

Erratum: Electron beams traversing spherical nanoparticles: Analytic and numerical treatment **[Phys. Rev. Research 6, 013239 (2024)]**

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We report an error in the formulas corresponding to the calculation of the electron energy-loss (EEL) probability presented in our original paper. In Sec. II A, we present the decomposition of the total EEL probability into three terms: the bulk, the surface, and the Begrenzung term. We point out an error in Eq. (7c) corresponding to the Begrenzung term, which should read

$$\Gamma_{\text{Begr}}(\omega) = \frac{e}{\pi \hbar \omega} \text{Re} \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{+\ell} \int_{-z_e}^{z_e} dz \frac{e^{-i\omega z/v}}{\sqrt{\ell(\ell+1)}} \left[mb_{\ell m}^{\text{I}} j_{\ell}(kr) Y_{\ell}^m(\theta, 0) - \frac{a_{\ell m}^{\text{I}}}{kb} \{ \mathcal{J}_{\ell m}^{+}(kz) + \mathcal{J}_{\ell m}^{-}(kz) \} \right. \\ \left. - mb_{\ell m}^{0,\text{II}} j_{\ell}(k_0 r) Y_{\ell}^m(\theta, 0) + \frac{a_{\ell m}^{0,\text{II}}}{k_0 b} \{ \mathcal{J}_{\ell m}^{+}(k_0 z) + \mathcal{J}_{\ell m}^{-}(k_0 z) \} \right]. \quad (1)$$

We further note an error in Eqs. (D8a) and (D8c) concerning the expansion coefficients entering the expressions for the cathodoluminescence (CL) and EEL probability, which should read as follows:

$$a_{\ell m}^{\text{I}} = T_{\text{El}}^{11} a_{\ell m}^{0,\text{I}} + T_{\text{El}}^{21} a_{\ell m}^{0,\text{II}}, \quad (2a)$$

$$a_{\ell m}^{\text{II}} = T_{\text{El}}^{12} a_{\ell m}^{0,\text{I}} + T_{\text{El}}^{22} a_{\ell m}^{0,\text{II}} - a_{\ell m}^{0,\text{I}}|_{\text{air}}, \quad (2b)$$

$$b_{\ell m}^{\text{I}} = T_{\text{Me}}^{11} b_{\ell m}^{0,\text{I}} + T_{\text{Me}}^{21} b_{\ell m}^{0,\text{II}}, \quad (2c)$$

$$b_{\ell m}^{\text{II}} = T_{\text{Me}}^{12} b_{\ell m}^{0,\text{I}} + T_{\text{Me}}^{22} b_{\ell m}^{0,\text{II}} - b_{\ell m}^{0,\text{I}}|_{\text{air}}, \quad (2d)$$

where the notation $a/b_{\ell m}^{0,\text{I}}|_{\text{air}}$ indicates evaluation of the terms given by Eq. (C4) in air ($\varepsilon = 1$, $k = k_0$). This correction ensures a zero EEL probability in the absence of the structure.

In what follows, we present a more rigorous formulation of the derivation outlined in Appendix D 1 of the original paper, providing a detailed account of how the above expressions are obtained. The EEL probability is given by Eq. (D1), as

$$\Gamma_{\text{EEL}}(\omega) = \frac{e}{\pi \hbar \omega} \int_{-\infty}^{+\infty} dz \text{Re} \{ e^{-i\omega z/v} \hat{\mathbf{z}} \cdot \mathbf{E}_{\text{ind}}(\mathbf{r}_e, \omega) \}. \quad (3)$$

The induced field can be found as the total field in the presence of the structure [Eq. (C5)] minus the total field $\mathbf{E}_{\text{tot}}|_{\text{air}}$ in the absence of the structure, given by the same expressions, evaluated in air with $\varepsilon = 1$ and $k = k_0$. Eventually, in the region outside (II) and inside (I) the nanosphere, the induced field is $\mathbf{E}_{\text{ind}}^{\text{II}} = \mathbf{E}_{\text{B}}^{\text{II}} - \mathbf{E}_{\text{B}}^{\text{II}}|_{\text{air}}$ and $\mathbf{E}_{\text{ind}}^{\text{I}} = \mathbf{E}_0^{\text{I}} + \mathbf{E}_{\text{B}}^{\text{I}} - \mathbf{E}_0^{\text{I}}|_{\text{air}} - \mathbf{E}_{\text{B}}^{\text{I}}|_{\text{air}}$, accordingly, and Eq. (3) yields

