

## Quantum convolutional neural network for phase recognition in two dimensions

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Quantum convolutional neural networks (QCNNs) are quantum circuits for characterizing complex quantum states. They have been proposed for recognizing quantum phases of matter at low sampling cost and have been designed for condensed matter systems in one dimension. Here we construct a QCNN that can perform phase recognition in two dimensions and correctly identify the phase transition from a toric code phase with  $\mathbb{Z}_2$  topological order to the paramagnetic phase. The network also exhibits a noise threshold up to which the topological order is recognized. Furthermore, it captures correlations between all stabilizer elements of the toric code, which cannot be accessed by direct measurements. This increases the threshold for errors leading to such correlations and allows for correctly identifying the topological phase in the presence of strong correlated errors. Our work generalizes phase recognition with QCNNs to higher spatial dimensions and intrinsic topological order, where exploration and characterization via classical numerics become challenging.

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With the remarkable progress in quantum computing technologies [1–4], the output states of quantum computers and simulations are becoming too complex to characterize them with direct measurements and classical computing. Although many local properties of quantum states can be efficiently characterized using shadow tomography [5], the characterization of global properties remains an open challenge. Direct processing of quantum data on quantum computers offers a potential solution to this problem, where recent developments include quantum autoencoders [6,7], quantum principal component analysis [8], certification of Hamiltonian dynamics [9,10], and quantum reservoir processing [11]. Particularly promising are quantum neural networks that use parameterized quantum circuits, measurement, and feedforward [12] to characterize large amounts of quantum data [13–17].

A central field for applications of quantum computing is condensed matter physics, where the characterization of quantum states is of central importance. In particular, the classification of phases of matter [18,19] is a key requirement for understanding strongly correlated materials [20]. While phases of matter in one-dimensional systems are well understood thanks to accurate classical numerical simulations [21], two- and higher-dimensional systems are notoriously hard to simulate on classical computers [22,23]. Of particular interest in this context are topological many-body phases

of matter, which have already been prepared on quantum computers [24–27], thus opening new avenues for their exploration. However, the characterization of these topological phases remains challenging because of the absence of local order parameters. Other characteristic quantities, such as fidelity susceptibility and entanglement, are hard to measure. Previous approaches addressing this challenge include applying classical machine learning techniques to data from classical numerical simulations [28–30] and classical shadows [31]. These approaches are, however, limited by the large amounts of data required for describing quantum states on classical computers.

Quantum convolutional neural networks (QCNNs) have been proposed to substantially reduce the sample complexity of recognizing symmetry-protected topological phases in one dimension [14,32] and were shown to uncover characteristic properties of such phases from training data via supervised learning [14,33,34]. Moreover, these QCNNs can be explicitly constructed by combining techniques from quantum error correction and renormalization-group flow [14,32,35,36], which is a well-established method for the classification of phases of matter [20]. Recently, QCNNs that are robust against errors in quantum data [36] were implemented on a superconducting quantum processor under noisy intermediate-scale quantum (NISQ) conditions [35].

So far, QCNNs have, however, only been applied to one-dimensional systems, which can be accurately simulated on classical numerical computers. Their extension to two- and higher-dimensional systems, where quantum computers can potentially provide a computational advantage, has remained largely unexplored. Existing work includes a method for detecting topological phases in two dimensions called locally error-corrected decoration (LED) [37], which is

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inspired by QCNNs. However, all current results are limited to snapshot-based LED, i.e., the classical postprocessing of direct measurements. Thus, in its current form, it faces limitations once the characterization of the quantum states requires more data than can be efficiently processed on classical computers. This can occur at interesting points, for example, close to the critical point of a quantum phase transition. Hence, extending QCNNs to two dimensions is a nontrivial generalization as quantum phases in two- and higher-dimensional systems are fundamentally different due to long-range entanglement, giving rise to topological order, which is absent in one dimension [38].

