

Operational models of temperature superpositions

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An interacting quantum system and thermal bath can reach thermal equilibrium, resulting in the system and bath acquiring the same temperature. But how does a delocalized quantum system thermalize with a bath whose local temperature varies as, for example, in the Tolman effect? Here, we formulate two scenarios in which the notion of a “superposition of temperatures” may arise: first, a probe interaction with two different baths dependent on the state of another quantum system (a control); and second, a probe interaction with a single bath whose purified state is a superposition of states that each correspond to different temperatures. We show that the two scenarios are fundamentally different and can be operationally distinguished. Moreover, we show that the final probe state is sensitive to the specific realization of the thermalizing channels, and that our results hold for partial and prethermalization cases. Our models may be applied to scenarios involving joint quantum and relativistic phenomena, and explain some recent results found in quantum interference of relativistic probes thermalizing with Unruh or Hawking radiation.

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I. INTRODUCTION

Temperature is well understood in a classical sense as a macroscopic property of the average kinetic energy of an ensemble of particles, while in quantum physics it is a key parameter defining the Gibbs state of a system—inextricably linked to the dynamics of thermalization, and the operation of the laws of thermodynamics in that regime [1]. Recent research into temperature’s role in quantum thermodynamics has mainly focused on sensing using quantum systems (i.e., thermometry) [2], or the complementarity relation between temperature and energy fluctuations [3]. There was even an argument made for describing temperature as a quantum operator [4].

In quantum thermometry, interferometry [5]—or some initial quantum coherence of a single-qubit probe [6]—appears to improve probe sensitivity. But can temperature itself exhibit quantum characteristics such as superposition or quantum coherence? It is natural to consider whether a situation exists where a quantum system could be delocalized and thus interact with baths of different temperatures. Alternatively, one could imagine a superposition of states that are each associated with a different temperature.

There are certain scenarios at the intersection of quantum theory and thermodynamics with relativity in which these abstract ideas do in fact become relevant. See, for example, the Tolman-Ehrenfest effect [7,8], in which locally measured temperatures within a gas in thermal equilibrium in a gravitational field vary with position due to gravitational redshift or time dilation. Because a quantum system cannot be infinitely well localized, it is implied that, fundamentally, such a system acting as a thermometer will be coherently spread over baths with different local temperatures. Another such scenario arises for thermalization of matter coupled to radiation, such as in the Unruh and Hawking effects. There, internal states of some system in noninertial motion thermalize to a temperature proportional to the system’s acceleration. Again, taking into account that classical trajectories are just an approximation, one fundamentally deals with a system that is coherently spread over trajectories with different local accelerations, each leading to a different thermal state of the probe system [9–11]. These scenarios suggest a need for a sharper understanding of thermal processes involving quantum systems: those where temperature does not take a fixed, classical value and thermalization can occur in superposition.

In this work, we outline a general approach by which to address this idea of quantum features of temperature, and identify two cases that describe how a “superposition of temperatures” might arise. In other words, we consider situations where the degrees of freedom of a system that are responsible for thermalization have some form of coherence. The first case is best visualized as a probe system (or “thermometer”) prepared in a superposition of two amplitudes and interacting with an independent thermal bath for each of the amplitudes.

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In the second scenario, a single bath is in a superposition of purified states, each of which yields a different thermal state when reduced to the bath alone. These models assume the usual idealized probe-bath dynamics driven by the zeroth law of thermodynamics, where an interaction between the two results in a thermal state of the probe at the same temperature as that of the bath. We use the framework of quantum channels to derive the outcomes of the processes in the two scenarios and discuss both the conditions under which thermalization can or cannot be achieved, and the sense in which a notion of “superposition of temperatures” might arise. We also study partial or prethermalization—where a quantum system far from equilibrium interacts with some environment for a finite time. We find, crucially, that the key features of the behavior of the probe and coherence between temperatures are reproduced in the prethermalization regime, showing that our results do not depend on full thermalization of the probe. Finally, we find that our two scenarios will naturally arise in the relativistic cases just discussed, and that the single-bath case in particular resolves an outstanding problem for relativistic quantum systems in superpositions of accelerations.

The paper is laid out as follows: In Sec. II, we set up the necessary background, addressing challenges in defining thermalization and superpositions in this context. In Sec. III, we consider how to produce appropriate models and then explore the two models described above: the first, in Sec. III A, involving a single probe in superposition interacting with two independent baths, and the second, in Sec. III B, a single probe interacting with a single bath that is in a superposition. We explore the partial or prethermalization case in Sec. IV, before briefly revisiting the relativistic scenarios just mentioned in Sec. V. We conclude by discussing the implications of the results in Sec. VI.

II. PRELIMINARIES AND DEFINITIONS

We assume a quantum system interacting with some environment, with temperature arising as a consequence of some thermalization process between the two. We can characterize the thermalization of an open quantum system as the action of a CPTP (completely positive trace-preserving) map between the system and environment, or the portion of the environment with which the probe interacts. For cases where full thermalization between the two constituents is reached, both the *initial* state of the thermal bath [12] (subscript B) and the *final* state of the probe system (subscript S) are Gibbs states with the same temperature T [13], i.e.,

$$\rho_{B(S)}^\beta = \frac{e^{-\beta \hat{H}_{B(S)}}}{\mathcal{Z}^\beta}, \quad (1)$$

where $\beta = 1/k_B T$ is the inverse temperature with k_B the Boltzmann constant, and $\mathcal{Z}^\beta = \text{Tr } e^{-\beta \hat{H}_{B(S)}}$ the normalizing partition function. $\hat{H}_{B(S)}$ is the Hamiltonian of the bath (probe).

Consider an initial input product state, in this case between our probe (S) and bath (B): $\rho_{\text{in}} = \rho_B \otimes \rho_S$. The general form of a quantum channel, e.g. [14], following action of the CPTP map $\mathcal{E}(\cdot)$, results in the following Kraus representation [15] of

the postinteraction state of the probe:

$$\mathcal{E}(\rho_S) = \text{Tr}_B\{U_{BS}\rho_{\text{in}}U_{BS}^\dagger\} \equiv \sum_{k,l} c_l^\beta M_{kl} \rho_S M_{kl}^\dagger, \quad (2)$$

where the Kraus operators arising from some unitary interaction between the bath and probe, U_{BS} , read $M_{kl} = \langle k|U_{BS}|l\rangle$, and we choose a basis in which ρ_B is diagonal: $\rho_B^\beta = \sum_l c_l^\beta |l\rangle\langle l|$, with $c_n^\beta := \frac{e^{-E_n\beta}}{\mathcal{Z}^\beta}$, and E_n (for $n = 0, 1, \dots$) the spectrum of the bath Hamiltonian.

For a thermalizing channel specifically, the probe-bath interaction represented by U_{BS} must leave the probe in a thermal state, i.e., $\rho_S^\beta = \sum_k c_k^\beta |k\rangle\langle k|$. A simple example would be a swap operator, $U_{BS} = S_{BS}$, which “swaps” the states of the bath and probe system. For a probe initially in some arbitrary pure state $\rho_S = |\eta\rangle\langle\eta|_S$ and under the swap operator, $\sum_{k,l} c_l^\beta M_{kl} \rho_S M_{kl}^\dagger = \rho_S^\beta$ —which is a thermal state on the probe at inverse temperature β . Note that we are agnostic toward the state in which the *bath* is left by the interaction. In the case of the swap operator example, it may indeed not be left in a thermal state, having acquired the initial state of the probe. One would need to take this into account when selecting an appropriate U_{BS} for the desired scenario. In order to describe a scenario where the bath effectively remains in a thermal state, one could consider the probe only interacting with a small portion of a much larger thermal environment, where that portion is then quickly rethermalized by its surroundings.

Here, however, we want to consider the most general dilation of a thermalization channel, not only to allow for freedom of choice of the unitary as just discussed, but because channels under quantum control are known to be sensitive to the choice of dilation [16–18]. We will discuss this point regarding controlled channels in Sec. III A, but here provide an overview of channel dilations.

Recall that Stinespring’s dilation theorem [19,20] states that all dilations can be expressed as isometries acting on an environment in a pure state, and that all such dilations are related to each other by isometries on the environment [21]. Therefore, to construct a general dilation, we need to address this point, and also purify the bath by entangling it with some ancillary system (denoted by A hereafter) to create an environment that is in a pure state [14]. These purified states are also known as thermofield double states in the context of finite-temperature quantum field theory [22,23]. We write a purification of a specifically thermal state with inverse temperature β , in the energy eigenbasis, as

$$|\theta^\beta\rangle = \sum_n \sqrt{c_n^\beta} |n, n\rangle. \quad (3)$$

The state of the bath alone can be recovered by tracing out the ancillary system: $\rho_B := \text{Tr}_A\{|\theta^\beta\rangle\langle\theta^\beta|_{AB}\}$, where $|\theta^\beta\rangle_{AB}$ is the joint pure state of the bath and ancilla.

