

Experimental evidence for invariant solutions mediating transitions from metastable turbulent regimes

B. Vishnu Kiran , Rajath Kashyap , and Balachandra Suri ^{*}

Department of Mechanical Engineering, *Indian Institute of Science, Bengaluru 560012, India*



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We present direct experimental evidence for turbulence visiting the neighborhoods of simple invariant solutions in phase space, immediately preceding and during rapid transitions between metastable flow regimes characterized by opposite large-scale circulations. Specifically, we identify the dynamical signature of an unstable time-periodic orbit that foreshadows these transitions, as well as brief visits to the neighborhoods of unstable equilibria during the transitions. Through direct numerical simulation of the flow, we show that these equilibria are edge states that reside on the boundary separating the two metastable regimes in phase space. Finally, recognizing turbulence approaching the unstable periodic orbit as a precursor, we predict and suppress imminent regime transitions through real-time closed-loop control in experiments.

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Introduction. An intriguing yet inadequately understood phenomenon in turbulence—and several other natural and human-made spatiotemporally complex systems—is their rapid and seemingly random transition between dynamically and/or statistically distinct regimes. Examples include extreme climatic events [1,2], geophysical dynamo reversals [3,4], zonal flow transitions [5], atmospheric blocking events [6,7], and relaminarization of turbulent flows [8–12]. Such rare transitions characterized by substantial changes in a dynamical system’s long-lived behavior are also classified as extreme events. Scientific understanding of why regime transitions occur, and whether they can be predicted and controlled when required, is quite nascent in high-dimensional complex systems [13–15], such as turbulence.

Sudden transitions between distinct flow regimes have previously been investigated in diverse canonical hydrodynamic systems [4,16–23]. In highly turbulent or stochastically driven flows, such events are rationalized as noise-induced transitions between different energy “wells” representing distinct metastable turbulent regimes [5,24–27]. While this mechanism has also been invoked to rationalize transitions in moderately turbulent flows driven by large-scale forcing, its appropriateness for such flows remains unclear, since velocity fluctuations (i.e., noise levels) are often much weaker in strength compared to the slow-evolving, energetically dominant large-scale flow features [28]. The framework of deterministic dynamical systems has recently enabled systematic numerical exploration of regime changes in transitional (i.e., moderately turbulent) flows [9,15,17,29,30]. It is this approach we employ to understand the appearance of char-

acteristic flow patterns enroute to regime transitions in a moderately turbulent laboratory flow.

Recent studies have demonstrated that coarse tiling the high-dimensional phase space of transitional turbulence using unstable invariant solutions, such as equilibria (EQ) [29,31,32], periodic orbits [33–36], and two-tori [37,38], offers profound insights into the dynamics and statistics of turbulent flows [20,33,36,39]. Invariant solutions, which cohabit the phase space with turbulence, are fleetingly visited by turbulent trajectories, which makes them dynamically relevant [40]. Furthermore, the unstable and stable manifolds of such solutions steer turbulent trajectories between different regions (even far separated) in the phase space, analogous to a network of highways that routes traffic between distant cities [17,37,41,42]. Consequently, scientists have long conjectured that invariant solutions could potentially serve as waypoints to track dramatic transitions between distinct flow regimes [7,43].

Prior numerical investigations of turbulent canonical flows have indeed demonstrated that laminar $\leftrightarrow$ turbulent flow transitions shadow select dynamical connections between invariant solutions far separated in phase space [17,29,37,44]. Dynamical signatures of such solutions have also been identified during transitions between symmetry-related turbulent metastable regimes in numerics [15,30,45]. Experimental evidence for turbulence approaching invariant solutions directly before and during regime transitions, however, is very scarce. The observation of a relative-equilibrium solution embedded within the laminar-turbulent boundary in pipe flow is an example [11]. Also, whether invariant solutions enable preemptive control of regime transitions in laboratory experiments has not been explored to date.

