

Characterizing skyrmion flow phases with principal component analysis

C. J. O. Reichhardt¹, D. McDermott², and C. Reichhardt^{1,*}

¹Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

²X-Theoretical Design Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA



(Received 30 October 2025; accepted 5 January 2026; published 16 January 2026)

Principal component analysis (PCA) is a powerful method that can identify patterns in large, complex data sets by constructing low-dimensional order parameters from higher-dimensional feature vectors. There are increasing efforts to use space-and-time-dependent PCA to detect transitions in nonequilibrium systems that are difficult to characterize with equilibrium methods. Here, we demonstrate that feature vectors incorporating the position and velocity information of driven skyrmions moving through random disorder permit PCA to resolve different types of disordered skyrmion motion as a function of driving force and the ratio of the Magnus force to the dissipation. Since the Magnus force creates gyroscopic motion and a finite Hall angle, skyrmions can exhibit a greater range of flow phases than what is observed in overdamped driven systems with quenched disorder. We show that in addition to identifying previously known skyrmion flow phases, PCA detects several additional phases, including different types of channel flow, moving fluids, and partially ordered states. Guided by the PCA analysis, we further characterize the disordered flow phases to elucidate the different microscopic dynamics and show that the changes in the PCA-derived order parameters can be connected to features in bulk transport measures, including the transverse and longitudinal velocity-force curves, differential conductivity, topological defect density, and changes in the skyrmion Hall angle as a function of drive. We discuss how asymmetric feature vectors can be used to improve the resolution of the PCA analysis, and how this technique can be extended to find disordered phases in other nonequilibrium systems with time-dependent dynamics.

DOI: [10.1103/wpkc-tvwt](https://doi.org/10.1103/wpkc-tvwt)

I. INTRODUCTION

Nonequilibrium systems can undergo spatial and temporal changes as a function of changing external parameters, and one of the biggest challenges in the field is to understand how to characterize the resulting different types of nonequilibrium phases and the transitions between them. In equilibrium systems, different states can be characterized by spatial symmetries, and transitions between these states can be understood in terms of the breaking of these symmetries. When structural disorder is present, however, even for equilibrium systems it can be difficult to determine what quantities to measure and how particular measures correlate with different states or transitions. In nonequilibrium systems, the problem becomes even more challenging since the states often do not have long-range order and can also change with time.

A powerful tool that can be used to identify patterns in large and complex data sets is principal component analysis (PCA), an unsupervised machine learning technique [1,2]. This method maps a high-dimensional set of feature vectors onto a much lower-dimensional set of order parameters

that best explain the variance present in the data. PCA has been applied to biological systems [3] and used for pattern recognition [4]. It has also been used to identify phases and phase transitions in condensed matter systems [5–8]. In three-dimensional systems of hard disks and of patchy colloidal particles, a PCA-derived order parameter showed sharp changes across equilibrium phase transitions [9], and could detect the different phases without using standard measures such as structure factors. PCA has also been used to find phase transitions in spin systems [8], in agreement with standard phase transition measures but again without using the standard measures. These results suggest that PCA could be used to find transitions among different states in equilibrium systems under situations where standard measures do not work or when it is not known *a priori* what a good order parameter is. In other work, PCA was shown to capture signatures of different nonequilibrium states and the transitions between them for several well-known models, including the random organization model and a driven transition from an isotropic to a nematic state in elliptical disks [10]. This again suggests that a PCA analysis could be extended to other nonequilibrium systems where different phases can arise. McDermott *et al.* used a position-based PCA approach to identify different types of flowing states for disks moving over random disorder [11]. A similar PCA analysis revealed the presence of distinct motility-induced phase separation states in active matter [12].

A particularly well-studied example of a system that undergoes transitions between different nonequilibrium states is interacting particles driven over random quenched disorder.

*Contact author: cjrx@lanl.gov

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

der. The transitions can occur between different types of pinned crystals, pinned glasses, heterogeneous plastic flow phases, moving fluids, and moving crystals [13,14]. For driven vortices in type-II superconductors [15–22], the depinning and sliding phases can be detected readily in the transport signatures, which are proportional to velocity versus force curves. The different dynamic states produce changes in the noise fluctuations and in the structure, such as by altering the number of particles with six neighbors and the orientation of the topological defects. Other systems that exhibit similar behavior include colloidal particles moving over random substrates [23–25], Wigner crystals in low-dimensional materials [26–28], active matter particles moving over disordered substrates [29,30], driven pattern-forming systems [31,32], driven emulsions [33], flow in porous media [34], and frictional models [35]. Related behavior can occur for sliding charge density waves [36,37], interface depinning [38], dislocation depinning and motion [39,40], and earthquake models [13]. There is strong evidence that depinning transitions can exhibit critical properties, associated with the scaling of the velocity-force curves and growing dynamical correlation lengths, and that the system can show different scaling depending on whether the depinning is plastic or elastic [13,14]. For drives above depinning, different types of flow states are possible, including motion in quasi-one-dimensional (1D) channels where pinned and moving particles coexist, solitonlike flow, fluidlike flow, and polycrystalline flow. At higher drives, the system can dynamically order into moving smectic or moving crystal phases. In general, the three most clearly identifiable phases are a pinned state at low drives, disordered or plastic flow at intermediate drives, and ordered flow at high drives. There is some evidence that the plastic flow regime is not a single type of flow but changes with increasing drive; however, due to the strong spatial disorder present in the system during plastic flow, traditional measures are unable to distinguish between different disordered flow phases.

Recently, Reichhardt *et al.* performed a particle-and-velocity-based (PVB) PCA of driven superconducting vortices moving over random disorder [41]. This system was chosen because it has been extensively studied in simulations and experiments and shown to have transitions among three well-known phases, pinned, plastic, and moving smectic, as a function of quenched disorder strength and external drive [15–22]. It also has weak and strong pinning regimes. For weak disorder, where the vortices or particles retain their triangular lattice structure, the depinning is elastic and the particles keep their same neighbors at depinning [13,42], while for strong disorder, the system breaks up at depinning. Both pinning regimes produce distinctive features in the transport curves and velocity distribution functions [14]. The three principal components of the PVB PCA were used to construct three order parameters that could identify the pinned, plastic, and moving smectic phases, as well as several distinct disordered flow regimes. In the plastic flow phase, PVB PCA was able to distinguish between what was termed nonergodic plastic flow, where the system is disordered but certain particles or regions remain immobile or inaccessible, and an ergodic flow regime, where the system is disordered but all particles participate in the motion over time. The ergodic phase could be further subdivided into a phase where some particles

are temporarily pinned and a phase where all the particles are constantly moving but where sufficiently strong velocity heterogeneities are present to maintain the system in a disordered state. In the nonergodic plastic flow regime, the PVB PCA identified a transition from quasi-1D plastic channels to fully two-dimensional (2D) filamentary plastic flow. The PVB PCA phase identification relied crucially on the fact that information about both the positions and velocities of the particles was included in the feature vector. Once the different flow states had been identified with PVB PCA, it was possible to compare the flow behavior with features in the transport curves.

Another type of particlelike system that can pass through different dynamical phases when driven over quenched disorder is magnetic skyrmions, which are topological magnetic textures that were discovered in chiral magnets in 2009 with neutron scattering measurements [43]. The skyrmions were subsequently observed using direct imaging techniques [44]. Skyrmions form a triangular lattice and can be driven with applied currents [45–52], interact with defects, and exhibit depinning thresholds [47,49,53–58]. Different types of sliding dynamic skyrmion phases have also been observed [49,56,59–63]. Due to their small size, stability, and ability to be manipulated with external drives, skyrmions are promising candidates for various applications [64–68]. As a result, there is significant interest in understanding the dynamics of skyrmions that are interacting with pinning sites or nanostructures [56,69].

Although magnetic skyrmions share many similarities with vortices in type-II superconductors, their dynamics can be significantly different due to the presence of a strong Magnus component that can induce gyroscopic motion [51,56,70,71]. The resulting Magnus force can cause the skyrmions to move at a finite Hall angle with respect to the driving force [51,70–72], in contrast to the zero Hall angle overdamped dynamics typically observed in previously studied driven particlelike systems with quenched disorder. The Magnus force can lead to interesting effects, such as a side-jump behavior for skyrmions moving over pinning sites. The side-jump distance increases as the skyrmions travel more slowly, resulting in a drive-dependent skyrmion Hall angle [59,73–80]. The Magnus force can also produce a swirling motion of the skyrmions around pinning sites or defects [56], leading to partial clustering effects in the presence of strong quenched disorder [56,60,81]. Simulations comparing the driven dynamics of skyrmions to that of vortices showed that each system exhibited a pinned state, plastic motion, and a high drive dynamically reordered phase [59,73,82]. The skyrmions dynamically reorder into a triangular lattice, rather than the moving smectic state found for the vortices. This was attributed to the Magnus force rotating the pinning-induced skyrmion velocity fluctuations into the direction perpendicular to the drive [82], giving a more isotropic effective drive-induced temperature compared to the strongly anisotropic velocity fluctuations along the driving direction caused by pinning in overdamped systems [19–22]. The presence of a Magnus force also modifies the velocity noise spectra [82,83]. When the Magnus-to-damping ratio is increased, a larger portion of the pinning-induced fluctuations get rotated perpendicular to the drive, increasing the extent

of the fluid flow regime [56,59,81] such that for drives at which overdamped vortices would form a smectic state, the skyrmions form a partially ordered fluid. The different phase transitions in skyrmions are associated with signatures in the velocity-force curves, and there can be distinct features not only in the velocity along the driving direction but also in the velocity perpendicular to the driving [56,59]. The behavior of the skyrmion Hall angle is also correlated with the different types of motion. The Hall angle is low or close to zero in the strongly plastic flow state, increases in the moving liquid phase, and saturates to a level close to the intrinsic skyrmion Hall angle in the driven crystal phase [56,59].

In this work, we construct position-and-velocity-based feature vectors and perform PVB PCA for driven skyrmions moving over random disorder at varied driving forces and different Magnus to damping force ratios. We use the first three principal components as order parameters and show that we can construct a dynamical phase diagram based on the PVB PCA results. Some of the features in the principal components, including peaks, dips, and zero crossings, are correlated with features in the longitudinal and transverse velocity-force curves, differential transport, fraction of topological defects, and changes in the skyrmion Hall effect. We found that by adjusting the relative amount of position and velocity information in the feature vector, we could improve the ability of PVB PCA to resolve different plastic flow states relative to our original studies on overdamped systems [41]. We show that PVB PCA successfully identifies the dynamic phases known from previous work on driven skyrmions, including a pinned phase, a plastic flow regime, an extended moving fluid regime, and a moving crystal phase. In addition, PVB PCA uncovers several regions inside the plastic flow state, including nonergodic and ergodic plastic flow phases where the flow channels change in structure and also undergo a tilting due to the skyrmion Hall effect. At higher drives, PVB PCA distinguishes two different states in the extended fluid regime: a dissipation-dominated fluid, and a Magnus-dominated fluid. Since the Magnus force generates local rotations that create topological defects, the system cannot completely recrystallize when the Magnus-to-damping force ratio is large, and the drive needed to reach the reordered state increases with increasing Magnus-to-damping force ratio. In contrast, at lower Magnus to damping force ratios, there is a competition between a tendency to form a smectic state that is aligned with the driving force and a tendency to form a crystal that is rotated slightly perpendicularly to the driving force, leading to the appearance of a partially ordered state.

II. METHODS

A. Simulation

We model N skyrmions in a 2D system of size $L \times L$ using a modified Thiele equation approach [56,59,84–87]. The system has periodic boundary conditions in the x and y directions with $L = 36\lambda$, where λ is the effective skyrmion diameter. The skyrmion density is given by $\rho = N/L^2$. We fix $N = 1216$ and $\rho = 1.0$ throughout this work. The motion for skyrmion i is numerically integrated using the following overdamped equation:

$$\alpha_d \mathbf{v}_i + \alpha_m \hat{\mathbf{z}} \times \mathbf{v}_i = \mathbf{F}_i^{ss} + \mathbf{F}_i^{sp} + \mathbf{F}^D. \quad (1)$$

Here, $\mathbf{v}_i = d\mathbf{r}_i/dt$, where $\mathbf{r}_i$ is the position of skyrmion i . The first term on the left represents the damping force, which aligns the velocity with the direction of the net external forces. The second term is the Magnus force, which aligns the velocity perpendicular to the net external forces. The repulsive skyrmion-skyrmion interaction force is given by $\mathbf{F}_i^{ss} = \sum_{j \neq i}^N F_0 K_1(r_{ij}/\lambda) \hat{\mathbf{r}}_{ij}$, where F_0 is a force coefficient that can depend on material parameters, $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ is the distance between particles i and j , and $\hat{\mathbf{r}}_{ij} = (\mathbf{r}_i - \mathbf{r}_j)/r_{ij}$. The interaction is described by the modified Bessel function of the first kind, $K_1(r)$, which decays exponentially at large R , as derived from continuum-based models [84].

The force from the quenched disorder, $\mathbf{F}^{sp}$, is produced by N_{pin} randomly placed nonoverlapping pinning sites modeled as attractive parabolic potential wells with a maximum range of $R_p = 0.3\lambda$ and a maximum strength of F_p . The pinning force is $\mathbf{F}_i^{sp} = \sum_{k=1}^{N_{\text{pin}}} (F_p/R_p) \Theta(|\mathbf{r}_{ik}^{(p)}| - R_p) \hat{\mathbf{r}}_{ik}^{(p)}$, where $\mathbf{r}_{ik}^{(p)} = \mathbf{r}_i - \mathbf{r}_k^{(p)}$ is the distance between vortex i and pin k , $\hat{\mathbf{r}}_{ik}^{(p)} = (\mathbf{r}_i - \mathbf{r}_k^{(p)})/|\mathbf{r}_{ik}^{(p)}|$, and Θ is the Heaviside step function. Throughout this work, we fix $F_p = 1.0$. The pinning density $\rho_p = N_{\text{pin}}/L^2$ is fixed at $\rho_p = 0.5$, the same value used in Ref. [41]. The external driving force is applied on all the skyrmions in the x direction, $\mathbf{F}^D = F_D \hat{\mathbf{x}}$. We start from $F_D = 0.0$ and increase the drive in increments of $\Delta F_D = 0.005 F_p$, spending 1×10^5 simulation time steps at each increment. The data is written out every 1000 simulation time steps, giving $N_f = 100$ frames of data for every value of the drive.