The general dilation of the channel is then given by

$$\mathcal{E}'(\rho_S) = \text{Tr}_{AB}\{U'_{A'B'S}\rho_{\text{in}}U_{A'B'S}^\dagger\}, \quad (4)$$

where primes indicate a larger Hilbert space such that $\dim(\mathcal{H}_{A'B'}) \geq \dim(\mathcal{H}_{AB})$, $\rho_{\text{in}} = |\theta^\beta\rangle\langle\theta^\beta|_{AB} \otimes \rho_S$, and

$$U'_{A'B'S} = (w \otimes \mathbb{I}_S) \cdot (\mathbb{I}_A \otimes U_{BS}). \quad (5)$$

The arbitrary isometry w is defined $w : \mathcal{H}_{AB} \rightarrow \mathcal{H}_{A'B'}$.

Hence, Eq. (4), dropping the identities in Eq. (5), will be

$$\mathcal{E}'(\rho_S) = \text{Tr}_{A'B'}\{w U_{BS} |\theta\rangle \langle\theta|_{AB} \rho_S U_{BS}^\dagger w^\dagger\}. \quad (6)$$

Cycling the isometries under the trace allows for the substitution $w^\dagger w = \mathbb{I}$. Hence, for the aforementioned swap operator, the probe is left in the thermal state ρ_S^β . The channel in Eq. (6) is a thermalizing one if we use the swap operator for U_{BS} , but due to Stinespring's dilation theorem, Eq. (6) also represents an *arbitrary* thermalizing channel in the sense that any two dilations are related by appropriate choice of the isometries.

If one recalls that an isometry can map a Hilbert space to another of a larger dimension (but not to a smaller one), one can see that the most general dilation can be written in the form of Eq. (6), where U_{BS} is a minimal dilation, namely, one with the smallest possible environment (here, systems A and B). For a thermalizing channel, a minimal dilation is given by the swap introduced above, so we do not lose generality by assuming that U_{BS} is the swap in Eq. (6) [26].

III. OPERATIONAL MODELS OF TEMPERATURE SUPERPOSITIONS

Ultimately, we are describing in this work a generalization of thermalization processes, motivated by the context of creating temperature superpositions. Of course, a system with some defined temperature is implied to be in a Gibbs state, whereas generally the notion of superposition only applies to two (or more) distinct pure states, for example, some $|\psi\rangle \neq |\phi\rangle$, placed in a linear combination, $|\chi\rangle = \alpha|\psi\rangle + \beta|\phi\rangle$ (where α and β are nonvanishing complex numbers). However, operationally, we can think of the preparation of a superposition as a process where a state of one or more different degrees of freedom (DoFs) is created in response to a quantum state of some other particular DoF, which we will later refer to as a control. By preparing and measuring the control in superposition, the other DoF(s) are prepared in the corresponding superposition.

Therefore, one operational context in which the notion of “superposition of temperatures” arises is when one has a superposition of purifications such as $|\theta^{\text{sup}}\rangle \propto |\theta^0\rangle + |\theta^1\rangle$, where, for $x = 0, 1$, the state $|\theta^x\rangle$ is a purification of a thermal state at a temperature T_x .

A second context is that of quantum controlled channels, e.g. [16], where in our case we have a system interacting with one of two baths each at a different temperature, depending on the state of some control DoF. We look at this scenario first.

A. Two baths: superpositions of thermal channels

For our first case of a superposition of temperatures, we define an interaction between a probe system ρ_S and a composite purified bath ρ_{AB} comprised of two subsystems in a product state $\rho_{AB} = \rho_{A_0 B_0} \otimes \rho_{A_1 B_1}$. This environment can arise, for example, if the subsystems are thermally isolated from each other, at least over the timescale of the interaction with the probe. Such a model would be relevant to a quantum system probing a Tolman-affected bath, as we discuss further in Sec. V.

We introduce a quantized control DoF ρ_C that determines which subsystem of the environment the probe thermalizes

with. We define this “quantum-controlled thermalization” as a process that admits an isometric dilation $U'_{A'B'CS}$ of the form

$$U'_{A'B'CS} = U'_0 \otimes |0\rangle \langle 0|_C + U'_1 \otimes |1\rangle \langle 1|_C, \quad (7)$$

where the control states $|i\rangle_C$, $i = 0, 1$, are orthonormal and determine that the probe interacts with the corresponding bath B_i through U_i . U'_i are two isometries through which the environments do not interact, and which are each dilations of a thermalizing map for the i th environment. Hence, by our discussion in the previous section, $U'_i = (w_i \otimes v_j \otimes \mathbb{I}_S)(\mathbb{I}_A \otimes U_{B_i S})$, where $U_{B_i S}$ is a minimal dilation (a dilation of the thermalization map for which the bath is equal in dimension to the probe system), w_i defines the dilation of the i th thermalizing channel (as discussed above), and, for $j \neq i$, $v_j : \mathcal{H}_{A_j B_j} \rightarrow \mathcal{H}_{A'_j B'_j}$ is an additional isometry describing the isolated evolution of the j th environment when it does not interact with the system [27].

The isometry for the entire controlled thermalization process is then

$$\begin{aligned} U'_{A'B'CS} = & (w_0 \otimes v_1 \otimes \mathbb{I}_S)(\mathbb{I}_{A_0} \otimes U_{B_0 S} \otimes \mathbb{I}_{A_1 B_1}) \otimes |0\rangle \langle 0|_C \\ & + (w_1 \otimes v_0 \otimes \mathbb{I}_S)(\mathbb{I}_{A_1} \otimes U_{B_1 S} \otimes \mathbb{I}_{A_0 B_0}) \otimes |1\rangle \langle 1|_C. \end{aligned} \quad (8)$$

As mentioned above, quantum-controlled channels are sensitive to the dilations on the individual channels, in the sense that the overall channel, acting on the system and the control after tracing out both environments, depends on the choice of isometries w_i , v_i . We stress that Stinespring's dilation theorem still applies to the overall channel, which remains independent of a global choice of dilation. In this respect, note that not all the dilations of the overall channel have the form of Eq. (8), because an additional isometry can let the two environments interact without observable consequences on the reduced channel. Our definition of a quantum-controlled thermalization channel is that it admits at least one dilation of the form of Eq. (8).

We take an initial state of the control to be $\rho_C = |+\rangle \langle +|$, where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, and measure it in the superposition $|\phi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)$, with relative phase ϕ assumed to be fully controllable.

While the treatment we are describing is independent of any specific framework, the Mach-Zehnder (MZ) interferometer of Fig. 1 is certainly a potential realization. In that example, the two states $|i\rangle_C$ of the control are identified with the two paths through the interferometer, the probe is an internal DoF of the interfering system (which could be, for example, a particle), and it thermalizes with subsystem B_0 or B_1 depending on the path taken.

For initial input state $\rho_{\text{in}} = \rho_{AB} \otimes \rho_C \otimes \rho_S$, the subnormalized output state of the probe, $\rho_S(\phi) = \text{Tr}_{A'B'}\{|\phi\rangle_C U' \rho_{\text{in}}(0) U'^\dagger |\phi\rangle_C\}$, then yields four terms:

$$\begin{aligned} \rho_S(\phi) = & \frac{1}{4} \text{Tr}_{A'B'}\{(w_0 U_{B_0 S} \otimes v_1) \rho_{ABS} (U_{B_0 S}^\dagger w_0^\dagger \otimes v_1^\dagger) \\ & + (w_1 U_{B_1 S} \otimes v_0) \rho_{ABS} (U_{B_1 S}^\dagger w_1^\dagger \otimes v_0^\dagger) \\ & + (e^{i\phi} (w_0 U_{B_0 S} \otimes v_1) \rho_{ABS} (U_{B_1 S}^\dagger w_1^\dagger \otimes v_0^\dagger) + \text{H.c.})\}, \end{aligned} \quad (9)$$

where H.c. refers to the Hermitian conjugate.

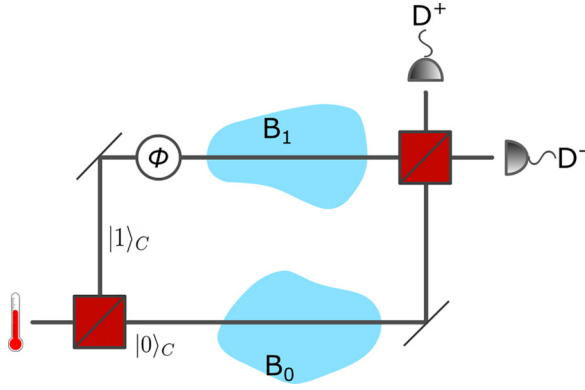


FIG. 1. A Mach-Zehnder interferometer as an example realization of the two-bath case. An input probe state enters from the left, is placed in a superposition, and travels along the two arms of the interferometer, on each of which is a bath thermalized to some temperature. The paths recombine at the second beam splitter, and the output state is then detected at D^+ , D^- .

