

Directional asymmetry in strong-field ionization induced by a terahertz-assisted laser pulse

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We investigate directional asymmetry in strong-field ionization driven by a few-cycle near-infrared laser pulse assisted by a time-delayed terahertz (THz) pulse. Within the strong-field approximation, we show that the relative delay between the laser and THz pulses provides an efficient control parameter for steering photoelectron emission along the laser polarization axis. Energy-resolved photoelectron spectra, momentum distributions, and asymmetry parameter demonstrate a pronounced and tunable left-right asymmetry for both direct and rescattered electrons, governed by the value of the THz vector potential during the ionization window. This control is significantly stronger than that achievable by varying the carrier-envelope phase of the laser pulse alone and remains robust with respect to variations of the THz-pulse parameters and focal averaging. Our results build upon the theoretical concept introduced earlier [Milošević, *Opt. Lett.* **47**, 1669 (2022)], whose predictions were subsequently confirmed by the experiment [Mitrofanov *et al.*, *Opt. Lett.* **50**, 6626 (2025)], establishing THz-assisted strong-field ionization as a reliable and experimentally validated approach for controlling photoelectron emission on ultrafast timescales.

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I. INTRODUCTION

When atoms or molecules are exposed to a strong laser field, various nonlinear phenomena may occur [1–10]. Prominent examples include above-threshold ionization (ATI), the process in which an atom is ionized by absorbing more laser-field photons than is necessary for ionization [11]. After ionization, the electron, freed by the strong laser field (these electrons are often denoted as the direct electrons), propagates toward the detector. However, the driving laser pulse oscillates in time so that the freed electron can be driven back to the vicinity of the parent ion where it may exhibit elastic scattering and gain additional energy before reaching the detector. These electrons are usually called the rescattered electrons, while the process is denoted as high-order ATI (HATI). The yield of rescattered electrons is approximately the same for a broad range of values of the photoelectron energy; i.e., it forms a plateau that ends with an abrupt cut-off after which the photoelectron yield drops exponentially [12–14]. The characteristics of the photoelectron spectrum depend on the driving-pulse parameters and the characteristics of the target. Particularly interesting situation occurs when the driving pulse consists of only a small number of optical cycles. These pulses are called the few-cycle pulses and the parameters such as the intensity, carrier wavelength, carrier-

envelope phase (CEP), and the type of the pulse envelope can be employed as control knobs [15].

Terahertz (THz) radiation, typically defined as electromagnetic waves spanning from approximately 0.1 to 30 THz, occupies the spectral region between microwaves and the infrared. Owing to its unique temporal and spectral characteristics, THz radiation has become a versatile tool across a wide range of scientific and technological applications [16–20]. In ultrafast spectroscopy and imaging, it enables chemical identification and time-domain analysis [21], while in applied contexts it has found use in security screening technologies [22]. Within strong-field and attosecond science, THz fields play an important role in CEP determination of few-cycle laser pulses via plasma-based down-conversion in ambient air [23,24], in the real-time probing of electron dynamics during high-harmonic generation [25], and in the manipulation and characterization of photoelectrons through THz streaking techniques [26–29]. Beyond spectroscopy, intense THz fields have opened pathways toward compact electron acceleration [28,30], molecular orientation control [31], and the development of integrated electronic and hybrid electronic-photonic systems [32]. Recent advances have led to the generation of THz pulses exceeding 1 MV/cm [17], and even to subcycle THz-driven electron emission from tungsten nanotips, where local peak fields approaching 0.4 GV/cm have been achieved [33]. Under such extreme conditions, electrons can be accelerated to kinetic energies of several keV, reaching up to 3.5 keV. A comprehensive overview of the latest progress in this rapidly evolving field can be found in the recent feature issue on ultrafast phenomena and THz waves [34].

The (H)ATI process induced by a strong laser pulse can be profoundly altered by the presence of auxiliary terahertz radiation with peak field strengths on the order of 1 MV/cm. This pronounced influence is determined by the THz-pulse

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vector potential, which increases with the wavelength and the field amplitude of the THz pulse. Because THz wavelengths exceed those of typical near-infrared laser pulses by several orders of magnitude, a comparatively weak THz pulse can generate a vector potential that rivals that of the intense laser pulse. Using this as our foundation, we have previously demonstrated that laser-induced ionization in the presence of a strong THz pulse exhibits substantial changes in the resulting photoelectron spectra caused by the presence of the THz pulse [35–39]. Moreover, we identified the time delay between the laser and THz pulses as a highly effective control parameter, enabling manipulation of both the photoelectron energy cutoff and the total ionization yield. The experimental verification of the theoretical results is presented in Ref. [40] and a good agreement was reported.

In the present paper, we investigate directional asymmetry in strong-field ionization driven by a combination of a few-cycle near-infrared laser pulse and a time-delayed THz pulse. Using a framework based on the strong-field approximation (SFA), we analyze how the time delay between the two pulses can be employed as an efficient control parameter for steering the photoelectron emission into opposite directions along the laser polarization axis. We systematically study energy-resolved photoelectron spectra, momentum distributions, and asymmetry parameter, demonstrating that the THz pulse induces a pronounced and tunable left-right asymmetry for both the direct and the rescattered electrons. Our results show that this form of control is significantly stronger than that achievable by varying the CEP of the laser pulse alone and persists over a broad photoelectron energy range. These findings establish THz-assisted strong-field ionization as a powerful scheme for manipulating electron emission dynamics on ultrafast timescales.

The paper is organized as follows. In Sec. II, we outline the theoretical framework used to describe strong-field ionization in the presence of a combined few-cycle laser pulse and a terahertz pulse. Section III presents and discusses our numerical results. In Sec. III A, we analyze the dependence of the photoelectron spectra and directional asymmetry on the time delay between the laser pulse and the THz pulse. Section III B is devoted to photoelectron momentum distributions and their interpretation in terms of the vector potential of the combined pulse. In Sec. III C, we examine the influence of the pulse parameters and the focal averaging on the asymmetry effects. Finally, Sec. IV summarizes the main conclusions of this work. We use atomic units.

II. THEORY

A. Differential ionization probability

Our theory is based on the improved SFA [5,11,15], which includes interaction with the parent ion during the rescattering (in the first Born approximation). This theory describes well the electron spectra for medium energies (plateau region) and high energies (cutoff region), while discrepancies may occur in the low-energy domain where the neglected influence of the Coulomb potential is important. To assess the reliability of this approximation, we refer to recent studies where the improved-SFA results have been systematically compared

with calculations that include Coulomb corrections [41] as well as with the results obtained from the full solution of the time-dependent Schrödinger equation [42]. In the medium- and high-energy spectral region, the use of the first Born approximation may be a limiting factor for angle-dependent spectra since this approximation does not adequately describe the exact scattering amplitude that may possess minima for particular scattering angles. In this case, one can use the quantitative rescattering theory [43], introduced in Ref. [44], or the low-frequency approximation [45,46], which leads to the results that agree well with the experiments [47–49]. In the present paper, we are interested in spectral asymmetries between the emission in the laser-field polarization direction and opposite to it, and not in the angular distribution, so that our improved SFA is adequate for our purpose. Furthermore, our improved SFA has been shown to be very successful in confirming experimental results for CEP-dependent asymmetry in strong-field ionization by few-cycle pulses [50,51].