To characterize the dynamic flow, we perform standard measures including the average velocity parallel, $\langle V_x \rangle = \langle N^{-1} \sum_i^N v_i \cdot \hat{\mathbf{x}} \rangle$, and perpendicular, $\langle V_y \rangle = \langle N^{-1} \sum_i^N v_i \cdot \hat{\mathbf{y}} \rangle$, to the driving direction, where the averaging is performed over each value of F_D . The average fraction of sixfold coordinated particles is $\langle p_6 \rangle = \langle N^{-1} \sum_i^N \delta(z_i - 6) \rangle$, where z_i is the coordination number of skyrmion i obtained from a Voronoi tessellation. For a perfect triangular lattice, $p_6 = 1.0$. Throughout this work, we fix $\alpha_d^2 + \alpha_m^2 = 1.0$ for varied values of α_d and α_m . This same normalization was used in previous work to allow for a comparison of the net velocity-force curves $\langle V \rangle$ versus F_D , where $\langle V \rangle = \sqrt{\langle V_x \rangle^2 + \langle V_y \rangle^2}$. For a system with no pinning, $\langle V \rangle = F_D$ for all drives and the velocity-force curve is linear. We also measure the differential velocities $d\langle V_x \rangle/dF_D$ and $d\langle V_y \rangle/dF_D$ versus F_D . For an overdamped system, features in the velocity-force and differential transport curves have been correlated with changes in the dynamical flow phases. The intrinsic skyrmion Hall angle is $\theta_{\text{intr}} = \tan^{-1}(\alpha_m/\alpha_d)$; however, when disorder is present, there is a finite driving threshold for motion, the velocity-force curves become nonlinear, and the measured skyrmion Hall angle θ_{sk} develops a drive dependence such that $\langle \theta_{sk} \rangle = \tan^{-1}(\langle V_y \rangle / \langle V_x \rangle)$.

B. Principal component analysis

For the principal component analysis, we use an improvement of the position-and-velocity-based feature vectors developed in our previous study [41]. PCA identifies the directions of maximum variance in the space defined by the feature vectors [1]. In this way, a large amount of data can be

reduced into a low-dimensional representation. The maximum variance can be associated with the physical features of the system and can show signatures when the spatial pattern and the dynamics are changing in character, which could correspond to a shift from one nonequilibrium phase to another. We use the particle-based PCA approach originally developed for off-lattice nonequilibrium systems [9], which has been used to identify a variety of phases and transitions [9,11,12]. The difference between the present work and the PVB PCA analysis applied to driven superconducting vortex systems [41] is a modification of the PVB PCA feature vector and the introduction of a Magnus force α_m . In the limit of $\alpha_m = 0.0$, the behavior becomes equivalent to that of a superconducting vortex system.

To construct our position-and-velocity-based feature vector, from individual frames of the simulation we first randomly select N_p probe skyrmions and begin with the position component. For each probe particle, we compute the distance from the probe particle to all of the other particles, $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$, and then sort these distances in ascending order. We then place only the n smallest distances into an array in increasing order of distance:

$$\vec{F}_i^{\text{raw}} = [r_{i0}, r_{i1}, r_{i2}, \dots, r_{ij}, \dots, r_{in}] \quad (2)$$

where $r_{i0} < r_{i1} < \dots < r_{in}$. The neighbor particles that appear in the array form the neighbor set $\mathcal{N}_i$ for probe particle i . Importantly, from the neighbor set $\mathcal{N}_i$, we also construct a reduced neighbor set $\mathcal{M}_i$ containing only the m smallest distances, with $m < n$. The second neighbor set will be used below. This procedure is repeated for all N_p probe particles. From the resulting arrays, we obtain the average distance to neighbor j in frame k of the movie as $\langle r_j \rangle = N_p^{-1} \sum_i r_{ij}$. We normalize the averaged distances according to $\bar{r}_j = (\langle r_j \rangle - \mu_r) / \sigma_r$ where μ_r is the mean and σ_r is the standard deviation of the N_p values of $\langle r_j \rangle$.

To construct the velocity component of the feature vector, for each probe particle we work only with the reduced neighbor set $\mathcal{M}_i$ identified above. We determine the net velocity $V_i = \sqrt{v_{xi}^2 + v_{yi}^2}$ of each neighbor particle, and sort the velocities into an array in increasing order:

$$\vec{G}_i^{\text{raw}} = [V_{i0}, V_{i1}, V_{i2}, \dots, V_{ij}, \dots, V_{im}] \quad (3)$$

where $V_{i0} < V_{i1} < \dots < V_{im}$. We repeat this procedure for all N_p probe particles and obtain the average velocity of neighbor j in frame k of the movie as $\langle V_j \rangle = N_p^{-1} \sum_i V_{ij}$. The average velocities are normalized according to $\bar{V}_j = (\langle V_j \rangle - \mu_V) / \sigma_V$, where μ_V is the mean and σ_V is the standard deviation of the N_p values of $\langle V_j \rangle$.

The position and velocity information is then assembled into an asymmetric feature vector for each frame, with the position information first, followed by the velocity information:

$$\vec{f}_k = [\bar{r}_{k0}, \bar{r}_{k1}, \bar{r}_{k2}, \dots, \bar{r}_{kj}, \dots, \bar{r}_{kn}, \bar{V}_{k0}, \bar{V}_{k1}, \bar{V}_{k2}, \dots, \bar{V}_{kj}, \dots, \bar{V}_{km}] \quad (4)$$

where $\bar{r}_{k0} < \bar{r}_{k1} < \dots < \bar{r}_{kn}$ and $\bar{V}_{k0} < \bar{V}_{k1} < \dots < \bar{V}_{km}$. We construct a matrix $\mathbf{M}$ in which each row is the vector $\vec{f}_k$ obtained by processing each frame from the entire simula-

tion data set. There are a total of 8×10^4 rows in matrix $\mathbf{M}$, since there are $N_f = 100$ frames per current and there are 800 current values in the velocity sweep. We find that for the skyrmion system, as for the superconducting vortex system, the relatively long-range particle-particle interactions smooth the density fluctuations enough that it is not necessary to prewhiten the feature vector, a procedure used in previous work for particles with short-range interactions to remove spurious geometric information caused by strong particle clustering [9,11,12].

We tested a variety of PVB feature vectors for the skyrmion system before determining that an asymmetric feature vector based on distances and absolute velocity gave the best resolution of the dynamic states. In our previous study [41], the feature vector was symmetric and contained n entries for each of the distance, x -velocity, and y -velocity components. The first issue with using a similar feature vector for the skyrmion system is the nonzero Magnus force on the skyrmions. In the overdamped superconducting vortex system, the particles always move parallel to the x -direction driving force, so a separation of the velocity into x and y components provided information on the motion parallel and perpendicular to the drive. In the skyrmion system, the particles move at a drive-dependent skyrmion Hall angle to the x -direction driving force, meaning that as the drive increases, the x and y components of the velocity contain blended amounts of the motion parallel and perpendicular to the effective driving direction. This blurred the reordering transition. The blurring is much more noticeable in the skyrmion system than in the superconducting vortex system, since the skyrmions have a relatively sharp reordering transition. To remedy this, we first tested breaking the velocities into the true parallel and perpendicular components at each drive. This was achieved by determining the average direction of travel in the frames belonging to a given current value, and then rotating our velocity reference frame into the travel direction. This rather complicated procedure gave some improvement but the reordering transition was still blurred. As will be described in Sec. III B, we later used the calculation of the average direction and speed of travel at each current for the purposes of image generation.

Still working with a symmetric feature vector, in which both the distances and velocities contained n entries, we tested a variety of choices of n and arrived at the following insight: Using larger n , which is equivalent to running the feature vector out to larger distances, gives improved resolution of the crystalline structure of the lattice, but washes out information on the plastic distortions of the moving particles. In contrast, using smaller n , which restricts the feature vector to smaller distances, gives good resolution of the local plastic distortions but blurs out information on the longer range crystalline ordering of the lattice. Since there is no rule stating that the feature vector must be symmetric, we settled on an asymmetric feature vector in which the number of distance components n is larger than the number of velocity components m . This gives us good resolution of the crystalline structure from the distance component, and good resolution of the plastic flow behavior from the velocity component. We also found that, with the asymmetric feature vector, it is sufficient to use only the average velocity V , and breaking the velocity into parallel

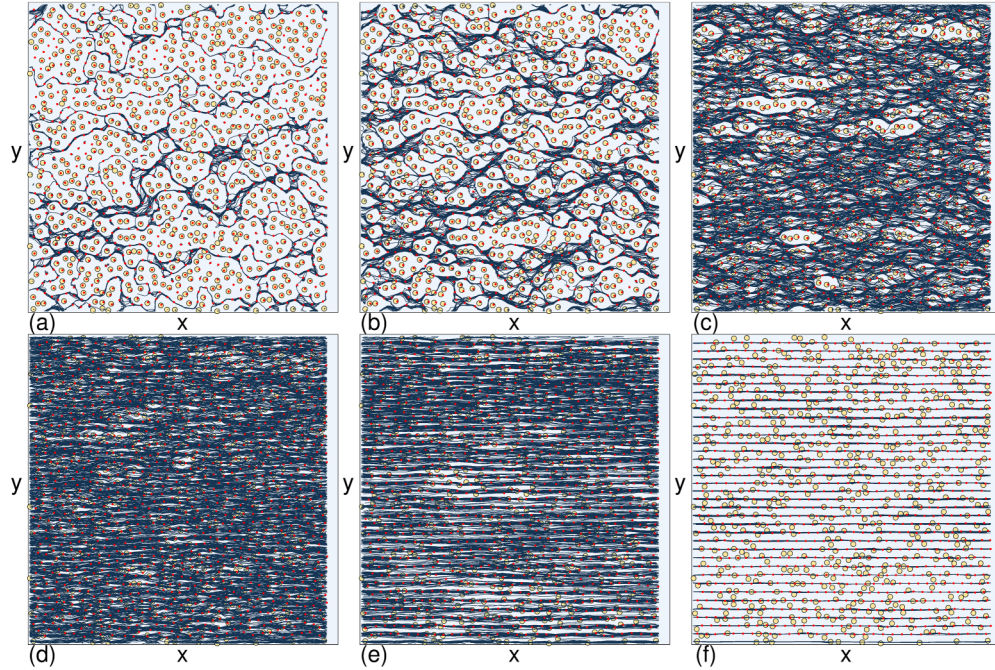


FIG. 1. Particle positions (dots), pinning sites (open circles), and trajectories (lines) for a system with $\alpha_m/\alpha_d = 0.0$ (the superconducting vortex limit) at different drives. The trajectories here and throughout this work are imaged over a time equal to that required for an individual particle moving through a pin-free sample to travel a distance of 50λ when subjected to the driving force F_D . Phase I, the pinned state, is not shown. (a) Phase II, the nonergodic isolated static channel flow phase, at $F_D = 0.165$. (b) Phase III, the lightly braided channel flow phase, at $F_D = 0.31$. (c) Phase IV, the heavily braided channel flow phase, at $F_D = 0.48$. (d) Phase V, the inhomogeneous ergodic plastic flow phase, at $F_D = 0.61$. (e) Phase VI, the emerging ordered flow phase, at $F_D = 0.85$. (f) DR, the dynamically reordered phase, at $F_D = 2.0$. Here, the vortices dynamically reorder into a moving smectic state.

and perpendicular components is not necessary. Finally, with our improved feature vector, we were able to train the PVB PCA model on only a single value of α_m/α_d and apply this pretrained model to all other values of α_m/α_d .

In this work, we use $n = 60$ distance neighbors, $m = 18$ velocity neighbors, and $N_p = 100$ probe particles. We use standard PCA techniques to compute the orthogonal transformation matrix $\mathbf{W}$ mapping our feature vectors of length $s = n + m$ to the principal components based on data from the entire simulated velocity-force sweep for a particular value of α_m/α_d ,

$$\vec{p}_s = \mathbf{W}\vec{f}_s. \quad (5)$$

Here, $\vec{p}_s$ are the principal components and $\mathbf{W} \equiv [\vec{q}_1, \dots, \vec{q}_m]^T$, where the unit vectors $\vec{q}_i$ define the directions of the principal components. We define the PCA order parameters for a current F_D from the average values of the principal component $\vec{p}_\alpha$ at that current according to

$$P_\alpha(F_D) = \langle \vec{p}_\alpha(F_D) \rangle / \sqrt{\lambda_\alpha} \quad (6)$$

$$= N_p^{-1} \sum_{k \in F_D^i} \vec{w}_\alpha^T \vec{f}_k / \sqrt{\lambda_\alpha}. \quad (7)$$

The normalization factor is the square root of λ_α , the eigenvalue associated with principal component α . We find that the first three principal components contain relevant information, so that $\alpha = \{1, 2, 3\}$. Note that the eigenvectors $\vec{q}_i$ are only defined to within a sign and remain eigenvectors if they are reflected across the origin. As a result, the order parameters

are *also only defined to within a sign*, and although there is significance to the order parameter passing through zero, since it means that the effective dimensionality of the data has been reduced by one, there is no special significance to the fact that the order parameter is positive or negative overall. The PVB PCA model is pretrained on the $\alpha_m/\alpha_d = 1.0$ system and then applied to all systems with different values of α_m/α_d . Pretraining is crucial for studying large values of α_m/α_d because in this regime, the dynamic reordering transition shifts to very high driving forces that fall outside of our simulation window.

III. RESULTS

A. Transport curves and topological defects

As a point of reference, we first illustrate the dynamic flow phases identified with the assistance of PVB PCA in the superconducting vortex system [41], in order to compare and contrast these states with what we observe for the skyrmion system. In Fig. 1, we plot the particle positions, trajectories, and pinning site locations for a system with $\alpha_m/\alpha_d = 0.0$, corresponding to the overdamped or superconducting vortex limit. Phase I (not shown) is the pinned phase in which there is no motion at a finite drive. Figure 1(a) shows the motion at $F_D = 0.165$ in phase II or the isolated channel flow state, a nonergodic flow regime where a large number of particles both at and between the pinning sites do not take part in the motion, and the channel structure is static. In Fig. 1(b) at $F_D = 0.31$ in phase III or the lightly braided channel flow phase, the motion again occurs through channels, but now the

channels change gradually over time. There are still particles both at the pinning sites and in the interstitial regions that do not participate in the motion. For $F_D = 0.48$ in Fig. 1(c) in the heavily braided channel flow phase IV, the flow is more 2D and fluctuating in nature, but there are still linear chains of pinned particles that do not take part in the motion. As a result, the dynamics remains nonergodic, and the velocity distribution function is bimodal with finite weight at $\langle V_x \rangle = 0.0$. Figure 1(d) shows $F_D = 0.61$ in phase V or the inhomogeneous ergodic plastic flow phase. All the particles take part in the motion over time; however, at any given moment there are still pinned particles present, so the velocity distribution function remains bimodal. The system can be regarded as a moving fluid. In Fig. 1(e) at $F_D/F_p = 0.85$ we show phase VI, the emerging ordered flow phase, where all the particles are moving and local patches of topological order are present. At $F_D/F_p = 2.0$ in Fig. 1(f), the system has reached a dynamically reordered (DR) state, which for the superconducting vortex system is a moving smectic. The particles move in well-defined 1D paths along the driving direction, and there are only a small number of topological defects present in the form of dislocations that have their Burgers vectors aligned parallel to the drive. For higher drives, the smectic regime persists since the particles remain locked in the flowing 1D channels. As has been shown previously, the sixfold order parameter $\langle p_6 \rangle$ most clearly captures the emergence of the moving smectic phase. The boundaries between phases II, III, IV, V, and VI are difficult to distinguish from transport and noise measures; however, we showed previously that PVB PCA produces three principal component order parameters that readily identify the distinct plastic flow states [41].

