only be revealed through a scan of electronic delay that takes huge data processing. Furthermore, the visibility of recovered interference fringes is limited to 50% [6]. So, information may not be fully recovered

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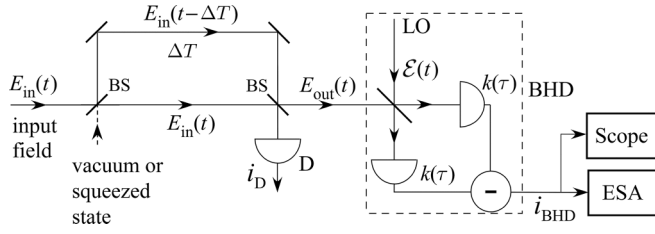


FIG. 1. Conceptual schematic for recovering interference in a path-unbalanced Mach-Zehnder interferometer by amplitude measurement with balanced homodyne detection. Interference fringes are revealed in spectral analysis of the output current i_{BHD} . BHD: balanced homodyne detection; LO: local oscillator for the BHD; $k(\tau)$: electronic response function of the detectors; ESA: electronic spectral analyzer; Scope: fast digital scope.

for the interference effect. Spectral analysis of electric current is a widely used technique in electrical engineering that can discriminate against unwanted low-frequency $1/f$ electronic noise and reveal more information about the electric current than time-domain analysis. Since the output photocurrent of the amplitude measurement contains complete information about the optical field, spectral analysis of the photocurrent is then equivalent to optical filtering, and interference can be recovered in the frequency domain, similar to the work with an optical spectrometer [1–3] but without the complicated and expensive optical elements. Compared to optical filtering, electronic filtering is much easier to handle and has much narrower bandwidth.

In this paper, we extend our previous work on recovering interference in the time domain in a path-unbalanced interferometer with homodyne measurement to frequency-domain analysis. The interference pattern as well as the path imbalance can be revealed in the spectrum of the photocurrent. The difference between our current work and the previous work by optical filtering [1–3] is the location where spectral analysis is performed: Previous work with intensity measurement requires optical spectral analysis *before detection*, while the current work with amplitude measurement has electronic spectral analysis *after detection*. Previous work performed optical spectral analysis, but ours is electronic. Spectral analysis also allows us to find the noise spectrum that is commonly used in electrical engineering for electric signal analysis and in quantum noise measurement of vacuum or squeezed states. This will enable us to find the sensitivity of the unbalanced interferometer and compare it with the balanced case.

The paper is organized as follows. In Sec. II, we discuss the general theory about the spectral analysis of the photocurrent from homodyne detection of the output of a path-unbalanced Mach-Zehnder interferometer. We consider the case where the unused port is either empty or in a broadband squeezed state for quantum noise analysis. We present the experimental verification of the theory in Sec. III and conclude with a discussion.

II. GENERAL THEORY

Let us start by considering the conceptual scheme in Fig. 1, where an input field $\hat{E}_{\text{in}}(t)$ enters an unbalanced

Mach-Zehnder interferometer with a large temporal delay ΔT in one arm. With the notations in Fig. 1 and the 50:50 beam splitters, the output field of the interferometer is related to the input fields as

$$\begin{aligned}\hat{E}_{\text{out}}(t) &= [\hat{E}_a(t) + \hat{E}_b(t - \Delta T)]/\sqrt{2} \\ &= [\hat{E}_{\text{in}}(t) - \hat{E}_0(t) \\ &\quad + \hat{E}_{\text{in}}(t - \Delta T) + \hat{E}_0(t - \Delta T)]/2,\end{aligned}\quad (1)$$

where $\hat{E}_0$ is the field operator from the unused port of the interferometer that is usually in vacuum. Here, we use full quantum theory of light because we apply the formalism to quantum states as input. A balanced homodyne detector (BHD) with a response function $k(\tau)$ and a quantum efficiency of η and a strong local oscillator $\mathcal{E}(t) = |\mathcal{E}|e^{i(\varphi - \omega_0 t)}$ is used to measure the quadrature-phase amplitude of the output field:

$$\hat{X}_{\text{out}}^\varphi(t) = \hat{E}_{\text{out}}(t)e^{-i(\varphi - \omega_0 t)} + \hat{E}_{\text{out}}^\dagger(t)e^{i(\varphi - \omega_0 t)}. \quad (2)$$

The photocurrent output i_{HD} from BHD is given by [7]

$$i_{\text{HD}}(t) = \eta|\mathcal{E}| \int d\tau k(t - \tau) \langle \hat{X}_{\text{out}}^\varphi(t) \rangle, \quad (3)$$

and is sent to an electronic spectral analyzer (ESA) for spectral analysis, or recorded by a fast digital oscilloscope (Scope) for further postdetection data processing.

The spectral density $S(\Omega)$ obtained by the ESA for the photocurrent i_{HD} of the BHD is given by [8–10]

$$S(\Omega) = \int d\tau \langle \Delta i_{\text{HD}}(t) \Delta i_{\text{HD}}(t + \tau) \rangle e^{-i\Omega\tau}, \quad (4)$$

with $\Delta i_{\text{HD}}(t) \equiv i_{\text{HD}}(t) - \langle i_{\text{HD}} \rangle$. It can be found [9,10] that the spectral density $S(\Omega)$ is related to $\hat{X}_{\text{out}}^\varphi(t)$ by

$$S(\Omega) = S_{\text{SN}}(\Omega)[1 + \eta\chi_{\text{out}}^\varphi(\Omega)], \quad (5)$$

where $S_{\text{SN}}(\Omega) = \eta|\mathcal{E}|^2|k(\Omega)|^2$ is the shot noise spectrum with $k(\Omega) \equiv \int d\tau k(\tau)e^{i\Omega\tau}$ and

$$\chi_{\text{out}}^\varphi(\Omega) \equiv \int d\tau \chi_{\text{out}}^\varphi(\tau)e^{i\Omega\tau}; \quad (6)$$

with $\chi_{\text{out}}^\varphi(\tau) \equiv \langle : \Delta \hat{X}_{\text{out}}^\varphi(t + \tau) \Delta \hat{X}_{\text{out}}^\varphi(t) : \rangle$. Notice that $\chi_{\text{out}}^\varphi(\tau)$ is independent of t because the field is stationary.

Since we are concerned with the spectral properties of the output, it is better to write the fields in terms of their frequency components:

$$\hat{E}_k(t) = \frac{1}{\sqrt{2\pi}} \int d\omega \hat{a}_k(\omega) e^{-i\omega t}, \quad (7)$$

with $k = \text{in}$ or 0 . Depending on the properties of $\hat{a}_{\text{in}}(\omega)$, $\hat{a}_0(\omega)$ in the input and unused ports, we can calculate $\chi_{\text{out}}^\varphi(\tau)$ and then $S(\Omega)$ as follows.