The differential ionization probability corresponding to the transition from the ground state ψ_0 to the continuum state with asymptotic momentum $\mathbf{p}$ and energy $E_{\mathbf{p}} = \mathbf{p}^2/2$ is $W_{\mathbf{p}} = p|M_{\mathbf{p}}|^2$, where $p = |\mathbf{p}|$ and the probability amplitude $M_{\mathbf{p}}$ is equal to the sum of the contributions related to the direct ($M_{\mathbf{p}}^D$) and the rescattered ($M_{\mathbf{p}}^R$) electrons. Within the framework of the SFA, the probability amplitude corresponding to the direct electrons is given by [15]

$$M_{\mathbf{p}}^D = -i \int_0^{T_{\text{pl}}} dt \langle \tilde{\mathbf{p}} + \mathbf{A}(t) | \mathbf{r} \cdot \mathbf{E}(t) | \psi_0 \rangle e^{iS(\tilde{\mathbf{p}};t)}, \quad (1)$$

where the electric field is the sum of the laser pulse and the time-delayed THz pulse $\mathbf{E}(t) = \mathbf{E}_L(t) + \mathbf{E}_T(t + \Delta t)$, with

$$\mathbf{E}_J(t) = E_{0J} \sin^2[\omega_J t / (2n_{pJ})] \cos(\omega_J t + \phi_J) \hat{\mathbf{e}}_x, \quad J = L, T. \quad (2)$$

Here, E_{0J} , ω_J , and ϕ_J are the electric-field amplitudes, carrier frequencies, and the carrier-envelope phases, respectively, $\hat{\mathbf{e}}_x$ is the unit polarization vector, while the pulses durations are $T_{pJ} = n_{pJ} T_J$, with $T_J = 2\pi/\omega_J$ and n_{pJ} integer. In addition, the vector potential is $\mathbf{A}(t) = \mathbf{A}_L(t) + \mathbf{A}_T(t + \Delta t)$, $\mathbf{A}_J(t) = -\int^t dt' \mathbf{E}_J(t')$, and $\tilde{\mathbf{p}} = \mathbf{p} - \mathbf{A}_L(T_{\text{pl}}) - \mathbf{A}_T(T_{\text{pT}})$ (see Refs. [15,36]). The field amplitude of the laser pulse is much stronger than the field amplitude of the THz pulse ($E_{0L} \gg E_{0T}$) so that the ionization happens only within the duration of the laser pulse. Also, we assume that the THz pulse is turned on earlier than the laser pulse and that the time delay is $\Delta t > 0$. Finally, the action corresponding to the direct electrons is $S(\tilde{\mathbf{p}};t) = I_p t + \int^t dt' [\tilde{\mathbf{p}} + \mathbf{A}(t')]^2/2$, with I_p being the ionization potential. The largest contribution to the probability amplitude $M_{\mathbf{p}}^D$ corresponds to the time t for which the action $S(\tilde{\mathbf{p}};t)$ is extremal, i.e., for which

$$\partial S(\tilde{\mathbf{p}};t)/\partial t = [\tilde{\mathbf{p}} + \mathbf{A}(t)]^2/2 + I_p = 0. \quad (3)$$

On the other hand, the probability amplitude corresponding to the rescattered electrons can be written as [15]

$$M_{\mathbf{p}}^R = - \int_0^{T_{\text{pl}}} dt e^{iS(\tilde{\mathbf{p}};t)} \int_0^t d\tau \left(\frac{2\pi}{i\tau} \right)^{3/2} \langle \tilde{\mathbf{p}} | V | \mathbf{k}_{\text{st}} \rangle \times e^{iS(\mathbf{k}_{\text{st}};t,\tau)} \langle \mathbf{k}_{\text{st}} + \mathbf{A}(t - \tau) | \mathbf{r} \cdot \mathbf{E}(t - \tau) | \psi_0 \rangle, \quad (4)$$

where $S(\mathbf{k}_{\text{st}}; t, \tau) = I_p \tau + \int_{t-\tau}^t dt' [\mathbf{k}_{\text{st}} + \mathbf{A}(t')]^2 / 2$ and $\mathbf{k}_{\text{st}} = -\int_{t-\tau}^t dt' \mathbf{A}(t') / \tau$ is the stationary momentum, with τ being the travel time, i.e., the time between the ionization and rescattering. For inert gases, the rescattering matrix element $\langle \tilde{\mathbf{p}} | V | \mathbf{k}_{\text{st}} \rangle$ [which is a Fourier transform of the scattering potential $V(r)$] is represented using the independent-particle-model potential in the form of a double Yukawa potential [52]

$$V(r) = -\frac{Z}{H} \frac{e^{-r/D}}{r} [1 + (H-1)e^{-Hr/D}], \quad (5)$$

where $H = DZ^{0.4}$ and the relevant values of D are given in Refs. [53,54]. The Fourier transform of this potential is

$$V_{\mathbf{K}} = -\frac{Z}{2\pi^2 H} \left[\frac{1}{K^2 + D^{-2}} + \frac{H-1}{K^2 + \left(\frac{H+1}{D}\right)^2} \right], \quad (6)$$

where $\mathbf{K} = \mathbf{k}_{\text{st}} - \tilde{\mathbf{p}}$. In addition, the ionization matrix element $\langle \mathbf{q} | \mathbf{r} | \psi_0 \rangle$ can be calculated analytically when the ground state ψ_0 is represented by the linear combination of the Slater-type orbitals.

The asymmetry between the photoelectron emission in the laser polarization direction and opposite to it can be quantified using the asymmetry parameter

$$A_p = \frac{W_{p\hat{e}_x} - W_{-p\hat{e}_x}}{W_{p\hat{e}_x} + W_{-p\hat{e}_x}}. \quad (7)$$

We define the emission angle θ as the angle between the unit vector $\hat{e}_x$ and the final photoelectron momentum $\mathbf{p}$.

B. Focal averaging

In realistic experimental conditions, the interaction between the laser pulse and the target does not occur at a single, well-defined intensity. Instead, the spatial profile of the focused laser beam leads to a continuous distribution of intensities across the interaction region. As a result, the measured photoelectron signal represents an average over contributions originating from different local field strengths rather than from the peak intensity alone. Properly accounting for this spatial averaging is therefore essential for a meaningful comparison between theoretical predictions and experimental observations. To incorporate this effect, we perform focal averaging over the laser intensity in the focal plane. Following the procedure described in Refs. [55–57], the intensity-averaged differential ionization probability is obtained by integrating the single-intensity probability $W_{\mathbf{p}}$ over all intensities up to the peak value I_0 ,

$$\langle W_{\mathbf{p}} \rangle = \int_0^{I_0} dI W_{\mathbf{p}} K(I, I_0), \quad (8)$$

where the weighting function $K(I, I_0)$ reflects the geometry of the laser focus. For a Gaussian beam, this kernel takes the form

$$K(I, I_0) \propto \frac{2I + I_0}{I^2} \sqrt{\frac{I_0 - I}{I}}, \quad (9)$$

with I_0 denoting the maximum intensity at the focal center. This kernel assigns larger weight to contributions originating from lower intensities, which occupy a larger spatial volume within the focus.