In this Letter, we provide clear experimental evidence for turbulence visiting neighborhoods of invariant solutions directly preceding and during regime transitions. Specifically, we focus on seemingly random reversals of metastable large-scale circulations (LSCs) in a moderately turbulent quasi-two-dimensional (Q2D) flow [15,16,21,28,46]. We

^{*}Contact author: balachandra@iisc.ac.in

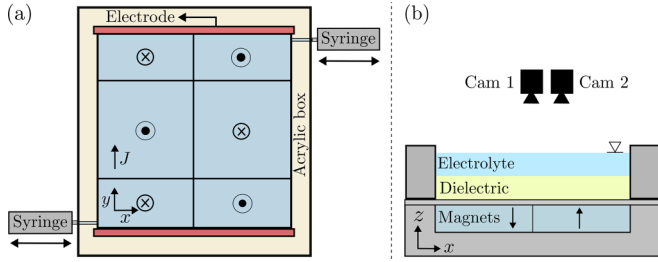


FIG. 1. Experimental setup employed to generate electromagnetically driven Q2D turbulent flow that undergoes regime transitions. (a) Top view illustrating the flow domain, direct current through the electrodes, and polarity of the magnets beneath. Flow control is achieved by actuating syringes filled with the electrolyte. (b) Side view showcasing the fluid bilayer configuration and camera setup.

show that dynamical signatures of an unstable periodic orbit (UPO) foreshadow such transitions. We then identify that, during transitions, turbulence approaches invariant solutions that reside on the boundary separating the metastable regimes. Finally, tracking the turbulent flow approaching a UPO in real time, we mitigate imminent regime transitions in the laboratory experiment using closed-loop control.

Experiments and direct numerical simulations. Quasi-two-dimensional LSCs in the laboratory are generated in a 100 mm $\times$ 100 mm $\times$ 10 mm flat-bottomed container, with its longitudinal and transverse walls made from copper and acrylic bars, respectively [see Fig. 1(a)]. A stably stratified 6.5 mm-thick bilayer, composed of a 2.5 mm-thick dielectric layer beneath a 4 mm-thick electrolyte layer, is realized in this container [cf. Fig. 1(b)]. A chessboardlike array of permanent magnets placed beneath the flow domain generates magnetic field directed predominantly perpendicular to the fluid layers. A steady electric current $\mathbf{J} = J \hat{\mathbf{y}}$ injected into the electrolyte through the copper bars interacts with this vertical magnetic field $\mathbf{B}$, resulting in a Lorentz force $\mathbf{F} \approx \mathbf{J} \times \mathbf{B} \propto e^{-\sqrt{2}\pi z/W} \sin(\pi x/W) \cos(\pi y/W) \hat{\mathbf{x}}$ that drives a nearly horizontal Q2D flow [47]. Here, $W = 50 \pm 0.5$ mm is the width of each magnet. The strength of electromagnetic forcing determines the spatiotemporally averaged root-mean-square (rms) velocity U , based on which we define flow Reynolds number as $Re = UW/\nu$, where $\nu = 1.2 \text{ mm}^2/\text{s}$ is the viscosity of the electrolyte. We note that U and other spatiotemporal averages are estimated only after the flow attains a statistically steady state, at least 20 min after turning the electric current on.

The acrylic side walls contain two millimetric through holes at a height of 4.5 mm above the bottom surface, at locations shown in Fig. 1(a). These holes are connected to electrolyte-filled syringes that can be actuated to provide external perturbations to the electromagnetically driven flow. The electrolyte-dielectric interface is seeded with glass microspheres and their positions are tracked using two digital cameras positioned side-by-side above the flow domain. Images from one camera are stored for a posteriori reconstruction of spatiotemporally finely resolved velocity fields using particle image velocimetry (PIV) [48]. Images from the second camera are processed on the fly using PIV with lower—but sufficient—spatiotemporal resolution, to enable real-time flow analysis and control.

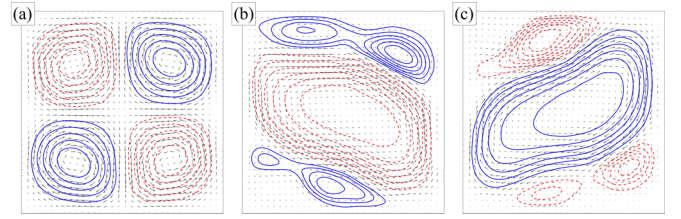


FIG. 2. Experimentally measured (a) laminar flow at $Re = 212$ and turbulent flows at $Re = 650$ characterized by (b) counterclockwise and (c) clockwise LSCs. Gray arrows represent instantaneous local velocity. Solid (blue) and dashed (red) curves represent clockwise and counterclockwise streamlines, respectively.