The first two terms in Eq. (9) are each a channel arising from the interaction U_{B_iS} between the system and bath B_i , with the form of a single quantum channel as in Sec. II. The other two terms are “cross terms” between channels, with the third term in Eq. (9), for arbitrary pure system state $\rho_S = |\eta\rangle_S \langle \eta|_S$:

$$\frac{1}{4} \text{Tr}_{A'B'} \{ e^{i\phi} w_0 U_{B_0S} (|\theta^{\beta_0}\rangle_{A_0B_0} |\eta\rangle_S) v_1 |\theta^{\beta_1}\rangle_{A_1B_1} \times \langle \theta^{\beta_0}|_{A_0B_0} v_0^\dagger (\langle \eta|_S \langle \theta^{\beta_1}|_{A_1B_1}) U_{B_1S}^\dagger w_1^\dagger \}. \quad (10)$$

The fourth term is of course its Hermitian conjugate.

As in Sec. II, we take the swap operator S_{B_iS} , which simply swaps the state of the i th bath with the state of the system. So, the first two terms of Eq. (9) are each thermal states on the probe $\rho_S^{\beta_i}$ at the respective temperatures of the baths and Eq. (10) becomes

$$\frac{1}{4} e^{i\phi} \langle \theta^{\beta_0}|_{A_0B_0} v_0^\dagger w_0 (\langle \theta^{\beta_0}\rangle_{A_0S} |\eta\rangle_{B_0}) \times (\langle \eta|_{B_1} \langle \theta^{\beta_1}|_{A_1S}) w_1^\dagger v_1 |\theta^{\beta_1}\rangle_{A_1B_1}, \quad (11)$$

with the fourth term again its Hermitian conjugate.

The theta kets and bras form partial inner products, but this is as far as we can reduce these cross terms without further specifying the involved systems.

Defining

$$|\Phi_i\rangle := \langle \theta^{\beta_i}|_{A_iB_i} v_i^\dagger w_i (|\theta^{\beta_i}\rangle_{A_iS} |\eta\rangle_{B_i}), \quad (12)$$

we can write the final (subnormalized) state of the probe as

$$\rho_S(\phi) = \frac{1}{4} [\rho_S^{\beta_0} + \rho_S^{\beta_1} + (e^{i\phi} |\Phi_0\rangle \langle \Phi_1| + \text{H.c.})]. \quad (13)$$

Note that Eq. (12) is also subnormalized, as $\langle \Phi_i|\Phi_i\rangle \leq 1$.

The third and fourth terms of Eq. (13) are not in general thermal, depending not just on the initial states of both baths and of the system, but also on the isometries associated with each channel dilation, as expected. It is also important to note that the isometries also ensure that we are free to choose any unitary to represent the probe-bath interaction, and that Eq. (13) does not depend on the swap operation. See the Appendix for the special case where the isometries are unitaries on the bath, which is similar to the result of

Ref. [16] for generic (i.e., not thermal) quantum channels in superposition.

One might have thought that if both baths are at the same temperature, i.e., for $\beta_0 = \beta_1 \equiv \beta$, the system simply thermalizes to that temperature; however, Eq. (13) explicitly demonstrates that this is not the case unless $v_i^\dagger w_i = \mathbb{I}$, or we tune ϕ to the particular value that destroys the interference of the control.

In cases where v and w do form an identity, we can further reduce the partial inner products of Eq. (12) and arrive at the corresponding special case of Eq. (13):

$$\rho_S(\phi) = \frac{1}{4} [\rho_S^{\beta_0} + \rho_S^{\beta_1} + (e^{i\phi} \rho_S^{\beta_0} |\eta\rangle_{B_0} \langle \eta|_{B_1} \rho_S^{\beta_1} + \text{H.c.})]. \quad (14)$$

So, we see that the form of Eq. (13) has crucial implications for the system’s coherence, and thus the extent to which one can speak about any kind of coherence in the final state of the probe (or of the temperature). This result can be understood through the complementarity between quantum coherence and which-path information—via the analogy with the Mach-Zehnder interferometer in Fig. 1. In an interferometric scenario like in Fig. 1, the visibility of the final interference pattern contains relevant information about the dynamics and correlations that develop between the involved systems. Recall that visibility is quantified by the magnitude of the off-diagonal elements of the final state [28,29]. Here, the final state is that of the control, and the visibility is equivalently the contrast of the interference pattern obtained by measuring the control in a superposition basis.

The probability of measuring the control in the state $|\phi\rangle$ is $P(\phi) = \text{Tr}\{\rho_S(\phi)\}$; explicitly,

$$P(\phi) = \frac{1}{2} + \frac{1}{2} |\text{Tr}\{|\Phi_0\rangle \langle \Phi_1|\}| \cos(\phi + \psi), \quad (15)$$

where ψ is a phase defined via $\text{Tr}_S\{|\Phi_0\rangle \langle \Phi_1|\} \equiv |\text{Tr}_S\{|\Phi_0\rangle \langle \Phi_1|\}| e^{-i\psi}$.

The visibility, therefore, is

$$\mathcal{V} = |\langle \Phi_1|\Phi_0\rangle|. \quad (16)$$

Crucially, this visibility depends not just on the temperatures of the two baths, but of course the isometries v and w too:

$$\langle \Phi_1|\Phi_0\rangle = \langle \theta^{\beta_0}|_{A_0B_0} v_0^\dagger w_0 (\sqrt{\rho^{\beta_0} \rho^{\beta_1}} \otimes |\eta\rangle_{B_0} \langle \eta|_{B_1}) \times w_1^\dagger v_1 |\theta^{\beta_1}\rangle_{A_1B_1}. \quad (17)$$

Again, if $v_i^\dagger w_i = \mathbb{I}$, for $\beta^0 = \beta_1 \equiv \beta$, the above reduces further to $\langle \eta|(\rho^\beta)^2|\eta\rangle = \text{Tr}(\rho^\beta)^2$.

Note that, in general, the visibility is less than 1. We know by the Cauchy-Schwarz inequality, $|\langle \Phi_1|\Phi_0\rangle| \leq \|\Phi_1\| \|\Phi_0\|$, that visibility can only equal 1 if the two states are normalized and equal (up to a phase). Taking Eq. (17),

$$\|\Phi_0\| = \langle \theta^{\beta_0}|_{A_0B_0} v_0^\dagger w_0 (\rho^{\beta_0} |\eta\rangle \langle \eta|_{B_0}) w_0^\dagger v_0 |\theta^{\beta_0}\rangle_{A_0B_0}, \quad (18)$$

consider the general structure of this expression: $\langle \psi|\rho|\psi\rangle$, for $\langle \psi|\psi\rangle \leq 1$. We know that since $\rho \geq 0$ and $\text{Tr}\rho = 1$, the eigenvalues of ρ cannot be larger than 1, so $\langle \psi|\rho|\psi\rangle = 1$ implies that $\rho = |\psi\rangle \langle \psi|$ and $\langle \psi|\psi\rangle = 1$.

This implies the condition

$$\rho^{\beta_0} |\eta\rangle \langle \eta|_{B_0} = w_0^\dagger v_0 |\theta^{\beta_0}\rangle \langle \theta^{\beta_0}| v_0^\dagger w_0, \quad (19)$$

and for this to hold the system and bath must both be in a pure state, which can only be achieved at zero temperature (and with a nondegenerate ground state). So, with $|\theta^{\beta_0}\rangle = |0\rangle|0\rangle$ signifying a state with these conditions, Eq. (17) becomes

$$\langle\Phi_1|\Phi_0\rangle = \langle 0|\langle 0|v_0^\dagger w_0|0\rangle|\eta\rangle_{B_0}\langle 0|\langle \eta|_{B_1}w_1^\dagger v_1|0\rangle|0\rangle. \quad (20)$$

This expression can only be equal to one when the combined isometries rotate the system to the vacuum state, i.e., $v_i^\dagger w_i|0\rangle|\eta\rangle = |0\rangle|0\rangle$.

Hence, in general, the visibility cannot reach unity, directly showing that the probe-bath(s) interactions do affect the coherence of the control even for equal bath temperatures. In other words, even for identical isometries and baths at the same temperature, the bath subsystems still store which-way information [29] about the control, except in the above special case of both baths at zero temperature and system S in a pure state.

The above also holds for the special case where the isometries are unitaries, as detailed in the Appendix. In that case, the visibility is

$$\mathcal{V} = |\text{Tr}\{u^{0\dagger}\rho_S^{\beta_0}\rho_S^{\beta_1}u^1\rho_S\}|, \quad (21)$$

where u^i are unitaries associated with each bath. The maximum visibility is

$$\mathcal{V}_{\max}^{\text{pure}} = c_0^{\beta_0}c_0^{\beta_1},$$

for an initial pure state of the probe. The intuition is again that visibility is maximized when the local unitaries effectively rotate the probe state to the energy ground state (up to a phase) as it has the highest overlap with a thermal state at any finite temperature. Thus, for an arbitrary mixed state $\rho_S = \sum_s p_s |s\rangle\langle s|$, the maximum visibility reads

$$\mathcal{V}_{\max} = \sum_s p_s c_s^{\beta_0}c_s^{\beta_1}. \quad (22)$$

Without loss of generality, the probabilities p_s are ordered decreasingly: $p_s \geq p_{s+1}$. This is because thermal weights are likewise ordered decreasingly, i.e., $c_k^{\beta_i} > c_{k+1}^{\beta_i}$, and due to the orthogonality of the states $|s\rangle$, a unitary u^i can only rotate one of the states from an orthogonal set to the energy ground state. That being said, the same strategy can be iterated to find a unitary mapping of the eigenstates of ρ_S to progressively higher-energy eigenstates.