We next enter the skyrmion regime by considering systems with nonzero α_m/α_d . In Figs. 2(a)–2(c), we plot $\langle V_x \rangle$, $\langle V_y \rangle$, and $\langle p_6 \rangle$, respectively, versus F_D at $\alpha_m/\alpha_d = 0.0, 0.5, 2.0, 4.0, 6.0$, and 8.5 . Figures 3(a)–3(c) show the corresponding $d\langle V_x \rangle/dF_D$, $d\langle V_y \rangle/dF_D$, and $\langle \theta_{sk} \rangle$ versus F_D curves. For $\alpha_m/\alpha_d = 0.5$, both V_x and V_y increase with increasing F_D , so there can be a finite Hall angle, as shown in Fig. 3(c). The threshold for motion is higher in the y direction than in the x direction, which is due to the fact that just above depinning, the slowly moving skyrmions initially remain confined by the filamentary 1D channels, giving a zero Hall angle. Previous simulation and experimental studies also showed that above the depinning transition, there is a range of drives for which the Hall angle is zero. This is more significant at finite temperatures in the creep regime, where the particles are able to hop between pinning sites; however, we remain in the zero temperature limit in the present work. The $\langle p_6 \rangle$ versus F_D curve at $\alpha_m/\alpha_d = 0.5$ follows the α_m/α_d curve relatively closely but shows two significant differences: For $\alpha_m/\alpha_d = 0.5$, there is a much sharper jump into the ordered phase, and $\langle p_6 \rangle$ reaches a value that is much closer to $\langle p_6 \rangle = 1.0$ in the DR state at higher drives. This indicates that the system is more topologically ordered in the DR phase compared to the $\alpha_m/\alpha_d = 0.0$ system, since the skyrmion system with $\alpha_m/\alpha_d = 0.5$ forms a reordered moving crystal rather than the reordered moving smectic that appears for the $\alpha_m/\alpha_d = 0.0$ vortex system.

At $\alpha_m/\alpha_d = 0.5$, both the $d\langle V_x \rangle/dF_D$ versus F_D curve in Fig. 3(a) and the $d\langle V_y \rangle/dF_D$ versus F_D curve in Fig. 3(b) pass through a peak or minimum, respectively, before reach-

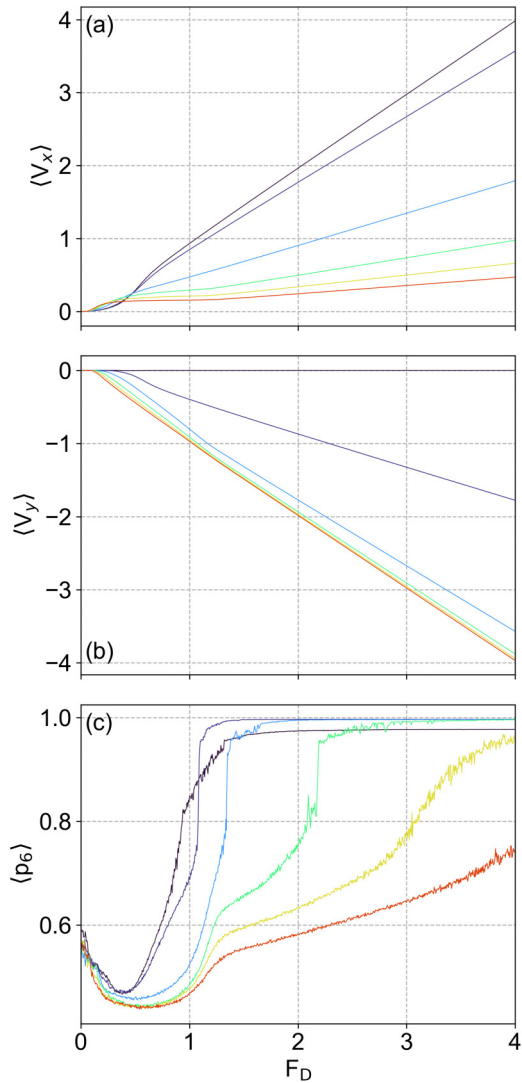


FIG. 2. (a) The longitudinal velocity $\langle V_x \rangle$ vs F_D for systems with $\alpha_m/\alpha_d = 0.0$ (black), 0.5 (purple), 2.0 (blue), 4.0 (green), 6.0 (yellow), and 8.5 (red). (b) The corresponding transverse velocity $\langle V_y \rangle$ vs F_D . (c) The corresponding fraction of particles with six neighbors $\langle p_6 \rangle$ vs F_D .

ing a saturation value at large drives, while in Fig. 3(c), the skyrmion Hall angle $\langle \theta_{sk} \rangle$ starts off at zero just above depinning and then linearly increases in magnitude with increasing F_D , showing a saturation that begins near $F_D = 0.8$ in which $\langle \theta_{sk} \rangle$ approaches the expected intrinsic value of $\theta_{intr} = -26.56^\circ$. The drive dependence of the skyrmion Hall angle in the plastic flow phase has been studied in previous experiments and simulations, where a similar linear increase of the Hall angle magnitude to the intrinsic Hall angle was observed as a function of drive over a similar range of values of α_m/α_d .

In Fig. 4(a), we show the skyrmion positions, trajectories, and pinning site locations in a system with $\alpha_m/\alpha_d = 0.5$ at $F_D = 0.15$ in phase II, where the motion is in isolated static channels. A large number of the particles are pinned at the pinning sites and in the interstitial regions between the pinning sites. At this drive, $\langle \theta_{sk} \rangle = 0.0^\circ$, and there is no indication of a

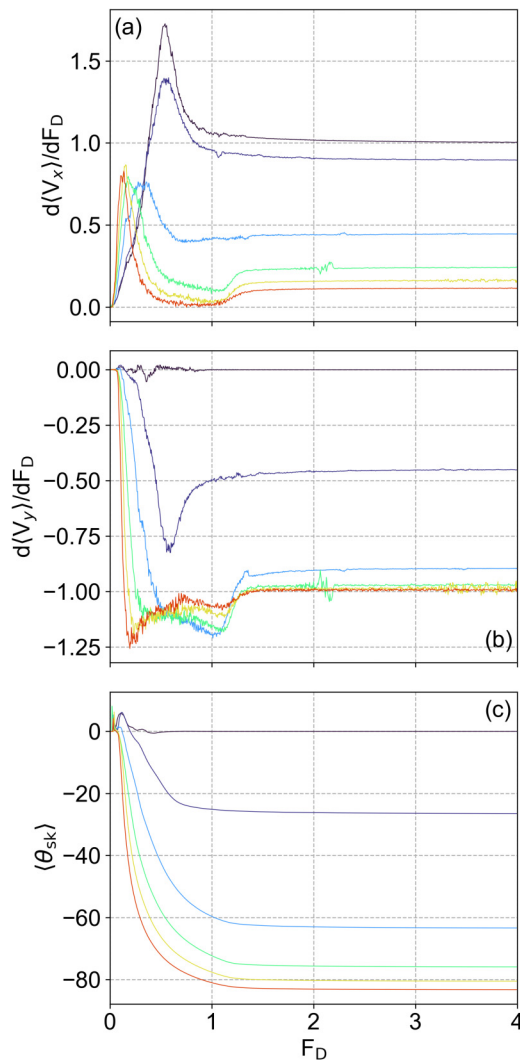


FIG. 3. (a) $d\langle V_x \rangle / dF_D$ vs F_D for the samples from Fig. 2 with $\alpha_m/\alpha_d = 0.0$ (black), 0.5 (purple), 2.0 (blue), 4.0 (green), 6.0 (yellow), and 8.5 (red). (b) The corresponding $d\langle V_y \rangle / dF_D$ vs F_D . (c) The corresponding average skyrmion Hall angle $\langle \theta_{sk} \rangle$ vs F_D .

nonzero Hall angle from the trajectories; all of the particles are moving in the direction of the driving force. Figure 4(b) shows phase III flow at $F_D = 0.31$. The Hall angle is now finite but small, much smaller than the intrinsic value of $\theta_{intr} = -26^\circ$. In phase IV, Fig. 4(c) shows that at $F_D = 0.48$, there are still some permanently pinned particles, but now the linear chains of pinned particles are tilted with respect to the driving or x direction as a result of the increasing Hall angle. At $F_D = 0.63$ in phase V, shown in Fig. 4(d), the system is topologically disordered and undergoes ergodic liquid like flow. In Fig. 4(e) at $F_D = 0.97$ we find phase VI flow, where there is more channeling of the particle motion. The flow clearly has a finite Hall angle. Finally in Fig. 4(f) we show the DR phase at $F_D = 2.0$, where the particles have reordered into a triangular lattice. In this case, we do not observe the well-defined 1D channels of flow that appear for $\alpha_m/\alpha_d = 0.0$. This is because the skyrmion system forms a triangular floating solid rather than a moving smectic state. The smectic phase is still strongly coupled to the pinning, while the moving crystal is far less

affected by the pinning. In addition, the linear change of the Hall angle with increasing drive tends to destabilize the formation of channels since the channels keep ceasing to be aligned with the direction of skyrmion motion as the drive becomes larger.

For $\alpha_m/\alpha_d = 2.0$, in Fig. 2(b) the $\langle V_y \rangle$ versus F_D curve becomes much more prominent, and there is an upward shift of the drive at which the system dynamically reorders, indicated by the point where $\langle p_6 \rangle$ approaches 1.0 in Fig. 2(c). A shoulder feature begins to emerge in $\langle p_6 \rangle$ above $F_D > 1.0$, and there is a sharp jump of $\langle p_6 \rangle$ when the system reaches the ordered phase. There are also changes in the $d\langle V_x \rangle / dF_D$ versus F_D curve shown in Fig. 3(a), including a clear shift in the peak to lower drives compared to the $\alpha_m/\alpha_d = 0.0$ system, as well as the first signs of a shoulder above $F_D > 1.0$. The $d\langle V_y \rangle / dF_D$ versus F_D curve in Fig. 3(b) now has a double dip feature, with a deeper dip following a shallow dip, while $\langle \theta_{sk} \rangle$ versus F_D shows a saturation to the intrinsic value of $\theta_{intr} = -63.43^\circ$ at higher drives. In general, for intrinsic Hall angles smaller in magnitude than $|\theta_{intr}| = 45^\circ$, there is a linear increase of $\langle \theta_{sk} \rangle$ with increasing F_D , while for higher intrinsic Hall angle magnitudes, the linearity is lost and the curve becomes more quadratic at low drives.

In Fig. 5(a), we show the particle positions and trajectories along with the pin locations for the $\alpha_m/\alpha_d = 2.0$ system in phase II at $F_D = 0.13$. This state resembles the phase II flow found at lower values of α_m/α_d . At $F_D = 0.36$ in Fig. 5(b), the system is in phase III, where the trajectories are more tilted due to the increasing Hall angle. Figure 5(c) shows phase IV at $F_D = 0.6$ with even stronger tilting of the trajectories, while a diminished number of particles remain pinned. Finally in Fig. 5(d), at $F_D = 1.11$ the system is in the DR or moving crystal phase, where there are no well-defined 1D flow channels but the particles have formed a triangular lattice.

The $\alpha_m/\alpha_d = 4.0$ curves show a strong deviation from the $\alpha_m/\alpha_d = 0.0$ curves in Figs. 2 and 3. For $\alpha_m/\alpha_d = 4.0$, the $\langle p_6 \rangle$ versus F_D curve shows a plateau-like region followed by a sharp transition to the dynamically ordered phase above $F_D = 2.0$. The differential transport curves $d\langle V_x \rangle / dF_D$ and $d\langle V_y \rangle / dF_D$ versus F_D have a two-step feature, and there is a small cusp near $F_D = 2.0$ that is associated with the reordering transition. The value of $\langle \theta_{sk} \rangle$ saturates near $F_D = 1.5$. For $\alpha_m/\alpha_d = 6.0$, a similar set of features appear in the transport curves, but now the plateau in $\langle p_6 \rangle$ extends out to higher drives. The plateau region occurs for drives above the reordering transition of the $\alpha_m/\alpha_d = 0.0$ system, and on the plateau, $\langle p_6 \rangle$ takes values between 0.55 and 0.9, indicating that the system is partially ordered but has not yet formed a moving crystal. The width of the plateau becomes larger as α_m/α_d increases, suggesting that it is produced by the Magnus force.

In Fig. 6(a), we show the particle positions and trajectories for the $\alpha_m/\alpha_d = 6.0$ system at $F_D = 0.12$ in phase II, where the Hall angle is much smaller than the intrinsic Hall angle. In this case, the channels have become less 1D-like due to the fact that the increased Magnus force creates more gyroscopic-like orbits. There are, however, still well-defined channels separated by large regions where motion does not occur. Figure 6(b) illustrates the same system at $F_D = 0.34$ in phase III, where the channels are now more strongly tilted and are also fatter than the phase III flow channels that form

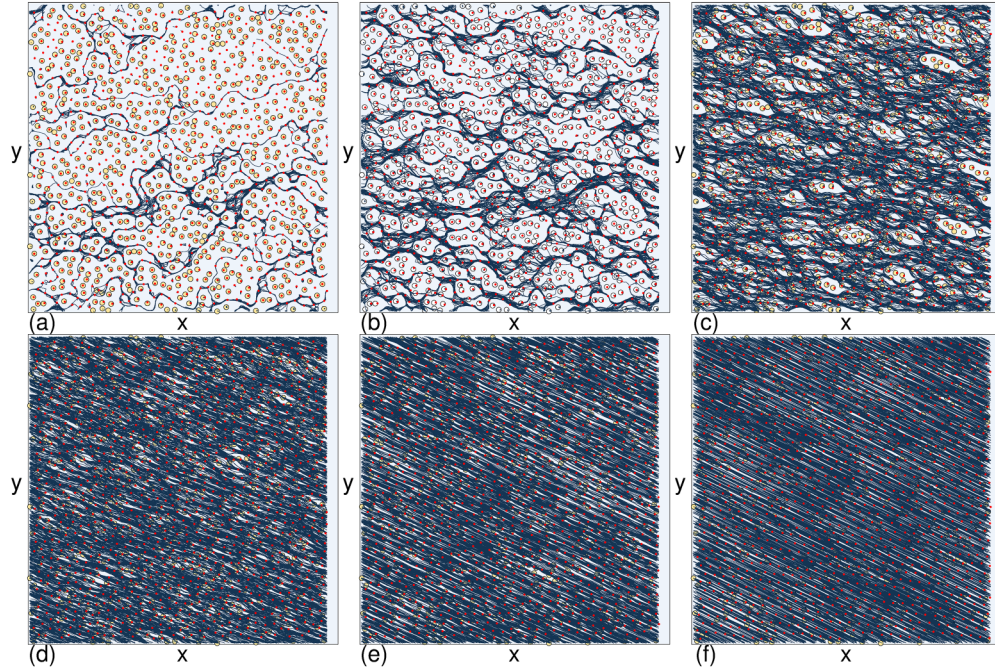


FIG. 4. Particle positions (dots), pinning sites (open circles), and trajectories for a system with $\alpha_m/\alpha_d = 0.5$. (a) Phase II, isolated channel flow, at $F_D = 0.15$. Here, the Hall angle is zero. (b) Phase III, lightly braided channel flow, at $F_D = 0.31$. There is now some evidence of tilt in the trajectories due to the emergence of a finite Hall angle. (c) Phase IV, heavily braided channel flow, at $F_D = 0.48$. The Hall angle has increased, and there are still some pinned particles present. (d) Phase V, inhomogeneous ergodic plastic flow, at $F_D = 0.63$. The system is in a moving fluid state, but some particles can be temporarily pinned. (e) Phase VI, emerging ordered flow, at $F_D = 0.97$, where all particles are flowing at all times but with varying speeds. (f) The DR phase at $F_D = 2.0$, where the system forms a triangular lattice moving at a finite Hall angle.

