

Crisis-induced intermittency in reservoir computing

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Reservoir computing enables embedding attractors into random neural networks (RNNs), often generating a “mirror” of a target attractor due to inherent symmetrical constraints in their architecture, which results in multistability. In this work, we report that a simple modulation of a global parameter—the scale of internal connectivity—can induce an attractor-merging crisis accompanied by intermittency, which is characterized by irregular alternation between staying within and escaping from the ruins of embedded attractors. We further reveal its underlying mechanism through a detailed analysis of the phase-space structure, and multiple case studies demonstrate that this bifurcation scenario is intrinsic to a general class of RNNs, independent of training data. This work bridges the gap between machine learning implementations and concepts in nonlinear dynamical systems, contributing to the design of systems that leverage the intrinsic properties of RNNs, as well as to a deeper understanding of biological neural dynamics.

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Introduction. Neural networks are crucial for modern machine learning (ML) and modeling. However, employing them requires careful recognition of their intrinsic properties that should be appropriately harnessed. Moreover, this perspective is also important for understanding biological neural systems.

Transient and intermittent dynamics arise not only in biological neural activity [1,2] but also in artificial intelligence [3,4]. Moreover, such dynamics have been applied in engineering fields, including information processing and optimization [5,6].

A particularly relevant concept is chaotic itinerancy (CI) [7–9], which refers to the dynamics that spontaneously switch between multiple quasistable states driven by their chaoticity. CI has been observed in various high-dimensional systems. Concerning ML, for instance, Iwami *et al.* [10] exploited laser-induced CI for reinforcement learning, while Inoue *et al.* [11] engineered CI in recurrent neural networks (RNNs) for spontaneous switching of motion primitives.

Within this context, reservoir computing (RC) [12–14] provides a compelling framework for investigating the dynamical properties of RNNs. In RC, an RNN called “reservoir” is employed. This has a D -dimensional input $\mathbf{u}_k$ and an N -dimensional state $\mathbf{x}_k$, evolving as

$$\mathbf{x}_{k+1} = \mathcal{R}(\mathbf{x}_k, \mathbf{u}_k). \quad (1)$$

By mapping $\mathbf{x}_k$ to a target output $\mathbf{y}_k$, RC conducts various signal processing of $\mathbf{u}_k$. The output $\mathbf{z}_k$ is given by

$$\mathbf{z}_k = \phi(\mathbf{W}_{\text{out}}^\top \psi(\mathbf{x}_k)), \quad (2)$$

where the functions ψ and ϕ are nonlinear functions, including the identity map. In RC, only the readout weights $\mathbf{W}_{\text{out}} \in \mathbb{R}^{N \times D}$ are subject to training, which reduces to a regression problem minimizing the following loss:

$$\mathcal{L} = \|\phi(\mathbf{X}\mathbf{W}_{\text{out}}) - \mathbf{Y}\|_F^2, \quad (3)$$

where

$$\mathbf{X} = [\psi(\mathbf{x}_{T_{\text{init}}}) \cdots \psi(\mathbf{x}_{T_{\text{init}}+T_{\text{train}}})]^\top, \quad (4)$$

$$\mathbf{Y} = [\mathbf{y}_{T_{\text{init}}} \cdots \mathbf{y}_{T_{\text{init}}+T_{\text{train}}}]^\top. \quad (5)$$

This enables efficient and lightweight training.

When $\mathbf{y}_k = \mathbf{u}_k$, RC performs a one-step-ahead prediction of the input and feeding back the predicted value $\mathbf{z}_k$ as an alternative input constructs an autonomous system:

$$\mathbf{x}_{k+1} = \mathcal{R}(\mathbf{x}_k, \mathbf{z}_k) = \hat{\mathcal{R}}(\mathbf{x}_k). \quad (6)$$

This closed-loop configuration enables the attractor reconstruction from a finite-length time series [15,16].

RC is not only a tool for time-series prediction but also a platform for designing dynamical systems via supervised learning. A key issue here is the nature of the generalization achieved by training. For example, in parameter-aware RC (PARC) [17], which learns multiple trajectories from a parametric family of dynamical systems along with their associated parameter values, it has been shown that the trained RC can reproduce the bifurcation structures of target systems—even beyond the range of training parameters. Conversely, the emergence of unintended properties is also important. Flynn *et al.* [18,19] showed that in multifunctional RC (MFRC),

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which learns multiple attractors to form a multistable system, attractors not in training data—referred to as “untrained attractor”—often appear.

In particular, concerning intermittency, Flynn [20] reported that placing the MFRC system at untrained parameter values led to the emergence of CI-like dynamics.

However, to the best of our knowledge, a comprehensive formalization of RNNs’ intermittent dynamics and a detailed analysis of their underlying mechanisms have not yet been fully developed. This study aims to address this gap by focusing on crisis-induced intermittency [21,22] in RC systems.