A. Classical stationary input field and vacuum at unused port

We first consider a traditional setting with a classical stationary field as input and the unused port in vacuum. So, there is no correlation between $\hat{E}_{\text{in}}$ and $\hat{E}_0$ and the stationary property of the input requires no correlation between different frequency components; that is, $\langle \hat{a}_{\text{in}}^\dagger(\omega) \hat{a}_{\text{in}}(\omega') \rangle = n(\omega) \delta(\omega - \omega')$, where $n(\omega)$ is the average photon number per frequency mode since the total photon rate

$R(t) \equiv \langle \hat{E}_{\text{in}}^\dagger(t) \hat{E}_{\text{in}}(t) \rangle = \frac{1}{2\pi} \int d\omega n(\omega) = \int d\nu n(2\pi\nu)$ ($\omega = 2\pi\nu$ is the angular frequency and ν is the frequency in unit of Hz = 1/s). Using $\hat{E}_{\text{out}}(t)$ given in Eq. (1) and assuming that the input field does not have a fixed phase with respect to the LO field so that $\langle \hat{X}^\varphi(t) \rangle = 0$, we obtain, after dropping out the vacuum field due to no contribution from vacuum to the normal ordered average,

$$\begin{aligned} \chi_{\text{out}}^\varphi(\tau) &= \langle : \Delta \hat{X}^\varphi(t + \tau) \Delta \hat{X}^\varphi(t) : \rangle \\ &= \chi_{\text{in}}^\varphi(\tau) \equiv \left\langle \frac{1}{4} [\hat{E}_{\text{in}}^\dagger(t) + \hat{E}_{\text{in}}^\dagger(t - \Delta T)] e^{i\omega_0\tau} \right. \\ &\quad \times [\hat{E}_{\text{in}}(t + \tau) + \hat{E}_{\text{in}}(t + \tau - \Delta T)] \\ &\quad + \frac{1}{4} [\hat{E}_{\text{in}}^\dagger(t + \tau) + \hat{E}_{\text{in}}^\dagger(t + \tau - \Delta T)] \\ &\quad \times [\hat{E}_{\text{in}}(t) + \hat{E}_{\text{in}}(t - \Delta T)] e^{-i\omega_0\tau} \Big\rangle \\ &= \frac{e^{-i\omega_0\tau}}{4} [2\Gamma(\tau) + \Gamma(\tau + \Delta T) + \Gamma(\tau - \Delta T)] \\ &\quad + \frac{e^{i\omega_0\tau}}{4} [2\Gamma^*(\tau) + \Gamma^*(\tau + \Delta T) + \Gamma^*(\tau - \Delta T)], \end{aligned} \quad (8)$$

where

$$\begin{aligned} \Gamma(\tau) &\equiv \langle \hat{E}_{\text{in}}^\dagger(t + \tau) \hat{E}_{\text{in}}(t) \rangle = \frac{1}{2\pi} \int d\omega n(\omega) e^{i\omega\tau} \\ &= \left[\frac{1}{2\pi} \int d\Omega n(\omega_0 + \Omega) e^{i\Omega\tau} \right] e^{i\omega_0\tau}. \end{aligned} \quad (9)$$

Using the following Fourier transformation of the Γ function,

$$\begin{aligned} \int d\tau \Gamma(\tau + \Delta T) e^{-i\omega_0\tau} e^{i\Omega\tau} \\ &= \frac{e^{i\omega_0\Delta T}}{2\pi} \int d\tau d\Omega' n(\omega_0 + \Omega') e^{i(\Omega + \Omega')\tau + \Omega'\Delta T} \\ &= n(\omega_0 - \Omega) e^{i(\omega_0 - \Omega)\Delta T}, \end{aligned} \quad (10)$$

$$\begin{aligned} \int d\tau \Gamma^*(\tau + \Delta T) e^{i\omega_0\tau} e^{i\Omega\tau} \\ &= \frac{e^{-i\omega_0\Delta T}}{2\pi} \int d\tau d\Omega' n(\omega_0 + \Omega') e^{i(\Omega - \Omega')\tau - \Omega'\Delta T} \\ &= n(\omega_0 + \Omega) e^{-i(\omega_0 + \Omega)\Delta T}. \end{aligned} \quad (11)$$

We arrive at the expression for $\chi_{\text{out}}^\varphi(\Omega)$ as follows:

$$\begin{aligned} \chi_{\text{out}}^\varphi(\Omega) &= \int d\tau \chi_{\text{out}}^\varphi(\tau) e^{i\Omega\tau} \\ &= \frac{1}{2} [n(\omega_0 - \Omega) + n(\omega_0 + \Omega)] \\ &\quad + \frac{1}{4} [n(\omega_0 - \Omega) e^{i(\omega_0 - \Omega)\Delta T} + \text{c.c.}] \\ &\quad + \frac{1}{4} [n(\omega_0 + \Omega) e^{-i(\omega_0 + \Omega)\Delta T} + \text{c.c.}] \\ &= \frac{1}{2} n(\omega_0 - \Omega) \{1 + \cos[(\omega_0 - \Omega)\Delta T]\} \end{aligned}$$

$$\begin{aligned} &+ \frac{1}{2} n(\omega_0 + \Omega) \{1 + \cos[(\omega_0 + \Omega)\Delta T]\} \\ &= \frac{1}{2} n_+ [1 + \cos(\Omega\Delta T) \cos \theta] \\ &\quad - \frac{1}{2} n_- \sin(\Omega\Delta T) \sin \theta, \end{aligned} \quad (12)$$

where $n_\pm = n(\omega_0 + \Omega) \pm n(\omega_0 - \Omega)$ and $\theta \equiv \omega_0\Delta T$ is the phase difference of the interferometer. Normally, we have $\omega_0 \gg \Omega$ so that $n_+ \approx 2n(\omega_0)$ and $n_- \approx 0$. Hence, we have

$$\chi_{\text{out}}^\varphi(\Omega) \approx n(\omega_0) [1 + \cos(\Omega\Delta T) \cos \theta]. \quad (13)$$

Substituting the above back into Eq. (5), we obtain the spectrum recorded in the ESA as normalized to the shot noise spectrum:

$$\begin{aligned} S(\Omega)/S_{\text{SN}}(\Omega) &\approx 1 + \eta n(\omega_0) [1 + \cos(\Omega\Delta T) \cos \theta] \\ &= [1 + \eta n(\omega_0)] [1 + V(\Omega) \cos \theta]. \end{aligned} \quad (14)$$

Notice that $\theta \equiv \omega_0\Delta T$ is the phase difference between the two arms, so $\cos \theta$ gives the interference fringes, and Eq. (14) shows an interference fringe in the frequency domain with visibility

$$V(\Omega) = \frac{\eta n(\omega_0)}{1 + \eta n(\omega_0)} |\cos(\Omega\Delta T)|, \quad (15)$$

which depends on the analyzing frequency Ω . In particular, $V(\Omega_m) = 0$ for $\Omega_m\Delta T = (m - 1/2)\pi$ or $\Omega_m = (m - 1/2)\pi/\Delta T$ ($m = 1, 2, 3, \dots$) but the maximum value $V(\Omega_M) = V_{\text{Max}} \equiv \eta n(\omega_0)/[1 + \eta n(\omega_0)]$ is reached for $\Omega_M\Delta T = M\pi$ or $\Omega_M = M\pi/\Delta T$ ($M = 1, 2, 3, \dots$).

Notice that the maximum visibility $V_{\text{Max}} \rightarrow 0$ when the average input photon number $n(\omega_0) \ll 1$. This is because vacuum noise or shot noise S_{SN} sets the minimum in Eq. (14) and dominates if the average input photon number $n(\omega_0)$ is much smaller than 1. It is well known that injection of the squeezed state in the unused port can reduce shot noise [11,12]. Let us consider this situation next.

B. Squeezed state input at the unused port

When a squeezed state is injected into the unused port as $\hat{E}_0$, it is usually independent of the input field, so the contribution from the input is the same as Eq. (13) but the contribution from $\hat{E}_0$ is not zero. We evaluate it using the frequency components in Eq. (7) as follows.