B. One bath: a superposition of purifications

The previous case used quantum control of thermal channels associated with two separate bath subsystems as an operational model of a temperature superposition. Here, we explore the other model identified at the beginning of this section—where the bath is a single system in a superposition of purifications, each corresponding to a different temperature. Figure 2 provides a sketch of such a situation. This scenario is highly relevant to accelerating quantum systems that experience the Unruh effect, as we discuss in Sec. V. The procedure also has some similarity with the “superposition of thermal states” in optics, considered in Ref. [30]; however, no superpositions of different temperatures were considered therein.

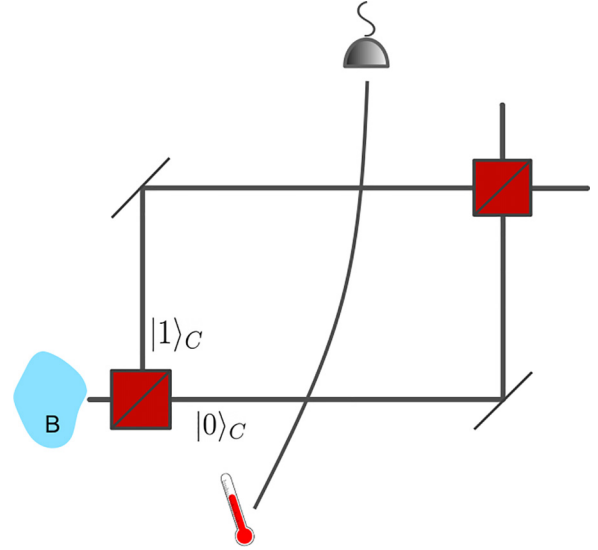


FIG. 2. A diagram for the one-bath case in analogy to Fig. 1. The bath is placed in a superposition and acquires a different temperature depending on the control state $|0\rangle_C$ or $|1\rangle_C$. The probe interacts/thermalizes with the bath while the bath is in superposition.

Purifications were defined in Sec. II and Eq. (3). We now take a more general form

$$|\theta^{\beta_x}(x)\rangle = \frac{1}{\sqrt{2}} \sum_b e^{-i\phi_x} \sqrt{c_b^{\beta_x}} |b, a(b, x)\rangle, \quad (23)$$

where x signifies that the purification gives rise to a thermal state at temperature β_x , and where $\{|a(b, x)\rangle\}_b$ is an orthonormal basis for the ancillary system. This basis can, in principle, be different for the different purifications $x = \{0, 1\}$; phase $\phi_x \in \mathbb{R}$.

An unnormalized superposition of Eq. (23) then reads $|\psi\rangle = \sum_{x=0,1} |\theta^{\beta_x}(x)\rangle$. One can also prepare the superposition of purifications using another DoF as a control, such that $|\tilde{\psi}\rangle = \frac{1}{\sqrt{2}} \sum_{x=0,1} |\theta^{\beta_x}(x)\rangle |x\rangle_C$. For Eq. (23), that is

$$|\tilde{\psi}\rangle = \frac{1}{\sqrt{2}} \sum_{x=0,1} \sum_b e^{-i\phi_x} \sqrt{c_b^{\beta_x}} |b, a(b, x)\rangle |x\rangle_C. \quad (24)$$

We could also choose to prepare an initial state for the bath by first measuring the control in the superposition $|\phi\rangle_C = \frac{1}{\sqrt{2}}(|0\rangle_C + e^{i\phi_C}|1\rangle_C)$ after tracing out the ancilla: $\rho_B^{\text{in}}(\phi) = \langle\phi|_C \text{Tr}_{A(x)}\{|\tilde{\psi}\rangle\langle\tilde{\psi}|\} |\phi\rangle_C$. This would yield a reduced *initial* state of the bath such as

$$\begin{aligned} \rho_B^{\text{in}}(\phi) &= \frac{1}{4} \left[\rho^{\beta_0} + \rho^{\beta_1} + \left(e^{-i\tilde{\phi}} \sum_{b,b'} \sqrt{c_b^{\beta_0} c_{b'}^{\beta_1}} V_{bb'}^{01} |b\rangle\langle b'| + \text{H.c.} \right) \right], \end{aligned} \quad (25)$$

where $\tilde{\phi} = \phi_0 - \phi_1 - \phi_C$, and we define $V_{bb'}^{xx'} := \langle a(b', x') | a(b, x) \rangle$. $V_{bb'}^{xx'}$ is a matrix element of a special unitary operator $V^{xx'}$ transforming between ancilla bases associated with control states x and x' . It satisfies $V_{bb'}^{xx} = V_{bb'}^{x'x'} = \delta_{bb'}$, and $V^{xx'} = (V^{x'x})^\dagger$. If the purifications are the same for both ancilla states, $V^{xx'}$, for all x, x' , reduces to a delta function $\delta_{bb'}$.

For the analysis here, however, we do not begin with the reduced state of the bath, instead using the joint state of bath, ancillary system, and control from Eq. (24):

$$\rho_{ABC}^{\text{in}} = \frac{1}{2} \sum_{x,x'=0,1} \sum_{b,b'} e^{-i(\phi_x - \phi_{x'})} \sqrt{c_b^{\beta_x} c_{b'}^{\beta_{x'}}} |a(b, x)\rangle |b\rangle |x\rangle \langle x'|_C \langle b'| \langle a(b'x')|, \quad (26)$$

where the entire initial state is $\rho^{\text{in}} = \rho_{ABC} \otimes \rho_S$. Again, we have a generic probe-bath unitary interaction U_{BS} . To generalize to arbitrary Kraus decompositions, we simply introduce additional unitary operators u_{AB}^x to the local interaction with the bath, which depend on the control and also act on both the bath and the ancillary system. Hence, the entire interaction, extended to include these “local unitaries”, takes the form

$$\tilde{U}_{ABCS} = \sum_{x=0,1} [(u_{AB}^x \otimes \mathbb{I}_S) \cdot (\mathbb{I}_A \otimes U_{BS})] \otimes |x\rangle \langle x|_C. \quad (27)$$

This $\tilde{U}_{ABCS}$ is already an arbitrary dilation as the ancillae themselves can be of arbitrary dimensions, so we do not need to employ the isometries as in the two-bath scenario.

The final probe state, conditioned on the control $|\phi\rangle_C = \frac{1}{\sqrt{2}}(|0\rangle_C + e^{i\phi_C}|1\rangle_C)$, is

$$\tilde{\rho}_S(\phi) = \langle \phi |_C \text{Tr}_{AB} \{ \tilde{U}_{ABCS} \rho_{ABC}^{\text{in}} \otimes \rho_S \tilde{U}_{ABCS}^\dagger \} | \phi \rangle_C. \quad (28)$$

Taking the swap operator once more to be our U_{BS} , this is, explicitly,

$$\tilde{\rho}_S(\phi) = \frac{1}{4} \left[\rho_S^{\beta_0} + \rho_S^{\beta_1} + \left(e^{-i\tilde{\phi}} \sum_{b,b'} \sqrt{c_b^{\beta_0} c_{b'}^{\beta_1}} W_{bb'}^{10} |b\rangle \langle b'| + \text{H.c.} \right) \right], \quad (29)$$

where $W_{bb'}^{xx'} := \langle a(b', x') | \text{Tr}_S \{ u_{AS}^{1\dagger} u_{AS}^0 \rho_S \} | a(b, x) \rangle$, similar to the $V_{bb'}^{xx}$ discussed above, but now postinteraction, and so dependent on the extra unitaries and the state of the probe.

We immediately see that, in this model too, the final probe state still depends on the temperatures of both baths, the initial state of the probe, and the interaction dynamics as represented by the local unitaries. Again, the first two terms correspond to thermal states at each of the involved temperatures, while the second two “cross terms” do not; however, it is in these two cross terms that there are marked differences to the model of Sec. III A.

Unlike in the two-bath case, here the probe *can* thermalize. Thermalization can occur when $\beta_0 = \beta_1$ and $W_{bb'}^{01} \propto \delta_{bb'}$. These criteria are met when (1) both amplitudes of the purification (or superposed purifications) correspond to the same thermal state of the bath, (2) the purification basis is the same for both amplitudes, and (3) $u_{AS}^{1\dagger} u_{AS}^0 = e^{i\psi} \mathbb{I}$ (in which case the matrix $W_{bb'}^{01}$ is the identity).