Here, we address this open challenge by generalizing QCNNs to detect  $\mathbb{Z}_2$  topological order in two-dimensional spin arrays. We analytically construct a QCNN inspired by quantum error correction and multiscale entanglement renormalization ansatz [39]. Using matrix-product-state (MPS) simulations, we show that the QCNN recognizes the topological phase of the toric code Hamiltonian in a magnetic field from a topologically trivial phase. In contrast to existing methods [37], which struggle to correctly classify complex quantum states close to phase boundaries, the QCNN precisely identifies the critical value of the magnetic field where the toric code undergoes the phase transition. Moreover, we show that the QCNN output is robust against incoherent errors below a threshold error probability allowing for the application of the QCNN on current and near-term quantum computers.

Classical approaches based on direct measurements, such as the snapshot-based LED method in Ref. [37], cannot fully capture quantum correlations. In the case of the toric code, the stabilizer elements would require readout in different Pauli bases and thus cannot be simultaneously measured via direct measurements on typical quantum processors. In contrast, in processing quantum states via the QCNN quantum circuit, correlations between all stabilizer elements can be taken into account, increasing the threshold probability for errors that lead to such correlations, similarly to correlated quantum error correction [40]. As a result, the QCNN correctly identifies the topological phase in the presence of strong correlated errors, where classical approaches misclassify topological states. Note that, in contrast to quantum error correction constructions [2], it does not require additional readout qubits.

The QCNN we consider is a quantum circuit that can be written as a unitary

$$U_{\text{QCNN}} = U_{\text{FC}} U_{\text{P}}^{(d)} \cdots U_{\text{P}}^{(2)} U_{\text{P}}^{(1)} U_{\text{C}}, \quad (1)$$

consisting of a convolutional layer  $U_{\text{C}}$ ,  $d$  pooling layers  $U_{\text{P}}^{(l)}$ , for  $l = 1, 2, \dots, d$ , and a fully connected layer  $U_{\text{FC}}$  [35,36]. The processing of quantum states in the QCNN is schematically depicted in Fig. 2. The convolutional layer  $U_{\text{C}}$  performs a translationally invariant transformation. In each pooling layer, the system size is reduced by discarding a fraction of qubits after applying gates on the remaining qubits. The fully connected layer  $U_{\text{FC}}$  prepares the output state of the remaining qubits, which are subsequently measured.

We construct the convolutional layer from Clifford gates and the pooling layers from single-qubit Pauli gates controlled by qubits that are subsequently discarded. In this case, the QCNN circuit [Eq. (1)] featuring only the single convolutional

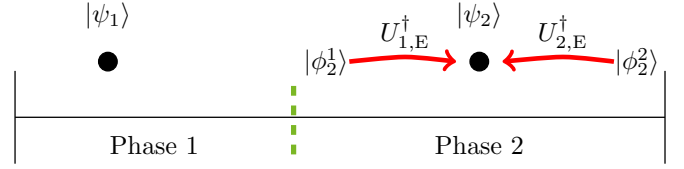


FIG. 1. Schematic of phase recognition via a QCNN for two phases with characteristic states  $|\psi_1\rangle$  and  $|\psi_2\rangle$ . Other states  $|\phi_j^k\rangle$  in the respective phases can be generated by local unitary perturbations  $|\phi_j^k\rangle = U_{k,E} |\psi_j\rangle$  and the phase can, for each input state, be identified by undoing the respective perturbation  $U_{k,E}$  via sufficient error correction.

layer is equivalent to a deeper structure proposed in Ref. [14], consisting of  $d$  interleaved convolutional and pooling layers [35,36]. The QCNN thus performs the same unitary with a substantially shorter circuit; see the Supplemental Material [41] for details. Moreover, all such pooling layers can be efficiently performed in classical postprocessing and only the initial convolutional layer  $U_{\text{C}}$  is executed on the quantum device.

The functionality of our QCNN is based on the fact that topological order is robust against local perturbations [18].