We first write Eq. (2) as

$$\begin{aligned} \hat{X}_{\text{out}}^\varphi(t) &= e^{-i(\varphi - \omega_0 t)} [\hat{E}_{\text{in}}(t) - \hat{E}_0(t) + \hat{E}_{\text{in}}(t - \Delta T) \\ &\quad + \hat{E}_0(t - \Delta T)]/2 + e^{i(\varphi - \omega_0 t)} [\hat{E}_{\text{in}}^\dagger(t) - \hat{E}_0^\dagger(t) \\ &\quad + \hat{E}_{\text{in}}^\dagger(t - \Delta T) + \hat{E}_0^\dagger(t - \Delta T)]/2 \\ &= [\hat{X}_{\text{in}}^\varphi(t) + \hat{X}_{\text{in}}^{(\varphi - \theta)}(t - \Delta T) \\ &\quad + \hat{X}_0^{(\varphi - \theta)}(t - \Delta T) - \hat{X}_0^\varphi(t)]/2, \end{aligned} \quad (16)$$

where $\theta \equiv \omega_0\Delta T$ and in terms of frequency-shifted

components,

$$\begin{aligned}\hat{X}_k^\varphi(t) &\equiv \hat{E}_k(t)e^{-i(\varphi-\omega_0 t)} + \hat{E}_k^\dagger(t)e^{i(\varphi-\omega_0 t)} \\ &= \frac{1}{\sqrt{2\pi}} \int d\Omega \hat{X}_k^\varphi(\Omega) e^{-i\Omega t},\end{aligned}\quad (17)$$

with

$$\hat{X}_k^\varphi(\Omega) \equiv \hat{a}_k(\omega_0 + \Omega)e^{-i\varphi} + \hat{a}_k^\dagger(\omega_0 - \Omega)e^{i\varphi}, \quad (18)$$

where $k = \text{in or } 0$. If the input field and the squeezed state are independent of each other, then we can write

$$\chi_{\text{out}}^\varphi(\tau) = \chi_{\text{in}}^\varphi(\tau) + \chi_0^\varphi(\tau), \quad (19)$$

with $\chi_{\text{in}}^\varphi(\tau)$ given in Eq. (8) and

$$\begin{aligned}\chi_0^\varphi(\tau) &\equiv \frac{1}{4} \langle : [\hat{X}_0^{(\varphi-\theta)}(t + \tau - \Delta T) - \hat{X}_0^\varphi(t + \tau)] \\ &\quad \times [\hat{X}_0^{(\varphi-\theta)}(t - \Delta T) - \hat{X}_0^\varphi(t)] : \rangle \\ &= \frac{1}{4} \langle [: \hat{X}_0^{(\varphi-\theta)}(t + \tau - \Delta T) \hat{X}_0^{(\varphi-\theta)}(t - \Delta T) : \rangle \\ &\quad - \langle : \hat{X}_0^{(\varphi-\theta)}(t + \tau - \Delta T) \hat{X}_0^\varphi(t) : \rangle \\ &\quad - \langle : \hat{X}_0^\varphi(t + \tau) \hat{X}_0^{(\varphi-\theta)}(t - \Delta T) : \rangle \\ &\quad + \langle : \hat{X}_0^\varphi(t + \tau) \hat{X}_0^\varphi(t) : \rangle]. \end{aligned}\quad (20)$$

To evaluate the four terms above, we simply calculate the general expression $\langle : \hat{X}_0^{\phi'}(t + \tau) \hat{X}_0^\phi(t) : \rangle$ with $\phi', \phi = \varphi - \theta, \varphi$. For this, we need the properties of the $\hat{E}_0$ field, which is in a squeezed state generated as the output field of a broadband parametric amplifier pumped at $2\omega_0$ with its signal field as the $\omega_0 + \Omega$ frequency component and its idler field as the $\omega_0 - \Omega$ frequency component:

$$\begin{aligned}\hat{a}_0(\omega_0 + \Omega) &= G(\Omega)\hat{c}(\omega_0 + \Omega) + g(\Omega)e^{i\beta}\hat{c}^\dagger(\omega_0 - \Omega), \\ \hat{a}_0(\omega_0 - \Omega) &= G(\Omega)\hat{c}(\omega_0 - \Omega) + g(\Omega)e^{i\beta}\hat{c}^\dagger(\omega_0 + \Omega),\end{aligned}\quad (21)$$

where $G(\Omega), g(\Omega)$, satisfying $G^2 - g^2 = 1$, are related to the pump power and are the amplitude gain parameters at frequency $\omega_0 \pm \Omega$ of the parametric amplifier. We assume that they are positive and that the pump field phase is denoted as β . Field $\hat{c}(\omega)$ is the vacuum input field of the parametric amplifier satisfying $[\hat{c}(\omega_1), \hat{c}^\dagger(\omega_2)] = \delta(\omega_1 - \omega_2)$ and $\langle \hat{c}(\omega) | \text{vacuum} \rangle = 0$. With these and using Eq. (17), we obtain

$$\begin{aligned}\langle : \hat{X}_0^{\phi'}(t + \tau) \hat{X}_0^\phi(t) : \rangle &= \frac{1}{2\pi} \int d\Omega_1 d\Omega_2 e^{-i\Omega_1(t+\tau)} e^{-i\Omega_2 t} \\ &\quad \times \langle : \hat{X}_0^{\phi'}(\Omega_1) \hat{X}_0^\phi(\Omega_2) : \rangle.\end{aligned}\quad (22)$$

From Eqs. (18) and (21), we have

$$\begin{aligned}\langle : \hat{X}_0^{\phi'}(\Omega_1) \hat{X}_0^\phi(\Omega_2) : \rangle &= \langle : [\hat{a}_0(\omega_0 + \Omega_1)e^{-i\phi'} + \hat{a}_0^\dagger(\omega_0 - \Omega_1)e^{i\phi'} \\ &\quad \times [\hat{a}_0(\omega_0 + \Omega_2)e^{-i\phi} + \hat{a}_0^\dagger(\omega_0 - \Omega_2)e^{i\phi}] : \rangle \\ &= \langle \hat{a}_0^\dagger(\omega_0 - \Omega_1) \hat{a}_0^\dagger(\omega_0 - \Omega_2) \rangle e^{i(\phi'+\phi)} \\ &\quad + \langle \hat{a}_0(\omega_0 + \Omega_1) \hat{a}_0(\omega_0 + \Omega_2) \rangle e^{-i(\phi+\phi')} \\ &\quad + \langle \hat{a}_0^\dagger(\omega_0 - \Omega_1) \hat{a}_0(\omega_0 + \Omega_2) \rangle e^{i(\phi'-\phi)} \\ &\quad + \langle \hat{a}_0^\dagger(\omega_0 - \Omega_2) \hat{a}_0(\omega_0 + \Omega_1) \rangle e^{i(\phi-\phi')}\end{aligned}$$

$$\begin{aligned}&= 2[G(\Omega_1)g(\Omega_1)\cos(\phi' + \phi - \beta) \\ &\quad + g^2(\Omega_1)\cos(\phi' - \phi)]\delta(\Omega_1 + \Omega_2).\end{aligned}\quad (23)$$