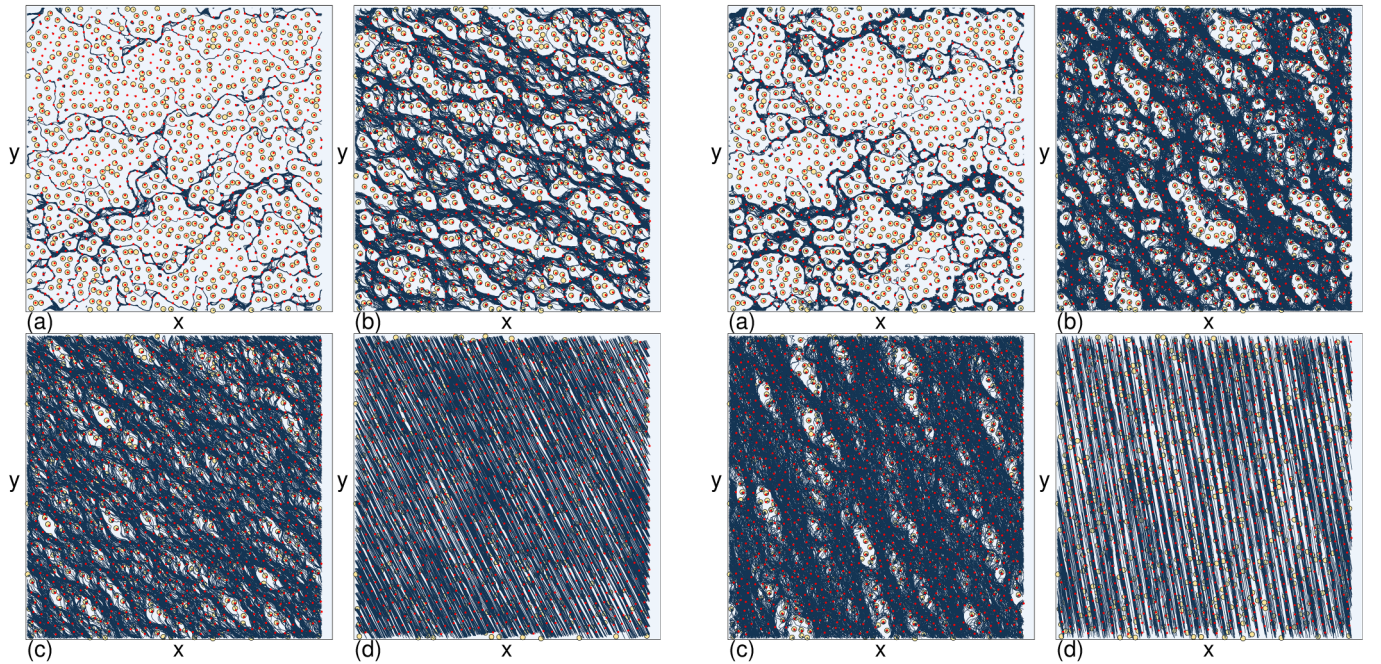


FIG. 5. Particle positions (dots), pinning site locations (open circles), and trajectories (lines) for a system with $\alpha_m/\alpha_d = 2.0$. (a) Phase II at $F_D = 0.13$. (b) Phase III at $F_D = 0.36$. (c) Phase IV at $F_D = 0.6$. (d) DR or moving crystal phase at $F_D = 1.11$.

FIG. 6. Particle positions (dots), pinning site locations (open circles), and trajectories (lines) for a system with $\alpha_m/\alpha_d = 6.0$. (a) Phase II at $F_D = 0.12$. (b) Phase III at $F_D = 0.34$. (c) Phase IV at $F_D = 0.64$. (d) DR or moving crystal phase at $F_D = 4.0$.

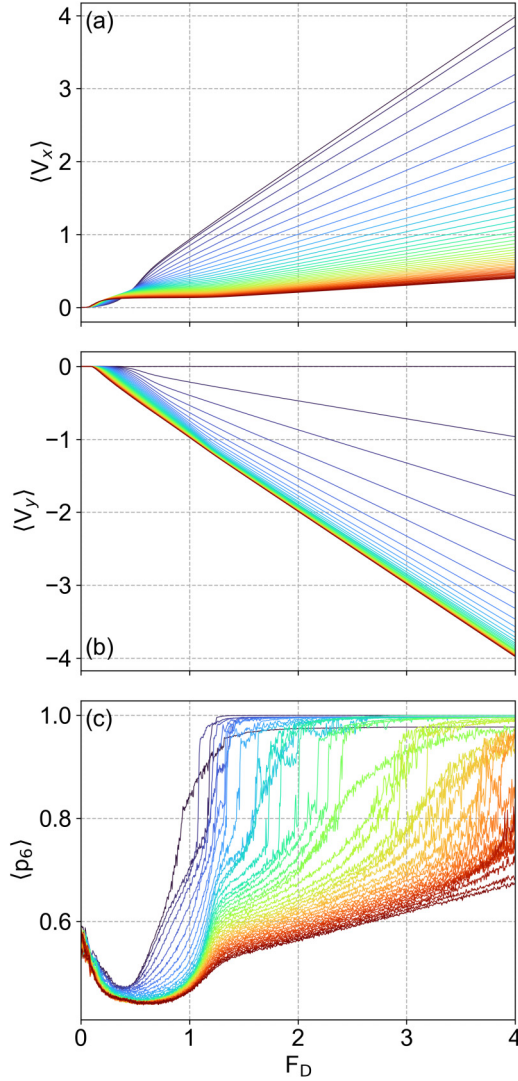


FIG. 7. (a) $\langle V_x \rangle$ vs F_D at varied α_m/α_d from $\alpha_m/\alpha_d = 0.0$ (black) to $\alpha_m/\alpha_d = 10.0$ (dark red) in intervals of 0.25. (b) The corresponding $\langle V_y \rangle$ vs F_D . (c) The corresponding $\langle p_6 \rangle$ vs F_D .

for lower values of α_m/α_d . In Fig. 6(c) at $F_D = 0.64$ in phase IV, there is stronger tilting of the channels, while at $F_D = 4.0$ in Fig. 6(d), the system is in the DR moving crystal state. The $\alpha_m/\alpha_d = 8.75$ system does not reach a dynamically ordered state over the range of drives we consider, and in Fig. 2(c) we find that $\langle p_6 \rangle = 0.73$ at $F_D = 4.0$ for this ratio of Magnus to damping force. The trajectories at $\alpha_m/\alpha_d = 8.75$ in the plastic flow states are similar to those found for $\alpha_m/\alpha_d = 6.0$.

In Fig. 7, we plot $\langle V_x \rangle$, $\langle V_y \rangle$, and $\langle p_6 \rangle$ versus F_D for varied α_m/α_d from 0.0 to 10.0 in increments of 0.25. Here, it can be seen that there are multiple crossings of the V_x curves as α_m/α_d increases, and the drive at which the reordering transition occurs shifts to higher values. The jump in $\langle p_6 \rangle$ accompanying the reordering transition is somewhat stochastic and can shift slightly from one disorder realization to another, but the trend of increasing drive for the reordering transition with increasing α_m/α_d remains robust. For the largest values of α_m/α_d , there is a rapid increase in $\langle p_6 \rangle$ just above $F_D = 1.0$,

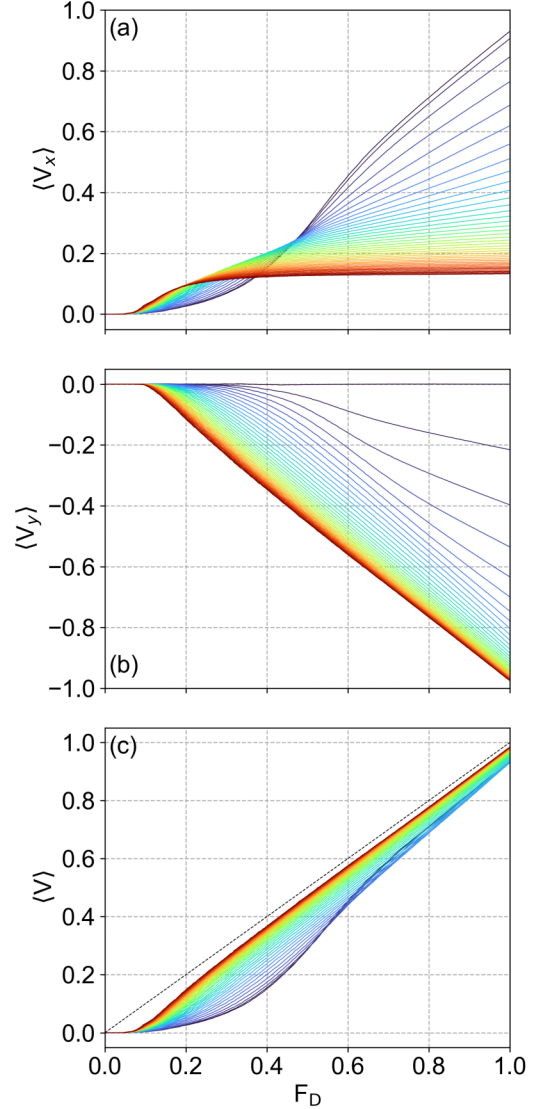


FIG. 8. (a) A blowup of $\langle V_x \rangle$ vs F_D for the system from Fig. 7, showing the crossing of the velocity-force curves with increasing α_m/α_d . Here, α_m/α_d ranges from 0 (black) to 10.0 (dark red) in intervals of 0.25. (b) The corresponding $\langle V_y \rangle$ vs F_D curves. (c) The net velocity $V = \sqrt{V_x^2 + V_y^2}$ vs F_D , where the dashed line indicates the response expected in the pin-free limit. There is a crossing of the curves for low α_m/α_d .

followed by a plateau region, and finally by a jump at higher drives corresponding to the point at which the system enters the moving crystal DR phase.

In Figs. 8(a) and 8(b), we show a blowup of the $\langle V_x \rangle$ and $\langle V_y \rangle$ versus F_D curves from Fig. 7 up to a maximum value of $F_D = 1.0$, which more clearly shows how the velocities evolve in the plastic flow regime. At lower α_m/α_d , $\langle V_x \rangle$ is low just above depinning and increases at higher drives. In contrast, at higher α_m/α_d , $\langle V_x \rangle$ increases rapidly above depinning but then saturates with increasing drive, which leads to a crossing of the velocity-force curves between the low and high values of α_m/α_d . The net velocity curves themselves cross above $F_D = 0.5$ for lower values of α_m/α_d , as shown in the plot of $\langle V \rangle = \sqrt{\langle V_x \rangle^2 + \langle V_y \rangle^2}$ versus F_D in Fig. 8(c). If increasing the

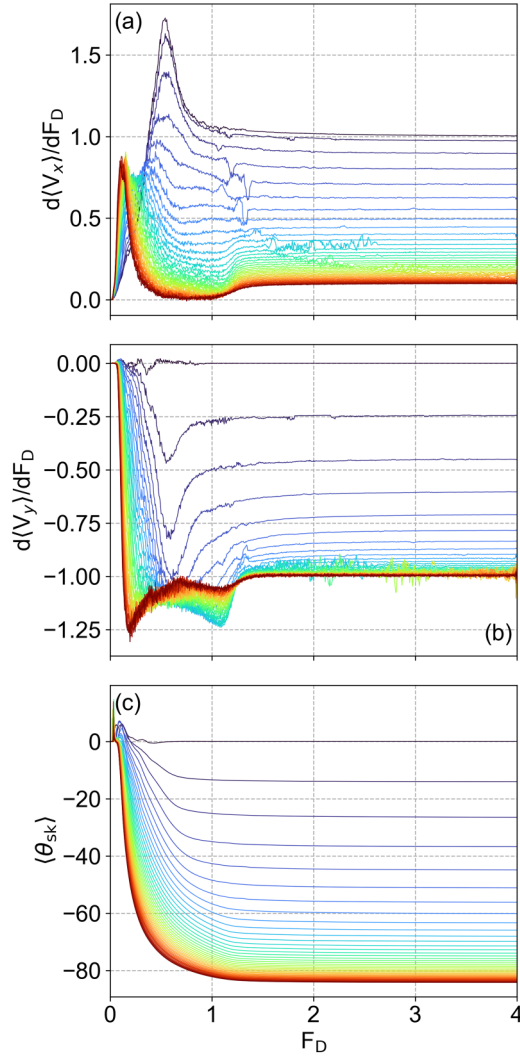


FIG. 9. (a) $d\langle V_x \rangle / dF_D$ vs F_D for varied α_m / α_d over the range $\alpha_m / \alpha_d = 0.0$ (black) to $\alpha_m / \alpha_d = 10.0$ (dark red) in intervals of 0.25. (b) The corresponding $d\langle V_y \rangle / dF_D$ vs F_D . (c) The corresponding $\langle \theta_{sk} \rangle$ vs F_D .

relative importance of the Magnus term simply weakened the effectiveness of the pinning, the velocity-force curves would gradually approach the pin-free limit, indicated by a dashed line in Fig. 8(c). This is true at low F_D , but in the region where the velocity-force curves cross, the pinning is effectively becoming stronger with increasing α_m / α_d , not weaker. The crossing occurs because the slope of $\langle V_x \rangle$ above $F_D = 0.5$ rapidly decreases with increasing α_m / α_d at low α_m / α_d , but the magnitude of the slope of $\langle V_y \rangle$ does not increase rapidly enough to offset this decrease. As a result, the net velocity above $F_D = 0.5$ initially decreases as α_m / α_d increases. Eventually, the slope of $\langle V_x \rangle$ begins to saturate toward a value of zero while the magnitude of the slope of $\langle V_y \rangle$ continues to increase with increasing α_m / α_d , and the net velocity begins to increase with increasing α_m / α_d .