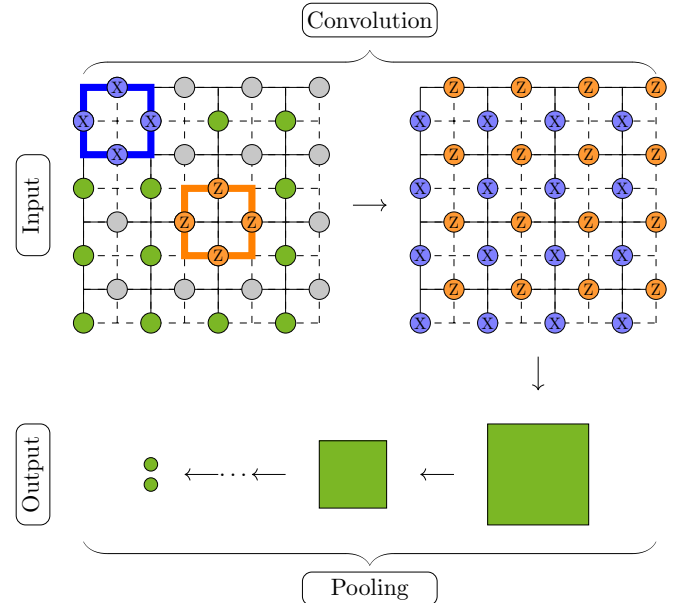


FIG. 2. QCNN structure. The first lattice on the input side shows the toric code lattice with dimension  $d = 4 \times 4$ , which is periodic in vertical and horizontal directions. The qubits on the respective sublattices are represented by gray (horizontal edges) and green circles (vertical edges). Examples of plaquette ( $A_p$ ) and vertex ( $B_v$ ) operators are highlighted in blue and orange. The circuit of the convolution maps the stabilizers of the input lattice to measurements on individual qubits of the respective colors; i.e., blue (green) qubits carry the measurement of the plaquette (vertex) stabilizer that is anchored on the qubits to their left. This convolution evolves error patterns on the input state and brings them into a form that can be corrected by the local operations in the pooling if the input was in the topological phase. Iterated pooling procedures reduce the lattice dimension (green squares) with every step until only two output qubits are left (green dots) and read out.

Hence, one can define characteristic states  $|\psi_i\rangle$  for each phase of the system and treat other states of the same phase as characteristic states masked by unitary perturbations  $|\phi_j^k\rangle = U_{k,E} |\psi_j\rangle$  [42], which we identify as errors. If one can recover the characteristic state by removing these errors, one has successfully identified the corresponding phase. In Fig. 1, we sketch such a phase space and the recovery of the reference state. The explicit structure of our QCNN can be found in the Supplemental Material [41].

At each layer  $l$ , we can generate an output by “measuring” the qubits in the computational basis,

$$M_{\text{QCNN}}^l = \frac{1}{N_l} \sum_{j=1}^{N_l} m_j, \quad (2)$$

where  $m_j \in \{-1, 1\}$  is the measurement outcome of qubit  $j$  and  $N_l$  is the number of qubits in layer  $l$ . The measurement outcomes  $m_j$  correspond to the values of plaquette and star operators on a lattice with the reduced system size  $N_l$ . The output  $M_{\text{QCNN}}^l$  thus measures how close the output state is to the error-free ground state of the toric code after having performed  $l$  pooling layers.

As the pooling layers are performed in classical postprocessing, their controlled Pauli gates are applied classically on the bit string of computational basis measurements for all qubits and the “measurements” correspond to simply recording the classical data. The support of the operator  $M_{\text{QCNN}}^l$  as defined in Eq. (2) will grow exponentially with the number of layers  $l$ . For our QCNN, we will obtain the following averages over a large number of samples:

$$\langle \text{QCNN} \rangle := \langle M_{\text{QCNN}}^d \rangle = \begin{cases} 1 & \text{if topological phase} \\ 0 & \text{if non-topological phase.} \end{cases} \quad (3)$$