Substituting into Eq. (22) and carrying out the integral, we obtain

$$\begin{aligned}\langle : \hat{X}_0^{\phi'}(t + \tau) \hat{X}_0^\phi(t) : \rangle &= \frac{1}{\pi} \int d\Omega e^{-i\Omega\tau} [g^2(\Omega)\cos(\phi' - \phi) \\ &\quad + G(\Omega)g(\Omega)\cos(\phi' + \phi - \beta)].\end{aligned}\quad (24)$$

Substituting the above into Eq. (20) with the appropriate ϕ', ϕ , we have

$$\begin{aligned}\chi_0^\varphi(\tau) &= \frac{1}{2\pi} \int d\Omega e^{-i\Omega\tau} g(\Omega)[g(\Omega)(1 - \cos\Omega\Delta T \cos\theta) \\ &\quad + G(\Omega)\cos(2\varphi - \theta - \beta)(\cos\theta - \cos\Omega\Delta T)].\end{aligned}\quad (25)$$

Performing the reverse Fourier transformation, we have

$$\begin{aligned}\chi_0^\varphi(\Omega) &= g(\Omega)[g(\Omega)(1 - \cos\Omega\Delta T \cos\theta) \\ &\quad + G(\Omega)\cos(2\varphi - \theta - \beta)(\cos\theta - \cos\Omega\Delta T)].\end{aligned}\quad (26)$$

Combining all results, we arrive at the final result:

$$\begin{aligned}S(\Omega)/S_{\text{SN}}(\Omega) &= \eta n(\omega_0)(1 + \cos\Omega\Delta T \cos\theta) + 1 \\ &\quad + \eta g(\Omega)[g(\Omega)(1 - \cos\Omega\Delta T \cos\theta) \\ &\quad + G(\Omega)\cos(2\varphi - \theta - \beta) \\ &\quad \times (\cos\theta - \cos\Omega\Delta T)].\end{aligned}\quad (27)$$

Obviously, the first term is from the input field and produces the phase signal of the interferometer, while the rest is the noise level and gives the shot noise if the squeezed state becomes vacuum ($g = 0$). Let us examine it in detail next.

We assume that maximum visibility is achieved with $\cos\Omega\Delta T = 1$. Usually, interferometers with homodyne detection operate in the dark mode; that is, the output signal term is zero with $\cos\theta = -1$. For the sake of clarity, we also assume an ideal quantum efficiency of $\eta = 1$. Then, the noise part becomes

$$N(\Omega) = 1 + 2g^2(\Omega) - 2G(\Omega)g(\Omega)\cos(2\varphi - \theta - \beta), \quad (28)$$

which is minimum when the phase of LO is adjusted so that $\cos(2\varphi - \theta - \beta) = 1$:

$$\begin{aligned}N_m(\Omega) &= 1 + 2g^2(\Omega) - 2G(\Omega)g(\Omega) \\ &= 1/[G(\Omega) + g(\Omega)]^2,\end{aligned}\quad (29)$$

where we use the identity $G^2 - g^2 = 1$. This is exactly the reduced noise level due to the squeezed state. Therefore, optimum performance with the squeezed state can be achieved even for an unbalanced interferometer, provided the analyzing frequency is set appropriately at $\Omega_M = M\pi/\Delta T$.

When calculating the visibility of the fringe, we need to be careful because the noise part in Eq. (27) also depends on the phase of the interferometer as well as the LO, which is different from the case of vacuum input when the noise is independent of the interferometer phase. Another reason to consider the phase-dependent noise as a part of the fringe

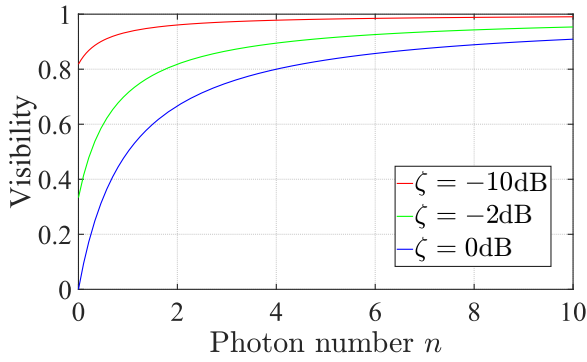


FIG. 2. Visibility as a function of the average input photon number n for various values of ζ .

is because the squeezed state has nonzero photon number, different from the vacuum input. To eliminate the LO phase dependence, we can adjust it to follow the phase of the interferometer in such a way that $\cos(2\phi - \theta - \beta) = -1$ and the noise part is in sync with the main fringe of the input field. For simplicity, we assume $\eta = 1$. Then, Eq. (27) becomes

$$\begin{aligned} S(\Omega)/S_{\text{SN}}(\Omega) &= n(\omega_0)(1 + \cos \theta) + 1 \\ &\quad + [g^2(\Omega) - g(\Omega)G(\Omega)](1 - \cos \theta) \\ &\propto 1 + V_{\text{sq}} \cos \theta, \end{aligned} \quad (30)$$

with

$$V_{\text{sq}} \equiv \frac{g + n(g + G)}{G + n(g + G)}, \quad (31)$$

where we used $G^2 - g^2 = 1$. We can rewrite the above in terms of the squeezing parameter $\zeta \equiv 1/(g + G)^2$, which is defined in Eq. (29), as follows:

$$V_{\text{sq}} = \frac{1 - \zeta + 2n}{1 + \zeta + 2n}, \quad (32)$$

which recovers Eq. (15) for vacuum with $\zeta = 1$. Notice that $V_{\text{sq}} = 1$ independent of n for an infinite amount of squeezing of $\zeta = 0$ when quantum noise is totally eliminated. Even for a finite amount of squeezing $0 < \zeta < 1$, we have a finite visibility of $V_{\text{sq}} = (1 - \zeta)/(1 + \zeta) \neq 0$ when there is no input field with $n = 0$. This is because the squeezed state has nonzero photon number. In order to see the effect of squeezing, we plot in Fig. 2 the visibility in Eq. (32) as a function of the input

photon number n for various amounts of squeezing ζ . As can be seen, visibility improves with the amount of squeezing.

III. EXPERIMENT

We next perform an experiment to verify some of the theoretical predictions above. The experimental schematic is shown in Fig. 3, where a path-unbalanced Mach-Zehnder interferometer has the input light field as a phase-randomized coherent state, which is realized by passing a laser beam through an electro-optic modulator (EOM) driven by a white noise source of bandwidth about 300 MHz. The extra delay in one arm is made through a 22-m-long single-mode fiber, which gives an optical delay of $\Delta T = n\Delta L/c = 1.5 \times 20/3 \times 10^8 = 110$ ns. The delay is chosen so that it is much longer than the coherence time $T_c \sim 3$ ns. This ensures that no interference can be observed using traditional intensity measurement. However, here we make an amplitude measurement by homodyne detection. The output of the interferometer is measured by mixing with a strong local oscillator obtained from the laser before detected by a fast photodetector (PD) of 10 GHz bandwidth. Attenuators (ATT) are used to control the intensity of the input field and the local oscillator. Polarization control units (FPC) are used to achieve a good mode match. The phase of interferometer (θ) is scanned by a piezo-mounted fiber stretcher (PFS). The output electric current from the detector (PD) is either analyzed by an ESA or recorded by a digital oscilloscope for postdetection data analysis. We first look at the results from the ESA.