The Q2D laboratory flow is theoretically modeled using a variant of the 2D Navier-Stokes equation with an additional linear friction term [16,49]:

$$\partial_t \mathbf{u} + \beta \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} (\nabla^2 \mathbf{u} - \gamma \mathbf{u}) + \mathbf{f}. \quad (1)$$

Here, $\mathbf{u} = u(x, y, t) \hat{\mathbf{x}} + v(x, y, t) \hat{\mathbf{y}}$ represents an incompressible ($\nabla \cdot \mathbf{u} = 0$) field that corresponds to the velocity at the electrolyte-dielectric interface in the experiment, $p(x, y, t)$ is the 2D kinematic pressure, and $\mathbf{f}(x, y)$ is the normalized depth-averaged forcing field. Parameters $\beta = 1.3$ and $\gamma = 241.8$ model the Q2D-ness of flow in the experiment [49,50]. Direct numerical simulation (DNS) of flow governed by Eq. (1) is performed using a second-order finite difference method, imposing zero-velocity boundary conditions and incorporating a numerical model for $\mathbf{f}$ induced by a homogeneously magnetized chessboardlike array of magnets [15,33,37,51,52]. Unstable equilibrium and time-periodic solutions of Eq. (1) are computed using a Newton-Krylov solver [53], such that $\partial_t \mathbf{u} = 0$ for EQs and $\mathbf{u}(t) = \mathbf{u}(t + T)$ for UPOs, where $T > 0$ is the (shortest) period of a solution. UPOs computed at $Re = 898$ as part of an earlier study [15] constitute the library of solutions we search for in the experiments. More details about the experiment and DNS are included in the Supplemental Material [54].

Results. Laminar flow in the experiment is composed of a 2×2 lattice of counter-rotating vortices, as shown in Fig. 2(a) [15,16,21]. This four-vortex flow is approximately invariant under reflections (R_x, R_y) in the horizontal and vertical axes passing through the domain center, as well as under rotation by π ($R_\pi = R_x R_y$) about the center. Deviation of the laminar flow from exact invariance, estimated as $\|\mathbf{u} - g\mathbf{u}\|/\|2\mathbf{u}\|$ for $g \in \{R_x, R_y, R_\pi\}$, is small (≈ 0.04 at $Re = 212$). Here, $\|\cdots\|$ represents spatial rms average. As Re increases, the flow undergoes a sequence of instabilities, ultimately transitioning to turbulence or spatiotemporal chaos (at $Re \lesssim 650$) characterized by asymmetric large-scale circulations, as shown in Figs. 2(b) and 2(c).

In the DNS, Eq. (1) is exactly equivariant under R_x, R_y , as well as R_π . Consequently, the flow converges equiprobably to opposite LSCs just above the onset of turbulence when initialized with different white noise realizations. The flow in the experiment, however, converges consistently to turbulent counterclockwise LSCs when the fluid layers are driven directly out of equilibrium. This bias arises from minor experimental imperfections, e.g., asymmetry in the magnetic field.

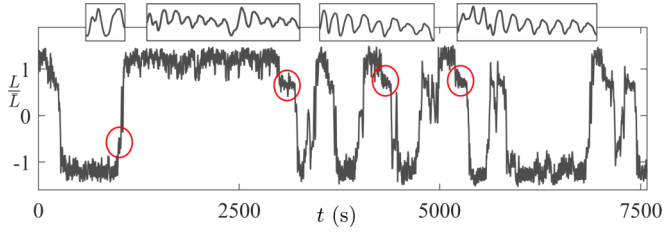


FIG. 3. Time series of normalized angular momentum L characterizing transitions between metastable turbulent regimes with opposite LSCs in the laboratory experiment. Intervals preceding these transitions highlighted in red showcase near-periodic dynamics (see insets), the signatures of UPOs.