We focus on the two-dimensional toric code [43] with periodic boundary conditions. It is a natural candidate for exploring QCNNs for two-dimensional systems as it shows intrinsic  $\mathbb{Z}_2$  topological order [43]. The toric code is a two-dimensional square lattice with the qubits located on its edges. For exploring its condensed matter physics, it can be described by the Hamiltonian

$$H_{\text{TC}} = - \sum_p A_p - \sum_v B_v, \quad (4)$$

with the plaquette operators  $A_p = \prod_{i=1}^4 X_i$  and the star operators  $B_v = \prod_{i=1}^4 Z_i$ , where  $X_i$  ( $Z_i$ ) is the Pauli- $X$  ( $Z$ ) operator on qubit  $i$  (see Fig. 2). Exposing the toric code to an external magnetic field gives rise to a rich phase diagram featuring the topologically ordered phase and a trivial phase, separated by first- or second-order phase transitions depending on the orientation of the field [44,45]. To explore these regimes, we modify Eq. (4) to include applied magnetic fields in  $X/Y/Z$  direction with strengths  $h_X$ ,  $h_Z$ , and  $h_Y$ ,

$$H_{\text{mag}} = H_{\text{TC}} - h_Z \sum_i Z_i - h_X \sum_i X_i - h_Y \sum_i Y_i. \quad (5)$$

The QCNN that we apply to (noisy) ground states of this model is sketched in Fig. 2 and its detailed description can be found in the Supplemental Material [41]. The QCNN we here propose is capable of detecting the topologically ordered

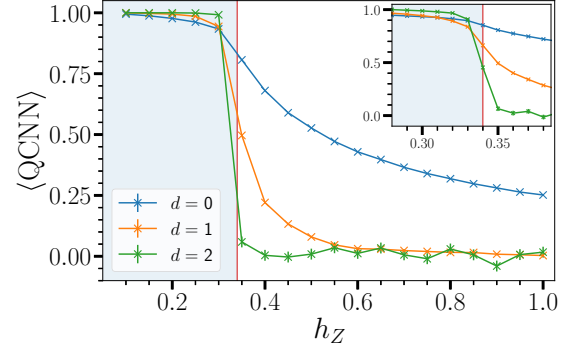


FIG. 3. QCNN output for a variation of the magnetic field strength  $h_Z$ , where  $h_X = 0$ , and for different depths  $d$  of the QCNN, corresponding to the number of pooling layers  $U_p^{(l)}$ . Each data point represents 2000 samples from the MPS representation of the corresponding ground state with the bond dimension  $\chi = 1250$  for an infinite cylinder with the periodic dimension  $l_2 = 9$ . The shaded region corresponds to the topologically ordered phase and the vertical line corresponds to critical magnetic field strength  $h_Z^* = 0.34$ . The inset shows the output for different samples with higher resolution around the phase transition, which are calculated with a bond dimension of  $\chi = 1500$ . For all samples, we observe increased steepness at the phase transition in the QCNN output with increasing QCNN depth corresponding to greater precision in the phase recognition.

phase with high accuracy. This means that the QCNN’s output converges to 1 with increasing depth  $d$  if the input state is in the topological phase and to 0 otherwise.

To evaluate its performance, we carried out several numerical simulations, where we use MPS simulations [46] to approximate ground states for large lattices [Eq. (5)] and classical simulations of Clifford gates on Pauli strings to explore the effects of incoherent Pauli noise. We show the technical details of our computations in the Supplemental Material [41].

We first test the QCNN on ground states of  $H_{\text{mag}}$  with a magnetic field in the  $Z$  direction,  $h_X = h_Y = 0$  in Eq. (5). Figure 3 shows the resulting phase diagram, where we consider field strengths  $0 \leq h_Z \leq 1$ . From iDMRG simulations for our cylindrical lattice, we determine the phase boundary at  $h_Z^* = 0.34$  as a peak in the second derivative  $d^2E/dh_Z^2$  of the ground-state energy with respect to  $h_Z$ , which we mark in the phase diagram. The plot of the QCNN output  $\langle M_{\text{QCNN}}^l \rangle$ , see Eq. (2), shows a transition at this expected point up to the sample resolution. Additionally, the sharpness of the transition increases with the layer number in the sense that the absolute value of the gradient of  $\langle M_{\text{QCNN}}^l \rangle$  increases, which makes the transition in the output more steplike with an increasing number of pooling layers, thus showing the successful recognition of the topological phase by the QCNN.