A. Spectral measurement by an electronic spectral analyzer

Figure 4 shows the interference fringe (brown) on logarithmic scale in a spectral span of 12 MHz as the interferometer phase θ is scanned in time (sweep time = 1 s). The blue trace is the shot noise level when the input field is blocked. The large bump near 1.5 MHz is from intensity noise of the laser because we do not use balanced homodyne detection scheme to eliminate the noise from local oscillator due to lack of extra fast detector. It can be seen that the visibility of interference is modulated as a function of frequency f and is zero at some specific frequencies of $f_m = \Omega_m/2\pi = (m - 1/2)/2\Delta T$, as predicted from Eq. (15). We focus at specific frequencies of 4.6 and 6.9 MHz and find fringes when the phase is swept in 100 ms, as shown in Figs. 5(a) and 5(b), from which we

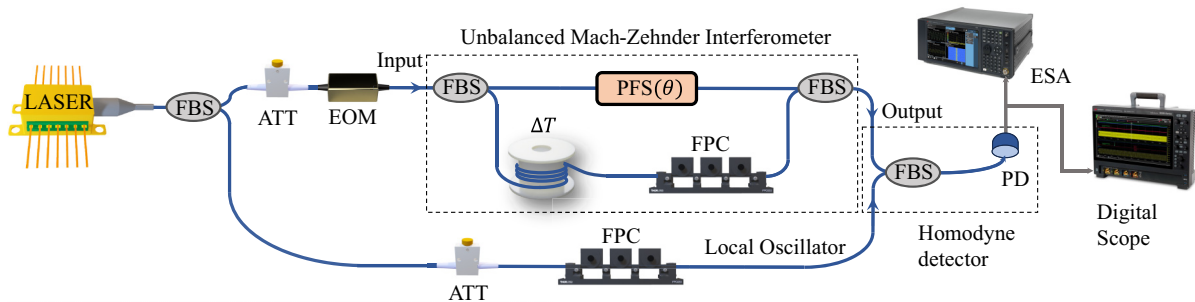


FIG. 3. Experimental layout. The dashed part is a path-unbalanced Mach-Zehnder interferometer with an optical delay much larger than the coherence time: $\Delta T \gg T_c$. ATT: attenuator; FPC: fiber polarization controller; FBS: fiber beam splitter; EOM: phase modulator; PD: photodetector; ESA: electronic spectral analyzer. PFS(θ): piezofiber stretcher for phase scan.

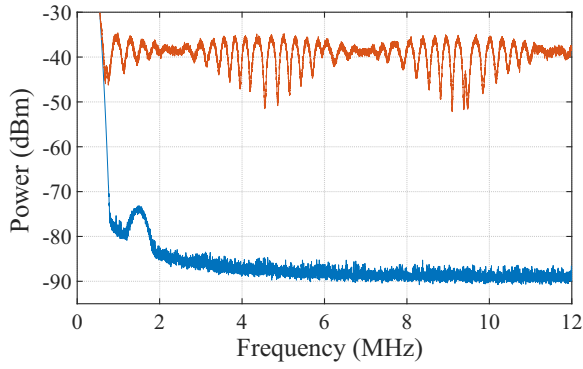


FIG. 4. Interference fringes (brown, in log scale) from an electronic spectral analyzer. Blue trace: shot noise level [resolution bandwidth (RBW) = 360 kHz, video bandwidth (VBW) = 1.2 kHz, sweep time = 1 s].

can extract visibility as a function of the analyzing center frequency. We plot this dependence in Fig. 5(c) and fit it to Eq. (15) under $\eta n \gg 1$ with a scaling factor as the free parameter to account for experimental imperfectness.

To fully verify Eq. (15) and find the influence of the shot noise, we change the power of the input field to the interferometer. Figure 6 shows the maximum visibility as a function of the input light power. It can be seen that visibility drops with the decrease in power, following the function given in Eq. (15), which is plotted as a solid curve. The drop in visibility at low power is due to the contribution of the shot noise.

B. Spectral measurement by postdetection data processing

A different approach to finding spectral information is by processing the output current $i_{\text{HD}}(t)$ of homodyne detection, which can be recorded with a fast digital oscilloscope. In the experiment, we record $i_{\text{HD}}(t)$ in a step of 6.25×10^{-10} s for a total of 100 ms, which is the time for one sweep of the phase θ as shown in Fig. 5(a). The step of 6.25×10^{-10} s is

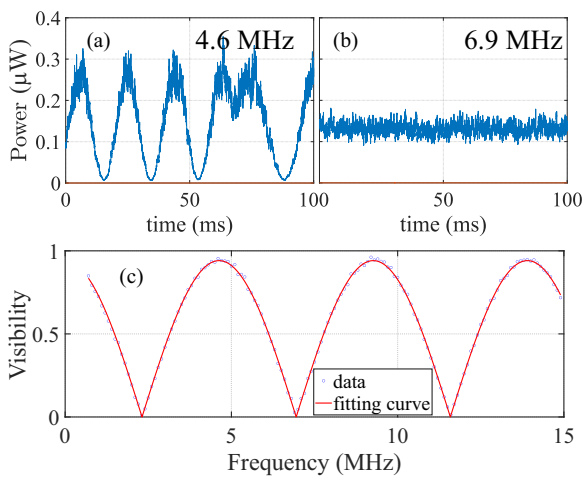


FIG. 5. Interference fringes shown in ESA at a center frequency of (a) $f = 4.6$ MHz and (b) $f = 6.9$ MHz (RBW = 360 kHz, VBW = 1.2 kHz). (c) Visibility as a function of analyzing center frequency f of ESA. Red solid trace is a fit to Eq. (15) with $\eta n \gg 1$.

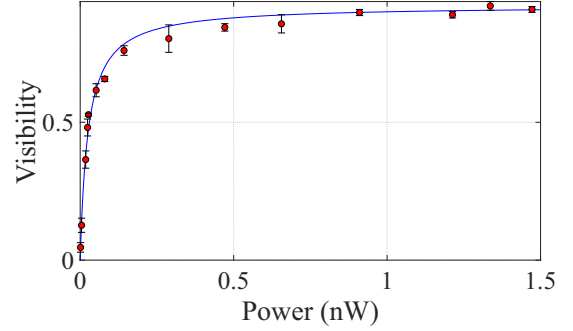


FIG. 6. Interference visibility as a function of input power at frequency of maximum visibility. The fitting curve is visibility = $\frac{a \times \text{power}}{\text{power} + b}$, with $a = 0.92$ and $b = 0.026$ nW in reference to Eq. (15).

much smaller than $T_c \sim 3$ ns to ensure that the fluctuations of the field are recorded. There are two methods that we can employ to perform postdetection data processing on $i_{\text{HD}}(t)$ using Fourier transformation to obtain spectral information.

In the first method, we resort to the formula in Eq. (4), where we first evaluate the autocorrelation function of $i_{\text{HD}}(t)$ and then make the Fourier transformation. Notice that the phase θ is a variable that slowly changes in our experiment to display the interference fringes [about four periods in 100 ms; see Fig. 5(a)], so the spectrum will also change over time. Because of this, unlike the result in Eq. (5) for stationary fields, t is also a variable and will appear in θ in Eq. (14). We need to find the spectrum at different times t or θ .