In Fig. 9(a)–9(c), we plot $d\langle V_x \rangle / dF_D$, $d\langle V_y \rangle / dF_D$, and $\langle \theta_{sk} \rangle$, respectively, versus F_D for varied α_m / α_d . For lower α_d / α_m , $d\langle V_x \rangle / dF_D$ has a single strong peak. As α_d / α_m increases, this first peak weakens and a second, smaller peak begins

to emerge near $F_D = 1.2$. The first peak shifts to lower F_D and the second weak peak shifts to higher F_D as α_m / α_d increases. At high values of α_m / α_d , there is a wide region where $d\langle V_x \rangle / dF_D = 0.0$ even though the drive is increasing, which indicates that the velocity in the x direction has ceased to increase with increasing F_D . The $d\langle V_y \rangle / dF_D$ curves in Fig. 9(b) all begin at a zero value and also show a single strong dip for small α_m / α_d , while a weaker second dip feature begins to emerge as α_m / α_d increases. At high α_m / α_d , this second peak reaches its maximum magnitude and then decreases in magnitude, while the original peak becomes more prominent again and drops to lower values of F_D . In Fig. 9(c), the skyrmion Hall angle $\langle \theta_{sk} \rangle$ shows a saturation for $F_D > 1.0$, indicating that the strong drive dependence of the Hall angle is the most pronounced when the maximum pinning force is greater than the driving force.

In Fig. 10(a), we plot a height field of the topological order $\langle p_6 \rangle$ as a function of α_m / α_d versus F_D . The red region indicates the formation of a dynamically ordered state in a transition that shifts to higher F_D with increasing α_m / α_d . The dark blue region is where the system is the most strongly disordered, which occurs for $F_D = F_D / F_p < 1.0$ in the regime where the pinning is dominant. There is also an intermediate or partially ordered regime with $0.6 < \langle p_6 \rangle < 0.8$ for $\alpha_m / \alpha_d > 2.0$. This regime emerges when the Magnus force becomes strong enough to induce local rotations and topological defects even when all of the skyrmions are flowing. Figure 10(b) shows the corresponding height field of $d\langle V_x \rangle / dF_D$ as a function of α_m / α_d versus F_D . There is a sharp peak above depinning when $\alpha_m / \alpha_d < 1.0$, similar to that observed in superconducting vortex systems with $\alpha_m / \alpha_d = 0.0$ [41]. For increasing α_m / α_d , the peak above depinning shifts to lower drives, and when $\alpha_m / \alpha_d > 2.0$, there is a region where $d\langle V_x \rangle / dF_D = 0.0$ in which the x -direction velocity does not increase with increasing F_D . In Fig. 10(c), the corresponding $d\langle V_y \rangle / dF_D$ height field shows that the trends reverse, with small minima appearing for low α_m / α_d that become stronger with increasing α_m / α_d . Two minima are present when $\alpha_m / \alpha_d > 1.0$, but at high α_m / α_d the minimum near $F_D = 1.0$ diminishes in size and the minimum near the depinning transition becomes dominant.

In Fig. 11(a)–11(c), we show the evolution of $\langle V_x \rangle$, $\langle |V_y| \rangle$, and the skyrmion Hall angle $\langle \theta_{sk} \rangle$, respectively, in the form of height fields as a function of α_m / α_d versus F_D . The skyrmion Hall angle is small for $F_D < 0.5$ where the pinning is dominant, while $\langle |V_y| \rangle$ increases and $\langle V_x \rangle$ decreases with increasing F_D .

B. PVB PCA order parameters

We next consider order parameters derived from the PVB PCA, where we specifically focus on averaged values of the first three principal components, $\langle P_1 \rangle$, $\langle P_2 \rangle$, and $\langle P_3 \rangle$. In Fig. 12(a)–12(c), we plot $\langle P_1 \rangle$, $\langle P_2 \rangle$, and $\langle P_3 \rangle$, respectively, versus F_D for systems with $\alpha_m / \alpha_d = 0.0, 0.5, 2.0, 4.0, 6.0$, and 8.5 . For $\alpha_m / \alpha_d = 0.0$ and 0.5 , $\langle P_1 \rangle$ drops from its initial peak at depinning, crosses zero at about $F_D = 0.7$ at the transition from phase V to phase VI, and continues to decrease until saturating at a drive near the crossover to the DR phase, which is a moving smectic for $\alpha_m / \alpha_d = 0.0$ and a dynamically re-

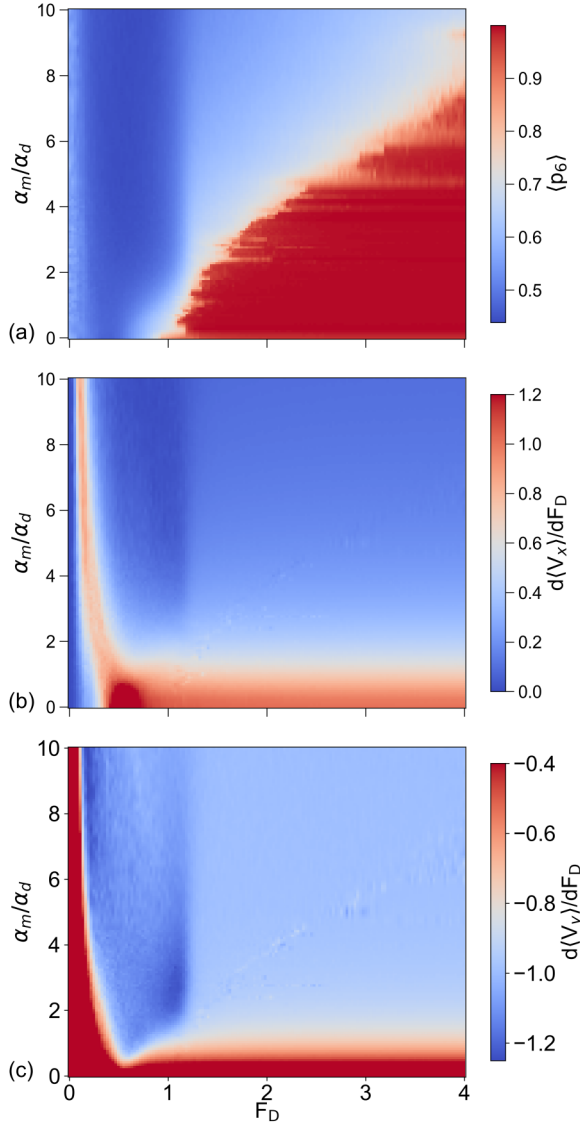


FIG. 10. Height fields as a function of α_m/α_d vs F_D : (a) $\langle p_6 \rangle$, where red indicates the dynamic reordering. (b) $d\langle V_x \rangle / dF_D$, showing the shifting of the main peak. (c) $d\langle V_y \rangle / dF_D$, showing the evolution of the two minima.

ordered lattice for $\alpha_m/\alpha_d = 0.5$. For the same systems, $\langle P_2 \rangle$ passes through a minimum at the boundary between phases IV and V and then crosses zero again at the onset of the DR state. $\langle P_3 \rangle$ shows a peak at the depinning transition from phase I to phase II, crosses zero at the crossover from phase II to phase III, and passes through a dip at the boundary between phase III and phase IV.

For $\alpha_m/\alpha_d = 2.0$ and $\alpha_m/\alpha_d = 4.0$ in Fig. 12, there is now a minimum in $\langle P_1 \rangle$ near $F_D = 1.0$ corresponding to a boundary between phase VI and a previously unobserved moving liquid (ML) state, described in further detail below. For $\alpha_m/\alpha_d = 2.0$, $\langle P_1 \rangle$ remains below zero at higher drives, but for $\alpha_m/\alpha_d = 4.0$, there is a higher drive zero crossing of $\langle P_1 \rangle$ near $F_D = 1.1$ that marks a boundary between the ML state and a second previously unobserved state, the clustered liquid (CL), where partial ordering has emerged but the system has not dynamically reordered. Additional details about

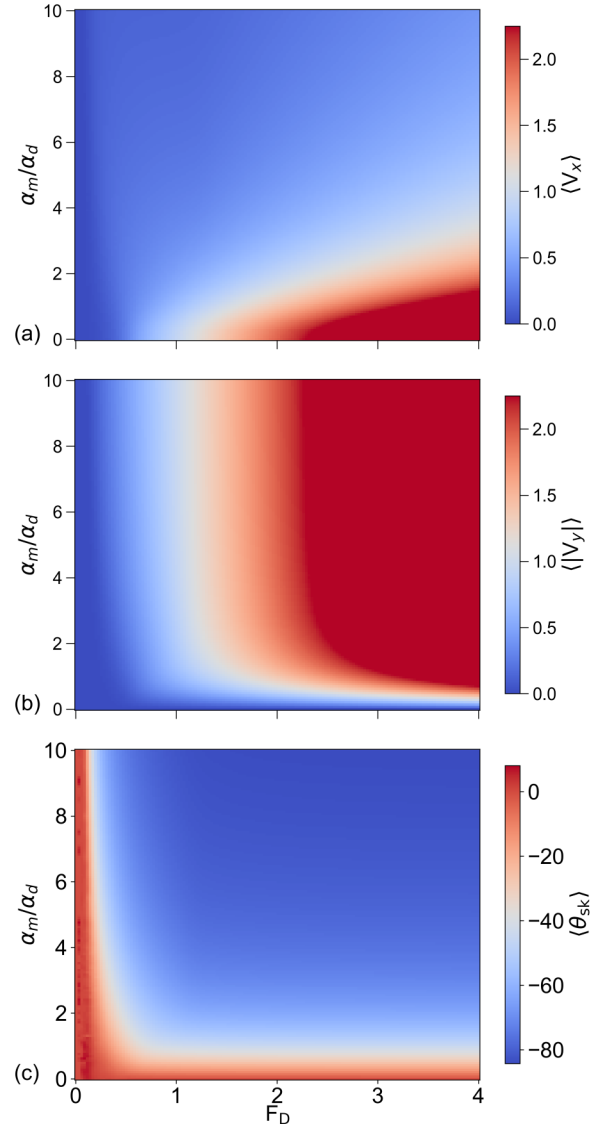


FIG. 11. Height fields as a function of α_m/α_d vs F_D : (a) $\langle V_x \rangle$. (b) $\langle |V_y| \rangle$. (c) The skyrmion Hall angle $\langle \theta_{sk} \rangle$ showing lower values of $\langle \theta_{sk} \rangle$ near depinning.

the CL state appear below. The minimum in $\langle P_2 \rangle$ marking the IV-V boundary deepens and continues to shift to slightly higher F_D with increasing α_m/α_d . At the same time, the zero crossing of $\langle P_2 \rangle$ that coincides with the appearance of the DR state, which had originally dropped to lower F_D between $\alpha_m/\alpha_d = 0.0$ and $\alpha_m/\alpha_d = 0.5$, now moves back up to higher F_D between $\alpha_m/\alpha_d = 2.0$ and $\alpha_m/\alpha_d = 4.0$, marking non-monotonic behavior of the DR boundary. The $\langle P_3 \rangle$ curve picks up a zero crossing and a peak that coincide with the features in $\langle P_1 \rangle$.

In the $\alpha_m/\alpha_d = 6.0$ and 8.5 curves in Fig. 12, $\langle P_1 \rangle$ retains a minimum near $F_D = 1.0$ but no longer crosses zero near the minimum. This corresponds to the disappearance of phase VI, which is replaced by a direct boundary between phase V and the ML state. The dip at the upper edge of phase V and the peak or shoulder at the ML-MC boundary become less pronounced with increasing α_m/α_d but remain at nearly

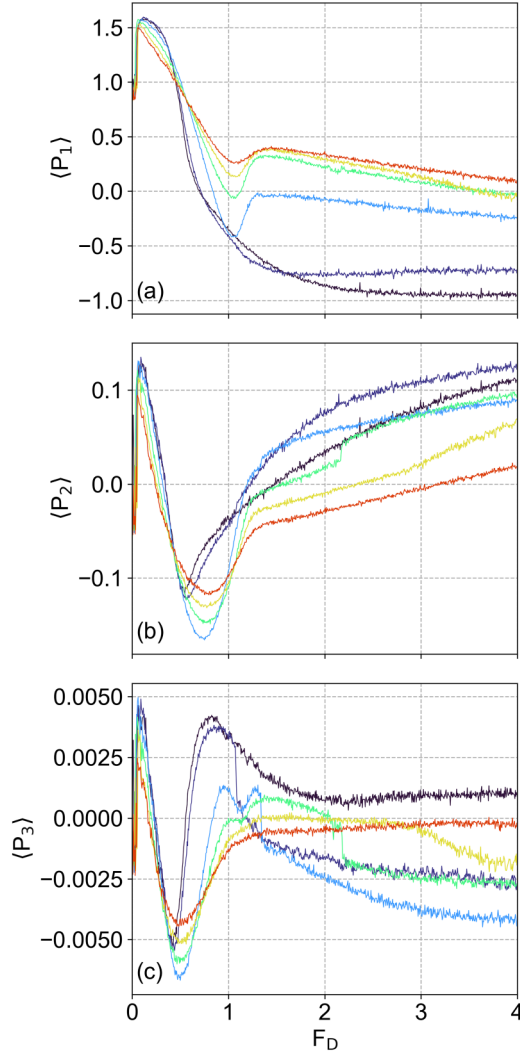


FIG. 12. Order parameters taken from the first three principal components for systems with $\alpha_m/\alpha_d = 0.0$ (black), 0.5 (purple), 2.0 (blue), 4.0 (green), 6.0 (yellow), and 8.5 (red). (a) $\langle P_1 \rangle$ vs F_D . (b) $\langle P_2 \rangle$ vs F_D . (c) $\langle P_3 \rangle$ vs F_D .

the same values of F_D . The minimum in $\langle P_2 \rangle$ at the IV-V boundary also remains at a nearly constant value of F_D for higher α_m/α_d , but the upper zero crossing of $\langle P_2 \rangle$ marking the CL-DR boundary continues to shift rapidly to higher drives as α_m/α_d increases. The features of $\langle P_3 \rangle$ show little dependence on α_m/α_d at the larger α_m/α_d values.