Crises are a typical bifurcation in chaotic systems, characterized by a discontinuous change of a chaotic attractor due to contact with an unstable periodic orbit [23]. In symmetric systems, multiple attractors can simultaneously touch an unstable orbit on their basin boundary, causing an attractor-merging crisis (AMC). The system then exhibits crisis-induced intermittency, alternating between staying within and escaping the attractor’s remnants. We refer to this quasistable structure as an “attractor ruin” [24], as AMC is closely associated with the emergence of CI [25,26].

Several previous studies have approached transient dynamics in RC. Kong *et al.* [27,28] utilized PARC to estimate crises from precrisis time series and reproduce plausible transients. About the itinerant dynamics in MFRC, Flynn and Amann [29] analyzed its underlying mechanisms in specific settings.

In contrast, we focus on cases where both multistability and transient responses emerge intrinsically—independent of learning—and investigate the general dynamical properties of RNNs. In the reservoir dynamics given by Eq. (1) and the readout defined in Eq. (2), if the symmetries $\mathcal{R}(-\mathbf{x}, -\mathbf{u}) = -\mathcal{R}(\mathbf{x}, \mathbf{u})$ and $\phi(-\mathbf{y}) = -\phi(\mathbf{y})$ hold, then the closed-loop system described in Eq. (6) satisfies the symmetry $\hat{\mathcal{R}}(-\mathbf{x}) = -\hat{\mathcal{R}}(\mathbf{x})$. As a result, when an attractor Λ^+ is embedded, a symmetric “mirror” attractor Λ^- necessarily exists, and the system becomes multistable if $\Lambda^+ \neq \Lambda^-$.

We demonstrate that this inherent bistability can collapse due to the AMC, which is induced by varying the reservoir parameters (Fig. 1). Furthermore, by conducting a detailed analysis of the phase space before and after the AMC, we elucidate the mechanism behind the resulting intermittency.

Method. As for the reservoir $\mathcal{R}$, we consider the following echo state network (ESN) [30]:

$$\mathcal{R}(\mathbf{x}, \mathbf{u}) = (1 - a)\mathbf{x} + a \tanh(\rho \mathbf{W} \mathbf{x} + \sigma \mathbf{W}_{\text{in}} \mathbf{u}), \quad (7)$$

where $\mathbf{W} \in \mathbb{R}^{N \times N}$ is a random matrix where the proportion of nonzero elements is p , and the values are drawn from a normal distribution and then scaled to have a unity spectral radius so that ρ is the spectral radius of the internal matrix $\rho \mathbf{W}$. Each element of the random matrix $\mathbf{W}_{\text{in}} \in \mathbb{R}^{N \times D}$ is drawn from a uniform distribution on $[-1, 1]$, and σ represents the input intensity. The parameter a is the leaky rate. Unless otherwise specified, we fix the parameters as $(N, p, a) = (1000, 0.01, 0.5)$ in this paper.

The parameter ρ is crucial for the reservoir’s convergence properties and memory capacity [30–33]. When σ is fixed, the conditional Lyapunov exponent of the input-driven reservoir increases with ρ [15,34]. In particular, when the input is constant, the attractor of $\mathbf{x}_{k+1} = \mathcal{R}(\mathbf{x}_k, c)$ transitions from a

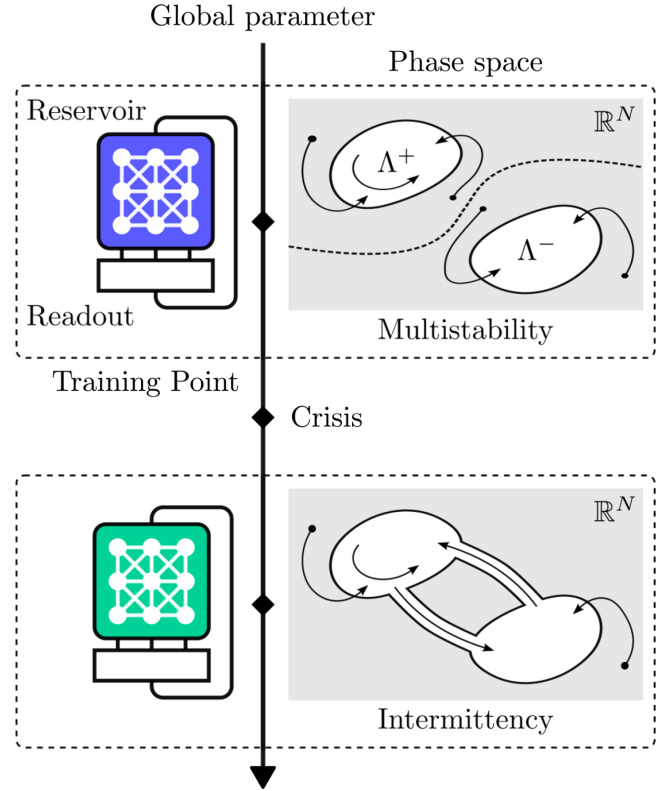


FIG. 1. Conceptual diagram of the target phenomenon.

fixed point to chaos, given a sufficiently large N (see Sec. IV of the Supplemental Material [35]).