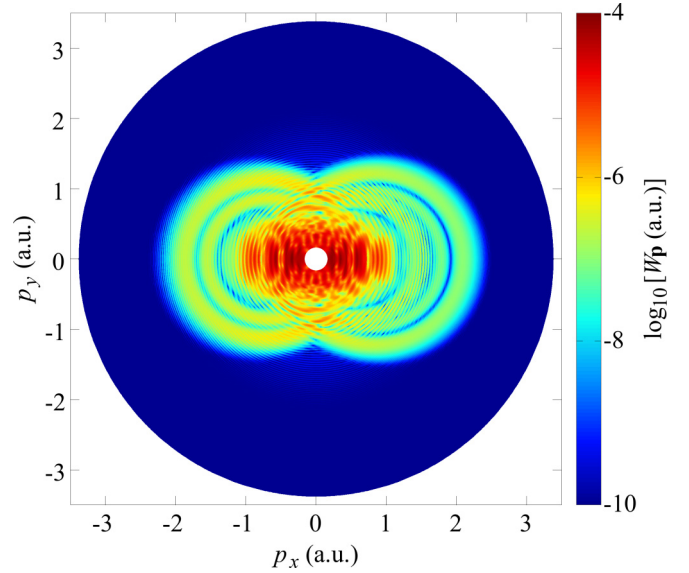


FIG. 1. Photoelectron momentum distribution of the Ar atom exposed to our six-cycle linearly polarized laser pulse in the absence of the THz pulse. The CEP of the laser pulse is $\phi_L = 0^\circ$.

III. NUMERICAL RESULTS

In this section, we present our numerical results using the example of the Ar atom (ionization potential $I_p = 15.76$ eV) exposed to the six-cycle linearly polarized laser pulse ($n_{pL} = 6$) with intensity $I_L = 10^{14}$ W/cm² and the carrier wavelength of 800 nm. The frequency of the assisting two-cycle THz pulse ($n_{pT} = 2$) is $\nu_T = c/\lambda_T = 1$ THz.

A. Dependence of the photoelectron spectra on the time delay between the laser pulse and the THz pulse

Before we analyze how the time delay between the laser pulse and the THz pulse influences the photoelectron spectra, we first recall what happens with the spectra in the absence of the THz pulse.

For illustration, in Fig. 1 we present the photoelectron momentum distribution calculated in the absence of the THz pulse and for the laser pulse with the CEP $\phi_L = 0^\circ$. Both the direct and the rescattered electrons are taken into account. The presented momentum distribution possesses the reflection symmetry $p_y \rightarrow -p_y$ due to the fact that the laser pulse is linearly polarized, while the reflection symmetry $p_x \rightarrow -p_x$ is broken by the ultrashort nature of the laser pulse. The corresponding asymmetry is usually denoted as the left-right asymmetry. The left-right asymmetry is particularly pronounced for the medium- and high-energy electrons, i.e., the electrons that exhibit rescattering (cf. the rescattering-electron clouds for $p_x > 0$ and $p_x < 0$ in Fig. 1), while it is weak for the direct electrons (see the low-energy part of the photoelectron momentum distribution in Fig. 1). In addition, for the ultrashort laser pulse, the combined transformation $p_x \rightarrow -p_x$ and $\phi_L \rightarrow \phi_L + \pi$ is the symmetry transformation, meaning that the photoelectron momentum distributions calculated for ϕ_L and $\phi_L + \pi$ are mutually related by the transformation $p_x \rightarrow -p_x$. The fact that the rescattered electrons are more sensitive on the value of the CEP than the

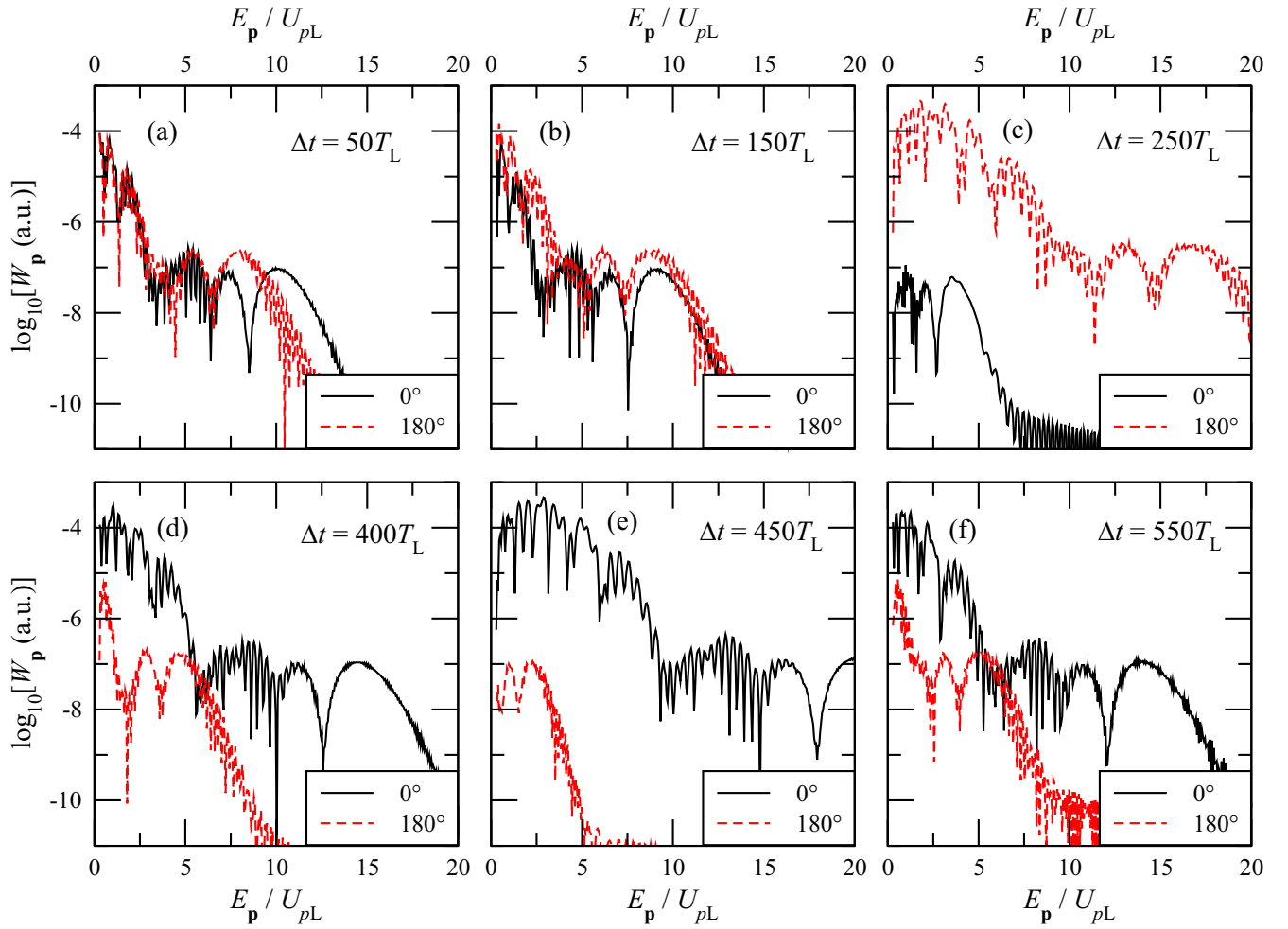


FIG. 2. Logarithm of the differential ionization probability for ionization of the Ar atom by the combined six-cycle linearly polarized laser pulse and the two-cycle THz pulse for the emission in the directions $\theta = 0^\circ$ (black solid lines) and $\theta = 180^\circ$ (red dashed lines) and for the values of the time delay as indicated in the panels. The carrier-envelope phases are set to zero.