To illustrate the full evolution of the PCA order parameters, in Fig. 13(a)–13(c), we plot $\langle P_1 \rangle$, $\langle P_2 \rangle$, and $\langle P_3 \rangle$, respectively, versus F_D for α_m/α_d ranging from 0 to 10.0 in intervals of 0.25. This information is also presented as height fields as a function of α_m/α_d versus F_D in Fig. 14. With increasing α_m/α_d , $\langle P_1 \rangle$ first develops a local minimum near $F_D = 1.0$ and then develops a local maximum at slightly higher drives, with both features becoming less prominent at higher α_m/α_d , and ceasing to cross zero for $\alpha_m/\alpha_d > 4.25$. The minimum in $\langle P_2 \rangle$ shifts to higher F_D with increasing α_m/α_d while first increasing and then decreasing in depth. At the same time, the higher drive zero crossing continues to climb to higher drives as α_m/α_d becomes large. The $\langle P_3 \rangle$ curves retain the steplike jump feature found in $\langle p_6 \rangle$ at the transition to a more

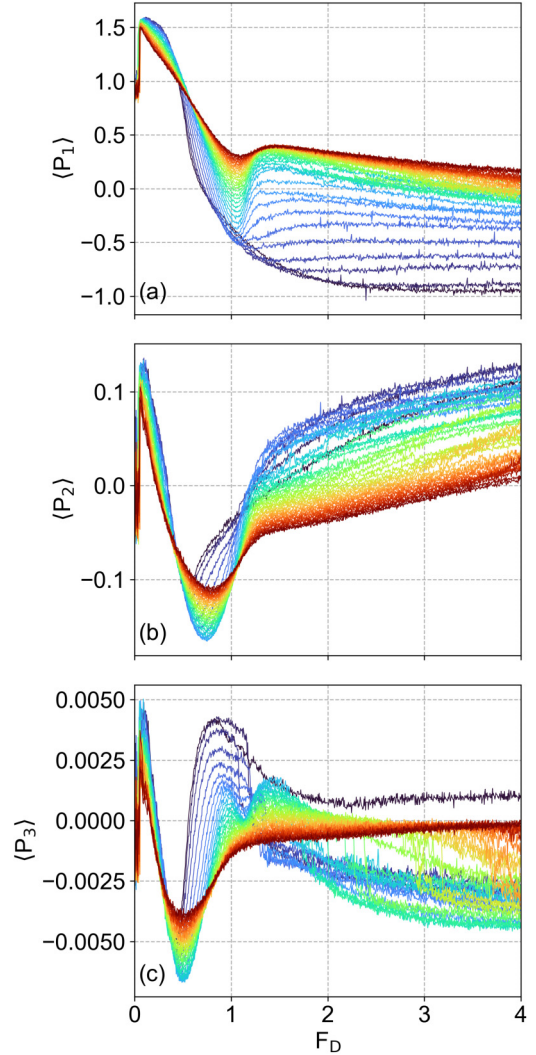


FIG. 13. Order parameters taken from the first three principal components at varied α_m/α_d from $\alpha_m/\alpha_d = 0.0$ (black) to $\alpha_m/\alpha_d = 10.0$ (dark red) in intervals of 0.25. (a) $\langle P_1 \rangle$ vs F_D . (b) $\langle P_2 \rangle$ vs F_D . (c) $\langle P_3 \rangle$ vs F_D .

ordered state, but also have a number of broader signatures that generally overlap with signatures in $\langle P_2 \rangle$ and $\langle P_1 \rangle$. The peak at depinning, which is followed by a zero crossing and then a dip, are all fairly insensitive to α_m/α_d , with the dip becoming more shallow as α_m/α_d increases.

In order to understand what is happening in the ML and CL states newly detected by PVB PCA, we first made trajectory plots similar to those shown in Figs. 4–6. We found, however, that unless the flow at higher drives is relatively ordered, as in the DR state, the trajectories present an indistinguishable blurred mass. Thus, we performed imaging in a moving reference frame in order to detect the deviations each particle makes around its original position as a function of time. This measure works badly when $F_D \leq 1.0$ because the pinning remains dominant and, in the moving reference frame, produces a series of streaks across the image. As the skyrmion structure increasingly begins to decouple from the pinning with increasing F_D , the moving reference frame images become more informative. Due to the rotation of the skyrmion direction of

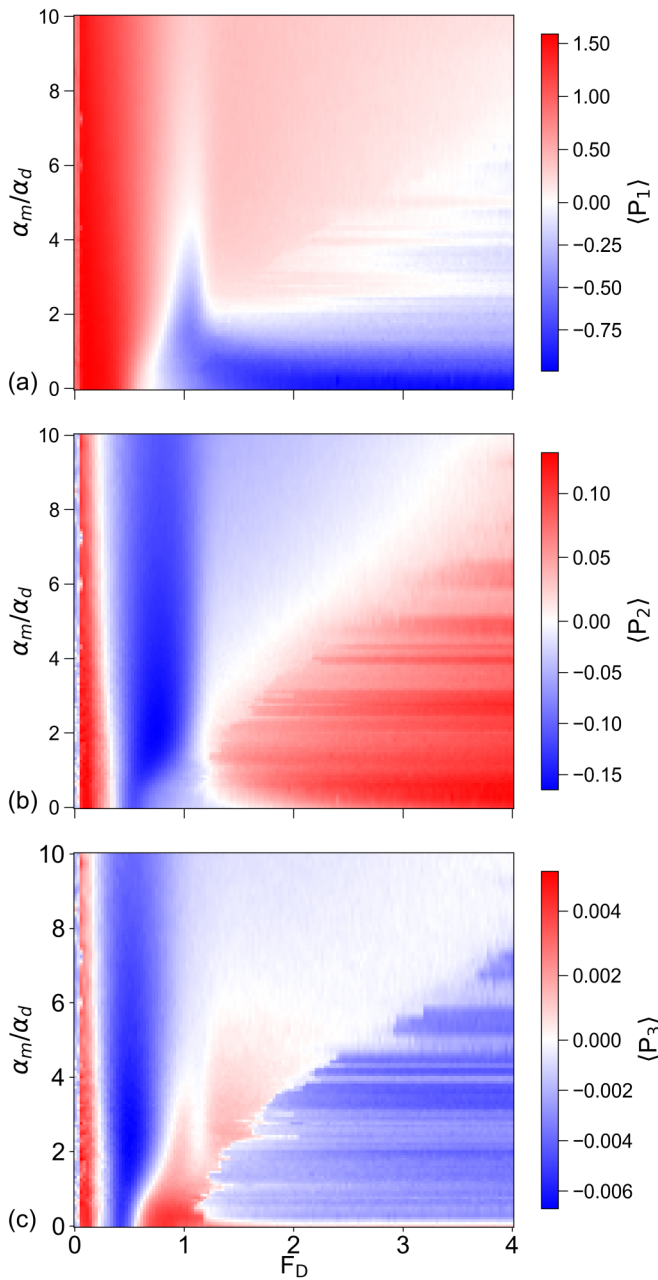


FIG. 14. Height fields as a function of α_m/α_d vs F_D : (a) $\langle P_1 \rangle$, (b) $\langle P_2 \rangle$, and (c) $\langle P_3 \rangle$.

travel by the Magnus force, we cannot simply subtract off the average velocity in the x direction with increasing F_D . Instead, we find the average motion of the center of mass across all frames for a given current, and subtract this average motion from all particles. In Fig. 15(a)–15(d), we show the moving frame images for $\alpha_m/\alpha_d = 0.0$ (the superconducting vortex limit) in phase IV at $F_D = 0.48$ in Fig. 15(a), phase V at $F_D = 0.61$ in Fig. 15(b), phase VI at $F_D = 0.85$ in Fig. 15(c), and the moving smectic or DR state at $F_D = 2.0$ in Fig. 15(d). For phase IV, the ordering is fluidlike but the diffusion is anisotropic and motion along the driving direction is favored. Some pinned particles are still present in phase IV and they produce streaks in the image. Phase V has a similar overall liquidlike ordering, but small patches of triangular ordering

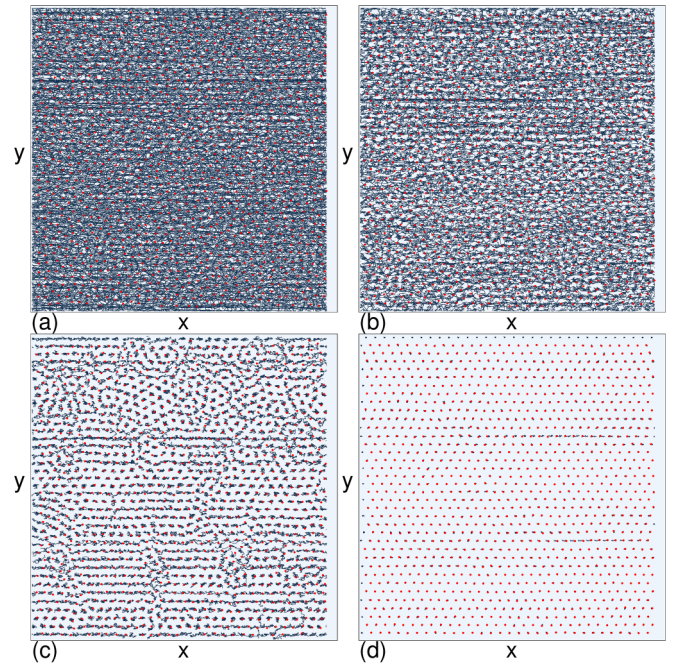


FIG. 15. Particle positions (dots) and trajectories (lines) in the moving reference frame for a system in the superconducting vortex limit at $\alpha_m/\alpha_d = 0.0$. As in the rest of this work, the trajectories are imaged over a time equal to that required for an individual particle moving through a pin-free sample to travel a distance of 50λ when subjected to the driving force F_D . (a) The heavily braided channel phase IV at $F_D = 0.48$, where the ordering is still fluidlike but the displacements of the particles are anisotropic and more prominent along the driving direction. (b) The inhomogeneous ergodic plastic flow phase V at $F_D = 0.61$. The system is still fluid but is developing small locally ordered patches. (c) The emerging ordered flow phase VI at $F_D = 0.85$, where there is more noticeable ordering and local regions exhibit largely 1D diffusion. (d) The moving smectic state or DR regime at $F_D = 2.0$, where the ordering is largely triangular but some aligned dislocations remain and produce strictly 1D motion along the rows.

are beginning to emerge. Here, the streaks are produced by the slowing of particles passing through pinning sites, rather than permanently pinned particles. In phase VI the particles are much more ordered and exhibit local regions of diffusion that has a strongly 1D character. The DR moving smectic state has well developed crystalline ordering and shows localized areas of 1D motion produced by the aligned dislocations that are present in the smectic structure.

Figure 16 shows the moving reference frame images at $\alpha_m/\alpha_d = 0.5$ for phase IV at $F_D = 0.48$ in Fig. 16(a), phase V at $F_D = 0.63$ in Fig. 16(b), phase VI at $F_D = 0.97$ in Fig. 16(c), and the moving crystal or DR phase at $F_D = 2.0$ in Fig. 16(d). In the presence of a finite Magnus term, phases IV and V show strong diffusion, while phase VI has more string-like diffusion, and the DR phase is now a fully ordered crystal. In Fig. 17 we show the $\alpha_m/\alpha_d = 4.0$ system in the moving reference frame in phase VI at $F_D = 1.02$ in Fig. 17(a), the moving liquid or ML phase at $F_D = 1.22$ in Fig. 17(b), the cluster liquid or CL phase at $F_D = 1.48$ in Fig. 17(c), and the DR phase at $F_D = 1.9$ in Fig. 17(d). This system is at the very edge of exhibiting phase IV; there is still

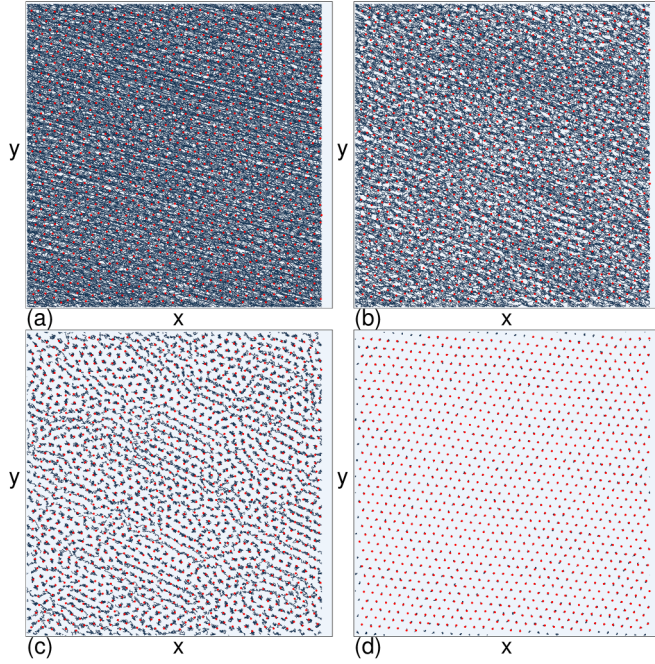


FIG. 16. Particle positions (dots) and trajectories (lines) in the moving reference frame for a system with $\alpha_m/\alpha_d = 0.5$. (a) The heavily braided channel phase IV at $F_D = 0.48$, with fluidlike structure. (b) The inhomogeneous ergodic plastic flow phase V at $F_D = 0.63$, where the motion resembles an anisotropic liquid. (c) The emerging ordered flow phase VI at $F_D = 0.97$, where greater order is present. (d) The moving crystal or DR phase at $F_D = 2.0$.

anisotropic transport present but the entire state is becoming increasingly fluidlike. In the ML phase the diffusion is now nearly isotropic, and the sequence ML-CL-DR resembles the freezing of an isotropic crystal. The distinction that PVB PCA draws between these three states is visible in the figure. The ML phase has fluidlike order and fluidlike isotropic diffusion. The CL phase has patches of local order and the diffusion has become heterogeneous, with percolating strings of diffusion occurring in all directions around small patches of more stationary particles. Finally in the DR phase, where some disorder persists due to the rotation of the particles caused by the Magnus force, the diffusing strings have become isolated and filamentary, and gradually pinch off as F_D increases further. Thus PVB PCA has successfully detected morphological changes in the nature of the moving reference frame diffusion.