Nevertheless, gently stirring the fluids clockwise during the initial spin-up phase drives the flow toward sustained turbulent clockwise LSCs. Thus, slightly above the onset of turbulence, two attracting regimes corresponding to turbulent LSCs with opposite senses of rotation are born. The time-averaged flow in each regime is approximately R_π invariant, while the reflection symmetries R_x and R_y are strongly broken. These broken symmetries approximately map the time-averaged flows, as well as the two turbulent regimes, onto each other (Fig. S1 in the Supplemental Material [54]).

Further increase in Re renders these regimes unstable, with the turbulent flow intermittently switching between opposite LSCs. We characterize this dynamical behavior using the time series of angular momentum $L = \langle xv - yu \rangle$, where $\langle \dots \rangle$ denotes spatial average [15,18]. A time series of L at $Re = 895$, normalized with the temporal mean $\bar{L}$ of its magnitude, is shown in Fig. 3. Here, temporal windows with $L \in (0.5, 1.5)\bar{L}$ and $L \in (-1.5, -0.5)\bar{L}$ represent turbulence exploring regimes characterized by counterclockwise and clockwise LSCs, respectively. Figure 3 illustrates that the two regimes are *metastable*, i.e., turbulence resides within each regime for a long duration between transitions, which themselves take a relatively short time [$O(10)$ s]. In the following, we explore whether signatures of invariant solutions manifest directly before and during such rapid regime transitions.

Close inspection of the time series in Fig. 3 reveals that L exhibits oscillations (highlighted with red circles) with a period of 14 ± 2 s for a brief duration preceding transitions from metastable regimes. This behavior is particularly pronounced just before transitions from counterclockwise to clockwise LSCs, and somewhat weaker preceding the reverse transitions. These near-periodic episodes, interrupting an otherwise chaotic evolution, are signatures of turbulent trajectories fleetingly shadowing UPOs in phase space [17,33,34]. Unlike turbulence—represented by a winding trajectory traversing the phase space—UPOs correspond to closed loops and a hierarchy of UPOs form the backbone of turbulent dynamics [33,55–57]. When a turbulent trajectory visits the neighborhood of a UPO, the flow exhibits almost periodic temporal dynamics, mimicking the spatiotemporal evolution of the nearby UPO.

To test if turbulence in the experiment visits the neighborhood of any UPO labeled PO_n for $n \in \{1, 2, \dots\}$ during these oscillatory precursors, we compute the distance between the instantaneous turbulent flow $\mathbf{u}(t)$ and PO_n as

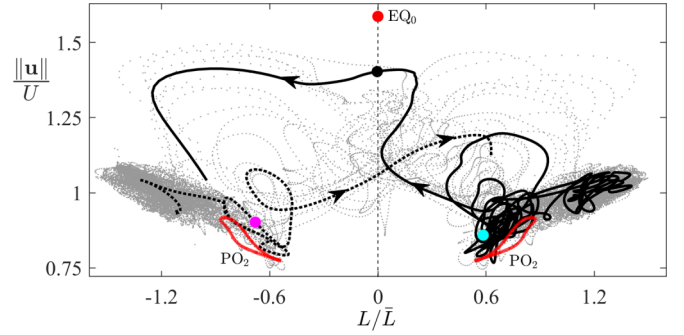


FIG. 4. Visualizing regime transitions in turbulent phase space using normalized $\|\mathbf{u}(t)\|$ vs $L(t)$ plot. Gray markers indicate turbulent flow states in the experiment, black curves represent turbulent trajectories, and red loops represent PO_2 and R_x -transformed PO_2 . Turbulent trajectories undergoing transitions between the two metastable regimes (gray clusters) approach PO_2 and then EQ_0 (red dot). Turbulent flow fields corresponding to cyan and black dots are shown in Fig. 5.

$D_n(t) = \min_{g, t'} \|\mathbf{u}(t) - g\mathbf{u}_{po}^n(t')\| / \|\mathbf{u}_{po}^n(t')\|$. $D_n(t)$ being small (ideally $D_n \approx 0$) suggests that the turbulent trajectory in phase space is near PO_n (or one of its symmetry-related copies) at the instant t . Remarkably, we find that a particular orbit PO_2 with a period $T = 14.07$ s is consistently visited just before regime transitions in the experiment. During such precursory visits, the distance D_2 drops noticeably compared to its temporal mean of $\bar{D}_2 \approx 0.6$, with $\min_t D_2 \approx 0.16$ (Fig. S2 in the Supplemental Material [54]).