We focus on qualitative changes in the dynamics of the closed-loop ESN (6), particularly those induced by varying ρ . To explore this, we consider three types of closed-loop models—binary-feedback system, switching-driven system, and Chaos-embedded system—as case studies.

Finally, we demonstrate that similar crisis-induced intermittency arises in a wide class of chaos-embedded models, confirming the generality of the observed phenomena.

Result: A. Case studies: a. Binary-feedback system. Owing to the symmetry of ESN, the attractors of $\mathcal{R}(\mathbf{x}, +c)$ and $\mathcal{R}(\mathbf{x}, -c)$ are symmetric about the origin. This property enables the construction of the following binary-feedback system:

$$\mathbf{x}_{k+1} = \mathcal{R}(\mathbf{x}_k, \text{sgn}(\mathbf{w}_{\text{out}}^\top \mathbf{x}_k)). \quad (8)$$

The vector $\mathbf{w}_{\text{out}} \in \mathbb{R}^N$ serves as a linear classifier that distinguishes between the attractors $\Lambda^\pm$ of $\mathcal{R}(\mathbf{x}, \pm 1)$ for a given ρ , and it is obtained using a support vector machine. If $\Lambda^\pm$ are linearly separable, system (8) becomes bistable. In particular, if the attractors are fixed points $\pm \mathbf{p}$, they are clearly linearly separable, and $\mathbf{w}_{\text{out}} = \mathbf{p} / \|\mathbf{p}\|^2$.

When $(\rho, \sigma) = (0.9, 0.2)$, the obtained system has two fixed points at $z = \mathbf{w}_{\text{out}}^\top \mathbf{x} \simeq \pm 1$. The system can be destabilized by fixing $\mathbf{w}_{\text{out}}$ and increasing ρ . To distinguish this bifurcation parameter from the hyperparameter in training, we denote it as $\hat{\rho}$. Figure 2(a) shows that as $\hat{\rho}$ increases, fixed points transition to periodic orbits and then to chaos around $\hat{\rho} = 1.3$, resulting in two coexisting chaotic attractors. A crisis