Furthermore, we test whether our QCNN is able to recognize the topological phase in the presence of incoherent noise. We do this by considering the toric code ground state perturbed by Pauli noise. Specifically, we apply random Pauli noise to each individual qubit in the form of the error channel

$$\rho = \mathcal{E}(|\psi\rangle \langle \psi|) = \sum_{l=0}^m K_l |\psi\rangle \langle \psi| K_l^\dagger, \quad (6)$$

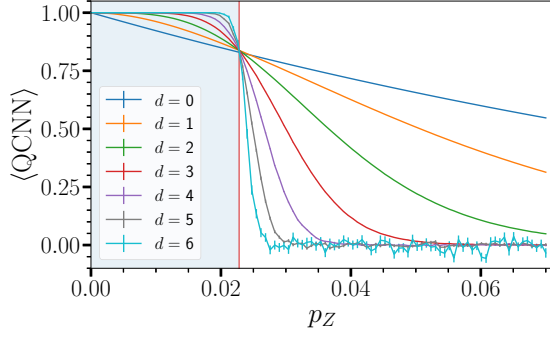


FIG. 4. QCNN output for incoherent Pauli noise perturbing the toric code ground state as in Eq. (6) with  $p_X = p_Y = 0$ . Via tracking the evolution of Pauli strings on the lattice under the convolution and the pooling layers, we can classically simulate the QCNN output for large system sizes. For this plot, we calculate the results for about 9.6 million qubits, which allows us to employ more pooling layers compared to the MPS samples. As for the case of the magnetic field, we find a transition in the QCNN output that becomes more pronounced with increasing depth of the QCNN. The red vertical line marks the error threshold of  $p_{\text{th}} = 2.28\%$ .

with  $K_l \in \{\sqrt{p_{\mathbb{I}}}\mathbb{I}, \sqrt{p_X}X, \sqrt{p_Y}Y, \sqrt{p_Z}Z\}^{\otimes N}$  to the input qubits, where  $p_E$  are probabilities of Pauli errors  $E = X, Y, Z$  and  $p_{\mathbb{I}} = 1 - p_X - p_Y - p_Z$ . As the QCNN only consists of Clifford and controlled Pauli gates, we can then subsequently track the evolution of the Pauli error strings on the lattice under the operations of the QCNN. It is sufficient to employ the following gate identities:  $X_j = H_j Z_j H_j$ ,  $\text{CNOT}_{i,j} X_i = X_i X_j \text{CNOT}_{i,j}$ , and  $\text{CNOT}_{i,j} Z_j = Z_i Z_j \text{CNOT}_{i,j}$ , where  $H_j$  is the Hadamard gate on qubit  $j$  and  $\text{CNOT}_{i,j}$  applies a Pauli  $X$  on qubit  $j$  conditioned on qubit  $i$ .

We present results for the QCNN output [Eq. (2)] for cases where the input state is the reference state of the topological phase, i.e., a ground state of  $H_{\text{TC}}$  as in Eq. (4), perturbed by Pauli-Z noise ( $p_X = p_Y = 0$ ) in Fig. 4, which shows a transition in the QCNN output over the Pauli-Z error probability  $p_Z$ . The QCNN output converges to the ideal noise-free value

$\langle M^d_{\text{QCNN}} \rangle = 1$  with the increasing number of pooling layers  $d$  below the threshold error probability  $p_{\text{th}} = 2.28\%$  (red vertical line). On the other hand, the output decreases to zero above this threshold. In the Supplemental Material [41], we also show results for simultaneous Pauli-Z and Pauli-X noise and demonstrate that the QCNN detects the phase transition at the critical strength  $h_Z^*$  of the magnetic field in the presence of noise.

We now discuss the key difference between the QCNN and approaches based on direct measurements and snapshot-based processing as in Ref. [37]. On most quantum processors, only measurements in local Pauli bases can be directly performed. In this case, plaquette operators  $A_p$  and star operators  $B_v$  cannot be simultaneously measured as they require readout in different Pauli bases. As a result, such direct measurements do not capture the correlations between these operators and only uncorrelated pooling, where the measurement outcomes of plaquette and star operators are processed separately, can be performed. In our QCNN, in contrast, all  $A_p$  and  $B_v$  operators are simultaneously measured after having applied the convolutional layer as a quantum circuit (see Fig. 2). Therefore, we can implement correlated pooling that takes into account correlations between these operators similarly to correlated quantum error correction [40]. See the Supplemental Material [41] for details about correlated pooling and its numerical simulation.