To visualize turbulence visiting neighborhoods of invariant solutions, we construct a two-dimensional projection of the phase space in Fig. 4, by plotting the normalized instantaneous rms velocity $\|\mathbf{u}(t)\|/U$ versus the angular momentum $L(t)/\bar{L}$. Turbulent flow at each instant in the experiment maps to a unique point (gray marker) in Fig. 4 and turbulent evolution maps to a winding trajectory (black curves) [29]. PO_2 and its reflection-symmetry-related copy are represented as red loops in the left and right halves of this projection. For the most part, turbulent trajectories explore the metastable regimes characterized by opposite LSCs represented by the densely populated gray regions around $(\pm 1.2, 1)$ in this projection. However, just before transitioning from one regime to the other, turbulent trajectories approach PO_2 , as illustrated in Fig. 4. Here, the dashed and solid black trajectories departing from the clockwise ($L < 0$) and counterclockwise ($L > 0$) regimes are chosen from the first and second insets highlighted in Fig. 3. The turbulent flow at the point highlighted using the cyan marker in Fig. 4 and the nearest snapshot along PO_2 are shown in Figs. 5(a) and 5(b), respectively. The distance between these two flow states is $D_2 \approx 0.21$. Analogous comparison for the state marked in magenta in Fig. 4 is shown in the Supplemental Material [54].

In the DNS of flow governed by Eq. (1) and equivariant under R_x , R_y , and R_π , flow states that map onto the $L = 0$ line in Fig. 4 reside on the boundary separating the two metastable regimes [58–61]. An R_π -invariant UPO (labeled PO_1) with just one unstable (real) direction resides on this boundary and its stable manifold constitutes the “edge” of the metastable regimes [15]. In the experiment, although PO_1 is approached

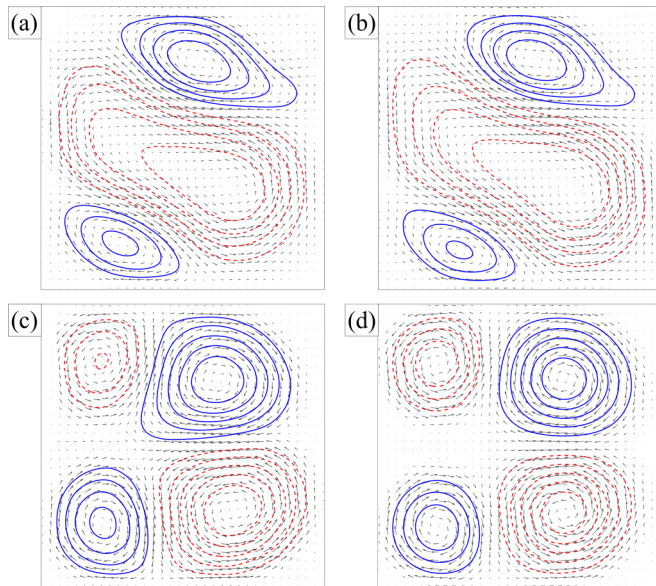


FIG. 5. Experimental flow snapshots [panels (a), (c)] at instants turbulence visits neighborhoods of invariant solutions (b) PO_2 and (d) EQ_0 computed in DNS. Snapshots (a) and (c) are marked using cyan and black markers, respectively, in Fig. 4.

during some regime transitions, its distance D_1 to the turbulent flow remains greater than 0.35 at all times (Fig. S2 in the Supplemental Material [54]). This raises the question of whether other invariant solutions that reside on the edge of metastable LSCs are approached more closely during regime transitions in the experiment.