From the different features in $\langle P_1 \rangle$, $\langle P_2 \rangle$, and $\langle P_3 \rangle$, we can construct a dynamical phase diagram as shown in Fig. 18 as a function of α_m/α_d versus F_D . Here, phase I is the pinned phase, phase II is the isolated 1D static channel phase, and phase III is the lightly braided quasi-2D channel phase. The skyrmion Hall angle is close to zero in phase II and low in phase III. Phase IV is a heavily braided channel flow state in which a portion of the particles remain permanently pinned, so that the flow is still nonergodic. Phase V is the disordered, inhomogeneous ergodic plastic flow phase, where some particles are temporarily pinned, but all of the particles take part in the motion. Phase VI is an emerging ordered flow state in which all of the particles are moving at close to the same speed but the system remains disordered. Phase ML is a disordered

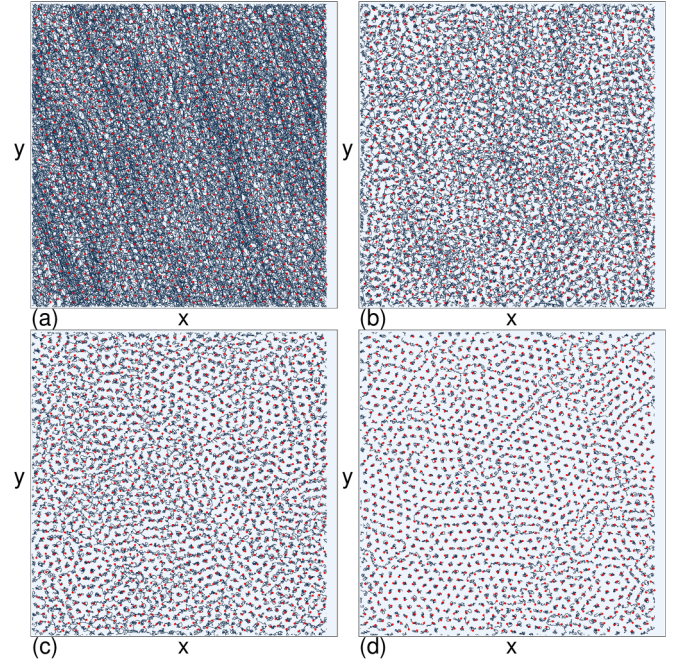


FIG. 17. Particle positions (dots) and trajectories (lines) in the moving reference frame for a system with $\alpha_m/\alpha_d = 4.0$. (a) The last vestiges of the emerging ordered flow phase VI at $F_D = 1.02$, where the motion has become relatively more fluid. (b) The moving liquid or ML phase at $F_D = 1.22$, where the diffusion is much more isotropic and the system in the moving frame resembles a fluid. (c) The clustered liquid or CL phase at $F_D = 1.48$, where patches of local triangular ordering are emerging and the diffusion resembles what would be expected in an isotropic crystal above melting. (d) Just into the DR state at $F_D = 1.9$, where the diffusion is becoming strongly localized and the crystalline ordering is improving.

moving liquid phase in which all of the particles are moving, the diffusion in the moving lattice frame is isotropic, and there is strong rotation of the particle trajectories due to the Magnus phase. Phase CL is the partially ordered cluster liquid phase where the system tries to form a triangular lattice, but the local velocity differences produced by the pinning and the strong Magnus force create local rotations which induce the formation of some topological defects. DR is the dynamically ordered phase, which is a crystal for finite Magnus force and a smectic for zero Magnus force.

IV. DISCUSSION

We have focused on a single value of the pinning force F_p which is large enough that the depinning process is plastic. For weaker pinning, the system should enter an elastic depinning regime in which each particle keeps the same neighbors during depinning and sliding [56]. In the elastic depinning regime, it has been shown that the skyrmion Hall angle still has a drive dependence, and that this drive dependence becomes more pronounced where thermal effects are introduced. For our stronger pinning, if thermal effects are included in the plastic depinning regime, it is likely that phase I would be replaced by phase II, and that phases III, IV, and V would be shifted to lower values of F_D .

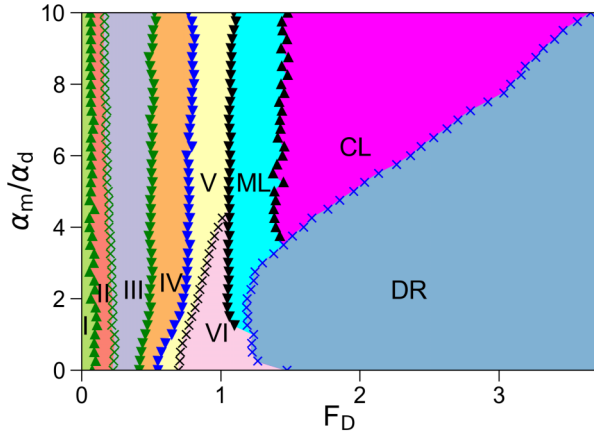


FIG. 18. Dynamic phase diagram as a function of α_m/α_d vs F_D constructed using the features identified by PVB PCA. Green: Pinned (phase I). Red: Isolated channel flow (phase II). Purple: Lightly braided channel flow (phase III). Orange: Heavily braided channel flow (phase IV) with a fraction of permanently pinned particles. Yellow: Inhomogeneous ergodic plastic flow (phase V) with no permanently pinned particles. Pink: Emerging ordered flow (phase VI). Blue: Dynamically reordered flow (phase DR), which is a moving crystal for finite Magnus force and a moving smectic for zero Magnus force. The phases newly detected by PVB PCA are as follows: Cyan: Moving liquid flow (phase ML) where all the particles are moving and there is strong rotation due to the Magnus term. Magenta: Clustered liquid flow (phase CL), a partially ordered phase where the system tries to form a triangular lattice but contains numerous topological defects. The phase boundaries are obtained from the following measures: I-II (green up triangles): peak of $\langle P_3 \rangle$, which also coincides with the lowest minimum of $\langle P_1 \rangle$. II-III (green crosses): lowest zero crossing of $\langle P_3 \rangle$. III-IV (green down triangles): minimum of $\langle P_3 \rangle$. IV-V (blue down triangles): minimum of $\langle P_2 \rangle$. V-VI (black crosses): zero crossing of $\langle P_1 \rangle$. VI-ML (black down triangles): minimum of $\langle P_1 \rangle$. ML-CL (black up triangles): upper maximum of $\langle P_1 \rangle$. Boundary of DR phase (blue crosses): upper zero crossing of $\langle P_2 \rangle$. Note that all order parameters are only defined to within a sign.

A broader question is whether the boundaries between the phases detected by PVB PCA are nonequilibrium phase transitions or crossovers. Previous work provided evidence that the plastic depinning transition is a nonequilibrium phase transition with critical properties [14], so it could be possible that at least some of the other phase boundaries are also marked by nonequilibrium phase transitions. In particular, the crossover from nonergodic to ergodic plastic flow at the IV-V boundary could fall in the class of absorbing phase transitions [88,89], and the dynamical reordering transitions could represent another type of nonequilibrium phase transition. It may be that the II-III or III-IV transitions are examples of different kinds of directed percolation with different quantities percolating. The percolation of diffusion in the moving reference frame could show similar transitions at the V-ML, ML-CL, and CL-DR boundaries. Although the PVB PCA can find evidence for the presence of different phases, it does not determine whether the boundaries it marks are nonequilibrium phase transitions. Future directions include determining whether the PVB PCA-derived order parameters themselves show scaling, and if they do, whether the exponents could be related to established universality classes.

V. SUMMARY

We have investigated the dynamics of driven skyrmions using a modified Thiele equation approach and a combination of standard measures such as features in the velocity-force curves, differential conductivity, and fraction of topological defects in combination with a PVB PCA. The PVB PCA combines information about both the spatial and velocity properties of the moving particles. The standard measures show evidence for plastic depinning and dynamic reordering, but from these measures alone, it is difficult to determine whether additional dynamic phases occur in the plastic flow regimes. Principal component analysis was previously used to distinguish different types of plastic flow regimes for driven overdamped systems moving over random disorder. Here, the strong Magnus component of the skyrmion dynamics produces previously unobserved higher velocity flow phases. For the skyrmions, we find that just above depinning, the flow follows static one-dimensional channels and that the skyrmion Hall angle is zero. At higher drives, the channels become lightly braided around permanently pinned particles, and the skyrmion Hall angle becomes finite but remains small. As the drive increases further, the channel flow becomes more two-dimensional and strongly fluctuating, but there are still some permanently pinned particles, so the system is in a nonergodic regime and the flow is described as heavily braided channel motion. Once the driving force exceeds the pinning force, we find a regime in which all of the particles take part in the motion over time, but some particles can be temporarily immobile, giving an inhomogeneous ergodic plastic flow state. At higher drives, in addition to the emerging ordered flow state found for overdamped particles, we observe two previously unreported plastic flow states: the moving liquid and the cluster liquid, in which the system when viewed in the moving reference frame takes on the appearance of a melted crystal that is being cooled. At the highest drives, the system can dynamically order into a moving crystal. Using the first three principal components as order parameters, we map out the evolution of these different phases as a function of the ratio of the Magnus to the damping force as well as the external drive. For increased Magnus force, there is an extended region of partially ordered flow phases that arise due to local rotations caused by the pinning. Our results indicate that PVB PCA, which takes into account both positions and velocities, can be used to distinguish different dynamical phases more accurately than standard transport measurements or changes in the defect density. The PVB PCA approach can be used as a general tool that is applicable to other types of driven systems that could exhibit different dynamical phases.

ACKNOWLEDGMENTS

We gratefully acknowledge the support of the U.S. Department of Energy through the LANL/LDRD program for this work. This work was supported by the U.S. Department of Energy through the Los Alamos National Laboratory. Los Alamos National Laboratory is operated by Triad National Security, LLC, for the National Nuclear Security Administration of the U.S. Department of Energy (Contract No. 892333218NCA000001).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not

technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

- [1] H. Abdi and L. J. Williams, Principal component analysis, *WIREs Comput. Stats.* **2**, 433 (2010).
- [2] J. Shlens, A tutorial on principal component analysis, [arXiv:1404.1100](https://arxiv.org/abs/1404.1100).
- [3] B. A. McKinney, D. M. Reif, M. D. Ritchie, and J. H. Moore, Machine learning for detecting gene-gene interactions: A review, *Appl. Bioinf.* **5**, 77 (2006).
- [4] C. Bishop, *Pattern Recognition and Machine Learning* (Springer-Verlag, New York, 2006).
- [5] L. Wang, Discovering phase transitions with unsupervised learning, *Phys. Rev. B* **94**, 195105 (2016).
- [6] J. Carrasquilla and R. G. Melko, Machine learning phases of matter, *Nat. Phys.* **13**, 431 (2017).
- [7] S. J. Wetzel, Unsupervised learning of phase transitions: From principal component analysis to variational autoencoders, *Phys. Rev. E* **96**, 022140 (2017).
- [8] W. Hu, R. R. P. Singh, and R. T. Scalettar, Discovering phases, phase transitions, and crossovers through unsupervised machine learning: A critical examination, *Phys. Rev. E* **95**, 062122 (2017).
- [9] R. B. Jadrich, B. A. Lindquist, and T. M. Truskett, Unsupervised machine learning for detection of phase transitions in off-lattice systems. I. Foundations, *J. Chem. Phys.* **149**, 194109 (2018).
- [10] R. B. Jadrich, B. A. Lindquist, W. D. Pineros, D. Banerjee, and T. M. Truskett, Unsupervised machine learning for detection of phase transitions in off-lattice systems. II. Applications, *J. Chem. Phys.* **149**, 194110 (2018).
- [11] D. McDermott, C. J. O. Reichhardt, and C. Reichhardt, Detecting depinning and nonequilibrium transitions with unsupervised machine learning, *Phys. Rev. E* **101**, 042101 (2020).
- [12] D. McDermott, C. Reichhardt, and C. J. O. Reichhardt, Characterizing different motility-induced regimes in active matter with machine learning and noise, *Phys. Rev. E* **108**, 064613 (2023).
- [13] D. S. Fisher, Collective transport in random media: From superconductors to earthquakes, *Phys. Rep.* **301**, 113 (1998).
- [14] C. Reichhardt and C. J. Olson Reichhardt, Depinning and nonequilibrium dynamic phases of particle assemblies driven over random and ordered substrates: A review, *Rep. Prog. Phys.* **80**, 026501 (2017).
- [15] H. J. Jensen, A. Brass, and A. J. Berlinsky, Lattice deformations and plastic flow through bottlenecks in a two-dimensional model for flux pinning in type-II superconductors, *Phys. Rev. Lett.* **60**, 1676 (1988).
- [16] S. Bhattacharya and M. J. Higgins, Dynamics of a disordered flux line lattice, *Phys. Rev. Lett.* **70**, 2617 (1993).
- [17] A. E. Koshelev and V. M. Vinokur, Dynamic melting of the vortex lattice, *Phys. Rev. Lett.* **73**, 3580 (1994).
- [18] M. J. Higgins and S. Bhattacharya, Varieties of dynamics in a disordered flux-line lattice, *Physica C* **257**, 232 (1996).
- [19] P. Le Doussal and T. Giamarchi, Moving glass theory of driven lattices with disorder, *Phys. Rev. B* **57**, 11356 (1998).
- [20] L. Balents, M. C. Marchetti, and L. Radzihovsky, Nonequilibrium steady states of driven periodic media, *Phys. Rev. B* **57**, 7705 (1998).
- [21] C. J. Olson, C. Reichhardt, and F. Nori, Nonequilibrium dynamic phase diagram for vortex lattices, *Phys. Rev. Lett.* **81**, 3757 (1998).
- [22] F. Pardo, F. de la Cruz, P. L. Gammel, E. Bucher, and D. J. Bishop, Observation of smectic and moving-Bragg-glass phases in flowing vortex lattices, *Nature (London)* **396**, 348 (1998).
- [23] C. Reichhardt and C. J. Olson, Colloidal dynamics on disordered substrates, *Phys. Rev. Lett.* **89**, 078301 (2002).
- [24] A. Pertsinidis and X. S. Ling, Statics and dynamics of 2D colloidal crystals in a random pinning potential, *Phys. Rev. Lett.* **100**, 028303 (2008).
- [25] P. Tierno, Depinning and collective dynamics of magnetically driven colloidal monolayers, *Phys. Rev. Lett.* **109**, 198304 (2012).
- [26] M.-C. Cha and H. A. Fertig, Orientational order and depinning of the disordered electron solid, *Phys. Rev. Lett.* **73**, 870 (1994).
- [27] C. Reichhardt, C. J. Olson, N. Grønbech-Jensen, and F. Nori, Moving Wigner glasses and smectics: Dynamics of disordered Wigner crystals, *Phys. Rev. Lett.* **86**, 4354 (2001).
- [28] P. T. Madathil, K. A. Villegas Rosales, Y. J. Chung, K. W. West, K. W. Baldwin, L. N. Pfeiffer, L. W. Engel, and M. Shayegan, Moving crystal phases of a quantum Wigner solid in an ultra-high-quality 2D electron system, *Phys. Rev. Lett.* **131**, 236501 (2023).
- [29] A. Morin, N. Desreumaux, J.-B. Caussin, and D. Bartolo, Distortion and destruction of colloidal flocks in disordered environments, *Nat. Phys.* **13**, 63 (2017).
- [30] C. Sándor, A. Libál, C. Reichhardt, and C. J. Olson Reichhardt, Dynamic phases of active matter systems with quenched disorder, *Phys. Rev. E* **95**, 032606 (2017).
- [31] C. Reichhardt, C. J. Olson Reichhardt, I. Martin, and A. R. Bishop, Dynamical ordering of driven stripe phases in quenched disorder, *Phys. Rev. Lett.* **90**, 026401 (2003).
- [32] H. J. Zhao, V. R. Misko, and F. M. Peeters, Dynamics of self-organized driven particles with competing range interaction, *Phys. Rev. E* **88**, 022914 (2013).
- [33] M. Le Blay, M. Adda-Bedia, and D. Bartolo, Emergence of scale-free smectic rivers and critical depinning in emulsions driven through disorder, *Proc. Natl. Acad. Sci. USA* **117**, 13914 (2020).
- [34] O. Narayan and D. S. Fisher, Nonlinear fluid flow in random media: Critical phenomena near threshold, *Phys. Rev. B* **49**, 9469 (1994).
- [35] A. Bylinskii, D. Gangloff, I. Counts, and V. Vuletić, Observation of Aubry-type transition in finite atom chains via friction, *Nat. Mater.* **15**, 717 (2016).
- [36] S. Bhattacharya, J. P. Stokes, M. J. Higgins, and R. A. Klemm, Temporal coherence in the sliding charge-density-wave condensate, *Phys. Rev. Lett.* **59**, 1849 (1987).
- [37] S. Brazovskii and T. Nattermann, Pinning and sliding of driven elastic systems: From domain walls to charge density waves, *Adv. Phys.* **53**, 177 (2004).