Since θ changes slowly with time, we can use data points around a specific time t to perform the average $\langle \dots \rangle$ in Eq. (4) for the correlation function. Here, we take 2×10^5 data points or a range of $[t - 62.5 \mu\text{s}, t + 62.5 \mu\text{s}]$ for the average and select $t = 100, 200, 300, \dots, 1000 \times 100 \mu\text{s}$. The selection of the average time of $62.5 \mu\text{s}$ is such that $62.5 \mu\text{s} \gg T_c \sim 3$ ns, the coherence time of the input field, to achieve the ensemble average of the field. It is also small enough that θ is approximately constant. A typical correlation function is shown in Fig. 7(a) for the range of interest (nonzero values), which shows three peaks. The one at the zero delay comes from regular autocorrelation of random amplitude in the photocurrent. Two adjacent peaks emerge at a symmetric delay of $\sim \pm 110$ ns corresponding to the optical delay we apply (ΔT). The maxima of the side peaks are smaller than the central peak at zero delay, indicating only partial correlation between two arms of the interferometer. Then, for each time t corresponding to different phase θ , we use Eq. (4) to take the fast Fourier transformation (FFT) of the autocorrelation function with integration variable τ also in the range of $[-62.5 \mu\text{s}, 62.5 \mu\text{s}]$. Figure 7(b) shows the spectra at three different times corresponding to $\theta = 0$ (blue), $\pi/2$ (orange), and π (yellow), respectively, which follow exactly the spectrum derived in Eq. (14). To see the interference fringes as θ is scanned, we pick out the spectral data at a specific frequency from the FFT results of different times t and plot the modulus of the specific frequency component of the FFT as a function of the time variable t . Figures 7(c) and 7(d) show two such plots at $f = 4.6$ and 6.9 MHz, respectively. This corresponds to zero frequency span of ESA [Figs. 5(a) and 5(b)]. From the

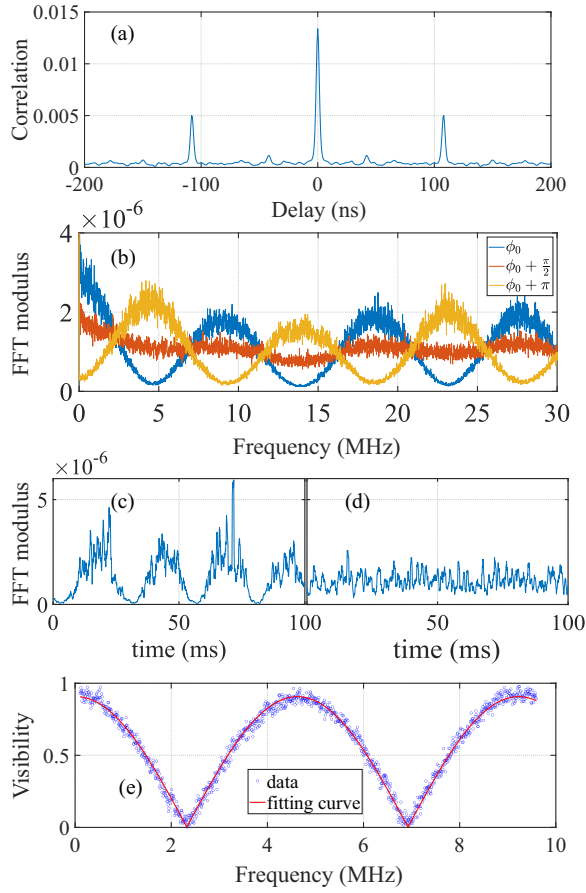


FIG. 7. Postdetection data analysis based on Eq. (4). (a) Autocorrelation function of the recorded photocurrent. (b) Spectra at various phases of $\theta = 0, \frac{\pi}{2}$, and π (average of 100 points is taken). Interference fringes at center analyzing frequency of (c) $f = 4.6$ MHz (left) and (d) 6.9 MHz (right) (average of ten points is taken). (e) Visibility as a function of center analyzing frequency.

interference fringes in Figs. 7(c) and 7(d), we extract visibility of interference and plot as a function of frequency in Fig. 7(e), and arrive at the expected visibility modulation behavior in the frequency domain, with the solid line as a fit to Eq. (15) with a period of $4.55 \text{ MHz} = 1/2\Delta T$. This is similar to the ESA result in Fig. 5(c).

We can also display interference fringes in a wide spectral range as shown in Fig. 8, similar to Fig. 4. However, the procedure to obtain Fig. 8 is not straightforward and takes more effort than those described above for Fig. 7. We present it in Appendix A.

Calculating the autocorrelation function can take a lot of computing power. A more practical method is to make a direct Fourier transformation of the recorded output current $i_{\text{HD}}(t)$. The theory of this approach is presented in Appendix B. The FFT simulation plays the same role of information extraction as correlation measurement, but the calculation burden for the computer is much smaller than the direct correlation function calculation. This method is practically the same as the correlation function method: We divide the photocurrent data recorded in a total time of 100 ms into segments of size $2 \times 62.5 \mu\text{s}$, with each corresponding to a different phase θ .

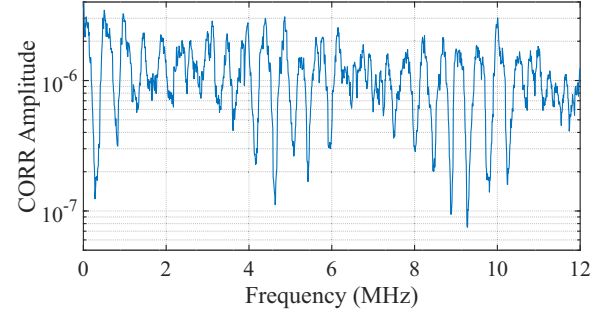


FIG. 8. Spectrum in a wide spectral range from the Fourier transformation of the autocorrelation function in Fig. 7(a).

Then, we apply the FFT algorithm for each segment and take the modulus square of the FFT result. The result is shown in Fig. 9, which shows interference fringes similar to the previous autocorrelation function method (Figs. 7 and 8).

IV. DISCUSSION

The amplitude-based interference technique offers us a picture to understand interference between distinguished spatial-temporal modes within the scheme of an unbalanced Mach-Zehnder interferometer (MZI). In this paper, we find that the phase information encoded in an optical field can also be fully recovered by spectral analysis. Compared to

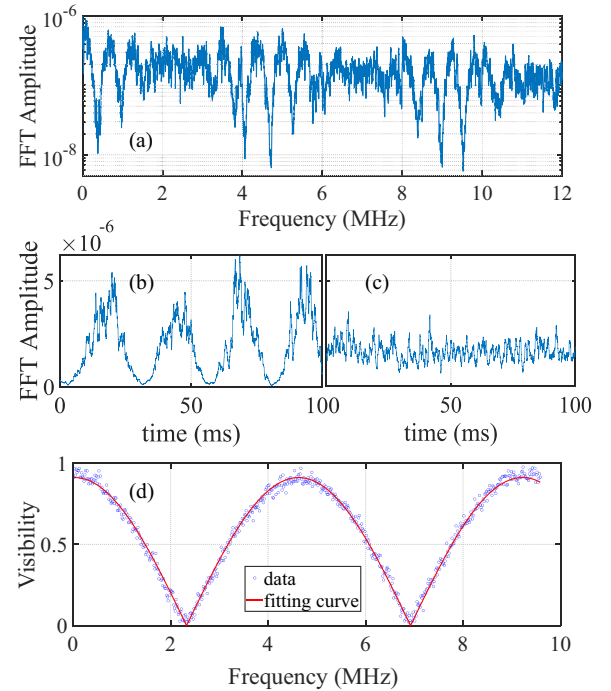


FIG. 9. Interference fringes by postdetection data analysis of direct FFT on the recorded photocurrent. (a) Full spectrum from 0 to 12 MHz (see Appendix A for details). (b) Fringes at center frequency of $f = 4.6$ MHz. (c) Fringes with near-zero visibility at center frequency of $f = 6.9$ MHz [panels (b) and (c) are both averaged over 100 points of total 13 991 sampling points for FFT analysis in 100 ms]. (d) Visibility as a function of center frequency with the solid red curve as a fit to Eq. (15).