To demonstrate the advantage of our QCNN, we consider incoherent Pauli-Y errors that simultaneously flip the parity of two plaquette and two star operators, introducing correlations between them. The error threshold for Pauli-Y errors is considerably increased to 3.48% for correlated pooling compared to 2.28% for uncorrelated pooling (see the Supplemental Material [41]). This shows that the error correction capability is boosted by taking correlations into account.

In addition to incoherent Pauli-Y errors, we assume a coherent magnetic field in the  $Y$  direction that drives the toric code across a first-order phase transition into the trivial phase [45]. We plot the output for uncorrelated and correlated pooling in Figs. 5(a) and 5(b) for an error probability  $p_Y = 3\%$ .

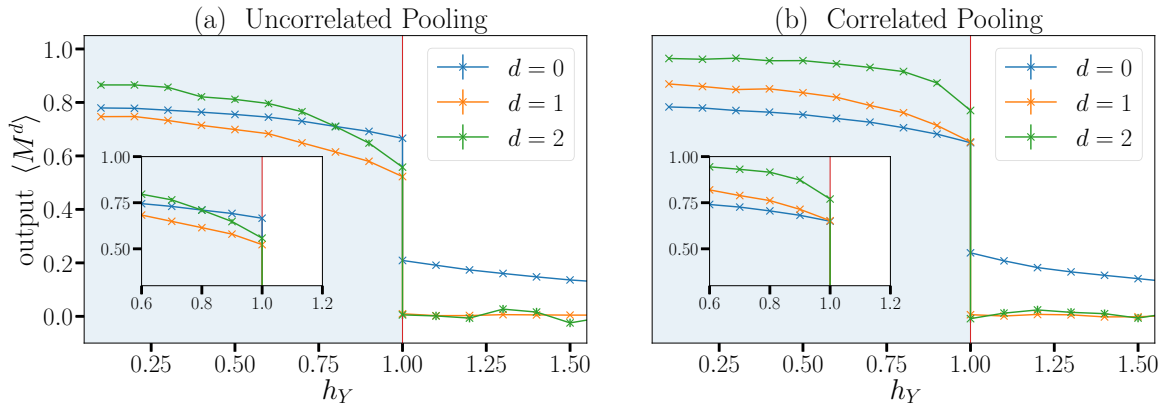


FIG. 5. Output of (a) uncorrelated pooling and (b) correlated pooling for the magnetic field strength  $h_Y$  in the presence of Pauli-Y noise with  $p_Y = 3\%$  and for different depths  $d$ . Each data point represents 2000 samples from the MPS representation of the corresponding ground state with (a) the bond dimension  $\chi = 1000$  and the periodic dimension  $l_2 = 9$ , and (b) the bond dimension  $\chi = 1200$  and the periodic dimension  $l_2 = 6$ . This comparison shows that the correlated pooling scheme can precisely detect the first-order phase transition (vertical red line), whereas topological states close the phase transition are misclassified for uncorrelated pooling (see the inset).