Looking at the visibility of the interference between the amplitudes of the control DoF, we find that the probability to measure the control in $|\phi\rangle_C$ is

$$\tilde{P}(\phi) = \frac{1}{4} \left[2 + \left(e^{-i\tilde{\phi}} \sum_b \sqrt{c_b^{\beta_0} c_b^{\beta_1}} W_{bb}^{01} + \text{H.c.} \right) \right], \quad (30)$$

which results in the visibility

$$\tilde{\mathcal{V}} = \left| \sum_b \sqrt{c_b^{\beta_0} c_b^{\beta_1}} W_{bb}^{01} \right|. \quad (31)$$

There always exist u_{AS}^x for which the visibility vanishes. Considering eigenstates $|s\rangle$ of ρ_S , it suffices to take u_{AS}^i that map each $|s\rangle$ to different energy eigenstates of the probe, i.e., such that $u_{AS}^1 |s\rangle$ is orthogonal to $u_{AS}^0 |s\rangle$ for every s . The same argument applies to the two-bath case under the conditions discussed in Sec. III A and the Appendix.

Considering maximum visibility, on the other hand, first, it is easy to see that

$$|W_{bb'}^{01}| = |\langle a(b', x') | \text{Tr}_S \{ u_{AS}^{0\dagger} u_{AS}^1 \rho_S \} | a(b, x) \rangle| \leq 1, \quad (32)$$

where the inequality is saturated—thus maximizing the visibility—when $u_{AS}^{0\dagger} u_{AS}^1 = e^{i\varphi} \mathbb{I}$, for arbitrary $\varphi \in \mathbb{R}$. This means the maximum visibility is, therefore,

$$\tilde{\mathcal{V}}_{\text{max}} = \sum_b \sqrt{c_b^{\beta_0} c_b^{\beta_1}}. \quad (33)$$

Compare this with the analogous scenario in the two-bath case where the isometries are assumed to be unitaries, as discussed in the previous section, and in the Appendix, where the maximum visibility, Eq. (22), is $\mathcal{V}_{\text{max}} = \sum_s p_s c_s^{\beta_0} c_s^{\beta_1}$. The terms p_s are probabilities arising from the diagonal form of the density matrix representation of the initial probe state. This is markedly different from the above one-bath case as it explicitly depends on the initial state of the probe.

Interestingly, Eq. (33) coincides with the fidelity between the two thermal states, $\tilde{\mathcal{V}}_{\text{max}} = \text{Tr} \sqrt{\sqrt{\rho^{\beta_0}} \rho^{\beta_1} \sqrt{\rho^{\beta_0}}} = \text{Tr} \sqrt{\rho^{\beta_0} \rho^{\beta_1}}$, where the latter simplification is possible because the two states commute [20].

When $\beta_0 = \beta_1 = \beta$, independent of the actual value of β , $\tilde{\mathcal{V}}_{\text{max}} = 1$. However, even when $\beta_0 = \beta_1$, the visibility in Eq. (31) will be less than 1 when $W_{bb}^{01} \neq 1$, that is, when the purification bases are not the same.

A further particular case of interest is probe-bath interactions, which are independent of the control. This renders u_{AB}^x independent of x , and the visibility is simply $\tilde{\mathcal{V}} = \sum_b \sqrt{c_b^{\beta_0} c_b^{\beta_1}} |\langle a(b, 1) | a(b, 0) \rangle|$. This corresponds to a situation where the probe interacts with the bath after it has been prepared in a superposition, that is, after a particular state of the control has been detected postinterference.

In summary, we generally find that in the two-bath scenario, full thermalization in general cannot be attained, while in this single-bath scenario it can, with particular conditions discussed above.

IV. PARTIAL THERMALIZATION

The two scenarios considered in the previous section assumed full thermalization, which can be seen as the asymptotic future of any physical process that would in practice take finite time. In this section, we look at partial or prethermalization processes corresponding to our two models, demonstrating that the general features identified in the previous section also hold for finite interaction times.

To model the process of prethermalization through a unitary interaction between the bath and the probe, we turn to

the theory of thermal attenuators (TAs) [31]. In general, TAs represent the effect of a (thermal) lossy environment acting on a probe. In the most generic case, this environment could be constituted by bosonic DoFs in a thermal state. However, for studying prethermalization, it will be sufficient for us to consider a qubit probe interacting with an environment also constituted by qubit(s) (we refer to these as “bath qubits” through this section). In this case, a TA channel is a generalized amplitude damping channel (GADC) on the probe.

The unitary interaction between the probe and bath that yields a GADC is of the form

$$U_{BS}^\eta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-\eta} & \sqrt{\eta} & 0 \\ 0 & -\sqrt{\eta} & \sqrt{1-\eta} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (34)$$

where η parametrizes the strength of the interaction. The action of a GADC on a probe can also be expressed in the form of Kraus operators, as per Eq. (2). We note that in that case, the purified bath state will consist of two qubits (one of them being the ancilla for purification). Such a purified state of the bath, constituted by qubits, has the form of Eq. (3) with $n = 0, 1$ and $\mathcal{Z}^\beta = e^{-E_0\beta} + e^{-E_1\beta}$. The unitary interaction between the (purified) bath and the probe is then given as follows:

$$U_{BS} = U_{BS}^\eta \otimes \mathbb{I}_A, \quad (35)$$

where U_{BS}^η acts on the probe and the bath qubit and the full interaction is an identity on the purification ancilla.

Hence, we model the process of thermalization of a finite-dimensional quantum system by the application of a GADC. If the interaction parameter is set to be the maximum ($\eta = 1$), the probe thermalizes after a single interaction with the bath, or equivalently with a single application of the GADC; thus, here we set $\eta < 1$. Therefore, after a single interaction with the bath, the probe acquires an intermediate state between the initial state and a thermal one.

To model the gradual approach to full thermalization, we further consider a collisional model [32], where our probe systems interact through the above-mentioned GADC with independent bath subsystems in the initial state $|\theta^\beta\rangle$. Specifically, the probe interacts with each of these subsystems individually and successively through the unitary operator U_{BS} . Notably, this corresponds to multiple consecutive applications of the GADC on the probe such that each application represents a time step. We generally expect that after a sufficient number of such steps, the probe assumes a thermal state and continues to remain in that state even if the process is continued. Indeed, this does hold, as we show in the following sections.

A. Collisional model with GADC

In this model of gradual thermalization, the state of a complete bath is of the form $|\theta^\beta\rangle^{\otimes \mathcal{M}}$, where $\mathcal{M}$ is the number of bath subsystems, and we have considered each of the bath subsystems to be in a purified state (for both one- and two-bath scenarios). We call an individual application of the unitary U_{BS} (on the probe and purified bath subsystem) a collision

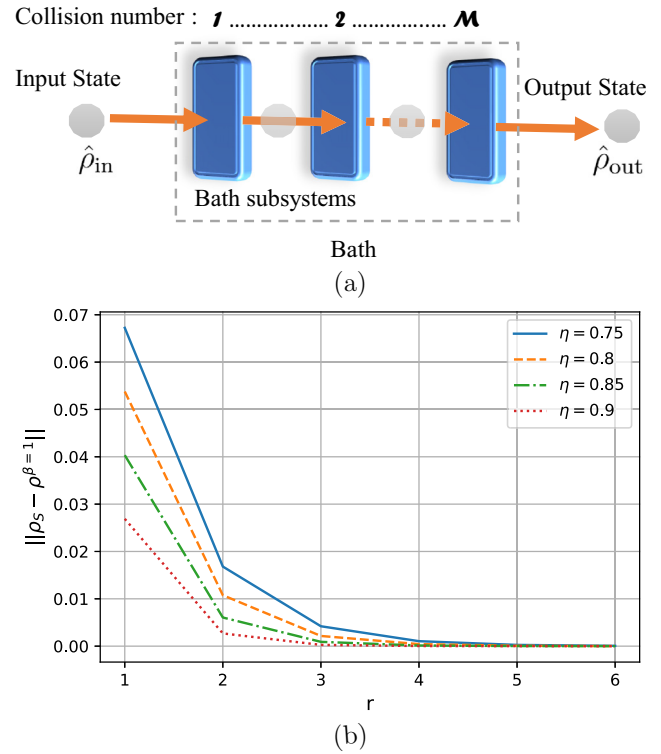


FIG. 3. (a) Schematic diagram showing the collisional model with each step consisting of the GADC unitary (U_{BS}^η) acting on the probe and the bath, each of whose subsystems are in a Gibbs state at a fixed temperature. Equivalently, one could consider the unitary interaction $U_{BS} = U_{BS}^\eta \otimes \mathbb{I}_A$ between one of the (purified) bath subsystems and the probe at each “collision.” (b) Trace distance between the state of the probe and a thermal state as a function of the collision number in the collisional GADC model. The probe is initialized in the state $|0\rangle$, whereas the initial state of each of the bath subsystems is defined as $|\theta^{\beta=1}\rangle$ corresponding to temperature $T = 1$. $\Delta E = 1$ for both probe and bath subsystems.

and assign a “collision number” to each such operation (see Fig. 3).

Such modeling of the thermalization process enables us to be in the prethermalization regime by controlling the strength and/or the number of collisions. Figure 3 shows the trace distance between the state of the probe, initialized as $|0\rangle$ (the ground state), and a thermal state at a temperature $T = 1$, as a function of the number of collisions. This distance approaches zero as the number of collisions increases. The number of collisions after which the trace norm falls below a fixed threshold $||\rho_S - \rho^{\beta=1}|| < \epsilon$, signaling that the probe approaches thermalization, depends on both the interaction strength and the initial state of the probe.