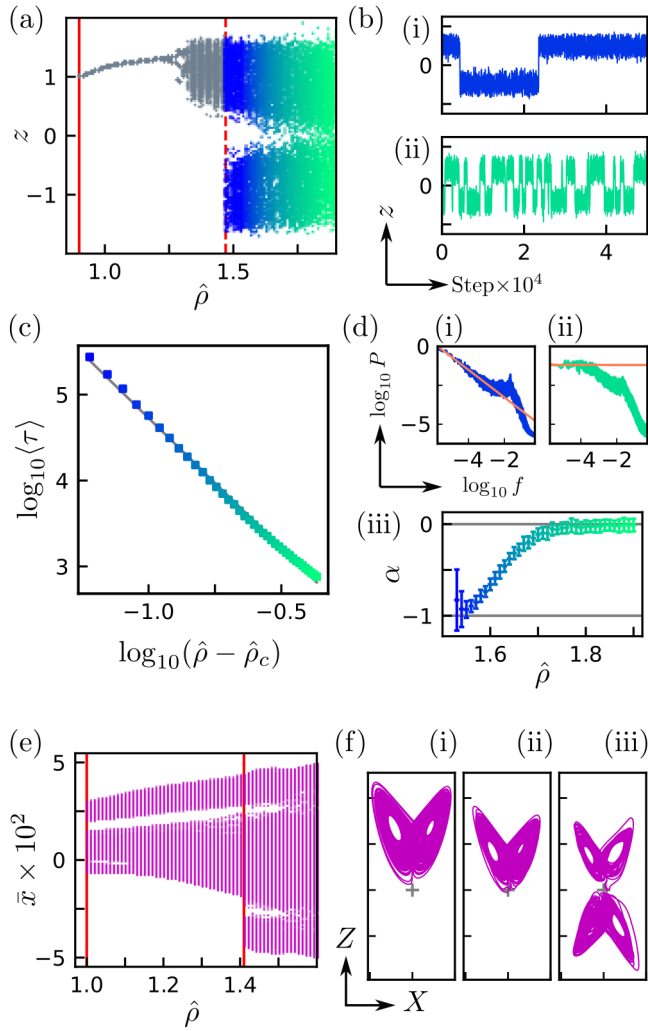


FIG. 2. (a)–(d) Bifurcation of a binary-feedback system induced by $\hat{\rho}$. (a) Bifurcation diagram plotting the extrema of $z = \mathbf{w}_{\text{out}}^T \mathbf{x}$. This system is trained at $\rho = 0.9$ (the solid line), and the critical point is $\hat{\rho}_c \approx 1.47$ (the dashed line). (b) z trajectories for (i) $\hat{\rho} = 1.56$ and (ii) $\hat{\rho} = 1.80$. (c), (d) Statistical analysis of z time series over 10^6 steps with 50 different initial states for each $\hat{\rho} > \hat{\rho}_c$. (c) Average interval $\langle \tau \rangle$ of sign changes, along with its power-law fitting, yielding $\gamma \simeq 2.83$. (d) (i), (ii) Power spectral density (PSD) at $\hat{\rho} = 1.56$ and $\hat{\rho} = 1.80$, with fitted lines for $f < 10^{-4}$, showing slopes α of -0.88 and 0.00 , respectively. (iii) Slope α at each $\hat{\rho}$. (e), (f) Bifurcation of the switching Lorenz-driven system trained at $\rho = 1.0$. The critical point is $\hat{\rho}_c \approx 1.41$. (e) Bifurcation diagram plotting the extrema of the mean internal activity $\bar{x}$. (f) (i)–(iii) Output z_k at $\hat{\rho} = 1.0, 1.4$, and 1.5 .

occurs at $\hat{\rho} = \hat{\rho}_c \simeq 1.47$, where the attractors merge. This is likely triggered by the expansion of both the attractor and the preimage set of the decision boundary $\mathbf{w}_{\text{out}}^T \mathbf{x}_k = 0$, leading to their collision at $\hat{\rho} = \hat{\rho}_c$.

When $\hat{\rho} > \hat{\rho}_c$, intermittency is observed as the alternating sign of z , and the frequency increases with $\hat{\rho}$ [Fig. 2(b)]. The mean span of sign reversals (i.e., the average residence time in attractor ruin) $\langle \tau \rangle$ decreases with a power law to $\hat{\rho} - \hat{\rho}_c$ [Fig. 2(c)]. This power law, $\langle \tau \rangle \propto (\hat{\rho} - \hat{\rho}_c)^{-\gamma}$, is a universal

characteristic of crisis-induced intermittency [22,26]. In this case, the estimated exponent is $\gamma \simeq 2.83$.

We analyzed the power spectral density of a 10^6 -step time series of z . For the distribution $P(f)$, we focused on low-frequency components ($f < 10^{-4}$) and estimated the slope on a log-log scale using linear regression: $\ln P(f) \sim \alpha \ln f + C$. The results for $\hat{\rho} = 1.56$ and $\hat{\rho} = 1.80$ are shown in Figs. 2(d)(i) and 2(d)(ii), respectively. As $\hat{\rho}$ increases, $|\alpha|$ decreases [Fig. 2(d)(iii)]. Notably, near $\hat{\rho}_c$, the low-frequency spectrum is well approximated by f^{-1} , and around $\hat{\rho} = 1.9$, the slope nearly flattens. This suggests that destabilization weakens trajectory autocorrelation, leading to a continuous transition from intermittency to fully merged chaos.

The same bifurcation occurs regardless of the training point ρ as long as $\Lambda^\pm$ are fixed points. However, ρ influences both $\hat{\rho}_c$ and γ by determining the distribution of the preimages of the decision boundary (Sec. IV of the Supplemental Material [35]).

b. Switching-driven system. Next, the constant input is extended to a time series $\{\mathbf{u}_k\}$ by constructing the switching-driven system:

$$\mathbf{z}_k = \mathbf{W}_{\text{out}}^T \mathbf{x}_k, \quad e_k^\pm = \|\pm \mathbf{u}_k - \mathbf{z}_k\|, \quad (9)$$

$$\mathbf{x}_{k+1} = \begin{cases} \mathcal{R}(\mathbf{x}_k, \mathbf{u}_k), & (e_k^+ < e_k^-), \\ \mathcal{R}(\mathbf{x}_k, \mathbf{0}), & (e_k^+ = e_k^-), \\ \mathcal{R}(\mathbf{x}_k, -\mathbf{u}_k), & (e_k^+ > e_k^-), \end{cases} \quad (10)$$

where $\mathbf{W}_{\text{out}}$ is trained for the one-step-ahead prediction.