$$\Gamma_{\text{EEL}}(\omega) = \frac{e}{\pi \hbar \omega} \text{Re} \left[\int_{-\infty}^{+\infty} dz e^{-i\omega z/v} \hat{\mathbf{z}} \cdot \mathbf{E}_{\text{ind}}^{\text{II}}(\mathbf{r}_e, \omega) - \int_{-z_e}^{z_e} dz e^{-i\omega z/v} \hat{\mathbf{z}} \cdot \mathbf{E}_{\text{ind}}^{\text{II}}(\mathbf{r}_e, \omega) + \int_{-z_e}^{z_e} dz e^{-i\omega z/v} \hat{\mathbf{z}} \cdot \mathbf{E}_{\text{ind}}^{\text{I}}(\mathbf{r}_e, \omega) \right] \\ = \frac{e}{\pi \hbar \omega} \text{Re} \left[\underbrace{\int_{-\infty}^{+\infty} dz e^{-i\omega z/v} \hat{\mathbf{z}} \cdot \{ \mathbf{E}_{\text{B}}^{\text{II}}(\mathbf{r}_e, \omega) - \mathbf{E}_{\text{B}}^{\text{II}}|_{\text{air}}(\mathbf{r}_e, \omega) \}}_{\Gamma_{\text{surf}}} - \underbrace{\int_{-z_e}^{z_e} dz e^{-i\omega z/v} \hat{\mathbf{z}} \cdot \{ \mathbf{E}_{\text{B}}^{\text{II}}(\mathbf{r}_e, \omega) - \mathbf{E}_{\text{B}}^{\text{II}}|_{\text{air}}(\mathbf{r}_e, \omega) \}}_{\Gamma_{\text{Begr}}} \right. \\ \left. + \underbrace{\int_{-z_e}^{z_e} dz e^{-i\omega z/v} \hat{\mathbf{z}} \cdot \{ \mathbf{E}_{\text{B}}^{\text{I}}(\mathbf{r}_e, \omega) - \mathbf{E}_{\text{B}}^{\text{I}}|_{\text{air}}(\mathbf{r}_e, \omega) \}}_{\Gamma_{\text{Begr}}} + \underbrace{\int_{-z_e}^{z_e} dz e^{-i\omega z/v} \hat{\mathbf{z}} \cdot \{ \mathbf{E}_0^{\text{I}}(\mathbf{r}_e, \omega) - \mathbf{E}_0^{\text{I}}|_{\text{air}}(\mathbf{r}_e, \omega) \}}_{\Gamma_{\text{bulk}}} \right] \\ = \Gamma_{\text{surf}} + \Gamma_{\text{Begr}} + \Gamma_{\text{bulk}}. \quad (4)$$

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To derive the bulk contribution, we use Eq. (B2) in the paper, which determines the z component of the electric field generated by the fast electron traveling a distance of $2z_e$ within an infinite medium of relative permittivity ε and relative permeability $\mu = 1$. The corresponding EEL probability $\Gamma_{\text{bulk}}(\omega)$ can be found via the momentum-resolved EEL probability $P_{\text{EEL}}(q, \omega)$ of Eq. (D4), as

$$P_{\text{EEL}}(q, \omega) = -\frac{e^2 q z_e}{\pi^2 \hbar \varepsilon_0 v^2} \text{Im} \left\{ \frac{1}{[q^2 - k^2 + (\omega/v)^2] \gamma^2 \varepsilon} - \frac{1}{[q^2 - k_0^2 + (\omega/v)^2] \gamma_0^2} \right\}. \quad (5)$$

The second term inside the curly brackets is real-valued and can thus be omitted. Then, the EEL probability $\Gamma_{\text{EEL}}(\omega)$ is obtained by

$$\Gamma_{\text{EEL}}(\omega) = \int_0^{q_c} dq P_{\text{EEL}}(q, \omega), \quad (6)$$

leading to

$$\Gamma_{\text{bulk}}(\omega) = -\frac{e^2 z_e}{2\pi^2 \varepsilon_0 \hbar v^2} \text{Im} \left\{ \frac{1}{\gamma^2 \varepsilon} \ln \left(\left[\frac{q_c \gamma v}{\omega} \right]^2 + 1 \right) \right\}. \quad (7)$$

This is the same as Eq. (7a) in the original paper, except that the first term in the curly brackets has been omitted, as it is identically zero.