In the following subsection, we use the above model to illustrate the drop in visibility in the pre/post-thermalization regime and the key differences while focusing on the two scenarios identified in the previous sections—the one-bath and two-bath cases. In all instances shown here, we consider any local unitaries or isometries acting on the bath(s) to be the identity. This, in particular, implies that the bath does not have its own intrinsic dynamics and that the purifications considered in the single-bath case are likewise independent of the control.

B. Two-bath case

In the two-bath case of Sec. III A, the probe interacts with two separate bath subsystems—for example, localized in the arms of an MZ interferometer (Fig. 1). For the study of prethermalization in this model, it will be convenient to use purified states of the baths, by which the initial state of both baths is given as

$$|B\rangle = |\theta^{\beta_0}\rangle^{\otimes \mathcal{M}} \otimes |\theta^{\beta_1}\rangle^{\otimes \mathcal{M}}, \quad (36)$$

where each state $|\theta^{\beta_i}\rangle$ is defined as in Eq. (3).

The interaction between the probe, the control, and the baths takes the form of Eq. (8), where the probe-bath unitaries are now additionally split into a tensor product of terms acting at subsequent collisions. Hence, for the r th collision, we have

$$\begin{aligned} U_{B_0S}^r &= (U_{BS})^r \otimes (\mathbb{I}_2 \otimes \mathbb{I}_2)^{\otimes \bar{r}}, \\ U_{B_1S}^r &= (U_{BS})^r \otimes (\mathbb{I}_2 \otimes \mathbb{I}_2)^{\otimes \bar{r}}, \end{aligned} \quad (37)$$

where U_{BS} denotes the unitary operator in Eq. (35), superscript r indicates the unitary being confined to the r th subsystem, and $\bar{r}$ refers to the remaining subsystems of $\mathcal{M}$. The subscript 2 on the identity operator reminds us that we are modeling each of the $\mathcal{M}$ subsystems of either of the baths as a qubit.

We compare the pre- and post-thermalization visibilities for the two-bath case (cf. Sec. III A) by changing the number of collisions (number of subsystems within the bath). The results are presented in Fig. 4. The interaction strength is kept constant and we take the energy ground state as the probe's initial state. Notably, we see a very good agreement between full- and prethermalization scenarios. In particular, we find that in the prethermalization case, even if the temperatures of the two baths are equal, the visibility is significantly reduced, with a stronger reduction for higher temperatures. This is in contrast with the one-bath case in Fig. 5, to be discussed in the next section, where for equal temperatures the visibility is maximal. Note also that the visibility generally falls with the number of collisions, while the high-visibility parameter region shrinks. These results are in agreement with the analytical results for full thermalization in Sec. III A.

C. One-bath case

As outlined in Sec. III B, in the one-bath case, the purified state of the bath is entangled with the control DoF, and its initial state is

$$|B\rangle = |0\rangle_C |\theta^{\beta_0}\rangle^{\otimes \mathcal{M}} + |1\rangle_C |\theta^{\beta_1}\rangle^{\otimes \mathcal{M}}. \quad (38)$$

Since here we assume that the purification of the bath is in the same basis, the operator $V_{bb'}^{01}$ is an identity. Note that in the r th collision, the probe interacts with the r th subsystem of the bath and thus the unitary operator is of the form $(U_{BS})^r \otimes (\mathbb{I}_2 \otimes \mathbb{I}_2)^{\otimes \bar{r}}$, as in Eq. (37).

The comparison between the visibilities obtained for partially thermalizing maps and completely thermalizing maps for the one-bath case is shown in Fig. 5. Once again, these results have been obtained by changing the number of collisions, keeping the interaction strength constant, and taking the probe in its energy ground state as the initial state. We find that the visibility is the greatest for similar temperatures of the baths and decreases when the temperatures are

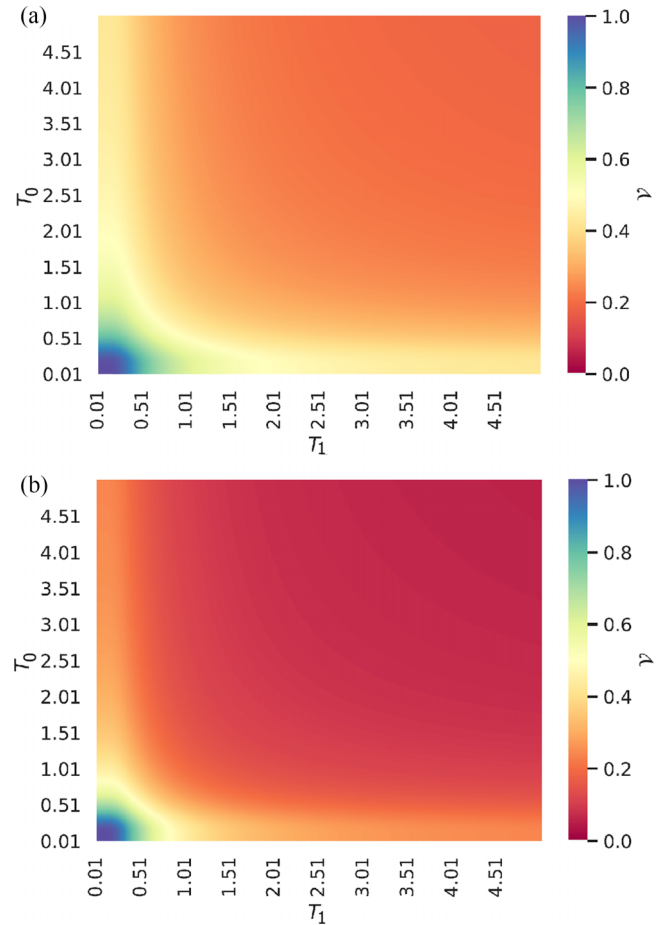


FIG. 4. Heat map of the visibility of the control as a function of the temperatures of the baths in the two-bath case for (a) prethermalization, $\mathcal{M} = 3$, and (b) post-thermalization, $\mathcal{M} = 5$. The interaction parameter is set as $\eta = 0.8$ to allow for partial thermalization.

different. This decrease is steeper for higher numbers of collisions and therefore depends on the fact that the probe is not fully thermalized. Furthermore, the parameter space of T_0 and T_1 , where the visibility is close to its maximum, shrinks as the number of collisions increases, signaling an increase in the distinguishability and the resulting loss of coherence.

V. RETURNING TO RELATIVITY

As discussed at the beginning of this work, there are certain highly relevant scenarios in relativistic quantum physics that naturally lead to the two cases we have studied here. For example, the two-bath model would arise in extended thermal systems in general relativity: Due to the Tolman-Ehrenfest effect [7,8], a bath in thermal equilibrium in a gravitational field has a nonuniform temperature T_i as measured by a local thermometer, where $T_i \sqrt{-g_{00}(r_i)} = \text{const}$. This is also true for extended quantum baths in equilibrium [33]. Therefore, a probe in a superposition of different locations would effectively interact with different baths at different temperatures, with the bath closest to the gravitational source experiencing greater time dilation, and therefore having higher temperature.

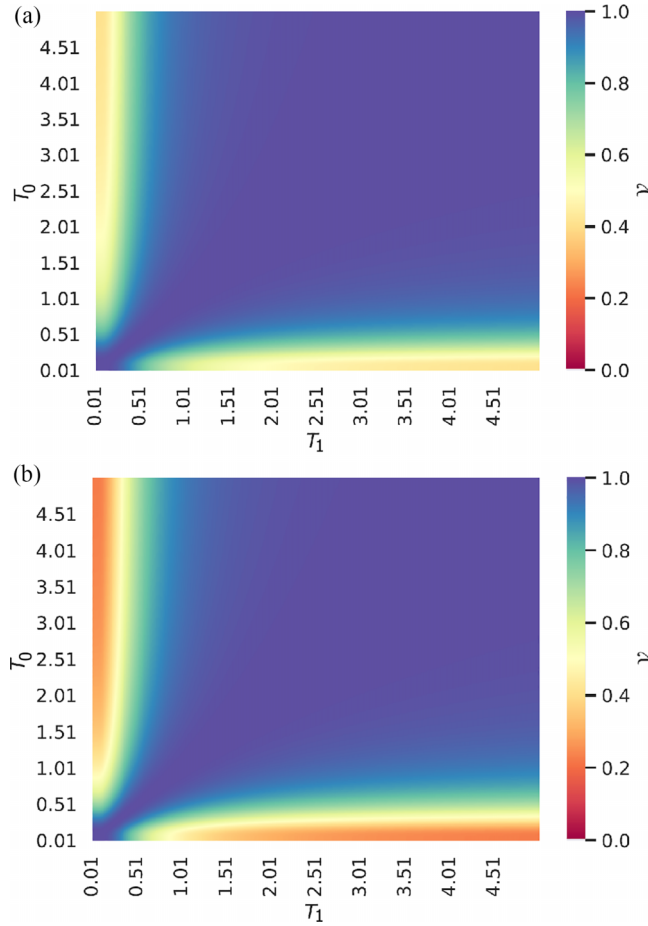


FIG. 5. Heat map of the visibility of the control as a function of temperatures of the bath in the one-bath case for (a) prethermalization, $\mathcal{M} = 3$, and (b) post-thermalization, $\mathcal{M} = 5$. The interaction parameter is set as $\eta = 0.8$ to allow for partial thermalization.