Now, a three-dimensional trajectory of the Lorenz system [36] is used as $\{\mathbf{u}_k\}$, focusing on the bifurcation induced by $\hat{\rho}$. Let $\mathbf{z}_k = [X_k, Y_k, Z_k]^T$. The readout is trained with the appropriate ρ , such that $e_k^+ < e_k^-$ always holds for $\hat{\rho} = \rho$. Here, the parameters are set to $(\rho, \sigma) = (1.0, 0.025)$. The attractor at $\hat{\rho} = \rho$ corresponds to the response attractor $\Lambda^\pm$ driven by $\{\pm \mathbf{u}_k\}$. As $\hat{\rho}$ increases, the attractors expand, as seen in input-driven ESNs, and eventually collide [Fig. 2(e)]. Consequently, z_k transitions irregularly between the two attractor ruins, which maintain the shape of the original Lorenz attractors to some extent [Fig. 2(f)]. The dynamics of this system is dominated by intermittent fluctuations in the prediction accuracy of $\mathbf{W}_{\text{out}}$. Two factors likely contribute to this phenomenon: the robustness of $\mathbf{W}_{\text{out}}$ and the robustness of generalized synchronization, which breaks down accompanied by intermittent desynchronization [37–39]. Therefore, the intermittency in this system is considered to arise from the destabilization of the input-driven ESN while maintaining the spatial structure of the synchronized state.

c. Lorenz-embedded system. By feeding back the above output z_k directly, this model becomes fully closed loop:

$$\hat{\mathcal{R}}(\mathbf{x}) = (1 - a)\mathbf{x} + a \tanh([\hat{\rho} \mathbf{W} + \sigma \mathbf{W}_{\text{in}} \mathbf{W}_{\text{out}}^T] \mathbf{x}), \quad (11)$$

which reconstructs the Lorenz attractor [Fig. 3(a)(i)]. Increasing $\hat{\rho}$ again induces an AMC accompanied by intermittency [20] [Figs. 3(a)(ii) and 3(a)(iii)]. We refer to this type of intermittency as “double Lorenz.”

Let $\Delta \rho = \hat{\rho} - \rho$ and $\Delta \rho_c = \hat{\rho}_c - \rho$. Based on the time-series observation for 10^7 steps, the estimated critical point is $-7 \times 10^{-8} < \Delta \rho_c < -6 \times 10^{-8}$. Therefore, the crisis has already occurred at $\hat{\rho} = \rho$. The statistical properties of