time-domain analysis, spectral analysis not only reveals the information in a more direct way: the optical delay and relative phase variation between delayed fields, but also shows the ability to obtain more complicated information with a simple analysis approach. These characteristics enable the application of the technique in some real scenarios: (1) The currently popular light detection and ranging (LIDAR) scheme usually requires a longer ranging distance $L_{\max}$ while achieving higher resolution ΔL at the same time: Traditional methods like frequency modulated continuous wave (FMCW) LIDAR offer flexible single-shot ranging and velocimetry. However, the coherence length of modulated continuous wave (CW) field becomes a limitation to $L_{\max}$ if the requirement for a larger modulation bandwidth is considered [13,14]. Our analysis approach could offer a solution to all these LIDAR systems to circumvent the coherence-length limitation. (2) Optical communication that relies on coherence of the field suffers from phase noise in long-distance transmission [15]. However, our method can overcome this problem by measuring the amplitude via homodyne detection with the aid of a narrow-band local oscillator. Spectral analysis easily avoids the $1/f$ electronic noise and operates at a higher bandwidth of the optoelectronic system, making full use of a broad optical spectrum and allowing for transmission of higher information capacity. In addition, a more profound implication in practical application is that it demands no coherence between the local oscillator and the signal field, which makes a big difference compared to coherent detection scheme.

As discussed in the theory part, spectral analysis can also be applied in quantum regime analysis when shot noise becomes dominant. One of the quantum advantages comes from the injection of squeezed vacuum into the unused input port of the unbalanced MZI, which offers phase measurement with the same noise suppression as a regular path-balanced interferometer. Therefore, our platform with spectrum analysis and the homodyne detection technique for amplitude measurement is naturally suitable for applications in quantum metrology. Further work on this will be presented elsewhere.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available because they contain commercially sensitive information. The data are available from the authors upon reasonable request.

APPENDIX A: PROCEDURE IN THE EXTRACTION OF FIG. 7(b)

The process of producing Fig. 7(b) is different from Fig. 4, which is obtained directly from the ESA. Data acquisition from ESA allows for a longer sweep time ($=1$ s) to record a wide spectrum during which the phase scan of θ runs

through many fringes as shown in Fig. 4. For the photocurrent analysis here, we only take data of 100 ms long, which covers about four interference fringes, and when spread over a wide spectral range of 12 MHz, it is hard to show visibility change of interference fringes over the spectral range. To show more fringes in a small spectral range as in Fig. 4, it would require recycling the data in 100 ms for different parts of the spectrum. The specific approach is by employing measured fringe once for a smaller frequency section [~ 2 MHz in Fig. 7(b)] to see the visibility variation with enough periods. Afterward, a sequential combination of these frequency sections (0–2 MHz, 2 MHz–4 MHz, 4 MHz–6 MHz, ...) to obtain the full frequency range of the spectrum is applied to see the visibility modulation. Through this approach, the actual “sweep time” of Fig. 7(b) is around $100 \text{ ms} \times \frac{12 \text{ MHz}}{2 \text{ MHz}} = 600 \text{ ms}$.

APPENDIX B: THEORY OF POSTDETECTION DATA ANALYSIS BY DIRECT FOURIER TRANSFORMATION OF THE PHOTOCURRENT

The theory presented in Sec. II is the output of an electronic spectral analyzer. What will be the result if we make a direct Fourier transformation of the photocurrent from the balanced homodyne detector? To find out, let us go back to Eq. (3) for the expression of the photocurrent of the homodyne detection of the output of an unbalanced MZI. The average over there is over the quantum state of the input field, which, in experiment, is equivalent to the average of the current over the time that is longer than the coherent time. However, we cannot make a Fourier transformation of the averaged current because it will lose the detailed fluctuations of the field. We consider instantaneous current, which corresponds to the operator before state average:

$$i_{\text{HD}}(t) \propto |\mathcal{E}| \int d\tau k(t - \tau) [\hat{X}_{\text{in}}^{\varphi}(\tau) + \hat{X}_{\text{in}}^{\varphi'}(\tau - \Delta T)], \quad (\text{B1})$$

where we include the finite response function $k(t)$ of the detectors and the quadrature-phase amplitudes are defined as

$$\begin{aligned} \hat{X}_{\text{in}}^{\varphi}(t) &= \hat{E}_{\text{in}}(t)e^{i\omega_0 t - i\varphi} + \hat{E}_{\text{in}}^{\dagger}(t)e^{-i\omega_0 t + i\varphi}, \\ \hat{X}_{\text{in}}^{\varphi'}(t) &= \hat{E}_{\text{in}}(t)e^{i\omega_0(t + \Delta T) - i\varphi} + \hat{E}_{\text{in}}^{\dagger}(t)e^{-i\omega_0(t + \Delta T) + i\varphi}. \end{aligned} \quad (\text{B2})$$

To prepare the Fourier transformation, we write the field operator of the input field in terms of the frequency components: $\hat{E}(t) = \frac{1}{\sqrt{2\pi}} \int d\omega \hat{a}(\omega)e^{-i\omega t}$. Then, we have

$$\begin{aligned} \hat{X}_{\text{in}}^{\varphi}(t) &= \frac{1}{\sqrt{2\pi}} \int d\omega \hat{a}(\omega)e^{-i\omega t + i\omega_0 t - i\varphi} + \text{H.c.} \\ &= \frac{1}{\sqrt{2\pi}} \int d\Omega \hat{a}(\omega_0 + \Omega)e^{-i\varphi - i\Omega t} + \text{H.c.} \\ &= \frac{1}{\sqrt{2\pi}} \int d\Omega [\hat{a}(\omega_0 + \Omega)e^{-i\varphi - i\Omega t} \\ &\quad + \hat{a}^{\dagger}(\omega_0 - \Omega)e^{i\varphi - i\Omega t}] \\ &\equiv \frac{1}{\sqrt{2\pi}} \int d\Omega e^{-i\Omega t} \hat{X}_{\text{in}}^{\varphi}(\Omega), \end{aligned} \quad (\text{B3})$$

where we made a change of the integration variables: $\omega - \omega_0 \rightarrow \Omega$ in the second line and $-\Omega \rightarrow \Omega$ in the second term

of the third line, and $\hat{X}_{\text{in}}^\varphi(\Omega) \equiv \hat{a}_{\text{in}}^\dagger(\omega_0 - \Omega)e^{i\varphi} + \hat{a}_{\text{in}}(\omega_0 + \Omega)e^{-i\varphi}$. Now, we apply the Fourier transform to the photocurrent. For fast detector assumption, the response function $k(t)$ is depicted as a δ function with a normalization factor $Q_1 = \int dt k(t)$:

$$\begin{aligned} i_{\text{HD}}(\Omega) &= \frac{1}{\sqrt{2\pi}} \int dt e^{i\Omega t} i_{\text{HD}}(t) \\ &\propto \frac{1}{\sqrt{2\pi}} \int dt e^{i\Omega t} |\mathcal{E}| \int d\tau [k(t - \tau) \hat{X}_{\text{in}}^{\varphi'}(\tau) \\ &\quad + k(t - \tau - \Delta T) \hat{X}_{\text{in}}^{\varphi'}(\tau)] \\ &\approx \frac{Q_1 |\mathcal{E}|}{\sqrt{2\pi}} \int dt e^{i\Omega t} [\hat{X}_{\text{in}}^\varphi(t) + \hat{X}_{\text{in}}^{\varphi'}(t - \Delta T)] \\ &= Q_1 |\mathcal{E}| [\hat{X}_{\text{in}}^\varphi(\Omega) + e^{i\Omega \Delta T} \hat{X}_{\text{in}}^{\varphi'}(\Omega)]. \end{aligned} \quad (\text{B4})$$