To answer this question, let us recall that symmetries R_x , R_y , and R_π are weakly broken in the experiment and, consequently, the boundary separating the two metastable regimes should deviate from the $L = 0$ line in Fig. 4. We find that, some turbulent trajectories undergoing transitions in the experiment remain close to the $L = 0$ line, e.g., the solid black curve in Fig. 4, which leads us to hypothesize that they are possibly shadowing the perturbed boundary between the metastable regimes [11,60]. Visual inspection of the turbulent flow from such episodes reveals a characteristic flow pattern that is nearly R_y invariant, as shown in Fig. 5(c). A Newton-Krylov solver initialized directly with this experimental flow field converged to an R_y -invariant equilibrium solution EQ_0 [cf. Fig. 5(d)] that resides on the edge of metastable turbulent LSCs by virtue of its symmetry. Flow snapshots in Figs. 5(c) and 5(d) are marked using black and red markers, respectively, in Fig. 4. The striking similarity between the experimental flow and EQ_0 confirms that turbulence approaches this invariant solution during the regime transition shown in Fig. 4. The distance between these states in the phase space, quantified as $D_0(t) = \min_g \|\mathbf{u}(t) - g\mathbf{u}_{eq}\| / \|\mathbf{u}_{eq}\|$ is $D_0 \approx 0.3$ [32], with $\min_t D_0 \approx 0.25$ for the time series in Fig. 3. Interestingly, we find that R_x -transformed EQ_0 , which also resides on the edge of metastable LSCs, is approached in the experiment during some transitions from clockwise to counterclockwise regimes. This observation aligns with the fact that forward and backward transition routes connecting the two metastable regimes should be related (approximately) via reflection symmetries.

Finally, DNS of the flow shows that EQ_0 (unlike PO_1) is not a local attractor on the edge of metastable regimes. A large portion of its unstable manifold lies on the edge, with the rest directed into either metastable regime [54,60,62].

Experimental evidence indicating that turbulence approaches invariant solutions just prior to regime transitions raises the question of whether these precursory signatures can be harnessed to forecast and control such transitions—a possibility long anticipated since the discovery of invariant solutions in experiments [43,63–66]. In the present study, we quantify flow energetics during regime transitions to motivate the need for controlling their occurrence. Figure 4 shows that spatial rms velocity of the turbulent flow can increase by nearly 50% during regime transitions. Concurrently, both kinetic energy of the flow and the energy dissipation rate exhibit sudden bursts, with instantaneous values exceeding twice their respective means [15]. The rapid progression of these transitions, coupled with substantial changes to flow energetics, justifies their classification as extreme events and scientific efforts for their preemptive mitigation [13]. In the following, we discuss closed-loop flow control that mitigates transitions from counterclockwise to clockwise metastable regimes. Details of controlling the reverse transitions are included in the Supplemental Material [54].

Let us recall that the flow domain is connected to two syringes driven by a motorized pump, as shown in Fig. 1(a). For this specific configuration, injecting the electrolyte into the flow domain concurrently leads to an influx of (positive) angular momentum, transiently increasing the strength of counterclockwise circulations. To prevent imminent transitions from counterclockwise to clockwise regimes, we continuously monitor the flow using the real-time PIV system for the precursory signature of turbulence approaching PO_2 , which we define empirically as D_2 averaged over 7 s (half the period of PO_2) decreasing below 0.32. Once a close pass is detected, we inject the electrolyte into the flow domain at a rate 20 ml/min from each syringe for a duration of 4 s. This perturbation steers the flow deeper into the metastable counterclockwise regime, mitigating an imminent transition to the clockwise regime. We then withdraw the syringes to their initial positions over a duration of 80 s, a weak actuation that practically leaves the turbulent dynamics unaffected. After this withdrawal phase, the flow is driven solely by the electromagnetic forcing. A specific example of flow control mitigating regime transitions in the experiment is detailed below.