- [38] H. Leschhorn, T. Nattermann, S. Stepanow, and L. H. Tang, Driven interface depinning in a disordered medium, *Ann. Phys.* **509**, 1 (1997).
- [39] P. Moretti, M.-C. Miguel, M. Zaiser, and S. Zapperi, Depinning transition of dislocation assemblies: Pileups and low-angle grain boundaries, *Phys. Rev. B* **69**, 214103 (2004).
- [40] A. Vanossi, N. Manini, and E. Tosatti, Static and dynamic friction in sliding colloidal monolayers, *Proc. Natl. Acad. Sci. USA* **109**, 16429 (2012).
- [41] C. J. O. Reichhardt, D. McDermott, and C. Reichhardt, Using principal component analysis to distinguish different dynamic phases in superconducting vortex matter, *Phys. Rev. B* **111**, 104508 (2025).
- [42] N. Di Scala, E. Olive, Y. Lansac, Y. Fily, and J. C. Soret, The elastic depinning transition of vortex lattices in two dimensions, *New J. Phys.* **14**, 123027 (2012).
- [43] S. Mülbauer, B. Binz, F. Jonietz, C. Pfleiderer, A. Rosch, A. Neubauer, R. Georgii, and P. Böni, Skyrmion lattice in a chiral magnet, *Science* **323**, 915 (2009).
- [44] X. Z. Yu, Y. Onose, N. Kanazawa, J. H. Park, J. H. Han, Y. Matsui, N. Nagaosa, and Y. Tokura, Real-space observation of a two-dimensional skyrmion crystal, *Nature (London)* **465**, 901 (2010).
- [45] F. Jonietz, S. Mülbauer, C. Pfleiderer, A. Neubauer, W. Münzer, A. Bauer, T. Adams, R. Georgii, P. Böni, R. A. Duine, K. Everschor, M. Garst, and A. Rosch, Spin transfer torques in MnSi at ultralow current densities, *Science* **330**, 1648 (2010).
- [46] J. Zang, M. Mostovoy, J. H. Han, and N. Nagaosa, Dynamics of Skyrmion crystals in metallic thin films, *Phys. Rev. Lett.* **107**, 136804 (2011).
- [47] T. Schulz, R. Ritz, A. Bauer, M. Halder, M. Wagner, C. Franz, C. Pfleiderer, K. Everschor, M. Garst, and A. Rosch, Emergent electrodynamics of skyrmions in a chiral magnet, *Nat. Phys.* **8**, 301 (2012).
- [48] X. Z. Yu, N. Kanazawa, W. Z. Zhang, T. Nagai, T. Hara, K. Kimoto, Y. Matsui, Y. Onose, and Y. Tokura, Skyrmion flow near room temperature in an ultralow current density, *Nat. Commun.* **3**, 988 (2012).
- [49] J. Iwasaki, M. Mochizuki, and N. Nagaosa, Universal current-velocity relation of skyrmion motion in chiral magnets, *Nat. Commun.* **4**, 1463 (2013).
- [50] S.-Z. Lin, C. Reichhardt, C. D. Batista, and A. Saxena, Driven skyrmions and dynamical transitions in chiral magnets, *Phys. Rev. Lett.* **110**, 207202 (2013).
- [51] N. Nagaosa and Y. Tokura, Topological properties and dynamics of magnetic skyrmions, *Nat. Nanotechnol.* **8**, 899 (2013).
- [52] W. Jiang, G. Chen, K. Liu, J. Zang, S. G. E. te Velthuis, and A. Hoffmann, Skyrmions in magnetic multilayers, *Phys. Rep.* **704**, 1 (2017).
- [53] I. L. Fernandes, J. Bouaziz, S. Blügel, and S. Lounis, Universality of defect-skyrmion interaction profiles, *Nat. Commun.* **9**, 4395 (2018).
- [54] L. Xiong, B. Zheng, M. H. Jin, and N. J. Zhou, Collective transport properties of skyrmions on the depinning phase transition, *Phys. Rev. B* **100**, 064426 (2019).
- [55] K. Litzius, J. Leliaert, P. Bassirian, D. Rodrigues, S. Kromin, I. Lemesch, J. Zázvorka, K.-J. Lee, J. Mulkers, N. Kerber, D. Heinze, N. Keil, R. M. Reeve, M. Weigand, B. Van Waeyenberge, G. Schütz, K. Everschor-Sitte, G. S. D. Beach, and M. Kläui, The role of temperature and drive current in skyrmion dynamics, *Nat. Electron.* **3**, 30 (2020).
- [56] C. Reichhardt, C. J. O. Reichhardt, and M. Milosevic, Statics and dynamics of skyrmions interacting with disorder and nanostructures, *Rev. Mod. Phys.* **94**, 035005 (2022).
- [57] R. Gruber, J. Zázvorka, M. A. Brems, D. R. Rodrigues, T. Dohi, N. Kerber, B. Seng, M. Vafaei, K. Everschor-Sitte, P. Virnau, and M. Kläui, Skyrmion pinning energetics in thin film systems, *Nat. Commun.* **13**, 3144 (2022).
- [58] Y.-M. Xie, Y. Liu, and N. Nagaosa, Sliding dynamics of current-driven skyrmion crystal and helix in chiral magnets, *Phys. Rev. Lett.* **133**, 096702 (2024).
- [59] C. Reichhardt, D. Ray, and C. J. Olson Reichhardt, Collective transport properties of driven skyrmions with random disorder, *Phys. Rev. Lett.* **114**, 217202 (2015).
- [60] W. Koshibae and N. Nagaosa, Theory of current-driven skyrmions in disordered magnets, *Sci. Rep.* **8**, 6328 (2018).
- [61] S. Mallick, Y. Sassi, N. F. Prestes, S. Krishnia, F. Gallego, L. M. V. Arche, T. Denneulin, S. Collin, K. Bouzehouane, A. Thiaville, R. E. Dunin-Borkowski, V. Jeudy, A. Fert, N. Feyren, and V. Cros, Driving skyrmions in flow regime in synthetic ferrimagnets, *Nat. Commun.* **15**, 8472 (2024).
- [62] D. Song, W. Wang, S. Zhang, Y. Liu, N. Wang, F. Zheng, M. Tian, R. E. Dunin-Borkowski, J. Zang, and H. Du, Steady motion of 80-nm-size skyrmions in a 100-nm-wide track, *Nat. Commun.* **15**, 5614 (2024).
- [63] K. Raab, M. Schmitt, M. A. Brems, J. Rothörl, F. Kammerbauer, S. Krishnia, M. Kläui, and P. Virnau, Skyrmion flow in periodically modulated channels, *Phys. Rev. E* **110**, L042601 (2024).
- [64] G. Finocchio, F. Büttner, R. Tomasello, M. Carpentieri, and M. Kläui, Magnetic skyrmions: From fundamental to applications, *J. Phys. D: Appl. Phys.* **49**, 423001 (2016).
- [65] A. Fert, N. Reyren, and V. Cros, Magnetic skyrmions: Advances in physics and potential applications, *Nat. Rev. Mater.* **2**, 17031 (2017).
- [66] K. Wang, V. Bheemarasetty, J. Duan, S. Zhou, and G. Xiao, Fundamental physics and applications of skyrmions: A review, *J. Magn. Magn. Mater.* **563**, 169905 (2022).
- [67] O. Lee, R. Msiska, M. A. Brems, M. Kläui, H. Kurebayashi, and K. Everschor-Sitte, Perspective on unconventional computing using magnetic skyrmions, *Appl. Phys. Lett.* **122**, 260501 (2023).
- [68] T. da Câmara Santa Clara Gomes, Y. Sassi, D. Sanz-Hernández, S. Krishnia, S. Collin, M.-B. Martin, P. Seneor, V. Cros, J. Grollier, and N. Reyren, Neuromorphic weighted sums with magnetic skyrmions, *Nat. Electron.* **8**, 204 (2025).
- [69] K. Everschor-Sitte, J. Masell, R. M. Reeve, and M. Kläui, Perspective: Magnetic skyrmions—Overview of recent progress in an active research field, *J. Appl. Phys.* **124**, 240901 (2018).
- [70] K. Everschor-Sitte and M. Sitte, Real-space Berry phases: Skyrmion soccer (invited), *J. Appl. Phys.* **115**, 172602 (2014).
- [71] S. Yang, Y. Zhao, X. Zhang, X. Xing, H. Du, X. Li, M. Mochizuki, X. Xu, J. Åkerman, and Y. Zhou, Fundamentals and applications of the skyrmion Hall effect, *Appl. Phys. Rev.* **11**, 041335 (2024).
- [72] R. Brearton, L. A. Turnbull, J. A. T. Verezhak, G. Balakrishnan, P. D. Hatton, G. van der Laan, and T. Hesjedal, Deriving the skyrmion Hall angle from skyrmion lattice dynamics, *Nat. Commun.* **12**, 2723 (2021).

- [73] C. Reichhardt and C. J. Olson Reichhardt, Noise fluctuations and drive dependence of the skyrmion Hall effect in disordered systems, *New J. Phys.* **18**, 095005 (2016).
- [74] W. Jiang, X. Zhang, G. Yu, W. Zhang, X. Wang, M. B. Jungfleisch, J. E. Pearson, X. Cheng, O. Heinonen, K. L. Wang, Y. Zhou, A. Hoffmann, and S. G. E. te Velthuis, Direct observation of the skyrmion Hall effect, *Nat. Phys.* **13**, 162 (2017).
- [75] K. Litzius, I. Lemes, B. Krüger, P. Bassirian, L. Caretta, K. Richter, F. Büttner, K. Sato, O. A. Tretiakov, J. Förster, R. M. Reeve, M. Weigand, I. Bykova, H. Stoll, G. Schütz, G. S. D. Beach, and M. Kläui, Skyrmion Hall effect revealed by direct time-resolved X-ray microscopy, *Nat. Phys.* **13**, 170 (2017).
- [76] W. Legrand, D. Maccariello, N. Reyren, K. Garcia, C. Moutafis, C. Moreau-Luchaire, S. Coffin, K. Bouzehouane, V. Cros, and A. Fert, Room-temperature current-induced generation and motion of sub-100 nm skyrmions, *Nano Lett.* **17**, 2703 (2017).
- [77] J.-V. Kim and M.-W. Yoo, Current-driven skyrmion dynamics in disordered films, *Appl. Phys. Lett.* **110**, 132404 (2017).
- [78] C. Reichhardt and C. J. O. Reichhardt, Thermal creep and the skyrmion Hall angle in driven skyrmion crystals, *J. Phys.: Condens. Matter* **31**, 07LT01 (2019).
- [79] R. Juge, *et al.*, Current-driven skyrmion dynamics and drive-dependent skyrmion Hall effect in an ultrathin film, *Phys. Rev. Appl.* **12**, 044007 (2019).
- [80] K. Zeissler, S. Finizio, C. Barton, A. J. Huxtable, J. Massey, J. Raabe, A. V. Sadovnikov, S. A. Nikitov, R. Brearton, T. Hesjedal, G. van der Laan, M. C. Rosamond, E. H. Linfield, G. Burnell, and C. H. Marrows, Diameter-independent skyrmion Hall angle observed in chiral magnetic multilayers, *Nat. Commun.* **11**, 428 (2020).
- [81] C. Reichhardt and C. J. O. Reichhardt, Nonlinear transport, dynamic ordering, and clustering for driven skyrmions on random pinning, *Phys. Rev. B* **99**, 104418 (2019).
- [82] S. A. Díaz, C. J. O. Reichhardt, D. P. Arovas, A. Saxena, and C. Reichhardt, Fluctuations and noise signatures of driven magnetic skyrmions, *Phys. Rev. B* **96**, 085106 (2017).
- [83] T. Sato, W. Koshibae, A. Kikkawa, T. Yokouchi, H. Oike, Y. Taguchi, N. Nagaosa, Y. Tokura, and F. Kagawa, Slow steady flow of a skyrmion lattice in a confined geometry probed by narrow-band resistance noise, *Phys. Rev. B* **100**, 094410 (2019).
- [84] S.-Z. Lin, C. Reichhardt, C. D. Batista, and A. Saxena, Particle model for skyrmions in metallic chiral magnets: Dynamics, pinning, and creep, *Phys. Rev. B* **87**, 214419 (2013).
- [85] J. Stidham and M. Pleimling, Late stages in the ordering of magnetic skyrmion lattices, *Phys. Rev. B* **102**, 144434 (2020).
- [86] C. Reichhardt and C. J. O. Reichhardt, Peak effect, melting, and transport in skyrmion crystals, *Phys. Rev. B* **108**, 014428 (2023).
- [87] M. A. Brems, T. Sparmann, S. M. Fröhlich, L.-C. Dany, J. Rothörl, F. Kammerbauer, E. M. Jefremovas, O. Farago, M. Kläui, and P. Virnau, Realizing quantitative quasiparticle modeling of skyrmion dynamics in arbitrary potentials, *Phys. Rev. Lett.* **134**, 046701 (2025).
- [88] H. Hinrichsen, Non-equilibrium critical phenomena and phase transitions into absorbing states, *Adv. Phys.* **49**, 815 (2000).
- [89] L. Corte, P. M. Chaikin, J. P. Gollub, and D. J. Pine, Random organization in periodically driven systems, *Nat. Phys.* **4**, 420 (2008).