direct electrons can be understood by noting that a substantial electron yield requires the laser field to be strong both at the moment of ionization and after the subsequent rescattering. By appropriately choosing the CEP, it can be achieved that the field at the ionization time is maximum while the field after rescattering time is still strong enough so that the photoelectron can achieve a high energy. The yield of the direct electrons is not significantly affected by the value of this parameter and other observables have to be employed to observe the CEP-dependent effects [58]. Finally, we mention that, in order to quantitatively analyze certain features of the photoelectron momentum distribution, more advanced theory (such as the quantitative rescattering theory or the theory based on the low-frequency approximation) might be necessary [43–49]. Now, we analyze how the presence of the THz pulse, in particular, the time delay between the laser pulse and the THz pulse, affects the photoelectron spectra. We use the THz pulse with the electric-field amplitude $E_{0T} = 0.732$ MV/cm. For this value of the THz-pulse amplitude, it is $A_{0T} = A_{0L}$ and thus $U_{pT} = U_{pL}$ [36], where U_{pL} (U_{pT}) is the contribution to the ponderomotive energy from the laser (THz) pulse. Figure 2 presents the photoelectron spectra for ionization of

the Ar atom by the combined six-cycle linearly polarized laser pulse and the two-cycle THz pulse for the emission in the directions $\theta = 0^\circ$ (black solid lines) and $\theta = 180^\circ$ (red dashed lines) and for the values of the time delay as indicated in the panels. The carrier-envelope phases are set to zero. The photoelectron spectrum is strongly influenced by the value of the time delay between the laser pulse and the THz pulse. For example, for the values of the time delay $\Delta t = 50T_L$ and $\Delta t = 150T_L$, the photoelectron spectra calculated for the emission in the directions $\theta = 0^\circ$ and $\theta = 180^\circ$ are qualitatively similar [see Figs. 2(a) and 2(b)]. In particular, the direct-electron yield (low-energy part of the photoelectron spectra) and the height of the rescattering plateau are comparable. For $\Delta t = 50T_L$, the lengths of the rescattering plateaus for $\theta = 0^\circ$ and $\theta = 180^\circ$ are slightly different. On the other hand, for the values of the time delay $\Delta t = 250T_L$ and $\Delta t = 450T_L$, the emission in one direction is strongly suppressed for both the direct and the rescattered electrons [see Figs. 2(c) and 2(e)]. More precisely, for $\Delta t = 250T_L$, the emission in the direction $\theta = 0^\circ$ is suppressed, while the photoelectron spectrum in the direction $\theta = 180^\circ$ exhibits long plateau and the differential ionization probability is significant. For $\Delta t = 450T_L$, the observations are

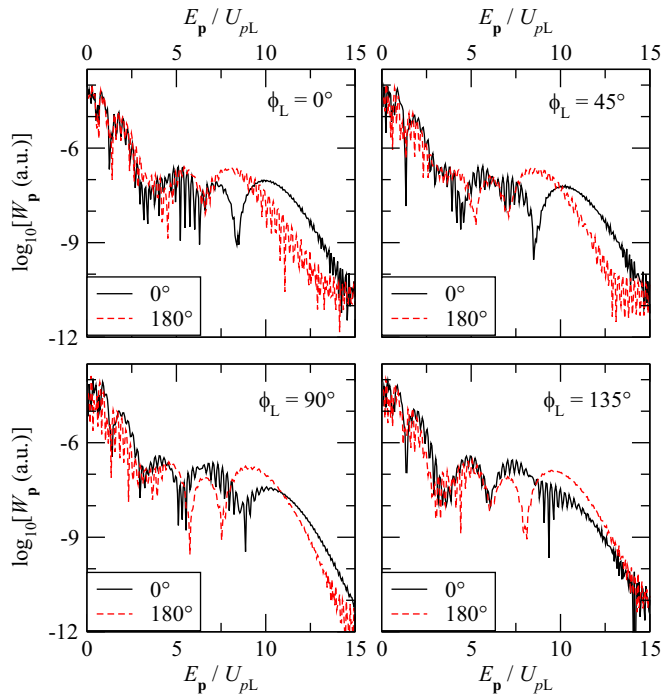


FIG. 3. Logarithm of the differential ionization probability for ionization of the Ar atom by our six-cycle linearly polarized laser pulse in the absence of the THz pulse for the emission in the directions $\theta = 0^\circ$ (black solid lines) and $\theta = 180^\circ$ (red dashed lines) and for several values of the CEP ϕ_L , as indicated in the panels.

reversed. Finally, for the values of the time delay $\Delta t = 400T_L$ and $\Delta t = 550T_L$, the direct electron spectrum is suppressed for the emission in the direction $\theta = 180^\circ$ in comparison with the spectrum corresponding to the emission in the direction $\theta = 0^\circ$ [see Figs. 2(d) and 2(f)]. Moreover, the yields corresponding to the rescattering electrons are comparable, but the rescattering plateau is much longer for the emission in the direction $\theta = 0^\circ$ than in the opposite direction. Here, we should recall that the control over the photoelectron emission in the two opposite directions can also be achieved using the laser-pulse CEP without the assisting THz pulse (see Fig. 1).

For illustration, in Fig. 3 we present the logarithm of the differential ionization probability for ionization of the Ar atom by our six-cycle linearly polarized laser pulse in the absence of the THz pulse for the emission in the directions $\theta = 0^\circ$ (black solid lines) and $\theta = 180^\circ$ (red dashed lines) and for several values of the CEP ϕ_L , as indicated in the panels. The photoelectron spectra exhibit a noticeable dependence on ϕ_L , particularly in the medium- and high-energy regions, while the low-energy part is less affected. However, the CEP-induced modulation is significantly weaker than the changes induced by varying the time delay in the combined laser-THz setup. As a result, the asymmetry between the photoelectron spectra for emission in the directions $\theta = 0^\circ$ and $\theta = 180^\circ$ is more pronounced in the presence of the THz pulse. Also, for a fixed value of the time delay, additional variation of the CEP leads only to minor modifications of the spectra (see Fig. 2 in the Supplemental Material of Ref. [35]). These findings clearly demonstrate that, in the combined-field configuration,

the time delay acts as the dominant control parameter for steering the directional asymmetry of the photoelectron emission.

In summary, for strong-field ionization by the combined laser pulse and the THz pulse, the time delay between these two pulses can successfully be employed to manipulate the photoelectron emission for both the direct and the rescattering electrons. This control is significantly stronger than the control by the laser-pulse CEP in the absence of the THz pulse.

A better way to quantify the asymmetry that appears for the photoelectron emission in the two opposite directions is by using the asymmetry parameter A_p [Eq. (7)]. Figure 4 displays the asymmetry parameter A_p for the same field configurations as in Fig. 2. The values of this parameter are bounded within the segment $A_p \in [-1, 1]$. The positive values of the parameter correspond to the case in which the emission in the direction $\theta = 0^\circ$ is stronger than the emission in the direction $\theta = 180^\circ$, while the opposite holds for the negative values of the parameter A_p . For the values of the time delay $\Delta t = 50T_L$ and $\Delta t = 150T_L$, the parameter A_p exhibits an oscillatory character [see Figs. 4(a) and 4(b)]. For $\Delta t = 50T_L$, the parameter has a large value (close to +1) for the photoelectron energy $E_p \in [10U_{pL}, 14U_{pL}]$, which is in agreement with the results presented in Fig. 2(a). For the time delay $\Delta t = 250T_L$, the parameter A_p is approximately equal to -1 [see Fig. 4(c)] indicating that, for this value of the time delay, the emission is dominant in the direction $\theta = 180^\circ$, while for $\Delta t = 450T_L$ the parameter A_p is approximately equal to +1 [see Fig. 4(e)] and the emission in the direction $\theta = 0^\circ$ dominates. Finally, for the values of the time delay $\Delta t = 400T_L$ and $\Delta t = 550T_L$, the parameter A_p is close to +1 (meaning that the emission in the direction $\theta = 180^\circ$ is suppressed) except for a narrow range of values of the photoelectron energy [see Figs. 4(d) and 4(f)]. This is again in agreement with the corresponding results presented in Fig. 2.