The modulus square is $|i_{\text{HD}}(\Omega)|^2$. Notice here that $\hat{X}_{\text{in}}^\varphi(t)$ satisfies $[\hat{X}_{\text{in}}^\varphi(\Omega)]^\dagger = \hat{X}_{\text{in}}^\varphi(-\Omega)$; then, we obtain the following expression:

$$\begin{aligned} |i_{\text{HD}}(\Omega)|^2 &\propto [\hat{X}_{\text{in}}^\varphi(-\Omega) + e^{-i\Omega \Delta T} \hat{X}_{\text{in}}^{\varphi'}(-\Omega)] \\ &\quad \times [\hat{X}_{\text{in}}^\varphi(\Omega) + e^{i\Omega \Delta T} \hat{X}_{\text{in}}^{\varphi'}(\Omega)] \\ &= \hat{X}_{\text{in}}^\varphi(-\Omega) \hat{X}_{\text{in}}^\varphi(\Omega) + \hat{X}_{\text{in}}^{\varphi'}(-\Omega) \hat{X}_{\text{in}}^{\varphi'}(\Omega) \\ &\quad + e^{-i\Omega \Delta T} \hat{X}_{\text{in}}^{\varphi'}(-\Omega) \hat{X}_{\text{in}}^\varphi(\Omega) \\ &\quad + e^{i\Omega \Delta T} \hat{X}_{\text{in}}^\varphi(-\Omega) \hat{X}_{\text{in}}^{\varphi'}(\Omega). \end{aligned} \quad (\text{B5})$$

Next, we evaluate each of the four terms. We start with the first term, which is from Eq. (B3):

$$\begin{aligned} \hat{X}_{\text{in}}^\varphi(\Omega) \hat{X}_{\text{in}}^\varphi(-\Omega) &= \frac{1}{2\pi} \int dt dt' e^{i\Omega(t-t')} \hat{X}_{\text{in}}^\varphi(t) \hat{X}_{\text{in}}^\varphi(t') \\ &= \frac{1}{2\pi} \int d\tau e^{i\Omega \tau} d\tau' \hat{X}_{\text{in}}^\varphi(t + \tau) \hat{X}_{\text{in}}^\varphi(t) \\ &= \frac{1}{2\pi} \int d\tau e^{i\Omega \tau} \langle \hat{X}_{\text{in}}^\varphi(t + \tau) \hat{X}_{\text{in}}^\varphi(t) \rangle_\Psi. \end{aligned} \quad (\text{B6})$$

Note that the time integral corresponds to the ensemble average over the input quantum state in the last line. The temporal cross-correlation above can be related to the field's second-order coherence function $\Gamma(\tau) = \langle \hat{E}_{\text{in}}^\dagger(t + \tau) \hat{E}_{\text{in}}(t) \rangle_\Psi$

as follows:

$$\begin{aligned} \langle \hat{X}_{\text{in}}^\varphi(t + \tau) \hat{X}_{\text{in}}^\varphi(t) \rangle_\Psi &= \langle : \hat{X}_{\text{in}}^\varphi(t + \tau) \hat{X}_{\text{in}}^\varphi(t) : \rangle_\Psi \\ &\quad + [\hat{E}_{\text{in}}(t + \tau), \hat{E}_{\text{in}}^\dagger(t)] e^{i\omega_0 \tau} \\ &= \Gamma(\tau) e^{-i\omega_0 \tau} + \Gamma(-\tau) e^{i\omega_0 \tau} + \delta(\tau) e^{i\omega_0 \tau}. \end{aligned} \quad (\text{B7})$$

As discussed in Sec. II A, the Fourier transform of the second-order coherence function $\Gamma(\tau)$ is the spectrum $n(\omega)$ of the field: $n(\omega) = \int d\tau \Gamma(\tau) e^{-i\omega \tau}$. So, we have

$$\int d\tau \Gamma(\pm \tau) e^{\mp i\omega_0 \tau} e^{i\Omega \tau} = n(\omega_0 \mp \Omega). \quad (\text{B8})$$

Hence,

$$\hat{X}_{\text{in}}^\varphi(\Omega) \hat{X}_{\text{in}}^\varphi(-\Omega) = \frac{1}{2\pi} [n(\omega_0 - \Omega) + n(\omega_0 + \Omega) + 1]. \quad (\text{B9})$$

Similarly,

$$\begin{aligned} \hat{X}_{\text{in}}^{\varphi'}(\Omega) \hat{X}_{\text{in}}^{\varphi'}(-\Omega) &= \frac{1}{2\pi} [n(\omega_0 - \Omega) + n(\omega_0 + \Omega) + 1], \\ \hat{X}_{\text{in}}^{\varphi'}(\Omega) \hat{X}_{\text{in}}^\varphi(-\Omega) &= \frac{1}{2\pi} [n(\omega_0 - \Omega) e^{-i\omega_0 \Delta T} + n(\omega_0 + \Omega) \\ &\quad \times e^{i\omega_0 \Delta T} + 1], \\ \hat{X}_{\text{in}}^\varphi(\Omega) \hat{X}_{\text{in}}^{\varphi'}(-\Omega) &= \frac{1}{2\pi} [n(\omega_0 - \Omega) e^{i\omega_0 \Delta T} + n(\omega_0 + \Omega) \\ &\quad \times e^{-i\omega_0 \Delta T} + 1]. \end{aligned} \quad (\text{B10})$$

Combining all the results, we have the modulus square of the Fourier transform as

$$\begin{aligned} |i_{\text{HD}}(\Omega)|^2 &\propto \frac{1}{\pi} n(\omega_0 - \Omega) \{1 + \cos[(\Omega - \omega_0) \Delta T]\} \\ &\quad + \frac{1}{\pi} n(\omega_0 + \Omega) \{1 + \cos[(\Omega + \omega_0) \Delta T]\} + \frac{2}{\pi} \\ &= \frac{2}{\pi} \{1 + 2n_+ [1 + \cos(\Omega \Delta T) \cos(\omega_0 \Delta T)] \\ &\quad + 2n_- [1 + \sin(\Omega \Delta T) \sin(\omega_0 \Delta T)]\}. \end{aligned} \quad (\text{B11})$$

With the first term in the expression above interpreted as the shot noise and $n_\pm \equiv n(\omega_0 + \Omega) \pm n(\omega_0 - \Omega)$, we recover Eqs. (5) and (12) for the spectrum of the photocurrent measured by an ESA. This confirms that direct FFT of the photocurrent is the same as the result by feeding the current to an electronic spectral analyzer, as demonstrated experimentally in Sec. III B.

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