So, if our two baths were at different heights in a gravitational field, e.g., due to a Schwarzschild black hole, fixed observers at radii r_0 and r_1 would see temperatures

$$T_0 = \frac{T_H}{\sqrt{-g_{00}(r_0)}}, \quad T_1 = \frac{T_H}{\sqrt{-g_{00}(r_1)}}, \quad (39)$$

where in this case the generalized constant is the Hawking temperature T_H , and if $r_0 < r_1$, then $T_0 > T_1$. We expect the final state of the probe after interacting with such an environment to be similar to that of our two-bath case [Eq. (13) or (A3)].

The second model would, in turn, arise in relativistic quantum systems subject to acceleration. As mentioned in our introduction, internal DoFs of a system on a trajectory with constant proper acceleration experience the Unruh effect [25,34]. They thermalize to an acceleration-dependent temperature upon interaction with a quantum field, even though the field is in a vacuum state according to any inertial observer. To derive this prediction, one can express a vacuum state in terms of the field modes in an accelerated reference frame (also called Rindler modes). The inertial vacuum is a two-mode squeezed state of these modes—a thermofield double state, or purification, as discussed in Sec. II. Rindler modes

can be divided into “left” and “right” modes, in reference to the two sides of a “Rindler horizon” [35]. These modes depend on where the acceleration takes place in addition to the magnitude of acceleration, since both criteria define the location of the horizon.

A superposition of trajectories, such as in Refs. [9–11], leads to the decomposition of the vacuum state as

$$\sum_{x=0,1} \prod_i C_i \sum_{n_i=0}^{+\infty} e^{-\pi n_i \omega_i / a} |n_i, R(x)\rangle |n_i, L(x)\rangle |x\rangle, \quad (40)$$

where $C_i = \sqrt{1 - e^{-2\pi\omega_i/a}}$ is a normalization factor, a is the magnitude of the acceleration, and $|n_i, R(L)\rangle$ is an n -particle state with frequency ω_i in the right (left) wedge. This corresponds to an Eq. (3) purification, where the field modes constitute the bath and the bath’s purified state is the above-mentioned vacuum of the relativistic field. The difference between the purifications is due to the difference between field modes associated with the Rindler wedges associated with different—e.g., rigidly translated—accelerated trajectories. One can also see the clear correspondence between Eq. (40) and the control-dependent ancillary system of Eq. (24), with $x = \{0, 1\}$ in Eq. (40) again the control—which also directly refers to the location of the accelerated trajectory—and the particle’s internal state playing the role of the probe. Importantly, the amplitude between the different purifications, e.g., $\langle n_i, L(0) | n_j, L(1) \rangle$, in general, depends on the overlap between the spacetime regions forming, in this example, the left Rindler wedge for the two different cases.

VI. DISCUSSION

Motivated by the question of whether it is even possible to meaningfully characterize a “superposition of temperatures,” we have explored here two ways to operationally make sense of such a notion. Both consist of some probe system undergoing thermalization, where one model places the probe in a controlled superposition of interactions with either of two independent thermal baths and the other model has the probe interacting with a single bath that is itself in a superposition of purifications of two thermal states.

We found that the two formulations lead to observably distinct behaviors, and so are not physically equivalent. Our results show that an extension of the notion of “superposition” beyond pure states requires a precise understanding of what physical scenario the superposition should capture. Specifically, we have found that the final state of systems undergoing thermalization in superposition is not, in general, thermal. Furthermore, the thermalization process depends on the physical implementation of the thermalization channels (e.g., on the particular type of system-bath interaction). Technically, the sensitivity to the channel’s implementation manifests as a dependence of observable properties (such as visibility) on the particular Kraus representation, or dilation, of the individual thermalization channels. This is a known feature in the general context of “superpositions of quantum channels”—that is, in scenarios where channels are applied to subspaces, rather than systems or subsystems [16–18,36–39]—but in the context of thermalization, only the consequences of applying two channels on the system in a control-state-dependent

order have been previously explored. Recent works have considered other thermodynamic effects in this so-called quantum switch scenario [40], claiming that indefinite causal order does [41–44] or does not [18,45,46] offer advantages in engine/cycle performance, ergotropy, etc., compared to “ordinary” superpositions (see also Ref. [47]).

Our results, beyond fundamental interest, show that experiments that probe thermal systems could be, by exploring quantum coherence as discussed in this work, sensitive to details of the thermal bath in ways ordinary thermometry cannot be.

Both of the scenarios we have considered feature a control system prepared in a superposition of two orthogonal states, each associated with one of the two temperatures. Intuitively, we think of different temperatures as distinct macroscopic variables, and one might expect that knowing the temperature could reveal enough information about the control to completely destroy its coherence. We have seen that this is generally not the case, and the precise prediction for the loss of coherence depends on the probe-bath interaction. Moreover, one can always find parameters for which the visibility vanishes, even for baths of equal temperatures, meaning that the which-way information leaking to the environment is not encoded in temperature alone. On the other hand, the *maximum* visibility remains strictly below 1 when the two temperatures in superposition are different, giving a meaningful measure of how temperature reveals which-way information.

As discussed in the previous section, the models we have considered here could arise in certain relativistic quantum physics scenarios, e.g., a quantum extension of the Tolman-Ehrenfest problem [8] or the Unruh and Hawking effects for particle detectors with quantized center of mass [9–11]. In fact, the one-bath model developed in this work explains previous results in which a local accelerating probe does not thermalize even if moving in a spatial superposition of trajectories with the exact same proper acceleration. As such trajectories are related by a rigid translation, and the global vacuum state of the field is translation invariant, such a result did not have an intuitive explanation and seemed at odds with the underlying translational symmetry of the problem. The present work shows that this result simply arises from the fact that the probe effectively interacts with different subsystems of the relativistic field and so thermalization does not necessarily follow from translation invariance.

Finally, as shown in Sec. IV, our results do not depend on full thermalization of the probe. Thus, they will be relevant for physical, nonidealized scenarios, including those relativistic scenarios discussed.

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APPENDIX: TWO-BATH SCENARIO WITH ISOMETRIES AS UNITARIES

In this Appendix, we consider the particular case where isometries between Kraus representations of the channel are unitaries, familiar from textbook descriptions of the Kraus, or operator-sum representation such as found in Ref. [14].

Recall the Kraus operator form of a regular quantum channel given in Eq. (2):

$$\text{Tr}_B\{U_{B,S}\rho_{BS}U_{B,S}^\dagger\} = \sum_k c_k^{\beta_i} M_{kk}^i \rho_S \sum_l c_l^{\beta_j} M_{ll}^{j\dagger}. \quad (\text{A1})$$

For the two-bath scenario of Sec. III A, the full final state of the probe in terms of the $M_{kl} = \langle k|U_{BS}|l\rangle$ Kraus operators is then

$$\begin{aligned} \rho_S(\phi) = & \frac{1}{4} \sum_{k,l} \left(c_l^{\beta_0} M_{kl}^0 \rho_S M_{kl}^{0\dagger} + c_l^{\beta_1} M_{kl}^1 \rho_S M_{kl}^{1\dagger} \right. \\ & \times e^{i\phi} c_k^{\beta_0} c_l^{\beta_1} M_{kk}^0 \rho_S M_{ll}^{1\dagger} + e^{-i\phi} c_k^{\beta_1} c_l^{\beta_0} M_{kk}^1 \rho_S M_{ll}^{0\dagger} \left. \right). \end{aligned} \quad (\text{A2})$$

We now introduce arbitrary unitary matrices u^i , with matrix elements u_{kl}^i , such that a different set of Kraus operators representing the channel are $M_{kl}^{i'} = \sum_s u_{ks}^i |l\rangle_S \langle s|_S$. Note that these are different for each of the two bath subsystems. Again, using the swap operator interaction discussed in the main text as an example of U_{BS} , we find a final state of the probe subject to a quantum superposition of thermal channels:

$$\rho_S(\phi) = \frac{1}{4} [\rho_S^{\beta_0} + \rho_S^{\beta_1} + (e^{i\phi} \rho_S^{\beta_0} u^0 \rho_S u^{1\dagger} \rho_S^{\beta_1} + \text{H.c.})]. \quad (\text{A3})$$

It is important to note that in this example case, the dimensions of the interacting systems are matched, as we are not utilizing the general channel dilations laid out in Eq. (8).

As in Sec. III A, even if both baths are at the same temperature, the system still does not thermalize, as explicitly seen from Eq. (A3) for $\beta_0 = \beta_1 \equiv \beta$.