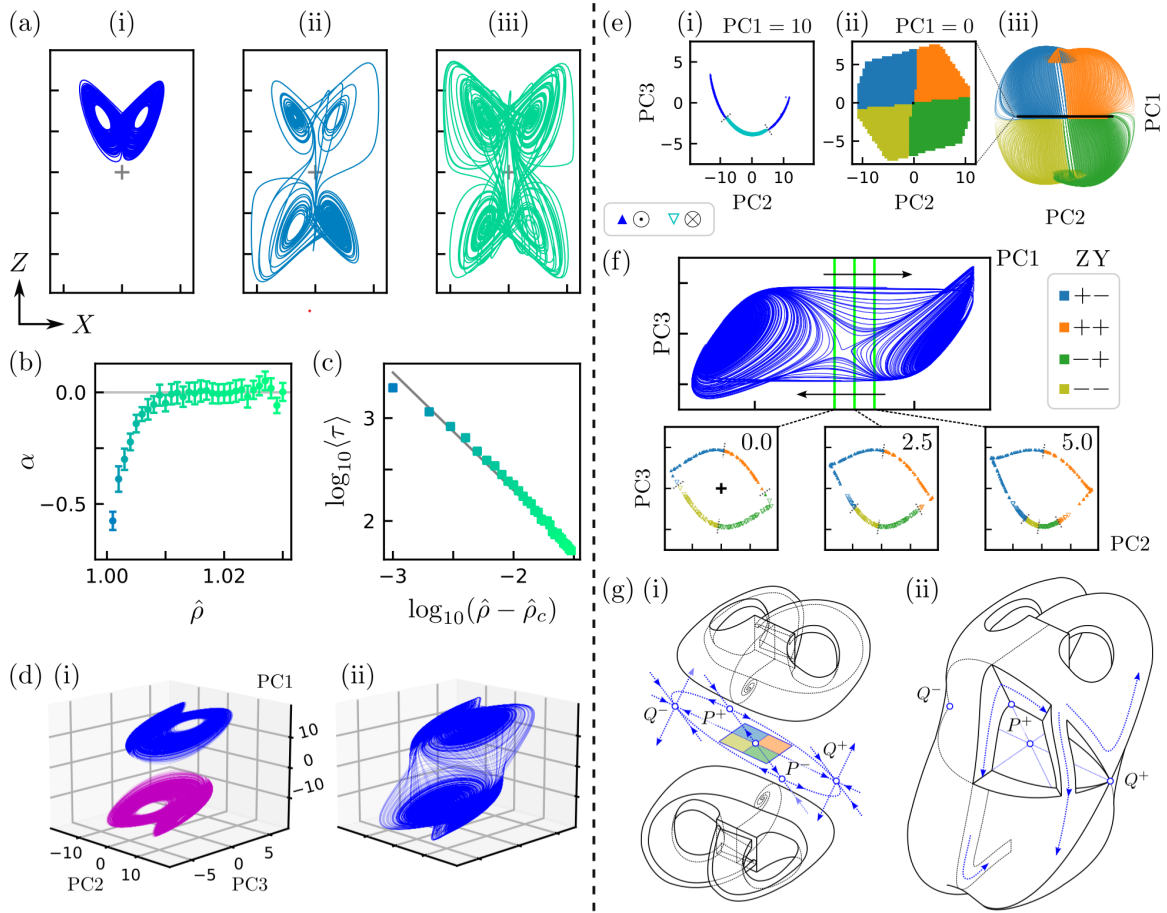


FIG. 3. AMC in the Lorenz-embedded model. (a) (i)–(iii) Output z_k at $\hat{\rho} = 1.0, 1.001$, and 1.01 , respectively. The critical point is $\hat{\rho}_c \approx \rho = 1.0$. (b), (c) Statistical analysis of the Z component time series over 10^5 steps from 20 different initial conditions; (b) slope α of the PSD for $f < 10^{-4}$ on a log-log scale and (c) the average interval $\langle \tau \rangle$ of the sign changes at each $\hat{\rho}$, along with its power-law fitting, yielding $\gamma \simeq 1.14$. (d) Orbit in the PC space at (i) $\Delta\rho = -10^{-7}$ with bistability and (ii) $\Delta\rho = 10^{-3}$ with intermittency. (e) For $\Delta\rho = -10^{-7}$: (i) Poincaré section at $PC1 = 10$; (ii) color-coded initial points on $PC1 = 0$ ($\max|x_i| < 1$) based on the sign of (Z, Y) when reaching $|Z| = 27$; and (iii) sample trajectories (200 steps). (f) For $\Delta\rho = 10^{-3}$: trajectories with $PC2 > 0$ in the $PC3$ - $PC1$ plane and Poincaré sections at $PC1 \in \{0.0, 2.5, 5.0\}$, with colored points as in panel (b)(ii). (g) Geometric flow model (i) before and (ii) after the crisis. The colored plane in panel (g)(i) corresponds to panel (e)(ii).

output Z are consistent; the slope α of the low-frequency PSD continuously changes from negative to zero, and the average residence time $\langle \tau \rangle$ decreases as a power law to $\hat{\rho} - \hat{\rho}_c$ [Figs. 3(b) and 3(c)].

In contrast to systems such as Eqs. (8) and (10), $\hat{\rho}$ directly affects the feedback value z in Eq. (11). Thus, understanding the emergence of the double-Lorenz attractor requires analyzing it as an autonomous system.

Figures 3(d)(i) and 3(d)(ii) show the inner activity $\mathbf{x}_k$ for $\Delta\rho = -10^{-7}$ and $\Delta\rho = 10^{-3}$, respectively, which are projected onto the space spanned by the first three principal components ($PC1$ – $PC3$), obtained at $\Delta\rho = -10^{-7}$, which we call “PC space.” The Poincaré section at $PC1 = 10$ shows parabolical curve [Fig. 3(e)(i)]. Trajectories are initially attracted to one of the four lobes of a pair of Lorenz attractors, distinguished by the signs of Z and Y . From this perspective, the state space can be divided into four regions, whose configuration on the $PC1 = 0$ plane is shown in Fig. 3(e)(ii). At the region boundaries, two pairs of saddles, $P^\pm$ and $Q^\pm$, exist. The points $P^\pm$ are located at the junctions of the four

regions, while $Q^\pm$ lie along the Z -sign boundary, visible at the left and right edges of Fig. 3(e)(iii). The upper panel of Fig. 3(f) depicts trajectory fragments with $PC2 > 0$ after the AMC, where the flow around Q^+ is observed. The lower panels present multiple Poincaré sections along $PC1$, with each point clustered according to its near-future state, similar to Figs. 3(e)(ii) and 3(e)(iii). An escape pocket from the attractor ruin is identified at the tip of the surface, which emerges from the contact between the attractor and the stable manifold of $P^\pm$. A schematic summary of the flow before and after AMC is presented in Fig. 3(g), which is based on the model by Guckenheimer and Williams [40]. For further details, see also Sec. V of the Supplemental Material [35]. This case study elucidated the details of a bifurcation scenario in which an acquired attractor comes into contact with a basin boundary that is intrinsically formed.