B. Dependence of the photoelectron momentum distributions on the time delay between the laser pulse and the THz pulse

Apart from the analysis of the dependence of the photoelectron spectra, calculated for fixed values of the emission angle, on the time delay between the laser pulse and the THz pulse, it is also possible to investigate how this parameter affects the whole momentum distributions. For this purpose, we chose the pulse configurations with $\Delta t = 250T_L$ and $\Delta t = 500T_L$, where the asymmetry should be particularly pronounced. The corresponding photoelectron momentum distributions are displayed in Fig. 5 [Fig. 5(a) corresponds to $\Delta t = 250T_L$, while Fig. 5(b) corresponds to $\Delta t = 500T_L$]. The photoelectron momentum distribution significantly depends on the time delay. Depending on the value of this parameter, the photoelectron momentum distribution can be shifted toward either the $p_x > 0$ or $p_x < 0$ region of the momentum plane. In this configuration, the yield is maximal in the populated region, and emission in the opposite half-plane is strongly suppressed. The observed shift is particularly pronounced for the direct electrons and cannot be achieved using linearly polarized laser pulse alone.

The shift of the direct-electron part of the photoelectron momentum distribution can be explained using the saddle-

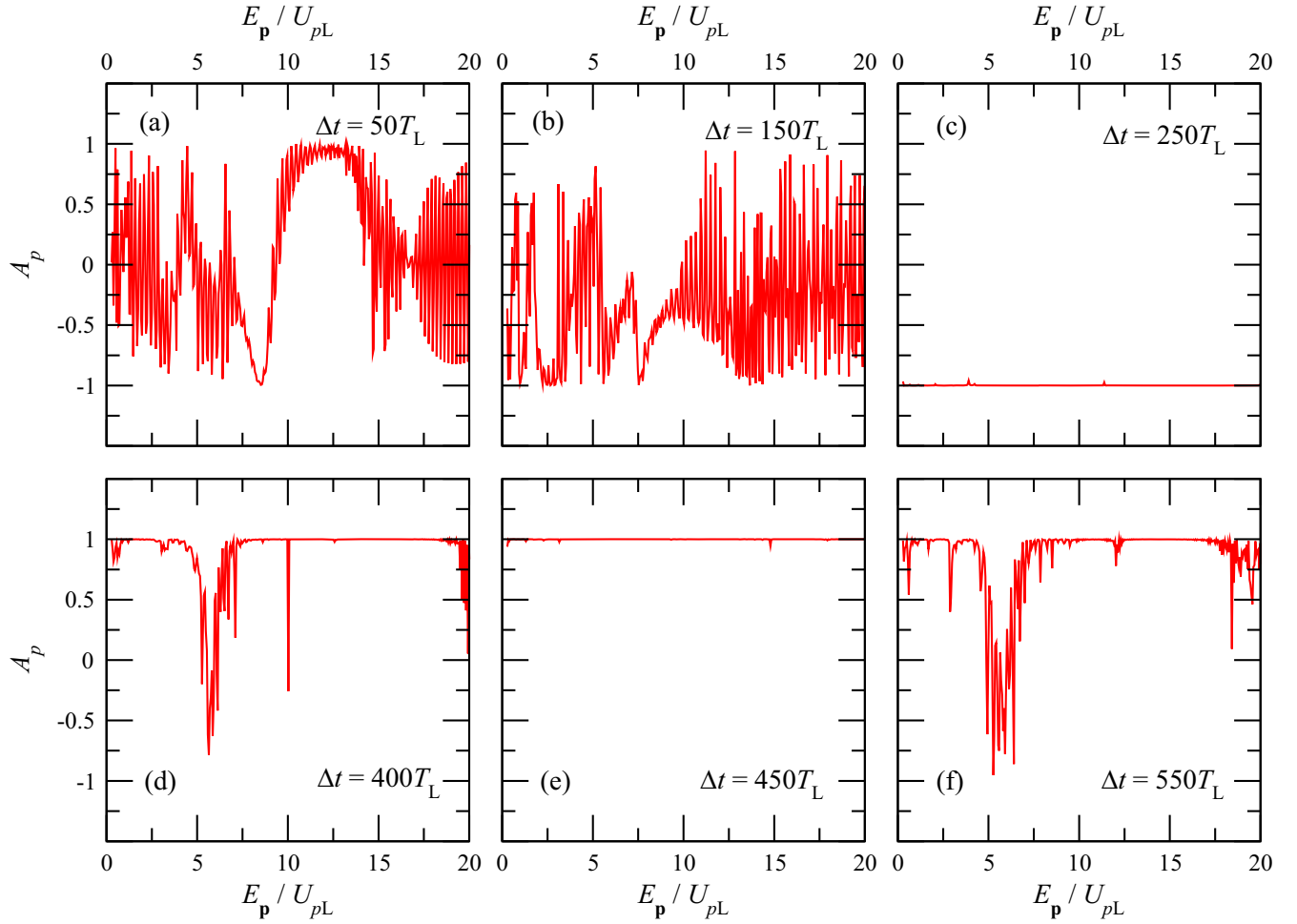


FIG. 4. Asymmetry parameter A_p calculated for the same field configurations as in Fig. 2.

point equation (3). If we neglect the ionization potential, the final photoelectron momentum follows the negative vector potential, i.e., $\mathbf{p} \approx -\mathbf{A}(t)$, where $\mathbf{A}(t)$ is the total vector potential of the combined laser and THz pulse at the ionization time t . In Fig. 6, we present the vector potentials of our linearly polarized six-cycle laser pulse (black solid line) and the two-cycle THz pulse with zero carrier-envelope phases and the time delay of $\Delta t = 250T_L$ (red dashed line) and $\Delta t = 500T_L$ (blue dotted line). Ionization occurs within the duration of the laser pulse. For both values of the time delay, the vector potential of the laser pulse at the moment of ionization is the same so that the final photoelectron momentum depends on the value of the vector potential of the THz pulse at the moment of ionization. For the value of the time delay $\Delta t = 250T_L$, the vector potential of the THz pulse is positive (see the red dashed line in Fig. 6) leading to a strong confinement of the photoelectron momentum distribution in the $p_x < 0$ part of the momentum plane [see Fig. 5(a)]. On the other hand, for $\Delta t = 500T_L$ the vector potential of the THz pulse is negative (see the blue dotted line in Fig. 6) and the photoelectron momentum distribution is restricted to the $p_x > 0$ part of the momentum plane [see Fig. 5(b)]. For the rescattered electrons, there is no simple relation between the final photoelectron momentum and the vector potential of

the field. However, it still holds that the value of the vector potential of the THz pulse within the duration of the laser pulse determines in which direction the emission of the photoelectrons is suppressed.