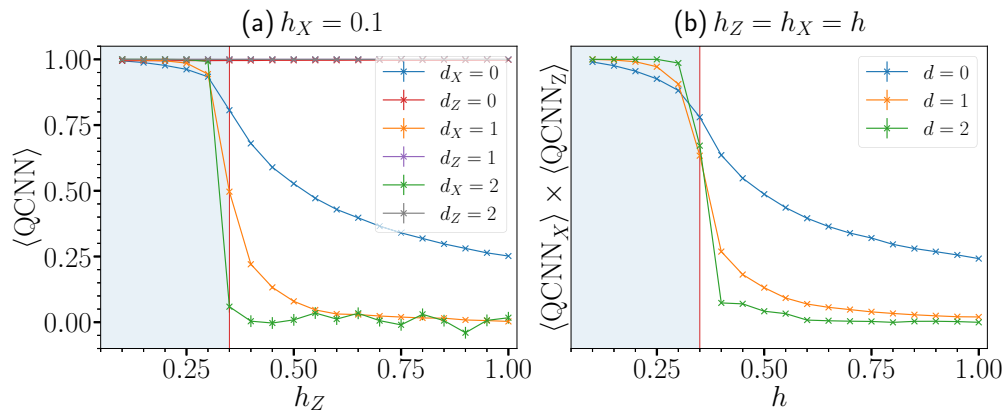


FIG. 6. (a) Output of the QCNN for the MPS samples under a variation of  $h_Z$  in the presence of a weak magnetic field in the  $X$  direction with  $h_X = 0.1$  with the bond dimension  $\chi = 1250$ . The output is displayed for simultaneous pooling in the  $X(Z)$  basis for the corresponding QCNN depth  $d_X(d_Z)$ . The phase transition for  $h_Z$  is correctly recognized in the presence of a field in the  $X$  direction. Furthermore, the output of the pooling layers  $d_Z$  quickly converges to unity due to the weak  $h_X = 0.1$ . (b) Detection of the multicritical point [47] in the presence of  $X$  and  $Z$  fields with  $h = h_X = h_Z$  for the bond dimension  $\chi = 1500$ . We take the product of the pooling in the  $X$  and  $Z$  bases for each QCNN layer to show the combined phase recognition along the multicritical line  $h_X = h_Z$ . A split plot can be found in Supplemental Material [41].

The output of the QCNN with correlated pooling is larger for all states in the topological phase than the output for uncorrelated pooling. Due to the limited ability to correct correlated errors, uncorrelated pooling misclassifies topological states close to the phase boundary as the output for  $d = 1, 2$  is smaller than for  $d = 0$  [see the inset of Fig. 5(a)]. In contrast, the QCNN with correlated pooling correctly recognizes all topological states. Hence, by exploiting correlations between measurement outcomes, the QCNN correctly recognizes the topological phase in the presence of strong correlated errors close to the phase transition, where classical approaches misclassify topological states.

Finally, we test the QCNN on ground states of  $H_{\text{mag}}$  with magnetic field simultaneously acting in both  $X$  and  $Z$  directions. First, we consider constant  $h_X = 0.1$  leading to a nonvanishing probability of error syndromes  $B_i = -1$  on vertices and thus  $M_{\text{QCNN}}^0 < 1$  [see Fig. 6(a)]. Second, we consider the transition into the trivial phase driven by perturbations in both  $X$  and  $Z$  directions for  $h_X = h_Z = h$  [47]. The QCNN output rapidly decreases with increasing depth  $d$  above the multicritical point  $h = 0.35$  [see Fig. 6(b)]. This provides clear evidence that the QCNN recognizes the phase transition also for these cases.

In conclusion, we generalized QCNNs to two-dimensional systems and showed that they recognize the topological phase of the toric code. The QCNN output is robust against errors below a threshold probability allowing for their realization on current quantum computers under NISQ conditions. In processing quantum states via the QCNN quantum circuit, correlations between all stabilizer elements of the toric code are taken into account. As a result, the threshold

probability for correlated errors is increased compared to classical approaches based on direct measurements as in Ref. [37]. Therefore, the QCNN correctly identifies the topological phase in the presence of strong correlated errors, where classical approaches misclassify topological states. Furthermore, all pooling layers can be efficiently performed in classical postprocessing to facilitate the QCNN implementation with restricted circuit depths. In future research, one can further investigate the error correction procedure in the pooling layers to increase the error threshold and enhance the phase detection by a faster convergence to a step function with the number of pooling layers. We expect that our work motivates the exploitation of QCNNs for the characterization of less-understood quantum phases of matter such as symmetry-enriched topological order [48] and quantum spin liquids [19]. Implementing the pooling layers as quantum circuits is an interesting possibility for characterizing such complex systems with reduced sample complexity, and allowing for the parametrization and training of the QCNNs [33,34,49].

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**Data availability.** The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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