Figure 6 shows the time series of distance D_2 and the angular momentum L of the turbulent flow. During the first ≈ 2000 s (blue window), the flow is driven solely by the electromagnetic forcing and undergoes two transitions between the metastable regimes. D_2 dips below 0.32 directly preceding these transitions, confirming turbulence approaching PO_2 . Once the flow reaches the counterclockwise regime after the second transition, we activate the control protocol wherein the actuation cycle is triggered only upon detecting a close pass to PO_2 , at instants highlighted by the red vertical lines. Remarkably, regime transitions are suppressed for the entire duration of 4000 s during which control protocol remains active. This (magenta) window is noticeably longer than the characteristic residence time ($\lesssim 1500$ s) of turbulence within a metastable regime. Additionally, the total duration over which the control

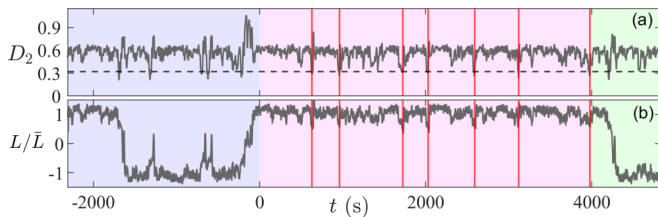


FIG. 6. Predictive control of regime transitions in turbulent laboratory flows. Time series of (a) the distance D_2 between PO_2 and turbulence (b) the angular momentum L . Blue/green windows represent turbulent evolution characterized by regime transitions, which are mitigated inside the magenta window via flow control triggered at instants marked using red lines.

actively influences the flow is a small fraction of the time for which the counterclockwise regime is stabilized. This demonstrates that regime transitions and extreme events can be averted preemptively, possibly indefinitely, through carefully designed control protocols. Finally, to assess whether repeated actuation cycles change the long-term flow dynamics, we perform a postcontrol test. In the green-shaded window, we withhold the control despite detecting turbulence approaching PO_2 . The flow undergoes a transition shortly thereafter, confirming that system's natural dynamics are not affected by earlier control events. Examples of flow control in the experiments are illustrated in Video 2 in the Supplemental Material [54].

It is worth mentioning that elimination of spatiotemporal chaos by stabilizing UPOs has been demonstrated previously in both numerical [67,68] and experimental [63,69,70] studies. The control we employ, however, is different in that it does not eliminate chaotic dynamics, but confines the dynamical system to a chaotic (saddle) metastable regime indefinitely, eliminating rapid transitions. Hence, it is also a demonstration of employing invariant solutions as waypoints in a predictive control framework mitigating extreme events [13,43]. Whether solutions with more complicated temporal dynamics, such as two-tori [37,38] and dynamical connections [37,71] whose computation in DNS and detection in experiments is more challenging, can be incorporated into such a framework remains open for exploration.

Conclusions. Several previous numerical studies have demonstrated that regime transitions in canonical turbulent

flows are organized around particular invariant solutions [15,17,29,30,43,45,58,60,72,73]. However, clear experimental evidence for a deterministic framework underpinning such transitions in laboratory flows remained outstanding. In this Letter, we reported the first experimental evidence for turbulent flows approaching simple invariant solutions, directly preceding and during transitions between statistically (and/or dynamically) distinct (metastable) regimes, an unambiguous signature of deterministic mechanisms driving such events. Furthermore, recognizing these regime transitions as extreme events marked by rapid and substantial changes to the flow energetics, we controlled their occurrence in experiments using a strategy that employs invariant solutions as waypoints en route to the transitions. This demonstration addressed the longstanding question of whether signatures of invariant solutions can be harnessed for predictive flow control in laboratory experiments.

Sudden transitions between symmetry-related turbulent flow regimes have been interpreted previously through both stochastic and deterministic frameworks, making them particularly distinctive cases for investigation [15,24,30,74,75]. Interestingly, similar efforts to bridge the stochastic and deterministic perspectives are currently underway in the context of climatic regime transitions. Some of these numerical studies have even employed the invariant solution-based dynamical systems framework to investigate such transitions [2,6,14,76]. Within this broader context, although the results of this study are obtained in a computationally and experimentally tractable moderately turbulent flow in a small box, they nonetheless support the possibility of deterministic mechanisms driving regime transitions in a wider range of spatiotemporally complex systems, even those inherently noisy (e.g., experiments), such as the climate [14], the human heart, and certain highly turbulent flows characterized by LSCs in fluid, thermal, and geophysical systems [16,24,77–79]. However, computing invariant solutions and experimentally characterizing regime transitions in such systems both remain highly challenging [80–83].

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Data availability. The data that support the findings of this article are openly available [84], embargo periods may apply.

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