To derive the remaining terms in Eq. (4), we insert the fields of Eq. (C5) and obtain

$$\Gamma_{\text{EEL}} = \Gamma_{\text{surf}}^{\text{M}} + \Gamma_{\text{surf}}^{\text{E}} + \Gamma_{\text{Begr}}^{\text{M}} + \Gamma_{\text{Begr}}^{\text{E}} + \Gamma_{\text{bulk}}, \quad (8)$$

where we have further decomposed the surface and Begrenzung terms into contributions from electric- (E) and magnetic-type (M) multipoles of the form

$$\Gamma_{\text{surf}}^{\text{M}}(\omega) = \frac{e}{\pi \hbar \omega} \text{Re} \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{+\ell} \frac{mb_{\ell m}^{\text{II}} e^{im\phi_0}}{\sqrt{\ell(\ell+1)}} \left[\frac{\mathcal{M}_{\ell m}^* K_m \left(\frac{\omega b}{v \gamma_0} \right)}{ik_0} - \int_{-z_e}^{z_e} dz e^{-i\omega z/v} h_{\ell}^+(k_0 r) Y_{\ell}^m(\theta, 0) \right], \quad (9a)$$

$$\Gamma_{\text{surf}}^{\text{E}}(\omega) = -\frac{e}{\pi \hbar \omega} \text{Re} \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{+\ell} \frac{a_{\ell m}^{\text{II}} e^{im\phi_0}}{\sqrt{\ell(\ell+1)}} \left[\frac{\mathcal{N}_{\ell m}^* K_m \left(\frac{\omega b}{v \gamma_0} \right)}{ik_0 \beta \gamma_0} - \frac{1}{k_0 b} \int_{-z_e}^{z_e} dz e^{-i\omega z/v} \{ \mathcal{H}_{\ell m}^+(k_0 z) + \mathcal{H}_{\ell m}^-(k_0 z) \} \right], \quad (9b)$$

$$\Gamma_{\text{Begr}}^{\text{M}}(\omega) = \frac{e}{\pi \hbar \omega} \text{Re} \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{+\ell} \frac{me^{im\phi_0}}{\sqrt{\ell(\ell+1)}} \left[b_{\ell m}^{\text{I}} \int_{-z_e}^{z_e} dz e^{-i\omega z/v} j_{\ell}(kr) Y_{\ell}^m(\theta, 0) - b_{\ell m}^{0, \text{II}} \int_{-z_e}^{z_e} dz e^{-i\omega z/v} j_{\ell}(k_0 r) Y_{\ell}^m(\theta, 0) \right], \quad (9c)$$

$$\Gamma_{\text{Begr}}^{\text{E}}(\omega) = -\frac{e}{\pi \hbar \omega} \text{Re} \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{+\ell} \frac{e^{im\phi_0}}{\sqrt{\ell(\ell+1)}} \left[\frac{a_{\ell m}^{\text{I}}}{kb} \int_{-z_e}^{z_e} dz e^{-i\omega z/v} \{ \mathcal{J}_{\ell m}^+(kz) + \mathcal{J}_{\ell m}^-(kz) \} \right. \\ \left. - \frac{a_{\ell m}^{0, \text{II}}}{k_0 b} \int_{-z_e}^{z_e} dz e^{-i\omega z/v} \{ \mathcal{J}_{\ell m}^+(k_0 z) + \mathcal{J}_{\ell m}^-(k_0 z) \} \right], \quad (9d)$$

with $\mathcal{H}_{\ell m}^{\pm}(k_0 z)$, $\mathcal{J}_{\ell m}^{\pm}(kz)$, and $\mathcal{J}_{\ell m}^{\pm}(k_0 z)$ given by Eq. (C2) in the original paper. We note here that the exponential term $e^{im\phi_0}$ in the formulas above need not be taken into account, since it is canceled out by the factor $e^{-im\phi_0}$ included in the expansion coefficients $a/b_{\ell m}^{0, \text{I/II}}$ of Eqs. (2). For notational simplicity, in Eqs. (9a) and (9b) we have redefined the expansion coefficients that emerged from the interface condition problem as

$$a_{\ell m}^{\text{II}} = T_{\text{E}\ell}^{12} a_{\ell m}^{0, \text{I}} + T_{\text{E}\ell}^{22} a_{\ell m}^{0, \text{II}}, \quad (10a)$$

$$b_{\ell m}^{\text{II}} = T_{\text{M}\ell}^{12} b_{\ell m}^{0, \text{I}} + T_{\text{M}\ell}^{22} b_{\ell m}^{0, \text{II}}, \quad (10b)$$

as those in Eqs. (2b) and (2d). The same modified expansion coefficients can be used to compute the CL probability, using the formula

$$\Gamma_{\text{CL}}(\omega) = \frac{1}{\pi \hbar \omega Z_0 k_0^2} \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{+\ell} [|b_{\ell m}^{\text{II}}|^2 + |a_{\ell m}^{\text{II}}|^2]. \quad (11)$$

Equation (11) with the modified expansion coefficients of Eqs. (2b) and (2d) agrees with the respective probability derived in Ref. [80] of the paper.

We note that all figures in the original publication were generated using the correct formulas, as presented here. Consequently, the reported errors do not affect the findings of our work.