Generalization: a. Designed chaos. The bistability of symmetric chaotic attractors realized by system (6) generally has the potential to induce intermittency via AMCs. While the emergence of the double-Lorenz attractor in this setup is

somewhat universal—observed in all 200 ESN with different random realizations (Sec. V of the Supplemental Material [35])—the induction of crisis by increasing $\hat{\rho}$ is not necessarily guaranteed. It may lead to other bifurcations, such as saddle-node or period-doubling bifurcations. If untrained attractors other than the targets exist, they can also be influential. Thus, when intentionally inducing intermittency in RC, it is essential to consider this uncertainty. We propose two approaches to address this issue. One approach is to explore other bifurcation parameters, such as the (ρ, σ) plane. The other involves intervention at the training stage—specifically ensuring the robustness of $\mathbf{W}_{\text{out}}$ so that the emergence of untrained periodic orbits is avoided. Therefore, potential countermeasures include introducing regularization (Method 1) or training $\mathbf{W}_{\text{out}}$ across multiple values of ρ as follows (Method 2):

$$[\mathbf{X}(\rho); \alpha \mathbf{X}(\rho')] \mathbf{W}_{\text{out}} \approx [\mathbf{Y}; \alpha \mathbf{Y}]. \quad (12)$$

Method 2 can be extended into a parameter-aware approach [17,27,28] by setting the training data $\mathbf{Y}$ to the trajectories $\mathbf{Y}'$.

Based on the above considerations, we examine several cases of AMC induction in RNNs. First, we consider an artificial case in which the governing equation of the target chaotic system is not explicitly defined. In RC-based embedding, training a periodic orbit with an excessively large ρ can form a chaotic attractor along the periodic orbit [41]. Figures 4(a)(i) and 4(a)(ii) depict a chaotic attractor formed along a Lissajous curve and crisis-induced intermittency induced by destabilization, respectively. A similar phenomenon can be observed even when attractors overlap in the output space; Fig. 4(b) shows an “at”-shaped chaotic attractor, constructed using a handwritten pattern, and the obtained intermittency. Here, the parameter is set to $p = 1.0$.

b. Bifurcation inference. Next, we explore applying this method to estimate AMC in target systems only by showing a single trajectory. Specifically, we use Chua’s circuit [42,43], which has an AMC scenario whereby two symmetric spiral attractors merge to form a double-scroll attractor. Here, a time series of a single spiral attractor is learned with Method 2; $(\rho, \rho', \alpha) = (1.5, 2.0, 0.25)$. At $\hat{\rho} = 1.5$, spiral attractors are observed [Fig. 4(c)(i)]. By increasing $\hat{\rho}$, the double-scroll attractor is successfully induced [Fig. 4(c)(ii)]. Even without Method 2—training with only one ρ —, the birth of a double-scroll attractor can be observed in the RC model, but it is induced by other parameter manipulations (Sec. VI of the Supplemental Material [35]).

c. Asymmetric combination. We also consider a case with an asymmetric combination of attractors. Figure 4(d-i) illustrates the case where a pair of Lorenz attractors is embedded using the MFRC with Method 2; $(\rho, \rho', \alpha) = (1.3, 1.31, 10^{-7})$ (Sec. VII of the Supplemental Material [35]). Due to symmetry, each attractor has a corresponding twin, making the system four-stable. These two pairs of attractors undergo crises and form a “quadruple-Lorenz” attractor [Fig. 4(d)(ii)]. Here, transitions occur between non-origin-symmetric (top-bottom and left-right) attractor ruins, indicating attractor merging similar to AMC, independent of symmetry.

Discussion. So far, we analyze AMCs and intermittency in random RNNs across multiple cases. Additional examples,