Another way of analyzing the dependence of the spectrum asymmetry on the time delay between the laser pulse and the THz pulse assumes that we examine the differential ionization probability as a function of both the time delay Δt and the photoelectron energy E_p for a fixed emission direction as shown in Fig. 7. In Fig. 7(a), we present this dependence for emission in the direction $\theta = 0^\circ$, while Fig. 7(b) shows the corresponding results for $\theta = 180^\circ$. The resulting patterns exhibit a pronounced sensitivity of the photoelectron emission on the value of the time delay, manifesting as alternating regions of enhanced and suppressed yield across the photoelectron energy range. This behavior reflects the strong temporal control exerted by the THz pulse on the ionization dynamics within the duration of the laser pulse. In particular, small variations of the time delay can lead to substantial redistributions of the photoelectron yield between the two opposite emission directions, in agreement with the asymmetry effects discussed above. These results further demonstrate that the time delay provides an efficient and robust control parameter for steering the directional photoelectron emission

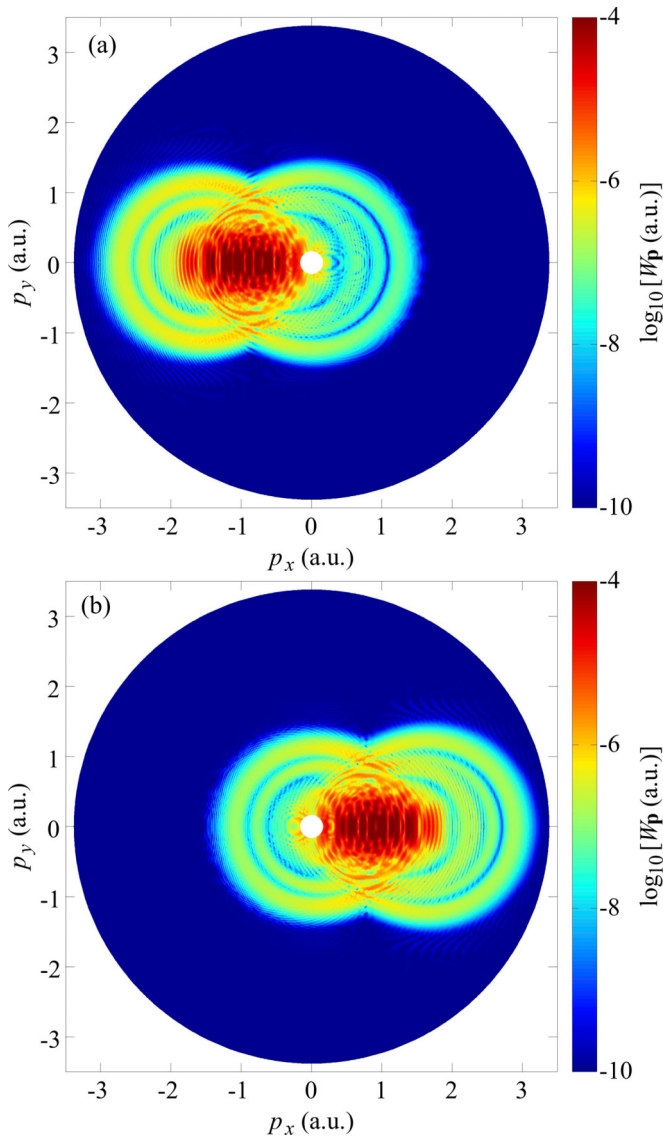


FIG. 5. Photoelectron momentum distribution calculated for the pulse configurations with $\Delta t = 250T_L$ [panel (a)] and $\Delta t = 500T_L$ [panel (b)].

over a broad energy range. Such a degree of control cannot be achieved in the absence of the THz pulse by varying only the CEP of the laser pulse. While the rescattered-electron contribution does exhibit a dependence on the CEP, this dependence is weaker than the dependence on the time delay in the combined laser and THz pulse, and the direct-electron contribution is largely insensitive to the CEP altogether. The present results therefore demonstrate that the addition of the THz field provides a qualitatively new and substantially more efficient mechanism for controlling the directional emission of photoelectrons over a broad energy range.

Finally, Fig. 8 provides a comprehensive overview of the directional asymmetry by displaying the asymmetry parameter A_p as a joint function of the time delay Δt and the photoelectron energy E_p . In contrast to Fig. 4, where Δt is fixed, Fig. 8 reveals the full two-dimensional structure of the asymmetry and clearly illustrates how the control of photo-

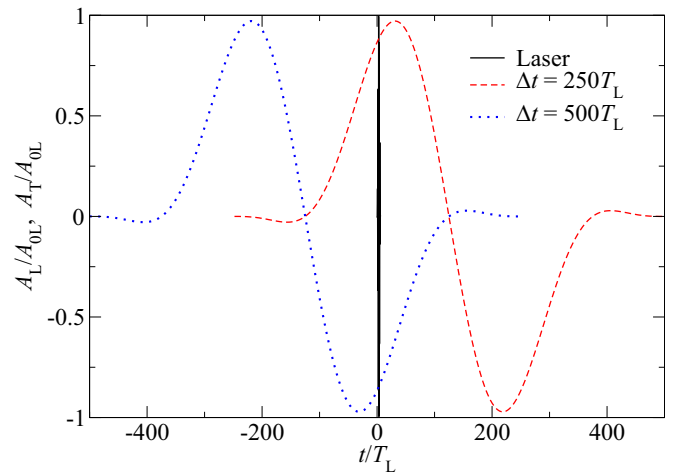


FIG. 6. Vector potential of the linearly polarized six-cycle laser pulse (black solid line) and the two-cycle THz pulse with zero carrier-envelope phases and the time delay of $\Delta t = 250T_L$ (red dashed line) and $\Delta t = 500T_L$ (blue dotted line).

electron emission emerges from the interplay between the time delay Δt and the photoelectron energy E_p . The pronounced alternating yellow and blue regions indicate that the dominant emission direction switches repeatedly as the time delay is varied, with this switching persisting over a broad energy range. Several important features can be identified. First, the asymmetry is not confined to the rescattering plateau but extends down to low photoelectron energies, confirming that the THz pulse efficiently breaks the left-right symmetry also for the direct electrons. This behavior is fundamentally different from the case of a laser pulse alone, where asymmetry effects are largely restricted to the medium- and high-energy parts of the spectrum. Second, the boundary between regions of opposite asymmetry is sharp over wide energy interval, indicating that relatively small change of the time delay can invert the preferred emission direction simultaneously for a broad range of values of the photoelectron energy. This highlights the role of the time delay as a highly effective control parameter. Moreover, the strongest asymmetries, with $|A_p|$ approaching unity, occur for time delays at which the vector potential of the THz pulse has a well-defined sign (either large positive or large negative value) during the laser pulse (cf. Fig. 6). In these regions, emission into one half of the momentum plane is almost completely suppressed, leading to a highly directional photoelectron beam. In addition, for some values of Δt the asymmetry exhibits a more complex, energy-dependent structure, reflecting the coexistence and competition of different ionization and rescattering trajectories that experience different effective streaking by the THz pulse.

Overall, Fig. 8 summarizes and unifies the main findings of this section: The combination of a few-cycle laser pulse with a time-delayed THz pulse enables a robust and tunable control of the directional photoelectron emission over a wide energy range. The resulting asymmetry patterns demonstrate that the time delay provides a level of control that is both qualitatively and quantitatively superior to that achievable by varying the CEP of the laser pulse alone.

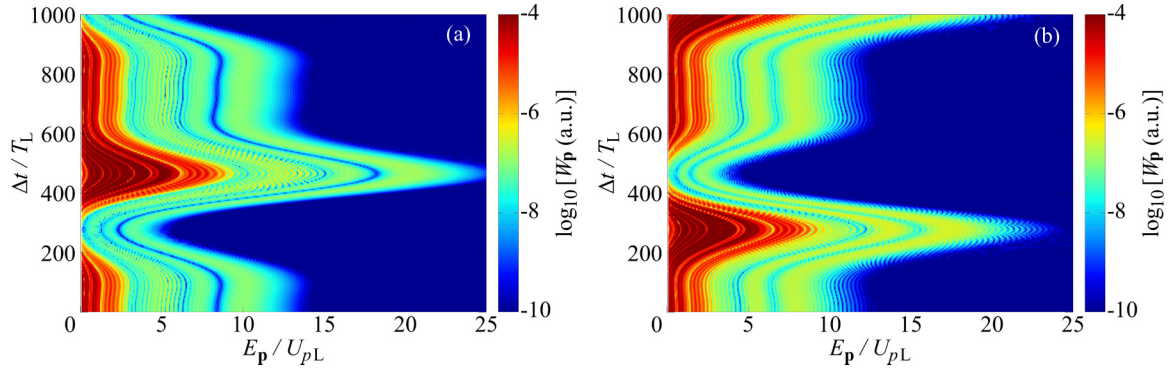


FIG. 7. Logarithm of the differential ionization probability as a function of the time delay Δt and the photoelectron energy E_p for the emission in the directions $\theta = 0^\circ$ [panel (a)] and $\theta = 180^\circ$ [panel (b)].