Indeed, for the final state to be thermal, we need $\rho_S^{\beta} u^1 \rho_S u^{0\dagger} \rho_S^{\beta} = e^{i\alpha} \rho_S^{\beta}$, where $\alpha \in \mathbb{R}$ is a possible additional phase. Taking matrix elements in the energy eigenbasis, the above requires $e^{i\alpha} c_k^{\beta} \delta_{k,l} = c_k^{\beta} c_l^{\beta} \langle k|u^1 \rho_S u^{0\dagger}|l\rangle$, for all k, l . However, due to the non-negativity of the thermal weights c_k^{β} , state normalization, and the unitarity of u^i , the modulus of the right-hand side is strictly less than c_k^{β} . That is to say, unless (1) $c_k^{\beta} = \delta_{k,0}$ (indicating both baths are at zero temperature), (2) the initial probe state ρ_S is pure, and (3) the unitaries u^i rotate

the probe state to the energy ground state, and are thus equal (up to a global phase α), the final state cannot be thermal.

As mentioned in the main text, the visibility is quantified by the magnitude of the off-diagonal elements of the final state, here of the control [28,29]. Equivalently, it is the contrast of the interference pattern obtained by measuring the control in a superposition basis. The probability of measuring the control in the state $|\phi\rangle$ is $P(\phi) = \text{Tr} \rho_S(\phi)$, and for our present case reads

$$P(\phi) = \frac{1}{2} + \frac{1}{2} |\text{Tr}\{u^{0\dagger} \rho_S^{\beta_0} \rho_S^{\beta_1} u^1 \rho_S\}| \cos(\phi + \psi), \quad (\text{A4})$$

where ψ is a phase defined via $\text{Tr}_S\{u^{0\dagger} \rho_S^{\beta_0} \rho_S^{\beta_1} u^1 \rho_S\} \equiv |\text{Tr}_S\{u^{0\dagger} \rho_S^{\beta_0} \rho_S^{\beta_1} u^1 \rho_S\}| e^{-i\psi}$. Hence, the visibility is

$$\mathcal{V} = |\text{Tr}\{u^{0\dagger} \rho_S^{\beta_0} \rho_S^{\beta_1} u^1 \rho_S\}|. \quad (\text{A5})$$

Crucially, it depends not just on the temperatures of the two baths, but also on the local unitaries.

Note also that the visibility can never reach its maximum value of 1, even if $\beta_0 = \beta_1$, except in the special case already discussed above; i.e., where both baths are at zero temperature, the system is in a pure state and the unitaries u^i rotate it to the ground state.

Moreover, for arbitrary fixed temperatures β_i , there exist u^i that yield $\mathcal{V} = 0$. Indeed, decomposing the probe state into its eigenbasis $\rho_S = \sum_s p_s |s\rangle \langle s|$, where $0 \leq p_s \leq 1$, and $\sum_s p_s = 1$, Eq. (16) reads

$$\mathcal{V} = \left| \sum_s p_s \sum_k c_k^{\beta_0} c_k^{\beta_1} \langle s | u^{0\dagger} | k \rangle \langle k | u^1 | s \rangle \right|. \quad (\text{A6})$$

Taking u^0 to map each state $|s\rangle$ to some energy eigenstate, and u^1 to be u^0 times a cyclic permutation of energy eigenstates, means that if $\langle k | u^0 | s \rangle = 1$, then $\langle k | u^1 | s \rangle = 0$ and vice versa, resulting in $\mathcal{V} = 0$.

The above means that the minimum visibility over the different physical realizations of the thermalization channel is zero. We have also seen that the coherence is, in general, not maximal. The question is therefore: What is the maximum of $\mathcal{V}$ in Eq. (A6) over local bath unitaries u^i for arbitrary but fixed temperatures of the baths and the probe initial state.

We let $\langle s | u^{0\dagger} | k \rangle = \alpha_k^0(s)$ and $\langle k | u^1 | s \rangle = \alpha_k^1(s)$ and because $c_k^{\beta_i}$ as well as p_s are non-negative, for maximum visibility, $\alpha_k^0(s)$ and $\alpha_k^1(s)$ must also be real and non-negative (up to a global phase). Hence, Eq. (A6) becomes $\mathcal{V} = \sum_s p_s \sum_k c_k^{\beta_0} c_k^{\beta_1} \alpha_k^0(s) \alpha_k^1(s)$.

Starting with a pure state $\rho_S = |s\rangle \langle s|$, we find the maximum visibility using the Lagrange multiplier method to find the optimum values of α_k^0 and α_k^1 .

The Lagrangian $\mathcal{L} = \mathcal{V}$ that we will optimize is

$$\bar{\mathcal{V}} = \mathcal{V} - \lambda_0 \left(1 - \sum_k |\alpha_k^0|^2 \right) - \lambda_1 \left(1 - \sum_k |\alpha_k^1|^2 \right),$$

where $\sum_k |\alpha_k^0|^2 = 1$ and $\sum_k |\alpha_k^1|^2 = 1$.

With $d\bar{\mathcal{V}} = 0$ as a constraint, we arrive at a set of four simultaneous equations:

$$\frac{d\bar{\mathcal{V}}}{d\alpha_l^0} = c_l^{\beta_0} c_l^{\beta_1} \alpha_l^1 + \lambda_0 2\alpha_l^0 = 0, \quad (\text{A7})$$

$$\frac{d\bar{\mathcal{V}}}{d\alpha_l^1} = c_l^{\beta_0} c_l^{\beta_1} \alpha_l^0 + \lambda_1 2\alpha_l^1 = 0, \quad (\text{A8})$$

$$\frac{d\bar{\mathcal{V}}}{d\lambda_0} = -1 + \sum_k |\alpha_k^0|^2 = 0 \Rightarrow \sum_k |\alpha_k^0|^2 = 1, \quad (\text{A9})$$

$$\frac{d\bar{\mathcal{V}}}{d\lambda_1} = -1 + \sum_k |\alpha_k^1|^2 = 0 \Rightarrow \sum_k |\alpha_k^1|^2 = 1. \quad (\text{A10})$$

Working with Eqs. (A7) and (A8),

$$\alpha_k^0 = \frac{c_k^{\beta_0} c_k^{\beta_1} \alpha_k^1}{2\lambda_0}, \quad (\text{A11})$$

$$\alpha_k^1 = \frac{c_k^{\beta_0} c_k^{\beta_1} \alpha_k^0}{2\lambda_1}. \quad (\text{A12})$$

Substituting Eq. (A12) into Eq. (A11),

$$\begin{aligned} \alpha_k^0 &= \frac{c_k^{\beta_0} c_k^{\beta_1}}{2\lambda_0} \left(\frac{c_k^{\beta_0} c_k^{\beta_1} \alpha_k^0}{2\lambda_1} \right) \\ &= \left(\frac{c_k^{\beta_0} c_k^{\beta_1}}{2} \right)^2 \frac{\alpha_k^0}{\lambda_0 \lambda_1}. \end{aligned}$$

This has the trivial solution $\alpha_k^0 = 0$, which would imply $\alpha_k^1 = 0$ too.

Otherwise, the above expression can also be rearranged for

$$\lambda_0 \lambda_1 = \left(\frac{c_k^{\beta_0} c_k^{\beta_1}}{2} \right)^2$$

and

$$(\alpha_l^0)^2 = \frac{\lambda_1}{\lambda_0} (\alpha_l^1)^2,$$

which implies $\lambda_1 = \lambda_0$, which in turn implies $\lambda_0 = \pm \frac{c_l^{\beta_0} c_l^{\beta_1}}{2}$. However, this means that $c_l^{\beta_0} c_l^{\beta_1} = \text{const}$, for all l . That is, $c_l^{\beta_0} = \text{const}$ and $c_l^{\beta_1} = \text{const}$.

Since $\lambda_0 \lambda_1 = \left(\frac{c_l^{\beta_0} c_l^{\beta_1}}{2} \right)^2$ is the solution in the case of nonzero α_l^0 , we can only have one nonzero alpha as a solution.

With $\alpha_l^0 = \pm \alpha_l^1 = \pm \delta_{l,l'}$, then $\mathcal{V}_{\max} = c_{l'}^{\beta_0} c_{l'}^{\beta_1}$.

Since for a thermal state $c_l^{\beta_i} > c_{l+1}^{\beta_i}$, the maximum visibility is found to be

$$\mathcal{V}_{\max}^{\text{pure}} = c_0^{\beta_0} c_0^{\beta_1}.$$

The intuition here, as mentioned in the main text, is that visibility is maximized when local unitaries effectively rotate the probe state $|s\rangle$ to the energy ground state (up to a phase), as it has the highest overlap with a thermal state at any finite temperature.

For an arbitrary mixed state, the maximum visibility then reads

$$\mathcal{V}_{\max} = \sum_s p_s c_s^{\beta_0} c_s^{\beta_1}, \quad (\text{A13})$$

where without loss of generality the probabilities p_s are ordered decreasingly, $p_s \geq p_{s+1}$. This is because thermal

weights are likewise ordered decreasingly, i.e., $c_k^{\beta_i} > c_{k+1}^{\beta_i}$, and due to the orthogonality of the states $|s\rangle$, a unitary u^i can only rotate one of the states from an orthogonal set to the energy ground state, but the same strategy can be iterated to find a unitary mapping of the eigenstates of ρ_s to progressively higher-energy eigenstates.

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