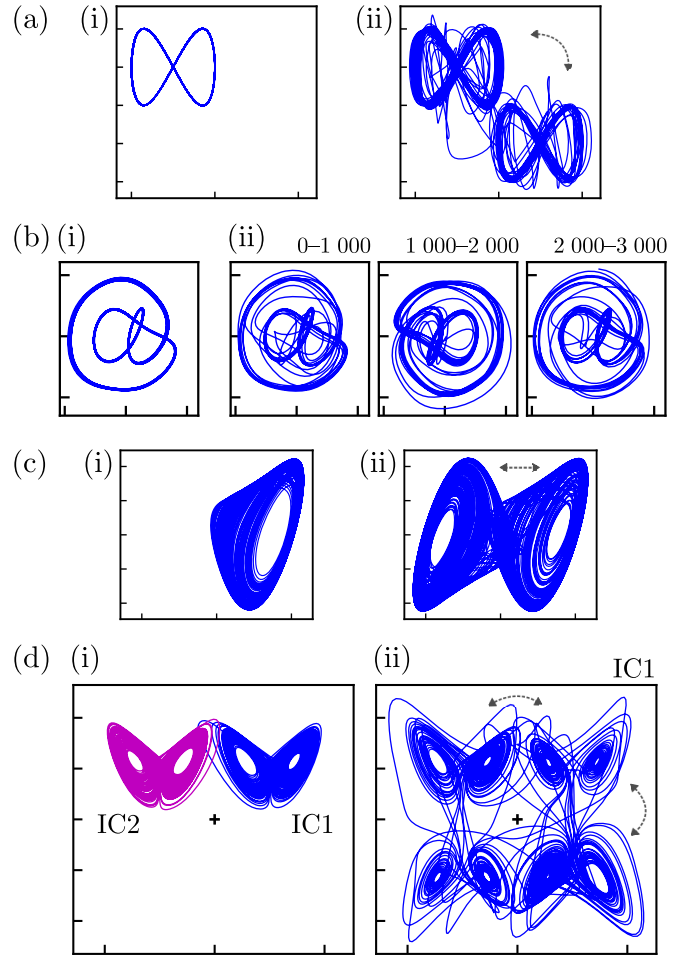


FIG. 4. (a) AMC of Lissajous curve-shaped chaos. (i) Chaotic attractor designed at $\rho = 1.4440$. (ii) Behavior at $\hat{\rho} = 1.4445$. (b) AMC of “at”-shaped chaos. (i) Chaotic attractor designed at $\rho = 1.300$. (ii) Behavior at $\hat{\rho} = 1.305$: Output in intervals $t \in [0, 1000]$, $[1000, 2000]$, and $[2000, 3000]$. (c) AMC of the RC system trained on Chua’s circuit spiral attractor. (i) Trajectory at $\hat{\rho} = \rho = 1.5$. (ii) Double-scroll attractor at $\hat{\rho} = 1.55$. (d) Crisis in the asymmetric case. (i) Two trajectories at $\hat{\rho} = \rho = 1.3$ with different initial conditions (IC1 and IC2) of the MFRC system trained on a pair of Lorenz attractors. (ii) A trajectory at $\rho = 1.301$ from IC1. See also movies in the Supplemental Material [35].

including training-free RNNs, are provided in Sec. III of the Supplemental Material [35].

The universality of AMCs in RNNs can be inferred from (1) the generality of crisis, (2) the typical bifurcation of ESNs, and (3) the robustness of generalized synchronization between chaotic systems. While these theoretical foundations support our observations, the evidence remains largely empirical, which constitutes a limitation of our work.

We compare our approach with the method proposed by Inoue *et al.* [11] for designing intermittent dynamics in RC, which enables the artificial implementation of CI by specifying attractor ruins and probabilistic transition rules among them. While this method offers high controllability, it requires extensive pretraining for the internal weights of the RNN [44]. In contrast, our approach only requires embedding precursors

of attractor ruins via RC. Thus, each trial operates at a low computational cost. Furthermore, due to the general properties of crises, the average residence time in the attractor ruin can be continuously adjusted by fine-tuning. This kind of controllability was not available in Ref. [11].

However, the response to parameter modulation exhibits variability across models, making it necessary to explore the parameter space to find AMC. As a solution, we propose explicitly embedding saddlelike structures into multistable attractors. For details, see Sec. IX of the Supplemental Material [35].

One potential application of our result is in designing itinerant dynamics via RC. This is compelling from a constructive perspective for understanding spontaneity. For example, Namikawa *et al.* [45] used a hierarchical chaotic RNN to generate intermittent sequences composed of imitated primitive actions on a humanoid robot, modeling cortical dynamics. Similarly, Steingrube *et al.* [46] employed a chaotic central pattern generator to realize diverse adaptive motion and exploratory chaotic behaviors, enabling fast and autonomous adaptation. Here, our study provides a framework for spontaneous behavior switching within a single RNN. A demonstration is presented in Sec. VIII of the Supplemental Material [35], in which a robot controlled by an RC system autonomously explores a simulated environment [47,48] by spontaneously switching among four motion primitives. In such a neurorobotics context, extending our framework to physical RC [49,50] represents an intriguing direction for future research.

The manifestation of crisis-induced intermittency as intrinsic bifurcation scenarios in RNNs is also of interest from the perspective of neuroscience. For example, Hadaeghi *et al.* [51,52] proposed a mathematical model for bipolar disorder

based on crisis-induced intermittency. In RC-based modeling, Yonekura *et al.* [53] modeled perceptual switching in binocular rivalry by using a reservoir whose internal parameters drift under respiratory influence. Flynn [20] further applied CI in destabilized MFRC to data-driven modeling of epilepsy. In the context of a constructive approach to neural dynamics, exploiting crisis-induced intermittency in RNNs may offer a promising option.

Conclusion. In this study, we formulate an emergent mechanism of intermittency in RC as crises. Through a series of case studies, we revealed how such crises emerge via tuning of a reservoir's global parameter and elucidated the structure underlying the observed dynamics. Moreover, we demonstrated that this AMC scenario is part of a universal bifurcation structure in typical random RNNs.

Our results can be informative for the design of spontaneously active dynamical systems, particularly in the context of robotics and artificial intelligence, as well as for modeling biological neural dynamics.

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Data availability. No data were created or analyzed in this study.

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