C. Influence of pulse parameters and focal averaging

After analyzing the asymmetry in the photoelectron spectra as a function of the time delay, we now investigate the robustness of the presented results with respect to the pulse parameters. The asymmetry observed in the photoelectron spectra is primarily governed by the value of the vector potential of the THz pulse, $A_T(t)$, during the temporal overlap with the laser pulse. The vector potential $A_T(t)$ depends on both the amplitude of the THz pulse, E_{0T} , and its frequency, ν_T . In this work, we vary the amplitude E_{0T} in order to modify the vector potential $A_T(t)$. Figure 9 shows the photoelectron momentum distributions calculated for THz pulse configurations with a fixed time delay of $\Delta t = 500T_L$ and amplitudes $E_{0T} = 0.3$ MV/cm [panel (a)], $E_{0T} = 0.6$ MV/cm [panel (b)], and $E_{0T} = 1$ MV/cm [panel (c)]. The asymmetry in the photoelectron momentum distribution is present for all three amplitudes E_{0T} and becomes increasingly pronounced as the amplitude increases. This behavior arises because the temporal positions of the extrema of the vector potential $A_T(t)$ (see Fig. 6) are determined solely by the time delay Δt and are independent of the amplitude E_{0T} . Consequently, if for a given time delay the photoelectron emission is dominant in one half of the momentum plane for a particular value of E_{0T} , the same qualitative behavior persists for other amplitudes. The

enhancement of the asymmetry with increasing E_{0T} is due to the corresponding increase in the extremal values of the vector potential $A_T(t)$. Analogous conclusions can be drawn when varying the THz-pulse frequency ν_T . In this case, the extremal values of $A_T(t)$ increase as the frequency ν_T decreases. This implies that effective control of the photoelectron emission can be achieved even in the absence of very strong THz fields by employing lower-frequency THz pulses. Finally, it is worth noting that a significant asymmetry is already observed for relatively modest THz-field amplitudes, as illustrated in Fig. 9(a) for $E_{0T} = 0.3$ MV/cm.

Finally, we analyze how the presented results depend on the spatial geometry of the laser pulse, which governs the ionization process. In particular, the spatial intensity profile of the laser beam requires averaging the results over the focal volume. Such focal averaging is numerically demanding. For this reason, we demonstrate the effects of focal averaging by analyzing photoelectron spectra calculated for fixed values of the emission angle θ . In Fig. 10, we present focal-averaged photoelectron spectra for the ionization of the Ar atom using the same pulse configuration as in Fig. 2 and for the time delays $\Delta t = 50T_L$ [panel (a)], $\Delta t = 250T_L$ [panel (b)], and $\Delta t = 550T_L$ [panel (c)]. The corresponding values of the asymmetry parameter A_p are shown in Figs. 10(d)–10(f). The dependence of the photoelectron spectra on the time delay Δt clearly survives focal averaging. For example, for $\Delta t = 50T_L$, the photoelectron spectrum extends to slightly higher energies for emission in the direction $\theta = 0^\circ$ than for emission in the direction $\theta = 180^\circ$, and the asymmetry parameter exhibits rapid oscillations as a function of the photoelectron energy [see Figs. 10(a) and 10(d), and cf. Figs. 2(a) and 4(a)]. Also, the value of the asymmetry parameter A_p is slightly smaller than that without focal averaging [cf. Figs. 10(d) and 4(a)]. In contrast, for $\Delta t = 250T_L$, emission in the direction $\theta = 180^\circ$ dominates, and the asymmetry parameter is close to -1 [see Figs. 10(b) and 10(e), and cf. Figs. 2(c) and 4(c)]. Finally, for $\Delta t = 550T_L$, the photoelectron spectrum corresponding to emission in the direction $\theta = 180^\circ$ is substantially longer than that obtained for emission in the direction $\theta = 0^\circ$ [see Figs. 10(c) and 10(f), and cf. Figs. 2(f) and 4(f)]. The asymmetry parameter A_p exhibits more oscillations than for the case where the focal averaging is not performed. However,

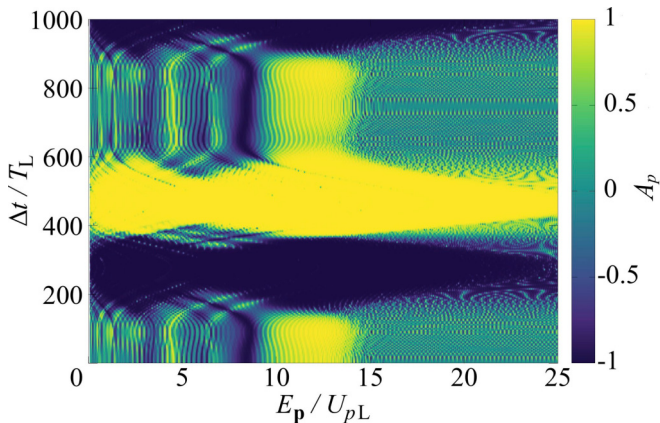


FIG. 8. Asymmetry parameter A_p as a function of the time delay Δt and the photoelectron energy E_p .

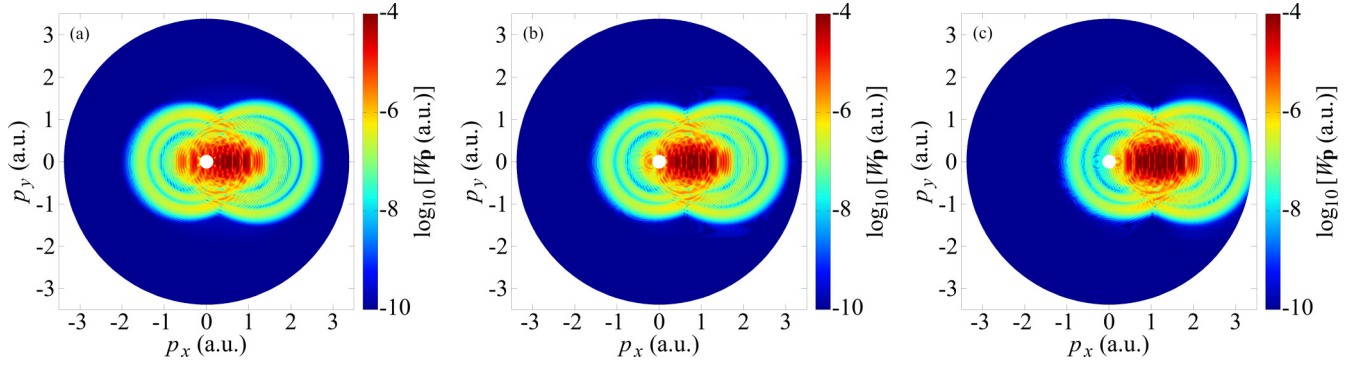


FIG. 9. Photoelectron momentum distribution calculated for the THz pulse configurations with $\Delta t = 500T_L$ and the amplitude $E_{OT} = 0.3$ MV/cm [panel (a)], $E_{OT} = 0.6$ MV/cm [panel (b)], and $E_{OT} = 1$ MV/cm [panel (c)].

on average, its value is similar to the one obtained without focal averaging. This demonstrates that our conclusions are robust with respect to focal averaging over the spatial intensity distribution of the laser pulse and therefore remain relevant under realistic experimental conditions.

IV. CONCLUSIONS

In this work, we have investigated directional asymmetry in strong-field ionization driven by a few-cycle near-infrared laser pulse in the presence of an auxiliary THz pulse. By analyzing energy-resolved photoelectron spectra, full momentum

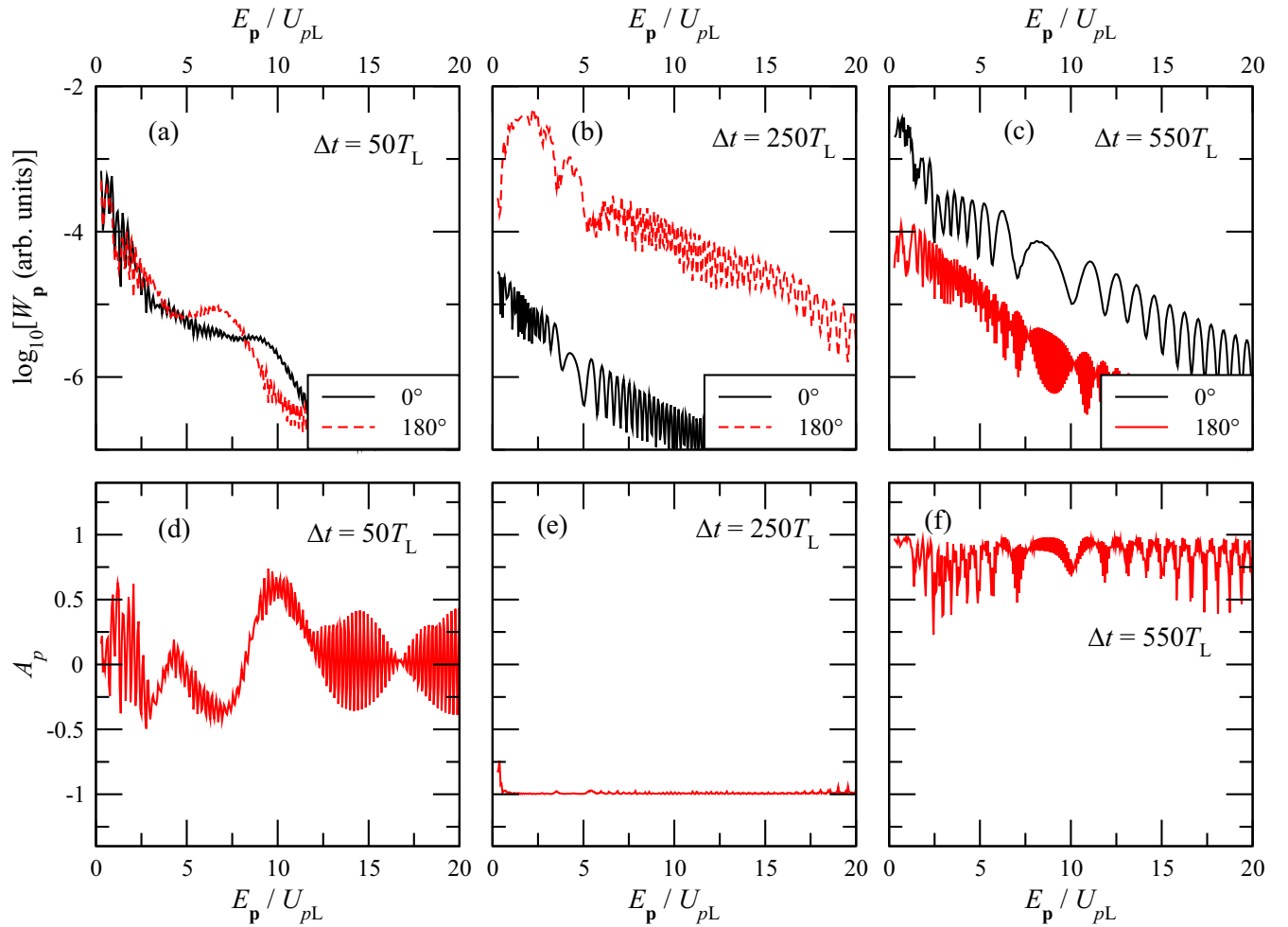


FIG. 10. The focal-averaged photoelectron spectra and the asymmetry parameter A_p for the ionization of the Ar atom using the same pulse configuration as in Fig. 2 and for the time delays $\Delta t = 50T_L$ [panels (a) and (d)], $\Delta t = 250T_L$ [panels (b) and (e)], and $\Delta t = 550T_L$ [panels (c) and (f)]. The values of the emission angle are indicated in the panels.

distributions, and the corresponding asymmetry parameter, we have demonstrated that the emission direction of photoelectrons can be efficiently controlled by adjusting the time delay between the laser and THz pulses.

We have shown that the underlying physical mechanism responsible for the observed asymmetry is governed by the value of the THz vector potential during the ionization window defined by the laser pulse. Depending on the relative timing of the two pulses, the THz pulse effectively influences the ionization process, leading to a pronounced left-right asymmetry that affects both direct and rescattered electrons. This behavior is fundamentally different from asymmetry effects induced solely by the CEP of a few-cycle laser pulse, which predominantly influence the rescattered-electron contribution and are generally weaker.

A systematic analysis of the dependence of the presented results on the pulse parameters demonstrates that the asymmetry is robust with respect to variations of the THz-pulse amplitude and frequency. While these parameters control the magnitude of the THz vector potential and hence the strength of the asymmetry, the qualitative features and the direction in which the emission is dominant are determined primarily by the time delay between the laser and THz pulses. Importantly, significant asymmetry persists even for relatively modest THz-field strengths or lower THz frequencies, indicating that the proposed control scheme is reliable for a broad range of values of the THz-pulse parameters. We have further shown that the delay-dependent asymmetry survives focal averaging over the spatial intensity distribution of the laser pulse. Although focal averaging slightly reduces the magnitude of the asymmetry and introduces additional structure

in the energy dependence of the asymmetry parameter, the dominant emission direction and its switching with the time delay remain clearly observable. This demonstrates that the predicted effects are not artifacts of an idealized single-intensity model and remain relevant under realistic experimental conditions.

The present work builds upon the theoretical concept introduced in Ref. [35], whose predictions were subsequently confirmed by the experimental results reported in Ref. [40]. By providing a detailed analysis of the directional asymmetry, its physical origin, and its robustness with respect to pulse parameters and focal averaging, our results further establish THz-assisted strong-field ionization as a powerful and reliable tool for controlling photoelectron emission dynamics on ultrafast timescales. We expect that these findings will stimulate further experimental and theoretical studies aimed at exploiting THz-assisted schemes for waveform-controlled electron dynamics in atoms and molecules. Method of stabilization and experimental determination of the CEP of an infrared laser pulse having duration of few femtoseconds is a veritable problem (see Ref. [15] and references therein). However, it is much easier to control the time delay between the THz and laser pulses since the corresponding time scale is two orders of magnitude larger (cf. Fig. 6). Such a control is achieved, for example, in Refs. [40,59]. This also emphasizes the advantage of the THz-time-delay control of the strong-field ionization dynamics.

DATA AVAILABILITY

The data are available from the authors upon reasonable request.

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